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The Design of a Multiple Intake Drainage Well

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<u>THE DESIGN OF A MULTIPLE INTAKE DRAINAGE WELL</u>

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INTRODUCTION

The reclamation of water-logged land by suitable yet
inexpensive forms of drainage is becoming a problem of con-
stantly increasing magnitude. A fairly large portion of the
extensive land areas, once considered as worthless has proved
to be valuable by the introduction of proper drainage systems.
Land areas, in which drainage in its simpler forms has proved
inadequate, could perhaps be made valuable were it possible
to develop an inexpensive yet effective form of drainage.

This paper has been prepared primarily for a consider-
ation of poorly drained land areas and a possible means of
their reclamation by a suitable choice and distribution of
wells. We shall review a few of the fundamental concepts of
soil hydro-dynamics and attempt to give to these concepts a
practical application in a special case described hereafter.

As a preface to the following developments it may be
instructive to consider, incidentally, the source and sphere
of ground water; also its intimate relation to surface sat-
uration of water-logged land.

SOURCE AND SPHERE OF GROUND-WATER

Many familiar facts demonstrate conclusively the pre-
sence of water beneath the surface of the land. The many
thousands of wells in lands peopled by civilized man and the

numerous springs issuing from mountain slopes and valleys
are sufficient proof of both the wide distribution and
abundance of ground water.

Certain well known facts make it clear that the supply
of ground water is intimately connected with rainfall. This
is evident from the change in water level in wells and the
discharge from springs in periods of draught and relatively
heavy rainfall - the sinking of the ground water table
being greater when the draught is long and the rise more
notable when the rainfall is heavy. Furthermore, rainfall is
observed to sink beneath the surface wherever the soil is
sufficiently porous, and thence pass to greater depths through
the various materials, clay, sand, gravel, etc., known col-
lectively as mantel rock. Even the solid rock beneath is
seldom so free from fracture as to prevent completely the per-
colation of water.

The amount of rainfall which actually contributes to the
water beneath the surface is governed by a number of factors,
such as, topography of surface, rate of rainfall (or rate at
which snow melts,) porosity of soil, amount of moisture already
present in soil, amount and character of vegetation, and re-
lative humidity.

The supply of ground-water in a given locality is not
restricted entirely to local precipitation. Ground-water is,

in general, in constant motion, but the amount of this move-
ment is greatly influenced by the porosity or perviousness
of the soil, the existing pressure gradient, and to a less
extent the temperature, the movement increasing with rise in
temperature. Subterranean passages are occasionally found in
limestone areas in which the water moves in regular channels
analogous to those upon the surface. In general, however, the
ground water moves by slow percolation, by far the greater
portion being not organized into definite streams. In regions
of undulating topography thewater table is usually higher
under an elevation than under surrounding lowlands while in
flat regions of uniform structure the ground water surface is
essentially level though it rises and sinks with the rainfall.

The depth to which ground water penetrates has not been
determined by actual observation nor does it concern us direct-
ly in the following developments. Suffice it to say that the
deepest borings or excavations extending more than a mile in
depth have not revealed any indication of an approaching limit.
However, thezone of fracture is believed not to exceed six
miles even for the more resistant rock - this depth being
probably far less than that at which the critical temperature
of water would be reached.

Most of the water which sinks into the earth reappears
at the surface after a longer or shorter journey. Some of it

is evaporated from the surface directly; some of it is absorbed by plant roots; some of it issues from the surface in the form of springs or seepage; some of it is drawn out through wells; a small portion enters into combination with mineral matter; and the remainder finds its way underground to lakes, seas or oceans.

FORMS OF DRAINAGE

Ground-water which reaches the surface in the form of seepage or which maintains a saturated surface, encompassing relatively large areas, is frequently encountered in agricultural districts. It is often possible to secure drainage of excessive moisture by ditches leading into some natural channel or stream, tile pipe laid below the surface, wells, etc. The first two methods though used extensively are not always sufficient to secure proper drainage, wells being the more practicable where the swampy surface is maintained from a perched water table in unconsolidated materials.

Though a knowledge of the component earth materials through which the water passes is of prime importance in any consideration or analysis of ground water movement and although this *structure*, in general, represents a rather systematic distribution of the various components into more or less distinct strata, we shall consider the more ideal case in which the water is confined chiefly to a single horizontal stratum of uniform

structure. Furthermore, we shall assume that this water
bearing stratum is sufficiently near the land surface to
facilitate the use of ordinary pumping equipment.

DARCY'S LAW

The velocity of flow of a liquid in a given horizontal
direction through a column of soil of uniform structure is
thought to vary directly as the pressure head difference and
inversely as the length of the column. This relation in the
form of an equation may be written:

(1)
$$V = -\kappa \frac{p}{h}$$

where v is the velocity, p the pressure head difference, and
h the length of the column, the minus sign merely indicating
that the direction of the velocity vector is taken opposite
to that of the increasing pressure function. The constant k,
known as the trans-$\underset{\wedge}{\text{mission}}$constant, depends upon the porosity of the
soil and viscosity of the liquid, and may be defined as the
quantity of liquid transmitted per unit of time through a tube
of soil of unit length and unit cross-section, under unit diff-
erence of head. Darcy's law has been shown to fail in the case
of high pressure gradients, however, for most practical pur-
poses the law may be assumed to apply.

Equation (1) may be written in the differential form

(2)
$$V_r = -\kappa \frac{\partial p}{\partial r} .$$

in which the liquid is not constrained to move in a single

direction but is at liberty to move in any horizontal direction r. If it were possible to suppress the external force upon the liquid particle due to gravity, equation (2) would express the motion in three dimensions.

It is assumed in this discussion that the fluid moves with "steady motion", in that all conditions remain constant with respect to time.

The motion of a fluid in a soil is analogous to the flow of a hypothetical fluid which we may suppose to replace both the fluid in the soil and the soil itself which has the law for steady flow given by the above equation,

$$V_r = -\kappa \frac{\partial P}{\partial r}$$

Assuming that no extraneous forces are operative, the equations of motion of the hypothetical fluid in three dimensions may be written

$$\text{(3)} \qquad u = \frac{dx}{dt} = -\kappa \frac{\partial P}{\partial x}$$

$$v = \frac{dy}{dt} = -\kappa \frac{\partial P}{\partial y}$$

$$\omega = \frac{dz}{dt} = -\kappa \frac{\partial P}{\partial z}$$

and equation (2) becomes,

$$\bar{V} = \bar{u} + \bar{v} + \bar{\omega}$$

$$= -\kappa \left[i \frac{\partial P}{\partial x} + j \frac{\partial P}{\partial y} + \kappa \frac{\partial P}{\partial z} \right]$$

$$\text{(4)} \qquad = -\kappa \nabla P$$

We may observe that for this special case the velocity is proportional to the force per unit of volume but inasmuch as the density of the liquid will be taken as constant in the

developments to follow, this may then be stated as being

proportional to the force per unit mass.[*]

Denoting by ϕ the external force due to gravity acting

on a unit mass of the fluid and by ρ the density of the fluid,

Equation (4) may be written in the more general form

$$V = -\kappa\rho\nabla\left(\frac{\rho}{\rho}+\phi\right)$$

(5)
$$= -\kappa'\nabla\Phi$$

where $k' = \kappa\rho$ and $\Phi = \frac{\rho}{\rho} + \phi$

The equation of continuity

$$\frac{\partial\rho}{\partial t} = -\nabla\cdot(\rho v)$$

states that the scalar product of the vector operator ∇ and the

flux density vector ρv is equal to the time rate of change of

density. By expanding the right side of this equation we obtain,

$$\frac{\partial\rho}{\partial t} = -\frac{\partial(\rho v)}{\partial x} - \frac{\partial(\rho v)}{\partial y} - \frac{\partial(\rho v)}{\partial z}$$

$$= -\rho\left[\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right] - v_x\frac{\partial\rho}{\partial x} - v_y\frac{\partial\rho}{\partial y} - v_z\frac{\partial\rho}{\partial z}$$

$$= -\rho\nabla\cdot v - v_x\frac{\partial\rho}{\partial x} - v_y\frac{\partial\rho}{\partial y} - v_z\frac{\partial\rho}{\partial z}$$

and $\frac{\partial\rho}{\partial t} + v_x\frac{\partial\rho}{\partial x} + v_y\frac{\partial\rho}{\partial y} + v_z\frac{\partial\rho}{\partial z} = -\rho\nabla\cdot v$

But the expansion on the left of this equation is the time rate

of change of density of an element of the fluid. Therefore,

the equation of continuity takes the form

$$\frac{\partial\rho}{\partial t} = -\rho\nabla\cdot v$$

If the fluid is incompressible ρ is constant wherein

(6)
$$\nabla\cdot v = 0$$

If, as in this case, the motion is irrotational v may be

[*] For a statement on the vector notation herein used
the reader is referred to "Vector and Tensor Analysis", by
Wills, and others.

expressed as the gradient of a scalar function of position as shown in equation (5), and we arrive at Laplaces equation given by

(7) $$\nabla \cdot \nabla \phi = o$$

In hydrodynamics this scalar function of position ϕ is known as the velocity potential since the velocity is obtained from it in the same manner as that used to derive the force from potential energy in the case of a conservative system.

While the solution of Laplaces equation given by equation (7) seems essential in any detailed mathematical development of ground water movement, it appears that, in the special case described in the following paragraphs, such an analytical proceedure is not imperative. The course of wisdom seems to indicate that the present problem involves but a simple direct application of Darcy's law.

APPLICATION OF DARCY'S LAW

In the case of a single cylindrical well penetrating completely a horizontal stratum of uniform homogeneous gravel filled with water under pressure, the stream lines are horizontal and the potential levels are vertical concentric circular cylinders whose centers lie in the axis of the well.

Since, in this case, the potential does not vary vertically in the stratum, it is possible to determine a so-called piezometric surface characteristic of the stratum. This

piezometric surface is represented by the imaginary surface
that would maintain the same conditions of flow with in the
stratum.

In order to reduce the cost of pumping the water from the
great depth that would be necessary from a well of small dia-
meter an important practical problem arises of increasing the
effective diameter, and it has been proposed to install a
series of wells from which water is to be pumped through air-
tight connecting pipes leading to a common center. For such a
series of wells located uniformly about the circumference of a
circle it is possible to compute the diameter of the equivalent
single well. By equivalent well is meant the well which when
pumped to the same depth will yield the same amount of water as
the entire battery with the same pressure distribution for the
remote potential levels, that is, with practically the same
piezometric surface at points remote from the well.

By means of the method of adding potentials for a series
of "sinks" it is possible to develop an expression for the potent-
ial for the general point, in the "two dimensional" water bearing
stratum, for the battery of wells thus determining the pie-
zometric surface, and by equating the expression to a similar
function for the equivalent single well, we succeed in establish-
ing the equation of the cylindrical surface which cuts the res-
pective piezometric surfaces in their curve of intersection.

A short development is given taken from a paper called "Drainage of Land Overlying Artesian Basins", by Willard Gardner and associates.

The equation taken from the above journal relative to the total flow Q, passing through equipotential cylindrical surfaces of radius r into a vertical well penetrating completely the water bearing stratum of thickness 1, is given by

(8)
$$Q = 2\pi r l k_1 e v$$

in which k is known as the porosity constant.

But, from equation (1)

$$v = K \frac{\partial p}{\partial r}$$

If this value of v, given by Darcy's Law, is substituted in equation (8) and the resulting expression integrated between the limits p and Po, and R and R, respectively, we obtain

(9)
$$P - P_o = \left[\frac{Q}{2\pi l e K_1 K} \right] ln \frac{R}{R_1}$$

If Po is taken as the pressure at the well, that is, zero or atmosphere pressure, this equation affords a means of determining the pressure at any random point P distant R from the well of radius $R_\bullet$. The constant quantity, within the brackets, may be replaced by C, wherein

$$C = \frac{Q}{2\pi l e K_1 K}$$

and Equation (9) becomes, simply

(10)
$$P = C \, ln \frac{R}{R_1}$$

Now it has been proposed, as previously stated, to introduce a battery of circular cylindrical wells uniformly about the circumference of a circle of radius Ra which would effect practically the same pressure distribution at the more remote potential levels as that of the equivalent single well. This is clearly illustrated by the graph, given on page (16), showing the potential levels of a battery of six wells.

By means of the source and sink method of adding potentials the total pressure effect or contribution of the entire battery is given by,

(11)
$$P = \Sigma p_i = \frac{q}{K} \sum_{i=o}^{i=n} \ln \frac{r}{r_i}$$

in which pi represents the change of potential due to the ith well of the n wells of radius r, comprising the battery, and r, the distance from this well to the point in question.

If we select any point P, as shown in Fig. 1, the value of r in terms of R, Ra, and Θ, may be determined from the familiar relation

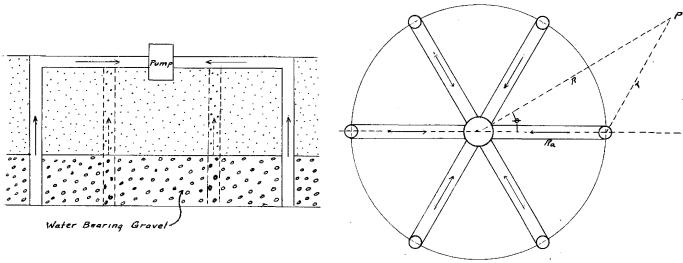$$r = \sqrt{R^2 + R_a^2 - 2 R R_a \cos \Theta}$$

Water Bearing Gravel

Fig. 1.

and Equation (11) may be written

$$P = \sum_{i=0}^{i=n} p_i = \frac{C}{n} \sum_{i=0}^{i=n} \ln \frac{\sqrt{R^2 + R_a^2 - 2RR_a\cos\theta}}{r_i}$$

or (12)

$$\frac{P}{C} = \frac{1}{n} \sum_{i=0}^{i=n} \ln \frac{\sqrt{R^2 + R_a^2 - 2RR_a\cos\theta}}{r_i}$$

By equating this value of $\frac{P}{C}$ to the similar function for the equivalent single well, we obtain the relation

$$\ln \frac{R}{R_1} = \frac{1}{n} \left\{ \sum \ln \frac{\sqrt{R^2 + R_a^2 - 2RR_a\cos(\theta + \frac{i2\pi}{n})}}{r_1} \right\}$$

$$= \frac{1}{n} \left\{ \sum \frac{1}{2} \ln \left[R^2 + R_a^2 - 2RR_a\cos(\theta + \frac{i2\pi}{n}) \right] - n\ln r_1 \right\}$$

$$= \frac{1}{n} \left\{ \frac{1}{2} \ln \left[R^2 + R_a^2 - 2RR_a\cos\theta \right] \right.$$

$$+ \frac{1}{2} \ln \left[R^2 + R_a^2 - 2RR_a\cos(\theta + \frac{2\pi}{n}) \right] + \cdots\cdots$$

$$\left. + \frac{1}{2} \ln \left[R^2 + R_a^2 - 2RR_a\cos(\theta + \frac{(n-1)2\pi}{n}) \right] - n\ln r_1 \right\}$$

Let $R = \alpha R_a$, in which α may assume any positive value, and the above expression may be reduced to,

$$\ln \frac{\alpha R_a}{R_1} = \frac{1}{n} \left\{ \frac{1}{2} \ln \left[\alpha^2 R_a^2 + R_a^2 - 2\alpha R_a^2 \cos\theta \right] \right.$$

$$+ \frac{1}{2} \ln \left[\alpha^2 R_a^2 + R_a^2 - 2\alpha R_a^2 \cos(\theta + \frac{2\pi}{n}) \right] + \cdots\cdots$$

$$\left. + \frac{1}{2} \ln \left[\alpha^2 R_a^2 + R_a^2 - 2\alpha R_a^2 \cos(\theta + \frac{(n-1)2\pi}{n}) \right] - n\ln r_1 \right\}$$

$$= \frac{1}{2n} \left\{ \ln R_a^2 [\alpha^2 + 1 - 2\alpha \cos\theta] \right.$$

$$+ \ln R_a^2 [\alpha^2 + 1 - 2\alpha \cos(\theta + \frac{2\pi}{n})] + \cdots\cdots$$

$$\left. + \ln R_a^2 [\alpha^2 + 1 - 2\alpha \cos(\theta + \frac{(n-1)2\pi}{n})] - 2n\ln r_1 \right\}$$

$$= \frac{1}{2n} \left\{ 2n\ln R_a - 2n\ln r_1 + \ln[\alpha^2 + 1 - 2\alpha\cos\theta][\cdots\cdots] \right.$$

$$\left. [\cdots\cdots][\alpha^2 + 1 - 2\alpha\cos(\theta + \frac{(n-1)2\pi}{n})] \right\}$$

(13)

$$= \ln R_a - \ln r_1 + \frac{1}{2n} \ln G_{\alpha\theta n}$$

where $G_{\alpha\theta n} = [\alpha^2 + 1 - 2\alpha\cos\theta][\alpha^2 + 1 - 2\alpha\cos(\theta + \frac{2\pi}{n})][\cdots\cdots][\alpha^2 + 1 - 2\alpha\cos(\theta + \frac{(n-1)2\pi}{n})]$

By expanding the left member of Equation (13) it is clear that $\ln R_a$ disappears, giving

$$\ln \alpha - \ln R_1 = -\ln r_1 + \frac{1}{2n} \ln G_{\alpha\theta n}$$

(14) $$Zn \frac{R_1}{r_1} = Zn\,\alpha - \frac{1}{2n} Zn\,G\alpha\theta n$$

For the values of $\alpha = 2$, $\theta = 0$ and $n = 6$ the equation (14) becomes,

$$Log \frac{R_1}{r_1} = Log\,2 - \frac{1}{12} Log\,(3969)$$

$$= Log\,\frac{2}{(3969)^{\frac{1}{12}}}$$

and

$$R_1 = \frac{2}{(3969)^{\frac{1}{12}}} r_1$$

(15) $$= \left[\frac{2}{1.9948}\right] r_1$$
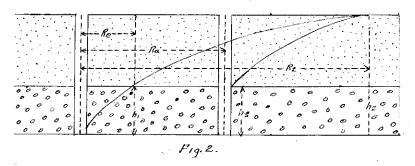
From this expression it is evident that the ratio $\frac{R_1}{r_1}$ is nearly equal to unity and it may be observed that as R_1 approaches r_1, the intersection of the corresponding piezometric surfaces approach a circle of infinite radius. If the cylindrical surface described by the intersection of the piezometric surfaces, falls at this point, the two such surfaces will practically coincide beyond this point.

The ratio of the potential to the constant c at the wells of the battery may be computed from the expression given above; also that corresponding to the point, $R = 2R_a$, $\theta = 0$. The ratio of these two values is equivalent to the ratio of the respective elevations h_1 and h_2 of the piezometric surfaces for the battery at the two points. The value of this ratio thus determined may be used to determine a third point in the stratum for which the elevation of the piezometric surface for the single central well will equal that of the battery piezometric surface at the battery well. The traces in the vertical plane of these two surfaces shown in

Fig. 2, will assist in making this clear.

Fig. 2.

The ratio of the potentials for the single central well may be written

(16)
$$\frac{P_1}{P_2} = \frac{\ln \frac{R_e}{R_1}}{\ln \frac{R_e}{R_1}}$$

and for the battery wells,

(17)
$$\frac{P_1}{P_2} = \frac{\left[\ln \frac{R_a}{r_1} + \frac{1}{2n} \ln G_{\alpha \theta n} \right]_1}{\left[\ln \frac{R_a}{r_1} + \frac{1}{2n} \ln G_{\alpha \theta n} \right]_2}$$

Now if $R_1 = r_1 = \beta R_a$; $\theta = 0$; $R_e = \alpha_e R_a$; $R_2 = \alpha_2 R_a$ and equate the right hand members, we obtain

$$\frac{\ln \alpha_e - \ln \beta}{\ln \alpha_2 - \ln \beta} = \frac{-\ln \beta + \frac{1}{2n} \ln G_{\alpha_1 \theta n}}{-\ln \beta + \frac{1}{2n} \ln G_{\alpha_2 \theta n}}$$

and (18)
$$\ln \alpha_e = \left(\frac{-\ln \beta + \frac{1}{2n} \ln G_{\alpha_1 \theta n}}{-\ln \beta + \frac{1}{2n} \ln G_{\alpha_2 \theta n}} \right)(\ln \alpha_2 - \ln \beta) + \ln \beta$$

For the specific case in which $\alpha_1 = \frac{121}{120}$; $\alpha_2 = 2$; $n = 6$; $\beta = \frac{1}{120}$ we have
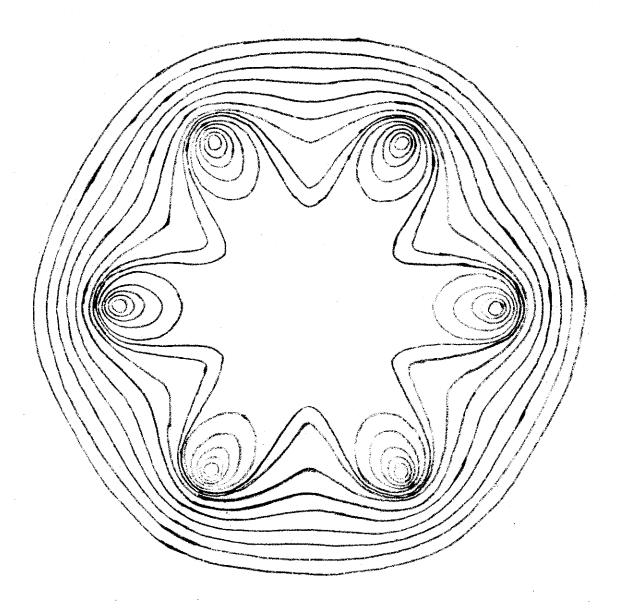
$$\log \alpha_e = \left[\frac{-\log \frac{1}{120} + \frac{1}{12} \log (\alpha_1^6 - 1)^2}{-\log \frac{1}{120} + \frac{1}{12} \log (\alpha_2^6 - 1)^2} (\log 240) - \log 120 \right]$$

$$= \left[\frac{\log 120 + \frac{1}{6} \log \left[\left(\frac{121}{120} \right)^6 - 1 \right]}{\log 120 + \frac{1}{6} \log [2^6 - 1]} (\log 240) - \log 120 \right]$$

wherein (approximately) $\alpha_e = .65$

and
$$R_e = \alpha_e R_a$$

$$= 13 \text{ feet}$$

We may observe that for this special case the diameter of the equivalent single well would approach 26 feet.

By means of Equation (18) it is possible to compute the size of the equivalent single well for any given circular battery of n equally spaced wells, in which the values of α_1, α_2, n and β are known.

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Schematic representation of the potential levels for a multiple (n=6) intake well.