Small Satellite Reaction Wheel Optimization

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ABSTRACT

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The very "smallness" of small satellites mandates mass minimization. This paper addresses minimization of overall reaction wheel mass, including the incremental mass of the power subsystem needed to support the reaction wheel. The results are applicable to a wide range of wheel sizes and are suitable for optimization at the configuration level. For an average momentum and torque operating point, the minimization process yields wheel radius and angular velocity, as well as, the masses associated with the motor, wheel, housing, and power subsystems. Only four parameters are needed: the power supply mass penalty, a generalized motor constant, the wheel form factor, and the housing mass penalty. Additionally, extremes where the momentum or the torque equals zero are examined in light of thermal and stress constraints, respectively. Excellent correlation with past tabulated momentum versus mass data is demonstrated. Finally, current Fairchild IR&D efforts on a small wheel concept for small satellites is described.

Introduction

This study addresses the problem of determining the lowest overall reaction wheel mass including the mass of that part of the spacecraft power system needed to support it.

Consider a given peak momentum and torque performance. If the cost of power is low (in terms of mass) then a large angular velocity, small inertia, small motor RWA would minimize the overall mass. This study examines the inter-relationships amongst the parameters and derives an equation for the minimum overall weight.

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Derivation

Parameters are defined and dimensions given in Table 1. Units are SI.

The fundamental motor equation is:

$$V = IR + k_B\dot{\Theta}$$

This is the static or steady state expression. For this study there is no interest in dynamic performance.

Multiplying by I,

$$VI = I^2R + k_B I\dot{\Theta} \quad (1)$$

$$P = I^2R + k_B I\dot{\Theta} \quad (2)$$

$$T = k_t I \quad (3)$$

where k_t is the torque constant.

Substituting from Equation (3) into Equation (2)

$$P = \frac{T^2}{k_t^2}R + \frac{k_B}{k_t}T\dot{\Theta} \quad (4)$$

In SI units

$$k_B = k_t = k_1 \quad (5)$$

Substituting from Equation (5) into Equation (4):

$$P = \frac{T^2}{k_1^2}R + T\dot{\Theta} \quad (6)$$

The first term is the power dissipated in the motor winding and the second term is the mechanical power delivered to the load.

The next step is to relate each of these terms to the motor mass and wheel mass, respectively.

It is instructive to first consider a simple relationship with the aid of Figure 1.

A motor is shown along with an enlarged version. How does the power dissipation change when the motor length is increased? In motor manufacturing parlance, this is called the stack length, or simply stack, because it consists of a stack of ferromagnetic laminations. Assume that the stack length is doubled. The wire size and number of turns remains the same. Neglect the resistance of the portions of the winding at the ends. Then the new motor winding will have twice the resistance of the old one. For the same winding current as before, the torque will obviously be doubled from Figure 1. Hence, the torque constant has doubled and the new power (for the same torque output) is:

$$P = \frac{T^2}{(2k_t)^2}(2R) = \frac{1}{2} * \frac{T^2}{k_t^2} R \qquad (7)$$

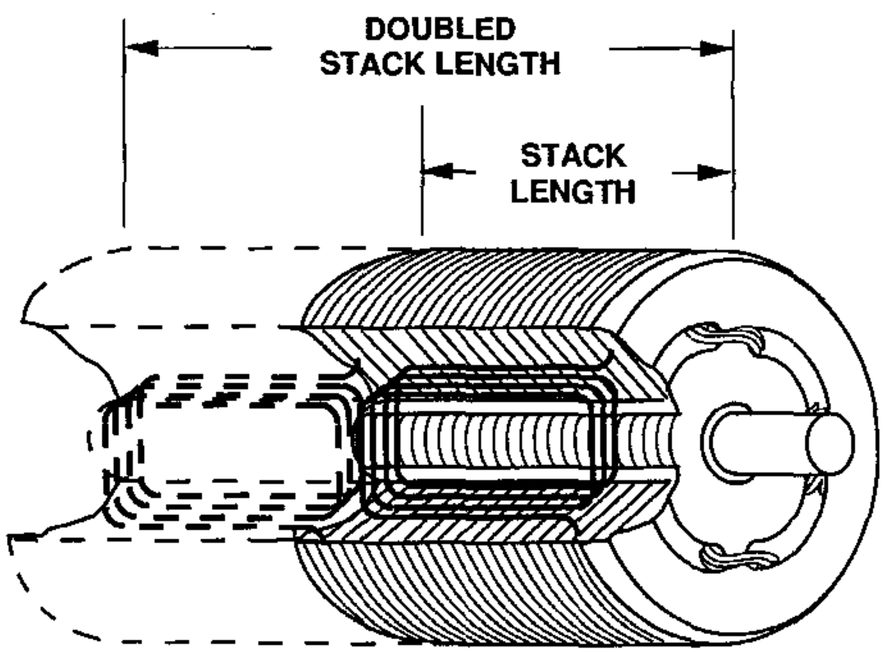

Figure 1: Motor Weight-Power Tradeoff

Therefore, if motor weight is doubled, power dissipation is halved.

The foregoing relative example needs to be replaced with an absolute model in order to obtain a generally applicable result.

$\frac{R}{k_1^2}$ is numerically equal to $\frac{1}{K_m^2}$

$$\frac{R}{k_1^2} = \frac{1}{K_m^2} \tag{8}$$

K_m is the motor constant given by manufacturers in their data sheets.

Substituting into Equation (6):

$$P = T(\frac{1}{K_m^2}T + \dot{\Theta}) \tag{9}$$

$$K_m = \frac{Nm}{\sqrt{watt}} \tag{10}$$

$$K_m^2 = \frac{(Nm)^2}{watt} \tag{11}$$

whereas R and k_1 depend upon the particular winding inpedance chosen, K_m is general.

Based upon the simple forgoing example (Figure 1) it can be appreciated that K_m increases with size, or mass. This gives the means to get motor mass into the equation. Using Inland Motor data sheets the ratio:

$$b = \frac{M_m}{K_m} \tag{12}$$

is, to a first approximation, independent of mass. From Reference 1, for 29 motors listed (45 grams to 8.3 Kg) the value of b is 5.23 with standard deviation 1.28.

Substituting from Equation (12) into Equation (9)

$$P = T(\frac{b^2}{M_m^2}T + \dot{\Theta}) \tag{13}$$

The value of b given above represents motor design based upon Alnico 5 technology. Equivalent values have been obtained using data from another manufacturer.

This overall figure of merit, b, depends upon many parameters such as copper resistivity and density, ferromagnetic material, permeability and density, air gap, and others. Motors designed with rare earth permanent magnets will have better (smaller numerical value) figures of merit.

The angular velocity must be related to momentum.

$$\dot{\Theta} = \frac{H}{J} \tag{14}$$

Substituting into Equation (13)

$$P = T\left(\frac{b^2}{M_m^2}T + \frac{H}{J}\right) \tag{15}$$

$$J = \frac{M_w r^2}{a} \qquad 1 < a < 2 \tag{16}$$

The parameter, a, depends upon inertia wheel configuration, 2 for a disc and 1 for a rim, the latter corresponding to conventional reaction wheel design.

Substituting into Equation (15)

$$P = T\left(\frac{b^2}{M_m^2}T + \frac{aH}{r^2 M_w}\right) \tag{17}$$

This gives the input power required to obtain a total torque at a given momentum value. It is composed of the mechanical power and motor loss.

It is obvious from Equation (17) that wheel radius should be large and an open wheel such as on TIROS allows this. A large open wheel is also applicable to a space station. However, for various reasons, an enclosed, evacuated reaction wheel assembly is the norm. The enclosure provides a constraint on r (another constraint is maximum wheel stress, to be considered later).

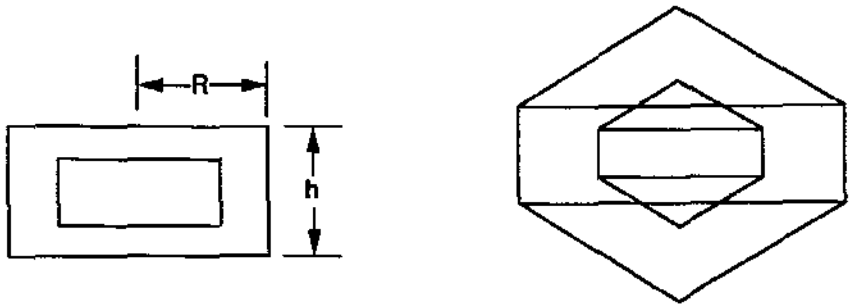

Figure 2: Reaction Wheel Housing

For simplicity, consider a pill box shaped evacuated enclosure as shown in Figure 2. The maximum stresses in the circular flat plates and cylindrical section are proportional to:

$$\frac{r^2}{t^2} \qquad \text{where} \begin{array}{lcl} t & = & \text{material thickness} \\ r & = & \text{case radius} \end{array}$$

Therefore, for a given working stress;

$$t = k_2 r$$

The flat plate volume is

$$\pi r^2 t = \pi k_2 r^3$$

The cylindrical section volume is:

$$2\pi r h t = 2\pi k_2 r^2 h$$

But note that h is proportional to r if the aspect ratio remains invarient with size. Then the cylindrical section volume also varies as r^3. Finally, it can be concluded that the mass of the housing is proportional to r^3. Replacement of flat plates with conical sections as well as internal strengthening webs in a real reaction wheel assembly will not alter the basic cubic dependency. Therefore:

$$M_H = kr^3 \tag{18}$$

Detailed design information on the TOPEX Honeywell wheel is available and it will be assumed that it represents a mature mass-efficient design.

From this;

$$k = 481 kg/m^3$$

In order to minimize the overall mass a power penalty is needed.

$$P = cM_p \tag{19}$$

M_p is the increment of power supply mass needed to obtain the power, P,

The overall mass is then

$$M_T = M_P + M_m + M_w + M_H \tag{20}$$

Substituting from Equation (17) into Equation (19) and then into Equation (20);

$$M_T = \frac{T}{c}(\frac{b^2}{M_m^2}T + \frac{aH}{r^2 M_w}) + M_m + M_w + M_H \tag{21}$$

This can be minimized by:

$$\frac{\partial M_t}{\partial M_m} = 0 \qquad \frac{\partial M_t}{\partial M_w} = 0 \qquad \frac{\partial M_t}{\partial r} = 0 \tag{22}$$

All second partial derivatives are greater than zero.

Solving Equation (21);

$$M_m = \left(\frac{2T^2b^2}{c}\right)^{\frac{1}{3}} \tag{23}$$

$$M_w = \frac{1}{r}\left(\frac{aHT}{c}\right)^{\frac{1}{2}} \tag{24}$$

$$r = \left(\frac{2aHT}{3ckMw}\right)^{\frac{1}{5}} \tag{25}$$

Substituting for r from Equation (25) into Equation (24);

$$M_w = \left[\left(\frac{aHT}{c}\right)^3\left(\frac{3k}{2}\right)^2\right]^{\frac{1}{8}} \tag{26}$$

and from Equation (25) into Equation (18);

$$M_H = \left[\frac{4ak^{\frac{2}{3}}HT}{9c}\right]^{\frac{3}{8}} \tag{27}$$

The 3 following equations have been derived from foregoing ones:

$$r = \left(\frac{2}{3k}\right)^{\frac{1}{4}}\left(\frac{aHT}{c}\right)^{\frac{1}{8}} \tag{28}$$

$$\dot{\theta} = \left[\left(\frac{3k}{2}\right)^2\frac{a^3c^5H^3}{T^5}\right]^{\frac{1}{8}} \tag{29}$$

$$\dot{\theta}r = \sqrt{\frac{acH}{T}} \tag{30}$$

Substituting from Equations (23), (25) and (27) into Equation (21);

$$M_{T_{MIN}} = \left(\frac{b^2T^2}{4c}\right)^{\frac{1}{3}} + \left(\frac{3}{2}\right)^{\frac{1}{4}} k^{\frac{1}{4}} \left(\frac{aHT}{c}\right)^{\frac{3}{8}} + \left(\frac{2T^2b^2}{c}\right)^{\frac{1}{3}} + \left(\frac{3}{2}\right)^{\frac{1}{4}} k^{\frac{1}{4}} \left(\frac{aHT}{c}\right)^{\frac{3}{8}} + \left(\frac{2}{3}\right)^{\frac{3}{4}} k^{\frac{1}{4}} \left(\frac{aHT}{c}\right)^{\frac{3}{8}} \tag{31}$$

This is the minimum overall mass.

The five terms are described below:

1) Mass of power supply required to provide motor I^2R power
2) Mass of power supply required to provide motor mechanical power
3) Motor mass
4) Wheel mass
5) Housing mass

Note the following:

a) The power supply mass needed to furnish mechanical power equals the wheel mass.
b) The ratio of housing mass to wheel mass is $\frac{2}{3}$.
c) The power supply mass needed to furnish motor I^2R loss is equal to one half the motor mass.

Equation (31) may be reduced to:

$$M_{T(MIN)} = \frac{3}{2^{\frac{2}{3}}} \left(\frac{b^2T^2}{c}\right)^{\frac{1}{3}} + \sqrt{2}\left(\frac{8}{3}\right)^{\frac{3}{4}} k^{\frac{1}{4}} \left(\frac{aHT}{c}\right)^{\frac{3}{8}} \tag{32}$$

This very general equation relates minimum overall mass to the two variables of interest using only four parameters. The foregoing equations provide all the other design points of interest as well. Moreover, they give a measure of sensitivity to the four key input parameters.

The reaction wheel assembly mass consists of the last three terms of Equation (31). Two terms are combined to yield Equation (33).

$$M_{RWA} = \left(\frac{2T^2b^2}{c}\right)^{\frac{1}{3}} + \frac{5}{3}\left(\frac{3}{2}k\right)^{\frac{1}{4}} \left(\frac{aHT}{c}\right)^{\frac{3}{8}} \tag{33}$$

Figure 3 shows a plot of T and H for constant M_{RWA}

As an example, consider that as a result of a spacecraft disturbance analysis a torque of 0.06 Nm was needed in combination with 1 Nms momentum as an average operating point. Then the optimum reaction wheel would have, from Figure 3, a mass of 2 Kg. (This is for the parameter values of Table 1.) From Figure 4, the wheel radius is 0.11 meter, wheel mass is 1 Kg and the power supply mass in 1.15 Kg. Wheel velocity is 86.5.

From Figure 4, the power supply mass is flat, a very weak function of momentum. This is because, for constant reaction wheel assembly mass, mechanical power and motor loss power vary inversely.

Note the effect of the parameter c in Figure 3. The performance of the 2 Kg RWA is markedly decreased when the power/mass ratio of the spacecraft power subsystem drops from 5.2 to 2.2.

These equations provide a top down system evaluation of a reaction wheel assembly, useful in preliminary design and proposal work.

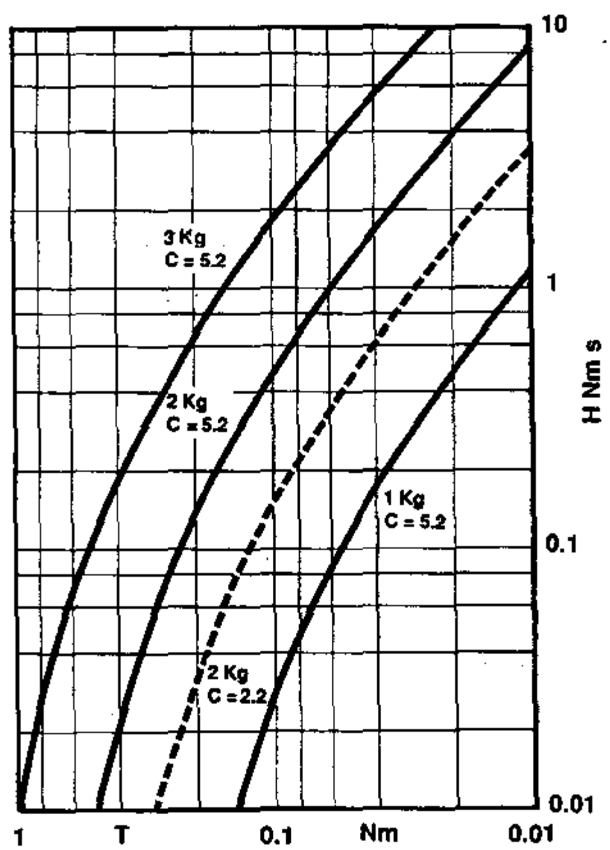

Figure 3: Optimum Mass

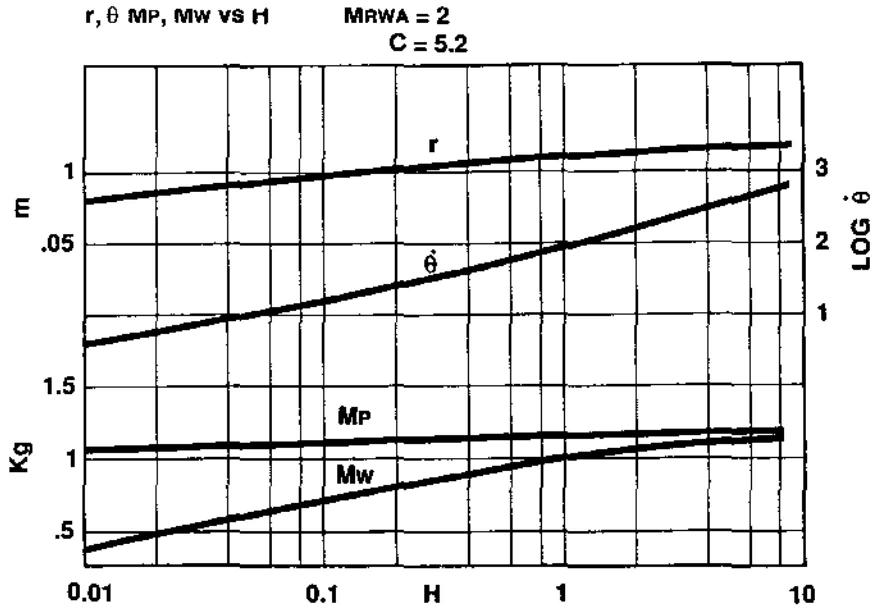

Figure 4: Wheel Parameters

Suppose the disturbance torque is orbital cyclic. Since the momentum is the time integral of torque it is easy to determine the product TH. In the cyclic case, one can consider energy exchange between battery and wheel (where the motor is capable of also acting as a generator) with the array furnishing only motor losses. There are probably two different power penalty values, c, in this case.

These equations were originally developed to size small RWA's for LWS (Light Weight Satellite), 40 kg. They are equally applicable to more normal size RWA's, predicting the TOPEX RWA mass closely, but showing less power. The TOPEX RWA, from a scale cross sectional diagram appears to have a small motor.

From a sales standpoint, it is advantageous for RWA manufacturers to advertize a large momentum to mass ratio. However, this superficial criteria obtained by a small motor may result in greater overall mass as can be seen from the foregoing development.

These system level equations cannot be used indiscriminantly. Constraints such as maximum tangential wheel velocity and motor temperature must be kept in mind.

Comparison: Momentum vs. Mass

It is of interest to compare the momentum wheel mass calculated from Equation (33) with various real wheels and previous work in this area. Ref 2 states: A rule of thumb relationship of wheel housing and associated electronics weight to the angular momentum for both reaction wheels and momentum wheels is:

$$W = 7h^{0.4} \qquad \text{(non SI)}$$

Converted to SI units the equation is:

$$M_{RWA} = 2.8H^{0.4} \tag{34}$$

This is independent of torque.

Figure 5 shows a comparison between Equations (33) and (34). Points corresponding to a number of Bendix and Honeywell wheels are also shown. Equation (33) is plotted for three values of torque. The "rule of thumb" slope of 0.4 compares with the theoretical slope of 0.375 for small and constant T.

At $H = 5$ the two masses are equal if $T = 0.18$. A logical assumption is that T increases with H. The slopes are equal (0.4) if T increases as $H^{0.067}$; a very weak function of H.

This paper provides a solid theoretical foundation for the rule of thumb cited above and moreover, shows the importance of torque and power penalty.

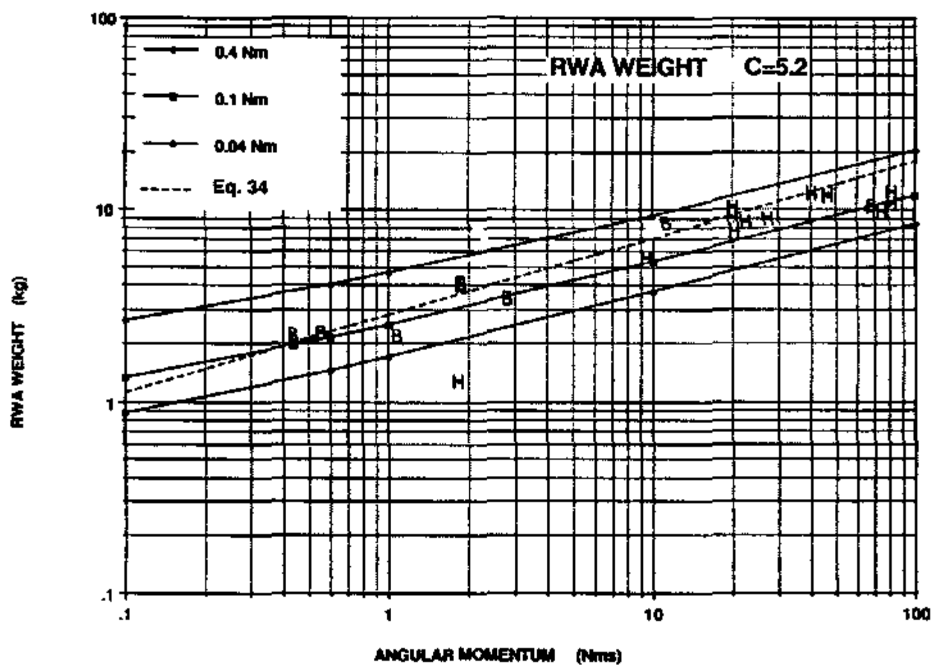

Figure 5: Comparison: Theory and Measured Data

Analysis for H or T = 0

It is useful and instructive to examine these equations for the singular points where either T or H approach zero.

When $H = 0$, Equations (26) thru (29) show that

$$r = \dot{\theta} = M_w = M_H = 0$$

and $$M_m = \left(\frac{2T^2b^2}{c}\right)^{\frac{1}{3}}$$ from Equation (23)

In effect, we have simply a motor producing a torque. An example of this is a motor working against a torsional spring to keep an optical shutter open, such as occurs on a star tracker. Equation (23) still gives the motor mass which minimizes overall mass. An examination of motor thermal data indicates that for motors of the size usually encountered in spacecraft, and for normal values of c, the optimum motor mass will be greater than that obtained considering thermal dissipation limits.

As torque approaches zero, Equations (28), (29), and (30) are useful. The torque from Equation (3), is the motor produced torque which will always be greater than zero due to bearing, windage and magnetic losses. However, it is still instructive to examine the equations.

The wheel radius approaches zero, and velocity approaches infinity as does the tangential rim velocity given by Equation (30). But the tangential rim velocity is related to the material stress.

For a thin rim the stress is simply:

$$\sigma = \rho_m(\dot{\theta}r)^2 \tag{35}$$

For a flat disc with no central hole the maximum tangential stress equals the maximum radial stress and occurs at the center.

The maximum stress is

$$\sigma = \frac{3+\gamma}{8}\rho_m(\dot{\theta}r)^2 \tag{36}$$

From Equation (30),

$$(\dot{\theta} r)^2 = \frac{acH}{T} \tag{37}$$

The maximum stress is then

$$\sigma = k_3 \rho_m \frac{acH}{T} \tag{38}$$

where $k_3 = f(\gamma)$, a function of geometry and Poisson's ratio.

Wheel stress limit will not be pursued for real rotor assemblies. Although important in energy storage applications, practical mechanical bearing life considerations, as well as a man-rated test environment, preclude operating near that limit. This exercise demonstrates parametrically how the limiting stress constrains wheel radius and velocity.

Equation (38) is solved for T:

$$T > \frac{acpmk_3 H}{\sigma} \tag{39}$$

where σ is interpreted as the maximum allowable stress. Substituting into Equations (28) and (29)

$$r > \left(\frac{2}{3k}\right)^{\frac{1}{4}} \left(\frac{a^2 k_3 \rho_m H^2}{\sigma}\right)^{\frac{1}{8}} \tag{40}$$

$$\dot{\theta} < \left[\left(\frac{3k}{2}\right)^2 \frac{\sigma^5}{a^2 k_3^5 \rho_m^5 H^2}\right]^{\frac{1}{8}} \tag{41}$$

Comparison: 3 axis (3DOF) Momentum Wheel versus 3 (SDOF) Reaction Wheels

We compare a single gimbaled momentum wheel which has three degrees of freedom with three reaction wheels each with a single degree of freedom. The comparison is based upon equal momentum storage capability of either sign on each axis. For fair comparison, the 3DOF wheel must be capable of encompassing the momentum operating envelope of the 3 SDOF wheels. This is a cube centered at the origin for the reaction wheels but displaced from the origin by an amount equal to the bias momentum for the 3DOF momentum wheel.

If the gimbal angle is $\pm 90°$ in each axis (see Figure 6) then the maximum momentum of the 3DOF wheel is:

$$\sqrt{6}H$$

and for the equivalent 3 wheels is:

$$3H$$

Using the relationship cited earlier the respective masses are:

$$\begin{aligned} (\sqrt{6}H)^{0.4} &= 1.43H^{0.4} \quad 1 \text{ 3DOF WHEEL} \\ 3(H)^{0.4} &= 3.00H^{0.4} \quad 3 \text{ SDOF WHEELS} \end{aligned}$$

Therefore, the 3DOF wheel is less than half the mass of 3 SDOF wheels. Even when the gimbal angle is restricted to $\pm 12°$ to accommodate Flex- Pivots, the 3DOF wheel mass is still only 73% of the 3 SDOF wheels. Gimbal mass must be included. This has not been studied in detail as yet, but a monocoque internal gimbal design should still retain the advantage for the 3DOF wheel.

Another advantage of the 3DOF wheel, especially during maneuvers, is its CMG quality of changing momentum direction at low power cost when compared to 3 SDOF wheels.

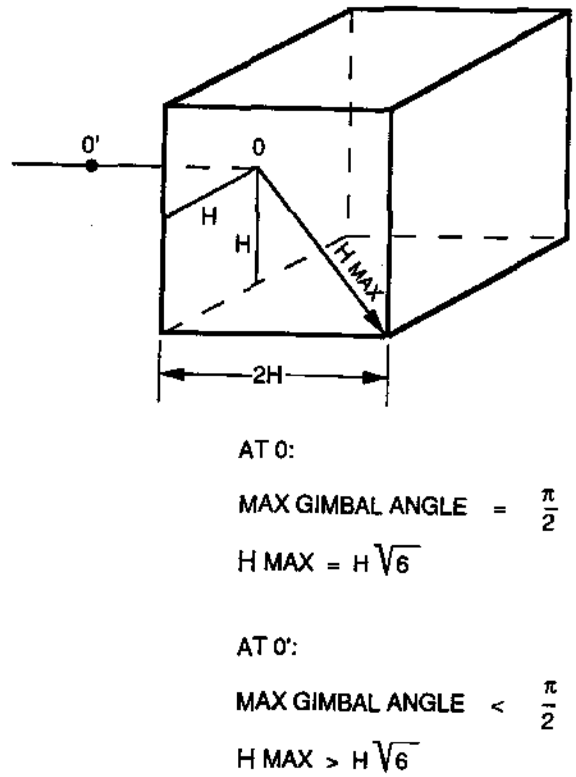

Figure 6: Momentum Envelope

References

1. Direct Drive DC Torque Motors, Inland Motor Corporation
2. J. Spacecraft, August 1971 Volume 8, No. 8, "Attitude Stabilization of Synchronous Communications Satellites Employing Narrow-Beam Antennas", H. J. Dougherty, K. L. Lebsock, J.J. Rodden

Acknowledgement: The assistance of R. Krysinski of Fairchild Space Company in obtaining a realistic value for the power penalty, c, and for generating Figure 5 is gratefully acknowledged.

Table 1
Symbols

Symbols	Description	Units	Value
V	motor voltage	volts	–
I	motor current	A (amperes)	–
P	motor power	watts	–
R	motor winding resistance	ohms	–
k_B	motor back emf constant	volt sec	–
k_t	motor torque constant	Nm/amp	–
k_1	$= k_B = k_t$	$\frac{Kgm^2}{As^2}$	–
k_2	housing constant	1	–
k	housing mass penalty	$\frac{Kg}{m^3}$	481
k_3	f(γ)	1	–
$\dot{\theta}$	motor velocity	sec^{-1}	–
T	motor torque	Nm	–
H	wheel momentum	Nm	–
K_m	motor constant	$\frac{Nm}{\sqrt{watt}}$	–
M_m	motor mass	Kg	–
M_w	wheel mass	Kg	–
M_H	housing mass	Kg	–
M_p	power source mass	Kg	–
M_T	total mass	Kg	–
Kg	kilogram	Kg	–
b	motor figure of merit	$\frac{Kg\sqrt{watt}}{Nm}$	$\sim$ 5.23
J	wheel inertia	Kgm^2	–
a	wheel geometry factor	1	1 to 2
t	housing thickness	m	–
r	wheel radius	m	–
c	power supply figure of merit	$\frac{watts}{Kg}$	$\sim$ 5.2
γ	poissons ratio	1	$\sim$ .33
ρ_m	mass density	$\frac{Kg}{m^3}$	–
σ	maximum material stress	$\frac{N}{m^2}$	–