2013

Development of the Multiple Use Plug Hybrid for Nanosats (Muphyn) Miniature Thruster

Shannon Dean Eilers
Utah State University

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DEVELOPMENT OF THE MULTIPLE USE PLUG HYBRID FOR NANOSATS
(MUPHYN) MINIATURE THRUSTER

by

Shannon Eilers

A dissertation submitted in partial fulfillment
of the requirements for the degree
of
DOCTOR OF PHILOSOPHY
in
Mechanical Engineering

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2013
Abstract

Development of the Multiple Use Plug Hybrid for Nanosats (MUPHyN) Miniature Thruster

by

Shannon Eilers, Doctor of Philosophy
Utah State University, 2013

Major Professor: Dr. Stephen A. Whitmore
Department: Mechanical and Aerospace Engineering

The Multiple Use Plug Hybrid for Nanosats (MUPHyN) prototype thruster incorporates solutions to several major challenges that have traditionally limited the deployment of chemical propulsion systems on small spacecraft. The MUPHyN thruster offers several features that are uniquely suited for NanoSat, and especially CubeSat, applications. These features include 1) a non-pyrotechnic ignition system, 2) non-mechanical thrust vectoring using secondary fluid injection on a regeneratively cooled aerospike nozzle, 3) non-toxic, chemically stable hybrid propellants, 4) a compact form factor enabled by the direct digital manufacture of the hybrid fuel grain. Hybrid rocket motors provide significant safety and reliability advantages over both solid composite and liquid propulsion systems; however, hybrid motors have found only limited use on operational vehicles due to 1) difficulty in modeling fuel regression rate mechanics 2) poor volumetric efficiency and/or form factor 3) significantly lower fuel flow rates than solid rocket motors 4) difficulty in obtaining high combustion efficiencies and therefore high specific impulses. The design features of the MUPHyN thruster are used to offset and/or overcome these shortcomings.

The MUPHyN thruster adapts a hybrid rocket motor to the scale and form factor required for CubeSat propulsion by employing a regeneratively cooled aerospike nozzle with
secondary gas injection thrust-vectoring and a direct-digitally manufactured acrylonitrile-
butadiene-styrene fuel grain with helical combustion ports. Aerospike nozzles have a shorter
form factor than an equivalent expansion-ratio traditional nozzle. This makes them well
suited to space applications where volumetric efficiency is critical. Aerospike nozzles can
provide multi-axis secondary-injection thrust-vectoring and also function as a cold gas re-
action control thruster without primary flow. To date, aerospike nozzles have not been
adopted on operational space vehicles due to several factors, including difficulty in cool-
ing the large nozzle throat area, an issue which is overcome on the MUPHyN motor with
nitrous-oxide regenerative cooling. The MUPHyN motor design represents a convergence of
technologies, including hybrid rocket regression rate modeling, aerospike secondary injection
thrust vectoring, multiphase injector modeling, non-pyrotechnic ignition, and nitrous oxide
regenerative cooling. This synthesis of technologies is unique to the MUPHyN thruster
design and no comparable work has been published in the open literature.
Public Abstract

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The MUPHyN motor design represents a convergence of technologies, including hybrid rocket regression rate modeling, aerospike secondary injection thrust vectoring, multiphase
injector modeling, non-pyrotechnic ignition, and nitrous oxide regenerative cooling that address the traditional challenges that limit the use of hybrid rocket motors and aerospike nozzles. This synthesis of technologies is unique to the MUPHyN thruster design and no comparable work has been published in the open literature.
Acknowledgments

I am very thankful for the advice, motivation, and support of my adviser, Dr. Stephen Whitmore, who enabled me greatly in my endeavor to work on propulsion-related topics at a university without a previously existing rocket propulsion program. I deeply appreciate the guidance of my committee members, Dr. Baron Smith, Dr. David Geller, Dr. Warren Phillips, and Dr. Doran Baker. I am also grateful for the fellowship from the Rocky Mountain Space Grant Consortium that funded my research during early years when no other funding was available.

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The construction of much of the hardware necessary for this project would not have been possible without the skill and dedication of our machinist, Terry Zollinger.

May all the fishes rest in peace.

Shannon Dean Eilers
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Nomenclature

\( \bar{c}_p \)  Specific heat

\( \Delta A \)  Area of intersection for high pressure region behind shock wave

\( \Delta H \)  Effective heat of gasification

\( \Delta H_r \)  Heat of reaction

\( \dot{r} \)  Solid fuel regression rate

\( \dot{m} \)  Mass flow rate

\( \dot{m}_j \)  Secondary injection mass flow rate

\( \dot{m}_s \)  Thrust vectoring injectant mass flow rate

\( \dot{Q} \)  Heat flux into the solid fuel surface

\( \dot{Q}_c \)  Convective heat transfer to fuel surface

\( \dot{Q}_{rad} \)  Radiative heat transfer to fuel surface

\( \Gamma \)  Boundary layer thickness

\( \gamma \)  Specific heat ratio

\( \kappa \)  Thermal conductivity

\( \mu \)  Fluid viscosity

\( \mu \)  Gas viscosity

\( \mu_{0s} \)  Reference viscosity for Sutherland’s law

\( \omega \)  Empirical constant

\( Pr \)  Prandtl number

\( \rho \)  Density

\( \rho_c \)  Density in combustion zone

\( \rho_e \)  Density at edge of boundary layer

\( \rho_s \)  Solid fuel density

\( \rho_w \)  Density at the wall

\( \rho_w \)  Density at the wall

\( \rho_w \)  Density of ablation products at fuel surface
<table>
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<th>Symbol</th>
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<td>$\rho_w$</td>
<td>Density of ablation products at fuel surface</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan–Boltzmann constant</td>
</tr>
<tr>
<td>$\tau_b$</td>
<td>Bubble formation time constant</td>
</tr>
<tr>
<td>$\tau_r$</td>
<td>Fluid residence time constant</td>
</tr>
<tr>
<td>$A$</td>
<td>Area</td>
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<td>$a$</td>
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<td>$C_s$</td>
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</tr>
<tr>
<td>$D$</td>
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</tr>
<tr>
<td>$D_h$</td>
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<tr>
<td>$d_i$</td>
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<tr>
<td>$d_p$</td>
<td>Material thermal diffusivity</td>
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<tr>
<td>$E_c$</td>
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<tr>
<td>$f(\text{Fr})$</td>
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<td>$F_s$</td>
<td>Amplified side force</td>
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<td>$G$</td>
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<tr>
<td>$g$</td>
<td>Acceleration of gravity</td>
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<tr>
<td>$G_{HEM}$</td>
<td>Mass flux from homogenous equilibrium model</td>
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\( G_{\text{NHNE}} \) Non-homogeneous, non-equilibrium mass flux

\( G_{\text{o}u} \) Initial oxidizer mass flux

\( G_{s,f} \) Material specific constant

\( G_{\text{SPI}} \) Mass flux from incompressible model

\( h \) Accommodation height

\( h_2 \) Heat transfer coefficient

\( h_3 \) Heat transfer coefficient corrected for curvature

\( h_{c_s} \) Stagnation enthalpy at the flame

\( h_d \) Heat of degradation of the material

\( h_{w_g} \) Gas phase enthalpy at the wall

\( I_{\text{sp}_s} \) Side-force specific impulse

\( k \) Non-equilibrium parameter

\( M \) Mach number

\( M_{\infty} \) Free stream Mach number

\( M_{j\infty} \) Mach number of injectant downstream of injection site

\( n \) Burning rate exponent for solid composite propellant regression

\( P \) Pressure

\( P_0 \) Chamber Pressure

\( P_2 \) Static pressure downstream of conical shock

\( P_{\infty} \) Static pressure of primary flow

\( P_s \) Static pressure in separated flow region

\( q'' \) Area specific heat transfer rate

\( q_w \) Heat flux into wall

\( r \) Local radius

\( R_{\text{ex}} \) Reynolds

\( R_g \) Gas constant

\( S \) Stanton number

\( s \) Curvilinear coordinate
\( S_0 \)  
Stanton number without wall blowing

\( St \)  
Stanton number

\( T \)  
Temperature

\( T_{0s} \)  
Reference temperature for Sutherland’s law

\( T_c \)  
Temperature in the combustion zone

\( T_i \)  
Fuel initial temperature

\( T_w \)  
Wall temperature

\( U \)  
Fluid velocity

\( u_c \)  
Velocity in combustion zone

\( u_e \)  
Velocity at edge of boundary layer

\( V_j \)  
Injectant velocity

\( v_{w} \)  
Velocity normal to the wall

\( v_{w} \)  
Velocity normal to the wall

\( X \)  
Distance between start of separation and injection point

\( X \)  
Fluid quality

\( x \)  
Longitudinal location

\( Y_{ps} \)  
Mass fraction of polymer near the outer edge of the degradation layer

\( C \)  
Thrust stand calibration matrix

\( F \)  
Thrust stand force and moment input vector

\( H \)  
Thrust stand calibration measurement matrix

\( h \)  
Thrust stand calibration measurement partial

\( J \)  
Jacobian for the thrust stand force/voltage system

\( V \)  
Load cell voltage output vector

\( V_0 \)  
Load cell voltage bias vector

\( \text{Re}_T \)  
Reynolds number based on boundary layer thickness

\( \text{Re}_D \)  
Reynolds number based on port diameter

\( 0 \)  
Static fluid conditions

\( h_{fg} \)  
Heat of vaporization
$l$ Liquid parameter
$v$ Vapor parameter
$\infty$ Local fluid conditions
$\alpha$ Conical nozzle half angle
$\delta$ Angle between outer boundary of separated region and nozzle wall
$\gamma$ Specific heat ratio
$\sigma$ Nondimensional pressure in cylindrical explosion
$\theta$ Angle between conical shock wave and nozzle wall
$\varepsilon$ Angle of injectant to nozzle radial vector
Chapter 1

Introduction

1.1 Research Overview

This research project has spanned five years and resulted in the design, development, fabrication, integration, and testing of a novel “CubeSat”-scale thruster that uses environmentally benign, safe-handling, and inexpensive nitrous oxide (N2O) and acrylonitrile-butadiene-styrene (ABS) as propellants. The resulting system offers the simplicity and reliability of a monopropellant system, but with the performance of a hybrid rocket propellant system. This novel hybrid propulsion system is specifically designed as a testbed for technologies critical to CubeSat and NanoSat sized spacecraft propulsion. Once commercially developed, this enhanced propulsive capability could enable multiple CubeSats to be deployed simultaneously by a single launch vehicle and then independently repositioned or re-boosted after orbital insertion, a capability that does not currently exist.

The MUPHyN thruster offers several features that are uniquely suited for CubeSat applications. These features include 1) a non-pyrotechnic ignitions system, 2) non-mechanical thrust vectoring using secondary fluid injection on a regeneratively cooled aerospike nozzle, 3) non-toxic, chemically stable hybrid propellants, 4) a compact form factor enabled by the direct digital manufacture of the ABS fuel grain. The resulting system is compact, non-toxic, non-explosive, and uses non-pyrotechnic means for reliable motor ignition.

Collectively, these features enable mechanical simplicity and low power requirements for motor ignition. Because the oxidizer and fuel components are mixed only within the combustion chamber once ignited, the system also retains the inherent safety of a hybrid rocket and can be piggy-backed with little to no overall mission risk increase to the primary
payload, an excellent characteristic for secondary launch systems.

The MUPHyN motor design represents a convergence of technologies including hybrid rocket regression rates scaling, aerospike secondary injection thrust vectoring, multiphase injector modeling, and nitrous oxide regenerative cooling – technologies there were all advanced over the course of this research project. The technology development tree for this research effort is shown in Fig. 1.1. This synthesis of technologies is unique to the MUPHyN thruster design, and no other commercial or government entity has produced comparable work that has been published in open literature.

1.2 Overview of Chemical Rocket Systems

There are three types of chemically propelled rocket engines: liquid, solid, and hybrid. Figure 1.2 shows these main engine types. Bipropellant liquid engines mix and burn highly volatile oxidizer and fuel components in the combustion chamber. Liquid monopropellant engines catalyze a volatile fuel or oxidizer before it is expelled through the nozzle. Solid rocket motors use a propellant grain that binds both the oxidizer and fuel in a hydrocarbon substrate. A hybrid rocket motor, or a solid-liquid rocket motor, typically burns a solid fuel and a liquid or gaseous oxidizer in the arrangement shown in Fig. 1.3.

Both bi-propellant liquid rocket engines and solid rocket motors have a potential for catastrophic explosion. NASA estimates that the Space Shuttle’s liquid fueled main engines will fail catastrophically once every 1530 sorties [1, 2], and the Space Shuttle solid rocket boosters will fail catastrophically once every 1550 sorties. Even small solid-propelled ordnance motors intended for non-crewed spacecraft fail approximately 1 in every 250 burns [3].

Traditional mono-propellant liquid engines are also not without their share of hazards. Hydrazine is by far the most commonly used monopropellant for primary spacecraft propulsion and attitude control thrusters [4, 5]. Hydrazine thrusters are relatively simple and consist of an electric solenoid valve, a pressurant tank, and a catalyst bed of alumina pellets impregnated with iridium or another transition metal. The catalyst initiates an exothermic decomposition of the hydrazine to produce ammonia, nitrogen, and hydrogen gases resulting in a vacuum specific impulse near 240s, depending on the level of ammonia
Fig. 1.1: MUPHyN motor development research tree.
Fig. 1.2: Types of chemical rocket engines.

Fig. 1.3: Typical hybrid rocket motor configuration.
decomposition. [4] Unfortunately, hydrazine is a powerful reducing agent that poses serious environmental and health concerns. Hydrazine is extremely destructive to living tissues, and is a known carcinogen. Exposure produces a variety of adverse systemic effects including damage to liver, kidneys, nervous system, and red blood cells [6]. In addition to these biological and toxicological impacts, hydrazine presents a level of environmental danger for the spacecraft and launch vehicle. Liquid hydrazine is capable of detonation if heated rapidly. Vapor and solid hydrazine are both capable of detonation given extreme shock, such as the nearby explosion of a linear charge or blasting cap. Linear charges and explosive bolts are common in large launch vehicles. Solid hydrazine buildup, possibly augmented by frozen oxygen, detonated by a linear charge in an adapter ring is suspected in the failure of an Atlas-Centaur upper stage [7].

During the past 50 years, conventional launch systems have been developed to a high state of capability; however, for a variety of reasons, these vehicles have become increasingly expensive to operate. Some of these reasons include manufacturing and operational complexity, safety and environmental regulations for dealing with hazardous materials, and the generally large “support army” required for flight preparation and operations [8,9]. Because of high launch performance demands including specific impulse and thrust-to-weight ratio, conventional liquid and solid-propelled rocket stages that employ highly energetic, explosive, or toxic propellants will likely remain the systems of choice for the lower stages of launch systems for large payloads. However, there exists an emerging commercial market that is willing to accept a lower system performance in exchange for reduced operational costs and lower environmental impact.

Hybrid motors that employ non-toxic, non-explosive, indefinitely storable propellants have the potential to fulfill this “market niche.” While hybrid systems based upon these low-risk propellants generally deliver lower specific impulses than conventional bi-propellant liquid and solid rockets of the same thrust level, because the propellant components remain inert until ignited within the motor chamber, hybrid rockets are inherently safer to transport, load, store and operate. This inherent safety greatly reduces ground handling and
transportation costs, and can potentially lead to an overall reduction in system operating costs.

In a hybrid rocket motor, liquid or gaseous oxidizer is stored in a tank separated by a valve from the solid fuel residing in the combustion chamber. The solid fuel also typically provides insulation for the walls of the combustion chamber. This arrangement segregates the fuel from the oxidizer until the valve is opened, and the reaction rate of the solid fuel and oxidizer is limited by oxidizer flow and the convection-based pyrolysis of the solid fuel grain. This is markedly different from the behavior of liquid bipropellant motors where the reaction rate is only limited by flow rate, diffusion, and kinetics or the behavior of solid rocket motors, where the reaction rate is limited by kinetics and heat transfer.

In contrast to traditional solid or liquid motors, hybrid rocket motors separate the liquid oxidizer from the solid fuel grain and present little to no risk of explosion while burning. Hybrid designs present near zero potential for explosion during storage and handling [10]. As a testament to the benign nature of hybrids, a plexiglass and gaseous oxygen hybrid demonstration model design developed by the United Technology Center has cumulatively been fired over 50,000 times by various organizations without ever experiencing a hazardous failure [11].

Other advantages of hybrid rockets when compared to solid rocket systems include the capability for in-flight restart, throttling, and ease of ground handling. Also unlike solid-propelled rockets, where fuel grain flaws and age-induced cracks present a significant safety issue, hybrid rockets exhibit a relative insusceptibility to fuel grain defects. Hybrid systems can offer greater performance than cold-gas, monopropellant, or solid rocket systems and can have a higher density-specific impulse than liquid bipropellant engines [5].

Hybrid motors are not without technical difficulties and operational shortcomings. Hybrid rocket motors also incur some unique design and modeling challenges that are not applicable solid or liquid rocket motors. Many of these challenges stem from the dependence of solid fuel regression, and therefore fuel flow rate, on the mass flux of the oxidizer through the fuel port. This critical parameter, which controls thrust and motor oxidizer to fuel ratio,
depends on heat transfer from a turbulent diffusion flame to the fuel surface. Heat transfer to the solid fuel surface is a complicated phenomenon that has no universally applicable models that work across a variety of fuel combinations and motor scales.

Hybrid rocket fuel regression mechanisms traditionally yield regression rates that are far lower than those seen in solid rocket motors. For a typical hybrid motor with polyurethane fuel, regression rates are often an order of magnitude lower than those seen by composite ammonium perchlorate propellants [5]. Thus, to achieve both desirable thrust levels and oxidizer-to-fuel (O/F) ratios, traditional hybrid motors have significantly long aspect ratios where the motor length is often 10 times the major cross section diameter. These long form factors are poorly suited to small-spacecraft applications. If hybrid motor designs are to be adapted for CubeSat applications, unique design features must be incorporated to overcome this volumetric inefficiency.

1.3 Overview of Aerospike Rocket Nozzles

The aerospike nozzle differs from a conventional rocket nozzle in that the propulsive fluid expands around a central plug or ramp and is not constrained by external solid boundaries as in a bell or conical nozzle. A full length aerospike nozzle is shown in Fig. 1.4. Because the external flow field is unconstrained, the pressure on the aerospike nozzle surface has the ability to adjust to changes in the surrounding free stream pressure.

In an aerospike nozzle, the primary fluid flow expands from the combustion chamber in a Prandtl–Meyer expansion fan until the fluid reaches the pressure of the ambient or free stream fluid surrounding the aerospike nozzle [5]. Figure 1.5 shows how these expansion fans adjust to changes in the free-stream or ambient pressure. When the nozzle is operating at the design pressure ratio, this expansion fan results in fluid streamlines that are parallel to the longitudinal axis of the aerospike nozzle. If the ambient pressure is lower than the design pressure, the expansion fan continues to expand outwards away from the nozzle, but the pressure on the aerospike nozzle surface does not change with decreasing external pressure. This is often referred to as a “closed wake” condition. However, if the ambient pressure is higher than the design operating conditions, the outer boundary of the Prandtl–
Fig. 1.4: Full length and truncated aerospike nozzles.

Fig. 1.5: Aerospike nozzle pressure compensation effect.
Meyer expansion fan leans in towards the aerospike nozzle axis. The ambient pressure conditions are translated towards the aerospike nozzle surface by compression waves. This is termed an “open wake” condition. This compression near the end of the nozzle avoids over-expanding the working fluid, which would lead to an area of the nozzle with lower than ambient pressure thereby reducing the total thrust. Thus, the pressure acting on the surface of the aerospike nozzle increases with increasing free stream pressure, leading to the “altitude compensation effect” for aerospike nozzles [12]. Figure 1.6 shows the the pressure profile in a bell nozzle for a corresponding change in ambient pressure. With a bell or conical nozzle, when the ambient pressure is high, the fluid either over-expands, reducing thrust, or separates from the nozzle wall which can cause excessive nozzle heating and efficiency losses. Aerospike nozzles mitigate this inefficiency, which has long been the primary driver for interest in aerospike nozzles.

Both the pressure and the surface area of aerospike nozzles decrease rapidly away from the throat as seen in Fig. 1.5. Because of these properties, aerospike nozzles can be, and usually are, truncated with little performance penalty [13]. For example, Rocketdyne, during testing in the 1960’s, demonstrated nozzle efficiencies of about 96 percent
for an aerospike nozzle truncated down to 12 percent of the full theoretical length. This performance loss can be compensated for by ejecting gas out of the truncated base region. The recirculation region behind a truncated aerospike nozzle augments the thrust of this “base bleed”, increasing overall efficiency. This technique effectively creates an “aerodynamic spike” from which the name “aerospike” originated [14]. This term has since been generalized for truncated and full length nozzles with and without base bleed.

While aerospike nozzles have long been known for their altitude compensation ability during endo-atmospheric flight, they also present significant potential advantages for purely in-space applications. Aerospike nozzles can be both more efficient and significantly smaller than conventional high expansion ratio bell nozzles. Given a fixed vehicle base area, an aerospike nozzle can present higher area expansion ratio than a bell nozzle, providing better performance in a space environment or near-vacuum environment like Mars. The increased specific impulse ($I_{sp}$) due to a higher possible expansion ratio using an aerospike nozzle translates to a 8–9% decrease in the propellant mass and total system weight for space and near-space applications [15].

Like traditional nozzles, aerospike nozzles can have the capability for fluidic thrust vectoring via secondary injection. A secondary fluid can be ejected near the end of the truncated aerospike nozzle, diverting the nozzle plume and gaining a net performance increase over ejecting the secondary fluid alone. Conventional de Laval nozzles also have this capability, but aerospike nozzles are unique in that these secondary orifices are external and can therefore be used as reaction control thrusters without the primary nozzle flow active as well as a mechanism for thrust vectoring when the main engine is firing.

1.4 Overview of Direct Digital Manufacturing

Direct digital manufacturing (DDM), also called rapid prototyping, additive manufacturing, or solid free-form fabrication is a process whereby three-dimensional parts are manufactured by a computer controlled machine directly from a digital model. Three common techniques for this process are selective laser sintering, stereolithography, and fused deposition modeling.
Selective laser sintering uses a laser to sinter either a powdered thermoplastic or metal into a solid piece. The powdered material is built up in layers and only part of each new layer is melted onto the layer below. In stereolithography, thin layers of a ultraviolet curable material are selectively cured out of a vat of liquid resin. Fused-deposition modeling (FDM) machines extrude a thin bead of heated thermoplastic onto a layer below, “3D printing” the model from the base up. All three of these methods can produce parts out of materials suitable for hybrid fuels although only stereolithography and fused deposition modeling have reportedly been used for this process in the open literature [16, 17].

Fused deposition modeling is an especially attractive technology for the production of hybrid rocket motor fuel grains. Fused deposition modeling was developed by and patented by Stratasys Inc in the late 1980’s and early 1990’s [18]. Figure 1.7 shows a basic schematic of an FDM machine. Stratasys’s technology uses ABS as a material and is executed on relatively low cost equipment [19]. Printed ABS has good structural properties and has been shown to perform relatively well in comparison to standard hybrid rocket fuels [17].

The application of DDM processes to hybrid rocket motor fuel grain fabrication allow for the creation of complex fuel grain designs that could not be cast using standard processes. This, in turn, allows for much greater flexibility in hybrid rocket motor chamber designs and configurations, and is critical to the design of the MUPHyN thruster. The use of DDM manufacturing for tailoring hybrid fuel grain flow paths is a very new technology, and the “evolutionary tree” of potential applications is only now starting to grow.

1.5 Research Motivation

There exists an emerging scientific, military, and commercial interest in constellations of small, inexpensive nano-scale spacecraft. Of particular interest are “NanoSats” that can be flown as secondary payloads. A particular NanoSat design that is seeing increasing popularity is a 10 by 10 cm cube (1U) form factor. Multiple 1U cubes can be coupled together to form “CubeSats.” Standard deployment systems for CubeSats as large as 6-U have been certified for flight on several USA and European launch vehicles.
If these small spacecraft can be deployed and organized into constellations to collectively perform a coordinated mission, they present distinct advantages not available to single larger-scale spacecraft that must be deployed one launch at a time. The distributed nature of this small spacecraft “swarm” offers a significant increase in mission reliability. A large constellation has built-in redundancy. Advanced space missions enabled by these orbiting constellations include 1) Sun-Earth Connection science missions that collect simultaneous multi-point spatial and temporal thermospheric and ionospheric data to analyze the causes and effects of space weather on the Earth, 2) persistent surveillance of Earth science targets, and 3) beyond-line of sight (BLOS) surface communications. For advanced mission concepts, providing a capability of approximately 800 m/sec allows a spacecraft to be deployed onto interplanetary trajectories from a standard Geostationary Transfer Orbit (GTO). This capability could enable NanoSat constellations to perform interplanetary missions.

Any of these missions require orbital maneuvers after the deployment of the launch vehicle’s primary payload. However, only a few specialized launch vehicles have upper
stages with the ability for in-space restarts [20]; these are typically reserved for expensive government-owned reconnaissance, communications, or command & control satellites. For existing rideshare launch opportunities, nano-scale spacecraft are delivered to orbit as passive secondary payloads and must accept whatever orbit they achieve during the deployment process.

Secondary payloads, especially in the NanoSat class, have no ability to modify their initial orbit by executing a propulsive maneuver and currently remain a novelty with little capability to accomplish serious scientific, strategic, or commercial missions. Thus, development of a propulsion unit that rides along with the secondary payload during launch, and then repositions or maintains the orbit after deployment is highly desirable. Such a device would benefit the entire small satellite industry.

However, if this device were constructed using conventional high-explosive propellants, the “ride-along” payloads – each with their own propulsion system – would dramatically increase the level of risk to the primary payload. Managing this risk would result in prohibitive launch costs [21]. Thus this “rideshare” propulsion unit must be developed using non-toxic propellants and feature inherently safe designs. The very significant explosion risk of liquid bi-propellant or solid composite propellants has traditionally banned spacecraft with propulsion systems from flying as secondary payloads. A lower risk propellant option is highly desirable. Hybrid rocket motors have the potential to fulfill this low-risk flight requirement.

Figure 1.8 a potential layout of a 6-U Nanosat with the MUPHyN thruster integrated. The design is based on the Space Dynamics Laboratory’s Picosatellite Exo-Atmospheric Research Laboratory (PEARL) spacecraft bus. This design incorporated a lower thrust version of the MUPHyN Thruster than the one used for testing in this project and could deliver approximately 160 m/s \( \Delta V \) to a 10 kg spacecraft.

1.6 Overview of the MUPHyN Motor Design

Figure 1.9 shows an exploded view of the prototype MUPHyN thruster assembly and Table 1.1 summarizes the primary design parameters. This prototype article was used
Fig. 1.8: A proposed 6-U CubeSat MUPHyN thruster propulsion system.

to perform ground tests on the USU campus. This prototype MUPHyN thruster design includes an FDM-manufactured fuel grain with an embedded helical fuel port, and an annular aerospike nozzle held by a central injector support fixture. The motor case is designed to fit within a 1U section of a CubeSat bus.

The aerospike nozzle contour was designed using the method of characteristics technique developed by Lee and Thompson [22]. The design nozzle expansion ratio is 2.25:1 and was selected as a compromise between performance, manufacturability, and heat transfer considerations. The 2.25:1 expansion ratio results in a nozzle that is slightly over expanded for the ambient pressure conditions at the test location in Logan UT, approximately 1300 meters above mean sea level (MSL). The nozzle was truncated to 70% of its full theoretical length.

The inner throat of the MUPHyN nozzle and nozzle plug are regenerative cooled and the outer throat is constructed from ablative high-density graphite. Nitrous oxide (the oxidizer) flows through the base of the MUPHyN, to the throat, and then down and out the tangential injectors into the combustion chamber. Figure 1.10 shows the oxidizer/coolant flow path. The walls of the combustion chamber are insulated with a phenolic liner on the sides and a graphite insert on the top (downstream near the nozzle exit). The outer casing of the test article is manufactured out of medium carbon steel. The base of the motor
Table 1.1: MUPHyN Motor Design Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Thrust</td>
<td>200 N</td>
</tr>
<tr>
<td>Chamber Pressure</td>
<td>690 kPa</td>
</tr>
<tr>
<td>Expansion Ratio</td>
<td>2.25</td>
</tr>
<tr>
<td>Throat Area</td>
<td>2.01 sq. cm</td>
</tr>
<tr>
<td>Oxidizer</td>
<td>Nitrous Oxide</td>
</tr>
<tr>
<td>Fuel</td>
<td>Acrylonitrile Butadiene Styrene</td>
</tr>
<tr>
<td>Design Specific Impulse</td>
<td>200s</td>
</tr>
<tr>
<td>Design Thrust Vectoring Side Force</td>
<td>10 N</td>
</tr>
<tr>
<td>Secondary Fluid</td>
<td>Helium, Nitrogen, or Oxygen</td>
</tr>
</tbody>
</table>

The case is aluminum and the aerospike components are copper to support heat transfer to the oxidizer. The prototype test article includes a single secondary injection port for testing effectiveness of secondary injection thrust vectoring during hot-fire test conditions.

Direct digital manufacturing of the ABS fuel grain is a key enabling technology for the MUPHyN design. The MUPHyN design uses FDM manufacturing techniques to embed a “multiple-helix” flow path into the ABS fuel grain. The embedded helical ports provide an extended length flow path with a large surface area in a short form factor. Additionally, the centrifugal forces created by the combustion gases rotating in the helical cores significantly increase the heat transfer to the fuel grain. This results in a significant increase in the fuel regression rates and propellant mass flow. Figure 1.11 shows a three-port helical fuel grain used for the prototype MUPHyN testing. This triple-helical port is more than 60-80% shorter than traditional cylindrical fuel ports. The entire MUPHyN motor and combustion chamber stack is only 5 cm in length, but features more than 30 cm of equivalent internal flow length.

1.7 Thesis of the Dissertation

As alluded to in the research overview, the thesis of this dissertation has many parts and involves the convergence of several technologies. The major components of this thesis are:
Fig. 1.9: Exploded view of prototype MUPHyN thruster assembly.

Fig. 1.10: MUPHyN coolant flow path.
A hybrid rocket thruster can be constructed to volumetrically fit into a CubeSat satellite bus using DDM techniques for fuel grain fabrication.

This hybrid thruster can achieve performance levels similar to those exhibited by conventional form factor hybrids with similar propellants.

Aerospike nozzles can use secondary fluid thrust vectoring and obtain similar efficiencies to conventional de Laval nozzles that use this technique.

The regression rate characteristics of hybrid rocket systems can be reasonably characterized using first-principle based heat transfer models and these models can be used to predict hybrid motor performance.

Regenerative cooling using nitrous oxide can provide sufficient cooling capacity for a small scale hybrid rocket thruster with an aerospike nozzle.

An igniter can be constructed for hybrid motors that is a sufficient source for ignition without the use of pyrotechnics or gaseous or liquid fuel/oxidizer mixtures.
All of these hypothesis are addressed by the design, construction, and evaluation of the MUPHyN thruster or by research directly supporting the MUPHyN design.
Chapter 2

Literature Review

2.1 Hybrid Rocket Development History

A very thorough overview of early hybrid rocket history is given by Altman [11], a summary of which is provided here. The beginning of hybrid rocket development dates back to the 1930’s but substantial sponsored research did not begin until the 1960s when several large institutions including the United Technology Center (UTC), Aerojet, Marquardt, Rocktdyne, Thiokol, Atlantic Research and the U.S. Naval Ordnance Test Station all began active hybrid rocket research programs. Perhaps the most significant research completed during this period was that undertaken by the the UTC which investigated hybrid rocket ballistics under a contract sponsored by the US Navy. This research effort provided the basics of hybrid motor ballistics theory.

After the development of the shuttle solid rocket boosters, research in hybrid rockets waned from the 1970s until the 1980s when the Space Shuttle Challenger disaster [23] and a Titan III failure motivated work towards replacement of large solid rocket boosters. The American Rocket Company (AMROC) worked towards developing large hybrid boosters in the 1980’s and 1990’s [24–28]. During this period, AMROC developed and tested motors up to 250,000 lbf thrust range. Unfortunately, AMROC’s attempts to prove the flight worthiness of their large hybrid designs stalled during the development of the Hybrid Technology Option Project, which experienced low frequency combustion instabilities. The financial burden of these problems and their proposed fixes eventually drove AMROC out of the project and large scale hybrid motor research declined shortly thereafter.

In 2004, hybrid motors enjoyed a flare of attention after SpaceShipOne, a rocket plane built by Scaled Composites and propelled by a hybrid rocket motor designed by SpaceDev, won the Ansari X Prize after launching a commercial vehicle to 103 km altitude [29]. The
inherent safety and low cost of hybrid motors was publicly demonstrated and continues to make hybrids an attractive choice for space tourism.

Leveraging the heritage of the successful SpaceShipOne motor, the Sierra Nevada Corporation (which acquired SpaceDev in 2008) is designing and testing a hybrid rocket propelled “Dream Chaser” vehicle as part of the Commercial Crew Development program (now called the Commercial Crew Integrated Capability initiative) [30, 31]. The Dream Chaser vehicle is a lifting-body design propelled by two 12,000 pound-force thrust nitrous oxide and hydroxy-terminated-polybutadiene (HTPB) motors. Dream Chaser is designed to be launched on top of an Atlas V and supply crew and cargo to low earth orbit, especially to the International Space Station. The Dream Chaser Program aims to have an orbital flight by 2014, an accomplishment which would undoubtedly create a surge of renewed interest in hybrids for crew and space applications.

Recently, efforts at Stanford University, NASA Ames Research Center, and the Space Propulsion Group (SPG) have significantly advanced the design and understanding of hybrid rocket motors using paraffin-based fuels. Stanford University and NASA Ames have been developing a nitrous oxide paraffin 100 km max altitude sounding rocket [32–34]. The SPG has designed a high performance hybrid upper stage motor designed to replace the Orion 38 solid rocket motor [35]. Their motor design, using liquid oxygen and a paraffin based fuel, shows significantly increased performance over the solid motor system it is designed to replace. The SPG, under a contract from the Air Force Research Labs, has contributed a great deal to the understanding of regression mechanisms for liquifying fuels, such as paraffin [36–39]. Their tests have shown regression rates for paraffin fuels several times those seen with conventional thermoset hybrid rocket fuels.

2.2 Regression Rate Modeling Development

One of the most significant challenges in the design of hybrid rocket motors lies in the prediction of the fuel flow rate. While the flow of oxidizer into a hybrid rocket motor can be precisely controlled by any of various throttling methods, the flow rate of the solid fuel depends upon the surface area open to combustion and the solid fuel regression rate.
The fuel regression rate is defined as the linear rate at which the fuel burns normal to the local surface. Figure 2.1 shows the progressive regression of a circular fuel port. The fuel regression rate and surface area determine the fuel mass flow rate and oxidizer-to-fuel (O/F) ratio for the motor. This O/F ratio typically varies throughout a hybrid motor burn and is a key parameter in obtaining a high performance motor design. If the O/F ratio is either too high or too low, combustion will generate less than the optimal specific impulse. In addition, if the O/F ratio is too high, the oxidizer rich combustion products can cause significant nozzle erosion. Hence, the accurate modeling of solid fuel regression is a crucial step in the design of hybrid rocket motors.

The regression of the solid fuel in a hybrid rocket motor is dominated by a substantially different mechanism than regression in solid rocket motors. Solid propellants are blended using a combination of oxidizer and fuel in a mass ratio that delivers the optimized performance for a given mission requirement. Because the propellant mixture ratio is set by the original formulation, the O/F remains constant. Propellant regression in solid rocket motors involves heat transfer from a combustion zone directly adjacent to the fuel surface and shows a strong pressure dependance. The regression rate in a solid motor is relatively independent of scale or geometry and is usually adequately described by a simple St. Robert’s law relationship specific to the fuel composition [5],
Fig. 2.2: Basic structure of hybrid rocket turbulent diffusion flame.

\[ \dot{r} = a P_0^n \]  

(2.1)

where \( \dot{r} \) is the propellant regression rate, \( P_0 \) is the motor chamber pressure and both \( a \) and \( n \) are empirical constants specific to propellant composition. In contrast, solid fuel regression in hybrid motors is limited by heat transfer from a turbulent diffusion flame located in a boundary layer to the fuel to the fuel surface. The basic geometry for this process is shown in Fig. 2.2. The complex fluid mechanics that drive this convection-dominated process make modeling regression in hybrid motors a great deal more complicated than modeling regression in solid rocket motors.

The first comprehensive theoretical work on modeling hybrid rocket fuel regression rates was completed in the 1960’s by Marxman and Gilbert [40,41] at the UTC. Marxman made several critical realizations about hybrid rocket fuel regression and combustion that enabled him and his collaborators to develop what is typically referred to as the classical hybrid rocket regression rate model. First, he noted that regression rates in most hybrid motors are heat transfer limited and not significantly influenced by chemical kinetics. He also noted that hybrid rocket motors are typically fully turbulent but seldom exhibit fully developed flow fields for traditional hybrid rocket form factors. Thus, he determined that regression rates should be well modeled by developing a Reynolds-analogy heat-transfer
model dependent upon the length-based Reynolds number. The resulting heat transfer model combined with the enthalpy of vaporization of the solid fuel results in a reasonable model for classical hybrid regression rates. For this analysis, Marxman assumed that the effective turbulent Prandtl number and Lewis number were equal to unity. Thus, the Stanton number was modeled as

\[ S = \frac{1}{2} C_f \left( \frac{\rho_e u_e^2}{\rho_c u_c^2} \right) = 0.03 R_{ex}^{-0.2} \left( \frac{\rho_e u_e^2}{\rho_c u_c^2} \right) \]  

(2.2)

where \( S \) is the Stanton number, \( C_f \) is the skin friction coefficient, \( \rho_e \) is the density at the boundary layer edge, \( \rho_c \) is the density in the combustion zone, \( u_e \) is the velocity at the edge of the boundary layer, \( u_c \) is the velocity at the combustion zone, and \( R_{ex} \) is the length-based Reynolds number.

Marxman and Gilbert noted that mass addition at the wall from fuel regression had a significant influence on the fuel regression rate and that the effective Stanton number would need to be adjusted for this “wall blowing” in the form

\[ S = S_0 \frac{S}{S_0} \]  

(2.3)

where \( S_0 \) is the Stanton number without mass addition at the wall.

Noting that the regression rate can be related to a simple energy balance,

\[ \rho_f \dot{r} \Delta H = \dot{Q} \]  

(2.4)

where \( \rho_f \) is the density of the solid fuel, \( \dot{Q} \) is the total heat flux into the solid fuel, and \( \Delta H \) is the effective heat of vaporization of the solid fuel, regression due to convective heat transfer was then modeled as

\[ \dot{r} = \frac{0.03 \rho_f R_{ex}^{-0.2} G S u_e}{S_0 u_c} \frac{h_{cs} - h_{w_g}}{\Delta H} \]  

(2.5)

where \( G \) is the oxidizer mass flux, \( h_{cs} \) is the enthalpy of the gas in the combustion zone, and \( h_{w_g} \) is the enthalpy of the gas at the fuel surface.
The flame height ratio, $\frac{u_c}{u_e}$, in eq. 2.5 accounts for the combustion zone lying between the wall and the edge of the boundary layer. The position of the flame sheet in the boundary layer determines the distance between the hot combustion zone and the fuel surface, significantly influencing the total heat transfer. This ratio is specific to fuel/oxidizer combinations and most of the data for this ratio was generated with HTPB fuel and gaseous oxygen. Marxman showed that this parameter can be estimated given the oxidizer to fuel ratio at the flame, $OF$ and the free-stream oxidizer concentration $K_{ox}e$,

$$\frac{u_c}{u_e} = \frac{OF(\Delta h_{cs} - \Delta h_{wg})}{K_{ox}e + (OF + K_{ox}e) \frac{(\Delta h_{cs} - \Delta h_{wg})}{\Delta H}}$$

Marxman noted that, in the plexiglass-oxygen system used for testing at the UTC, the flame zone generally resided at a slightly fuel-rich oxygen to fuel ratio of about 1.5. The oxygen to fuel ratio should not be confused with an oxidizer to fuel ratio as it only includes the mass fraction of elemental oxygen to the hydrocarbon fuel. During tests with nitrogen dilution, the flame sheet still occurred when the ratio of oxygen to fuel vapor was approximately 1.5. Hence, the effective oxidizer to fuel ratio for oxidizers that are only partially oxygen will be much higher than 1.5, depending on their oxygen concentration.

In later work [41], Marxman and Muzzy completed this model by employing a relationship for the ratio of Stanton numbers that accounts for the “wall blowing” effect caused by the vaporizing fuel surface limiting convective heat transfer. This was inspired by previous work by Lees [42] with flows with mass addition and chemical reactions,

$$\frac{S}{S_0} = 1.2B^{-0.77}$$

where

$$B = \frac{\rho_w v_w}{\rho_e u_c C_f}$$

and $\rho_w$ is the density at the wall, and $v_w$ is the velocity of the pyrolysis products normal to the wall. The blowing parameter often related to the thermodynamic properties of the
fuel and combustion products,

\[ B = \frac{u_e}{u_b} \frac{h_{cs} - h_{w_g}}{\Delta H} \quad (2.9) \]

The blowing parameter, \( B \), is extremely important to hybrid rocket heat transfer. Wall blowing of the pyrolysis products at the fuel surface provides a significant blocking effect to heat transfer. The wall blowing reduces the overall sensitivity of the regression rate model to the properties of the core fluid flow. Marxman noted in his work that this ratio could be as low as 0.2, considerably lowering the total heat transfer. The collected regression rate relationship for convection is then

\[ \dot{r} = \frac{0.036G}{\rho_f} \left( \frac{Gx}{\mu} \right)^{-0.2} \left( \frac{u_e}{u_c} \frac{h_{cs} - h_{w_g}}{\Delta H} \right)^{0.23} \quad (2.10) \]

where \( \mu \) is the gas viscosity of the core flow. This expression depends on the local total mass flux, \( G \), and the axial location along the port, \( x \). Assuming a constant diameter circular port, this expression was later integrated along the port length to yield an expression for the average regression rate [43],

\[ \dot{r} = \frac{G_o}{\rho_f} \left( 0.06 \left( \frac{G_o L}{\mu} \right)^{-0.2} \right) \times \]

\[ \left[ 1.25 + 2.5 \left( 0.06 \left( \frac{G_o L}{\mu} \right)^{-0.2} \right) \frac{L}{D_0} + 2.18 \left( 0.06 \left( \frac{G_o L}{\mu} \right)^{-0.2} \right) \left( \frac{L}{D_0} \right)^2 \right] \quad (2.11) \]

where \( L \) is the motor length, \( D_0 \) is the initial port diameter and \( G_o \) is the initial oxidizer mass flux.

Several researchers including Marxman and his colleges have examined the influence of radiation on hybrid rocket regression. Marxman correlated his regression rate measurements with experimental data from plexiglass combustion, for which he determined that radiant heat transfer was only 5-10% of the convective heat flux [41], although this fraction is sometimes significantly higher for other fuels. In Marxman’s original work, radiation was
accounted for by the addition of a second, radiative term to the right hand side of this regression relationship,

\[ \dot{r} = 0.036G \left( \frac{Gx}{\mu} \right)^{-0.2} \left( \frac{u_c}{u_e} \frac{h_{c_s} - h_{w_g}}{\Delta H} \right)^{0.23} + \frac{\sigma \varepsilon_w (\varepsilon_g T_c^4 - \alpha_g T_w^4)}{\rho_f \Delta H} \]  

\[ (2.12) \]

where \( T_c \) is the temperature of the combustion zone, \( T_w \) is the wall temperature, \( \varepsilon_g \) is the emissivity of the combustion products, \( \sigma \) is the Stefan–Boltzmann constant, \( \varepsilon_w \) is the emissivity and absorptivity of the wall, and \( \alpha_g \) is the absorptivity of the gas at the wall.

Later, Marxman and his colleges noted that the blowing parameter could strongly couple radiative and convective heat transfer to the grain surface. Increased radiant heat transfer causes increased wall blowing, and decreased convective heat transfer. Marxman and Wooldridge modeled the change in the blowing parameter with the relationship

\[ \frac{B_{rad}}{B} = \varepsilon^{1.3 \frac{\dot{Q}_{rad}}{\dot{Q}_c}} \]  

\[ (2.13) \]

where \( \dot{Q}_{rad} \) is the radiant heat transfer and \( \dot{Q}_c \) is the convective heat transfer.

In the 1980’s Paul and his colleagues at the Indian Institute of Science examined the effects of the density of fuel ablation products on the blockage effect created by fuel ablation [44]. They argued that the significant differences in measured regression rates between fuels could not be explained by the original relationships derived by Marxman. A series of experiments was then designed in an attempt to directly measure the effects of the blowing parameter on regression. With this data, Paul et al. produced a regression rate relation that depends on the density of the gaseous fuel products at the fuel surface,

\[ \dot{r} = \frac{G^{0.8}}{\rho_f^{0.056}} \left( \frac{x}{\mu} \right)^{-0.2} \left( \frac{\rho_f}{\rho_e} \right)^{0.71} \left( \frac{\rho_w}{\rho_e} \right)^{0.14} B (1 + B)^{-0.73 + 0.002 \frac{\varepsilon_w}{\varepsilon_e}} \]  

\[ (2.14) \]

They suggested that this regression rate significantly reduced data scatter in their experiments, but the data analyzed was limited and this analysis ignored the effects of radiant heat transfer, based upon the negligible radiant heat transfer seen by Marxman et al. in their original research with PMMA. However, it should be noted that the fuel used in Paul’s
experiment was a non-traditional rubber which may have had very different regression characteristics than conventional fuels. Additionally, radiant heat transfer has been shown to be significant for many hybrid motor fuels, especially at the relatively low oxidizer mass fluxes used in Paul’s experiments [45].

Later researchers have mostly worked to modify the classic form of the regression rate model proposed by Marxman and his colleagues. Altman [46] and his coworkers at the American Rocket Company, noted that the effect of blowing on the Stanton number in eq. 2.7 was more accurately described by

$$\frac{S}{S_0} = 1.0B^{-0.68}$$  \hspace{1cm} (2.15)

This correction results in a change in exponents for the classical regression rate theory,

$$\dot{r} = \frac{0.036}{\rho_f} G \left( \frac{Gx}{\mu} \right)^{-0.2} \left( \frac{u_e h_{cs} - h_{w_g}}{u_c \Delta H} \right)^{0.32}$$ \hspace{1cm} (2.16)

In the 1990’s, Estey, Altman, and others examined the error from nine separate empirical fits to a range of hybrid motor regression rate data. Their analysis included data from hot firings of motors with 1.5 inch to 10.5 inch port diameters [47]. Their work showed that oxidizer mass flux based correlations worked best on conventional hybrids using pure hydrocarbon fuels. They also noted that empirical expressions created in this way often do not extrapolate well to larger motors, which has been a significant issue throughout hybrid motor history.

In contrast to Estey and Altman’s findings that radiation was not significant for fuels not loaded with metals, researchers at the Joint Propulsion Laboratory in the 1990s determined that the radiative component of total heat flux can be significant for commonly used pure hydrocarbon fuels such as HTPB [45]. Their tests showed radiative heat fluxes on the order of 30% of the total heat flux for pure HTPB fuel burned with gaseous oxygen. They noted that the soot formed by hydrocarbon combustion is an effective radiator. This soot concentration will vary due to pressure and oxidizer to fuel ratio which will cause data scat-
between tests with similar fuels but different geometry or flow rates. They also noted that oxidizer mass flux based hybrid rocket regression rate correlations are often insufficient for predicting regression rates a priori, and frequently will have significant error even when applied a posteriori.

More recently, Chiaverini and his colleagues at Pennsylvania State University have revisited the classical regression rate theorems that were constructed in the 1960s [48]. They conducted a series of slab-motor tests capable of instantaneous regression rate measurements which they used to determine total heat flux. Their tests found that radiative heat transfer was an important phenomenon. They also obtained semi-empirical regression rate correlations of the form developed by Marxman and his colleges. Their analysis examined the effects of wall blowing and found that the ratio of Stanton numbers was well modeled as

\[
\frac{S}{S_0} = \left(0.65 + 9.56B_{mod}^{1.45}\right) \left(\frac{D_h}{L}\right)^0.3
\]  

before boundary layers in their combustor merged and

\[
\frac{S}{S_0} = \left(0.73 + 9.16B_{mod}^{1.6}\right) \left(\frac{D_h}{L}\right)
\]

afterward. In these relations Chiaverini et al. defined the blowing parameter using the average local mass flux and not the core oxidizer mass flux as Marxman and his colleges had done before,

\[
B_{mod} \equiv \frac{\Delta H_r + h_{bulk} - h_{wg}}{\Delta H}
\]

where \(\Delta H_r\) is the heat of reaction and \(h_{bulk}\) is the enthalpy of the combustion reactants. By employing a curve fit between the relations for un-merged and merged boundary layers, Chiaverini et al. constructed the regression rate correlation
\[ \dot{r} = \frac{G}{\rho_f} 0.0155 \left( \frac{D_h}{L} \right)^{0.3} \left( \frac{T_c}{T_w} \right)^{0.6} Re_D^{-0.2} \left( 0.73B + 9.16B_{mod} \left[ \frac{q_{rad}}{q_{c}} \right]^{-1.6} \right) \times 1.3 \left( \frac{q_{rad}}{q_{c}} \right)^{0.75} + e \left( -1.3 \left( \frac{q_{rad}}{q_{c}} \right)^{0.75} \right) \]  

(2.20)

It should be noted that the ratio of radiative to convective heat transfer, \( \frac{q_{rad}}{q_{c}} \) in this model must be evaluated based upon specific fuel and oxidizer combinations and may also vary with O/F ratio.

Carmicino and Sorge at the University of Naples investigated the effects of oxidizer injection geometry on regression rate behavior [49]. While most of the studies on hybrid rocket regression rates use a flow conditioner to provide uniform oxidizer flow at the head end of the fuel grain, practical hybrid rocket motors almost universally use injectors with a diameter much smaller than the combustion port. This injection scheme creates an oxidizer jet that Carmincio et al. found to substantially influence the longitudinal regression rate distribution as well as the overall mean regression rate. Through a least squares regression analysis, they found that the regression rate for their polyethylene fuel and gaseous oxygen hybrid obeyed the relationship

\[ \dot{r} = 0.716 \frac{G}{\rho_f} Re_D^{0.596} \left( \frac{d_i}{D_h} \right)^{-0.85} B^{-0.16} \]  

(2.21)

where \( Re_D \) is the port-diameter based Reynolds number and \( d_i \) is the injector diameter. The authors give no explanation for the use of a diameter-based Reynolds number in this correlation as opposed to the conventional length based models. Additionally, the injector amplification term in this equation, \( \left( \frac{d_i}{D_h} \right)^{-0.85} \), generates extreme results for small injector diameters. For example, using this model, an injector 1/10 the diameter of the port with would yield seven times the regression rate of an injector with a diameter equal to the port diameter, an amplification factor certainly not found in the literature. Although the authors showed a strong dependence for regression rate on injector geometry, clearly a more
an accurate model for this phenomenon is required.

In general, much of the early work on regression rate modeling focused on creating regression rate models using parameters that could be determined a priori. Marxman’s relation made a great deal of headway towards this goal, but still included terms, especially the ratio \( \frac{u_e}{u_c} \), which were specific to oxidizer fuel combinations and must be determined empirically. The free stream viscosity, enthalpy and other fluid properties in the classical relationship also have traditionally required empirical data. This has generally restricted the use of these relations to performance prediction with motors that use fuel/oxidizer configurations similar or identical to those used to derive the relationship.

Later work tended to focus much more on empirical fits that are not easily applied to different oxidizer/fuel combinations or widely different motor form factors. Much of the hybrid rocket industry continues to use to a simple, but inaccurate and not extensively applicable, Saint Robert’s Law style empirical correlations of the form [5]

\[
\dot{r} = aG^n
\]  

(2.22)

Recent work, including work by Chiaverini, has created much more accurate fits of empirical data using Marxman-style correlations, but are still only applicable to similar fuel combinations and motor configurations to that for which data has already been obtained.

More recently, chemical analysis tools such as Chemical Equilibrium Analysis with Applications (CEA) [50, 51] allow for prediction of many of the parameters in the original regression rate equations. The regression rate correlations developed by Eilers and Whitmore [52], to be described in chapter 3, use these tools to predict regression rates using Marxman-style enthalpy balance models. The closed form of these models opens the door for the prediction of hybrid motor performance in a truly a priori fashion, a feat not generally achievable with previously existing regression rate models.

### 2.3 Recent Advanced Developments in Hybrid Rocket Motors

Hybrid rocket motors have traditionally been designed with long, thin form factors in
order to facilitate enough fuel mass flow from the solid fuel grain. This is necessary to achieve an oxidizer to fuel ratio low enough to have both good performance characteristics and to prevent excessive erosion in ablative nozzles. This form factor has long prevented the application of hybrid rocket motors to vehicles that cannot accommodate a long form factor. This shortcoming has long been acknowledged by the hybrid rocket community. As a result, there has been some notable work completed on non-traditional hybrid motor designs with other form factors.

Surrey Satellite Technology worked on a compact hybrid motor concept, dubbed a “pancake” hybrid, in 2001 [53]. Their design “sandwiches” two fuel grain discs into a motor casing with the combustion port in between. Figure 2.3 shows this configuration. Oxidizer is injected tangentially from the outside of the combustion port and combustion products exit through a nozzle mounted in the center of motor case. They demonstrated relatively high combustion efficiencies compared to standard hybrid motor designs. They attributed this property to centrifugal forces keeping unburned pieces of fuel away from the nozzle exit in the center of the motor.

Knuth, Chiaverini and his colleagues at Orbital Technologies Corporation designed a “vortex hybrid” rocket motor [54]. Their design injects oxidizer from the outer edge of the motor case in a similar fashion to the Surrey design, but injectors and the motor case are sized such that co-axial vortices form in the motor port. Figure 2.4 shows this design. The tangential velocity of the injected flow keeps the injected oxidizer pressed against the fuel surface on the outer walls of the motor until the oxidizer flow reaches the top of the combustion chamber. At this point, the oxidizer flow spirals inwards and the combustion products flow through the center of the motor out the nozzle. The increased density of reactants or fuel slivers compared to completed combustion products, coupled with the spiraling flow, help ensure that only the lighter combustion products exit out of the central nozzle. The vortex hybrid design significantly increased the effective oxidizer mass flux due to the convoluted fluid flow path where combustion products travel through the combustion chamber twice, up and then down, before exiting the motor. This design
showed high regression rates as well as high combustion efficiencies.

Caravella et al. at Purdue University tested a radial flow hybrid burning polyethylene and hydrogen peroxide [55]. Their motor used oxidizer injected radially outward from the center between two flat fuel discs, similar to the “pancake” hybrid developed by Surrey Satellite Technology. Their tests demonstrated larger regression rates than would have been predicted through classical regression rate theory, but were conducted at oxidizer mass fluxes far below those experienced in traditional hybrid motors.

The Aerospace Corporation has recently investigated the benefits of rapid prototyping for small scale hybrid motor fuel grains, producing some non-traditional designs and form factors [16]. Their fuel grain designs were meant to showcase the flexibility of grains produced by stereolithography or fused deposition modeling. They designed and tested grains with triple helices, blind ports, and cells containing paraffin. Their tests exhibited stable combustion although combustion efficiencies and specific impulses were not reported. A fuel grain with a triple helix printed in clear material is shown in Fig. 2.5. Unfortunately, the Aerospace Corporation did not publish regression rate or performance data for these helical fuel port configurations in the open literature.

2.4 Aerospike Nozzle Development History

Significant development on aerospike nozzles began in the 1950s and 1960s when truncated plug nozzles were being considered for the Saturn V upper stages [56] due partially to the large form factor advantage of aerospike engines. Figure 2.6 shows a size comparison of an aerospike replacement for the J2 upper stage motor. It was initially determined that a toroidal aerospike engine could have resulted in a two to five percent increase in Saturn V payload capacity. Aerospike nozzles were later examined as a possible choice for the Space Shuttle’s main engine [57–59]. During this period, Rocketdyne conducted extensive research into both aerospike performance and liquid injection thrust vectoring on aerospike nozzles [14,60]. Rocketdyne examined the effects of aerospike nozzle truncation, base bleed, and general performance over a series of cold flow and hot flow test fires. The test fire of one of these engines, a 250,000 lbf thrust annular aerospike, is shown in Fig. 2.7. During
Fig. 2.3: Surrey Satellite Technology “pancake” hybrid motor. [53]

Fig. 2.4: Pennsylvania State University “vortex injection” hybrid motor. [54]
their thrust vectoring efforts, Rocketdyne concluded that aerospike nozzles had less or equal thrust vectoring capability than bell nozzle counterparts. However, their tests were limited to liquid injection. Rocketdyne did not perform cold flow thrust vectoring tests and hot gas injection hardware was not then available.

After a conventional bell nozzle was chosen for the Space Shuttle Main Engine, work on aerospike nozzles was reduced significantly until the 1990s when work began on the X-33 single stage to orbit (SSTO) vehicle [61]. In support of this effort, significant testing was performed by Rocketdyne for Lockheed during the development of the RS-2200 linear aerospike [62–64].

After the X-33 and the Venture Star programs were canceled, aerospike nozzle development once again became sporadic. In the USA, NASA Langley explored parametric modeling and optimization of aerospike nozzles and created a database and methodology for aerospike engine optimization [65]. Simultaneously, computational algorithms to evaluate thrust vector control for aerospike nozzles were developed at the University of Alabama in Huntsville [66]. Differential plenum throttling research was completed at the Marshall Space Flight Center [67]. During this test series, differential throttling was found to have little effect on total nozzle efficiency, but side force was highly dependent on the total nozzle pressure ratio.

In 2005, engineers from NASA Dryden Flight Research Center and the US Air Force Research Laboratory, Propulsion Directorate designed and flew an aerospike nozzle on a
Fig. 2.6: Size comparison of the Saturn V J2 engine and a replacement aerospike engine. [56]

Fig. 2.7: Rocketdyne test fire of a 250,000 lbf annular truncated annular aerospike engine.
high power rocket [15]. Their tests only indirectly measured nozzle performance – they measured vehicle acceleration and position – and showed somewhat decreased performance for their annular aerospike nozzle over a conventional nozzle. However, this performance reduction probably resulted from the larger throat area of the aerospike nozzle altering motor performance.

Several universities in the United States have had active aerospike nozzle research programs. Research on adapting annular aerospike nozzles for hybrid rockets was performed at Arizona State University [68] and the University of Washington [69]. There were notable challenges due to erosion of the nozzle support structure in the former, and nozzle ablation rates were not presented in the latter. California Polytechnic University has also investigated coupling an aerospike nozzle with a hybrid rocket motor. Their efforts centered on non-regenerative active cooling techniques [70–72]. Their tests showed that nitrous oxide, a common hybrid oxidizer, is an acceptable coolant fluid for aerospike nozzles. California State University, Long Beach in association with the Garvey Spacecraft Corporation has also completed extensive testing of liquid, clustered aerospike engines. Their efforts have culminated in the launch of several sounding rockets [13,73–76].

Outside of the United States, aerospike nozzles have enjoyed a large amount of attention in recent decades. The European Space Agency has investigated the relative effectiveness of various aerospike thrust vector control techniques [77] although little in the way of hard conclusions were drawn. The French Aerospace Lab performed a large bank of tests on clustered aerospike nozzles and generated a large database of pressure, temperature, heat flux, Schlieren photography, oil visualizations, pressure sensitive paint, and IR images [78]. In the mid 1990’s The Technical University of Munich conducted analytical research on performance aspects of aerospike nozzles including performance losses due to nozzle clustering [79,80]. Their research mostly focused on performance of aerospike cluster configurations as they would apply to single-stage-to-orbit vehicles. They concluded that aerospike nozzle configurations would probably experience a weight penalty over conventional bell nozzle configurations, but this efficiency loss could be made up for by savings
to the vehicle thrust frame, aerospike nozzle altitude compensation, and, especially, the efficiencies that come from the higher area ratios that are practical on aerospike nozzles. They also noted that flow losses due to plug cluster designs were often on the same order as efficiency gains due to altitude compensation, and thus these losses require detailed analysis. Some research has also been completed at the German Aerospace Center on CFD simulation of cluster aerospike configurations [81–83]. Their worked examined flow structures exhibited by plenum exhaust interaction in cluster configurations and examined the mechanics of closed and open wake aerospike nozzle configurations. They achieved CFD solutions for some configurations that showed good agreement with pressure profiles on experimental nozzles. The same research group also looked at tradeoffs for advanced nozzle concepts including aerospike nozzles [82]. In Italy, computational work has been done to characterize the heat flux into aerospike nozzles [84]. They computed a maximum heat flux of approximately $10 \text{ MW/m}^2$ for a 0.16 meter long polyethylene-oxygen aerospike motor.

A substantial amount of work has also been completed in Italy on performance validation, flight behavior, and motor cluster performance for aerospike nozzles [85–88]. Computational studies concluded that the gaps between plenum exhaust ports for cluster configurations on linear aerospike nozzles can significantly reduce nozzle performance. Their work has also shown that free-stream interaction effects can significantly reduce altitude compensation effects on aerospike nozzles in supersonic flight at low or medium altitudes through effectively influencing the “closed wake” transition point where the nozzle no longer adjusts to ambient pressure.

Numerous analytical studies have been performed at several Universities in Japan on aerospike nozzles involving slipstream effects, slipstream effect mitigation, and base bleed injection [89–97]. Much of their work looked into computational modeling of aerospike nozzle base drag and drag mitigation through base bleed injection, resulting in approximations for optimal base bleed mass flow rates, angles, and orifice locations.

Experimental work has also been completed in Japan to investigate the flow field of clustered linear aerospike nozzles [98]. They noted the importance of sidewalls on linear
aerospike nozzle efficiency and examined effectiveness of base bleed injection. Researchers at the Japan Aerospace Exploration Agency computationally and experimentally investigated thrust loses in clustered linear aerospike nozzles, most notably including the test fire of a clustered 14kN linear aerospike engine [99–101] which showed a total thruster efficiency of about 87%. More recently, these researchers examined the conceptual design of a SSTO vehicle aerospike nozzle [100]. Beijing University in China has performed computational analysis as well as cold flow tests on aerospike nozzles. These tests investigated nozzle performance, base bleed effects, and thrust vectoring [102–106] mostly in regard to linear aerospike engines.

The National Aerospace Laboratories in Bangalore, India have investigated the acoustics of aerospike nozzles [107] and performance characteristics of conical aerospike nozzle contours [108]. In the latter research they showed that their conical nozzle performed about 3% to 4% less effectively than ideal expansion and truncating the conical nozzle at half length further reduced performance on the order of 3%.

Some analytical work has also been completed in Russia on optimal aerospike contours [109]. These researchers created a mathematical model for total thrust optimization of both the nozzle contour and the angle of the throat to the aerospike nozzle surface. The Aerospace Research Institute in Iran has also completed some work on base bleed performance, finding that the optimal base bleed for under-expanded conditions was about 2% of the main flow, and about 5% for over-expanded, open-wake conditions [110].

### 2.5 Secondary Injection Thrust Vectoring Development History

The use of fluid injection into a convergent/divergent nozzle for attitude control on a rocket was originally patented by Arthur Weatherbee of the United Aircraft Corporation in 1960. Secondary injection thrust vector control (SITVC) techniques have been a subject of major research since the 1950’s and 1960’s. For SITVC, a secondary fluid is injected into the divergent section of the primary nozzle. The primary flow features involved in SITVC are shown in Fig. 2.8. The injectant displaces fluid in the nozzle flow field which creates a separation region before the injection location as well as a shock wave. The separation
region and the bow shock create a high pressure region in front of the secondary injection orifice, which adds to the side force provided by the momentum of the injected fluid.

These thrust vector techniques have been especially attractive for solid propellant rocket engines which, due to their size, are substantially more difficult to gimbal than liquid engines [111]. Originally, secondary injectants were liquid instead of hot gas because high temperature valves had not yet been developed [5]. Early liquid injectants were often water, freon, or a reactive fluid such as hydrazine. The performance of liquid secondary injection stems from the vaporization (and reaction, in the case of hydrazine) of the liquid injectant in the high temperature gas flow [112]. Typical side force specific impulses for freon ranged from 45s–70s and in the upper 230s for decomposing hydrazine [112]. Hot gas injection systems have much higher performance than liquid injection systems, which has largely eliminated liquid injection thrust vector control from consideration on modern vehicles.

Thrust vectoring effectiveness for gaseous injection is commonly defined in terms of a ratio of either side force specific impulse to axial force specific impulse, or the side force specific impulse to the side force specific impulse without axial flow. For this research
project, gaseous injection side force amplification factor, $A_f$, is defined as the ratio of side force produced with a main axial flow active, $F_i$, (and corresponding amplifying flow effects) to the side force with secondary injection but without primary flow $F_v$,

$$A_f = \frac{F_i}{F_v} \quad (2.23)$$

This is similar to the definition used by Walker [113] in thrust vectoring research performed at Johns Hopkins University. Using this definition for side force amplification factor avoids the dependence on the arbitrary efficiency of the primary thruster.

Thrust vectoring efficiency for gas injection into conical nozzles has been well established although data for gaseous injection is not as available as that for liquid injection. Work performed by Gunter and Farenholz on cold flow tests with a conical nozzle yielded amplification factors of approximately 2.0 [114]. Walker, Stone and Shandler also performed cold flow tests, including some with carbon dioxide as a working fluid, and had side force amplification factors that ranged from approximately 1.8 to 3.0, with the highest amplification factors gained by the smallest orifices [113]. Inouye performed a series of hot gas injection tests and produced amplification factors generally between about 1.2 and 1.8 for a motor and secondary injection motor using red fuming nitric acid and unsymmetrical dimethylhydrazine [115].

Significant attempts have also been made to create theoretical models for the side forces generated by secondary fluid injection. One of these models is the blast wave theory analogy promoted by Broadwell in the 1960’s [116]. Broadwell noted the similarity between the shock wave shapes exhibited by secondary injection into supersonic flow and the shock waves generated by linear charges. This approach neglects any dependence on boundary layer effects at the injection site. Unfortunately, these effects are not insignificant. Hence, this, like the other theoretical models proposed in the 1960s, should be used to predict trends only and do not make good predictors of actual side force amplification factors. The defects of theoretical models for side force injection were examined in detail by Guhse [117]. With the aforementioned note, the blast wave theory model predicts that side force amplification
should generally obey the relation

\[
\frac{F_i}{F_v} = \frac{C\sigma (\gamma) M_\infty V_\infty}{(V_j)} \left[ 1 + \frac{2 + (\gamma - 1) M_\infty^2}{2(\gamma - 1) M_\infty} \right] \tag{2.24}
\]

where \(M_\infty\) is the free stream Mach number, \(V_\infty\) is the free stream velocity, \(\sigma\) is the nondimensional pressure in a cylindrical explosion, \(\gamma\) is the specific heat ratio, and \(V_j\) is the injectant velocity. It is noteworthy that this relationship has a strong positive dependence on Mach number. Walker, Stone, and Shander [113] modified a linear flow relation by Vinson, Amik and Liepman [118] that assumed that the side force could be approximated by linear supersonic flow analysis over the area displaced by a secondary flow injectant expanded to the pressure of the primary flow. Their relation, once solved for the amplification factor defined above, yields

\[
\frac{F_i}{F_v} = \frac{1}{(1 + \gamma)} \frac{\gamma M_\infty^2}{(M_\infty^2 - 1)^{\frac{1}{2}}} \left[ \frac{1 + \gamma}{M_{j_\infty}^2} \left[ 2 + (\gamma - 1) M_{j_\infty}^2 \right] \right] \tag{2.25}
\]

where \(M_{j_\infty}\) is the injectant Mach number downstream of the injection site.

Another prominent model which seems to reproduce trends for secondary injection for conical nozzles is the relation proposed by Wu et al. [119]. Their method uses conical shock analysis to approximate the pressure distribution behind the primary bow shock with a parabolic fit. They adjust for geometry but presume that the effects of the bow shock and the over expansion region behind the injection point cancel downstream of the injection orifice. In their model side force can be determined from the relation

\[
F_i = \left[ \left( \frac{P_2}{P_\infty} - 1 \right) (\Delta A - X h_a) + \left( \frac{P_s}{P_\infty} - 1 \right) \left( X h - \frac{A_j}{2} \right) \right] P_\infty \cos (\alpha) + P_\infty A_j \cos (\varepsilon) \left( \frac{P_j}{P_\infty} - 1 \right) + \dot{m}_j V_j \tag{2.26}
\]

where \(P_2\) is the static pressure downstream of the conical shock wave, \(P_\infty\) is the static pressure of the primary flow, \(\Delta A\) is the area behind the shock wave, \(X\) is the distance
between the start of separation and the injection point, \( h_a \) is the accommodation height, \( P_s \) is the static pressure behind the shock wave, \( A_j \) is the area of the secondary injectant, \( \alpha \) conical nozzle half area, \( \dot{m}_j \) is the mass flow rate of the secondary injectant, and \( \varepsilon \) is the angle of the injectant with respect to the nozzle radial direction. The accommodation height, separation distance, and area behind the shock can be estimated via

\[
h_a = \left[ \frac{2A_\infty}{\pi} \right]^{\frac{1}{2}} \left[ \frac{2\gamma^2 M_j^2 \left( 1 + \frac{\gamma - 1}{2} M_j^2 \right)}{\left( \frac{A_\infty}{A_j} \right)^2 \left( \frac{P_\infty}{P_j} \right)^2 \left( \frac{P_s}{P_\infty} - 1 \right) \left[ \gamma + 1 + \frac{\gamma - 1}{\gamma + 1} \frac{P_s}{P_\infty} \right]} \right]^{\frac{3}{4}} \tag{2.27}
\]

\[
X = h_a [\cot (\delta) + \tan (\alpha + \varepsilon)] \tag{2.28}
\]

\[
\Delta A = h_a^2 [\cot (\delta) + \tan (\alpha + \varepsilon)]^2 \tan \theta \tag{2.29}
\]

where \( \theta \) is the angle between the conical shock wave and the nozzle wall, and \( \delta \) is the angle between the outer boundary of the separation region and the nozzle wall.

Unfortunately, the models described above predict widely varying forces for the same operating conditions. The wide divergence in trends predicted by these models tends to limit confidence in their use as prediction tools. For this reason, experimental characterization of secondary injection thrust vectoring continues to be an important step early in the design phase for systems using fluidic thrust vector control.

It is noteworthy that LITVC systems have been designed for hybrid motor thrust vectoring in the past. The Aquila series of launch vehicles designed, but never produced, by AMROC incorporated a liquid oxygen LITVC system [28]. Their design could deflect thrust 6 degree by injecting liquid oxygen approximately 1/3 of the distance between the nozzle throat and exit plane.

2.6 Hybrid Rocket Motor Ignition System Development History

The potential for starting, stopping, and restarting hybrids has been known since their inception, and has often been touted as a key selling point of hybrid motors. Any form
of hybrid used for on-orbit station keeping or orbital maneuvers will need an efficient and reliable method for restarting a cold motor. However, there has been almost no investigation into practical methods for in-flight restart. Even the conventional methods for single ignition of hybrids, namely secondary hypergolics [120], small solid propellant igniters [121], and propane-sparker systems [122] were developed and patented between 1964 and 1970. The idea of using a pyrotechnic valve to initiate oxidizer flow and ignite the motor [123] is relatively recent, as it was patented in 1995, but this is the extent of the current technology. None of these methods combine both simplicity and re-ignition capability. A hypergolic ignition system defeats the safety advantage of nitrous oxide, solid propellant igniters are not reusable, propane sparker systems require a secondary tank and piping path for propane and significantly increases risk, and a pyrotechnic valve is a one-use ignition method. Clearly a new system is needed, preferably one that maintains the safety of the system, with a minimum of added complexity.
Chapter 3

Development of the Longitudinally Averaged, Variable Prandtl Number Hybrid Rocket Regression Rate Model

For the design of any operational hybrid motor, accurate regression rate prediction is of extraordinary importance. The fuel regression rate determines the fuel mass flow which control the motor’s O/F ratio which greatly impacts motor performance. Also, the fuel in a hybrid rocket motor is often used as insulation between the hot combustion gases and the chamber walls. If the fuel burns to the wall long before motor termination, the hot combustion gases can quickly lead to catastrophic failure of the motor case. Thus the prediction of fuel regression rate is extremely import also from a motor health perspective. A high oxidizer to fuel ratio also allows oxidizing species to travel through the nozzle. If the nozzle is an ablative material such as graphite, this can dramatically increase nozzle erosion.

The regression rate in a hybrid motor is driven by heat transfer to the fuel wall. Heat transfer in a hybrid motor is usually dominated by turbulent convection. This means that regression rate models must deal with the same amount of uncertainty and lack of closure as would any turbulent convective heat transfer model. Additionally, hybrid rocket heat transfer is driven by the combustion of gases in the boundary layer. This makes matters far more complicated. As put by Lester Lees [42], a former professor of Aeronautics at the California Institute of Technology, “Our understanding of the mechanism of turbulent transport in non-uniform flow is still very limited even for pure gases, and the situation is hardly likely to be improved by adding chemical reactions and mass transfer to the problem.” Even with this inherent difficulty, however, hybrid regression rate models can and have been made that show reasonable agreement with experimental data over certain flow regimes. Most commonly, these models are derived for simple cylindrical port hybrids
where the internal boundary layer development is relatively simple to model.

Even though the regression rate in the MUPHyN motor is a great deal more complicated than that predicted for a simple cylindrical grain, the derivation of a closed form hybrid model still yields valuable insight into the regression processes. Even though they were eventually proven to be very low, this model was also used for baseline regression rate predictions in the MUPHyN motor. Thus a development for a closed-form hybrid rocket system model is included here. A discussion on the development of this model was also published in the Journal of Spacecraft and Rockets [52].

3.1 Motor Combustion and Regression Model Development

This section develops an end-to-end hybrid motor regression and combustion model, the Longitudinally Averaged, Variable Prandtl number (LAVP) model. A regression model based on an enthalpy balance from the surface of the fuel grain to the flame zone is developed first. This regression model is used to develop the chamber pressure response equation based on choking mass flow at the throat and the propellant mass flow through the combustion chamber. Finally, a thermochemical model for combustion is developed and used to generate look-up tables for use in the regression and combustion pressure models.

3.1.1 Fuel Regression Model

Sutton and Biblarz [5] outlines the basic structure of the enthalpy-balance regression rate model used for this analysis. For normal operating pressures, fuel surface regression rate has been shown to primarily be a function of convective heat transfer across a turbulent boundary layer to the fuel surface [5,40,41,124]. Mass transport across the boundary layer creates a region in which oxidizer flow from the center of the motor combustion port mixes with vaporizing solid fuel leaving the fuel wall. Inside this boundary layer is the flame zone, where this mixture of fuel and oxidizer reaches a ratio that supports combustion. Heat transfer from this zone to the solid fuel grain drives the regression rate behavior of hybrid rocket motors. This process is depicted in Fig. 2 3.1. The enthalpy-based regression model starts with the examination of heat transfer in a control unit on the fuel-grain surface. Heat
transfer from convection is directly related to the fuel regression rate through

\[ \dot{Q} = \rho_f \dot{r} \Delta H \]  

(3.1)

A simple energy balance means heat transfer from convection is equal to the heat transfer into the solid fuel grain at the fuel-grain surface. Assuming an approximately cylindrical port geometry, this enthalpy balance yields

\[ \rho_f \dot{r} h_v = H (T_f - T_s) \]  

(3.2)

In Eq. (3.2), \( \dot{r} \) is the linear rate of change of the fuel port radius, equivalent to the rate of fuel-grain regression. Figure 3.2 illustrates the fuel regression for a circular cross section within the motor.
The convective heat-transfer coefficient in Eq. (3.2) can be expressed in terms of the Stanton number:

\[ H = c_p \rho_e u_e S \]  

(3.3)

Equation 3.3 is substituted into Eq. (3.2) to yield

\[ \rho \dot{f} \Delta H = c_p \rho_e u_e S (T_f - T_s) = \rho_e u_e S (h_{c_s} - h_{w_g}) \]  

(3.4)

where \( u_e, \rho_e, c_p \) are the oxidizer velocity, density, and specific heat at the edge of the flame boundary, respectively. The \( h_{c_s} - h_{w_g} \) term is the enthalpy difference between the gas at the surface of the fuel grain and the gas within the flame zone. Because the internal flow in a hybrid motor of this size is highly turbulent, the Stanton number can be related to the local skin friction coefficient using the Reynolds-Colburn heat transfer / skin friction analogy for non-unity Prandtl number,

\[ S = C_f \frac{2}{Pr} \]  

(3.5)

Equation (3.5) is a modification of the original Reynolds analogy (valid only for unity Prandtl number) and is derived from the assumption that the velocity and temperature profiles within the laminar sub-layer (within the fuel ablation zone) are linear with height above the surface. Depending on the working fluid, the turbulent Prandtl number is a weak function of the fluid Prandtl number and has a magnitude ranging from approximately 0.5 to 1.0 [125]. For the hybrid motor, the properties of the combustion products products within the chamber are strongly influenced by the propellant mixture ratio and the chamber pressure. A 50–100% variation in fluidic Prandtl number can be observed over the operating range of the motor. Thus, for this combustion model, the Prandtl number is considered a function of motor mixture ratio and chamber pressure. Estimation of the Prandtl number and other fluid properties will be discussed in detail later in the Modeling of the Combustion Product Properties subsection. Substituting eq. 3.5 into eq. 3.4 and
solving for the regression rate yields

\[
\dot{r} = \frac{C_f}{2} Pr^{-\frac{2}{3}} \Delta h \rho_e u_e \rho_f h_v
\]  

(3.6)

The Reynolds–Colburn analogy is strictly valid only for fixed wall boundaries with no out-gassing, and a correction factor for surface blowing is needed to account for the mass addition of the vaporized fuel. This correction for an actively evaporating fuel surface is presented by Lees \cite{42} as

\[
(c_f)_{\text{blowing}} = 1.27 \left( \frac{\Delta h}{h_v} \right)^{-0.77} C_f
\]  

(3.7)

Substituting Eq. (3.7) into Eq. 3.6 and collecting terms gives the corrected form of the regression rate equation,

\[
\dot{r} = \frac{0.635 \left( \frac{\Delta h}{h_v} \right)^{0.23} C_f Pr^{-\frac{2}{3}} \rho_e u_e}{\rho_f}
\]  

(3.8)

Equation 3.8 includes the longitudinal development of the boundary layer within the fuel grain. Clearly the boundary layer within the tube would grow until fully developed channel flow is reached. This boundary layer growth process is depicted in Fig. 3.3.

Typical hybrid rocket grain geometries have length to diameter ratios less than 20, and fully developed flow is not generally reached in the port length. Thus an empirical skin friction model based on flat plate boundary layer theory should be used in lieu of a fully developed model for pipe-flow skin friction. Thus, the model used for this analysis was
developed based on an empirical relationship for boundary layer thickness,

$$\delta_x = \frac{0.38 x}{(Re_x)^{0.2}} = \frac{0.38 x}{(\rho u_x \delta \mu)^{0.2}}$$  \hfill (3.9)

and the Blasius formula for turbulent wall shear stress,

$$C_f = \frac{\tau_{wall}}{\frac{1}{2} \rho u_e^2} = \frac{0.0465}{Re_\delta^{\frac{1}{4}}} = \frac{0.0465}{(\rho \rho \mu \delta \mu)^{\frac{1}{4}}}$$  \hfill (3.10)

In Eq. (3.10) $Re_\delta$ is the Reynolds number based on the longitudinal distance down the chamber, $\delta_x$ is the local boundary layer thickness, $u_e$ is the velocity at the edge of the boundary layer, and $\rho_e$ and $\mu$ are the density and dynamic viscosity of the combustion products. Equations 3.9 and 3.10 are valid for $10^6 < Re_x < 10^7$. Substituting Eq. 3.9 into Eq. 3.10 and collecting terms yields

$$C_f = \frac{\tau_{wall}}{\frac{1}{2} \rho u_e^2} = \frac{0.0592}{(Re_x)^{\frac{1}{5}}}$$  \hfill (3.11)

Equation 3.11 is integrated along the length of the port to give an averaged value for the skin friction coefficient,

$$C_f = \frac{1}{L} \int_0^L \frac{0.0592}{(Re_x)^{\frac{1}{5}}} dx = \frac{0.074}{(Re_x)^{\frac{1}{5}}}$$  \hfill (3.12)

The resulting longitudinal mean skin friction model is

$$C_f = \frac{0.074}{(Re_L)^{\frac{1}{5}}}$$  \hfill (3.13)

where, $Re_L$ is Reynolds number based on fuel port length which is typically on the order of $10^6$ for lab-scale hybrid motors and $10^4 - 10^5$ for miniature thrusters. The oxidizer mass flux term, $\rho_e u_e$, is related to the instantaneous cross-sectional combustion chamber area and the mass flow rate of oxidizer.
\[ \rho_e u_e = \frac{\dot{m}_{ox}}{A_c} \]  

(3.14)

Assuming pseudo-incompressible flow through the injector, the mass flow rate of oxidizer can also be related to the chamber pressure.

\[ \dot{m}_{ox} = A_{ox} C_{d_{ox}} \sqrt{2\rho_{ox} (P_{ox} - P_0)} \]  

(3.15)

This relationship can be adjusted for multiphase flow regimes that no longer accurately obey this relationship through the adjustment of the discharge coefficient, \( C_{d_{ox}} \).

Substituting Eqs. 3.13, 3.14, and 3.15 for the skin friction coefficient and oxidizer mass flux into Eq. 3.8 yields an approximate model for the longitudinally averaged regression rate along the length of the fuel port as a function of injector parameters, combustion parameters, and port diameter,

\[ \dot{\hat{r}} = \frac{0.047}{Pr^{\frac{2}{3}}} \rho_f \left( \frac{c_p (T_f - T_s)}{h_v} \right)^{0.23} \left( \frac{A_{ox} C_{d_{ox}}}{A_c} \sqrt{2\rho_{ox} (P_{ox} - P_0)} \right)^{\frac{4}{5}} \left( \frac{\mu_{ox}}{L} \right)^{\frac{1}{5}} \]  

(3.16)

The total mass flow fuel into the combustion chamber is simply

\[ \dot{m}_f = A_{burn} \rho_f \dot{\hat{r}} \]  

(3.17)

where the fuel surface area, \( A_{burn} \), is a function of the total fuel regression.

### 3.1.2 Chamber Pressure Model

After ignition, the combustion process produces high temperature gases that escape through the nozzle throat. The mass flow through the nozzle will lag the mass flow into the chamber from the fuel and injector so the and pressure within the combustion chamber builds. The time response of this chamber pressure growth can be calculated by a balance between the gases coming into the fuel port and the gases leaving through the choked nozzle,
\[
\frac{\partial P_0}{\partial t} = \frac{R_g T_0}{V_c} \left( \dot{m}_f + \dot{m}_{ox} - \frac{R_g T_0}{V_c} \dot{m}_{\text{nozzle}} \right) - \frac{P_0}{V_c} \frac{\partial}{\partial t} V_c \tag{3.19}
\]

Assuming the nozzle chokes immediately after the initiation of combustion, the change in mass of the combustion chamber can be expressed in terms of the general nozzle mass flow equation, the regression rate, and the oxidizer mass flow,

\[
\frac{\partial P_0}{\partial t} = \frac{R_g T_0}{V_c} \left( A_{\text{burn}} \rho_f \dot{r} + A_{\text{ox}} C_d_{\text{ox}} \sqrt{2 \rho_{\text{ox}} (P_{\text{ox}} - P_0)} \right) \tag{3.20}
\]

Noting that the time derivative of the chamber volume is simply equal to the regression rate multiplied by the instantaneous burn area, this becomes

\[
\frac{\partial P_0}{\partial t} = \frac{R_g T_0}{V_c} \left( A_{\text{burn}} \rho_f \dot{r} + A_{\text{ox}} C_d_{\text{ox}} \sqrt{2 \rho_{\text{ox}} (P_{\text{ox}} - P_0)} \right) - \frac{P_0}{V_c} \frac{\partial}{\partial t} V_c \tag{3.21}
\]

The growth of the port radius is described by

\[
\frac{\partial R_{\text{port}}}{\partial t} = \dot{r} \tag{3.22}
\]

and the instantaneous chamber volume is

\[
V_c = \pi R_{\text{port}}^2 L \tag{3.23}
\]

The instantaneous burn area and combustion port area can be calculated in a similar fashion from

\[
A_{\text{burn}} = 2\pi R_{\text{port}} L \tag{3.24}
\]
\[ A_c = \pi R_{\text{port}}^2 \] (3.25)

Equations 3.15, 3.17, 3.21, 3.22 are integrated as a system with the state vector containing combustion pressure, instantaneous combustion port radius, and the total masses of fuel and oxidizer consumed by the system. These equations allow the chamber fill dynamics to be modeled as a function of time along with the regression rate.

For a fixed oxidizer mass flow, the properties of the combustion gasses will change as the port radius grows larger and the mixture ratio becomes increasingly richer. For a fixed oxidizer mass flow, the properties of the combustion gasses will change as the port radius grows larger. Based on Eqs. 3.15 and 3.17, it can be shown that the effective mixture ratio of the hybrid motor is

\[
M_R = \frac{\dot{m}_{\text{ox}}}{\dot{m}_{\text{fuel}}} = \frac{A_{\text{ox}} C_{\text{dox}} \sqrt{2p_{\text{ox}} (p_{\text{ox}} - p_0)}}{A_{\text{burn}} \rho_{\text{fuel}} \rho_{\text{ox}} \sqrt{2p_{\text{ox}} (p_{\text{ox}} - p_0)}} = A_{\text{ox}} C_{\text{dox}} \sqrt{2p_{\text{ox}} (p_{\text{ox}} - p_0)} \rho_{\text{ox}} (p_{\text{ox}} - p_0)
\]

\[
A_{\text{burn}} \rho_{\text{fuel}} \rho_{\text{ox}} (p_{\text{ox}} - p_0)
\]

\[
21.28 P_{\text{m}}^{\frac{4}{3}} \left[ A_{\text{ox}} C_{\text{dox}} \right]^{\frac{1}{3}} \left[ \frac{A_{\text{burn}}}{A_{\text{ox}}} \right]^{\frac{1}{3}} \left( \frac{C_{\text{dox}} \sqrt{2p_{\text{ox}} (p_{\text{ox}} - p_0)}}{C_{\text{fuel}} \sqrt{p_{\text{fuel}}} \sqrt{p_{\text{ox}}}} \right)^{\frac{1}{3}}
\]

Modeling the mixture ratio in this form allows for the non-iterative computation of combustion product properties at every time step using an equilibrium analysis method.

3.1.3 Modeling of the Combustion Product Properties

The motor modeled in this analysis uses a combination of liquid nitrous oxide at saturation conditions as an oxidizer and hydroxyl-terminated polybutadiene (HTPB) as a fuel. For this analysis, the equilibrium gas-chemistry code Chemical Equilibrium with Applications (CEA) [50, 51] was used to model the combustion products. The CEA code was developed at NASA Glenn Research Center, and has been successfully applied for the analysis of rocket combustion, detonation, and flow across non-adiabatic shock waves. The code posits chemical reactions and then minimizes the Gibbs free energy in order to reach
thermodynamic and transport properties at chemical equilibrium. The CEA code has extensive internal libraries for gas thermodynamic and transport properties including standard and non-standard temperature and pressure conditions. Unfortunately, the CEA thermochemical database does not have an entry for HTPB, and the fuel grain properties including the atomic mole-fraction formula and enthalpy of formation must be externally input to the program. Further complicating the problem is that polymer properties including enthalpy of formation, density, tensile strength, and ablation temperature depend on the molecule chain length and shape, and these variables can be significantly different for each polymer molecule [126]. Even the effects of the “cooking” process for the same material brand name can result in two samples with drastically different properties. A typical molecular formula for the cured HTPB polymer is approximated by [127]

\[
HTPB = C_{4.9872}H_{5.03162}O_{0.03162}
\]  

(3.27)

These atomic mole fractions are in contrast to the pure butadiene gas molecular formula of C4H6.

For this analysis, the thermodynamic properties of the combustion products including the specific heat ratio, molecular weight, adiabatic flame temperature, viscosity, Prandtl number, and characteristic exhaust velocity, output by CEA are stored as a function of chamber pressure and mixture ratio, and were evaluated at the beginning of each data frame using a two dimensional table look-up. The flame temperature was scaled by the square root of an assumed combustion efficiency (~ 0.98 nominal) to get the combustion chamber temperature. The gas constant and mixture specific heats were computed from the results of these table look-ups. Figure 5 shows these properties plotted as a function of mixture ratio for combustor pressures varying from 100 kPa to 10,000 kPa. Clearly, the optimal mixture ratio for this propellant combination is in the vicinity of 6.0. The inflection points on the specific heat ratio and Prandtl number curves are a result of the formation of complex alcohol molecules in the combustion products at very fuel rich mixture ratios.
Fig. 3.4: Thermodynamic and transport properties for HTPB/Nitrous Oxide combustion.

\[
\dot{r}_{avg} = \frac{1}{L_{chamber}} \int_{0}^{L_{chamber}} \left[ \sum_{i=1}^{2} a_i x^i \right] dx = \sum_{i=1}^{2} \frac{a_i (L_{chamber})^i}{i + 1} \quad (3.28)
\]

3.2 Experimental Data Collection

Experimental data for model comparison was collected from two separate burns of a small-scale 10.2 cm diameter hybrid rocket motor. Motor construction, instrumentation, test procedures, and results from two test firings will be presented in this section. Results from the two tests will be compared to performance predictions based on the combustion model developed in the previous section.
3.2.1 Motor Construction

The 10.2 cm motors use nitrous oxide and HTPB as propellants. The HTPB fuel was manufactured using commercially available products: Arco R45M polybutadiene resin and PAPI 94\textsuperscript{®} curative. Arco R45M is polybutadiene diol manufactured by Sinclair Petrochemicals Inc.’s Arco Division. The resin has a polymerization factor of approximately 45 and a molecular weight of 2745 kg/kg-mol [128].

PAPI 94 is a polymethylene polyphenylisocyanate produced by Dow\textsuperscript{®} Plastics Inc. The formulation contains methylene diphenylene diisocyanate (MDI) in proprietary proportions. The curative has an average molecular weight of 290 kg/kg-mol. The nitrogen, carbon, oxygen (NCO) bonds in the MDI react with the hydroxyl (OH) terminations in the polybutadiene resin to cure the fuel grain. The curative and resin are mixed in a 1:8 weight ratio. The resin and curative were mixed in a commercial paint mixer that was sealed and fitted so that the fuel mixture could be placed under a vacuum during the mixing process. The vacuum was used to remove gas bubbles created in the fuel grain during the mixing process.

The motor casing used for these test fires was constructed of an aluminum tube, 1.016 meters in length and 10.2 cm in diameter. The HTPB propellant was cast into a thin-walled (0.28 cm) PVC pipe that served as a removable motor cartridge. The propellant in the motor cartridge could be cast externally to the motor casing, and then inserted when the fuel grain was completely cured. This cartridge design allows for quick removal of both the nozzle and the fuel cartridge so that multiple grain geometries can be tested in a single day. After each test firing, the motor cartridge was removed and cut length wise for burn-pattern inspection. The internal propellant grain is approximately 0.84 meters long.

The nozzle was manufactured from a single piece of graphite. The nozzle throat diameter is 2.76 cm and has an expansion ratio of 5.566. The nozzle contour follows a partial bell, with an end plane divergence half-angle of 14 degrees. The nozzle fits internally into the fuel cartridge that fits inside the motor casing. The nozzle was held in place by a retaining ring at the end of the motor cartridge. The N2O injector was an impinging “showerhead” design.
In total, 18-injector ports approximately 0.16 cm in diameter were drilled into the injector head. This port configuration resulted in an oxidizer injection area of approximately 0.356 cm². The oxidizer system was not actively pressurized and instead relied on the natural vapor pressure of the nitrous oxide in the propellant tank.

### 3.2.2 Measurements and Instrumentation System

For the first test firing, the initial port diameter was approximately 2.6 cm. As will be discussed later, this initial diameter was expanded to approximately 5.1 cm for the second test firing. For the first test firing, combustion pressure, oxidizer pressure, thrust, case temperature and exhaust flame temperature were measured. The test setup is depicted in Fig. 3.5 and the corresponding sensors are listed in Table 3.1. Oxidizer pressure was measured upstream of the main oxidizer valve. Combustion pressure was measured from a port in the motor cap on the forward end of the combustion chamber. The motor was fixed to a test sled on a portable test trailer and thrust was measured with a strain gauge load cell at the forward end of this test sled. Thermocouples were fastened to the outside of the motor casing and to a steel spar positioned in the exhaust plume. The thermocouple (TC) on the motor case was used to detect the presence of any abnormal temperature spikes during the burn, an indication of possible imminent burn through of the motor casing. The thermocouple situated in the exhaust plume was positioned in an attempt to gauge the exit temperature of the motor configuration before the destruction of the thermocouple. Reference temperature was sensed using a 3-wire Resistance temperature detector (RTD) inside of the TC connector block.
Table 3.1: Instrumentation System Components.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Sensor</th>
<th>Fieldpoint Module</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combustion Pressure</td>
<td>1k MSI 600 Pressure Transducer</td>
<td>cFP-AI118</td>
</tr>
<tr>
<td>Oxidizer Pressure</td>
<td>2.5k MSI 600 Pressure Transducer</td>
<td>cFP-AI118</td>
</tr>
<tr>
<td>Thrust</td>
<td>OMEGA LCCB01K</td>
<td>cFP-AI118</td>
</tr>
<tr>
<td>Case Temperature</td>
<td>Type K Thermocouple</td>
<td>cFP-AI112</td>
</tr>
<tr>
<td>Exhaust Flame Temperature</td>
<td>Type K Thermocouple</td>
<td>cFP-AI112</td>
</tr>
<tr>
<td>Reference Temperature</td>
<td>3-Wire RTD</td>
<td>cFP-RTD122</td>
</tr>
</tbody>
</table>

Fig. 3.6: Thermocouple placement in fuel grain.

For the second test fire, 12 type K thermocouples were embedded in the fuel grain to retrieve instantaneous regression rate measurements over the burn duration. Groups of 24-gauge, fine bead, type K thermocouples were placed at intervals of 10 inches measured axially along the combustion chamber. Three thermocouples each were placed on the forward and aft positions and six thermocouples were placed in the center. The thermocouple wires were run along the inside of the motor casing through a simple sealed pass-through installed in the motor cap. Figure 3.6 shows the general placement of the fuel-grain thermocouples, and the thermocouples as attached to the mounting bracket cast into the fuel grain.

The data acquisition system that was chosen for this endeavor is a National Instruments Compact Fieldpoint® intelligent control and acquisition system. Two separate Fieldpoint modules, the cFP-AI-118 and the cFP-AI-112, were used for data collection. The 16-bit cFP-AI-118 was used to collect outputs from the MSI 600 pressure transducers and the
OMEGA LLCB 1000 lbf load cell. The 16 bit, lower range cFP-AI-112s were used to retrieve thermocouple voltage. The Fieldpoint system was situated on the test trailer and data was broadcast back over Ethernet to a remote, data-logging laptop. The instrumentation set up for the second test fire was similar to that to the first test fire except that the 12 thermocouples occupied the AI-118 modules and the pressure transducers and load cells were measured with an AI-112. Additionally, the Fieldpoint system was connected to a data-logging laptop directly with a crossover cable rather than through a hub and avoided reliability issues previously experienced with a commercial Ethernet hub.

3.2.3 Motor Fire Control System

Motor control was managed via a second laptop and an RF-link to a custom-built controller. The motor combustion was initiated using one of two small solid ammonium nitrate/magnesium rockets imbedded in the motor injector cap. The ignition motors were initiated via an electrical pulse across a nichrome ignition wire. The test sequence would open the oxidizer control valve, and then initiate the igniter. For a typical test fire, the control system would be verified using an inert gas and a spare igniter wire. Once proper operation was confirmed, the system was loaded with nitrous oxide and the firing sequence was repeated. Only one igniter was needed to start the motor. The second ignition motor was held in reserve for redundancy purposes. Figure 3.7 shows a collage of a typical 10.2 cm motor test firing including the rocket control system arrangement.

3.2.4 Test Results

Figure 3.8 shows a typical test fire data time history for oxidizer pressure (kPa) measured at the tank, combustion pressure (kPa) and thrust (N). Two distinct phases of combustion can be identified. The first stage is characterized by high combustion pressure and thrust. During this period, there is liquid oxidizer flowing through the injector, resulting in a high oxidizer mass velocity and relatively lean combustion mixture ratio. After approximately 15 seconds, the liquid nitrous oxide is consumed and the remaining nitrous oxide vapor in the tank begins to flow into the chamber. A dramatically reduced oxidizer
Fig. 3.7: Typical test firing of the USU 10.2 cm N2O/HTPB hybrid rocket motor.
mass velocity and atypically rich mixture ratio during this period results in dramatically reduced thrust and chamber pressure. Combustion of this form continued for approximately 12 seconds before the main oxidizer valve was closed, terminating combustion. The large pressure loss between the oxidizer tank pressure measurement and the chamber pressure is a result of a quick release fitting that was in line with the oxidizer feed line. This fitting resulted in an effective injector discharge coefficient of approximately 0.27. After the fitting was discovered, it was removed for the second test firing.

3.2.5 Fuel Grain Burn Patterns

Although the first test fire yielded good data for combustion temperature and thrust, examination of the combustion chamber after the test revealed deep pitting and channeling along the length of the fuel grain. This erosive burning made direct linear measurement of the total fuel regression impossible and posed a potential hazard to the structural integrity of the motor. This chaotic burn pattern was likely caused by a combination of two factors:
1) air bubbles in the fuel grain, 2) high Mach numbers during the initial few seconds of burn time in the fuel port. The bubbles in the fuel grain were a result of the fuel mixing and curing process that was employed for the first test. The vacuum pump that was used to degas the fuel had insufficient power to completely degas the mixture. The high initial fuel combustion port mach numbers were a result of the small initial port diameter (2.5 cm) for the first test firing. This port diameter was actually slightly smaller than the nozzle than the throat and resulted in a short period of choked flow near the aft end of the fuel grain. This high initial Mach number resulted in compressible boundary layer heating and produced excessive frictional shear and heat transfer at the fuel grain walls. This high initial heat transfer coupled with the weakened fuel grain caused by the imbedded gas bubbles resulted in chaotic erosive burning.

In an attempt to remedy this issue, two solutions were employed simultaneously. First a better fuel degassing procedure was employed during the casting process. To aid the degassing procedure, a commercial H-VAC pump was used in place of an older legacy vacuum pump and a much deeper vacuum on the mixing chamber was achieved. The new pump supplied a vacuum to a depth of almost 30 inches of mercury, while the older pump was only capable of a vacuum to 28 inches of mercury. Using this method, a fuel casting visually free of bubbles or voids and a density of approximately 930 kg/m$^3$ was produced. The fuel used in the first casting had a density of approximately 860 kg/m$^3$ and had visible bubbles and defects. Second, and likely more important, a larger initial port diameter was used for the second test fire. Increasing the combustion chamber diameter substantially lowered the initial chamber Mach number. For the second test the initial port diameter was increased to approximately 5.1 cm. Figure 3.9 shows the calculated initial Mach numbers for the 2.5 cm and 5.1 cm port diameters. With the smaller port the initial flow is choked at ignition until the fuel regression results in a larger port area. Eventually the Mach number approaches that of the larger port after approximately 5-seconds of burn time. However, by this time the “damage has been done,” and the erosive burning pattern has already begun. At this point, the process appears to irreversible and the regression patterns in the fuel
Fig. 3.9: Estimated port Mach number at end of fuel grain for both test 1 and test 2.

grain increase heat transfer asymmetrically to the fuel surface.

Figure 3.10 shows the burn patterns for the first and second test firings. The fuel grain changes between the two test firings resulted in dramatic improvement in the uniformity of the fuel grain regression. In the first burn the fuel shows deep fissures for nearly 50% of the motor length. In contrast, the burn pattern generated by the second test fire exhibited a smooth and uniform burn free of the chaotic and erosive pattern experienced by the first burn. Also interesting in the second burn are the various stages made evident by the post-fire combustion port from the second test fire. For approximately the first 20 cm the fuel surface is smooth and covered with a thin layer of soot. For the next 15 cm, the surface has a wavy appearance and the soot layer diminishes. After this point, the fuel surface once again becomes smooth, and the surface is soot free.

3.2.6 Fuel Grain Regression Rate Measurements

The post burn “fuel carcass” for the 5.1 cm initial fuel port had sufficiently “clean” burn pattern that mean regression rate estimates could be calculated from the post-test fuel grain using direct measurements of the residual HTPB propellant in the motor casing. Figure 3.11 shows a cross section of the fuel grain showing the fuel port, and the residual propellant. Measurements stations as well as the combustion chamber entrance and exit
The total fuel regression during the burn was calculated by measuring the residual HTPB thickness on the upper and lower sides of the chamber cross-section, and the total chamber width every 10 cm down the length of the fuel grain. The upper and lower surface propellant thicknesses were summed and then subtracted from the chamber thickness at each measurement section. Subtracting the original port diameter (5.1 cm) gives the total fuel regression at each station. Finally, dividing the total regression data by the motor burn time (6.25 sec) calculates the time-averaged rate of regression. Figures 3.12 and 3.13 show these regression measurements plotted against longitudinal distance down the chamber along with mean second order trend lines. Figure 3.12 shows the total regression data. Figure 3.13 plots the regression rate data. By integrating the second order trend-line curve fit along the length of the motor, and then normalizing by the motor length, the longitudinally averaged regression rate was calculated.
Table 3.2: Total Regression and Regression Rate Statistical Data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Longitudinal Mean Value</th>
<th>Standard Error Trend Line</th>
<th>95% Confidence Interval about Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Regression, cm</td>
<td>0.854</td>
<td>0.0736</td>
<td>±0.0560</td>
</tr>
<tr>
<td>Regression Rate, cm/s</td>
<td>0.1299</td>
<td>0.0119</td>
<td>±0.0091</td>
</tr>
</tbody>
</table>

\[
\dot{r}_{avg} = \frac{1}{L_{chamber}} \int_{0}^{L_{chamber}} \left[ \sum_{i=1}^{2} a_i x^i \right] dx = \sum_{i=1}^{2} \frac{a_i (L_{chamber})^i}{i + 1} \tag{3.29}
\]

Table 3.2 shows the calculated mean regression and regression rate, the standard errors in the curve fits, and the 95% confidence interval for the data based on Student’s t-distribution for 8 degrees of freedom.

In contrast to the well defined time-averaged regression rates presented in Table 3.2, interpretation of the instantaneous regression rates from the imbedded thermocouples is more ambiguous and the results must be presented with some level of reservation. As mentioned earlier, there were three thermocouple regression arrays embedded in the fuel grain, unfortunately, only the densely populated array near the center of the motor experienced sufficient regression so that multiple TC’s were exposed to the flow. Additionally,
the embedded TC array induced some localized erosion in the combustion port, leading to an increased regression at the fuel surface near the thermocouples. The channel eroded by these thermocouples can be seen in Fig 3.14. Due to the localized regression around the thermocouples in the motor, the instantaneous regression rates calculated from this TC sensor array will not be representative of the total regression and must be presented with some level of reservation. Clearly this localized erosion issue emphasizes the need to create a non-intrusive regression rate sensor.

Figure 3.15 presents the time histories of the three mid-motor thermocouples that were exposed to the flow by the fuel grain regression. Figure 3.16 shows the relative positions
of the thermocouples within the sensor array. Interestingly, both the first and second thermocouples become an open circuit when first exposed to the flow, only to “re-weld” at some point later on. The third thermocouple did not create an open circuit because the motor was shut down at 6.25 seconds. The time intervals between the thermocouple traces was evaluated using by taking the numerical cross correlation of the time history traces for various lag intervals and selecting the lag interval with the maximum correlation. For correlations of the first and second thermocouple with the third thermocouple, the time histories were truncated for temperatures above 760 C. The regression rate was calculated by dividing the lag correlation parameter by the distances between the thermocouples in the regression array.

Table 3.3 summarizes the mid-motor regression sensor parameters. The regression rates are calculated in three ways; 1) using the maximum correlation time lag (12) for TC’s 1 and 2 and the distance separating these sensors, 2) using time lag (23) for TC’s 2 and 3 and the distance separating these sensors, and 3) using time lag (13) for TC’s 1 and 3 and the distance separating these sensors. Each of the measurements produces remarkably similar results, and these results agree reasonably well with the mean regression rate data presented in Table 3.3. Thus it appears that the localized erosion near the mid-motor TC array did.
Table 3.3: Mid-Motor Regression Sensor Data Summary.

<table>
<thead>
<tr>
<th>Thermocouple Pair</th>
<th>$\Delta t,s$</th>
<th>Absolute Position in Sensor Array, cm</th>
<th>$\Delta r$, from First TC om Array, cm</th>
<th>Regression rate, cm/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2</td>
<td>2.1667</td>
<td>0.000</td>
<td>0.2667</td>
<td>0.1260</td>
</tr>
<tr>
<td>2,3</td>
<td>2.8143</td>
<td>0.2667</td>
<td>0.3556</td>
<td>0.1264</td>
</tr>
<tr>
<td>1,3</td>
<td>4.0267</td>
<td>0.3223</td>
<td>0.6223</td>
<td>0.1275</td>
</tr>
</tbody>
</table>

not substantially affect the relative regression rate between the individual TC’s embedded in the motor grain.

3.3 Model Comparisons with Experimental Results

As mentioned in the previous section, the first and second test firings mentioned above burned fuel grains with an initial port diameter of approximately 2.5 cm and 5.1 cm, respectively. Although the regression rate model used for this analysis is longitudinally averaged, predictions from the numerical model characterize the motor performance reasonably well. However, before the behavior of the model could be validated, specific parameters of the relevant motor system had to be evaluated. First, the injector discharge coefficient for both
Table 3.4: Parameters Used for Model Calculations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>First Test Fire</th>
<th>Second Test Fire</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combustion Port Length</td>
<td>86.36 cm</td>
<td>86.36 cm</td>
</tr>
<tr>
<td>Combustion Efficiency</td>
<td>95.0%</td>
<td>95.0%</td>
</tr>
<tr>
<td>Fuel Density</td>
<td>866.0 kg/m³</td>
<td>930.0 kg/m³</td>
</tr>
<tr>
<td>Grain Temperature</td>
<td>300 K</td>
<td>300 K</td>
</tr>
<tr>
<td>Heat of Vaporization</td>
<td>1.8 MJ/kg</td>
<td>1.8 MJ/kg</td>
</tr>
<tr>
<td>Injector Area</td>
<td>0.35628 cm²</td>
<td>0.35628 cm²</td>
</tr>
<tr>
<td>Injector Discharge Coefficient</td>
<td>0.27</td>
<td>0.8 + pipe losses</td>
</tr>
<tr>
<td>Initial Port Diameter</td>
<td>2.54 cm</td>
<td>5.08 cm</td>
</tr>
<tr>
<td>Nozzle Throat Area</td>
<td>5.067 cm²</td>
<td>6.0 cm²</td>
</tr>
<tr>
<td>Nozzle Exit Divergence Angle</td>
<td>28 degrees</td>
<td>28 degrees</td>
</tr>
<tr>
<td>Nozzle Expansion Ratio</td>
<td>6.45</td>
<td>5.569</td>
</tr>
<tr>
<td>Oxidizer Liquid Density</td>
<td>800 kg/m³</td>
<td>800 kg/m³</td>
</tr>
</tbody>
</table>

firings had to be calculated. Although the theoretical discharge coefficient was known, fittings and plumbing losses lead to an effective discharge coefficient substantially different from that estimated. For the first firing, the discharge coefficient was evaluated by integrating eq. 3.15, the oxidizer mass flow rate for an injector, over a time-logged pressure differential with a known total mass discharge. Using this method, a discharge coefficient of 0.27 was obtained for the injector. As mentioned in the earlier discussion, this very low discharge coefficient was the result of a quick release fitting in the oxidizer flow path. For the second test fire, less restrictive fittings were used and the discharge coefficient was estimated using turbulent head loss based on the known geometry of the oxidizer feed line. Table 3.4 lists the relevant parameters used for the combustor model calculations.

### 3.3.1 Chamber Pressure, Thrust, and Mixture Ratio Comparisons

The oxidizer system for this test motor relied on the natural vapor pressure of the nitrous oxide in the propellant tank. As a result, a decaying oxidizer pressure drove the upstream injector pressure. This measured oxidizer pressure time history was imported into the motor algorithm and used to calculate the oxidizer mass flow across the injector.
Fig. 3.17: Comparison of model predictions and measured value, combustion pressure, test 1.

and the resulting combustor pressure. The predicted motor chamber pressure and thrust are compared with the measured values in Figs. 3.17-3.20. Figure 3.17 shows the chamber pressure comparison for test 1, and Fig. 3.18 shows the chamber pressure comparisons for test 2. Figure 3.19 plots the motor thrust comparison for test 1, and Fig. 3.20 plots the thrust comparison for test 2.

Whereas the predicted chamber pressure and thrust values for test 1 show some considerable disagreement with the measured values, the predictions for test 2 are nearly “dead-on.” Clearly the improved burn pattern also improved the model’s ability to characterize the combustion process.

Figure 3.21 compares the predicted burn mixture ratios for the two tests. In both cases the motors are burning at a richer mixture ratio than is predicted as optimal. However, this is desirable for hybrid motors where nozzle erosion is a potential issue (Fig. 3.4).

3.3.2 Regression Rate Comparisons

Only data from the second motor test firing are available for comparison with the enthalpy balance model predictions. The chaotic grain regression in the first test firing made regression rate correlation impossible. Both the time averaged and instantaneous
Fig. 3.18: Comparison of model predictions and measured value, combustion pressure, test 2.

Fig. 3.19: Comparison of model predictions and measured value, thrust, test 1.
Fig. 3.20: Comparison of model predictions and measured value, thrust, test 2.

Fig. 3.21: Modeled mixture ratio from regression rate for both test 1 and test 2.
regression rate data from Table 3.2 and Table 3.3 will be presented here. Figure 3.22 presents the regression rate comparisons. In this figure the hybrid regression rate is plotted as a function of oxidizer mass flux. The enthalpy-balance model predictions are plotted as dashed gray lines with the lower dashed-line using a constant Prandtl number of 1.0 and the upper dashed-line using a table look-up of the Prandtl numbers predicted by CEA (Fig. 3.4). For the measured data, the oxidizer mass flow is calculated using the pressure difference across the injector assuming a discharge coefficient of 0.8. For the time-averaged regression rate data, the longitudinal average of the initial and final port cross-sectional area is used to normalize the mass flow calculation. For the instantaneous regression rate data, the cross-sectional area is calculated using the averaged regression rate from Table 3.3 integrated over time. The regression data from Table 3.2 is plotted as rectangular gray symbol, and the instantaneous mid-motor regression data are plotted as open circles. Also plotted are test data for two motors that burn gaseous oxygen (Gox) and HTPB. These data are taken directly from Sutton and Biblarz [5]. The data are plotted here using English units so that the data from Sutton can be presented unaltered. Figure 3.22 shows that the regression rate model, especially when using the Prandtl number generated by CEA, shows remarkable agreement with the previously existing empirical data for the GOX/HTPB lab-scale motor and agrees well with the data obtained from the second N2O/HTPB test firing.

3.4 Summary and Concluding Remarks

This section developed a refined model for predicting hybrid motor fuel regression rates using an enthalpy-balance approach where the ablation heat of the fuel grain is balanced by the convective heat transfer from the combustion flame zone to the fuel grain surface. The original enthalpy-balance regression model developed by Marxman was modified to allow for non-unity Prandtl number and the model was “closed” using CEA to predict combustion chamber properties and the creation of a chamber pressure model. For this model formulation, the variation in fluid Prandtl number has non negligible effects on the predicted regression rates. When compared to experimental data, this model (the LAVP model) is shown to accurately predict fuel regression rates for the 10.2 cm Nitrous/HTPB
motor. This model also generally agrees well with data for GOX/HTPB motors in the literature. Given the regression rate prediction, the model also satisfactorily predicts the time varying chamber pressure, thrust and specific impulse of the 10.2 cm motor.

Fig. 3.22: Comparison of empirical and modeled HTPB fuel regression rate.
Chapter 4

Extensions to the Longitudinally Averaged Variable-Prandtl-Number Regression Rate Model

Although the LAVP model described in Chapter 3 shows reasonable results for the specific experiments with HTPB and nitrous oxide with which it was compared, there are several simplifications used in the model that reduce the general applicability of this model to other fuel/oxidizer combinations or fuel grain geometries. The primary assumptions in this model are

- The addition of fuel mass flux into the chamber only influences the regression rate through the chamber temperature.
- The flame layer is modeled by the combustion temperature for the total mixture ratio at the edge of the boundary layer.
- The change in fuel surface temperature with regression does not substantially influence heat transfer.
- The boundary layer heat transfer is influenced by the fluid Prandtl number.
- Radiant heat transfer is negligible.

These assumptions create a closed-form, explicit, model for regression well suited to integration over time while including a differential equation for chamber pressure. However, the solution of the differential equation for chamber pressure necessitated the use of very small time steps which in turn made a computationally simple model desirable.

The updated model extends the LAVP model to include the effects of fuel surface temperature, radiation, the flame height in the boundary layer, the increase in total mass flux along the fuel grain, and variable fluid properties. This model will be referred to as the
“updated variable surface temperature” model, or UPVST. The UPVST model generally predicts regression rates for a range of hybrid configurations more accurately then the LAVP model. This model also acts as a test bed for modeling heat transfer phenomenon that influence regression rates in hybrid rocket motors in order to facilitate the computational fluid dynamics based regression rate model presented in Chapter 5.

A similar predictive model that includes the parameters discussed in this section is not known to exist in the open literature.

4.1 Discussion of Wall Blowing and the Blowing Parameter

In hybrid rocket regression, the gaseous fuel vapor leaving the fuel wall has a significant effect on the heat transfer from the flame zone to the fuel surface. The mechanism responsible for this effect is the very same phenomenon that blocks heat transfer to surfaces in transpiration cooling. The transport of heat towards the fuel surface is impeded by the flow of the gaseous fuel produced by pyrolysis of the fuel grain surface. This tends to significantly inhibit heat transfer from the flame zone to the fuel, typically on the order of 30-70 percent. Despite the critical importance of this parameter, a great deal of variation exists in its treatment throughout hybrid literature. In order to justify a choice of a definition for the blowing parameter and a corresponding model for it’s effect on convective heat transfer, a brief discussion of the models often used in the hybrid literature is warranted.

For convective heat transfer scenarios outside of hybrid rocketry, the blowing parameter is often defined as the momentum ratio between the gas leaving the fuel surface to that in the free stream over the Stanton number without wall blowing [125],

$$B = \frac{\rho_w v_w}{\rho_e u_e S_0}$$  \hspace{1cm} (4.1)

For the creation of closed form regression rate solutions from Reynolds analogy type heat transfer correlations, this definition is inconvenient, as the Stanton number is not known a priori and is usually estimated with the application of empirical skin friction models. Thus, the blowing parameter is often expressed as a function of the skin friction...
coefficient by application of the Reynolds analogy,

\[ B = \frac{\rho_w v_w}{\rho_e u_e C_f^2} \]  

(4.2)

This expression is sometimes “corrected” for the effects of non-unity Prandtl number with

\[ B = \frac{\rho_w v_w}{\rho_e u_e C_f^2 \Pr^{-\frac{2}{3}}} \]  

(4.3)

however, caution must be used with this approach. A final expression for the adjustment to the Stanton number due to wall blowing will be required in any regression rate model, and often these expressions are derived in terms of a blowing parameter as defined by eq. 4.1. Any “adjustment” for non-unity Prandtl number for a relationship using this definition would be redundant, as no unity Prandtl number assumptions may have been made.

If radiant heat transfer is assumed to be negligible (which is sometimes not the case) the blowing parameter can be modeled as a thermochemical fuel parameter by using the definition of the Stanton number,

\[ B = \frac{\dot{\rho}_f}{\rho_e u_e S} = \frac{\dot{Q}}{\Delta H \rho u_e} = \frac{S \rho_e u_e (h_{c_s} - h_{w_g})}{\Delta H \rho_e u_e} = \frac{(h_{c_s} - h_{w_g})}{\Delta H} \]  

(4.4)

The original Marxman correlation incorporated the position of the flame height in the boundary layer. The Stanton number relationship used was then

\[ \dot{Q}_c = S \rho_e u_e \left( \frac{u_g}{u_e} \right) (h_{c_s} - h_{w_g}) \]  

(4.5)

The corresponding blowing parameter was then

\[ B = \left( \frac{u_e}{u_c} \right) \frac{(h_{c_s} - h_{w_g})}{\Delta H} \]  

(4.6)

For any definition of the blowing parameter, a suitable model for the heat transfer adjustment from wall blowing must be formulated. Obtaining inspiration from previous work by Lees [42], a semi-emperical relationship was derived by Marxman [43],
\[
\frac{C_f}{C_{f0}} = \left( \frac{\ln (1 + B)}{B} \right)^{0.8} \left( \frac{1 + 1.3B + 0.364B^2}{(1 + \frac{B}{2})^2 (1 + B)} \right)^{0.2}
\]  

(4.7)

based upon the Prandtl mixing length model and an experimentally derived relationship for turbulent heat transfer.

This relationship, being unwieldy for manual computation before the era of the digital computer, was fit with a much simpler relationship [43],

\[
\frac{S}{S_0} = 1.2B^{-0.77}
\]  

(4.8)

It should also be noted that Altman [46] showed that this relationship would be better fit over typical ranges of interest in hybrid motors by

\[
\frac{S}{S_0} = B^{-0.68}
\]  

(4.9)

Either of these relationships have appeared numerous times in the hybrid rocket literature [129].

The blowing parameter has shown a strong dependence on the specific heat of the injected fluid [125]. Data for transpiration cooling for injected fluids with various specific heats is shown in Fig. 4.1. There is a substantial change in Stanton number ratio with specific heat for the same blowing parameter. The fluids with the lower specific heats and therefore lower molecular weights have a dramatically increased blocking effect over higher molecular weight fluids. Paul et al. noted that fuel pyrolysis products can vary extensively for fuel that produce pyrolysis products with different molecular weights and examined this effect in detail.

Based on their data, Paul and his colleagues [44] created a rather complicated model for the blowing parameter based upon a semi-emperical fit for the densities of the vaporizing fuel, the free stream, and the flame,
Fig. 4.1: Stanton number ratio in a turbulent boundary layer with transpiration cooling for a free stream fluid with $c_p = 1.01$. [125]

\[
\frac{S}{S_0} = \left( \frac{\rho_f}{\rho_e} \right)^{0.71} \left( \frac{\rho_w}{\rho_e} \right)^{0.14} B (1 + B)^{-0.73+0.002\frac{\rho_w}{\rho_e}}
\]

(4.10)

Instead of Paul’s inexplicably complicated formula, the data presented in Fig. 4.1 can be extrapolated for typical molecular weights experienced in HTPB combustion. Chiaverini examined the pyrolysis products of HTPB in the typical range of hybrid motor operation [130]. In this study, the pyrolysis products consisted mostly of Butadiene gas with much lower amounts of other hydrocarbons. The molecular weight of the products for a range of temperatures was near 60 g/mol. CEA estimates that nitrous oxide combustion with HTPB yields an average molecular weight of about 27 g/mol for a range of oxidizer to fuel ratios. Presuming that the specific heat of these products is well modeled by ideal gas relationships, the inverse of the ratio of molecular weights can be used to approximate the specific heat ratio. Thus the ratio of free stream specific heat to the specific heat of the pyrolysis products should be about 2.2. Using this assumption, the results given by Kays and Crawford were then extrapolated for the specific heat ratio commonly seen in hybrid...
rocket motor operation using a parabolic fit for Stanton number ratio vs specific heat ratio for a range of blowing parameters. This result is shown in Fig. 4.2. It is noteworthy that over this range, the Stanton number ratio appears rather insensitive to the factor of 2 change in specific heat ratio. This implies that more accurate modeling of the specific heat of pyrolysis products or the boundary layer will have very limited total effect on the net heat transfer relationship.

The extrapolated correlation created with data from Kays and Crawford is well fit by

\[
\frac{S}{S_0} = 0.439e^{-0.589B} + 0.553e^{-0.0678B}
\]

(4.11)

This fit and the associated fit error is shown in Fig. 4.3. For other hybrid fuels, a similar procedure could be used to formulate a blowing parameter relationship given applicable pyrolysis products.

Chiaverini obtained drastically different correlations for the blowing parameter from experiments using a hybrid slab burner [48], however his results are not really comparable with traditional correlations because of the choice of Stanton number relationships used in
this work. Chiaverini, in a marked departure from standard hybrid rocket literature used a Stanton number correlation for turbulent pipe flow,

\[ S_0 = 0.023 \text{Re}^{-0.2} \left( \frac{D}{L} \right)^{-0.7} \]  \(\text{(4.12)}\)

Using this correlation, Chiaverini experimentally obtained blowing parameter relationships,

\[ \frac{S}{S_0} = \left( 0.65 + 9.56 B \right)^{1.45} \left( \frac{D_h}{L} \right)^{0.3} \]  \(\text{(4.13)}\)

for flow near the beginning of the fuel port and

\[ \frac{S}{S_0} = \left( 0.73 + 9.16 B \right)^{1.6} \left( \frac{D_h}{L} \right)^{0.3} \]  \(\text{(4.14)}\)

for flow near the end of the fuel port.

There are several noteworthy features in these relationships. First, hybrid rocket regression has long been shown to generally obey Stanton number relationships that account
for boundary layer growth along the fuel port. Thus, it is not surprising that a diameter dependent term shows up in the experimentally derived relationships with an exponent of 0.3, which nearly cancels the diameter term in the Reynolds number which has the power of −0.2. However, the resulting term does not contain a term that accounts for position in the fuel grain, which means the use of multiple relationships for different axial positions is not surprising.

Secondly, the Stanton number definition used by Chaiverini does not account for the flame height in the boundary layer, which can account for a substantial increase in total heat transfer. Thus, it is not surprising that the Chiaverini’s results predict Stanton number ratios above one for low blowing numbers. This is easily explained by the low prediction in heat transfer from the Stanton number correlation used to obtain a Stanton number without wall blowing.

Chiaverini’s results along with the experimental results in Kays and Marxman’s relationship are shown in Fig. 4.4. Due to the use of a non-typical Stanton number correlation, Chiaverini’s results are probably best eliminated from consideration for a regression rate model that will not be simply be empirically fit to regression rate data. The extrapolated results from Kays and Crawford experimental data however show remarkable similarity to the semi-analytical results generated by Marxman in the 1960’s. As the Kays and Crawford data is experimentally derived, the updated model uses the fit correlation described above as a fit for the blowing parameter.

4.2 Inclusion of Length Variation and Total Mass Flux for Simple Port Geometries

The LAVP regression rate model used a longitudinally averaged skin friction model to compute a longitudinally averaged regression rate as a function of the total motor length, \( L \). The collected LAVP regression rate model, expressed in terms of the oxidizer mass flux, instead of injector parameters is,
As the regression rate is length dependent, a model of this form will lose applicability as the fuel port regresses in a non-uniform manner. Thus, it makes sense to integrate the regression rate expression numerically along the port length. The length-varying form of eq. (4.15) is

\[
\dot{r} = \frac{1.27}{\rho_f} \left( \frac{c_p(T_0 - T_{fuel})}{h_v Pr_1^{3/2}} \right)^{0.23} \left[ 0.0296 G_0^{4/3} \left( \frac{\mu}{\chi} \right)^{3/5} \right]
\]  

One of the simplifying assumptions in the original LAVP regression rate model is that the total mass flux is equal to the oxidizer mass flux. For high oxidizer to fuel ratios, this assumption is probably reasonable as the mass flow rate of fuel added to the port will be small compared to the oxidizer mass flow. However, for lower oxidizer to fuel ratios this assumption becomes less valid. The inclusion of the total mass flux requires the computation of the total fuel flux which either requires analytic or numerical integration of eq. (4.15)
along the fuel grain. Marxman integrated his original relationship analytically to yield an average regression rate expression [43], however this approach is only correct for straight cylindrical ports.

It should be noted that this relationship is singular at the front end of the fuel grain, i.e., when \( x = 0 \). However, the net regression rate varies only slowly with axial length, so the motor can be discretized with a fairly small number of elements. Thus, if the regression rates are computed at the element centers, this singularity is seldom a problem in practice. This singularity could also be avoided by adding a short distance for boundary layer growth before the fuel edge. The net mass flux at any axial location is then

\[
G_i = \frac{\dot{m}_{ox} + \sum_{j=1}^{i-1} \dot{m}_{f_j}}{A_{c_i}} = \frac{\dot{m}_{ox} + \sum_{j=1}^{i-1} \dot{r}_j A_{s_j} \rho_f}{A_{c_i}}
\]

(4.17)

where the subscript \( i \) denotes the axial index of the element.

The longitudinal variation predicted by the UPVST model is shown in Fig. 4.5 for cases both including and neglecting the addition of fuel mass flux into the fuel port for the mass flux computation. Clearly, the effects of mass flux addition into the fuel port have a strong effect near the end of the fuel grain, but a very small effect near the fuel port entrance. For longer motors or lower O/F ratio motors, this effect will be further accentuated.

### 4.3 Discussion of Injector and Macroscopic Flow Field Effects

The UPVST model discussed in this chapter does not include a model for oxidizer injector effects, however, the results of oxidizer injector effects are not insignificant and thus warrant discussion.

The length-dependent flat-plate based Stanton number relationships discussed in section 4.2 predict very high heat transfer and regression rates near the leading edge of the fuel port where the boundary layer should be thin. This result has not been noted in data collected at USU. The final fuel port profile after a 10s burn for both a longitudinally varying theoretical model and experimental results from a test fire of a 98mm motor at USU are shown in Fig. 4.6.
Fig. 4.5: Evolution of the predicted fuel port profile over time for a constant oxidizer flow rate of 0.3 kg/s in the USU 98 mm hybrid motor. Cases are shown that both include and neglect fuel addition in the total mass flux.
The cause of this divergence is almost certainly the formation of counter-rotating vortices near the fuel port entrance. For most operation hybrid motors, the oxidizer injector orifice is significantly smaller than the fuel port and, there is a substantial pressure drop across the injector. For many hybrids this pressure drop is desirable for stability reasons and can also be important for droplet atomization of liquid oxidizers. Provided that there is not a very long pre-combustion chamber, for either liquid or gaseous oxidizers, the expanding oxidizer plume from the injector will not immediately attach to the fuel walls. This is very analogous to flow separation behind a rearward facing step. Qualitative recirculation regions for the USU 98mm motor are shown in Fig. 4.7.

Because the boundary layer thickness is not thin, the separated flow region will not exhibit the high heat transfer predicted for the leading edge of a flat plate with a uniform flow inlet. At the reattachment zone, heat transfer will increase due to flow impingement. Downstream of this zone, the boundary layer grows similar to flat-plate based predictions. Thus, given sufficient length, the role of oxidizer injector effects should diminish with distance down the fuel port. The strength of this effect should be proportional to the velocity of the injected fluid, which will also directly depend on the oxidizer density at the injector and the diameter ratio between the injector and the fuel port.

In one of the few studies in the open literature devoted to the subject, Carcimino and Sorge also showed very strong dependence of regression rate on injector geometry [49]. Their tests used gaseous oxygen as an oxidizer and they showed that the max regression rate exhibited during their tests corresponded to jet impingement on the fuel surface given an assumed jet divergence angle between 6 and 8 degrees, corresponding to the reattachment zone. Their models predict a nearly seven fold increase in regression rate for short motors with high injection velocities. An increase of this magnitude was not seen in the USU motors, which illustrates the complexity of this phenomenon and the variability of this effect between motors that use liquid and gaseous oxidizers.

A gaseous oxidizer with a high pressure ratio across the injector will for very high velocities downstream of the orifice, eventually sonically choking the orifice. Another way
to approach this idea is that the oxidizer density at the injector will be approximately the same order of magnitude as the density of the fluid flow through the chamber. Thus, because most of the fluid flow in the chamber also flows through the injector, the flow velocities at the injector must be very high. This will drive very strong recirculation regions near the injector, resulting in high heat transfer.

For a liquid oxidizer, the density of the oxidizer immediately before injection is often several orders of magnitude higher than the core fluid flow. For example, the density of nitrous oxide is on the order of 800 kg/m$^3$ and the density of combustion products for a typical USU 98mm motor firing is on the order of 30 kg/m$^3$. Thus, the velocities at the injector will not necessarily be very high, resulting in weaker vortices and lower heat transfer. Eventually, the oxidizer will vaporize and the density will decrease. For oxidizers near saturation pressure, this process will start as soon as the static fluid pressure is lower than the supersaturation pressure of the fluid, which often occurs at or slightly before the injection orifice. Unfortunately, this is not an equilibrium process as both droplet vaporization and bubble growth have non-negligible time constants. This makes analytical prediction of the oxidizer velocity near the injector extremely difficult.

For this reason, a satisfactory oxidizer injector effect model suitable for incorporation into the UPVST model that would be applicable across test conditions and motor configurations was not found. It is anticipated that this parameter would either need to be found via experimentation for a given motor configuration or computational fluid dynamics solutions. Unfortunately, due to the multiphase nature of the oxidizer injection plume in the USU 98 mm motor, even computational fluid dynamics solutions of this phenomenon are far from trivial. As shown in section 4.12, the UPVST model still predicts total regression rates that agree closely with data from the USU 98mm motor even without a model for oxidizer injector effects. This may, however, be a source of error for other test configurations.

4.4 Discussion of the Flame Temperature

The fluid parameters $c_p$, $\mu$, $T_0$, and Pr in the LAVP model are all evaluated using CEA, the chamber pressure, and the total oxidizer to fuel ratio in the original LAVP model. This
Fig. 4.6: Theoretical and experimental radii vs axial location for a 10s hybrid motor burn of a 98 mm motor at USU.

Fig. 4.7: Qualitative oxidizer flow lines and recirculation regions in the forward end of the USU 98mm motor.
assumption allows regression to scale the combustion temperature generated by different fuel/oxidizer combinations. However, the total oxidizer to fuel ratio only really applies to fluid properties in the post-combustion chamber. The original Marxman relation used the flame temperature, $T_f$, instead of the combustion temperature to calculate the enthalpy difference between the combustion gasses and the fuel surface. With a length-varying regression rate model, it once again makes sense to use the flame temperature for this calculation.

Much of the early work on hybrid regression rate models used gaseous oxygen for an oxidizer. It was demonstrated early on that the hybrid diffusion flame typically burns at an $O/F$ ratio well below stoichiometric and this ratio remains unchanged over the port length [40]. For the gaseous oxygen / plexiglass systems that were popular in early research, the oxidizer to fuel ratio at the flame was about 1.5, whereas the optimal oxidizer to fuel ratio is near 2.0. Tests with oxidizer diluted with nitrogen showed that the flame existed at a concentration of oxygen to fuel at a ratio of 1.5, despite the diluent. This should not be confused with the oxidizer to fuel ratio commonly referred to in rocketry literature, which includes the mass of both oxygen and other elements in the oxidizer in the mass of the oxidizer. It should be noted that decomposed nitrous oxide is chemically identical to oxygen diluted to 36.3% concentration by mass with nitrogen. Hence, to obtain a oxygen to fuel concentration of 1.5, the nitrous oxide oxidizer to fuel ratio would be 4.13.

4.5 Location of the Flame Height in the Boundary Layer

The original regression rate law derived by Marxman and his colleagues included the effect of the flame height in the boundary layer, rather than assuming that heat transfer occurred from the inner edge of the boundary layer [40, 41]. Tests were completed at the UTC with different oxidizer dilutions which verified that the flame height in the boundary layer closely obeyed the theoretical relationship,

\[
\frac{u_c}{u_e} = \frac{OF(h_{cs} - h_{wg})}{\Delta H} \frac{1}{K_{oxe} + (OF + K_{oxe}) \frac{(h_{cs} - h_{wg})}{\Delta H}}
\]

(4.18)

with an oxygen to fuel ratio, $O/F$, near 1.5. For typical test conditions with nitrous oxide,
the value of the flame height ratio is usually close to 0.6, increasing the value of the blowing parameter by about 67%. Although, the oxygen to fuel ratio should be close to the value determined by the UTC, it should be noted that several parameters in the regression rate expression are reasonably sensitive to this variable. The variability of flame temperature and the flame height ratio as well as the resulting blowing parameter are shown in Fig. 4.8 for an oxidizer mass flux of about 395 kilograms per square meter.

4.6 Inclusion of the Fuel Surface Temperature

The original LAVP regression relationship used the fuel temperature as the fuel surface temperature. However, the fuel surface temperature is a function of the fuel pyrolysis properties. The fuel surface temperature for typical hybrid fuels at normal hybrid regression rates will generally be substantially higher than the initial fuel temperature. This will have several effects on the regression rate. First, the elevation of surface temperatures reduces the enthalpy difference between the flame layer and the fuel surface, lowering total heat transfer to the fuel surface. Secondly, heating the fuel requires energy, which increases the
effective heat of gasification of the fuel, further reducing the regression rate. Thirdly, the
temperature increase of the fuel surface translates into an increase in temperature of the
pyrolysis products, which will have some impact on the equilibrium combustion temperature
in the flame zone.

The fuel surface temperature can be estimated by using classical Arrhenius reaction
kinetics parameters. By assuming that the pyrolysis reaction is first order and examining
the heat transfer and the thickness of the pyrolyzing degradation layer, the regression rate
can be expressed in terms of pyrolysis kinetics constants and the surface temperature [131],

\[
\dot{r}^2 = A_c \frac{e^{-J_c}}{J_c} d_p R^2
\]  

(4.19)

where

\[
R^2 = \frac{1}{-\ln (Y_{ps}) \left(1 - \frac{T_i}{T_s} + \frac{h_d}{c_{pf}} T_s \right) - \frac{h_d}{c_{pf}} T_s} 
\]  

(4.20)

\[
J_c = \frac{E_a}{RT_s}
\]  

(4.21)

\[A_c\] is the pyrolysis pre-exponential factor, \[d_p\] is the material thermal diffusivity, \[h_d\] is the
heat of degradation, \[T_i\] is the initial temperature of the fuel, \[E_a\] is the activation energy, and
\[Y_{ps}\] is the mass fraction of polymer near the outer edge of the degradation layer. Chiaverini
et al. showed that this phenomenon can be approximated by a much simpler relationship,

\[
\dot{r} = A_c e^{-\frac{E_a}{RT_s}}
\]  

(4.22)

hence,

\[
T_s = \frac{E_a}{R_u \left(\ln (A_c) - \ln (\dot{r})\right)}
\]  

(4.23)

Often pyrolysis parameters are obtained through low heating rate experiments such as
thermogravimetry that often had heating rates on the order of 1 degree kelvin per second.
Chiaverini performed pyrolysis experiments using a pre-heated copper rod dropped onto samples of HTPB. This yielded heating rates on the order of 1000 degrees Kelvin per second and surface temperatures on the order of 760 K, parameters which are comparable to those experienced in a hybrid motor. For surface temperatures above 722 K, Chiaverini found that HTPB pyrolysis had an activation energy of 4.91kcal/mole and a rate constant of 11.04 mm/s. For temperatures below this, he found that had an activation energy of 13.35 kcal/mole and a rate constant of 3.965 mm/s. This yields the pyrolysis parameters shown in Fig. 4.9. It is clear that the pyrolysis parameters derived at low surface temperatures would yield a great deal of error if extrapolated to the higher temperature operating regimes seen by conventional hybrid motors.

For this algorithm, the pyrolysis parameters are appropriately selected based upon the local, instantaneous rate. The large changes in surface temperature have a significant effect on total heat transfer. High regression rates correspond to high surface temperatures, which will substantially lower the value of the heat transfer to the solid fuel compared to a model where the fuel surface is considered to be equal to the initial fuel temperature. Additionally, the surface temperature has a large effect on the effective heat of vaporization of the fuel. The effective heat of vaporization is a sum of the heat of degradation and the energy required to raise the fuel from it’s initial temperature to the surface temperature,

\[ \Delta H = c_{pf} (T_s - T_0) + h_D \]  

where \( c_{pf} \) is the specific heat of the solid fuel. For HTPB, the effective heat of vaporization at typical operating temperatures is often about 70% higher than the heat of degradation alone due to the inclusion of this term. The incorporation of eq. (4.23) into a model means that regression rate is an implicit function so that the surface temperature and regression rate must be solved for iteratively.

It should be noted that the energy “consumed” in heating the fuel before pyrolysis is not lost from the system. The resulting pyrolysis products will be at a higher temperature as well as the fuel surface. The temperature of the pyrolysis products influences the final
flame temperature and the combustion products, as the enthalpy of the fuel is higher at elevated temperatures (higher regression rates) than at lower temperatures (lower regression rates). However, this is far from a one-to-one correlation. The flame temperature is highly constrained by chemical processes which limit the formation of reactants at elevated temperatures. For example, if the flame temperature for the products of a reaction is 2500 K when the reactants start at 500 K, this does not mean that the adiabatic flame temperature would be about 4500 K if the reactants started at 2000K. At elevated temperatures, the products of the reaction change to accommodate for the increased enthalpy of the reactants and less energy can be extracted without heat transfer out of the system. For instance, combustion of hydrogen and oxygen will combine to form water, releasing energy, but at sufficient temperatures water splits into hydrogen and oxygen. Thus, at high enough temperatures, hydrogen and oxygen will not sufficiently combine into water for a net release of energy.

The pyrolysis products of HTPB consist mostly of butadiene gas [130]. The heat capacity of butadiene gas is around 0.079 kJ/mol-K [132]. Thus, an increase in fuel temperature
of 800 K (a reasonable increase for typical hybrid operation) results in an increase in the
total enthalpy of the pyrolysis products of about 63 kJ/mol. Using CEA, and gaseous oxy-
gen as an oxidizer at a mixture ratio of 2.0, and a chamber pressure of 2 MPa, this results
in an change in flame temperature of about 62 K, a change of less than two percent. For
Nitrous Oxide/HTPB combustion at an oxidier to fuel ratio of 4.5, the flame temperature
is somewhat lower and more amenable to increase due to reactant enthalpy. For this case
the resulting increase in flame temperature is about 88 K, an increase of just under three
percent. Considering the relative magnitude of this effect, and the fact that blowing reduces
the impact of the enthalpy difference between the fuel grain and the flame, including this
effect would result in a substantial increase in complexity and/or overhead for computing
combustion product properties for a very minimal increase in regression rate prediction
accuracy. Thus, the increase in flame temperature due to increased surface temperature is
neglected for the UPVST model but the decrease in heat transfer caused by the elevated
surface temperature as well as the increase in the effective heat of gasification is included.

4.7 Reference Viscosity

The original LAVP model used the viscosity of the combustion products to approx-
imate the viscosity in the boundary layer. However, the viscosity will vary substantially
throughout the boundary layer. The fluid near the fuel wall is nearly all pyrolyzed fuel and is
at temperatures on the order of approximately 1000 K. Midway through the boundary layer
is the flame layer which is below, but relatively near, stoichiometric oxidizer to fuel ratios.
The flame layer is the hottest part of the boundary layer and is typically between 1000K
and 2000 K higher in temperature than the fuel surface. On the inner edge of the bound-
ary layer, the fluid consists mostly of oxidizer and is near the oxidizer inlet temperature,
which is much colder than the rest of the boundary layer. The oxidizer inlet temperature
is typically on the order of 300 K or lower.

The byproducts of HTPB pyrolysis have been shown to consist mostly of butadiene gas
at normal hybrid motor operating conditions [130], so it is probable that the viscosity of the
pyrolysis gas is near the viscosity of butadiene gas. Unfortunately, little data exists for the
Table 4.1: Butadiene Gas Viscosity [133].

<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>Viscosity (Pa-s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>298.15</td>
<td>8.12E-6</td>
</tr>
<tr>
<td>351.75</td>
<td>9.98E-6</td>
</tr>
</tbody>
</table>

viscosity of butadiene gas at elevated temperatures. This dearth of data is presumably due to the fact that it is primarily used in rubber synthesis and therefore is seldom used or stored in gaseous form in industry except at saturation properties. However low temperature gas phase data was collected by Lambert and his colleagues [133]. This data is listed in Table 4.1.

Sutherland’s formula has been shown to accurately predict the viscosity of gases over a range of temperatures [134]. If one assumes butadiene gas adheres well to Sutherland’s formula and the gas near the surface is mostly butadiene, the viscosity at the fuel surface should be reasonable modeled by

\[ \mu = \mu_0 s \left( \frac{T_{0s} + C_s}{T_s + C_s} \right) \left( \frac{T_s}{T_{0s}} \right)^{\frac{3}{2}} \]  \quad (4.25)

where \( T_{0s} \) is the reference temperature for the reference viscosity, \( \mu_0 s \), and \( C_s \) is an empirically derived constant with units of temperature. Unfortunately no direct data for \( C_s \) for butadiene could be found, but this constant can be solved for explicitly from eq. (4.25) given two data points for viscosity and temperature. Using the data in Table 4.1, this results in \( C_s = 960K \). The typical surface temperatures in hybrid rockets are far above the temperatures at which data for butadiene gas viscosity has been documented, so the application of Sutherland’s formula will require extrapolation. As with the extrapolation of any data, this must be completed with some level of caution. Thus, the examination of some measure of the sensitivity of eq. (4.25) to the emperical constant \( C_s \) is warranted. The viscosity curves for the value calculated from the data in Table 4.1 along with this value ±200 K are shown in Fig. 4.10. At typical hybrid fuel temperatures of around 1000K, this approximately 20 percent variation in \( C_s \) yields about a 15 percent variation in viscosity. Fortunately, the
Fig. 4.10: Viscosity of butadiene gas estimated with Sutherland’s law for several values of $C_s$.

regression rate formulation is only a weak function of viscosity, so this level of inaccuracy in viscosity is tolerable. It is noteworthy that this level of error assumes the no error from the application of Sutherland’s formula. The validity of Sutherland’s formula for butadiene could only be verified by additional experimental data in the operating temperature range, which could not be found. Due to the absence of a more precise model, Sutherland’s formula is used to calculate the viscosity at the fuel surface in the updated model.

The outer edge of the boundary layer will be nearly completely oxidizer, in this case nitrous oxide, and it should be relatively cold. Unfortunately, there is a dearth of data on the viscosity of nitrous oxide. However, nitrous oxide generally has very similar fluid properties to carbon dioxide, a fluid which is often used as a surrogate for cold flow testing instead of the much more expensive and potentially dangerous nitrous oxide. For example, the viscosity of nitrous oxide and carbon dioxide gas are only 0.87% different at atmospheric pressure and 0 C [132]. The viscosity properties of carbon dioxide are well known, and the viscosity of carbon dioxide gas at 300 K is $1.55E^{-5}$ Pa-s [135–137].
The actual temperature of the core nitrous oxide flow depends on several factors including pre-combustion chamber recirculation, nitrous oxide vapor/liquid mass fraction at the injector, nitrous droplet vaporization time, and the speed of exothermic nitrous oxide decomposition in the chamber. All of these phenomenon would be extremely difficult to model. However, it seems reasonable to assume that some of the core nitrous oxide flow will still be multiphase for a significant length of the chamber, and thus the temperature of the nitrous should be somewhat higher than the saturation temperature which range between about 224 K and 280 K for conventional motor operating pressures. Hence, 300 K should be a reasonable approximation for the fluid temperature at the edge of the boundary layer.

As the flame height can be calculated from the previously discussed relationship derived by Marxman, these three viscosities can be used in a weighted average to compute an average viscosity over the boundary layer,

\[
\mu_{\text{avg}} = \left( \frac{\mu_f}{2} + \frac{\mu_s}{2} \right) \frac{u_e}{u_c} + \left( \frac{\mu_f}{2} + \frac{\mu_e}{2} \right) \left( 1 - \frac{u_e}{u_c} \right)
\]

\[= \left( \frac{\mu_f}{2} + \frac{\mu_s}{2} \right) + \left( \frac{\mu_s}{2} - \frac{\mu_e}{2} \right) \frac{u_e}{u_c} \]

(4.26)

where \(\mu_f\) is the viscosity at the flame, \(\mu_s\) is the viscosity at the gas surface, and \(\mu_e\) is the viscosity of the core gas flow. A typical average viscosity computed in this manner is about 5.8E⁻⁵ Pa·s, which is about 30% lower than the viscosity at the flame sheet and about 40% lower than a viscosity computed at the total oxidizer to fuel ratio.

### 4.8 Effect of Variable Fluid Properties

Unfortunately, the effects of property variation throughout the boundary layer are neither constant nor linear. There is some contention in the literature about how to deal with variable fluid properties, which Lees [42] refers to as “the ancient controversy over the effect of variable fluid properties on turbulent heat transfer.” For flows with temperature variation across the boundary layer, Kays and Crawford [125] suggest a Stanton number correction of the form
where $St_0$ is the uncorrected Stanton number and $n_{st}$ is an empirically derived exponent. Kays et al. suggest a value for of about -0.55 for this exponent for external, turbulent flows where $T_\infty > T_s$. Chiaverini et al. found a formula of this form with an exponent of -0.60 fit their data well [48]. The reference properties for this relation are suggested to be calculated at a temperature determined via

$$T_{ref} = T_\infty + 0.5 (T_s - T_\infty)$$  \hspace{1cm} (4.28)

However, this relationship was derived assuming that the temperature varies across the boundary layer monotonically. The boundary layer in a hybrid motor has clear maximum temperature at the flame layer. Thus, it is projected that calculation of the average viscosity via eq. (4.26) should yield a more accurate result. The updated LAVP model uses a correction of the form of eq. (4.27) with an exponent of -0.55 and an average viscosity computed via eq. (4.26).

### 4.9 Radiation

Marxman and his colleagues noted early on that radiative and convective heat transfer were linked in the hybrid rocket. Radiative heat transfer increases wall blowing, which reduces convective heat transfer [41]. They showed that a regression rate model including the effects of radiation would have the form

$$\dot{r} = \frac{1}{\rho_f \Delta H} \left( \dot{Q}_{c'} e^{-\dot{Q}_{rad} / \dot{Q}_{c'}} + \dot{Q}_{rad} \right)$$  \hspace{1cm} (4.29)

where the convective heat transfer can be computed via

$$\dot{Q}_{c'} = \dot{r}_c \Delta H \rho_f$$  \hspace{1cm} (4.30)

It should be noted that the convective heat transfer in these relationships is not an estimate.
for the actual convective heat transfer, but an estimate for the convective heat transfer without radiation. This estimate of convective heat transfer is then modified in eq. (4.29) to adjust for the coupling between radiation and convection, presuming the form of this relationship is well modeled by the exponential term.

However, an explicit regression rate relationship including both radiation and convective heat transfer is not required if numerical iteration is allowable. In this case, it is much more straightforward to leave the regression rate in the form

\[
\dot{r} = \frac{1}{\Delta H \rho_f} (\dot{Q}_{rad} + \dot{Q}_c)
\]  

(4.31)

where the convective heat transfer is a function of the blowing parameter defined as a function of the total regression rate including radiant heat transfer,

\[
B = \frac{\dot{r} \rho_f}{\rho_e u_e \frac{C_f}{2} \Pr^{-\frac{2}{3}}}
\]  

(4.32)

Given a model for radiant heat transfer, these two expression can then be solved numerically for the regression rate as a function of radiant and convective heat transfer simultaneously. As the model described in this chapter uses an iterative procedure to solve for fuel surface temperature, iteration overhead for this solution is minimal.

Radiation in the hybrid motor is a complicated phenomenon and direct data for radiant heat transfer in hybrids is scarce. Perhaps the most significant source of radiative heat transfer data is work by Strand et al. at the Jet Propulsion Laboratory. Strand and his colleagues performed tests with gaseous oxygen and HTPB [45] in a slab burner. Using direct measurements of thermal radiation from the flame layer through quartz windows, their tests showed that radiation accounted for approximately a third of the total heat transfer to the fuel surface. It should be noted that their tests were generally operated at oxidizer mass fluxes below about 50 kg/m²s, far lower than typical hybrid operating ranges. (The mean mass flux over a 10 second burn for the 98mm motor firings at USU were in the range of 300kg/m²s.) Thus, it is not surprising that radiant heat transfer had
a substantial effect, as convective heat transfer would be very low for that range of mass fluxes. Thus, radiation is probably still a marginal contributor to heat transfer at normal operating conditions for non-metalized fuels that do not contain strong radiative sources. However, the inclusion of radiation still extend the applicability of the regression rate model, and thus it’s inclusion is still warranted.

Strand’s research found that gas phase emissivity was very low and that radiative heat transfer was dominated by radiation from soot particles created by incomplete combustion and the pyrolysis process. They found that radiative heat from soot was modeled by

\[ \dot{Q}_{\text{rad}} = \sigma T_f^4 \left(1 - e^{-a_p N_p}\right) \] (4.33)

where \( \sigma \) is the Stefan–Boltzman constant, and \( a_p N_p \) was a constant related to the soot density.

The soot density was found to be a function of oxidizer to fuel ratio and, unsurprisingly, more soot was created at lower oxidizer contents. They found this soot density constant was well fit by

\[ a_p N_p = 0.134 \left(\frac{\alpha_p P_0}{1 + OF - \alpha_p}\right) \] (4.34)

where \( \alpha_p = 0.045 \) for their test conditions although they noted that this parameter was as low as 0.01 for some tests. Noting that Strands tests typically operated at very low mass fluxes that are more likely to be influenced by radiation, the UPVST model uses a value on the lower end of this scale, 0.015.

4.10 Model Summary and Solution Methodology

The original LAVP regression formulation modeled the inlet and outlet flow flow rates of the combustion chamber separately to determine the mass in the chamber and operating pressure as a function of time. However, the inclusion of the chamber pressure model adds a great deal of stiffness to the combined differential equations. The chamber pressure in a typical motor configuration can double on the order of milliseconds, while the port
cross sectional area, the other major time dependent phenomenon, doubles on the order of several seconds. Thus, the inclusion of differential equations for chamber pressure requires the use of much smaller time steps than would be needed if a pressure term were not included. Although the inclusion of a chamber pressure model allows for the examination of certain transient phenomenon, an accurate modeling of hybrid transients would require the addition of differential equations responsible for said transients, such as oxidizer feed response or boundary layer delay transients. As this section provides a model intended to estimate regression rates during pseudo steady state behavior, differential equations for chamber pressure will not be used here.

As an alternative to directly integrating differential equations governing chamber pressure, the chamber pressure can be readily solved for by assuming steady state operation and choked flow at the nozzle exit. In practice, this situation will be reached in a fraction of a second under normal operating conditions. Thus, the chamber pressure at steady state conditions is

\[ P_0 = (\dot{m}_{ox} + \dot{m}_{fuel}) \left( A_{star} \sqrt{T_0} \left( \frac{R_g}{\gamma} \frac{\gamma + 1}{2} \right)^{\frac{\gamma + 1}{\gamma - 1}} \right)^{-1} \]  

(4.35)

where all fluid properties are evaluated using chemical equilibrium analysis based upon the total oxidizer to fuel ratio. Unlike solid motors, hybrid motor reactions generally do not operate in a regime where the pressure dependence of chemical reaction kinetics influence motor performance. However, chamber pressure does influence fluid properties that do influence regression rate. The regression rate, in turn, changes the mass flow rate through the nozzle and so this process must be iterated. It was found that convergence was both fast and stable if this method was slightly under-relaxed such that

\[ P_{0i+1} = P_{0i-1} + 0.8 \left( P_{0i} - P_{0i-1} \right) \]  

(4.36)

As described in section 4.2, the fuel port is separated into sections longitudinally and the total mass flux at any location is a function of the total mass flux and therefore, the
regression rate at all upstream sections. Thus, the regression rate must be computed for each longitudinal location in series, starting at the forward end of the motor. The updated model does not assume a constant fuel surface temperature along the length of the fuel port. Thus, the surface temperature must also be computed for each longitudinal step. The surface temperature is a weak function of regression rate, so the surface temperature and resulting regression rate must be solved for iteratively at a given longitudinal fuel section. The net solution process for computing regression rates along the length of the fuel port is shown in Fig. 4.11. A MATLAB function for the LAVP model is presented in Appendix A and a MATLAB function for the UPVST model is included in Appendix B.

4.11 Regression Rate Model Summary and Comparison

Many changes were made to the original LAVP model in the formulation of the UPVST model. A summary of the changes made to the original LAVP model in the UPVST model is listed in Table 4.2. In general, the sensitivity of the UPVST to each parameter is not easily characterized on an individual level as each configuration depends on operating conditions and there is a high degree of cross-coupling between parameters. However, general trends are noteworthy and the general order of the effect each model is included in Table 4.3.

4.12 Results for the UPVST Regression Rate Model

Although regression rates are sometimes proprietary and often remain unpublished, regression data from several tests with both nitrous oxide and gaseous oxygen are available in the open literature. Additionally, a great deal of data for both HTPB and ABS regression with nitrous oxide has been collected during a parallel research effort at Utah State University to evaluate ABS as a hybrid rocket fuel [17]. Generally, regression rate data is presented as average regression rate vs oxidizer mass flux. The average oxidizer mass flux is somewhat ambiguous and can be calculated several ways, however the standard in the literature for circular ports is the average mass flow divided by the area based on the average diameter [138]. Thus, this is how the regression rate data obtained at Utah State University
Guess regression rates at all longitudinal sections.

Guess chamber pressure.

Compute fluid properties at the flame and near the wall using chamber pressure estimate.

Compute regression rates at all longitudinal sections.

Compute fuel surface temperature at longitudinal section using Arrhenius relationship and regression rate estimate.

Compute the enthalpy difference between the flame and the fuel surface.

Compute the flame height ratio.

Compute the effective boundary layer viscosity.

Compute the blowing parameter from regression rate guess.

Calculate heat transfer from convection.

Calculate heat transfer from radiation.

Update regression rate estimate for longitudinal section.

Update fuel flow rate for downstream mass flux calculations.

If regression rate change is below threshold, advance to next longitudinal section. Otherwise recalculate fuel surface temperature.

After last longitudinal section, update chamber pressure estimate from combustion parameters and total propellant flow rate.

If chamber pressure change is below threshold, exit. Otherwise, recalculate fluid properties.

Fig. 4.11: Combined regression rate algorithm.
Table 4.2: Summary of Models Used in UPVST and LAPV Regression Rate Algorithms.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>UPVST</th>
<th>LAPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stanton Number</td>
<td>Flat-plat skin friction and Reynolds analogy with longitudinal variation of mass flux</td>
<td>Longitudinally averaged flat-plate skin friction with Reynolds analogy</td>
</tr>
<tr>
<td>Blowing Number Adjustment</td>
<td>Extrapolated empirical transpiration cooling data for butadiene heat capacity</td>
<td>Marxman-style exponential simple fit</td>
</tr>
<tr>
<td>Fuel Surface Temperature</td>
<td>Arrhenius model from Chiaverini data</td>
<td>Assumes constant temperature fuel</td>
</tr>
<tr>
<td>Blowing Parameter</td>
<td>Full mass flux included in blowing parameter, not exponential approximation</td>
<td>Uses enthalpy-based expression</td>
</tr>
<tr>
<td>Flame Properties</td>
<td>CEA data with constant O/F at flame layer</td>
<td>CEA from total O/F ratio</td>
</tr>
<tr>
<td>Flame Height</td>
<td>Classical Marxman flame height ratio based on oxygen (not oxidizer) to fuel ratio</td>
<td>Inner edge of boundary layer</td>
</tr>
<tr>
<td>Effective Heat of Gasification</td>
<td>Chiaverini pyrolysis data with heat capacity of fuel</td>
<td>Constant, uses heat of degradation</td>
</tr>
<tr>
<td>Radiative Heat Transfer</td>
<td>Strand Soot Model based on flame O/F ratio, flame temperature and surface temperature</td>
<td>None</td>
</tr>
<tr>
<td>Reference Viscosity</td>
<td>Weighted average based on flame height. Sutherland’s model for butadiene viscosity near fuel surface.</td>
<td>Assumes constant viscosity equal to flame properties at total O/F ratio</td>
</tr>
<tr>
<td>Property Variation in Boundary Layer</td>
<td>$(\frac{T}{T_0})^n$ correction using reference viscosity</td>
<td>None</td>
</tr>
<tr>
<td>Prandtl Number</td>
<td>Constant, turbulent Prandtl number of ~ 0.85</td>
<td>Fluid Prandtl number from flame properties at total O/F ratio</td>
</tr>
<tr>
<td>Chamber Pressure Model</td>
<td>Pseudo-steady state</td>
<td>Differential equations for chamber fill</td>
</tr>
<tr>
<td>Solution Methodology</td>
<td>Iterative</td>
<td>Explicit</td>
</tr>
</tbody>
</table>
Table 4.3: Summary of Effect of Changes from LAVP Regression Model to UPVST Model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model</th>
<th>Approximate Relative Effect Compared to LAVP Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stanton Number</td>
<td>Flat-plat skin friction and Reynolds analogy with longitudinal variation of mass flux</td>
<td>Order of +10% for moderate O/F ratios, increases regression as O/F decreases</td>
</tr>
<tr>
<td>Blowing Number Adjustment</td>
<td>Extrapolated empirical transpiration cooling data for butadiene heat capacity</td>
<td>Order of -10% from Marxman Fit</td>
</tr>
<tr>
<td>Fuel Surface Temperature</td>
<td>Arrhenius model from Chiaverini data</td>
<td>Order of -30%, increases relative sensitivity to flame temperature</td>
</tr>
<tr>
<td>Blowing Parameter</td>
<td>Full mass flux included in blowing parameter, not exponential approximation</td>
<td>Limits effect of radiation model inclusion, by coupling radiation and convection</td>
</tr>
<tr>
<td>Flame Properties</td>
<td>CEA data with constant O/F at flame layer</td>
<td>Little change, reduces sensitivity to net O/F ratio</td>
</tr>
<tr>
<td>Flame Height</td>
<td>Classical Marxman flame height ratio based on oxygen (not oxidizer) to fuel ratio</td>
<td>Order of +25%</td>
</tr>
<tr>
<td>Effective Heat of Gasification</td>
<td>Chiaverini pyrolysis data with heat capacity of fuel</td>
<td>Order of -25%, highly coupled with surface temperature</td>
</tr>
<tr>
<td>Radiative Heat Transfer</td>
<td>Strand Soot Model based on flame O/F ratio, flame temperature and surface temperature</td>
<td>Order of +10%, much higher for very low mass fluxes</td>
</tr>
<tr>
<td>Reference Viscosity</td>
<td>Weighted average based on flame height. Sutherland’s model for butadiene viscosity near fuel surface</td>
<td>Little change, configuration dependent</td>
</tr>
<tr>
<td>Property Variation in Boundary Layer</td>
<td>$(\frac{T}{T_s})^n$ correction using reference viscosity</td>
<td>Order of +30%</td>
</tr>
<tr>
<td>Prandtl Number</td>
<td>Constant, turbulent Prandtl number of ~ 0.85</td>
<td>Order of -10%, decreases sensitivity to flame properties</td>
</tr>
</tbody>
</table>
Table 4.4: Regression Data Summary.

<table>
<thead>
<tr>
<th>Data Source</th>
<th>Oxidizer</th>
<th>Fuel</th>
<th>Port Length</th>
<th>Representative Oxidizer Flow Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>USU HTPB [17]</td>
<td>N2O</td>
<td>HTPB</td>
<td>0.57 m</td>
<td>0.3 kg/s</td>
</tr>
<tr>
<td>USU ABS [17]</td>
<td>N2O</td>
<td>HTPB</td>
<td>0.57 m</td>
<td>0.3 kg/s</td>
</tr>
<tr>
<td>Thiokol</td>
<td>O2</td>
<td>HTPB</td>
<td>2.6 m</td>
<td>3 kg/s</td>
</tr>
<tr>
<td>JIRAD [139]</td>
<td>O2</td>
<td>HTPB</td>
<td>2.6 m</td>
<td>3 kg/s</td>
</tr>
<tr>
<td>Strand [140]</td>
<td>O2</td>
<td>HTPB</td>
<td>0.41 m</td>
<td>0.1 kg/s</td>
</tr>
<tr>
<td>Chiaverini Slab</td>
<td>O2</td>
<td>HTPB</td>
<td>0.58 m</td>
<td>0.2 kg/s</td>
</tr>
<tr>
<td>Burner [48]</td>
<td>O2</td>
<td>HTPB</td>
<td>0.56 m</td>
<td>0.22 kg/s</td>
</tr>
<tr>
<td>OrbiTEC</td>
<td>O2</td>
<td>HTPB</td>
<td>0.56 m</td>
<td>0.22 kg/s</td>
</tr>
<tr>
<td>Mirras [141]</td>
<td>O2</td>
<td>HTPB</td>
<td>0.56 m</td>
<td>0.22 kg/s</td>
</tr>
<tr>
<td>Stanford</td>
<td>N20</td>
<td>HTPB</td>
<td>0.15 m</td>
<td>0.1 kg/s</td>
</tr>
<tr>
<td>Nitrous [142]</td>
<td>O2</td>
<td>HTPB</td>
<td>0.25 m</td>
<td>0.25 kg/s</td>
</tr>
</tbody>
</table>

was reduced, and it was presumed that other data sources used the same method (although the specific method for calculating oxidizer mass flux is not always reported). Including data from USU, nine sources of regress rate data were considered for comparison to the original LAVP model as well as the updated model. These data sources are listed in Table 4.4 and are plotted in Fig. 4.12.

There is obviously a high degree of data scatter for most of the regression rate data sources shown in Fig. 4.12. A large amount of this scatter is due to the use of different oxidizers. Hybrid rocket motors that use gaseous oxygen will have a higher flame temperature and therefore regression rate than motors that use nitrous oxide. Additionally, the regression rate scales inversely with Reynolds number, so shorter motors will have a much higher overall regression rate. The data scatter for identical motors could possibly be caused by experimental uncertainty or inconsistent experimental procedures, including fuel preparation.

There are some motors that do not follow the overall trends that would be expected due to oxidizer use and motor length. Both of the Thiokol data sets, the JIRAD motor and the lab-scale motors have much lower regression than would be expected for a motors of that
size. However, the fuel used in these motors was not a standard formulation of HTPB, but included additives which increased the strength of the material. These additives increased the density of the fuel over standard HTPB densities by approximately 8 percent. However, this increase in density is not enough to explain the low regression rates. It is unknown how the fuel additives used by Thiokol change the heat capacity and the heat of degradation of the fuel material. Any change in these parameters will directly influence the effective heat of gasification of the material and will influence the overall regression rate of the material. As these parameters are not presented in the literature, these data cannot be used to compare with the predictions from regression rate models.

The data by Strand exhibit substantially higher regression rate than would be expected for the corresponding oxidizer mass fluxes. However, these mass fluxes are substantially lower than would be expected for normal hybrid rocket operating conditions. During these tests Strand showed a high dependence on radiation and development of substantial char layers, both conditions not normally seen during typical hybrid rocket observation. Thus, these data are also not good candidates for comparison with the regression rate models.
In order to match regression rates for a given oxidizer mass flux, motor operating parameters must be matched to the mass flux. Typical hybrid motors have a nearly fixed oxidizer mass flow so that the change in oxidizer mass flux is a function of port cross sectional area. For experimental motors, the oxidizer mass flux is sometimes changed by varying the oxidizer mass flow rate in between tests in lieu of the cross sectional port area. However, as actual mass flow rates and cross sectional areas are generally not reported with regression rate data, a constant oxidizer mass flow rate was assumed the mode of operation for all of the data sets. The port radius was adjusted to yield the required oxidizer mass flux for the nominal flow rate. Additionally, the updated regression rate model can include longitudinal variation in regression rate, but this is also not generally reported in the literature, so all motor sections were assumed to have a constant cross sectional area over the axial length of the fuel port.

Some datasets report typical chamber pressures but not nozzle exit areas or fuel surface area. Thus, the motor operating pressure cannot be calculated with the method described in section 4.10. For the experiments where the pressure is prescribed this can simply be used directly to calculate flow properties which eliminates the pressure solution loop in the updated algorithm.

The predicted and experimental data for the original LAVP correlation is shown in Fig. 4.13 and the predicted and experimental regression rates for the updated correlation is shown in Fig. 4.14. The original LAVP model tends to under predict the regression rate for the regression rate for gaseous oxygen motors. The flame temperature for gaseous oxygen-HTPB hybrid motors is higher than that for nitrous oxide motors. The original LAVP model does not include the effects of surface temperature, which tends to increase the predicted LAVP regression rate, or the flame height in the boundary layer, which decreases the predicted regression rate over models that do. For nitrous oxide motors, such as the motors tested at Utah State University, these effects tend to cancel out. However, the LAVP regression rate is less sensitive to changes in flame temperature than the updated
model, because the fluid enthalpy difference is measured from the flame temperature to the original fuel temperature whereas the updated model uses the difference between the wall temperature and the flame temperature. Thus, an increase in the flame temperature will have a lower percentile effect on the LAVP model than on the updated model.

In general, the updated model predicts regression rates better than the original LAVP model for these data sets. The mean error and the norm of the error for all data points considered is shown in Table 4.5.

Although a great deal of data scatter exists in the original data set, the updated model does significantly decrease the data scatter as well as the regression rate bias. A representation of the degree of data scatter is the minor axis of the covariance ellipse for the data shown in figures 4.13 and 4.14. Table 4.5 contains these data. The updated model shows a reduction in the minor axis of the covariance ellipse of approximately 35 percent.

Some additional insight into the performance of both the LAVP regression rate model and the updated model can be found from comparing the regression model predictions to the data obtained for nitrous-oxide motors burned at Utah State University. These data
Fig. 4.14: Updated model predictions vs experimental data.

Table 4.5: Regression Correlations Summary.

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Mean Error</th>
<th>Norm Fractional Error</th>
<th>Minor axis of covariance ellipsoid</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAVP Model</td>
<td>−22.3%</td>
<td>2.63</td>
<td>0.0103 ·10⁻⁶ m</td>
</tr>
<tr>
<td>UPVST Model</td>
<td>2.6%</td>
<td>1.05</td>
<td>0.0064·10⁻⁶ m</td>
</tr>
</tbody>
</table>
and the analytical curves for these burns is shown in Fig. 4.15. Several of the regression rates on this curve come from fuel grains that were fired multiple in support of electrical discharge ignition capability. One HTPB motor grain (“Carbon Black is People”) was fired four times for 2.5 seconds each and an ABS fuel grain (“The White Whale”) was fired twice for 2.5 seconds each. These tests provide regression rate data for a range of oxidizer mass fluxes instead of simply the average oxidizer mass flux obtained for the vast majority of the motor firings at Utah State University.

The updated regression model predicts the regression rate at the lower mass fluxes more accurately than the original LAVP model. There are several reasons for this effect. Note that lower oxidizer mass fluxes in a typical hybrid will result in low oxidizer to fuel ratios. The updated regression rate model uses the local flame temperature instead of the combustion temperature. As the combustion temperature will decrease at very low mixture ratios, the LAVP model will predict a corresponding decrease in regression rate with lower combustion temperature. Additionally, the updated model adjusts for the net mass additional along the fuel grain. The net mass flux will substantially increase over the oxidizer mass flux for low oxidizer to-fuel ratios, which will tend to increase the predicted regression rate in the updated model for low oxidizer to fuel ratios.

The effective mass flux exponent for the updated model is also obviously lower than that predicted for the LAVP model, which closely follows a $aG^{0.8}$ curve. This is mostly due to the inclusion of the fuel surface temperature. The fuel surface temperature increases with increasing regression rate, which tends to lower heat transfer to the fuel surface, effectively reducing the mass flux sensitivity below the classic 0.8 predictions. It is noteworthy that empirical fits also tend to exhibit mass flux exponents on the order of 0.65 to 0.77, generally below the classical 0.8 [5, 129].
Fig. 4.15: Data from USU hybrid motor firings and analytical predictions for both LAVP and updated models.
Chapter 5

A Computational Fluid Dynamics Based Regression Model for Complex Fuel Port Geometries

This section develops a regression algorithm that uses many of the models developed in Chapter 4 but applies them to three dimensional fuel ports based upon a computational fluid dynamics (CFD) derived Stanton number correlation. This model is referred to as the computational fluids based Stanton number variable surface temperature (CSVST ) model.

5.1 Theoretical Motivation

Helical fuel ports such as the geometries used in the MUPHyN or other non-typical hybrid motor fuel ports will exhibit flow fields and thus heat transfer distributions that differ substantially from those in the short, cylindrical port hybrids assumed in the basic LAVP regression rate model and the UPVST regression rate model. Unlike simple cylindrical port hybrids, combustion ports with helical structures, mixing devices, bound vortices or any other complex geometric features are generally not going to have an accurate empirical or analytical relationship for either the skin friction or Stanton number. However, even the most complicated hybrid fuel grain configurations generally have subsonic flow fields and geometries that are easily numerically meshed, which makes heat transfer amenable to solution through CFD. Some CFD solutions for hybrid rocket motors including fuel vaporization and combustion have been documented in the literature, but these solutions have been limited to the simplest of axisymmetric or two dimensional cases [129]. Typically the computational fluid dynamics approach to modeling of hybrid rocket motor heat transfer is avoided due to the great deal of difficulty involved in modeling heat-transfer-dependent wall blowing, radiation, multiphase fluid flow, and chemical reactions.

Fortunately, the deviation between the heat transfer mechanism for hybrid motors and
noncombustible surfaces is mostly contained within the boundary layer and does not necessarily influence macroscopic flow field structures. This is assumption is the bases for nearly all hybrid rocket motor regression rate correlations. Indeed, the classical Marxman regression relationship and the LAVP model both use empirical skin friction correlations and then invoke the Reynolds analogy to create a Stanton number model for the fuel port. This Stanton number relationship is then adjusted locally for wall blowing or other phenomena to yield an applicable hybrid rocket regression rate model.

Unfortunately, the complicated flow fields generated by unconventional hybrid fuel grains, as well as not having simple empirical relationships for skin friction, often will not adhere to the Reynolds analogy. Many of these fuel grains will have flow reattachment or impingement points that will have zero net skin friction, but will have substantial heat transfer. This is is a clear deviation from the standard Reynolds analogies. Thus, it is unlikely that a model of only skin friction for a complex grain geometry will yield an accurate heat transfer distribution. However, a Stanton number correlation could be obtained given a mass flux and a temperature differential for a given grain configuration.

Following the success in applying skin friction based correlations in typical hybrid to obtain a Stanton number, it is reasonable that if a relevant Stanton number correlation can be obtained directly from either computational fluid dynamics or complicated empirical correlations these relationships may lead to reasonable regression rate distributions. This procedure does not involve modeling of combustion, flow injection, radiation, or any of the other complicated phenomena in hybrids that makes CFD-based modeling and solutions difficult. These effects can then be modeled using conventional hybrid rocket relationships to modify the heat transfer distribution to correct for hybrid combustion phenomena, yielding a regression rate that accounts for three dimensional geometric effects.

5.2 Solution Methodology

Boundary heat transfer into a fuel port can be modeled as heat transfer out of a hot fuel wall into the cooler core fluid flow. This approach was taken instead of the using a hot core fluid and a cold fuel surface because the temperature generation in a hybrid motor lies very
close to the fuel surface, within the boundary layer. Due to this temperature distribution, the density profile in a hybrid fuel port should closely resemble the density profile created from a hot wall and a cool core fluid. Although the method described in this chapter does not include variable density, this approach was used to enable the examination of density variation on heat transfer in the future.

The local Stanton number for a fuel grain geometry without combustion or wall blowing can be calculated with respect to a nominal mass flux with

$$S_n = \frac{Q_n}{(\rho_c u_c \Delta h)_n}$$  \hspace{1cm} (5.1)

As was shown for the LAVP and the updated model, this relationship works well for hybrids with cylindrical ports. As the skin friction in a configuration will scale inversely with boundary layer growth, it makes sense that the Stanton number will still scale proportionally with $R_{e_x}^{-0.2}$. As length scales will remain fixed between the computational solution and a regression rate model, it is proposed that the Stanton number will scale with viscosity and mass flux according to

$$\frac{S_0}{S_c} = \left(\frac{\rho_c V_c}{\mu_{avg}}\right)^{-0.2} \left(\frac{G_C}{\mu_C}\right)^{-0.2}$$  \hspace{1cm} (5.2)

This correlation can then be adjusted for wall blowing, radiation, viscosity and the height of the flame layer through the same methods previously described for cylindrical port hybrids. A MATLAB function for the CSVST model is included in Appendix C.

### 5.3 Stanton Number Correlations for Flat Plates and Cylindrical Ports

As a simple illustration and “proof of concept” of the method described in this chapter, Stanton number distributions were obtained for flat plate and axisymmetric scenarios and compared to the empirical correlations commonly used for these cases. In this effort, CD-adapco’s StarCCM+\(^1\) was used to solve for heat flux from a heated wall into a turbu-

\(^1\)http://www.cd-adapco.com/products/star_ccm_plus/
Compute fluid properties at the flame and near the wall from chamber pressure

Guess regression rates at all fuel surface nodes

Compute the enthalpy difference between the flame and the fuel surface.

Compute the flame height ratio.

Compute the effective boundary layer viscosity.

Compute the blowing parameter from regression rate guess.

Calculate heat transfer from convection using Stanton number solution.

Calculate heat transfer from radiation.

Update regression rate estimate for longitudinal section.

Update fuel flow rate for downstream mass flux calculations.

If the matrix norm of the regression rate change is below threshold, exit

Compute Stanton number at nodes on fuel surface from computational fluid dynamics solution.

Fig. 5.1: Combined, computationally based regression rate algorithm.
lent, incompressible, constant density fluid using the finite volume method. Although it is understood that the fluid in a hybrid does not have a constant density, a constant density solution significantly simplifies numerical solution and should still generate flow fields with macroscopic flow structures that will heavily influence convective heat transfer.

For both the flat plate and the axisymmetric scenarios, the models used for this solution are listed in Table 5.1 and corresponding parameters are in Table 5.2. The mesh for the flat plate solution has 94,000 cells, clustered mostly near the wall. The mesh near the inlet is shown in Fig. 5.2. The strange pattern of mesh refinement in this model is an artifact of StarCCM+ grid sizing and refinement tools, however, the mesh is much finer near the wall and is generally coarser farther away from the wall. The domain’s height was sized such that the boundary layer had negligible effect on the main stream velocity. The domain for the asymmetric solution, however, corresponds to the initial port length for a typical 98 mm motor firing at Utah State University, which has a length to diameter ratio of about 22. This scenario corresponds to the case where the flow field would be closest to full development and divergence from the flat plate scenario would be most apparent. The mesh near the inlet for the axisymmetric solution is shown in Fig. 5.3.

Solutions for each of these cases were iterated until the norm of the solution residuals had decreased by at least four orders of magnitude. The Stanton number correlations resulting from eq. (5.1) for these solutions is shown in Fig. 5.4. Both solutions lie closely to the flat plate empirical correlation. The agreement of the flat plate and the axisymmetric solutions tends to add support for the often used assumption that flow fields in hybrid motors are more closely modeled by flat plate correlations than fully developed tube models.

In general, the close agreement of the empirical and numerical solutions in Fig. 5.4 illustrates the applicability of the numerical-based regression rate method discussed in this section. Obviously, the three dimensional analog for this method for the MUPHyN fuel grain is a great deal more complicated, but the overall methodology and theoretical backing remains very similar.
Fig. 5.2: Mesh for flat-plate solution.

Fig. 5.3: Mesh for axisymmetric solution.

<table>
<thead>
<tr>
<th>Turbulence Model</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>Segregated Fluid Temperature</td>
</tr>
<tr>
<td>Density</td>
<td>Constant Density</td>
</tr>
</tbody>
</table>
Table 5.2: CFD Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>20 $kg/m^3$</td>
</tr>
<tr>
<td>Viscosity</td>
<td>1.0E$-4 \ Pa \ - \ s$</td>
</tr>
<tr>
<td>Specific Heat</td>
<td>1000 $J/kg - K$</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>0.1$W/m - K$</td>
</tr>
<tr>
<td>Turbulent Prandtl Number</td>
<td>0.9</td>
</tr>
<tr>
<td>Inlet Velocity</td>
<td>100 $m/s$</td>
</tr>
<tr>
<td>Fluid Inlet Temperature</td>
<td>300 $K$</td>
</tr>
<tr>
<td>Length of Domain</td>
<td>0.55 $m$</td>
</tr>
<tr>
<td>Wall Temperature</td>
<td>1000 $K$</td>
</tr>
</tbody>
</table>

Fig. 5.4: Numerical solutions for Stanton number with axisymmetric fuel grains and flat plates shown with with the analytical correlation for flat plates.
5.4 Stanton Number Correlations for the MUPHyN Motor

Many of the MUPHyN fuel grains burned too completely to reasonably compare to analytical regression rates. The fuel grain for MUPHyN HF2, “Boysenberry” was relatively intact compared to the other burned fuel grains. Thus, the geometry for the HF2 configuration was chosen to demonstrate the computational-fluids based regression rate methodology.

The MUPHyN HF2 fuel grain contained two helical combustion ports. To reduce computational demand, only a single port was modeled in this analysis. The MUPHyN motor contains four injectors that spray multiphase nitrous oxide into the pre-combustion chamber. Modeling this injection process would significantly increase the complexity of the computational models. Instead of modeling the multiphase injection, an inlet with a uniform velocity was used at a cross section of the pre-combustion chamber. The computation mesh for this domain is shown in Fig. 5.5. A summary of the parameters used for this solution is listed in Table 5.3. The models used for this solution are the same models listed in Table 5.1.

It is noteworthy that in a typical hybrid motor, the flame temperature resides near the fuel surface at a near constant position in the boundary layer. This results in a relatively constant temperature difference between the fuel surface and the flame zone. This analysis used a constant temperature wall and a set inlet fluid temperature, so long flow fields will start to see a decrease in temperature difference between the wall and the core flow near the end of the port. To limit this effect, Stanton number correlations based on numerical models can be completed piecewise. This analysis examined the Stanton number in the helical flow port, so both the pre-combustion chamber and the post combustion chamber were modeled with adiabatic walls.

5.5 Regression Rate Results for the MUPHyN Motor

The numerically generated Stanton number correlation for the fuel port used in test fire HF2 (“Boysenberry”) was used to calculate regression rates on the fuel port surface assuming a chamber pressure of 1 MPa. An iterative method was used to simultaneously calculate regression rate and the fuel surface temperature based upon the models discussed
Table 5.3: MUPHyN Port CFD Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>20 kg/m$^3$</td>
</tr>
<tr>
<td>Viscosity</td>
<td>1.0E−4 Pa − s</td>
</tr>
<tr>
<td>Specific Heat</td>
<td>1000 J/kg − K</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>0.1W/m − K</td>
</tr>
<tr>
<td>Turbulent Prandtl Number</td>
<td>0.9</td>
</tr>
<tr>
<td>Reference Mass Flux</td>
<td>185kg/m$^2$ − s</td>
</tr>
<tr>
<td>Fluid Inlet Temperature</td>
<td>300 K</td>
</tr>
<tr>
<td>Wall Temperature</td>
<td>2000 K</td>
</tr>
</tbody>
</table>

Fig. 5.5: Computational grid for MUPHyN fuel grain “Boysenberry.”
in Chapter 4. The mass flux increase along the fuel port due to fuel addition was neglected for this analysis. The calculated regression rates for the fuel ports are shown in Figs. 5.6 through 5.9.

It is difficult to compare the absolute magnitude of the regression rates calculated numerical to experimental values from post-fire fuel grains due to fuel grain burn-throughs and the general complexity of the fuel grains ports. It is noteworthy that the maximum predicted regression rates for this configuration are about 6 mm/s, which is about six times higher than the average regression rates for the 98mm motor firings. Likewise, the minimum regression rates predicted are about half of the average seen in the 98mm motor firings. Given the complexity of the MUPHyN motor fuel grains, these values are not unreasonable but cannot really be used to validate or discredit the method. For comparison, the empirical skin-friction based methods presented in Chapter 3 and Chapter 4 predict regression rates near 1.0 mm/s.

Although the global regression values are difficult to validate, the locations of “hot spots” and “cold spots” corresponding to high and low regress rates be directly compared
Fig. 5.7: Predicted regression rates in MUPHyN Motor HF2 port, bottom view.

Fig. 5.8: Predicted regression rates in MUPHyN Motor HF2 port, inner view.
to channels and protrusion in the post-burn fuel grain. These hot and cold spots are shown in Fig. 5.6 and Fig. 5.8. Indeed, the flow features show remarkable correlation to fuel surface features in the HF2 fuel grain. The post burn HF2 fuel grain is shown in Fig. 5.10. There are two distinct hot spots on the “top” of the fuel grain port, “Hot Spot 1” and “Hot Spot 2.” These spots correlate to reattachment points after the rearward facing step at the entrance to the fuel grain and correspond to burned-away sections in the post-burn fuel grain.

There is also a hot spot near the entrance of the fuel grain port, “Hot Spot 3” where flow accelerates around the sharp lower edge of the helical fuel port. This location corresponds to a channel in the fuel grain wall in the post-burn fuel grain. The bottom of the numerical solution has a cool ridge “Cold Spot 1” corresponding to a separation region as flow accelerates round the helical flow channel. This corresponds to an obvious ridge in the post-burn fuel grain also.
5.6 Numerical Regression Rate Model Conclusions

The computation fluids approach to solving for regression rates for complex port geometries shows a great deal of promise and could certainly be used in the design of future fuel grains. This problem, however, is far from fully solved. Additional validation and verification would be required before a great deal of trust could be put into this method.

Ideally, this method would also be extended to predict the regression rate in the motor as a function of time. The regression rates could be used to compute a new fuel port surface, and CFD could be used to generate a new Stanton number distribution. This process could be iterated over time to produce time histories of hybrid combustion for complex fuel grains.
Chapter 6

Cold Flow Experimental Characterization of Aerodynamic Thrust Vectoring for an Annular Aerospike Nozzle

6.1 Introduction

The incorporation of attitude control with aerospike thrust vectoring into a small satellite propulsion system has the potential to increase overall system performance by decreasing the required propellant for attitude control. Although the mission trade study to examine the trade-offs involved with thrust-vectoring attitude control was not within the scope of this project, experimental values for the thrust vectoring ineffective on aerospike nozzles that support this form of a study were a focus of investigation. This chapter presents an experimental study on aerodynamic thrust vectoring on an aerospike nozzle in cold flow conditions. This research was also published in the Journal of Propulsion and Power [144].

6.2 Experimental Apparatus, Setup and Test Procedure

A series of cold-flow tests were performed to examine the viability of fluidic thrust vectoring by gas injection on a truncated annular aerospike nozzle in near-optimally expanded conditions. Although the final aerospike nozzle was slightly over expanded at the test conditions, it was not sufficiently over expanded to change the near-surface flow field. As the pressure and velocity distribution near the secondary injection orifice are nearly identical to the flow field experienced by an under-expanded or optimally expanded nozzle, the thrust vectoring test results are applicable to high altitude or in-space conditions. Research focused on the effects of injection port location on vectoring effectiveness and side force fidelity. Side force dependence on nozzle pressure ratio was also investigated.
6.2.1 Test Stand Description

All aerospike static tests were performed in the Engineering Technology Department’s jet engine test cell on the Utah State University (USU) campus. For static thrust tests, commercially available test stands were examined and found to be excessively expensive and have structural support mechanisms that were unsuitable for mounting the aerospike prototype. Consequently, a custom-made, portable, test stand was designed and built to support the needs of the aerospike project.

The test stand features a six-degree-of-freedom load balance with type S load cells configured as shown in Fig. 6.1. Three 100 lbf-range (445 newtons) axial and three 25 lbf-range (111 newtons) lateral load cells are arranged such that six-degree-of-freedom force and moment measurements can be resolved. The thrust stand is designed so that the nozzle exhaust plume exits vertically, and the thrust acts downward onto the test cart. The thrust stand coordinate system, also pictured in Fig. 6.1, is defined with x-axis aligned vertically upward along the axial centerline of the nozzle. The test stand was calibrated in-situ with a simultaneously multi-axial calibration method. The total resultant uncertainty (to 95% confidence) for forces using this calibration method was statistically determined as approximately 0.25 newtons for side forces and 1.75 newtons for axial loading with nominal values of 15 newtons and approximately 400 newtons, respectively [145].

For ease of storage and handling, carbon dioxide was chosen for a working fluid. Figure 6.2 presents a plumbing and instrumentation diagram of the associated cold-gas feed system. Saturated liquid carbon dioxide is stored in standard K-sized storage tanks, with each tank having a storage capacity of approximately 25 kg. Multiple tanks were manifolded to assure that the required mass flow levels and run times can be achieved. Flow out of the tanks is controlled via a pneumatic ball valve. The pneumatic valve actuator is controlled with a 12-volt direct current solenoid valve. Beyond the ball valve, carbon dioxide flows through a manually set needle valve that drops the pressure from the saturation pressure of carbon dioxide, 4825-5515 kPa (700-800 psia) at room temperature, to approximately 1035 kPa (150 psi). Carbon dioxide then flows into a water-bath heat exchanger which raises
the temperature of the expanded carbon dioxide by approximately 25 C. The pressure downstream of the needle valve is controlled using a back-flow pressure regulator and a primary regulator in parallel. The needle valve and the back-flow regulator maintains approximately 1034 kPa (150 psi) upstream of the primary regulator. The primary flow regulator further drops the feed pressure to approximately 690 kPa (100 psi) at the plenum inlet.

At full pressure, the primary regulator is set to allow approximately one kilogram per second mass flow through the aerospike nozzle throat. The back-pressure regulator will vent approximately half that flow rate at startup. As the tanks evacuate and the overall system pressure drops, flow through the back-pressure regulator diminishes to zero. An additional electronic regulator in parallel with the main flow regulator controls the upstream pressure of the secondary (thrust vectoring and base-bleed) flow injection ports.

Type K thermocouples and pressure transducers are used to monitor temperatures and pressures throughout the flow system. A custom manufactured Venturi flow meter, also using pressure transducers to measure the pressure differential, is situated upstream of the electronic regulator. Although a differential pressure transducer was not used, the pressure transducer voltage bias is removed at full operating pressure when the secondary flow injection is turned fully off. This results in a differential pressure measurement accurate to within about 0.1 percent of differential reading. The Venturi was calibrated in-situ using high flow coefficient sonic orifices. In this manner, the flow coefficient for the Venturi was calculated to be 0.980 which is very near the expected result for Venturi flow meters of this design. The typical mass flow rate uncertainty with this Venturi was about 1.0 percent of the measurement.

6.2.2 Test Article Description

Due to manufacturing considerations (high expansion ratios yield larger aerospike nozzles which are easier to manufacture), the aerospike nozzle used for cold flow experimentation is sized to be slightly over expanded for operating conditions at the test altitude, 1450 m (about 86.2 kPa), in Logan, Utah. The resulting expansion ratio is 2.47. It is desirable
Fig. 6.1: Six degree of freedom test stand.

Fig. 6.2: Aerospike propellant feed system.
Table 6.1: Cold-Flow Aerospike Parameters

<table>
<thead>
<tr>
<th>Aerospike Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plug Diameter</td>
<td>3.2 cm</td>
</tr>
<tr>
<td>Outer Throat Diameter</td>
<td>3.86 cm</td>
</tr>
<tr>
<td>Truncated Length</td>
<td>2.54 cm</td>
</tr>
<tr>
<td>Full Isentropic Spike Length</td>
<td>4.31 cm</td>
</tr>
<tr>
<td>Truncation Ratio</td>
<td>57%</td>
</tr>
<tr>
<td>Throat Diameter</td>
<td>0.29 cm</td>
</tr>
<tr>
<td>Operating Stagnation Pressure</td>
<td>775 kPa</td>
</tr>
<tr>
<td>Nozzle Expansion Ratio</td>
<td>2.47</td>
</tr>
<tr>
<td>Plenum Exit Throat Area</td>
<td>4.73 cm³</td>
</tr>
<tr>
<td>Secondary Injection Port Diameter</td>
<td>0.3175 cm</td>
</tr>
<tr>
<td>Design Altitude</td>
<td>4206 m MSL</td>
</tr>
<tr>
<td>Design Thrust</td>
<td>454 N</td>
</tr>
<tr>
<td>Design Mass Flow Rate</td>
<td>1.0 kg/s</td>
</tr>
</tbody>
</table>

to keep the near surface flow field close to what would be experienced in in-space or under expanded conditions. To accomplish this, the aerospike was designed using a method of characteristics code to verify that compression waves generated by over expansion would not intersect the end of the truncated spike at full chamber pressure. The resulting plug was truncated such that the test article was 57% of the length of a full spike. This design results in an aerospike pressure distribution roughly independent of atmospheric pressure except for the base area and the very end of the spike length. This effect was confirmed through the use of computational fluid dynamics. The salient aerospike characteristics are shown in Table 6.1 and the aerospike and plenum geometries are shown in Fig. 6.3 and Fig. 6.4.

6.3 Experimental Results

Aerospike nozzle configurations with secondary injection ports located at 20%, 80% and 90% axial position along the truncated spike were tested with secondary mass flow rates between 0.005 kg/s and 0.016 kg/s and primary mass flow rates between 0.70 and 0.95 kg/s. These flow rates correspond to secondary flow inlet pressures between approximately 400 and 800 kPa and primary plenum pressures between about 350 and 600 kPa. The secondary injection orifices were machined such that they injected fluid normal to the aerospike’s
Fig. 6.3: Cold flow aerospike test configuration.

Fig. 6.4: Cold flow aerospike profile.
Fig. 6.5: Side force and secondary injection pressure for 90% injection point.

longitudinal axis. These port locations are shown in Fig. 6.3. Lateral force, secondary injection pressure, mass flow rate and temperatures for two typical tests are shown in Fig. 6.5. The side force, specific impulse, and secondary flow pressure for 90% secondary injection location for both main flow on and main flow off is shown in Fig. 6.6. The response fidelity between the electronic regulator control and the output size force is clearly shown in both of these figures. A high degree of repeatability and crisp thrust vectoring response was typical of the entire test series.

The resulting side force amplification factor and specific impulse for each configuration is shown in Table 6.2 and Fig. 6.7. An additional configuration with a larger diameter injection orifice and at approximately 90% the length of the truncated spike was also tested to examine side force scaling. These results are shown in Fig. 6.8.

The use of carbon dioxide as a operating fluid allowed for excellent test flow visualization when the fluid crystallizes near the end of the aerospike contour. This phase change
Fig. 6.6: Side force and secondary injection for 90% injection point for both primary flow on and secondary flow only configurations.

Table 6.2: Cold Flow Test Specific Impulse Results.

<table>
<thead>
<tr>
<th>Test Series</th>
<th>$I_{sp}$ (s)</th>
<th>$I_{sp}$ Uncertainty (s, 95%)</th>
<th>Amplification Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injection Location at 90%</td>
<td>54.8</td>
<td>± 1.9</td>
<td>1.39</td>
</tr>
<tr>
<td>Injection Location at 80%</td>
<td>47.0</td>
<td>± 1.9</td>
<td>1.19</td>
</tr>
<tr>
<td>Injection Location at 20%</td>
<td>21.2</td>
<td>± 1.7</td>
<td>0.54</td>
</tr>
<tr>
<td>Secondary Flow Only</td>
<td>39.5</td>
<td>± 1.8</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 6.7: Cold flow secondary injection results and regressed specific impulses for various hole locations.

Fig. 6.8: Cold flow secondary injection with increased secondary orifice diameter results.
creates a semi-opaque cloud that is readily visible. The temperature increase caused by shock waves resulting from secondary fluid injection create clear areas in the flow field that are easily distinguishable. The leading edge bow shock caused by fluid injection for a high flow rate test is clearly seen in Fig. 6.9.

During the cold flow test series, the ratio of the chamber pressure to ambient pressure was varied from approximately 5.0 to 8.0. No meaningful correlation between side force specific impulse and chamber pressure was observed over this range. The side force $I_{sp}$ for this range with the 90% injection orifice location is shown in Fig. 6.10. Near the upper part of this range, the nozzle surface pressure is effectively independent of ambient pressure. At lower pressure ratios, aerospike altitude compensation influences local ambient Mach number and density around the secondary flow orifice. It is probable that the variation of these two parameters have counterbalancing influences on the side force specific impulse over the range of pressure ratios examined during cold flow testing.

It is notable that the secondary injectant does not reach sonic velocity at the immediate exit of the injection orifice. The bow shock caused by primary flow results in an effectively reduced orifice exit area for the injectant immediately downstream of the orifice. This reduced area results in a drop in discharge coefficient of approximately five percent between tests with secondary injection only and secondary injection with active primary flow.

For aerospike configurations with the secondary injection point near the end of the aerospike, the effect of secondary fluid injection on axial thrust was small enough such that it was not detectable by the current testing apparatus. The maximum side force for the larger orifice is approximately 14 N. For these tests, the average primary thrust level is 343 N. This results in a total thrust vector deflection of about 2.3 degrees or a side force level of 4.1%. Resultant cosine losses for this thrust angle are therefore less than 0.1 percent so it is not surprising that no net effect on axial thrust is detected at these side force levels. Similarly, maximum side force levels for the smaller, more efficient orifice size were about 8 N. This yields a net side force level of about 2.3% or about 1.3 degrees. Cosine losses for this configuration would be on the order of 0.03%.
Fig. 6.9: Aerospike cold flow test with 4.4 mm diameter orifice located at 90% of the length of the truncated aerospike. a) Thrust vectoring on, showing clear bow shock. b) Thrust vectoring off.

Fig. 6.10: Cold flow specific impulse vs. nozzle pressure ratio.
6.4 Effect of longitudinal injection location

The test series showed a marked dependence of side force amplification factor on longitudinal hole location. Over the range of locations examined in this test series, the optimal injection location was at the aft edge of the truncated aerospike length. This is clearly shown in Fig. 6.11. This trend is in direct contradiction to side force relations historically obtained on conical nozzles. For lab scale tests on conical nozzles, the optimum injection point for gas injection has been found to be nearest the throat where the resulting bow shock does not impinge on the opposite nozzle wall [114]. Two explanations for this optimal port location are diminishing the effect of the low pressure, over-expanded region directly downstream of the injection location and the effect of local primary flow Mach number at the injection location.

The mechanism for side force generation by secondary injection on an aerospike nozzle differs significantly from those generated by side injection on a conventional nozzle. Figure 6.12 compares the side-injection flow patterns on conventional and aerospike nozzles. In both cases the injected flow produces a strong shock wave and a significant pro-verse pressure.
increase behind the shock wave. In both cases there is also a low pressure region caused by over expansion of the secondary injectant into the primary flow field. These flow regions have been amply studied during secondary injection experiments completed on flat plates [117,146]. In a conventional nozzle, the flow aft of this injection point follows a concave path away from the centerline. This limits the deterioration of the high pressure region caused by the shock wave as the entire shock wave is “captured” by the nozzle geometry. Thus, the effects of the leading shock wave and the low pressure region due to over expansion tend to cancel out in a bell nozzle [119].

On an aerospike nozzle, however, the flow behind the injection point follows a convex path. Directly downstream of the injection site there occurs a predicted drop in Mach number and a corresponding pressure increase. Because of the convex aerospike nozzle shape, the secondary injection disturbance propagates across the entire upper spike surface downstream of the injection site, and the convex surface contour results in a flow expansion on the injectant side of the nozzle. When the injection occurs on the upstream portions of the nozzle, the resulting expansion region offsets any gain achieved by the high-pressure compression behind the shock wave. The net result is a total side force that is less than what would be produced by the injected pulse alone.

When the injection location is near the end of the aerospike, the effect of the low pressure region is diminished which results in large efficiency gains. As the aerospike flow is sufficiently expanded such that the base of the aerospike “feels” atmospheric pressure, the secondary injection entrains flow from the base region without causing a significant reduction in base pressure. This drives a complex flow field of counter rotating vortexes that “drag” fluid from the separated base towards the secondary injection jet. This fluid flow keeps the secondary flow from significantly over expanding after the secondary injection orifice, increasing the thrust vectoring efficiency. As the aerospike is operating in “open wake” conditions where the aerospike base adjusts to ambient pressure, this does not adversely impact thrust in the longitudinal direction. Future computational and/or experimental results will be required to examine if thruster efficiency in the primary direction is at all
compromised in low ambient pressure or “closed wake” conditions when the aerospike base is independent of ambient pressure.

### 6.5 Cold Flow Test Conclusions

Fluidic thrust vectoring on a truncated aerospike nozzle was performed with carbon dioxide as a working fluid. A strong dependence of thrust vectoring efficiency on longitudinal orifice location was discovered. In order to enhance the thrust vectoring effectiveness of side-force injection on a three-dimensional aerospike nozzle, the injection site must be moved aft so that flow over-expansion does not occur on the surface of the physical spike surface. This assertion is in direct contrast to what was previously known about side-injection on conventional nozzles. Data were collected for configurations with side injection port locations at 20%, 80%, and 90% of the nozzle length; and significant force amplification factors were observed for orifices near the end of the nozzle. The side-force specific impulse at the
90% port location is enhanced by nearly 40%. The enhanced side force $I_{sp}$ means that the same control impulse can be achieved for significantly less propellant than would be used by a stand-alone reaction control thruster.

Although the amplification factors generated for the cold flow aerospike in this test are somewhat lower than for conical nozzles, it should be noted at all of the conical nozzle test series described above involved a much higher primary flow pressure ratio than those examined in the cold flow aerospike tests for this test series. Additionally, the high end amplification factors generated for conical nozzles generally corresponded to very small secondary orifice diameters. It is expected that variation of orifice size on an aerospike nozzle would likewise show a maximum at some orifice diameter.

Due to the ability to use thrust vectoring ports on an aerospike nozzle for small impulse attitude control maneuvers without primary flow active, it also provides the possibility to replace conventional reaction control thrusters.

The primary gain from thrust vectoring on an aerospike nozzle is the ability to use secondary injection jets as stand alone reaction control without use of the primary engine. When the primary thruster is fired then the additional benefits of flow amplification are gained. A jet internal to a conventional nozzle would obviously not share this same operational advantage. This, coupled with the volumetric efficiency gains of aerospike nozzles makes aerospike nozzles with thrust vectoring a strong option for small satellite missions.
Chapter 7

Development of Heat Transfer Models for Nitrous Oxide

Regenerative Cooling on an Aerospike Nozzle

7.1 Introduction

The MUPHyN motor uses an aerospike nozzle to take advantage of the short-form factor benefits provided by aerospike nozzles and the utility provided by aerospike nozzle thrust vectoring. However, the implementation of an aerospike nozzle in a hot-gas environment brings along a significant technological challenge: the management of the high thermal load imparted to the nozzle surface by the high-velocity, high-temperature combustion products. Aerospike nozzles with high expansion ratios have a far larger throat surface area than a bell or conical nozzle with the same throat exit area and imparted heat loads are significantly higher. A tolerable amount of erosion in a conical nozzle can translate into a very large increase in throat area on an equivalent expansion ratio annular aerospike nozzle due to the very thin throat annulus. Additionally, the aerospike “plug” is nearly completely surrounded by hot gas so there is not a strong mechanism for passive radiative or conductive heat transfer out of the system. Thus, ablative cooling and radiative cooling solutions commonly used for conventional nozzles are seldom applicable to aerospike nozzles. Fortunately the compact design of the MUPyN thruster allows for relatively straight-forward application of regenerative cooling using the oxidizer flow.

The MUPHyN coolant system design is inspired by research performed by Lemieux at California Polytechnic State University where nitrous oxide was used to cool a copper throated conical nozzle [70, 71] and was later designed to cool an aerospike nozzle in a traditionally long form hybrid motor [72]. The authors found that saturated nitrous oxide – when care is taken not to allow the liquid phase to fully boil off – is an effective regenerative
coolant. If the liquid phase is allowed to fully boil off, heat transfer to the coolant reduces significantly. If heat transfer is high enough, the resulting vapor could reach temperatures that would support exothermic decomposition, an event that could produce catastrophic failure of the aerospike nozzle.

The MUPHyN motor shape, with its compact longitudinal form factor, allows oxidizer to be passed through coolant channels near the throat and then down back down to an injector near the bottom of the combustion chamber with no external plumbing. Figure 7.1 shows the cooling channels on the MUPHyN nozzle.

### 7.2 Analysis of the Convective Heat Transfer from the Combustion Flame Zone to the Aerospike Nozzle

Convective heat transfer from the nozzle flow field to the nozzle surface in traditional deLaval rocket nozzles is generally predicted with correlations derived for fully developed pipe flow [5]. Convective heat transfer in an aerospike nozzle is non-fully developed and the axisymmetric model developed by Mayer [147] for external expansion, spike, and other novel rocket nozzle configurations is more applicable. Instead of a hydraulic diameter based correlation, the model created by Mayer uses a thermal Reynolds number of the form

$$\text{Re}_\Gamma = \frac{\rho_\infty U_\infty \Gamma}{\mu_\infty}$$

(7.1)
where $\Gamma$ is the thermal boundary layer thickness. Assuming a control volume inside this thermal boundary layer, an energy balance yields

$$q_w = \mu_\infty \bar{c}_p T_{0,\infty} \frac{d}{ds} \text{Re}_\Gamma$$

(7.2)

A modified form of the Reynold’s analogy correlation,

$$St_\infty = B Pr \text{Re}_{x,\infty}^{-b}$$

(7.3)

is applied to relate the thermal Reynolds number to the traditional fluid-dynamic Reynolds number,

$$\text{Re}_{x,\infty} = \int_0^s \frac{\rho_\infty U_\infty}{\mu_\infty} ds$$

(7.4)

In Eqs. (7.3) and (7.4) $St_\infty$ is the Stanton number, $B$ and $b$ are empirical constants, and Pr is the Prandtl number of the core fluid flow. The heat transfer coefficient $h$, is expressed in terms of the integral,

$$h_2(s) = \rho_\infty \bar{c}_p U_\infty St = \frac{\beta \frac{1}{(1-b)} B Pr^{-\frac{2}{3}} \rho_\infty \bar{c}_p U_\infty}{\left[ \int_0^s \beta \frac{1}{(1-b)} \rho_\infty \bar{c}_p U_\infty \mu_\infty^{-1} ds \right]^b}$$

(7.5)

where the parameters in Eq. (7.5) are defined as

$$\beta = \left( \frac{T_{\text{surf}}}{T_\ast} \right)^{(1-b(1+\omega))}$$

$$B = 0.0296$$

$$b = \frac{1}{5}$$

$$\omega = \frac{3}{2}$$

(7.6)

Equation (7.5) is corrected for annular surfaces to yield the relationship

$$h_3(s) = h_2(s) \left[ \frac{T_\ast^{\frac{1}{5}} \int_0^s \beta^{\frac{1}{1-b}} \rho_\infty \bar{c}_p U_\infty \mu_\infty^{-1} ds}{\int_0^s \left( \beta r \right)^{\frac{1}{1-b}} \rho_\infty \bar{c}_p U_\infty \mu_\infty^{-1} } \right]^b$$

(7.7)
Table 7.1 lists combustion and nozzle parameters used to calculate fluid properties for this model. The combustion products were computed with the NASA code Chemical Equilibrium Analysis with Applications [50, 51].

For this analysis, a uniform aerospike surface temperature of 400 K was assumed. Although the actual surface temperature will vary with both heat transfer rate and location, this surface variation should be small compared to the difference between the surface temperature and the far higher combustion gas flame temperature. This effect is expected to be much lower than the general accuracy of this model and is thus neglected. The local mean cross section combustion gas temperature ($T(s)$), pressure ($P(s)$), and sonic velocity ($U(s)$) were calculated using local isentropic flow relationships,

\[
T(s) = \frac{T_0}{(1 + \frac{1}{2}(\gamma - 1)M(s)^2)^{\frac{\gamma}{\gamma - 1}}}
\]

\[
P(s) = \frac{P_0}{(1 + \frac{1}{2}(\gamma - 1)M(s)^2)^{\frac{\gamma - 1}{\gamma - 1}}}
\]

\[
U(s) = M(s) \sqrt{\gamma R_g T(s)}
\]

The convective heat transfer to the nozzle was calculated by breaking the surface into a series of local nodes. A 0.75 cm convergent section was chosen to model boundary layer growth before the throat. Cosine clustering towards the throat was used to place nodes along the convergent section and the nodes created by a method of characteristics solver were used for the divergent section. Conical frustum areas between nodes and trapezoidal integration were used for surface integration of total heat transfer rates. Because of the significantly lower surface heating rates, the base region was not included in this analysis. Figure 7.2 plots the resulting convective heat transfer coefficients and area specific heat transfer rates. The resultant total heat transfer computed via this method is about 3500 watts.

As noted previously, the 2.25:1 expansion ratio on the MUPHyN prototype was significantly lower than would be desirable for a space thruster. Assuming a fixed throat area (and thus exit mass flow) for an aerospike nozzle, the exposed surface area increases proportionately with nozzle expansion ratio. A high expansion ratio nozzle will experience a
Table 7.1: MUPHyN Motor Combustion and Nozzle Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer Throat Radius</td>
<td>1.2 cm</td>
</tr>
<tr>
<td>Chamber Pressure</td>
<td>775.6 kPa</td>
</tr>
<tr>
<td>Specific Heat Ratio</td>
<td>1.27</td>
</tr>
<tr>
<td>Molecular Weight</td>
<td>24.247</td>
</tr>
<tr>
<td>Expansion Ratio</td>
<td>2.25</td>
</tr>
<tr>
<td>Viscosity</td>
<td>0.844 mP</td>
</tr>
<tr>
<td>Chamber Temperature</td>
<td>3046 K</td>
</tr>
<tr>
<td>Viscosity Temperature Exponent</td>
<td>1.5</td>
</tr>
<tr>
<td>Convergent Surface Length</td>
<td>0.75 cm</td>
</tr>
<tr>
<td>Aerospike Surface Temperature</td>
<td>400 K</td>
</tr>
</tbody>
</table>

Fig. 7.2: Heat transfer coefficient and heat transfer rate for aerospike surface.
significantly higher convective heating load than will a low expansion ratio nozzle even though mass flow, and thus the oxidizer coolant flow, remains the same. Thus, the low expansion ratio of the MUPHyN prototype was selected to allow a significant heating margin of safety for the preliminary rounds of testing. Once the precise convective heating levels are better understood, future MUPHyN development tests could scale up the expansion ratio into a region more desirable for vacuum operation. Additionally, some combination of high temperature insulative material could be used in conjunction with regenerative cooling to extend the available operating envelope to higher expansion ratios.

7.3 Analysis of the Regenerative Cooling Nitrous Oxide Heat Transfer Rate

The coolant side heat transfer can be modeled with relations originally developed for boiling in smooth circular tubes [148, 149]. Although, as can be clearly seen in Fig. 7.1, the coolant channels in the MUPHyN are not circular tubes, the flow in the impinging jet channels with fins should facilitate even higher heat transfer. Thus, it is believed that this will yield a conservative estimate.

Nitrous oxide is expanded through an orifice before reaching the cooling channels. This expansion drops the fluid pressure below the initial saturation pressure. This results in multiphase heat transfer. Because the phase change removes significantly more heat than convection to liquid flow alone, the multiphase heat transfer is expressed as in terms of a ratio relative to heat transfer fore pure liquid, and empirical fits exist in the literature for these ratios based upon fluid quality and other parameters,

\[
\frac{h}{h_l} = 0.6683 \left( \frac{\rho_l}{\rho_v} \right)^{0.1} X^{0.16} (1 - X)^{0.64} f(Fr) + 1058 \left( \frac{q'' \dot{m}'' h_{fg}}{\rho_v} \right)^{0.7} (1 - X)^{0.8} G_{s,f} \tag{7.9}
\]

\[
\frac{h}{h_l} = 1.136 \left( \frac{\rho_l}{\rho_v} \right)^{0.45} X^{0.72} (1 - X)^{0.08} f(Fr) + 667.2 \left( \frac{q'' \dot{m}'' h_{fg}}{\rho_v} \right)^{0.7} (1 - X)^{0.8} G_{s,f} \tag{7.10}
\]
Table 7.2: Boiling Heat Transfer Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>specific heat transfer rate, (q'')</td>
<td>7430 kW/sq meter</td>
</tr>
<tr>
<td>total heat transfer rate</td>
<td>3500 W</td>
</tr>
<tr>
<td>mass flow rate</td>
<td>0.08 kg/s (total)</td>
</tr>
</tbody>
</table>

In Eq. (7.9), the term \(G\) is a constant related to the specific materials and coolants used, but generally ranges around 1.0. The stratification parameter, \(f (Fr)\), was assumed to be unity also, as it is doubtful that the coolant will have time to experience buoyancy effects over the extremely short coolant channel length. Equation 7.9 models heat transfer for convective dominated flow regimes and Eq. (7.10) models heat transfer in nucleate boiling regimes. The larger of the two values determines which effect dominates and, therefore, which relationship should be used. Table 7.2 lists the other relevant parameters used in this calculation. For this analysis, the total heat transfer rate was rounded up from the hot gas side heat transfer calculated above. Fluid specific properties were computed using Helmholtz relations for real fluids [135–137].

State properties for nitrous oxide at different coolant pressures were calculated assuming isenthalpic expansion across the orifice before the coolant channels. Any heat transfer to the fluid was assumed to happen after this initial expansion. Depending on the coolant pressure, the ratio of multiphase heat transfer to liquid only heat transfer predicted by Eq. (7.9) ranges between 10 and 20 for this configuration.

To complete the heat transfer model, a heat transfer relationship for a pure liquid phase is then required. The liquid heat transfer coefficient is modeled by [5,149]

\[
h_l = 0.023c_p \frac{\dot{m}}{A} \left( \frac{DVU \rho_l}{\mu_l} \right)^{-0.2} \left( \frac{\mu_l C_p}{\kappa_l} \right)^{-\frac{2}{3}}
\]

(7.11)

Figure 7.3 show the heat transfer coefficients computed using this method for a range of coolant pressures along with the average fluid quality in the coolant channels. Fig. 7.4 shows the predicted nozzle surface temperatures along with the nitrous oxide temperatures.
Fig. 7.3: Coolant side heat transfer coefficient and average coolant quality.

Fig. 7.4: Coolant side aerospike surface and coolant temperature for heat transfer of 3500 W.
7.4 Non-Homogeneous, Non-Equilibrium Two Phase Mass Flow Model

Due to regenerative cooling requirements and the phase change of nitrous oxide in the cooling channels due to heat addition, the injector orifice sizing for the MUPHyN motor is a great deal more complicated than traditional injector sizing in hybrid motors. The pressure, and therefore the quality and heat transfer coefficient of the nitrous oxide coolant in the cooling channels depends on the size ratio between the expansion orifice upstream of the cooling channels and the injector which sprays oxidizer into the motor downstream of the coolant channels.

A modified version of the non-homogeneous, non-equilibrium (NHNE) model developed by Dyer, et al at Stanford University was used for injector size calculation [150]. This model uses a weighted average of the homogeneous equilibrium (HEM) mass flux,

\[ G_{HEM} = \frac{\dot{m}}{A} = \rho_2 \sqrt{2 (h_1 - h_2)} \tag{7.12} \]

and the incompressible mass flux (SPI),

\[ G_{SPI} = \frac{\dot{m}}{A} = \sqrt{2 \rho_1 (P_1 - P_2)} \tag{7.13} \]

to compute a single mass flux using a weighted “non equilibrium parameter” \( k \),

\[ k = \frac{\tau_b}{\tau_r} = \sqrt{\frac{P_1 - P_2}{P_v - P_2}} \tag{7.14} \]

The two-phase mass flux is calculated as a weighted average of the incompressible and HEM mass fluxes,

\[ G_{NHNE} = C_d \left( \frac{1}{1 + k} G_{HEM} + \left( 1 - \frac{1}{1 + k} \right) G_{SPI} \right) \tag{7.15} \]

In these relations the subscript 1 represents the conditions at the orifice inlet, and the subscript 2 represents the conditions at the outlet. This same relationship, with different

\[ \text{It should be noted that in the paper cited above the } G_{HEM} \text{ and } G_{SPI} \text{ terms are reversed in eq (7.15) and would lead to the mass flux tending towards } G_{HEM} \text{ when the vapor pressure is very low, which is incorrect.} \]
pressure drops and initial qualities, applies to both the expansion orifice positioned before
the coolant channels and the injector orifice that sprays into the combustion chamber.

The parameter $k$ is the inverse square root of the cavitation number and expresses the
ratio of the difference between the upstream total pressure and the downstream pressure,
and the vapor pressure and the downstream pressure. Small values for $k$ demonstrate a
high degree of cavitation in the flow and an increase in fluid quality in the injector orifice.
When $k$ is large, the incompressible SPI model is weighted heavily. When $k$ is small, the
two-phase HEM model is weighted heavily. The combined model of Eq. (7.15) allows for
two-phase flow effects that plateau the mass flux as the downstream pressure is lowered.
This is consistent with observed two-phase mass flow properties.

The model proposed by Dyer was further extrapolated to incorporate choking mass
flow. For very small exit pressures, the mass fluxes predicted by the NHNE model decrease
with decreasing exit pressure, a trend unlikely to exist in reality. Thus, a model was used
that uses the maximum flow rate predicted by NHNE model for any downstream pressure
between the upstream pressure and the exit pressure. Figure 7.5 shows mass fluxes predicted
by the SPI model, the HEM, the NHNE model, and the choked NHEM model (CNHNE) for
nitrous that is slightly sub-cooled upstream of the injector. It is noteworthy that the SPI
model and HEM are identical if the downstream fluid is still sub-cooled and the CHNHE
and HNHE model are identical above about 1 MPa.

### 7.5 Injector and Expansion Orifice Size Calculation

As shown in Fig. 7.6, the nitrous oxide flow through the cooling channels can be broken
down into four fluid states. Nitrous oxide enters the MUPHyN motor regenerative cooling
paths in liquid form at slightly above saturation pressure. As the fluid enters the cooling
channels, it encounters a constrictive orifice that quasi-adiabatically expands the flow to a
significantly lower pressure. Between states 2 and 3, external energy is added through heat
transfer from the external combustor flow, and finally at the injector (state 4) resulting
multiphase fluid adiabatically expands to the combustor chamber pressure.

In order to maintain the desired coolant pressure and mass flow rates, the orifice before
the coolant channels as well as the injector orifice must be correctly sized. Clearly, increasing the pressure drop across the initial orifice decreases the pressure and therefore the fluid temperature in the coolant channels, which aids in heat transfer. However, reducing the coolant pressure increases the fluid vapor-to-liquid ratio (quality) of the fluid in the coolant channels. This increase in fluid quality significantly decreases the overall heat transfer coefficient. If the heat transfer coefficient were the only parameter of interest it would therefore be desirable to maximize the heat transfer coefficient by minimizing the coolant quality.

However, heat transfer into the fluid along the regenerative cooling channels can significantly influence the exit fluid state properties (including density) which changes the pressure drop across the injector, and will significantly affect the mass flow rate into the motor. Thus, it is desirable to have a large pressure drop before the coolant channels to help minimize mass flow variation during motor startup. The flow rate across an expansions orifice upstream of the coolant channels will not significantly vary with heat transfer into the coolant. The flow rate across the injector orifice downstream of the coolant channels is

Fig. 7.5: Mass fluxes predicted by the SPI model, the HEM, the NHNE model, and the CNHNE model for nitrous oxide with various downstream pressures and an upstream pressure of 5.58 MPa and temperature of 295 K.
highly dependent upon density, which is tightly coupled to heat transfer into the coolant. If the pressure drop across the injector orifice is comparatively large, this means the injector orifice, and therefore density and heat transfer, will have a large effect on the total mass flow rate. Hence, an upstream pressure drop has the effect of decoupling the total mass flow rate from the heat transfer into the fluid. If the fluid mass flow rate were significantly affected by the amount of regenerative heat transfer and the orifice sizes were designed for the steady state operational condition, a substantially higher mass flow rate would exist during the start up transient. This could result in a potential combustion chamber over pressurization during the start-up thermal transient for the motor.

To meet desired thrust levels, orifice sizes between states one and two and also three and four must be chosen such that the total mass flow rate aligns with requirements. A reasonable bound for the pressure drop across the initial expansion orifice was estimated to be about half of the initial fluid pressure, which prescribes the pressure at state two and three as 2750 kPa (400 psi). Pertinent fluid parameters are listed in Table 7.3 below based upon isenthalpic expansion described above and 3500 watts of heat addition between states 2 and 3. An incompressible discharge coefficient of 0.85 was assumed for this analysis. This should be a reasonable number for square edged orifices.

The achieve the design thrust level of 125 N for the prototype MUPHyN thruster, the injection and throttling orifices were sized to achieve a mass flow rate of approximately 0.08 kg/s with a oxidizer inlet pressure is approximately 5500 kPa. The resulting pressure is approximately 2750 kPa for each of the four coolant channels, and the design chamber pressure is approximately 690 kPa. Table 7.3 and Table 7.4 show the corresponding fluid properties and coolant flux rates at each of the state-points 1-4. These states are also shown on the phase diagram for nitrous oxide in Fig. 7.8 and on the temperature-enthalpy diagram in Fig. 7.9. Table 7.5 lists the area requirements derived from these mass fluxes and the final as-built orifice diameters.

Figure 7.7 illustrates the change in mass flow rate with heat transfer for MUPHyN design configuration and a configuration with heat transfer into the fluid before a single
The design operating condition yields an estimated 3500 watts of heat transfer. The two orifice configuration described above will have a flow rate about 2% lower during motor operation than startup or operation without heat transfer. If there was not a stabilizing initial pressure drop, the total mass flow rate would drop by nearly 21% between startup and steady state heat transfer.

Table 7.4: Nitrous Oxide Mass Flux Predictions

<table>
<thead>
<tr>
<th>Orifice</th>
<th>$G$</th>
<th>$G_{HEM}$</th>
<th>$G_{SPI}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>39,840 kg/m$^2$-s</td>
<td>27,741 kg/m$^2$-s</td>
<td>66,035 kg/m$^2$-s</td>
</tr>
<tr>
<td>3-4</td>
<td>14,460 kg/m$^2$-s</td>
<td>8,739 kg/m$^2$-s</td>
<td>25,283 kg/m$^2$-s</td>
</tr>
</tbody>
</table>
Fig. 7.7: Mass flow rate variation with heat transfer into oxidizer flow for single and double pressure drop configurations.

Fig. 7.8: Nitrous oxide coolant states on nitrous oxide phase diagram.
Fig. 7.9: Nitrous oxide coolant states on nitrous oxide temperature-enthalpy diagram.

Table 7.5: Orifice Area Requirements

<table>
<thead>
<tr>
<th>Orifice</th>
<th>Total Area Required</th>
<th>Final Orifice Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>2.008E-6 m²</td>
<td>0.8 mm (1/32 drill)</td>
</tr>
<tr>
<td>3-4</td>
<td>5.533E-6 m²</td>
<td>1.3 mm (#55 drill)</td>
</tr>
</tbody>
</table>
7.6 Heat Transfer Summary and Conclusions

Using convective heat transfer models developed by Mayer, the steady state heat transfer into the MUPHyN motor aerospike surface was estimated at about 3500 watts. This heat loading is removed through the use of regenerative cooling. To prevent substantial coupling between heat transfer and mass flow rate, an orifice was designed to expand nitrous oxide before entering the cooling channels, thereby mitigating the sensitivity of the flow rate to changes in heat transfer (such as the transient due to motor startup). The mass flow rates through these orifices were predicted using non-equilibrium multiphase heat transfer models. MATLAB code for the hot-gas heat transfer algorithm is included in Appendix D. The multi-phase mass flux algorithms are presented in Appendix E, and the coolant side heat transfer algorithm is included in Appendix F.
Chapter 8

Integration and Testing of the MUPHyN Motor Prototype

8.1 Introduction

The MUPHyN motor prototype was designed, constructed in tested to fulfill several research goals:

- Demonstrate the validity of hybrid rocket propulsion that fits in a 1 U cubesat
- Characterize aerodynamic thrust vectoring in hot flow conditions
- Evaluate the combustion stability of this non-traditional fuel port design
- Characterize the performance of this very short form factor hybrid motor
- Demonstrate the feasibility of nitrous oxide regenerative cooling on an aerospike nozzle

These objectives were completed through a series of hot fire tests of the MUPHyN motor prototype. Additionally, a non-pyrotechnic ignitions system was developed suitable for use on small satellites in a parallel. This chapter discusses the integration and testing of the MUPHyN motor prototype as well as the development of the non-pyrotechnic igniter.

8.2 Experimental Apparatus used for MUPHyN Tests

The MUPHyN hot-fire static tests used a legacy test stand developed at USU for larger scale hybrid motor tests but modified to accomplish the MUPHyN test objectives. This system features the Mobile Nitrous oxide Supply and Test Resource (MoNSTeR) cart that contains a run tank which is preloaded with nitrous oxide and then top pressured with helium for the duration of the burn. Figure 8.1 shows the Piping and Instrumentation Diagram (P&ID) for the MoNSTeR cart oxidizer delivery system. Primary flow is controlled
Fig. 8.1: Plumbing and instrumentation diagram for “MoNSTeR Cart” supply system.

via a binary, pneumatic operated ball valve and secondary flow is controlled via a fast-
response solenoid valve.

A custom designed Venturi flow meter measures primary oxidizer flow and another sim-
ilar but smaller Venturi is used to measure the flow rate of the thrust vectoring fluid. For
these measurements, the Venturi discharge coefficient was assumed equal to 0.985 which cor-
responds to high Reynolds number number flow. From calibration and comparison to tank
weight data, the estimated 95% confidence interval for these flow meters is approximately
0.5% of the flow rate.

8.3 Thrust Stand Development and Calibration

To measure both axial thrust and side force, a four degree of freedom thrust balance
was designed specifically for MUPHyN testing. Two axial load cells are used to measure
axial thrust and a two side load cell measures the much smaller side forces as well as axial
torque. The test stand features custom-engineered three axis flexures in the vertical and axial directions to limit frictional load losses and ball-and-clevis joints on the side load cells. Figure 8.2 shows the MUPHyN thruster mounted in the 4-DOF test stand. The axial load cells on the MUPHyN test stand were calibrated using conventional single axis methods. However, the test stand was calibrated for side force, roll, and yaw using a simultaneously multivariable calibration method similar to the one previously described by Eilers et al. [151]. The method was modified from previous methods by allowing drift of the bias during calibration whereas the previous method assumed zeroed reference data. As this method was designed specifically for use on multi-axial tests stands such as the MUPHyN, is not presented elsewhere in the open literature, and played substantial role in the collection of accurate side force data, a brief discussion of this method is presented here.

The load cell system can be modeled as a linear system with an input force vector that generates an output voltage vector of the form

$$ V = JF + V_0 $$  \hspace{1cm} (8.1)

Where $V$ is the output voltage of the system, $V_0$ is a vector of load cell voltage biases at zero load, $F$ is the input force and moment vector and $J$ is the vector partial

$$ J = \frac{\partial V}{\partial F} $$  \hspace{1cm} (8.2)

Herein, the voltages were defined as each of the side load cell voltages and the difference between the axial load cells, yielding a 3x3 matrix $J$.

The unknown quantities in Eq. (8.1) can be rearranged into a vector,
\[ s = \begin{bmatrix} J_{1,1} \\ J_{1,2} \\ \vdots \\ J_{3,3} \\ V_0, \\ \vdots \\ V_{02} \end{bmatrix} \] (8.3)

Using applied calibration loads, the measurement partial of these unknown quantities is

\[ \mathbf{h}^T = \frac{\partial \mathbf{V}}{\partial \mathbf{s}} = \begin{bmatrix} \mathbf{F}^T & 0 & 0 & 1 & 0 \\ 0 & \mathbf{F}^T & 0 & 0 & 1 \\ 0 & 0 & \mathbf{F}^T & 0 & 0 \end{bmatrix} \] (8.4)

So that the measurement matrix is

\[ \mathbf{H} = \begin{bmatrix} \mathbf{h}_1 & \ldots & \mathbf{h}_m \end{bmatrix} \] (8.5)

where the subscript “m” denotes each set of applied calibration loads. The voltage outputs from each of these sets of calibration loads is

\[ \tilde{\mathbf{V}} = \begin{bmatrix} \mathbf{V}^1 \\ \mathbf{V}^2 \\ \vdots \\ \mathbf{V}^m \end{bmatrix}_{3m \times 1} \] (8.6)

The unknown states in Eq. (8.1) can then be computed with a simple pseudo inverse,

\[ s = (\mathbf{H}^T \mathbf{H})^{-1} \tilde{\mathbf{V}} \] (8.7)

The loads given a set of voltage inputs is then
A typical calibration matrix for the MUPHyN system is

\[
\mathbf{C} = \mathbf{J}^{-1} = \begin{bmatrix}
-1448.2 & -1337.5 & 1185.0 \\
-83.1 & 80.8 & 218.1 \\
12.7 & -4.1 & -884.2
\end{bmatrix}
\]  

(8.9)

where \( \mathbf{C} \) is the calibration matrix. In Eq. (8.8), the rows in the output state vector to side force, yaw, and roll (in that order), and the three voltage inputs correspond to voltage from the side load cells and then the difference between the axial load cells. Thus terms in \( \mathbf{C}(2,3), \mathbf{C}(3,1), \) and \( \mathbf{C}(3,2) \) all represent cross coupling parameters that would be neglected (for a loss of accuracy) using conventional geometry-based calibration methods.

The resulting side force calibration using this method has a 95% uncertainty estimate of approximately +0.038 N or about +0.5% of the nominal side force value.
8.4 Reusable Igniter Development

A prototype reusable igniter was developed in the spring of 2012. This motor used ABS fuel grains approximately 1.6 cm in diameter printed on a “Maker Bot”\(^1\) FDM machine with a conductive fuel layer cast out of HTPB and 5% carbon black by weight. Gaseous oxygen at approximately 500 kPa was used as the oxidizer. A commercial “stun gun” was used for spark ignition. This motor was tested successfully on the Utah State University campus and has demonstrated up to 27 ignitions with the same igniter fuel grain. The igniter prototype used for this testing is shown in Fig. 8.3.

An alternative series of tests is currently being performed to develop and refine the non-pyrotechnic igniter concept. These tests are beyond the scope of the current research activities and will be reported in additional publications.

8.5 Hot Fire Test Results

A total of 7 hot-fire static tests have been performed on the MUPHyN prototype; 6 “data gathering” test fires and an instrumentation system checkout hot fire. Several cold flow “system check” tests were performed before these the hot fire tests with helical fuel grains began. These tests are listed in Table 8.1 and Table 8.2 summarizes the results of the hot fire tests.

8.5.1 Primary Plenum Flow Test Results

Figure 8.4 presents pressure and thrust time-history profiles for a typical MUPHyN

\(^1\)http://www.makerbot.com/
Table 8.1: MUPHyN Hot Flow Test Matrix.

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Fuel Grain Port Configuration</th>
<th>Burn Time</th>
<th>Secondary Injectant</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC1</td>
<td>Annular</td>
<td>-</td>
<td>-</td>
<td>System Test</td>
</tr>
<tr>
<td>SC2</td>
<td>Annular</td>
<td>-</td>
<td>-</td>
<td>System Test</td>
</tr>
<tr>
<td>SC3</td>
<td>Annular</td>
<td>-</td>
<td>-</td>
<td>System Test</td>
</tr>
<tr>
<td>SC4</td>
<td>Annular</td>
<td>3s</td>
<td>-</td>
<td>System Test</td>
</tr>
<tr>
<td>HF1</td>
<td>Double Helix</td>
<td>3s</td>
<td>-</td>
<td>Coolant Test</td>
</tr>
<tr>
<td>HF2</td>
<td>Double Helix</td>
<td>3s</td>
<td>Helium</td>
<td>Hot Fire/ Thrust Vectoring</td>
</tr>
<tr>
<td>HF3</td>
<td>Double Helix</td>
<td>3s</td>
<td>Helium</td>
<td>Flame Out</td>
</tr>
<tr>
<td>HF4</td>
<td>Double Helix</td>
<td>3s</td>
<td>Helium</td>
<td>Hot Fire/ Thrust Vectoring</td>
</tr>
<tr>
<td>HF5</td>
<td>Double Helix</td>
<td>3s</td>
<td>Nitrogen</td>
<td>Hot Fire/ Thrust Vectoring</td>
</tr>
<tr>
<td>HF6</td>
<td>Triple Helix</td>
<td>4s</td>
<td>Nitrogen</td>
<td>Hot Fire/ Thrust Vectoring</td>
</tr>
<tr>
<td>HF7</td>
<td>Triple Helix</td>
<td>4s</td>
<td>Oxygen</td>
<td>Hot Fire/ Thrust Vectoring</td>
</tr>
</tbody>
</table>

burn. After the initial startup transient, the motor achieves a steady-state thrust level that is within 5% of the design value of 120 N. Obviously, the Isp’s listed in Table 8.2 are significantly lower than would be expected for a well tuned hybrid rocket motor. Additionally, the characteristic velocities are far below those expected for this fuel combination. For comparison, tests completed with nitrous-oxide and HTPB at USU in a traditional hybrid motor yielded specific impulses of about 195 seconds with an expansion ratio of 4.5 [17] and an average characteristic velocity of about 1450 m/s. There are two plausible explanations for this lowered performance: 1) this initial series of tests was designed to have a higher than desirable oxidizer mass flow rate of oxidizer to ensure sufficient cooling, and 2) the fuel regression rate was much higher than initially anticipated. The high total mass flow rates result in very low chamber dwell times, potentially lowering combustion efficiency. The high regression rates also decreased the OF ratio well below optimum for many of the tests, also decreasing overall performance. The high regression rate is presumably due to centrifugal flow effects produced by the helical port in the ABS fuel grains. Although the high regression rate is not by itself undesirable, the lower OF ratio tends to both decrease
the performance directly and lower the flame temperature. As noticed by Strand, a decrease in the flame temperature could lower the rate of chemical kinetics, yielding lower flame temperatures and efficiencies in a viscous-circle type process [152]. It should be noted that the later fuel grains were designed to increase mixing, and these fuel grains do indeed show a substantial increase in characteristic velocity. Figure 8.5 contains photographs of each of the burned fuel grains for each of the test fires listed in Table 8.2.

8.5.2 Regenerative Cooling Test Results

During each of the MUPHyN test firings there was no notable erosion on the aerospike surface, and the regenerative cooling system maintained the aerospike and the supporting injector structure well within material temperature limits. The combustion temperature for MUPHyN tests is estimated to exceed 2800 C.

Figure 8.6 presents temperature profiles from two hot-fire tests performed with a thermocouple embedded just inside of the nozzle coolant channels. The location of this thermocouple in relation to the coolant channels is shown in Fig. 8.7. A large nozzle temperature
Fig. 8.5: Fuel grains after test firings.

Table 8.2: MUPHyN Test Fire Summary.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Burn Time (s)</th>
<th>Isp (s)</th>
<th>Total Impulse (Ns)</th>
<th>Characteristic Velocity (m/s)</th>
<th>O/F Ratio</th>
<th>Secondary Injectant</th>
<th>Average Ox. Flow Rate (kg/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HF1</td>
<td>3</td>
<td>137</td>
<td>487</td>
<td>1367</td>
<td>3.16</td>
<td>none</td>
<td>0.088</td>
</tr>
<tr>
<td>HF2</td>
<td>3</td>
<td>122</td>
<td>370</td>
<td>na</td>
<td>4.14</td>
<td>Helium</td>
<td>0.077</td>
</tr>
<tr>
<td>HF4</td>
<td>3</td>
<td>128</td>
<td>400</td>
<td>1325</td>
<td>3.13</td>
<td>Helium</td>
<td>0.077</td>
</tr>
<tr>
<td>HF5</td>
<td>3</td>
<td>106</td>
<td>320</td>
<td>1195</td>
<td>3.16</td>
<td>Nitrogen</td>
<td>0.072</td>
</tr>
<tr>
<td>HF6</td>
<td>4</td>
<td>144</td>
<td>450</td>
<td>1473</td>
<td>3.35</td>
<td>Nitrogen</td>
<td>0.060</td>
</tr>
<tr>
<td>HF7</td>
<td>4</td>
<td>142</td>
<td>469</td>
<td>1451</td>
<td>3.38</td>
<td>Oxygen</td>
<td>0.063</td>
</tr>
</tbody>
</table>

Measurement Uncertainty (95%): ±2.4%, ±0.4%, ±3.4%, ±1.5%, ±0.05%
difference between the two tests is noted. The initial MUPHyN tests used a graphite insulator below the aerospike nozzle. In later tests, this insert was replaced with ABS fuel, which substantially lowered the total heat transfer into the fuel grain. Heat transfer through the graphite insert was neglected for the heat transfer analysis, which may be the source of the under-predicted nozzle temperature.

For the tests with ABS insulation, the aerospike temperature presented in 8.6 shows reasonable agreement with the predicted aerospike temperatures discussed in Chapter 7 and shown in Fig. 7.4 for a net heat flux of about 3500 watts. Although no precise estimates for the model error can be extrapolated from results without direct heat transfer measurement, these temperature result suggests that the heat transfer models used in this analysis are at least a reasonable approximation.
8.5.3 Effects of Fuel Grain Geometry on Fuel Regression Rate and Motor Performance

Using typical skin-friction based hybrid motor regression-rate prediction models [52], the average regression rates in the MUPHyN motor were expected to be near 1.0 mm/s and the chamber ports/test fire durations were designed such that it would require over twice this regression rate to burn through the 6.6 mm fuel port wall during a motor firing. As seen in Fig. 8.5, the original MUPHyN helix demonstrated much higher fuel regression rates than expected (at least 2.0 mm/s as evidenced by burn-throughs) and low combustion efficiencies (as seen by the low specific impulses). In addition, the oxidizer flow rate was constrained by requirements to maintain a high safety factor on coolant capacity, not thrust level or desired oxidizer mass flux. The test HF5 showed ample cooling capacity once the center aerospike support was insulated with ABS instead of graphite. This allowed more flexibility in nitrous oxide flow rates. For the next two tests, the main oxidizer flow rate was decreased by approximately 25% which allowed for lower oxidizer mass fluxes in the fuel grain. This, in turn, allowed for greater flexibility in fuel grain design.

For HF6 and HF7, the double helix design was replaced with a triple helix design.
Table 8.3: Fuel Grain Geometry Summary

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Port Area (cm$^2$)</th>
<th>Helical Radius (cm)</th>
<th>Pitch (cm)</th>
<th>Initial Surface Area (cm$^2$)</th>
<th>Number of Ports</th>
</tr>
</thead>
<tbody>
<tr>
<td>HF1</td>
<td>1.59</td>
<td>2.74</td>
<td>3.81</td>
<td>222</td>
<td>2</td>
</tr>
<tr>
<td>HF2</td>
<td>1.54</td>
<td>2.73</td>
<td>3.81</td>
<td>211</td>
<td>2</td>
</tr>
<tr>
<td>HF4, HF5</td>
<td>1.59</td>
<td>2.74</td>
<td>6.35</td>
<td>190</td>
<td>2</td>
</tr>
<tr>
<td>HF6, HF7</td>
<td>55.2</td>
<td>2.91</td>
<td>12.7</td>
<td>194</td>
<td>3</td>
</tr>
</tbody>
</table>

with much thinner and taller combustion chambers. This geometry is shown in Fig. 8.8 and the geometry for all of the test fires is listed in Table 8.3. The thinner triple helix promotes more mixing of the center port than the double helix design and results in more fuel between the combustion chamber and the motor wall, which allows for longer burn times. The pre-combustion chamber was also designed with fuel structures designed to promote flame holding and to turn the oxidizer streams, preventing their direct impingement on the opposite fuel wall. Figure 8.9 shows these fuel structures.

As a result of this redesign, the specific impulse for HF6 and HF7 increased by approximately 16% over the previous test fires. The motor plume for these tests was also distinctly different from the previous tests. Figure 8.11 shows the differences in flow features between test HF5 and HF7. Although it is difficult to illustrate with a still photograph, the flame pattern on the bottom of Fig. 8.11 is indeed helical, and not the result of Mach diamonds. The plume in HF7 is much more uniform and the unmixed helical flow pattern exhibited by the previous tests is absent. It is believed by the authors that further reduction in the oxidizer mass flow rate would continue this trend, further increasing the MUPHyN specific impulse.

8.5.4 Thrust Vectoring Test Results

Thrust vectoring tests have been completed with nitrogen, helium, and oxygen as secondary injectants. Table 8.4 summarizes the thrust vectoring test results with parameters including side-force specific impulse, amplification factor, and equivalent thrust vector angle for these tests with the MUPHyN motor. The side force specific impulse is defined as
Fig. 8.8: Fuel grain geometry for HF6 and HF7.

Fig. 8.9: Fuel grain geometry in pre-combustion chamber for HF6 and HF7.
Fig. 8.10: Plumes for MUPHyN HF5 (double helix) and MUPHyN HF7 (triple helix).
\[ I_{sp_s} = \frac{F_s}{\dot{m}_s g} \]  \hfill (8.10)

where \( F_s \) is the side force and \( \dot{m}_s \) is the mass flow rate of the secondary injectant.

As discussed previously, secondary injection on an aerospike nozzle creates a localized bow shock in front of the injection site and increases the total generated side force. Figure 8.11 shows the MUPHyN plume with and without secondary injection active. When the secondary injection port is active, the shock waves created by secondary flow interaction ahead of the injection site are clearly visible.

Figures 8.12 through 8.14 plots the side force, specific impulse, and mass flow rates achieved using gaseous nitrogen, helium, and oxygen, respectively. The side force impulses appear to be both crisp and repeatable. The total thrust vector angle for tests with helium was substantially higher than those with nitrogen and oxygen due to higher total mass flow rates. The higher achieved side-force specific impulse for helium is likely a result of the significantly lower molecular weight and/or the higher specific heat ratio of the injectant. The amplification factor for oxygen was not substantially higher than that shown for nitrogen, which implies that combustion of the oxygen with unreacted fuel in the separated region before the secondary injection port does not significantly influence thrust vectoring efficiency. The estimated uncertainty in side-force specific impulse calculations is approximately 2.0 seconds.

The hot-gas side force amplification factor (132% for nitrogen/oxygen, 136% for helium) is only slightly lower than the 139% amplification factor demonstrated by Eilers et. al [151] for cold flow tests using CO2 gas. The collected specific impulses from both MUPHyN and CO2 gas tests are shown in Fig. 8.15. The uncertainties for the specific impulse measurements with CO2 gas are approximately 3.5% of the measurement and 1.1% of the measurement for tests with the MUPHyN configuration.

8.6 MUPHyN Integration and Testing Conclusions

The initial series of MUPHyN motor test fires have demonstrated stable combustion
Fig. 8.11: MUPHyN motor plume with and without active secondary injection.

Fig. 8.12: Secondary flow side force, mass flow rate, and Isp with nitrogen secondary injection.

Table 8.4: Thrust Vectoring Test Summary.

<table>
<thead>
<tr>
<th>Injectant</th>
<th>Secondary Flow Only Isp (s)</th>
<th>Isp with Primary Flow (s)</th>
<th>Amplification Factor</th>
<th>Thrust Vectoring Angle (deg)</th>
<th>Injectant Static Pressure (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrogen</td>
<td>51.0</td>
<td>67.1</td>
<td>1.32</td>
<td>1.95</td>
<td>3.5</td>
</tr>
<tr>
<td>Helium</td>
<td>121.3</td>
<td>165.5</td>
<td>1.36</td>
<td>3.63</td>
<td>5.7</td>
</tr>
<tr>
<td>Oxygen</td>
<td>55.2</td>
<td>73.1</td>
<td>1.32</td>
<td>2.63</td>
<td>3.5</td>
</tr>
</tbody>
</table>
Fig. 8.13: Secondary flow side force, mass flow rate, and Isp with helium secondary injection.

Fig. 8.14: Secondary flow side force, mass flow rate, and Isp with oxygen secondary injection.
Fig. 8.15: Thrust vectoring specific impulse test results for both cold flow (CF) and MUPHyN tests each with primary flow active and secondary only flow configurations.

and shown thrust vectoring effectiveness that closely reproduces previously demonstrated results achieved during cold flow testing. The regenerative cooling system has performed effectively in all test fires to date.

The achieved main flow specific impulses were lower than expected. There are two plausible explanations for this lowered performance: 1) this initial series of tests was designed to have a higher than desirable oxidizer mass flow rate of oxidizer to ensure sufficient cooling, and 2) the fuel regression rate was much higher than initially anticipated. The high regression rate is presumably due to centrifugal flow effects produced by the helical port in the ABS fuel grains. These higher-than-expected regression rates resulted in O/F ratios significantly lower than the levels desired for good combustion efficiency. A MUPHyN design with lower oxidizer flow rates and a fuel grain with geometry that induced additional mixing showed significant improvement in specific impulse and it is believed that this trend would continue for even lower flow rates.
Chapter 9
Conclusions

9.1 Project Summary

The development and analysis of the MUPHyN motor was a multi-year project. The project objectives were designed to incrementally advance enabling technologies for small satellite propulsion. These technologies span small-scale hybrid rocket motor propulsion, integration, and modeling. This program involved work on several technological fronts that appear somewhat eclectic when taken individually, but all parts played critical roles in the MUPyN motor development and analysis. In the end, the MUPHyN motor experimental results and development processes were designed to be a technological stepping stone for small satellite propulsion development.

The MUPHyN system provides attitude and velocity control using secondary-injection thrust vectoring without mechanical nozzle gimbals or additional reaction control thrusters. Both larger impulse $\Delta V$ and small impulse attitude control and proximity operations burns could be performed with the same system.

This synthesis of technologies is unique to the MYPHyN thruster design and no other commercial or government entity has produced comparable work that has been published in open literature. The resulting system is compact, non-toxic, non-explosive, and uses non-pyrotechnic means for reliable motor ignition.

When fully developed, this enhanced propulsive capability could enable multiple Cube-Sats to be deployed simultaneously by a single launch vehicle and independently repositioned, a key enabling technology for multi-point measurement science missions.

During this process, the theses listed in Chapter 1 were evaluated. The results from this are

- A hybrid rocket was successfully designed and manufactured to fit inside a 1 U of a
cubesat satellite using an FDM manufactured fuel grain. The performance levels of this motor were on the low end of hybrid motors, and suggestions were made that could remedy this issue.

- The aerodynamic thrust vectoring efficiency on aerospike nozzles was found to be in the same range as thrust vectoring effectiveness for conventional bell or conical nozzles published in the open literature.

- Regression rate characteristics of conventional hybrid systems were reasonably well predicted by the models developed. The UPVST model was shown to predict regression rates better than the previous LAVST model. The CSVST model was also shown to at least qualitatively predict regression rates in the MUPHyN motor.

- The MUPHyN motor was designed with a regenerative cooling using nitrous oxide. Sufficient cooling capacity was demonstrated over a series of hot fires by the fact that the nozzle did not melt, deform, or erode.

- A non-pyrotechnic igniter was designed, constructed, and tested although it was never used to ignite the MUPHyN motor, it was used to ignite a larger 98mm motor.

Results in specific areas are discussed in more detail below.

9.2 Summary of Results

9.2.1 Regression Rate Modeling

Much of the work in this project involved regression rate prediction in hybrid rocket motors. Due to the critical nature of this parameter for hybrid rocket motor operation, three separate chapters were presented on this topic. First, the derivation of a simple longitudinally averaged regression rate model, the LAVP model, was presented that illustrates the basic dependence of hybrid rocket regression on fluid parameters and convective heat transfer through a turbulent boundary layer. This model is effectively “closed” and is suited to integration for full system performance prediction.
The LAVP model was an initial step for the more complicated but theoretically rigorous UPVST model presented in the next chapter. This model corrects for several parameters and effects not included in the LAVP model. These adjustments include:

- An experimentally based relationship for wall blowing that accounts for the molecular weight of the pyrolysis products
- Length variation of skin friction and the evolution of the total mass flux along the length of the fuel port
- A flame temperature based on boundary layer phenomena, not mean combustion properties
- The flame height ratio
- The fuel surface temperature as a function of regression rate
- A more complete model of the boundary layer viscosity
- Variable fluid properties across the boundary layer
- Radiative heat transfer

The inclusion of many of these models required the use of iterative solution methods which, while allowing greater flexibility for formulation, tend to obscure the significance of the many parameters. Thus, for direct comparison to other models the UPVST model was compared to a great deal of data in the literature and was shown to perform better than the original LAVP model. The LAVP model is also more amenable to adjustment for other hybrid motor fuel combinations or operating conditions because it contains “theoretically correct knobs” that adjust the regression rate based upon physical processes.

As the LAVP model was the foundation for the UPVST model, the UPVST model was the foundation for a model based upon computational fluid dynamics solutions for surface heat transfer, the CSVST model. This final regression modeling step collects the regression rate models used in the UPVST algorithm into a form that can be applied to complicated
fuel port geometries like those used in the MUPHyN motor prototype. Although this model was not used to design fuel grains of the MUPHyN motor prototype, it aided significantly in explaining increased regression rates and “hot spots” seen in the post-burn MUPHyN motor fuel grains. A model of this form certainly could, however, be used to design more efficient fuel grains for complex form factors like those required by any small satellite propulsion device in the future.

9.2.2 Aerodynamic Thrust Vectoring

Aerospike nozzles and aerodynamic thrust vectoring form a promising pair of technologies that hold promise for small satellite applications. Chapter 6 was devoted to the characterization of thrust vectoring on annular aerospike nozzles in a series cold flow experiments. These experiments showed sizable gain in side force specific impulse when the secondary fluid was injected into the main flow plume. Most importantly, a strong dependence of efficiency on the axial location of the injection port was discovered, with locations near the end of the aerospike nozzle showing the highest amplification factors. The data and relationships uncovered by this research were later used in the design of the MUPHyN motor prototype.

9.2.3 Regenerative Cooling Heat Transfer Modeling

A principal challenge in the use of annular aerospike nozzles is the high heat loading imparted to the nozzle from the hot exit gases. For this reason, heat transfer relationships and regenerative cooling models were discussed in Chapter 7. Analytical heat transfer models were developed and applied to the MUPHyN motor design to estimate the total heat transfer into the MUPHyN motor aerospike. These models were used in a study to size the aerospike nozzle. This study illuminated the interdependence of cooling requirements and expansion ratio for annular aerospike nozzles. Heat transfer models for multiphase nitrous oxide were also developed and used to select an appropriate coolant pressure and cooling channel design. Unfortunately monetary and size constraints on the MUPHYN motor did not allow the direct experimental measurement of heat transfer into the nitrous
oxide coolant so verification of these models could only be used as a bound on the limits of regenerative cooling for nitrous oxide motors as evidenced by the lack of catastrophic failure of the test apparatus.

9.2.4 Final Integration and Testing of the MUPHyN Prototype

Finally, the integration and testing of the MUPHyN motor prototype was discussed in Chapter 8. The performance of this demonstrator-prototype was characterized in a series of test fires conducted at USU. In addition, a non pyrotechnic ignition device was developed suitable for use on small satellites, although this igniter was not used on the MUPHyN motor. Several interesting trends were noted during the testing of the MUPHyN motor. First, the regression rate in the helical MUPHyN fuel ports was much higher than expected. Secondly, the combustion efficiency of the prototype was a strong function of fuel port geometry and initially significantly lower than expected, although later tests did substantially improve this parameter. Finally, “hot flow” test fire results for aerodynamic thrust vectoring were obtained that compared very well with cold flow results.

9.3 Suggestions for Future Work

As the MUPHyN motor development involved several technologies, it is not surprising that several interesting areas that warrant future research were found. Some of these topics are discussed in this section.

9.3.1 Regression Rate Characterization in Helical Fuel Ports

The MUPHyN motor helical fuel ports regressed several times faster than would be predicted by standard regression rate models. Additionally, the surface area in a helical fuel port is substantially higher than in a straight port for the same motor length. This leads to dramatically increased fuel flow rates for very short form factors. Currently, traditionally low hybrid rocket regression rates limit hybrid rocket application to a variety of fields. To further examine this phenomenon, several helical grains were burned in a 75mm diameter motor. These test fires also obtained dramatically increased regression rates, however a
rigorous theoretical explanation for this phenomenon has not yet been completed. Along with increased skin friction due to rotation of the combustion products in the fuel port, it is possible that secondary flows driven by density variation in the fuel port along with the high radial acceleration also increase mixing and heat transfer to the fuel surface. The CSVST model would be a useful analytical tool for validating fluid dynamic parameters than have an effect on this regression rate, and characterizing scaling effects for other motor configurations. A reasonable data set already exists from the tests of helical fuel grains for this effort.

9.3.2 Fuel Surface Prediction with the CSVST Model

The CSVST model has currently only been used to generate regression rates for the initial hybrid motor fuel port. Ideally, this estimate would be used to create a new three dimensional fuel port surface corresponding to a step forward in time. This new fuel port could be used to obtain a new computational fluid dynamics solution for the flow field in the fuel port, that could then be used with the CSVST model to estimate the regression rate for this time step. In this manner, the CSVST model could be used to model regression of a complicated fuel port over time, a step that would save a great deal of expensive experimental iteration for complicated fuel grains.

9.3.3 Experimental Characterization of Regenerative Cooling with Nitrous Oxide

The heat transfer rates in the MUPHyN motor could not be experimentally characterized due to monetary constraints and the very compact form factor of the prototype. Detailed characterization of both heat transfer into the nozzle surface and heat transfer into nitrous oxide cooling in an apparatus without these constraints would make development of regeneratively cooled aerospike systems in the future easier.

9.3.4 Development of a Faceted Aerospike or “Aerostar” Nozzle

Because of the constraints placed upon nozzle expansion ratio in an annular aerospike
nozzle by the regenerative cooling limits and the machining accuracy, an alternative high expansion ratio nozzle solution would be desirable. Aerospike nozzles with circular ports that direct combustion gases onto a central aerospike nozzle have documented in the literature. These configurations have dramatically lower throat surface area and thus significantly lower heat loadings than annular nozzles. Additionally, any nozzle regression in the throat will not substantially change the nozzle throat area because the throat is no longer an annulus. These attempts have often not included any effort to optimize the nozzle surface between nozzle “plenums” however, and have shown substantial flow losses due this issue. If a faceted aerospike nozzle could be optimized to expand the flow from each separate plenum onto a nearly annular external aerospike, the cooling advantages to a multiple plenum design would be maintained along with nozzle performance. Such a configuration, dubbed an “aerostar” by this research project, could provide great benefit to small satellite propulsion.

9.3.5 Scale-Up Testing of Hybrid Motor Production Through Fused Deposition Modeling

The testing of FDM fuel grains in a university setting is somewhat limited by scale and production cost. The price of $1 kg of fuel in a cartridge configuration for a typical “3D printer” is on the order of $270, about 100 to 200 times the price of stock ABS material. Additionally, conventional 3D printers are slow due to the small diameter of extruded filaments sized in an effort to increase accuracy. The technology involved in FMD manufacturing is neither complicated nor limited to small extrusion rates. Thus, a “industrial scale” printer could be manufactured with dramatically higher extrusion rates at some expense in dimensional accuracy. If larger fuel grains with complicated fuel ports were desirable, however, the development of such a process would almost certainly be required in order to minimize manufacturing time and material cost.

9.3.6 Characterization of ABS Fuel Pyrolysis

The UPVST regression rate model seems to predict the regression rate for ABS fuel grains with some degree of accuracy, even when using HTPB fuel pyrolysis parameters to
predict fuel surface temperatures. HTPB parameters were used because Arrhenius type regression parameters could not be found in the open literature for the regression rates and surface temperatures expected for typical motor operation. However, it is highly possible that this is purely fortuitous due to cancellation of errors. As other motor configurations could see a substantial departure from the operational characteristics that produced this performance, it would be beneficial if ABS pyrolysis properties were characterized in a manner similar to models that exist for HTPB pyrolysis.

9.3.7 Characterization of Non-Pyrotechnic Ignition System Reliability and Hybrid Rocket Minimum Ignition Energy Requirements

The “Multiple Use” goal of the MUPHyN motor motivated development of a non-pyrotechnic igniter, the “micro hybrid” igniter. As part of a master’s thesis, igniter development continued after the work discussed in Chapter 8, culminating in multiple ignitions of a 98 mm diameter hybrid motor with this device. Despite this success, the reliability of this form of igniter is not well understood and effort to statistically characterize this parameter would not be wasted. Additionally, the flow rates used in this igniter were obviously far in excess of what would be required to actually ignite a hybrid motor of this size. No work was found in the literature characterizing ignition power source requirements for hybrid motors. A study in this area could be easily completed by adjusting flow rates in the micro-hybrid igniter.
References


Appendices
Appendix A

The LAVP Algorithm

The following is a MATLAB function for the LAVP regression rate model, adjusted for steady state operation for easier comparison to other models. This function computes regression rates and combustion properties provided data structures containing chemical, flow rate, and geometric parameters. Additionally several modifications to the original algorithm can be enabled through the use of input flags.

function [rdot, P0, MR, CProps, B] = rdotEilersWhitmoreSS(PropData,...
    MotorProps,TsFlag,AltFlag,RadFlag)

%This function computes length-averaged regression rate based on the
%LAVP regression rate model
%G Mass flux, kg/m^3
%Viscosity Pa*s
%T0 K
%hv heat of vaporization
%Tfuel K
%Cp Specific heat, J/KgK
%Pr Prandtl number
%TsFlag flag 1- compute hv and Ts from Chaiverini
%Alt flag, use Altman correction for wall blowing
%Rad flag, include radiation

if MotorProps.ASurf>0
    Asurf=MotorProps.ASurf;
else
    Asurf=2*MotorProps.r*pi*MotorProps.L;
end

P0=3*10^-6;
noP0iter=0;
if isfield(MotorProps,'P0')
    if MotorProps.P0>0
        P0=MotorProps.P0;
        noP0iter=1;
    end
end
end
end
MR=3;
rdot0=0.001;
error=1;
sigma=5.67*10^-8;
psi2Pa=6894.75729;
ap=0.045;
Gox=MotorProps.mdotOx/(MotorProps.r^2*pi);
while abs(error) > 10^-9
    CProps=CombProps(P0,MR,PropData);
    if TsFlag==1
        Ru=1.9858*10^-3; %kCal/K/mol;
        if rdot0 <0.00033
            Ea=13.35; %kCal/mol;
            A=3.9648; % m/s;
        else
            Ea=4.91; %kCal/mol;
            A=.0104; % m/s;
        end
        Ts=-Ea/Ru/log(rdot0/A); %pyrolosys law from Chaiverini
        hve=MotorProps.hv+MotorProps.Cp*(Ts-MotorProps.Tfuel);
    else
        Ts=MotorProps.Tfuel;
        hve=MotorProps.hv;
    end
if AltFlag==1
    rdot = 0.0392/(MotorProps.rho_fuel)*...
            ((CProps.Cp*(CProps.T0-Ts))/hve/CProps.Pr^(-2/3))^0.32...
            *(Gox)^(-4/5)*(CProps.Visc/MotorProps.L)^.2;
    qconv=rdot*MotorProps.rho_fuel*hve;
else
    rdot = 0.047/(MotorProps.rho_fuel)*...
            ((CProps.Cp*(CProps.T0-Ts))/hve/CProps.Pr^(-2/3))^0.23...
            *(Gox)^(-4/5)*(CProps.Visc/MotorProps.L)^.2;
    qconv=rdot*MotorProps.rho_fuel*hve;
end
B=(CProps.Cp*(CProps.T0-Ts))/hve/CProps.Pr^(-2/3);
if RadFlag==1
    apnp=0.134*ap*P0/psi2Pa/(1+MotorProps.OFN2O-ap);
    qrad=sigma*CProps.T0^4*(1-exp(-apnp));

    rdot=(qconv*exp(-qrad/qconv)+qrad)/(hve*MotorProps.rho_fuel);
end

mdotFuel=rdot*Asurf*MotorProps.rho_fuel;

%New MR and P0
MRn=MotorProps.mdotOx/mdotFuel;

P0n=(MotorProps.mdotOx+mdotFuel)/...
    MotorProps.Astar*sqrt(CProps.T0)*sqrt(CProps.Rg/CProps.gamma)... *
    *((CProps.gamma+1)/2)*)((CProps.gamma+1)/(2*CProps.gamma-2));

MR=MR+0.5*(MRn-MR);

if noP0iter==1
    rerror=0;
else
    rerror=rdot-rdot0;
    P0=P0+0.5*(P0n-P0);
end

rdot0=rdot;

end

end
Appendix B

The UPVST Algorithm

The following is a MATLAB function for the UPVST regression rate model. This function computes regression rates and combustion properties provided data structures containing chemical, flow rate, and geometric parameters. Inputs also include flags used to enable or disable various parts of the model.

```matlab
function [rdotm rdot G mdotF P0 Bsav Tf UrSav qrad Re Ts Sm] = ... 
rdotIterativeRad(PropData,MotorProps,TsFlag,BLFlag,RadFlag,ViscFlag,...
VarPropFlag,injflag,PrFlag,P0g)

%This function computes length-averaged regression rate based on the
%relationship from
%Marxman
%\( G_{ox} \) Oxidizer Mass flux, kg/m^3
%\( \eta \) Viscosity Pa*s
%\( T_f \) K
%\( h_v \) heat of vaporization
%\( T_{fuel} \) K
%\( C_p \) Specific heat, J/KgK
%Pr Prandtl number
if nargin<8
    P0=3*10^-5;
else
    P0=P0g;
end

noP0iter=0;
if isfield(MotorProps,'P0')
    if MotorProps.P0>0
        P0=MotorProps.P0;
        noP0iter=1;
    end
end

end
```
psi2Pa=6894.75729;
Pr=0.85;
rdot0=0.0015;
sigma=5.67*10^-8;
ap=0.015;
%Area Properties
dx=MotorProps.L/length(MotorProps.rv);

if MotorProps.ASurf>0
    Asurfx=ones(size(MotorProps.rv))*MotorProps.ASurf/...
    length(MotorProps.rv);
else
    Asurfx=2*MotorProps.rv*pi*dx;
end

if strcmp(PropData.Ox,'O2')
    Kox=1;
else
    Kox=0.364;
end

%Outer loop solves for P0;
P0err=100;
while P0err>1;
    i=1;
    mdotFTot=0;

    FlProps=CombProps(P0,MotorProps.OFN2O/Kox,PropData);
    LowOFProps=CombProps(P0,0.25,PropData);
    Tf=FlProps.T0;
    for x=dx/2:dx:MotorProps.L;
        G(i)=(MotorProps.mdotOx+mdotFTot)/(MotorProps.rv(i)^2*pi);
        %Inner loop solves for surface temperature/Blowing parameter
        rerr=1;
        riter=1;
        while rerr>10^-6
            if TsFlag==1
                Ru=1.9858*10^-3; %kCal/K/mol;
                if rdot0 <0.00033
                    Ea=13.35; %kCal/mol;
A=3.9648; % m/s;
else
    Ea=4.91; %kCal/mol;
    A=.0104; % m/s;
end

Ts=-Ea/Ru/log(rdot0/A); %pyrolosys law from Chaiverini
if Ts<MotorProps.Tfuel
    Ts=MotorProps.Tfuel;
end
hve=MotorProps.hv+MotorProps.Cp*(Ts-MotorProps.Tfuel);
else
    Ts=MotorProps.Tfuel;
    hve=MotorProps.hv;
end

dh=FlProps.Cp*(FlProps.T0-Ts);
Ur=MotorProps.OFN2O*dh/hve/(Kox+(MotorProps.OFN2O+Kox)*dh/hve);
Csu=960;
WallVisc=0.00000812*(298.15+Csu)/(Ts+Csu)*(Ts/298.15)^(3/2);
if ViscFlag==1
    %Calculate the average viscosity in the boundary layer, %based on
    %the flame temperature and position
    mu0x=MotorProps.muOx;%1.5500*10^-5;% Pa*s
    ViscUP=(mu0x+FlProps.Visc)/2;
    ViscLw=(WallVisc+FlProps.Visc)/2;
    Visc=Ur*ViscUP+(1-Ur)*ViscLw;
else
    %Use the viscosity at the flame
    Visc=FlProps.Visc;
end
%Blowing parameter, as defined by Lees.
B=rdot0*MotorProps.rho_fuel/...
    (G(i)*0.03*(G(i)*x/Visc)^(-0.2)*Pr^(-2/3));
S0=(0.03*(G(i)*x/Visc)^(-0.2)*Pr^(-2/3));
Sm(i)=S0/(MotorProps.Gox0/Visc)^(-0.2);
%Full marxman blowing coefficient correction. Often fit with
%1.2B^-0.77
%Various Blowing Parameter Correlations
\[
\begin{align*}
    f_{B\_MM} &= (\log(1+B)/B)^{0.8} \times ((1+1.3B+0.364B^2)/(1+B/2)^2(1+B))^{0.2}, \\
    f_{B\_Chia} &= Ur \times (0.65+9.56B^{-1.45}) \times (MotorProps.rv(i) \times 2/MotorProps.L)^{0.3}, \\
    f_{B\_MMFit} &= 1.2B^{-0.77}, \\
    f_{b\_Alt} &= B^{-0.68}, \\
    f_{b\_Kays} &= 0.439 \times \exp(-0.589B) + 0.553 \times \exp(-0.0678B).
\end{align*}
\]

switch BLFlag

    case 0
        f_B = f_{B\_MM};
    case 1
        f_B = f_{B\_Chia};
    case 2
        f_B = f_{B\_MMFit};
    case 3
        f_B = f_{b\_Alt};
    case 4
        f_B = f_{b\_Kays};
    otherwise
        disp('Incorrect Blowing Flag')
end

\[
\begin{align*}
    qconv &= 0.03 \times (G(i) \times x / \text{Visc})^{-0.2} \times G(i) / Ur \times dh \times Pr^{-2/3}, \\
    Re(i) &= G(i) \times x / \text{Visc}, \\
    \text{if VarPropFlag==1,} \\
        PropTerm &= (FlProps.T0/Ts)^{0.55}, \\
        qconv &= qconv \times PropTerm; \\
    \text{end}
\end{align*}
\]

\[
\begin{align*}
    qrad &= \sigma \times FlProps.T0^4 \times (1 - \exp(-apnp)), \\
    \text{if RadFlag==1} \\
        OFrad &= MotorProps.OFN2O; \\
        \text{apnp} &= 0.02 \times \text{ap} \times P0/\text{psi2Pa}/(1+OFrad-\text{ap}); \\
        \%apnp &= 0.134 \times \text{ap} \times P0/\text{psi2Pa}/(1+OFrad-\text{ap}); \\
        \text{qrad} &= \sigma \times FlProps.T0^4 \times (1 - \exp(-apnp));
    \text{end}
\end{align*}
\]

dot(i) = (qconv + qrad) / (hve * MotorProps.rho_fuel);
err = abs(rdot(i) - rdot0);
rdot0 = rdot0 + 0.4 * (rdot(i) - rdot0);
riter = riter + 1;
UrSav = Ur;

Bsav(i) = B;
end

if injflag == 1
    a = -0.27;
    r1 = 3.08;
    r2 = 4.92;
    injcoef = 1 + a * exp(-(r1 + r2 * 1i) * x / MotorProps.L) +
               a * exp(-(r1 - r2 * 1i) * x / MotorProps.L);
    rdot(i) = rdot(i) * injcoef;
end

rdot0 = rdot(i);
if injflag == 1
    mdotF(i) = 0;
else
    mdotF(i) = rdot(i) * Asurfx(i) * MotorProps.rho_fuel;
end
mdotFTot = sum(mdotF(1:i));
G = MotorProps.Gox + mdotFTot / (MotorProps.r^2 * pi);
i = i + 1;
end

rdotm = mean(rdot);
% New MR and PO
MR = MotorProps.mdotOx / mdotFTot;
CProps = CombProps(P0, MR, PropData);
P0Old = P0;
POn = (MotorProps.mdotOx + mdotFTot) / ...
    MotorProps.Astar * sqrt(CProps.T0) * sqrt(CProps.Rg / CProps.gamma) ... *
    ((CProps.gamma + 1) / 2) * ((CProps.gamma + 1) / (2 * CProps.gamma - 2)));

if noP0iter == 1
    P0err = 0;
else
    P0err = abs(POn - P0Old);
P0=P0+0.8*(P0n-P0);  
end  

end
Appendix C

The UPVST Algorithm

The following is a MATLAB function for the CSVST regression rate model. This function computes regression rates and combustion properties provided data structures containing chemical, flow rate, and geometric parameters as well as data for CFD-generated Stanton numbers. Inputs also include the operating pressure, mass flux, and Stanton number adjustment for operating conditions.

```matlab
function [rdotV ] = rdotVector(PropData,MotorProps,P0,G,Sm)

%This fucntion computues length-averaged regression rate based on the relationship from Marxman
%Gox Oxidizer Mass flux, kg/m^-3
%Viscosity Pa*s
%Tf K
%hv heat of vaporization
%Tfuel K
%Cp Specific heat, J/KgK
%Pr Prandtl number

psi2Pa=6894.75729;
Pr=0.85;
rdot0=0.0015;
sigma=5.67*10^-8;
ap=0.015;

if strcmp(PropData.Ox,'O2')
    Kox=1;
else
    Kox=0.364;
end

%This loop solves for the surface temperature and regression rate
```

FlProps=CombProps(P0,MotorProps.OFN2O/Kox,PropData);
LowOFFProps=CombProps(P0,0.25,PropData);
Tf=FlProps.T0;

%Inner loop solves for surface temperature/Blowing parameter
rerr=1;
riter=1;
while rerr>10^-6

Ru=1.9858*10^-3; %kCal/K/mol;
if rdot0 <0.00033
   Ea=13.35; %kCal/mol;
   A=3.9648; % m/s;
else
   Ea=4.91; %kCal/mol;
   A=.0104; % m/s;
end

Ts=-Ea./Ru./log(rdot0./A); %pyrolosys law from Chaiverini
dh=FlProps.Cp*(FlProps.T0-Ts);
hve=MotorProps.hv+MotorProps.Cp.*(Ts-MotorProps.Tfuel);
Ur=MotorProps.OFN2O.*dh./hve./(Kox+(MotorProps.OFN2O+Kox).*dh./hve);

%Calculate the average viscosity in the boundary layer, based on
%the flame temperature and position

Csu=960;
WallVisc=0.00000812*(298.15+Csu)/(Ts+Csu)*(Ts/298.15)^(3/2);

%Calculate the average viscosity in the boundary layer, based on
%the flame temperature and position
mu0x=MotorProps.muOx;%1.5500*10^-5;% Pa*s
ViscUP=(mu0x+FlProps.Visc)/2;
ViscLw=(WallVisc+FlProps.Visc)/2;
Visc=Ur*ViscUP+(1-Ur)*ViscLw;

%Use the viscosity at the flame
%Visc=FlProps.Visc;

S0=Sm.*(G./Visc).^(-0.2);

%Blowing parameter, as defined by Lees.
B = rdot0 .* MotorProps.rho_fuel ./ (G .* S0);

% Full marxman blowing coefficient correction. Often fit with
% 1.2B^-0.77
f_B = (log(1+B) ./ B) .^ 0.8 .* ((1+1.3 .* B+0.364 .* B.^2)./((1+B./2).^2.*(1+B))).^0.2;
% f_B=1.2*B^(-0.77);
S = S0 .* f_B;
qconv = S .* G .* dh .* 1 ./ Ur;

PropTerm = (FlProps.T0 ./ Ts) .^ 0.55;
qconv = qconv .* PropTerm;

OFrad = MotorProps.OFN2O;
% OFrad = 3.4;
apnp = 0.02 * ap * P0 / psi2Pa / (1 + OFrad - ap);
% apnp = 0.134 * ap * P0 / psi2Pa / (1 + OFrad - ap);
qrad = sigma * FlProps.T0^4 * (1 - exp(-apnp));

rdotV = (qconv + qrad) / (hve * MotorProps.rho_fuel);

err = abs(rdotV - rdot0);
rdotV = rdot0 + 0.7 * (rdotV - rdot0);
riter = riter + 1;

rdot0 = rdotV;
end
Appendix D
Aerospike Hot Gas Heat Transfer Algorithm

The following is a MATLAB script that computes heat transfer into the nozzle surface given external flow conditions with the Mayer algorithm. Input fluid properties and velocities were calculated with a separate method of characteristics code based upon the method presented by Lee and Thompson [22].

```
clc
clear all
close all

m2in=39.3700787;
in2m=0.0254;
fid = fopen('AerospikeInputHybridSmall.dat');
if fid == -1
    throw(MException('myerror:badData','...
        'Error: Cannot find AerospikeInput.dat input file.'))
end

while 1
    str = fgetl(fid);
    if str == -1; break; end
    str = strtrim(str);
    if isempty(str) || strcmp(str(1),'%'); continue; end
    eval(str);
end
fclose(fid);

if exist('g','var'); temp.g = g; end
if exist('P0','var'); temp.P0 = P0; end
if exist('T0','var'); temp.T0 = T0; end
if exist('Rc','var'); temp.Rc = Rc; end
if exist('Pa','var'); temp.Pa = Pa; end
if exist('Rg','var'); temp.Rg = Rg; end
```
if exist('MW','var'); temp.MW = MW; end
if exist('npts','var'); temp.npts = npts; end
if exist('gamma','var'); temp.gamma = gamma; end
if exist('exp_ratio','var'); temp.exp_ratio = exp_ratio; end
if exist('truncation','var'); temp.truncation = truncation; end

temp.truncation = 70.0;

%Mixture Ratio of 4 for ABS/HTPB
% 100 psi
temp.P0 = 775.602;
 temp.gamma = 1.27;
temp.MW = 24.247;
temp.T0 = 3046;
temp.exp_ratio = 2.25;
temp.Pa=86.186;
temp.npts=1500;
viscosity = 0.84419; %mP


 temp.Rc = .012;

[geom design actual trunc] = aerospike_external(temp);
drawplot=0;
if exist('drawplot','var') && drawplot
    hf1=figure(1);
    hold on
    title('Geometric aerospike nozzle')
    plot(geom.Xthroat,geom.Rthroat,'k','linewidth',1.5)
    plot(geom.Xthroat,-geom.Rthroat,'k','linewidth',1.5)
    %plot(geom.X,geom.R,'m--','linewidth',1.5)
    %plot(geom.X,-geom.R,'m--','linewidth',1.5)
    %plot(actual.Xflow,actual.Rflow,'c:','linewidth',1.5)
    %plot(actual.Xflow,-actual.Rflow,'c:','linewidth',1.5)
    plot([trunc.X trunc.X(end)],[trunc.R 0],'k','linewidth',1.5)
    plot([trunc.X trunc.X(end)],[trunc.R 0],'k','linewidth',1.5)
    %plot(trunc.Xflow,trunc.Rflow,'b','linewidth',1.5)
    %plot(trunc.Xflow,-trunc.Rflow,'b','linewidth',1.5)
    xlabel('X (m)')
    ylabel('Y (m)')
    axis equal
    grid on

    %saveas(hf1,'PanCakeSpikeProf.jpg')

figure(2)
plot(geom.X,design.M,'--r','linewidth',1.5)
hold on
plot(geom.X, actual.M, 'b', 'linewidth', 1.5)
plot(trunc.X, trunc.M, '--c', 'linewidth', 1.5)

title('Mach vs. axial distance')
xlabel('Axial distance X along spike (m)')
ylabel('Mach number at spike surface')
legend('Mach at Design Altitude', 'Actual Mach', 'Truncated Mach')

figure(3)
title('Pressure vs. axial distance')
xlabel('Axial distance X along spike (m)')
ylabel('Pressure at spike surface (kpa)')
legend('Pressure at Design Altitude', 'Actual Pressure',...
      'Truncated Pressure')
end

FA = zeros(1, npts);
MA = zeros(1, npts);
for i = 2:npts
    xArea = abs(2*((geom.X(i-1)-geom.X(i))*(geom.R(i-1)-geom.R(i))));
    xP = (actual.P(i-1)+actual.P(i))/2 - actual.P(end);
    FA(i) = FA(i-1) + xP*xArea;
    MA(i) = MA(i-1) + FA(i)*(geom.X(i-1)+geom.X(i))/2;
end

aerospike_fileout

maxpts = 200;
xArr = linspace(trunc.X(1), trunc.length, maxpts);
rArr = nan(1, maxpts);
fid = fopen('SpikeProfileSmallSat.txt', 'wt+');
fprintf(fid, ' %012.5E	%012.5E	%012.5E
', geom.Xthroat(2), ..., geom.Xthroat(2), 0.0);
for i = 1:maxpts
    rArr(i) = interp1(trunc.X, trunc.R, xArr(i), 'spline');
    fprintf(fid, ' %012.5E	%012.5E	%012.5E
', xArr(i), rArr(i), 0.0);
end
fclose(fid);

flowAngle=actual.flowangle

mdot=geom.mdot
thrust=design.Thrust

dThroat_in= sqrt((geom.Xthroat(2)-geom.Xthroat(1))^2+...
    (geom.Rthroat(2)-geom.Rthroat(1))^2)*m2in
lthrt=min(dThroat_in*10/m2in, .0075);

thrtvec=[geom.Xthroat(2) geom.Rthroat(2)]-[trunc.X(1) trunc.R(1)];

thetaTrt=atan2(thrtvec(2),thrtvec(1));
theta1=thetaTrt+pi/2+20*pi/180;
theta2=thetaTrt+pi/2-20*pi/180;

thrtst1=[trunc.X(1) trunc.R(1)]+[lthrt*cos(theta1) lthrt*sin(theta1) ];
thrtst2=[geom.Xthroat(2) geom.Rthroat(2)]+[lthrt*cos(theta2)...
    lthrt*sin(theta2) ];

profspike=[
    thrtst1 0
    thrtst2 0
    geom.Xthroat(2) geom.Rthroat(2) 0
    trunc.X' trunc.R' 0*trunc.R'
    trunc.X(end) 0 0
    trunc.X(end)*2 geom.Rthroat(2) 0
    trunc.X(end)*2 0 0];

xlswrite(['MuffinSurfExp2_25_50trunc'],profspike(4:end-4,:)/in2m)

profspikein=profspike/in2m;

save MuffinSpike2.txt profspike -ASCII

figure(4)
plot(profspike(3:end-4,1),profspike(3:end-4,2),'*')

%mdot2approxarea=mdot/(geom.Rthroat(1)*geom.length)
figure(12)
plot(profspike(:,1),profspike(:,2),'*b')
for i=[1 2 3 4 [6:20:66]]
    text ( profspike(i,1),profspike(i,2),...
        [num2str(profspikein(i,1)) ' ' num2str(profspikein(i,2)) ]); end
axis equal
%xlim([-01 .006])
%ylim([0.007 0.016 ])

%Regression Stuff
%Internal Fuel Surface Area (approx)
AFuel = 0.03 * 0.07 * pi; \% m^2
rhofuel = 900;

mdotfuel = 0.2 * mdot;
rdotfuel = mdotfuel / (rhofuel * AFuel);

i = 1;
thetaCluster = pi / 2 : -pi / 50 : pi / 500;
for i = 1:length(thetaCluster)
    ltemp = lthrt * (1 - cos(thetaCluster(i)));
    CnvgLower(i,:) = [trunc.X(1) trunc.R(1) 0] + [ltemp * cos(theta1) ...
                       ltemp * sin(theta1) 0];
    CnvgUpper(i,:) = [geom.Xthroat(2) geom.Rthroat(2) 0] + [ltemp * cos(theta2) ...
                       ltemp * sin(theta2) 0];
    T = T0 / (1 + (gamma - 1) / 2);
    P = P0 / (1 + (gamma - 1) / 2)^(gamma / (gamma - 1));
end

% profspike_HeatTransfer = [ CnvgLower
%   CnvgUpper
%   geom.Xthroat(2) geom.Rthroat(2) 0
%   trunc.X' trunc.R' 0*trunc.R'
%   trunc.X(end) 0 0
%   trunc.X(end)*2 geom.Rthroat(2) 0
%   trunc.X(end)*2 0 0];

%% Heat Transfer Properties
mu = 0.859/1000/10; \% From mP to Pa*S (from CEA)
L = 0.01; \% M... approximately centimeter length scale
Ru = 8.314;
T0 = temp.T0;
P0 = temp.P0*1000;
MW = temp.MW;
gamma = temp.gamma;
Tsurf = 400;
Rg = Ru/MW*1000;

% Heat Transfer Parameters
B = 0.0296; \% Blasius
b = 1/5;
omega = 3/2;
T_star = 2680; \% Reference Temperature
Pr = 0.6266;
Cp = 2278.1; \% J/Kg*K

% Compute Conical Frustum Area, and velocity and pressure in convergent
for i=1:size(CnvgLower,1)
    A_convg(i)=pi*norm(CnvgLower(i,:)-CnvgUpper(i,:))*(CnvgLower(i,2)+... 
        CnvgUpper(i,2));
    d_convg(i)=norm(CnvgLower(i,:)-CnvgUpper(i,:));
    ConvgMach(i)=c_mach(A_convg(i)/geom.Astar,gamma,0);
end

%Ccreate Array of Mach number along entire spike
Mach=[ConvgMach actual.M(1:length(trunc.X))];

%Compute other flow properties along spike
Ts= T0./(1+(gamma-1)*Mach.^2/2);
Ps= P0./(1+(gamma-1)*Mach.^2/2).^(gamma/(gamma-1));
Vs=Mach.*sqrt(gamma.*Rg.*Ts);
Rhos=Ps./(Ts*Rg);
SpikeSurf=[ CnvgLower(:,1:2) 
        trunc.X' trunc.R' ];
ds=sqrt(diff(SpikeSurf(:,1)).^2+diff(SpikeSurf(:,2)).^2);
s=[0; cumsum(ds)];

%Compute eq
Beta=(Tsurf/T_star)^(1-b*(1+omega));
Integrand_2=Beta^(1/(1-b)).*Rhos.*Vs/mu;

%Compute integral... this is really a Reynolds number
Integral_2=cumtrapz(s,Integrand_2);

Integrand_3=(Beta.*SpikeSurf(:,2)').^(1/(1-b)).*Rhos.*Vs/mu;
Integral_3=cumtrapz(s,Integrand_3)+1;
%Start the Re at 1000 as the flow will never be completely 
%undeveloped...this gets rid of oddity at beginning of flow...
Integral_2=Integral_2+1;

%Check boundary layer size .. ensure it isn’t larger than 50% of allowable 
%thickness...
thrtIndx=length(A_convg)+1;
blHeight=Integral_2.*mu./(Rhos.*Vs);
d_convg(end+1:length(blHeight))=.1;

AdjustDev=0;
if AdjustDev==1
\(\text{blHeight} = \text{blHeight} > 0.5 \times \text{d_convg} = 0.5 \times \text{d_convg} \times (\text{blHeight} > 0.5 \times \text{d_convg});\)

\(\text{blHeight}(\text{thrtIndx} : \text{end}) = \text{blHeight}(\text{thrtIndx} : \text{end}) - (\text{blHeight}(\text{thrtIndx}) - \text{blHeight}(\text{thrtIndx} - 1));\)

\(\text{Integral}_2 = (\text{Rhos} \times \text{Vs} \times \text{blHeight}) / \mu;\)

\(\% \text{BLratio} = \text{delThrt} / (\text{dThroat_in/m2in});\)

\(\% \text{Compute h2 for a "curved plate"}\)

\(h2 = \text{Beta}^{-(1/(1-b))} \times B \times \text{Pr}^{-(-2/3)} \times \text{Rhos} \times \text{Cp} \times \text{Vs} / \text{Integral}_2^{-b};\)

\(h2(1) = 0;\)

\(h3 = h2 \times (\text{SpikeSurf(:,2)}^{(1/(1-b))} \times \text{Integral}_2 / \text{Integral}_3)^{-b};\)

\(\% \text{Check surface integral with connical frustrum relation}\)

\(\% A_{\text{LowCong}} = \pi \times \text{norm}((\text{CnvgLower(end,:) - CnvgUpper(1,:))} \times (\text{CnvgLower(end,2)} \times \text{CnvgLower}(1,2)));\)

\(\% \text{Specific heat transfer rate as a function of location...}\)

\(q = h2 \times (\text{T_s} - \text{Tsurf});\)

\(q2 = h3 \times (\text{T_s} - \text{Tsurf});\)

\(\% \text{Compute differential "skirt" area}\)

\(dS = \text{SpikeSurf(:,2)} \times \pi \times 2 \times \sqrt{1 + ([\text{diff}(\text{SpikeSurf(1:2,2)); \text{diff}(\text{SpikeSurf(:,2))} / [\text{diff}(\text{SpikeSurf(1:2,1)); \text{diff}(\text{SpikeSurf(:,1))}]^2);\}

\(\% \text{total heat flux as a function of location is then}\)

\(\text{cumQ} = \text{cumtrapz}(\text{SpikeSurf(:,1)}, q' \times dS);\)

\(\text{cumQ}_2 = \text{cumtrapz}(\text{SpikeSurf(:,1)}, q2' \times dS);\)

\(\% \text{Heat transfer per unit length}\)

\(q_p = dS \times q';\)

\(\% \text{Total heat transfer}\)

\(Q_{\text{total}} = \text{cumQ}(\text{end});\)

\(Q_{\text{total,2}} = \text{cumQ}_2(\text{end});\)

\(\% \text{Cooling Calcs... from nitrous properties 800 psi liq in, 200 psi vapor out}\)

\(\text{hout} = 399.47 \times 1000;\)

\(\% \text{hout} = 660 \times 1000;\)

\(\text{hin} = 225.80 \times 1000;\)

\(Q_{\text{coolant}} = (\text{hout} - \text{hin}) \times \text{mdot} \times 4 / 5 \% \text{Assume a mixture ratio of about 4:1}\)

\(\% \text{Coolant "saftey factor"}\)

\(\text{SF}_{\text{Coolant}} = Q_{\text{coolant}} / Q_{\text{total}}\)

\(h1 = \text{figure}(5)\)
plot(SpikeSurf(:,1), SpikeSurf(:,2), '*')
xlabel('Longitudinal Position (m)')
ylabel('Radial Position (m)')
saveas(h1, 'GridLocation.emf')

h6 = figure(6);
subplot(3,1,1)
plot(SpikeSurf(2:end,1), Ts(2:end), 'linewidth', 2)
grid on
subplot(3,1,2)
plot(SpikeSurf(2:end,1), Rhos(2:end), 'linewidth', 2)
ylabel('T (W/m^2)')

hnd7 = figure(7)
subplot(2,1,1)
% plot(SpikeSurf(2:end,1), h2(2:end), 'linewidth', 2)
hold all
plot(SpikeSurf(2:end,1), h3(2:end), 'k', 'linewidth', 2)
grid on
% legend('Heat Transfer Coefficient')
ylabel('h (W/m^2*K)')
v = get(gca, 'Position');
set(gca, 'Position', [v(1)*1.2 v(2) v(3:4)])

subplot(2,1,2)
plot(SpikeSurf(2:end,1), q(2:end), 'k', 'linewidth', 2)
grid on
ylabel('q (W/m^2)')
set(gcf, 'PaperPositionMode', 'auto')
    set(gcf, 'Units', 'inches');
    set(gcf, 'PaperSize', [4 3.5]);
    set(gcf, 'Position', [95 4 3.5]);

xlabel('Axial Position from Throat (m)')
v = get(gca, 'Position');
set(gca, 'Position', [v(1)*1.2 v(2) v(3:4)])
saveas(hnd7, 'HeatTransferCoeffMayer.pdf')

% figure(8)
% plot(SpikeSurf(2:end,1), q_p(2:end)/1000)
% ylabel('q_p (W/mm)')
\% figure(10)
\% plot(SpikeSurf(:,1),[0; ds])

\% figure(11)
\% plot(SpikeSurf(:,1),blHeight)
\% hold all
\% plot(SpikeSurf(1:length(d_convg),1),d_convg)
\% ylim([0 .01])
\% ylabel('BLHeigh')
\%
\% ABase=.02^2*pi;
\% lbase=.03;
\% k=390;
\% dTbase=300;
\%
\% Qspikebase=ABase*k*dTbase/lbase

%%Vapor Heat Transfer 425 K and 785 Kpa
Dp=.125*in2m; \%cooling pipe diameter.. assuming tubular coolant channels
\%for now

\%All properties below are nitrous at 425K and 785 kPa
Ap=Dp^2*pi/4; \%Area
Ap=2.52*10^{-6}; \%Actual
mdotv=0.08; \%kg/s
rhov=9.90; \%kg/m\(^3\) (vapor) from NIST
muv= 2.07* 10^{-5}; \%N2O viscosity, Pa*s via wolfram alpha
kv=.0288; \%N2O thermal conductivity W/m*K via Wolfram Alpha
Cpv=1.0006*1000; \% N2O J/KgK from NIST
Pr=Cpv*muv/kv

Lpipe=.06; \% Assume about 6cm total coolant channel length
AsurfCool=pi*Dp*Lpipe; \%Cooling channels surface area
AsurfCool=0.00038003061; \%Actual

\%Velocity assuming 4 cooling channels
Vv=mdotv/4/(Ap*rhov) \%veloity (m/s)

\%Reynolds number
Red=rhov*Dp*Vv/muv

\%vapor Nusselt number is then (Dittus Boelter)
Nu=0.023*Red^{-4/5}*Pr^{-0.3}
%Vapor only Heat transfer coefficient is
hvap=\textit{Nu}\*kv/Dp

%Delta T required between channel surface and vapor (\textit{Q}\textsubscript{total 2} is total %hot gas side heat transfer... about 3000 to 4000 Watts)
dTreq\textsubscript{vap}=\textit{Q}\textsubscript{total 2}/(A\textsubscript{surf Cool}\*hvap)

%Cal-Poly long beach "measured" nitrous boiling heat transfer
hboil=84000
%Delta T required between channel surface and boiling
dTreq\textsubscript{boil}=\textit{Q}\textsubscript{total 2}/(A\textsubscript{surf Cool}\*hboil)

% s0=0.957
% sl=0.497
% sv=1.691
%
% X=(s0-sl)/(sv-sl)

%All properties below are nitrous at 425K and 785 kPa
Ap=\textit{Dp}^2\pi/4; %Area
\textit{mdot l}=0.08; %kg/s
rhol=752.95; %Saturated liquid at 800 psi (NIST)
\textit{mul}=6.26\textasteriskcentered10^{-5}; %N2O visocisty, Pa\*s via wolfram alpha
kl=.0836; % N2O thermal conductivity W/m\*K via Wolfram Alpha
Cpl=3.7095\textasteriskcentered1000; % N2O J/KgK from NIST
Prl=Cpl*\textit{mul}/kl

%Velocity assuming 4 cooling channels
Vl=\textit{mdot l}/4/(Ap*rhol) %veloity (m/s)

%Reynolds number
\textit{Rel}=rhol*Dp*Vl/\textit{mul}

%liquid Nusselt number is then (Dittus Boelter)
Nul=0.023*\textit{Rel}^{-\text{4/5}}*Prl^{-0.3}

%Vapor only Heat transfer coefficient is
%\textit{hl}=Nul*\textit{kl}/Dp

%From sutton
\textit{hl}=0.023*Cpl*\textit{mdot l}/4/\textit{Ap}*(Dp*Vl*rhol/\textit{mul})^{-0.2}*(\textit{mul*Cpl/kl})^{-2/3}
Appendix E

Multi-Phase Fluid Flow Algorithm

The following is a MATLAB script used to calculate the mass fluxes and flow rates in the MUPHyN orifices using the modified non-homogenous, non-equilibrium method presented by Dyer [150], given nitrous oxide fluid properties and a known total heat transfer rate into the fluid.

```matlab
close all
clear all
clc

psi2Pa=6894.75729;
in2m=0.0254;
Cd=0.85;
A1=1.97932*10^-6;
%A1=1.1675*10^-6;
A2=5.48*10^-6;

Cd2=0.85;

T1=295;
P1=810*psi2Pa;
P2=400*psi2Pa;
P0=112*psi2Pa;
mdot=0.08;
Q=3.500;

[Props1]= N2OStateSolver(P1,[],[],T1);
GTOT1=ChokedMassFlux(Props1,P2);
Props2=N2OStateSolver(P2,Props1.h,[],[]);

h3=Props2.h+Q/mdot;
Props3=N2OStateSolver(P2,h3,[],[]);
```
GTOT2=ChokedMassFlux(Props3,P0);

Props4=N2OStateSolver(P0,h3,[],[]);

Pvec=[0.01:.01:0.99 0.999]*P1;
for i=1:length(Pvec)

[GTOT(i) GHEM(i) GSPI(i) GTOT_UNC(i)]=ChokedMassFlux(Props1,Pvec(i));
end

figure(1)
hold all
plot(Pvec,GTOT)
plot(Pvec,GHEM)
plot(Pvec,GSPI)
plot(Pvec,GTOT_UNC)
Appendix F

Multi-Phase Fluid Heat Transfer Algorithm

The following is a MATLAB script used to calculate the heat transfer coefficient for heat transfer from a hot wall into saturated nitrous oxide in a coolant channel provided fluid data for nitrous oxide and coolant channel parameters.

```matlab
close all
clear all
clc

psi2Pa=6894.75729;
in2m=0.0254;

%Temperatures (K) these correspond to saturation pressures of 200 to 800 psi
T=[244.57 257.96 268.39 277.07 284.54 291.14 297.05];

rhoSat=400; %this is a saturated density, this will cause the N2O props %code %return properties for both vapor and liquid

%initial entropy... for isentropic expansion calcs, rounded slightly up to %avoid rounding errors to negative qualites at 800 psi
s0=0.9165;

%Pull nitrous properties as calculated via Helmholtz relations
for i=1:length(T)
    [Props] = N2OProps(T(i),rhoSat) ;
    P(i)=Props.P;
    CpL(i)=Props.cp_l;
    CpV(i)=Props.cp_v;
    rhoL(i)=Props.rho_l;
    rhoV(i)=Props.rho_v;
    hL(i)=Props.h_l;
    hV(i)=Props.h_v;
    hfg(i)=Props.h_v-Props.h_l;
    sL(i)=Props.s_l;
    sV(i)=Props.s_v;
```
%Calculate quality based on isentropic expansion
X(i)=(s0-sL(i))/(sV(i)-sL(i));

%Viscosity and thermal conductivty data from via Wolfram Alpha via... one
%of about 20 listed possible
%sources... not sure which one this actually came from (Nist does not list
%k or mu for nitrous)
muL=[1.29*10^{-4} 1.09*10^{-4} 9.43*10^{-5} 8.33*10^{-5} 7.45*10^{-5} ...
   6.67*10^{-5} 5.94*10^{-5}]; %Pa*s
muV=[1.28*10^{-5} 1.38*10^{-5} 1.47*10^{-5} 1.55*10^{-5} 1.63*10^{-5} ...
   1.72*10^{-5}]; %Pa*s
kL=[0.136 0.1218 0.1108 0.1017 0.09383 0.087 0.0811]; %W/m*K
kV=[0.0154 0.01738 0.01937 0.02155 0.02412 0.02736 0.03184]; %W/m*K

Dp=.125*in2m; %cooling pipe diameter... assuming tubular coolant channels
%for now
Ap=(1/16*in2m)^2; %Cross sectional coolant Area

As=(1/16*in2m)*6*(0.31*2*pi*in2m);
mdotl=0.08; %kg/s
%rhol=752.95; %Saturated liquid at 800 psi (NIST)

PrL=CpL.*muL./kL;

%Velocity (assuming 4 cooling channels)
Vl=mdotl./4./(Ap.*rhol); %veloity (m/s)

%Reynolds number
Rel=rhol.*Dp.*Vl./muL;

%liquid Nusselt number is then (Dittus Boelter)
Nul=0.023.*Rel.^(4/5).*PrL.^0.3;

Q=3500; %total Watts used 3k as about an average from our analysis
qs=Q/(As); %Specific heat transfer in W/m^-2
G=1.0; % This is a fluid and wall material coefficient that is generally
%around 1.0
mVap=Q./hfg; %total mass of liquid N2O that will need to be vaporized for
%heat transfer

%Average quality... based on inlet and outlet quality
Xchange=mVap./(mdotl*1000);
Xout=Xchange+X;
Xbar=((X+Xout))./2;
CpLkg=CpL*1000; %Matthew's N20 code has density in kg, but Cp is in J/g
%not J/kg, thus
%conversion is necessary

%From Sutton, liquid heat transfer coefficient
hl=0.023.*CpLkg.*mdotl./4./Ap.*(Dp.*Vl.*rhoL./muL).^(-0.2).*
(muL.*CpLkg./kL).^(-2/3);

% From Incropera "Fundamentals of Heat and Mass Transfer" correlation comes
% from Kandlikar heat transfer book.. these are the boiling to liquid heat
% transfer ratios. I assumed no stratification for the phases, figure it
% will be moving fast enough and lines will be too short for liquid to move
% to bottom of channel thus Froude number is 1. This relation is
% theoretically applicable for qualities from 0 to 0.8... we are well
% within this range (between .12 and .42)

hr1=0.6683.*(rhoL./rhoV).^(0.1).*Xbar.^(0.16).*(1-Xbar).^(0.64).*
+1058*(qs./((mdotl*1000/4./Ap).*hfg)).^0.7.*(1-Xbar).^0.8.*G

hr2=1.136.*(rhoL./rhoV).^(0.45).*Xbar.^(0.72).*(1-Xbar).^(0.08).*
+667.2*(qs./((mdotl*1000/4./Ap).*hfg)).^0.7.*(1-Xbar).^0.8.*G

%total boiling heat transfer coefficint W/m^2K
hboil=hl.*max([hr1;hr2])

%Average outer coolant channel temperature required for heat transfer
tSpike=T+Q./(As.*hboil)

h1=figure(1)
plot(P/psi2Pa*10^6,tSpike-273.15,'linewidth',2)
hold all
plot(P/psi2Pa*10^-6,T-273.15,'r','linewidth',2)
grid on
xlabel('N_2O coolant pressure (psi)')
ylabel('Temperature (C)')
legend('Surface Temperature','Coolant Temperature',2)
set(gcf, 'Units', 'inches');
set(gcf, 'PaperSize', [3.5 3.5]);
set(gcf,'Position',[-9 5 4 4]);
saveas(h1,'SpikeSurfaceTemperature.emf')

h2=figure(2)
plot(P/psi2Pa*10^-6,hboil,'linewidth',2)
grid on
hold all
xlabel('N_2O coolant pressure (psi)')
ylabel('Heat transfer coefficient (W/m^2K)')
set(gcf, 'Units', 'inches');
set(gcf, 'PaperSize', [3.5 3.5]);
set(gcf,'Position',[-9 5 4 4]);
saveas(h2,'SpikeHeatTransferCoeff.emf')
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RESEARCH/CAREER INTERESTS

Development, application, and/or testing in the fields of propulsion, guidance, navigation, control, flight dynamics and/or aerodynamics for aerospace applications

EDUCATION

Utah State University, Logan Utah
PhD in Mechanical Engineering  May 2013
Dissertation: Development of the Multiple Use Plug Hybrid for NanoSats (MUPHyN) Miniature Thruster
M.S. in Mechanical Engineering  May 2008
B.S. in Mechanical Engineering-Aerospace Emphasis  May 2008
• Graduate GPA 3.97
• Undergraduate GPA 3.85

PROJECT, RESEARCH, AND TEACHING EXPERIENCE

Engineering Research Assistant  5/06-Present
• Led design and experimental work on CubeSat scale hybrid motor prototype with a regeneratively cooled aerospike nozzle and secondary injection thrust vector control. Motor burned FDM manufactured fuel grain with novel fuel port
• Led design and testing of lab-scale hybrid motors to investigate hybrid motor thrust variability
• Led design and experimental work on cold flow aerospike nozzle with fluidic thrust vectoring
• Led design and construction of several complete multi-DOF thrust stand and test apparatuses
• Design and testing of hybrid rocket closed-loop throttle controller
• Design and testing of nitrous oxide decomposition reactor
• Design and testing of multi-fuel hybrid rocket fuel grain configurations
• Design and testing of multi-phase flow rate models
• Design of hydroxyl ammonium nitrate decomposition test apparatus
• Design and testing of thrust vector control algorithm of hobby-scale jet engine
• Software design for acoustics testing in preparation for AFRL MKV tests

Instructor of Flight Mechanics Course  1/13-5/13
• Taught graduate-level course on aircraft stability, dynamics, and control

Instructor Aerospace Senior Design Course  8/08-5/10
• Instructor and principal advisor to senior design course that won NASA University Student Launch Initiative competition in both 2009 and 2010. Team designed and built rocket that flew a closed-loop energy management system using pneumatically deployed airbrakes and an on-board Kalman filter to reach but not exceed a target altitude. The 2010 design also included 3 DOF aerodynamic attitude control.

Engineering Intern at SpaceDev  7/06-9/06
• Designed software for small-scale hybrid motor test fires.
SKILLS

MATLAB/SIMULINK   LabVIEW   FORTRAN
FLUENT   StarCCM   FEMAP
Solid Edge   LyX   Instrumentation
Piping and Fluid Control   Metal Shop Experience

ACADEMIC HONORS/AWARDS/ACTIVITIES

College of Engineering Graduate Student Researcher of the Year 2013, Utah State
Mechanical and Aerospace Engineering Outstanding PhD Researcher 2013, Utah State
MAE Department Outstanding Graduate Teaching Assistant 2009, Utah State
MAE Department Outstanding MS Researcher 2008, Utah State
Undergraduate Presidential Scholarship Recipient, Utah State
Active member of Utah State University Engineers Without Borders Program
NASA Rocket Mountain Space Grant Fellow 2006-2009

PEER REVIEWED PUBLICATIONS


PRESENTATIONS AND PAPERS PUBLISHED IN CONFERENCE PROCEEDINGS


