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A CONTROLLED STUDY OF THE FLIPPED CLASSROOM WITH  
NUMERICAL METHODS FOR ENGINEERS

by

Jacob L. Bishop

A dissertation submitted in partial fulfillment  
of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Engineering Education

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UTAH STATE UNIVERSITY  
Logan, Utah

2013

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## Abstract

A Controlled Study of the Flipped Classroom with Numerical Methods for Engineers

by

Jacob L. Bishop, Doctor of Philosophy

Utah State University, 2013

Major Professor: Dr. Gilberto E. Urroz  
Department: Engineering Education

Recent advances in technology and ideology have unlocked entirely new directions for education research. Mounting pressure from increasing tuition costs and free, online course offerings are opening discussion and catalyzing change in the physical classroom. The flipped classroom is at the center of this discussion. The flipped classroom is a new pedagogical method, which employs asynchronous video lectures, practice problems as homework, and active, group-based problem-solving activities in the classroom. It represents a unique combination of learning theories once thought to be incompatible—active, problem-based learning activities founded upon constructivist schema and instructional lectures derived from direct instruction methods founded upon behaviorist principles. The primary reason for examining this teaching method is that it holds the promise of delivering the best from both worlds. A controlled study of a sophomore-level numerical methods course was conducted using video lectures and model-eliciting activities (MEAs) in one section (treatment) and traditional group lecture-based teaching in the other (comparison). This study compared knowledge-based outcomes on two dimensions: conceptual understanding and conventional problem-solving ability. Homework and unit exams were used to assess conventional problem-solving ability, while quizzes and a conceptual test were used to measure

conceptual understanding. There was no difference between sections on conceptual understanding as measured by quizzes and concept test scores. The difference between average exam scores was also not significant. However, homework scores were significantly lower by 15.5 percentage points (out of 100), which was equivalent to an effect size of 0.70. This difference appears to be due to the fact that students in the MEA/video lecture section had a higher workload than students in the comparison section and consequently neglected to do some of the homework because it was not heavily weighted in the final course grade. A comparison of student evaluations across the sections of this course revealed that perceptions were significantly lower for the MEA/video lecture section on 3 items (out of 18). Based on student feedback, it is recommended that future implementations ensure tighter integration between MEAs and other required course assignments. This could involve using a higher number of shorter MEAs and more focus on the early introduction of MEAs to students.

(283 pages)

## Public Abstract

A Controlled Study of the Flipped Classroom with Numerical Methods for Engineers

by

Jacob L. Bishop, Doctor of Philosophy

Utah State University, 2013

Major Professor: Dr. Gilberto E. Urroz

Department: Engineering Education

Recent advances in technology and new ways of using it have led to new possibilities for education research. Increasing tuition costs and free, online course offerings are two influences that have led researchers to re-consider the wisdom of conventional teaching methods and to consider alternatives. The flipped classroom is a new teaching method, which uses video lectures and practice problems as homework, while group-based problem-solving activities are used in the classroom. It combines aspects of two learning theories once thought to be incompatible—constructivism and behaviorism. Active, problem-based learning activities are based on the theories of constructivism, and direct instructional (video) lectures are based on behaviorist principles. The main reason for studying the flipped classroom is that it can potentially deliver the best from both worlds. A controlled study of students taking a second-year university course in numerical methods was conducted that used video lectures and model-eliciting activities (MEAs) in one section (treatment) and traditional group lecture-based teaching in the other (comparison). This study compared knowledge in two areas: conceptual understanding and conventional problem-solving ability. Homework and unit exams were used to measure conventional problem-solving ability, while quizzes and a conceptual test were used to measure conceptual understanding. No difference was found between the two sections on conceptual understanding (measured by quiz and concept test

scores). No difference in exam scores was found, either. However, homework scores were 15.5 percentage points (out of 100) lower for the comparison section, which is considered to be a large difference. This difference is probably due to the fact that students in the treatment section had a higher workload than students in the comparison section and did not complete some of the homework because it did not count very much toward the final course grade (5% out of 100%). Student responses to an opinion-based survey of the class were also compared. Students in the treatment section gave lower ratings for the course than students in the comparison section on 3 out of 18 items. The responses on the remaining 15 items were indistinguishable. Based on student comments about the course, it was recommended that future studies make sure there is tighter integration between in-class group activities (MEAs) and other assignments. This might involve shortening the length of group problems so that more problems can be solved in the same time. It should also include more guidance for students during early stages of group problem solving, and a better explanation of why solving MEAs will help students in their future careers.

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Jacob L. Bishop



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# Chapter 1

## Introduction

### 1.1 Motivation

There are currently two related movements that are combining to change the face of education. The first of these is a technological movement. This technological movement has enabled the amplification and duplication of information at an extremely low-cost. It started with the printing press in the 1400s, and has continued at an ever-increasing rate. The electronic telegraph came in the 1830s, wireless radio in the late 1800s and early 1900s, television in the 1920s, computers in the 1940s, the internet in the 1960s, and the world-wide web in the 1990s.

As these technologies have been adopted, the ideas that have been spread through their channels have enabled a second movement. Whereas the technological movement sought to overcome real physical barriers to the free and open flow of information, this ideological movement seeks to remove the artificial, man-made barriers. This is epitomized in the free-software movement, although this movement is certainly not limited to software.

A good example of this can be seen from the encyclopedia. Encyclopedia Britannica has been continuously published for nearly 250 years<sup>1</sup> (Encyclopaedia Britannica, 2012a). Although Encyclopedia Britannica content has existed digitally since 1981, it was not until the advent of Wikipedia in 2001 that open access to encyclopedic content became available to users worldwide. Access to Encyclopedia Britannica remains restricted to a limited number of paid subscribers (Encyclopaedia Britannica, 2012b), but access to Wikipedia is open, and the website receives over 2.7 billion US monthly page views. Thus, although the technology and digital content was available to enable free access to encyclopedic content, ideological

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<sup>1</sup>Since 1768.

roadblocks prevented this from happening. It was not until these ideologies had been overcome that humanity was empowered to create what has become the world's largest, most up-to-date encyclopedia (Wikipedia, 2012).

In a similar way, the combined effects of these two movements on higher education are becoming evident. Educational research on instructional technology has made significant advances. Studies show that video lectures (slightly) outperform in-person lectures (Cohen, Ebeling, & Kulik, 1981), with interactive online videos doing even better<sup>2</sup> (McNeil, 1989; Zhang, Zhou, Briggs, & Nunamaker, 2006). Online homework can be just as effective as paper-and-pencil homework (Bonham, Deardorff, & Beichner, 2003; Fynewever, 2008), and carefully developed intelligent tutoring systems have been shown to be just as effective as human tutors (VanLehn, 2011). Despite these advancements, their adoption has been slow, as the development of good educational systems can be prohibitively expensive. However, the corresponding ideological movement is beginning to break down these financial barriers.

Ideologically, MIT took a significant step forward when it announced its OpenCourseWare (OCW) initiative in 2001 (MIT, 2012b). This opened access to information that had previously only been available to students who paid university tuition, which is over \$40,000 per year at MIT (MIT, 2012a). Continuing this trend, MIT alum Salman Khan founded the Khan Academy in 2006, which has released a library of over 3200 videos and 350 practice exercises (Khan Academy, 2012). The stated mission of the Khan Academy is to provide “a free world-class education to anyone anywhere.” In the past year, this movement has rapidly gained momentum. Inspired by Khan's efforts, Stanford professors Sebastian Thrun and Andrew Ng opened access to their online courses in Fall 2011. Thrun taught artificial intelligence with Peter Norvig, attracting over 160,000 students to their free online course. Subsequently, Thrun left the university and founded Udacity, which is now hosting 11 free courses (Udacity, 2012). With support from Stanford, Ng also started his own open online educational initiative, Coursera. Princeton, University of Pennsylvania, and University of Michigan have joined the Coursera partnership, which has expanded its offerings to 42 courses (Coursera, 2012). MIT has also upgraded its open educational initiative, and joined

---

<sup>2</sup>Effect size=0.5.

with Harvard in a \$60 million dollar venture, edX (2012). EdX will “offer Harvard and MIT classes online for free.”

While online education is improving, expanding, and becoming openly available for free, university tuition at brick-and-mortar schools is rapidly rising (National Center for Educational Statistics, 2012). Tuition in the University of California system has nearly tripled since 2000 (Gollan, 2011). Naturally, this tuition increase is not being received well by university students in California (Asimov, 2012). Likewise, students in Quebec are actively protesting planned tuition hikes (Delange, 2012). In resistance to planned tuition hikes, student protestors at Rutgers interrupted<sup>3</sup> a board meeting to make their voices heard (Heyboer, 2012). Adding fuel to the fire, results from a recent study by Gillen, Denhart, and Robe (2011) indicate that undergraduate student tuition is used to subsidize research. As a result, the natural question being asked by both students and educational institutions is exactly what students are getting for their money. This is applying a certain pressure on physical academic institutions to improve and enhance the in-person educational experience of their students.

Students are not the only ones demanding higher outcomes from educational institutions. There is also increasing pressure from accreditation institutions. In particular, the Accreditation Board for Engineering and Technology (ABET) specifies outcomes that university graduates in engineering and technology must meet for their programs to be accredited (ABET, 2013). Commonly referred to as outcomes 3a-k, these criteria include, “an ability to communicate effectively,” and “an ability to identify, formulate, and solve engineering problems,” as well as, “an ability to function on multidisciplinary teams.” Many of these criterion are generally difficult to teach and assess effectively with informative lectures and closed form questions.

Problem-based learning methods, however, can be much more effective at achieving these goals. Felder and Brent (2003) survey research indicating that problem-based learning methods can be used to fulfill many ABET 3a-k outcomes. In engineering, model-eliciting activities (MEAs) have recently surfaced as a promising problem-based approach for fulfilling

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<sup>3</sup>On June 20, 2012



these outcomes (Diefes-Dux, Moore, Zawojewski, Imbrie, & Follman, 2004). Model-eliciting activities are realistic, open-ended, client-driven engineering problems designed to foster students' mathematical modeling abilities (Diefes-Dux et al., 2004; Lesh & Doerr, 2003). Focused around six guiding principles, MEAs have the potential for improving student performance on all eleven ABET 3a-k outcomes (Diefes-Dux et al., 2004).

Adoption of MEAs and problem-based learning is hindered by the fact that the curriculum for engineering programs is already tightly packed. Cramming even more into these programs may seem impossible. Although computer technology is to blame for at least a portion of the uncomfortable situation in which educational institutions find themselves, it may also form a key part of the solution. Since the stone age, man has used tools to improve the effectiveness and efficiency of his efforts. In modern industry, this is accomplished by automating tasks that can be automated, and focusing human effort on those that cannot. Although group lectures have been sharply criticized in a portion of the educational literature (e.g., Paul, 1993), there seems to be little convincing evidence to support these criticisms. However, since video lectures can be as effective as in-person lectures at conveying basic information (Cohen et al., 1981; McNeil, 1989; Zhang et al., 2006), the wisdom of using student and instructor time for live lectures is questionable. Rather, pre-recorded lectures can be assigned to students as homework, leaving class time open for interactive learning activities—activities that cannot be automated or computerized. This is the key concept behind what is becoming the new buzzword in educational circles: the flipped classroom.

While the flipped classroom represents an exciting new topic in educational research, there is a lack of consensus on what exactly the flipped classroom is, and a corresponding lack of good research on its effectiveness. Thus, it is proposed that a specific version of the flipped classroom be studied that utilizes model-eliciting activities (MEAs) as the primary in-class activity, with video lectures and practice problems as assigned homework. There are three key features of this study that distinguish it from existing research:

- The use of Model-Eliciting activities as the primary in-class activity.
- Inclusion of a concurrent comparison group in the study.

- Objective comparison of student learning outcomes.

## 1.2 Purpose Statement

The purpose of this research is to compare the educational effectiveness of flipped classroom instruction consisting of model-eliciting activities and video lectures to traditional classroom instruction in a university-level introductory numerical methods course for engineers. Using a controlled research design, educational effectiveness will be evaluated along two dimensions: conventional problem-solving ability and conceptual understanding. Student perceptions will also be compared, which will help identify ways to improve on existing methods and evaluate the similarity of the current study with prior flipped classroom research.

## 1.3 Defining the Flipped Classroom

Perhaps the simplest definition of the *flipped*, (or *inverted*) classroom is given by Lage, Platt, and Treglia (2000). “Inverting the classroom means that events that have traditionally taken place inside the classroom now take place outside the classroom and vice versa” (p. 32). This *flipping* is demonstrated in the first two rows of Table 1.1. Note that there are two other possible permutations of lecture and homework. Both may take place in class, or both may take place outside class. These might be referred to as boarding school and independent study, respectively. While this explanation captures the rationale for using the terminology *inverted* or *flipped*, it does not adequately represent the practice of what researchers are calling the flipped classroom. This definition would imply that the flipped classroom merely represents a re-ordering of classroom and at-home activities. In practice, however, this is not the case (Demetry, 2010; Foertsch, Moses, Strikwerda, & Litzkow, 2002; Lage et al., 2000; Toto & Nguyen, 2009; Warter-Perez & Dong, 2012; Zappe, Leicht, Messner, Litzinger, & Lee, 2009).

Most research on the flipped classroom employs group-based interactive learning activities inside the classroom, citing student-centered learning theories based on the works of Piaget (1964/1967) and Vygotsky (1978). The exact nature of these activities varies widely

Table 1.1: Simplified Definition of the Flipped Classroom

Style	Inside class	Outside class
Traditional	Lectures	Practice exercises & problem solving
Flipped	Practice exercises & problem solving	Video lectures
De facto flipped	Questions & answers, group-based/open-ended problem-solving	Video lectures, closed-ended quizzes & practice exercises

between studies. Similarly, there is wide variation in what is being assigned as “homework.” The flipped classroom label is more often assigned to courses that use activities made up of asynchronous web-based video lectures and closed-ended problems or quizzes. In many traditional courses, this represents all the instruction students ever get. Thus, the flipped classroom actually represents an expansion of the curriculum, rather than a mere re-arrangement of activities. A simplified depiction of this is shown in the last row of Table 1.1.

The flipped classroom is an educational technique that consists of two parts: interactive group learning activities inside the classroom, and direct computer-based individual instruction outside the classroom. A graphic representation of this definition is shown in Figure 1.1.

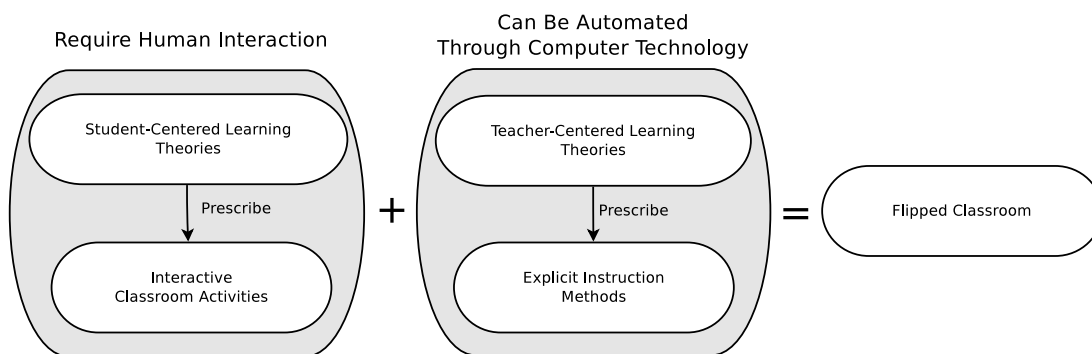


Fig. 1.1: Flipped classroom.

#### 1.4 Educational Effectiveness and the Flipped Classroom

The Merriam-Webster Online dictionary defines education as, “the process of receiving or giving systematic instruction, especially at a school or university.” Effectiveness is defined as “the degree to which something is successful in producing a desired result; success.” Thus, educational effectiveness is the degree to which educational experiences [facilitated by a program and/or instructor(s)] are successful in producing the desired results.

This raises the question of what the *desired results* consist of. To address this question, two sources will be examined. First, the mission statement of the College of Engineering at Utah State University, which is: “To foster a diverse and creative learning environment that will empower students and faculty with the necessary knowledge and facilities to be international leaders in creating new technologies and services that will improve tomorrow’s economy and environment.”

Second, the Accreditation Board for Engineering and Technology (ABET), which sets forth several broad “outcomes 3a-k” that university programs should strive to achieve (ABET, 2013). These include

- (a) an ability to apply knowledge of mathematics, science and engineering
- (b) an ability to design and conduct experiments, as well as to analyze and interpret data
- (c) an ability to design a system, component, or process to meet desired needs within realistic constraints such as economic, environmental, social, political, ethical, health and safety, manufacturability, and sustainability
- (d) an ability to function on multidisciplinary teams
- (e) an ability to identify, formulate and solve engineering problems
- (f) an understanding of professional and ethical responsibility
- (g) an ability to communicate effectively
- (h) the broad education necessary to understand the impact of engineering solutions in a global, economic, environmental, and societal context

- (i) a recognition of the need for, and an ability to engage in life-long learning
- (j) a knowledge of contemporary issues
- (k) an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.

An essential first step is to select an educational method that shows promise in being able to achieve success on these criteria, which constitute the *desired results*. Several publications have already been mentioned that illustrate how MEAs (and other similar student-centered learning approaches) are uniquely positioned to help students attain these learning goals (e.g., Diefes-Dux et al., 2004; Felder & Brent, 2003).

One of the major challenges, however, is that progress on these outcomes is difficult to measure objectively. Further, a tradition has been established to almost exclusively examine student performance on solving pre-formulated closed-ended problems (problems with one known answer). Traditions such as these are not easily changed. In fact, one of the major criticisms of problem-based learning is that when it is implemented, knowledge outcomes tend to be lower than conventionally taught courses, even though skills are usually higher (Dochy, Segers, Van den Bossche, & Gijbels, 2003; Gijbels, Dochy, Van den Bossche, & Segers, 2005).

Because it represents a unique combination of both student-centered and conventional lecture-based teaching methods, the flipped classroom may perform better than purely problem-based approaches on knowledge outcomes, while still working toward broad learning goals that conventional teaching methods usually ignore. This statement represents the central motivating hypothesis of the present work. Successful evaluation of this theory will allow educators and future researchers to make more informed decisions that will hopefully lead to higher overall educational effectiveness.

#### **1.4.1 Knowledge Outcomes of Student Learning**

Two dimensions of objective student performance were identified, which will be used in this study as indicators of knowledge gained by students: conventional problem-solving

ability, and conceptual understanding. Conventional problem-solving ability refers to how well students perform on closed-ended quantitative problem-solving tasks. An example of such problems within the context of the current work can be found at the end of each chapter in Chapra and Canale (2009). Conventional problem-solving ability is often contrasted with conceptual understanding (e.g., Mazur, 1997). As stated by Novak (1996), “concepts are packages of meaning; they capture regularities, patterns, or relationships among objects, events, and other concepts.” Thus, a concept is not unlike a *schema* as used by Piaget (1964/1967). The notion of conceptual understanding stems from the idea that knowledge is not merely a quantification of declarative facts that are stored in isolation from each other, but is also contained in the connections or relationships of facts and ideas to each other. Not surprisingly, conceptual understanding is measured by student responses to conceptual questions. Mazur (1997) contains numerous illustrative examples of that highlight the distinction between conventional and conceptual questions, but perhaps the best concrete example is the multiple-choice force concept inventory (Hestenes, Wells, & Swackhamer, 1992).

#### **1.4.2 Student Perceptions**

In addition, student perceptions of the course were also examined. Maintaining positive student perceptions is not to a stated goal or objective of the learning process, at least this is not specified as a desired outcome by ABET nor by the College of Engineering at Utah State University. Nevertheless, student perceptions of the learning process are regularly measured each semester by course evaluations. The results of the course evaluations were used to help establish whether the current study was similar in this regard to other studies of the flipped classroom, most of which focused exclusively on student perceptions.

Although it will have no bearing on the central focus of this research, which is to compare the educational effectiveness of the flipped classroom to the traditional classroom, a survey of student perceptions will help identify ways to improve the teaching and learning process.

### 1.5 Research Questions

Based on a thorough review of the literature, the primary hypothesis of this research is that a numerical methods course taught using the video lecture/MEA format will be as or more educationally effective than a traditional lecture-based course, on two dimensions: conventional problem-solving ability and conceptual understanding. Student perceptions between the two sections will also be compared. Based on prior research on the flipped classroom, it is anticipated that student perceptions in the section taught using the video lecture/MEA format will be equal or greater than those in the traditional lecture-based section. This leads to three specific research questions:

1. Will students in a numerical methods course taught using the video lecture/MEA format attain equal or higher conventional problem-solving performance than students in a traditional lecture-based course, as measured by student homework and exam scores?
2. Will students in a numerical methods course taught using the video lecture/MEA format attain equal or higher conceptual understanding than students in a traditional lecture-based course, as measured by conceptual quizzes and a concept test?
3. Will students in a numerical methods course taught using the video lecture/MEA format attain equal or higher opinions of the learning experience as students in a traditional lecture-based course, as measured by students' self-reported attitudes?

### 1.6 Dissertation Outline

The organization of the remainder of this dissertation is as follows: Chapter 2 provides a detailed review of the literature for each of the key areas of this research. That is, model-eliciting activities (within the broader psycho-educational context of student-centered learning theories), interactive video lectures (within the broader literature of computer-aided instruction), the flipped classroom (which combines the previous areas), and teaching numerical methods (which provides the context for the current study). Chapter 3 presents the details of the research design and methods used for this study. In particular, the analysis

variables, measurement instruments, participants, and analysis procedures are described. The results of the present study are presented in Chapter 4, and a discussion is given of these results in Chapter 5. Finally, conclusions are summarized in Chapter 6, along with recommendations for future work.





## Chapter 2

### Literature Review

#### 2.1 Literature Map

There are five major components to this literature review. These include:

1. A brief overview of student-centered learning theories, and the corresponding classroom activities they prescribe. This includes an overview of model-eliciting activities, and rationale for using these in the current work. It also establishes the necessary background to understand the motives of different authors for the activities used as the in-class segment of the flipped classroom. This is presented in Section 2.2.
2. Research on how computer technology has enabled the *automation*<sup>1</sup> of what makes up the *out-of-class* activities in the flipped classroom—in particular, lectures and closed-ended homework exercises. To adequately accomplish this task, it is necessary to locate this research within the larger context of computer-aided instruction. This is presented in Section 2.3.
3. Detailed analysis of existing research on the flipped classroom. This is presented in Section 2.4.
4. An overview of research on teaching numerical methods for engineers, which establishes the subject-specific context in which the study took place. This is presented in Section 2.5.
5. A discussion of how the preceding four areas are combined to form the basis for the current study. This is presented in Section 2.6.

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<sup>1</sup>*Automation* refers to learning tasks that do not require direct intervention on the part of an instructor. Of course, these tasks still involve significant effort on the part of the students.

A broad graphical overview corresponding to the five major components of this literature review is provided in Figure 2.1. It should be noted that the first three of these were presented previously as given in Figure 1.1. As depicted, student-centered teaching methods are combined with computer-aided instruction to yield the flipped classroom. The flipped classroom has its own growing corpus of dedicated literature. The literature map for student-centered teaching methods is shown in the upper-left quadrant of Figure 2.1, computer-aided instruction is shown in the upper-right, and the flipped classroom is depicted in the lower-right. The context-specific area of numerical methods is shown in the lower-left quadrant. Literature from all four of these areas combined forms the basis for the present study, which is depicted in the center of Figure 2.1. In particular, model-eliciting activities are used from student-centered learning methods, asynchronous video lectures from computer-aided instruction, and a concept test from numerical methods research. Flipped classroom research provided both context and direction, and the present review exposed the lack of controlled studies in this area. While Figure 2.1 is useful for situating the current work within the literature on the highest level, it is difficult for a single graphic to include sufficient detail. Thus, a more detailed graphic map or table for each component area will be presented as it is discussed.

## 2.2 Student-Centered Learning Theories & Methods

This section provides an introduction to several different student-centered learning theories and methods. The origins of these theories and methods are traced over the last  $\sim 100$  years. Major figures who influenced the development of these ideas are identified, and the resulting methods and theories are briefly described. To guide this discussion, two graphics are used. Figure 2.3 traces the intellectual origins of these learning theories and methods. Loosely speaking, this figure is organized chronologically with the earliest published writings at the top and the newest near the bottom, while major links between theories, theorists, or methods are shown as directional arrows. These several theories can also be described with a set relation, which is shown in Figure 2.2. As far as possible, the reader is guided through Figure 2.3 from top to bottom, and then Figure 2.2 from the inside out.

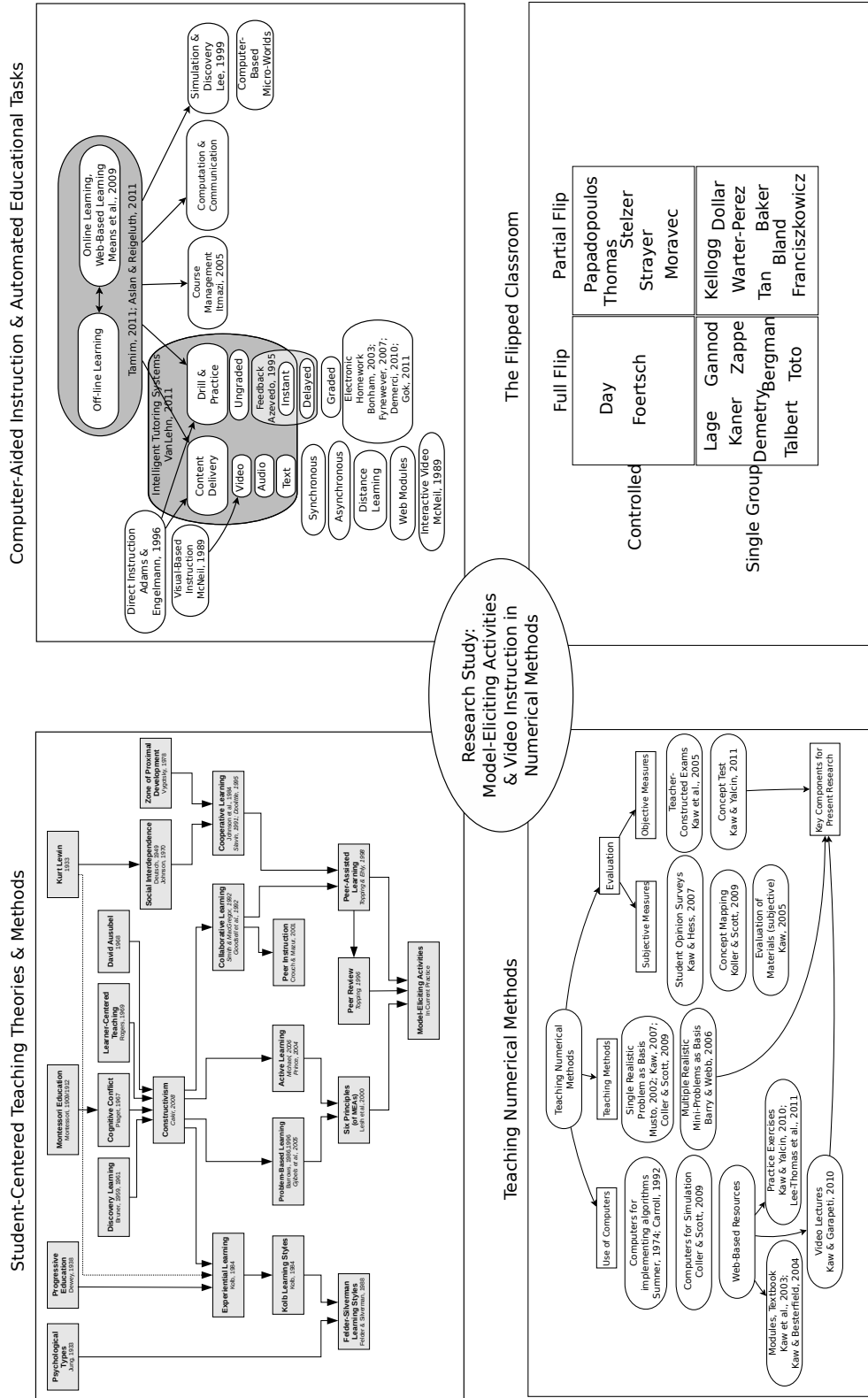


Fig. 2.1: Main literature map.

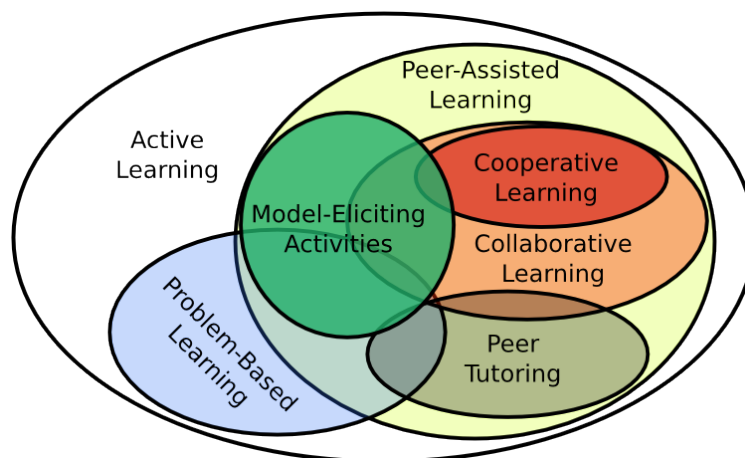


Fig. 2.2: Venn diagram of several student-centered learning theories and methods.

### 2.2.1 Origins

Figure 2.3 traces the foundations for several psycho-educational learning theories that form the basis for student-centered classroom activities prescribed by proponents of the flipped classroom. Although these foundations could possibly be traced earlier, 100 years of history is sufficient for the current discussion. The basis for most student-centered learning theories ultimately stem from the writings of four early researchers. Specifically, Carl Jung on psychological types (1933/2001), John Dewey on progressive education (1938), and Kurt Lewin on personality (1935), action research (1946), and group dynamics (1947). The influential work of Montessori (1909/1912) is also of this era, but it is more difficult to draw direct connections between Montessori’s research and later theories. Thayer-Bacon (2012) suggests this is because of her gender, which led to the publication of a detrimental “biased analysis of the Montessori method,” (p. 15) by Dewey’s protégé, Kilpatrick (1914). Nevertheless, Thayer-Bacon (2012) identifies Montessori as a “significant influence” (p. 11) on Jean Piaget, who is widely recognized as the primary architect of constructivism (Piaget, 1964/1967).

### 2.2.2 Constructivism

Piaget (1964/1967) describes the process of intellectual development in terms of *schema*, *assimilation*, and *accommodation*. The term *schema* is used to represent an individual’s men-

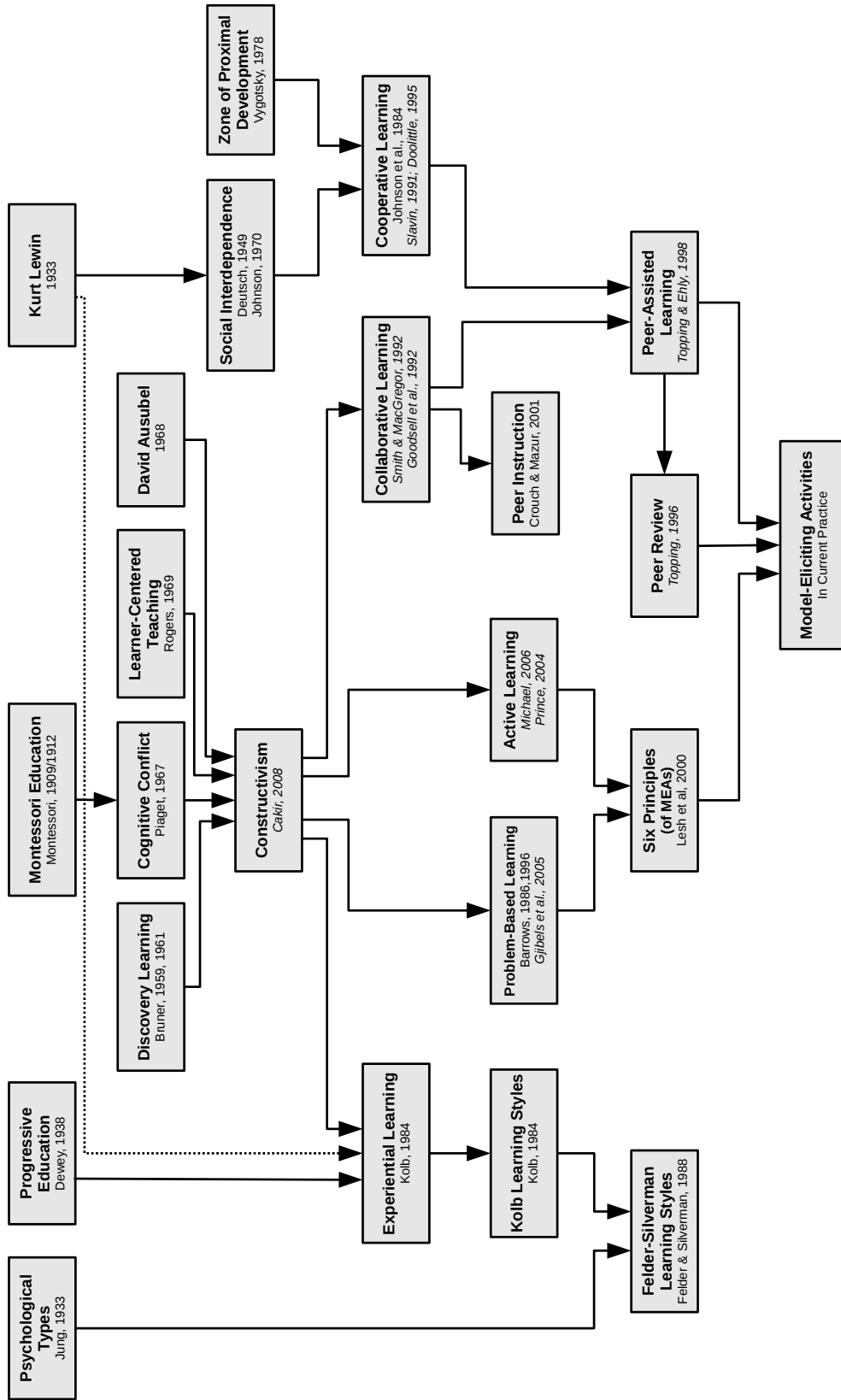


Fig. 2-3: Psycho-educational origins of student-centered learning theories. Seminal works are shown in regular font, meta-analyses and research survey articles are shown in italics.

tal structures (or conceptual understanding), which form the basis for a class of actions. A schema is the package of meaning used to represent a generalization of the patterns, relationships that are posited to exist among objects. An individual's schema is said to be constructed, or developed through interaction with others and the surrounding environment. When observations do not fit within existing schema, the schema is either augmented (assimilation), or modified to create a new schema (accommodation). These concepts emphasize the central role of the student in actively constructing his or her own understanding through schema development.

In addition to Piaget, Gijbels, Watering, Dochy, and Bossche (2006) identified several other researchers whose writings also helped shape the constructivist perspective, including Bruner (1959, 1961), Rogers (1969), and Ausubel (1968). Bruner (1959, 1961) developed a method known as discovery learning, which emphasizes the importance of learning by doing. Rogers (1969) argued that the student should be at the center of the learning process, and emphasized that subject matter should have personal meaning, feeling and relevance to the learner. Influenced by Piaget, Ausubel (1968) emphasized meaningful symbolic learning, in which novel ideas (usually in the form of verbal propositions) are assimilated into existing cognitive structures. This is consistent with Ausubel's (1960) research on advance organizers, which were seen as a method for helping learners activate mental structures or present information relevant to developing new schemas.

### **2.2.3 Zone of Proximal Development**

Meanwhile, Vygotsky (1978) theorized that learning and assessment should focus on the ability of students to solve problems either alone or with help. He coined the term, *zone of proximal development* to describe the range of problems that could be solved under adult or peer guidance, which is greater than the range of problems that can be solved alone. Thus, while the primary focus of Vygotsky was on learning and development through interactions with adults and more capable peers, the focus of constructivism is on the active construction of meaningful schemas within an individual. Although it sometimes receives less focus than the other elements of his theories, Piaget also developed the theory of cognitive conflict,

which is based on the idea that a difference in perspective can produce discussion with others to resolve the conflict and restore equilibrium. For Piaget, these are typically symmetrical, reciprocal relationships, whereas Vygotsky focused on what could be accomplished in the presence of more capable peers. Tudge and Winterhoff (1993) provided a detailed analysis of the similarities and differences between the theories of Piaget (1964/1967) and Vygotsky (1978).

#### **2.2.4 Social Interdependence**

Lewin's theories, particularly group dynamics (1947) and force field analysis (1943) led to the formulation of the theory of social interdependence by Deutsch (1949). Social interdependence theory posits that individual goals determine interactions, and these interactions determine outcomes. Further contributions to this theory were made by Johnson (1970) and others (for recent developments, see Johnson & Johnson, 2005).

#### **2.2.5 Cooperative Learning**

The theories of social interdependence and the zone of proximal development taken together form the theoretical basis for cooperative learning. In tracing these influences, Johnson and Johnson (2009) point mainly to the line of intellectual development through Lewin, Deutsch, and social interdependence, while Doolittle (1995) emphasizes the role of Vygotsky as the primary inspirational figure. Foot and Howe (1998) describe cooperative learning as including three key parts:

1. Students work in teams toward the attainment of some superordinate goal.
2. Labor is divided between team members, such that each individual takes responsibility for a different sub-goal.
3. Individual contributions are pooled into a composite product to ensure that the goal is reached.



Synthesizing the views of several theorists (Johnson, 1984; Ormrod, 1995; Rottier & Ogan, 1991; Sharan, 1990), Doolittle (1995) notes that while there is not perfect consensus on what constitutes cooperative learning, five factors are paramount:

1. Positive interdependence
2. Face-to-face interaction
3. Individual accountability
4. Small group & interpersonal skills
5. Group self-evaluation

Meta-analysis of the effects of cooperative learning methods are given by Slavin (1991), Johnson, Johnson, and Smith (1998), and Johnson, Johnson, and Stanne (2000). Of these, Johnson et al. (2000) provide the most detail, ranking specific cooperative learning methods by effectiveness. These have broad variation. The magnitude of effect sizes range from low (0.18) to high (0.85). The most effective of these is *Learning Together & Alone*, which focuses on “the integrated use of cooperative, competitive and individualistic learning” (Johnson & Johnson, 2002). In a more recent publication, Johnson and Johnson (2002) give a dedicated meta-analysis of this particular cooperative learning method. Smith and MacGregor (1992) indicate that Lewin (1935) and Deutsch (1949) were important influences in cooperative learning through their social interdependence theories.

### **2.2.6 Collaborative Learning and Peer-assisted Learning**

Collaborative learning is perhaps best understood by contrast with cooperative learning (discussed previously). Collaborative learning stems from Piaget’s theory of conflict (Foot & Howe, 1998), while cooperative learning stems from both Vygotsky’s zone of proximal development (Doolittle, 1995) and social interdependence theory (Johnson & Johnson, 2009). Smith and MacGregor (1992) further explain, “cooperative learning represents the most carefully structured end of the collaborative learning continuum” (p.15). Thus, while Figure

2.3 is useful for tracing the origins and influences of peer-assisted learning, it does not adequately represent the relationship of this with other closely-related learning theories. Before attempting to construct a diagram to represent this relation, peer-assisted learning is also considered.

Topping and Ehly (1998) define peer-assisted learning as “the acquisition of knowledge and skill through active helping and supporting among status equals or matched companions” (p.1). This broad definition prepares us for the view that peer-assisted learning is an umbrella term large enough to accommodate many theories and methods. This includes both collaborative and cooperative learning, as well as peer tutoring, peer modeling, peer monitoring, peer instruction, peer assessment, peer review (Foot & Howe, 1998; Topping & Ehly, 1998). Thus, the preceding statements lead to a set relationship akin to the Venn diagram given in Figure 2.2.

### **2.2.7 Peer instruction**

In an effort led by Eric Mazur, a method dubbed *peer instruction* has been used to teach introductory physics courses for non-majors at Harvard University (Crouch & Mazur, 2001; Mazur, 1997). Peer instruction is a structured method that uses short lecture segments, interspersed with interactive conceptual questions. After responding to questions, students are given a few minutes to discuss their answers with each other. Because all students have not all responded the same, cognitive conflict produced by this difference in perspectives. This conflict is often resolved through discussion with others, after which the instructor re-polls the students and explains the correct answer before moving to the next topic. Since this method is both specialized, and closely aligned with the theory of cognitive conflict (Piaget, 1964/1967), it is placed under collaborative learning, which serves as a broader category, and stems from the same theoretical source. The effectiveness of peer instruction has been evaluated using both teacher-constructed tests and the Force Concept Inventory (FCI). Hake’s normalized gain scores on the FCI for traditionally taught courses ranged from 0.25 to 0.40, while courses taught with peer instruction saw gain scores from 0.49 to 0.74 (Hake, 1998).

### 2.2.8 Problem-Based Learning

Constructivism is considered the source for the theories of both problem-based and active learning (Grabinger & Dunlap, 1995). Problem-based learning was originally developed the 1950s and 1960s at the medical school at McMaster University in Ontario, Canada because of dissatisfaction with teaching practices at the time Gijbels et al. (2005). Problem-based learning has since spread to other universities and disciplines, including engineering. Hmelo-Silver (2004) lays out five goals of problem-based learning. These include helping students develop

1. Flexible knowledge
2. Effective problem-solving skills
3. Self-directed learning skills
4. Effective collaboration skills
5. Intrinsic motivation

Barrows (1996) describes six characteristics of problem-based learning, running somewhat parallel to these goals:

1. Learning is Student-Centered.
2. Learning Occurs in Small Student Groups.
3. Teachers are Facilitators or Guides.
4. Problems Form the Organizing Focus and Stimulus for Learning.
5. Problems are a vehicle for the development of clinical problem-solving skills.
6. New information is acquired through self-directed learning.

Dochy et al. (2003) and Gijbels et al. (2005) both present meta-analytic results on the effectiveness of problem-based learning (PBL). These indicate that the effect of PBL on skills is positive, while its effect on knowledge is negative. Combined results indicate an overall

negative effect for problem-based learning. Consequently, Gijbels et al. (2005) recommend careful consideration of assessment methods in measuring problem-based learning outcomes.

### 2.2.9 Active Learning

Prince (2004) defines active learning broadly as, “any instructional method that engages students in the learning process” (p. 1). This definition is itself broad enough to include many traditional classroom activities such as lectures (provided students are reflecting, taking notes, or asking questions). However, in an effort to maintain contrast with traditional *teacher-centered*<sup>2</sup> approaches, these methods are systematically dismissed by explicit exclusion. Thus, active learning acts as a superset for both peer-assisted and problem-based learning approaches. Prince (2004) also clarifies the relationship between these two, indicating that problem-based learning is, “always active and usually (but not necessarily) collaborative or cooperative” (p. 1). This leads to a revision of the previous diagram, resulting in the relationship shown in Figure 2.2. The reviews of Prince (2004) and Michael (2006) provide a broad sweep of the literature, highlighting the evidence and methods of active learning.

### 2.2.10 Model-Eliciting Activities

Model-Eliciting Activities (MEAs) are realistic, open-ended, client-driven engineering problems designed to foster students’ mathematical modeling abilities. Drawing from decades of research on student-centered approaches, MEAs represent a latest generation development. MEAs are based firmly on the concepts of constructivism and active learning, but are established with a clear focus and purpose relevant to mathematics and engineering education. There are six guiding principles that undergird the development of MEAs (Lesh & Doerr, 2003; Lesh, Hoover, Hole, Kelly, & Post, 2000):

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<sup>2</sup>The label *student-centered* is carefully crafted for linguistic effect. This is much like the label *pro-choice*, which is crafted to portray an opposite of *anti-choice* or *pro-slavery*. Correspondingly, the label *pro-life* suggests an opposite of *pro-death* or *anti-life*. Educators do not typically self-identify as having a *teacher-centered* or *non-student focused* teaching philosophy. Like its political counterpart, the educational debate is both complex and highly polarizing.

1. Model-Construction Principle: The activity must require explicit construction, description, explanation, or prediction of a mathematically significant situation. This is usually accomplished through descriptive prompts and by selecting an appropriate base problem.
2. Reality Principle: The activity must present a problem that is posed in a realistic context. This helps students understand the importance of what they are studying and serves to enable students of all skill levels to actively engage in the problem. To facilitate this, realistic problems may be posed as a request from a client.
3. Self-Assessment Principle: Students must be enabled to self-check their method. This can be facilitated by presenting sample data, which students use to test their model or procedure.
4. Model-Documentation Principle: Students must be required to communicate the mathematical model or procedure developed in response to the problem. This may be facilitated by having students prepare a written response memorandum.
5. Construct Share-Ability and Re-Usability Principle: Students must be required to create a generalizable procedure that is applicable to other problems similar to the one given. The intent of teaching mathematics is typically to provide general tools that can be used to solve many problems. This allows the passing on (sharing) of the solution for application by someone else, or the re-use of the same basic procedure to other situations. Since students often find it hard to address this issue, it can be useful to provide additional datasets for which a specific model would fail, but a generalizable model would be easily adapted.
6. Effective Prototype Principle: This principle requires the solution to represent a metaphor, or learning prototype for interpreting other structurally similar problems. This ensures that the activity has been beneficial. Solutions that serve as effective prototypes are usually structurally simple, based on a basic underlying concept, as

opposed to complex sets of instructions and arbitrary chains of decisions with no clear guiding logic.

These principles establish a clear definition for both what MEAs are and are not. In addition to the explicit criterion set forth here, there are a number of informal implementation principles that have been developed and refined through the repeated implementation of MEAs in the classroom. These include:

1. **Group-Based Activities:** While not explicitly listed among the six original principles of MEAs, the use of small groups of about three students is common with other student-centered learning methods. The activities lend themselves well to this implementation, and it also increases the effectiveness of MEAs in fulfilling ABET standards (ABET, 2013; Felder & Brent, 2003; Moore, Diefes-Dux, & Imbrie, 2006). The group basis of MEAs implies that they form a part of the broader category of peer-assisted learning methods.
2. **Peer Review Process:** The use of peer review not only makes the implementation of MEAs with large numbers of students more practical, it also serves to help students reflect on their own work, as well as communicate their solution to others. This is in accordance with the self-assessment, documentation, and share-ability principles. Further, peer review can be seen as a special case of peer assessment, which is also part of peer-assisted learning.
3. **Iteration:** Allowing for multiple revisions works in tandem with the peer review process, and also helps encourage self-assessment, clear communication, and simplification (related to producing an effective prototype) of the solution. Due to the realistic context required for the MEA problems, students may develop interpretations of the problem that are not congruent with the desired activity goals (or client needs). Iteration helps identify and correct these inaccurate interpretations.

Under these criterion alone, model-eliciting activities meet at least five of the six characteristics of PBL and share at least three of the five PBL goals. However, MEAs also differ in

significant ways. The use of cooperative group-based activities and the peer review process places MEAs firmly within the realm of peer-assisted learning. This trait is not generally shared by PBL, which can be seen from Figure 2.3. The documentation principle of MEAs requires that students communicate their ideas to others, which is also implicitly required from the review and iteration processes. These features imply that MEAs are better suited than PBL methods to help students develop the ability to function on multi-disciplinary teams (ABET outcome 3d), and also to communicate effectively (ABET outcome 3g). This leads to the conclusion that MEAs are particularly well-suited when working within an engineering education context.

An analysis of the MEA literature reflects the evolution and current state of MEAs as an instructional tool. A significant portion of this literature is presented in Table 2.1. This table provides a categorization for each work, grouped by three general categories: (a) MEA theory, (b) MEA development, and (c) MEA implementation, and additionally define sub-categories depending on the focus of the publication.

Note the progression of MEA research, and how topics have been shifting over time. Early research focused on developing the six principles of MEAs and defining what MEAs are (Diefes-Dux et al., 2004; Lesh & Doerr, 2003; Lesh et al., 2000; Moore, 2008). Later research focused on development of MEAs for different scenarios (Moore & Diefes-Dux, 2004; Yildirim et al., 2010), and developing rubric and evaluation tools (Diefes-Dux et al., 2009; Moore et al., 2006). This step necessarily precedes MEA implementation, which has evolved over time, but has also settled on several recommendations. These recommendations include using teams to solve an MEA, using peers and teaching assistants (TAs) to provide feedback to fellow-students, and having a good TA training program.

### **2.2.11 Summary of Student-Centered Learning Literature**

In summary, the psycho-educational foundations for a broad range of student-centered learning theories have been outlined, and each of these has been described in some detail. This is important to the present work not only for establishing a theoretical basis for model-eliciting activities (which were used in this research), but also to give context to the

Table 2.1: An Overview of Literature on Model-Eliciting Activities

Authors	Year	Primary Author Institution	Category	Subcategory
Lesh et al.	2000	Purdue University	Theory	Six principles
Lesh and Doerr	2003	Purdue University	Theory	Six principles
Diefes-Dux et al.	2004	Purdue University	Theory	Intro. to MEAs
Moore and Diefes-Dux	2004	Purdue University	Development	Scenario development
Moore, Diefes-Dux, and Imbrie	2005	Purdue University	Development	Student reflection
Moore et al.	2006	Purdue University	Development	Evaluation tools
Verleger and Diefes-Dux	2008	Purdue University	Implementation	Feedback & peer review
Moore	2008	University of Minnesota	Theory	Intro. to MEAs
Zawojewski, Diefes-Dux, and Bowman	2008	Illinois Institute of Technology	Theory	-
			Development	-
			Implementation	-
Diefes-Dux and Verleger	2009	Purdue University	Implementation	Peer review
Diefes-Dux, Verleger, Zawojewski, and Hjalmarson	2009	Purdue University	Implementation	Evaluation tools
Verleger	2009	Purdue University	Implementation	Peer review
Yildirim, Shuman, Besterfield-Sacre, and Yildirim	2010	University of Pittsburgh	Development	Scenario development
Verleger, Diefes-Dux, Ohland, Besterfield-Sacre, and Brophy	2010	Purdue University	Implementation	Peer review
Carnes, Cardella, and Diefes-Dux	2010	Purdue University	Implementation	Peer review



subsequent review of the flipped classroom. The connections between constructivism, the zone of proximal development, social interdependence, cooperative learning, collaborative learning, peer assisted learning, peer instruction, learning styles, problem-based learning, active learning, and model-eliciting activities were tied to the educational philosophies of Jung, Dewey, Montessori, Lewin, Bruner, Piaget, Rogers, Ausubel, Vygotsky, Deutsch, Kolb, and others. The six principles of MEAs (model construction, reality, self-assessment, documentation, share/reuse-ability, effective prototype), and also three implementation principles based on peer-assisted learning (using peer review, working in small groups, and iterating through problem solutions) were also outlined.

Model-eliciting activities represent a latest-generation development in student-centered teaching methods, specifically within the context of engineering. MEAs leverage the lessons from nearly a century of research, including the appropriate use of cooperative groups, peer review, and active, problem-based learning methods. The past decade of research on MEAs has yielded important lessons for implementation and best practice. MEAs have been designed specifically to help students activate prior knowledge by connecting it with realistic situations, work collaboratively and interactively, and communicate that knowledge to others. These characteristics of MEAs fit the demands of recently required ABET (2013) outcomes that current programs lack (Diefes-Dux et al., 2004). By providing a rich interactive experience, MEAs have the potential to help satisfy the needs of students demanding value for their tuition dollars.

### **2.3 Computer-Aided Instruction**

Computer-aided instruction (CAI), computer-based instruction, computer-assisted instruction, computer-based learning, computer-assisted learning, computer based teaching, computer-managed instruction, online learning, web-based learning, and distance learning are some of the many broad categories used to describe the use of computers in education (Itmazi & Megías, 2005; Means, Toyama, Murphy, Bakia, & Jones, 2009; VanLehn, 2011).

There are numerous meta-analytic studies of computer-aided instruction. J. A. Kulik, Kulik, and Cohen (1980) provide a meta-analysis of computer-aided instruction in college

students, finding an overall effect size of 0.25.<sup>3</sup> In a later analysis examining elementary school, J. A. Kulik, Kulik, and Bangert-Drowns (1985) found an effect size of 0.47. Findings are slightly better for statistics (Larwin & Larwin, 2011; Sosa, Berger, Saw, & Mary, 2011), but overall seem to indicate similar results (C. L. C. Kulik & Kulik, 1991). They do not seem to vary significantly with country either, as indicated by a meta-analysis of CAI in Taiwan by Liao (2007) and one of Physics in Turkey by Yesilyurt (2011). In a report commissioned by the US Department of Education, Means et al. (2009) examined online learning, and found an overall effect size of 0.2. Tamim, Bernard, Borokhovski, Abrami, and Schmid (2011) performed a second-order meta-analysis of computer-aided instruction, combining the results of the meta-analysis mentioned, as well as many others. The overall finding was an effect size of 0.35. Despite this robust positive effect, the act of simply placing a computer in the classroom does not produce a magical aura that improves student learning. As Cook (2005) points out, it is essential to consider *how* computers are being used.

Particular functions of computers in education include: simulation and discovery, computation and communication, course management, drill and practice, and content delivery (Bayraktar, 2002; Itmazi & Megías, 2005). The combination of content delivery with drill and practice constitutes an intelligent tutoring system. These functions are shown emanating from the main CAI region in Figure 2.4. These functional purposes of computers are used to organize the present review of CAI literature, with the subsequent sub-sections given in the order mentioned.

Although CAI can be placed into functional categories (which serve as the organizing basis for this review), it is also important to consider a historical perspective of computers. Computers have been used to some extent within education for over 50 years. During this time, both the form and function of computers have evolved rapidly. Aslan and Reigeluth (2011) present an overview of CAI from this perspective. They define three periods, which they name the “mainframe period,” “microcomputer period,” and “internet period.”

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<sup>3</sup>Effect size (ES) is an index that indicates the magnitude of a treatment effect (for an introduction to effect size see, e.g., Ellis, 2010).

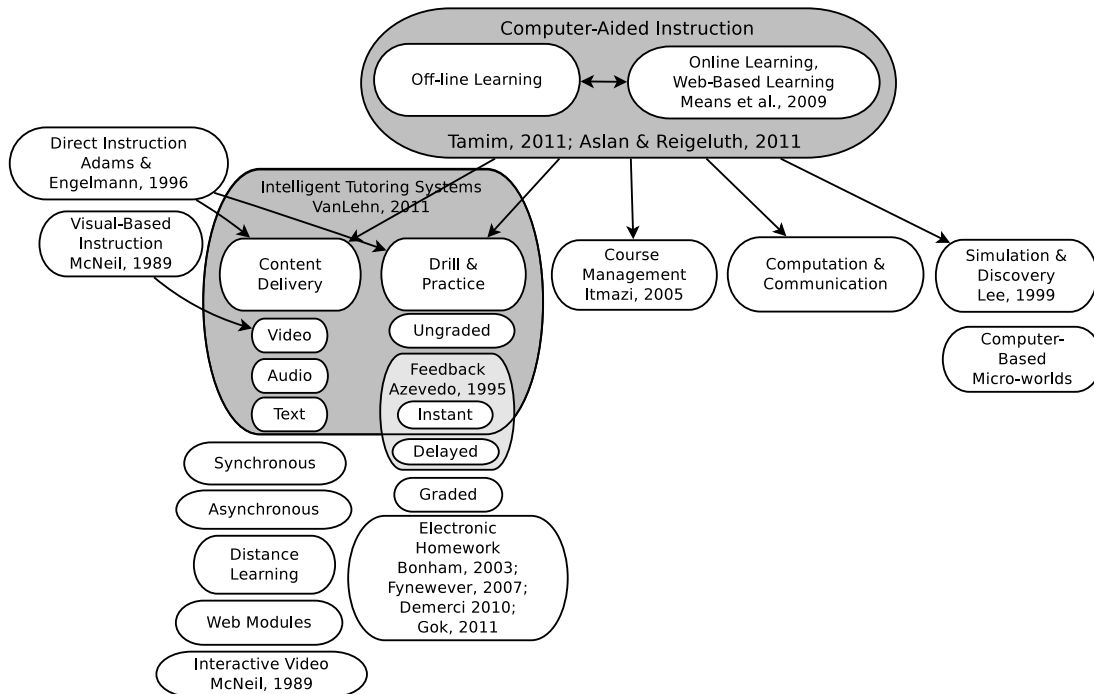


Fig. 2.4: Computer-aided instruction.

### 2.3.1 Simulation & Discovery

Just as there is variation within teaching and learning theories which generally deal with personal interactions, this variation also exists within computer-aided instruction. The simulation and discovery segment of computer-aided instruction represents the least structured field of CAI, and has much in common with principles of constructivism, active learning, discovery, and self-regulated learning. One goal of simulation in CAI is to provide students with freedom and control to explore and discover. Thus, researchers often seek to avoid using measurement outcomes based on behavioral objectives to measure the success of simulation in this area of CAI. This point is highlighted in a meta-analysis of computer-based instructional simulation by Lee (1999). It is also emphasized in an article by Rieber (1992), who mentions the behavior-oriented tradition of CAI, and argues that computer-based micro-worlds (a sub-area within the realm of simulation & discovery) could form the basis for a bridge between direct instruction and constructivism. The computer is the physical embodiment of that bridge.

### 2.3.2 Computation & Communication

This category covers the use of computers as a tool to facilitate work that would otherwise be done by hand, just as a calculator may be used if the focus is not on the arithmetic, but on the procedure for solving a problem. Although the computation role of computers is important, most quantitative research-oriented literature has focused on the use of computers in other roles. Computers nevertheless have developed important roles as tools within education. Examples of this include electronic word processors, spreadsheets, and drawing tools (Aslan & Reigeluth, 2011). More recently, the internet emerged, which opened access to educational content which was previously unavailable. Aslan and Reigeluth (2011) suggest that to some extent, this has undercut the monopoly of teachers and textbooks on education. Jonassen, Peck, and Wilson (1999) tie this to explorative learning and constructivism. Within this context, computers can also act as tools to facilitate communication, which may in turn lead to self-directed education through gathering (online) information. Under a broad definition of computer, this also covers the use of *clickers* within the classroom.

### 2.3.3 Course Management

Course management systems, learning management systems, and learning content management systems are several names used for software platforms designed to aid in the educational process. The category of course management is intended to cover the use of computers in facilitating management activities for a course or program. Itmazi and Megías (2005) survey 58 studies of these systems, giving a high-level overview with comparisons and evaluations. The management purpose is really the use of the computer as a tool to reduce the workload of teachers and (sometimes) students. Educational outcomes are not the key focus for this category. However, many content management systems are expanding functionality beyond management to fulfill content delivery, practice (Kortemeyer, Kashy, Benenson, & Bauer, 2008), or even tutorial roles. Thus, the line between course management systems and the other categories of computer-aided instruction presented in Figure 2.4 is becoming increasingly blurred.

#### **2.3.4 Drill & Practice**

Practice exercises may be graded or ungraded, and feedback may be instant or delayed. A meta-analysis of feedback in computer-aided instruction conducted by Azevedo and Bernard (1995) resulted in a mean effect size of 0.8 for immediate achievement results, and 0.35 for delayed posttest achievement. Corbalan, Paas, and Cuypers (2010) found that stepwise feedback was most effective. If graded, computer-administered exercises may also be referred to as electronic homework (Fynnewever, 2008). Studies comparing electronic homework to paper-and-pencil homework repeatedly show no significant difference between the two (Bonham et al., 2003; Demirci, 2010; Fynnewever, 2008; Gok, 2011).

#### **2.3.5 Content Delivery**

Content may be delivered as text, video, audio, or some combination of these. The term web modules is typically used to refer to a self-contained unit designed to deliver specific content over the web. In a meta-analysis of computer-aided instruction, Timmerman and Kruepke (2006) found that media richness was an important factor in determining the effectiveness of computer-aided instruction. In particular, benefits were greatest when an audio channel was included in the delivery of content.

Since the experience is similar in many cases, research on computer-based delivery of audiovisual content borrows from earlier work on video and visual-based instruction. As Cohen et al. (1981) showed in a meta-analysis, visual-based instruction is just as effective as in-person instruction. McNeil (1989) and Zhang et al. (2006) found video-based instruction to be more effective if it is interactive (effect size=0.5).

#### **2.3.6 Intelligent Tutoring Systems**

Intelligent tutoring systems combine the functions of content delivery, practice exercises, and feedback (VanLehn, 2011). This is represented by the shaded region including these as shown in Figure 2.4. VanLehn (2011) performed a meta-analysis of intelligent tutoring, along with a comparison to human tutoring and found that when stepwise tutoring is employed both human and computer tutors produced similar results, an effect size of about 0.79.

Direct instruction is an instructional strategy for content delivery that is carefully planned and often heavily scripted (G. L. Adams & Engelmann, 1996). Careful planning principles of direct instruction are typically used for computer-based content delivery and for determining the appropriate sequence in computer-based practice exercises.

### **2.3.7 Summary of Computer-Aided Instruction Literature**

Computer-aided instruction can be organized into functional groups defined by how computers are used to facilitate education. These include simulation and discovery, computation and communication, course management, drill and practice, content delivery, and intelligent tutoring. Although the effectiveness of computers is robust and positive, four important observations can additionally be made.

First, an important result highlighted by Means et al. (2009) is that CAI results are stronger when there is a blending of computer-aided and in-person instruction. A second important observation made by J. A. Kulik et al. (1980), is that while the use of computers only marginally improves the performance of students, it substantially reduces the amount of time needed for instruction. Third, videos on You-Tube or other similar web-based players that allow pausing and scanning should be classified as interactive according to the levels-of-interactivity scale presented by McNeil (1989). Finally, as Mayo (2007) points out, in addition to the benefits afforded by the use of computers in instruction (Mayo is speaking directly of educational games), there are also development costs.

Asynchronous (interactive) video lectures were used in this research because although they do not provide quite the same level of benefit as intelligent tutoring systems, the development costs are significantly less. Thus, the payoff is potentially substantial.

## **2.4 The Flipped Classroom**

A search of the literature through June 2012 revealed 22 studies related to the flipped classroom. A spreadsheet with a complete encoding of study features was created, including the publication type, year of publication, course, educational institution, study type, sample

size, measurement instruments, theoretical framework, in-class activities, and out-of-class activities. A limited subset of this information is listed in Table 2.2.

The last column in Table 2.2 the theoretical framework used by the authors to situate their research and guide the design of the classroom activities. This is of particular interest, because it connects these studies with student-centered learning theories discussed previously. Active learning was cited eight times, learning styles five times, problem-based learning four times, cooperative learning two times, and other methods (constructivism, Piagetian, collaborative learning, peer-assisted learning) once each. This is troublesome, since these can be used to describe or support an extremely wide range of in-class activities. This makes replication and meaningful comparison across studies more difficult. One advantage of the present study is that it uses MEAs, which use specific principles and established implementation practices.

For all studies, the combination of in-class and out-of-class activities was evaluated to determine whether the study actually represented a flipped classroom. To meet the criteria, out-of-class activities must include required video lectures; in-class activities must be required, and must involve interactive learning activities—specifically, the primary in-class component could not be lectures. This eliminated 12 studies (of 22). These are labeled *partial flip* in Table 2.2. Some of these studies required students to read material before class, rather than having it presented in an audiovisual format (e.g., Papadopoulos & Santiago-Román, 2010; Papadopoulos, Santiago-Román, & Portela, 2010), others maintained that either video lectures or in-class activities were optional (e.g., Thomas & Philpot, 2012). Of the remaining studies, all but two either formally or informally examined only student perceptions and did not consider objective performance measures.

Despite differences among studies, general reports of student perceptions were relatively consistent. Opinions tended to be positive, but there were invariably a few students who strongly disliked the change. Students did tend to watch the videos when assigned, and even when they were not. DeGrazia, Falconer, Nicodemus, and Medlin (2012) noted that students supplied with optional video lectures came to class much better prepared than when they had

Table 2.2: Published Studies of the Flipped Classroom

Study Class, Primary Author	Grade Level	In-Class Activities	Out-of-Class Activities	$N$ , Treatment	$N$ , Control	Instrument Type	Test Structure	Theoretical Framework
Full Flip, Single-Group								
Lage	Fr	SGA	VL	40	-	O	O-X	LS
Kaner	U	SGA	VL	-	-	O	O-X	LS
Bergmann	HS	-	-	-	-	-	-	-
Talbert	U	SGA;Q	VL	7	-	O	O-X	PBL
Gannod	Fr-Gr	HW;SGA	VL	20;160 IP	-	O	X-X	COOP
Toto	Jr	-	VL;Q	74	-	O	X <sub>1</sub> -X <sub>2</sub>	AL, LS
Zappe	U	SGA	VL;Q	77	-	O	-XX	AL
Demetry	U	SGA	VL;Q	125	-	-	-	PAL
Full Flip, Controlled								
Day	So-Gr	SGA	VL;HW	28	18	O;P	O-X	Constr.
Foertsch	So-Jr	SGA	VL	415	234	O	O-X	COOP
Partial Flip, Single-Group								
Kellogg	U	-	CM	-	-	-	-	LS
Warter-Perez	Fr-So	L;SGA	VL	25-30	-	O	X-X	PBL, CL
Dollar	U	SGA	CM	-	-	P	X-X	LS
Tan	Fr	L;VL	HW?	75	-	O	O-X	AL
Baker	U	SGA	RA;CM	-	-	-	O-X	AL
Bland	So-Jr	HW	HW	-	-	O	O-X	AL
Franciszkowicz	Fr-So	HW	HW	1074	-	O	-XX	AL, PBL
Partial Flip, Controlled								
Thomas	U	HW	VL	405	275-668	P	O-X	-
Stelzer	U	L;SGA	VL;CM	500+	500+	O;P	O-X	AL
Moravec	Fr-So	L;SGA	VL	795	1310	O;P	O-X	AL
Strayer	U	SGA	CM	23	26	O	O-X	Piagetian
Papadopoulos	U	L?	CM;HW	43	11	O;P	X-X	PBL

*Note.* Grade Level: U = Undergraduate; Fr,So,Jr,Sr = First, Second, Third, and Fourth Year Undergraduate; HS = High School. In-Class and Out-of-Class Activities: L = Lecture; VL = Video Lecture; HW = Homework; Q = Quizzes; SGA = Small-Group Activities; CM = Computer Modules (text-based). Number of Participants: IP = In Progress. Instrument Type: O = Subjective Opinion Survey or Informal Assessment; P = Objective Performance Test. Test Structure: O-X = Posttest Only; X-X = Matched Pretest-Posttest; X<sub>1</sub>-X<sub>2</sub> = Unmatched Pre- and Post Measures; -XX = Mid- and Post- Semester Measures. Theoretical Framework: AL = Active learning; LS = learning styles; PBL = Problem-based learning; COOP = Cooperative (and collaborative) learning; CL = collaborative learning; Constr. = constructivism. Information for entries marked with - was missing or not available.



been given textbook readings. This observation is encouraging because although learning gains are high for information presented textually, Sappington, Kinsey, and Munsayac (2002) shows that college students do not generally complete reading assignments. Nevertheless, upon recommendation by students, many instructors instituted a required pre-class quiz on the lecture material. This was touted as a highly successful practice. Students preferred live in-person lectures to video lectures, but also liked interactive class time more than in-person lectures (Toto & Nguyen, 2009). Shorter, rather than longer videos were preferred (Zappe et al., 2009).

The two remaining results that both qualified as flipped classroom studies and examined objective student performance are those by Moravec, Williams, Aguilar-Roca, and O'Dowd (2010) and Day and Foley (2006). Moravec et al. (2010) modified the presentation method for three lectures in an introductory biology course. Students were required to watch narrated PowerPoint videos and complete a worksheet before class time. In class, students participated in alternating 10-minute mini-lectures and 5- to 7- minute active learning exercises. This led to a performance increase of 21% on exam questions related to the topics introduced outside class with videos. While these results are encouraging, there are several shortcomings to this study. First, in-class activities still carried a lecture component, even though time was provided for interactive activities. Second, the duration of the treatment was very short, and topics on both sides of the flipped topics were still taught with traditional methods. This leaves open the question of whether similar results would be achieved across all topics if the entire class were flipped.

Day and Foley (2006) conducted their study in a senior-level computer interaction course. They taught concurrent experimental and comparison sections of the course, and matched sections on topics, assignments, and time on task. Students in the experimental section watched narrated PowerPoint videos outside of class, and participated in interactive learning activities inside class. Students in the flipped environment scored significantly higher on all homework assignments, projects, and tests.

In summary, of all the studies on the flipped classroom, there is only one (Day & Foley,

2006) that has examined student performance throughout a semester. While the results from this study are encouraging, this is not sufficient evidence to warrant generalization far beyond that situation. Further, the solution was very specific, rather than being based on established principles to guide adaptation. Thus, additional research is needed to examine the influence of flipped classroom instruction on objective learning outcomes. The present study differs from that of Day and Foley (2006) in two important ways. First, students will be assessed with both pre- and post- measures. Day and Foley (2006) examined only post- intervention scores. Second, the present study will use conventional measures such as homework and tests, as well as a concept test to measure conceptual understanding.

## **2.5 Teaching Numerical Methods for Engineers**

Two concepts central to this work—the use of computers, and the use of a realistic organizing problem as the basis for teaching numerical methods to engineering students—have, at least to some extent, been explored in previous research. As early as 1974, Sumner prescribed the use of computers to help teach numerical methods concepts. These sentiments have also been echoed more recently by science and engineering educators (Carroll, 1992). Barry and Webb (2006) also support the use of programming to teach numerical methods to engineers of many disciplines. However, in a recent restructuring of their program, they organized this teaching around six two-week engineering problems. Others have also adopted this problem focus, but prefer to organize all activities around a single, multi-part semester-long problem (Coller & Scott, 2009; Coller & Shernoff, 2010; Musto, 2002). Musto calls this effort, “popular and successful” (p. 245). At Northern Illinois University, the central problem was based on an open-source race-car simulator, and students were taught numerical methods concepts as they were given a series of different challenges related to controlling and racing their virtual vehicle. Coller and Scott (2009) report that students gained a deeper understanding of numerical methods concepts using this approach. Additionally, students in the project-based course spent nearly twice as much time as students in the traditional course. Coller and Shernoff (2010) observe that student engagement in this course is much higher than their other engineering courses.

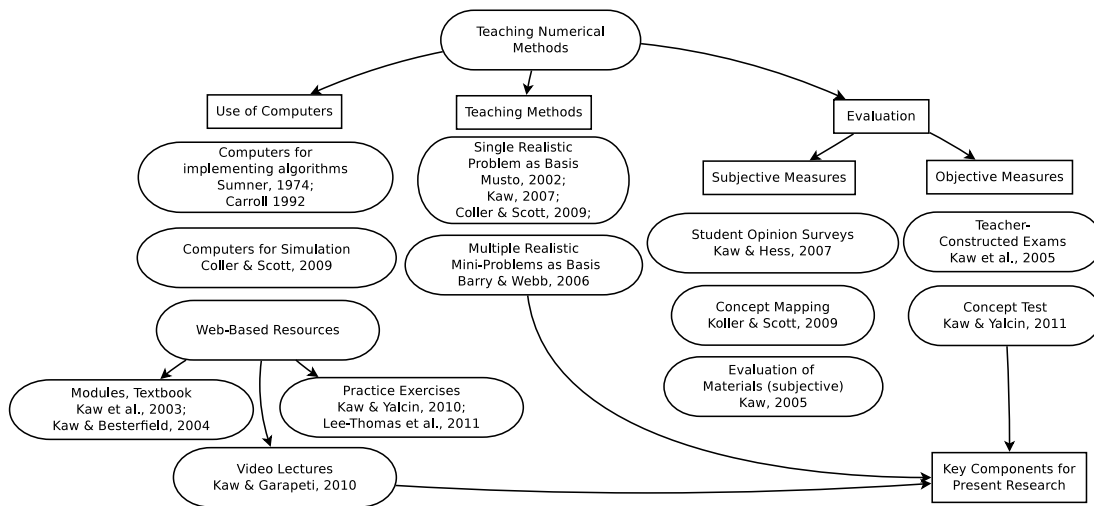


Fig. 2.5: Research on teaching numerical methods.

The largest body of relevant research in teaching Numerical Methods for Engineers comes from a research group at the University of South Florida (USF). Led by Autar Kaw, the *Holistic Numerical Methods Institute* began developing web-based course materials with funding from the National Science Foundation in 2001 (Kaw et al., 2012). In the subsequent 11 years, and with additional NSF funding, the online resources developed by this group have gradually expanded and grown in popularity (Hess, Kaw, & Owens, 2009; Owens, Kaw, & Hess, 2010). These materials now include digital audiovisual lectures (Kaw & Garapati, 2010, 2011), slide presentations, worksheets, textbook chapters (Kaw & Besterfield, 2004; Kaw, Collier, Keteltas, Paul, & Besterfield, 2003; Kaw, Keteltas, Paul, Eison, & Besterfield, 2003), realistic application problems (Kaw, 2007; Kaw & Yalcin, 2008), and multiple-choice quizzes (Kaw & Yalcin, 2010; Lee-Thomas, Kaw, & Yalcin, 2011). A recent publication from this research group (Kaw et al., 2012) chronicles the history of their work.

Of particular interest to the current work is the progress this group has made in developing instruments for measuring the effectiveness of the methods. An overview of these is given in Table 2.3. Instruments were developed to measure student learning, student satisfaction, and to evaluate the effectiveness of web resources. Web resources were evaluated by both students and teachers (Kaw, Besterfield, & Eison, 2005), and their popularity was examined using Google Analytics (Hess et al., 2009; Owens et al., 2010). A student

satisfaction survey was used to assess student reactions to learning experiences (Kaw et al., 2005; Kaw & Hess, 2006, 2007). Finally, two different methods were used to measure student learning. First, a teacher-constructed multiple-choice examination (Kaw et al., 2005; Kaw & Hess, 2006, 2007). Second, a more carefully constructed concept *test* designed to function as a concept inventory, but developed using less rigorous standards (Kaw & Yalcin, 2011). The group has indicated (Kaw et al., 2012) their intention of producing a revised version of this concept inventory using the Delphi method (Streveler et al., 2011).

Figure 2.5 provides a graphical depiction of the research on teaching numerical methods as just presented. There are three key components from previous work that the present research builds on. First, video lectures were used to present numerical methods principles outside of class. Although Kaw and Garapeti (2010, 2011) detailed the development and popular appeal of these video lectures developed for numerical methods, in their studies these materials served as an optional supplement, rather than the primary method of delivering new content. Second, several realistic problems were used for the in-class activities, similar to Barry and Webb (2006), except these experiences comply with the structured practices of model-eliciting activities. Third, the concept test developed by Kaw and Yalcin (2011) was used to measure conceptual gains for students in the course. This decision is directly supported by the recommendations of Gijbels et al. (2005).

Table 2.3: References and Instruments Used with Numerical Methods at USF.

Publication	Instruments	Purpose of Publication
Kaw et al. 2005	SRC, MCX, SRW	Compare 2 delivery modalities, 1 unit
Kaw and Hess 2006	SRC, MCX	Compare 4 delivery modalities, 1 unit
Kaw and Hess 2007	SRC, MCX	Compare 4 delivery modalities, 1 unit
Hess et al. 2009	SRW, TRW, GA	Evaluate web resources
Owens et al. 2010	SRW, TRW, GA	Evaluate web resources
Kaw and Yalcin 2011	CT	Instrument Development

*Note.* SRC = student review of the course; MCX = multiple choice exam; SRW = student review of web materials; TRW = teacher review of web materials; GA = google analytics. CT = concept test.

## 2.6 Summary

In this chapter, the research literature was reviewed in four key areas: student-centered learning, computer-aided instruction, the flipped classroom, and teaching numerical methods. Student-centered learning theories focus to provide the basis for the in-class portion of flipped classroom activities, while computer-aided instruction provides a similar foundation for out-of-class activities. This research study used MEAs, which are realistic, team-based, client-driven engineering problems designed to foster students' mathematical modeling abilities (e.g., Diefes-Dux et al., 2004). MEAs are based on over a century of research that suggests that because knowledge is constructed rather than transferred, students learn more effectively when they are actively and collaboratively engaged in solving realistic problems.

Interactive video lectures were also used in this study, which can be more effective than in-person lectures (e.g., Cohen et al., 1981; McNeil, 1989; Zhang et al., 2006), as the primary delivery method of new content for the course. These two components—interactive student-centered learning activities and interactive video lectures—make up the flipped classroom. A review of the flipped classroom research revealed several recommendations from previous research that were used to guide the design and implementation of the present research. These include recommendations to implement pre-class quizzes (e.g., Toto & Nguyen, 2009), and keeping video length to less than 20 minutes (e.g., Zappe et al., 2009).

Given that the subject-specific context for this research was numerical methods, published studies on teaching numerical methods were also reviewed. Previous research suggests that implementation of problem-based learning has led to increased student engagement (e.g., Barry & Webb, 2006; Coller & Scott, 2009; Musto, 2002), and that introducing short quizzes (e.g., Kaw & Yalcin, 2010; Lee-Thomas et al., 2011) and online video lectures (e.g., Kaw & Garapati, 2010, 2011) have also been successful.

Based on this information, the implementation of the flipped classroom model through the use of MEAs and interactive video lectures is a promising method, which may improve educational effectiveness in the classroom. The current study was designed to help evaluate this conjecture. The affordable state of recording technology and ubiquity web-based dissemination tools also make this research both timely and cost-effective.

## Chapter 3

### Methods

In Section 1.2, the purpose of this research is described, which was to compare the educational effectiveness of flipped classroom instruction consisting of model-eliciting activities and video lectures to traditional classroom instruction in a university-level introductory numerical methods course for engineers. Educational effectiveness was evaluated along three dimensions: conventional problem-solving ability, conceptual understanding, and student perceptions. This led to three specific research questions, which were stated previously in Section 1.5. Based on these research questions, the independent and dependent variables are described in Section 3.1. Section 3.2 sets forth the measurement instruments used, and Section 3.3 describes the research design. Considerations for the preparation of videos and other logistics are discussed in Section 3.4. Finally, Section 3.5 presents the data collection and analysis procedures.

#### 3.1 Variables

##### 3.1.1 Independent variable

The independent variable in this research was the type of instruction for the numerical methods course. This variable has two levels. Traditional instruction represents one level of the independent variable, and video lecture/MEA classroom instruction represents the second level. Each of these is described in detail in subsequent sections.

##### 3.1.2 Dependent Variables

There were two general constructs being evaluated as part of this research. The first construct is knowledge of numerical methods. This was measured along two dimensions, conventional problem-solving ability and conceptual understanding. Two instruments were

employed in an effort to measure conventional problem-solving ability, and two were used to measure conceptual understanding. Conventional problem-solving ability was measured using homework scores and exam scores, while conceptual understanding was assessed with short quizzes and a numerical methods concept test. Homework scores acted as a formative indicator of conventional problem-solving ability, and exams served as a summative measure. Similarly, quizzes served as a formative measure of conceptual understanding, while the concept test fulfilled a summative evaluation. Together, these four measures (homework, exams, quizzes, and the concept test) constitute the first four dependent variables in this study.

The second construct is student perceptions of the course. Due to an abundance of research on student perceptions of the flipped classroom, measurement of this is secondary, and is aimed more at validating the implementation than contributing new knowledge to the corpus of research. This was measured with two instruments: a student perceptions survey, and routine course evaluations.

## **3.2 Instruments**

Five instruments were used to evaluate student perceptions and learning: routine course evaluations, homework assignments, examinations, conceptual quizzes, and a numerical methods concept test.

### **3.2.1 Homework Assignments**

Seven teacher-constructed homework assignments were given throughout the semester. These assignments generally included three to six quantitative numerical methods problems based on textbook exercises (Chapra & Canale, 2009). The exact number of problems varied based on the topic-specific considerations.

Because it is possible for students to collaborate or cheat on homework, this measure had carry limited weight in the final evaluation of educational effectiveness. The low relative homework weight in final grading (5%), as indicated in Table 3.3 was meant to discourage cheating, but not collaboration. Despite their inherent limitations, it would not be wise to

completely ignore homework scores in evaluating the results of the study. Homework assignments are given in Appendix A. Peer grading was used (on all except the first assignment) in order to provide rapid feedback on assignments and to allow students to see, and have an opportunity to discuss, the correct answers to all homework questions before an exam was administered on the given material.

### **3.2.2 Examinations**

Seven examinations based on the content from each section were administered throughout the course. Six examinations were topic-specific, administered at approximately two-week intervals. These examinations are constrained in length because they must be given during the regularly scheduled class time (50 minutes). During the (110-minute) final examination period, the concept posttest was administered, along with a nine-question comprehensive test. These exams typically required students to solve relatively simple computational problems related from the material for each section. The examinations (with solutions) are given in Appendix B.

### **3.2.3 Quizzes**

The recommendations for best practice noted in the flipped classroom literature, suggest using a quiz or pre-class activity to provide motivation to watch the video lectures (see e.g., Day & Foley, 2006; Zappe et al., 2009). Therefore, a quiz was due every day before class on the material from the previously scheduled class. Classroom attendance was required. The quizzes were designed to test conceptual understanding rather than the computational skills required by conventional quantitative problems. The quizzes are given in Appendix C.

### **3.2.4 Concept Test**

The concept test is an assessment tool for numerical methods, developed at the University of South Florida. The development and properties of this test are reported by Kaw and Yalcin (2011). It was designed using the principles of a concept inventory, such as the well-known force concept inventory used in physics Hestenes et al. (1992), but has not yet



undergone the same level of rigorous expert review. The purpose of concept inventories as a measure of student learning is distinct from other measures. It is intended to probe how well instruction causes students to develop deep conceptual understanding and think like experts (W. K. Adams & Wieman, 2011).

The numerical methods concept test consists of 16 multiple-choice questions, with two questions from each of eight numerical methods topics:

- Fundamentals of scientific computing,
- differentiation,
- nonlinear equations,
- simultaneous linear equations,
- interpolation,
- regression,
- integration, and
- ordinary differential equations.

It is designed to be administered as a pre-post test pair. In evaluating the instrument, Hake's gain index scores of 0.36 to 0.43 were recorded. Reliability coefficients, as measured with Cronbach's alpha were 0.5 to 0.6. This means that 40% to 50% of the observed variability in scores is due to measurement error, while the remaining portion is variability of the construct. Criterion-related validity was evaluated by comparing results to those of an instructor-created exam administered to the same students. Results of a Pearson product-moment correlation were:  $r = 0.47$ ,  $n = 136$ ,  $p < 0.001$ . This strong positive relationship between concept test and exam scores supports the appropriateness of this test to the numerical methods context. A copy of the numerical methods concept test was obtained from Dr. Kaw and permission to use the test for this research was granted under a strict non-disclosure agreement (it cannot be included in an appendix).

### 3.2.5 Course Evaluations

At Utah State University, course evaluations are a routine part of the final activities in every class. A recent study of the psychometric properties of course evaluations at USU was conducted by Drysdale (2010). Three-week test-retest reliability coefficients ranged from 0.64 to 0.94 and correlation coefficients for validity ranged -0.39 to 0.71. The mean instructor rating-final exam correlation was 0.14, and the mean course rating-final exam correlation was 0.11. This suggests that while course evaluations are relatively consistent with themselves, there is virtually no relationship between rated instruction or course quality and examination scores. Full psychometric properties, and a copy of the USU course evaluation sheet are given by Drysdale (2010).

In 2011, Utah State University instituted a new course evaluation system, managed by IDEA, a nonprofit organization that services colleges and universities. While it is possible that the results from the currently used IDEA evaluations have different properties from those studied by Drysdale (2010), it is unlikely. This is supported by a recent meta-analysis of student evaluations of teaching conducted by Clayson (2009). The overall finding of Clayson's meta-analysis was that while there was a small positive relationship between objective measures of student learning evaluations of teaching, this value was not significantly different from zero. Further, they found that the more objective the measurement process became, the more the learning/evaluation association was reduced. Although the course evaluations were administered online, a copy of the equivalent paper form with the evaluation questions is given in Appendix G.

### 3.3 Research Design

This study was conducted as a controlled quasi-experiment, since a single-group design would not have been sufficient to respond to the research questions. Two sections of ENGR 2450 were offered concurrently, and no instructor was listed when students initially signed up for the course. While a true experimental design would have been preferred, randomly assigning students to a section was not logistically possible. Although students did self-select, the two choices were indistinguishable, so randomization of groups in this study

was closer to full randomization than a typical quasi-experiment. Because students of one particular major may have different course schedules, holding both sections of the course concurrently was important because it ensured the maximum opportunity for mixing, thus avoiding one section consisting of students of all the same major. A coin flip was used to randomly designate one section as the comparison (control) group, and the other as the treatment (experimental) group.

### 3.3.1 Course & Participants

ENGR 2450 - Numerical Methods for Engineers is a sophomore-level engineering course at Utah State University in Logan, Utah. It is required for engineering students in five majors: Biological Engineering, Environmental Engineering, Civil Engineering, Computer Engineering, and Electrical Engineering. ENGR 2450 is usually offered once a year, during the spring semester. Total combined enrollment for ENGR 2450 Spring 2013 was 164. Forty-six students were excluded from the analysis, of whom 8 dropped the course, 9 switched sections, and the remainder did not return a signed consent form. Of the remaining students, 63 were in the comparison section (Section 001), and 55 were in the treatment section (Section 002).

Information regarding the declared major for students participating in this study is shown in Figure 3.1. Ethnicity and gender data for students enrolled in the course was not directly available. In 2012, however, 88.9% of students in the the College of Engineering at Utah State University, were white non-Hispanic, and 11.1% were female.<sup>1</sup>

Three courses are required as pre-requisites to ENGR 2450: Linear Algebra and Differential Equations, Introductory Programming, and Calculus II. However, ENGR 2450 and Linear Algebra and Differential Equations may be taken concurrently. ENGR 2450 is a three-credit course. It met three times a week for 50 minutes each period. There were a total of 45 class periods over the course of the 15-week semester. One of these was a 110-minute period reserved for the final exam.

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<sup>1</sup>[http://usu.edu/aaa/stem\\_analysis.cfm](http://usu.edu/aaa/stem_analysis.cfm)

Table 3.1: Declared Field of Study Within Engineering for Students in ENGR 2450 Spring 2013

Major	Section		Total	Percent
	Comparison	Treatment		
Biological Engineering	19	4	23	19.5
Civil Engineering	21	20	41	34.7
Computer Engineering	9	4	13	11.0
Electrical Engineering	10	19	29	24.6
Environmental Engineering	1	4	5	4.2
Other	3	4	7	5.9
Total	63	55	118	99.9

*Note.* Total is less than 100% due to rounding.

Seven general topics mirroring those presented in the required text by Chapra and Canale (2009) were introduced in this course. These included number representation & error analysis, nonlinear equations (root-finding), linear systems (linear algebra), interpolation/polynomial approximation & curve fitting, integration & differentiation, ordinary differential equations, and partial differential equations.

### 3.3.2 Treatment Section

The treatment section of the course was redesigned to integrate both web-based video lectures and model-eliciting activities. For both sections, the same topics were taught in the same order, except that all in-class lectures for the treatment section were supplanted by web lectures. As recommended in the flipped classroom literature (Day & Foley, 2006; Zappe et al., 2009), a quiz on the lecture material was required before class started. Classroom attendance was required.

Two MEAs were used throughout the semester, and each MEA took approximately 6 weeks. The first MEA used just-in-time manufacturing as a motivating problem, while the second was based on a challenge to re-assemble shredded documents. The just-in-time manufacturing context was chosen for the first MEA because it does not require a deep understanding of numerical methods concepts, which would not be introduced until later in the course. However, it serves to introduce MEAs with an engineering context that is

interesting and engaging. The shredded document MEA was chosen for the second activity because regression methods are well-suited to this problem, and this topic would coincide with the introduction of regression and curve fitting methods in the course. A copy of the MEA assignments is given in Appendix F.

Students were given 12 in-class days to work on each MEA. Working in groups of 3-4, students were required to complete a total of three drafts of their work for each MEA. Each group provided and received peer feedback between each draft of the MEA memo. A detailed schedule of the activities for this section is provided in Table D.2 of the appendix. For the first two weeks of the course, both sections were combined in a single lecture hall and covered common material (Part 1 of Chapra & Canale, 2009).

### **3.3.3 Comparison Section**

The comparison section of ENGR 2450 was taught in the typical manner for a numerical methods course, with a few modifications to maintain equivalence on non-treatment aspects between the sections. Specifically, 35 class periods were used for in-class lectures and review. Nine class periods were used for administering the concept test and exams. Quizzes identical to those given to the treatment section were administered through the same online learning management system (CANVAS). Lecture materials such as PowerPoint slides and other supplementary materials were available to both sections. Homework, exams, and quizzes were administered or collected at the same time for both sections. For the first two weeks of the course, both sections were combined in a single lecture hall and covered common material (Part 1 of Chapra & Canale, 2009). This portion of the course was taught by both instructors. From week two onward, the instructors alternated between sections as shown in Table 3.2. Instructors alternated sections in order to ensure equal exposure to both instructors. This was so that if differences were found between groups, these could be attributed to differences in instruction methods rather than instructor-specific factors (e.g., instruction styles or prior teaching experience).

Table 3.2: Side-by-side Comparison of the Instructor for Each Section

Time	Section		Course material
	Treatment	Comparison	
Weeks 1-2	Both	Both	Part 1
Weeks 3-4	Instructor 1	Instructor 2	Part 2
Weeks 5-6	Instructor 2	Instructor 1	Part 3
Weeks 7-8	Instructor 1	Instructor 2	Part 5
Weeks 9-10	Instructor 2	Instructor 1	Part 6
Weeks 11-12	Instructor 1	Instructor 2	Part 7
Weeks 13-14	Instructor 2	Instructor 1	Part 8
Week 15	Instructor 1	Instructor 2	Review

### 3.3.4 Validity Threats & Countermeasures

Maintaining control is particularly difficult for a semester-long course. However, the treatment and comparison sections were matched on as many factors as possible (i.e., ethical). The following factors were held constant for both sections of the course.

- Same instructor(s) taught both sections of the course.
- Presentation of material were similar.
- The same topics were covered concurrently in both sections.
- Written notes and supplemental materials (except MEA and video lectures) were common.
- Recommended textbook readings were the same.
- Attendance was required for both sections.
- Lecture quizzes were identical, assigned after lecture material is covered.
- Exams were identical, and were administered concurrently.
- Grade weighting was comparable, see Table 3.3.
- Treatment section: In-class time + web lecture length + est. external MEA time.
- Comparison section: In-class time + est. time external programming assignments.

Table 3.3: Side-by-side Comparison of the Grading for Each Section

Activity	Grade Weight	
	Treatment	Comparison
Prog. Assignments	20%	-
MEAs	-	20%
CT-Pre	2.5%	2.5%
Quizzes	10%	10%
Homework	5%	5%
Exam 2	10%	10%
Exam 3	10%	10%
Exam 5	10%	10%
Exam 6	10%	10%
Exam 7	10%	10%
Exam 8	10%	10%
Final Exam	10%	10%
CT-Post	2.5%	2.5%

*Note.* The lowest score from among exams 2-8 was dropped for each student.

The attendance policy for both sections was given as follows: Attendance will be taken, and while two absences will be dismissed without explanation, further absences will be penalized by subtracting 1% from the students' grade. Access to web-based video lectures was limited to students in the treatment section. This measure was intended to avoid treatment diffusion, which is a concern in a course this long with participants in close proximity.

In spite of the researchers' best efforts, there are some threats to validity that remained. These may be caused by:

- Personality differences
- Learning styles
- Hawthorne effect (knowing that they are in a study)
- Differences in class size
- Influences of the concept pretest
- Differences in aptitude

- Differences in major
- Instrument (in)validity

Despite these weaknesses, this study still represents a sufficiently strong design to enable valid comparison between groups.

### **3.4 Preparation & Logistics**

#### **3.4.1 IRB & Informed Consent**

The institutional review board (IRB) at Utah State University reviewed the proposal for this research and approval was granted to proceed as outlined in this paper.

Participants in this study were presented with a consent form to permit research and publication of anonymized information from course-related materials. Data from minors or from participants not willing to grant consent were excluded from the analysis. A copy of the consent form is given in Appendix H.

#### **3.4.2 Video Production**

Web lectures were produced by instructor 1 using a Wacom Create pen tablet and the drawing software SmoothDraw3. They were recorded with the Camtasia Studio 8 screen capture software. Following recommended best-practice, target running time for the videos was 10 minutes or less. Longer topics were usually split across multiple videos. A total of 197 videos were produced on 7 major topics and 28 sub-topics. The total video length was 21 hours, 9 minutes, 58 seconds. The average video length was 6 minutes, 27 seconds. This resulted in an average of 45 minutes of video per scheduled lecture (4 lectures per two-week unit). Videos for the course material from part 1 were recorded but not used. A hyperlinked index of the video lectures, with unit, title, and video length is given in Appendix E.

Kaw and Garapati (2010) recorded 200 ten-minute video lectures on numerical methods topics in 2009. Recordings were made during two 75-minute sections each week for 14 weeks. Two technicians recorded the lectures, and a third performed editing and processing.



Estimated expense was \$7,000. Video lectures were recorded with a single person, a graphics tablet, and a desktop computer. Production, editing, and processing were also done by a single person, and since three technicians were eliminated from the process, production costs were approximately one-fourth those reported by Kaw and Garapati (2010).

### **3.4.3 MEA Training**

Prior research (Diefes-Dux et al., 2004) indicates the importance of training before the administration of MEAs. Thus, an 8-hour training was required of teaching assistants facilitating MEA-related activities in the treatment section. This training was split over the course of 2 days. It included actually solving an MEA, instruction and practice in using the MEA scoring rubric, discussion of teamwork-related issues, and best practices on providing feedback to students.

## **3.5 Data Collection & Analysis**

### **3.5.1 Data Collection Procedures**

Homework assignment 1 was collected and scored by teaching assistants. All other homework assignments were graded in class by peers, and instructors or teaching assistants recorded scores in the CANVAS learning management system. Examinations were collected after being administered in class and were scored by teaching assistants based on the instructor's rubric. After being recorded, scores were entered into the CANVAS learning management system.

Quizzes were administered through the CANVAS learning management system, and were usually due before the class period following the lecture. Electronic administration of the quizzes helped to ensure that closing time limits were enforced. The concept pretest was administered after the first week of class. It was not administered earlier due to the transient nature of the first week. The concept posttest was administered during the final testing period. This test was administered, collected, and scored manually, since electronic testing was not allowed (as per the NDA). Work for students who contested scores on homework,

quizzes, or exams because of grading related errors was reviewed by the instructor and scores were adjusted when necessary.

Course evaluations were also collected electronically, since this is the new routine delivery method using the IDEA system. The schedule for exams and other course activities is given in Table D.1 of Appendix D.

### 3.5.2 Data Analysis

Quantitative data for student learning measures was analyzed using the multivariate analysis of variance (MANOVA), and  $t$  test procedures in the MATLAB computation environment. A cutoff value of 0.05 for the Type I error rate was used in all tests of significance. Student evaluation scores were analyzed independently from knowledge measures, due to the demonstrated lack of correlation between the two (Drysdale, 2010), and also the fact that course evaluations are conducted anonymously. Differences in student perceptions between sections were analyzed using  $t$  tests on an item-level basis, using the Bonferroni adjustment to control the family-wise error rate. Forty-six students were excluded from the analysis, of whom 8 dropped the course, 9 switched sections, and the remainder did not return a signed consent form.

This study design presented represents a calculated tradeoff between time and resources. It will allow for a careful data-driven examination of the core hypothesis guiding this research: that a numerical methods course taught using the video lecture/MEA format will be as or more educationally effective than a traditional lecture-based course, as measured by a concept test, quizzes, homework, and exam scores.



## Chapter 4

### Results

This chapter presents the results from the five measurement instruments used in this course: homework assignments, examinations, quizzes, a conceptual test, and course evaluations. In Section 4.1, results are presented of a one-way MANOVA with treatment group as the independent variable, and four dependent variables: (1) average homework assignment score, (2) average examination score, (3) average quiz score, and (4) concept test difference score. This is followed by a detailed comparison of homework scores in Section 4.2, exam scores in Section 4.3, quiz scores in Section 4.4, and concept test scores in Section 4.5. Finally, course evaluation results are presented in Section 4.6.

#### 4.1 MANOVA Results

The data summarized in Table 4.1 was analyzed using the multivariate analysis of variance (MANOVA) procedure `manova1` in the MATLAB computing environment. The analysis examined differences on objective measures of student performance, as a function of section membership (comparison or treatment section). The four response variables included average scores for homework, examinations, and quizzes, and difference scores for the conceptual test. This MANOVA concurrently tests research questions 1 and 2, which are concerned with whether students taught using video lectures and MEAs performed as good or better than students taught with conventional group lectures. The effect of section was significant:  $F(4, 107) = 4.81, p < .01$ . Interpretation of this effect is more clear in the context of the follow-up tests.

#### 4.2 Homework

The distribution of average homework scores is shown in Figure 4.1. Comparison of

Table 4.1: Mean Scores on Objective Measures of Student Performance

Section	$N$	Homework		Exams		Quizzes		Concept Test	
		Mean	$SD$	Mean	$SD$	Mean	$SD$	Mean	$SD$
Comparison	63	83.39	19.57	85.14	9.81	74.90	16.08	13.79	14.38
Treatment	55	67.88	25.07	84.98	8.26	74.96	17.09	14.82	11.54

*Note.* Scores for homework, exams, and quizzes represent average percent correct, with missing scores treated as zeros. Concept test scores represent the difference between average percent correct on the posttest minus average percent correct on the pretest, with missing scores excluded.  $N$  = Sample Size.

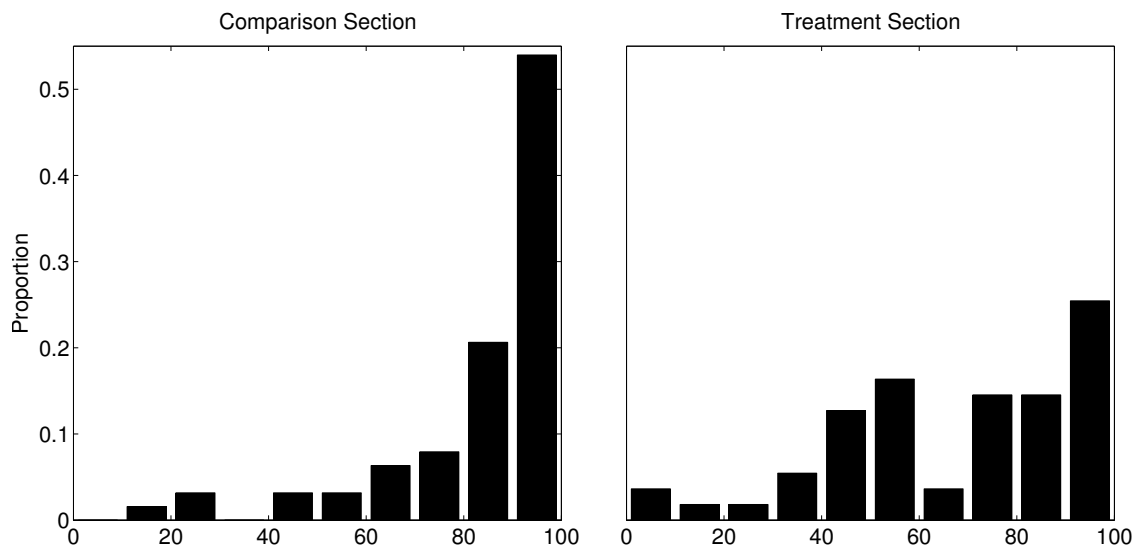


Fig. 4.1: Average homework score distribution by section.

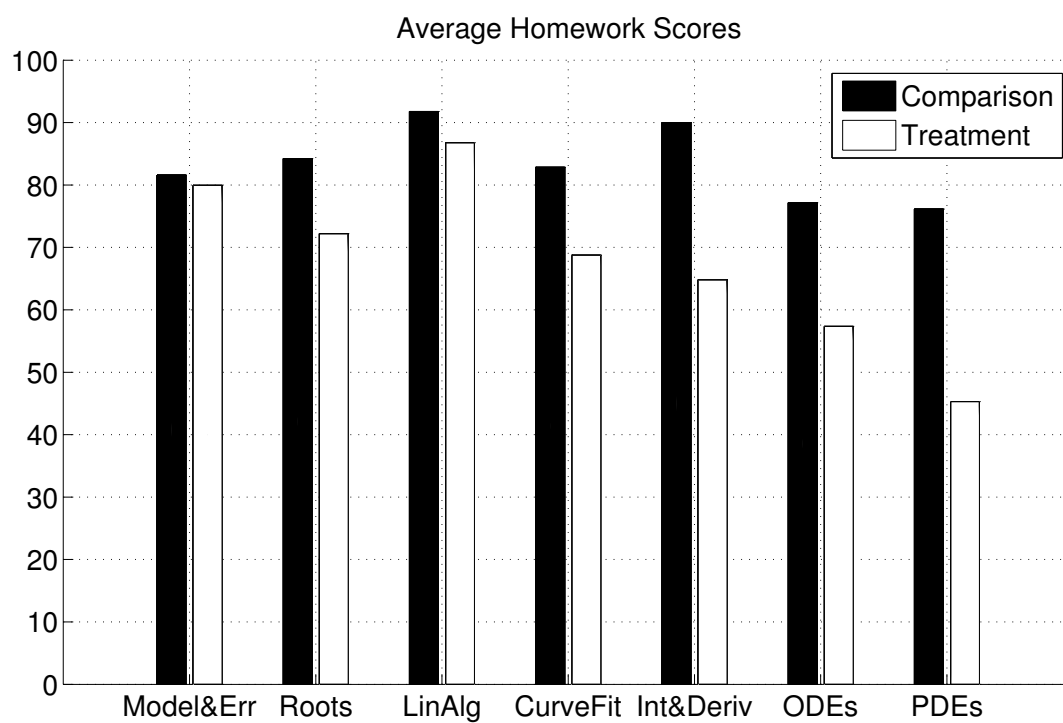


Fig. 4.2: Homework scores by topic.

these scores with a  $t$  test resulted in a significant difference:  $t(116) = 3.77, p < 0.001, SD = 22.30$ . Average homework scores were lower for the treatment section than for the comparison section. This did not confirm the research hypothesis, which was that students in the treatment section would attain equal or higher scores than students in the comparison section. When homework scores are separated by topic, shown in Figure 4.2, it is apparent that the difference in scores between sections was larger for later units in the course.

### 4.3 Examinations

The distribution of average exam scores is shown in Figure 4.3. These scores were not significantly different. Specifically,  $t(116) = 0.092, p = 0.93, SD = 9.12$ . This result confirmed the research hypothesis that students in the treatment section would attain equal or higher scores than students in the comparison section. This is in contrast to the previous finding with homework scores. Further, examining the test scores for each unit, shown in Figure 4.4, shows that the difference between scores (while not significant) was sometimes higher for the comparison section, and sometimes for the treatment section.

### 4.4 Quizzes

The distribution of average quiz scores is shown in Figure 4.5. The difference in scores between sections was not significant,  $t(116) = -0.02, p = 0.98, SD = 16.56$ . This confirmed the research hypothesis that students in the treatment section would attain equal or higher scores than students in the comparison section. The equivalence of these scores also evident when examining the quiz scores for each unit, shown in Figure 4.6.

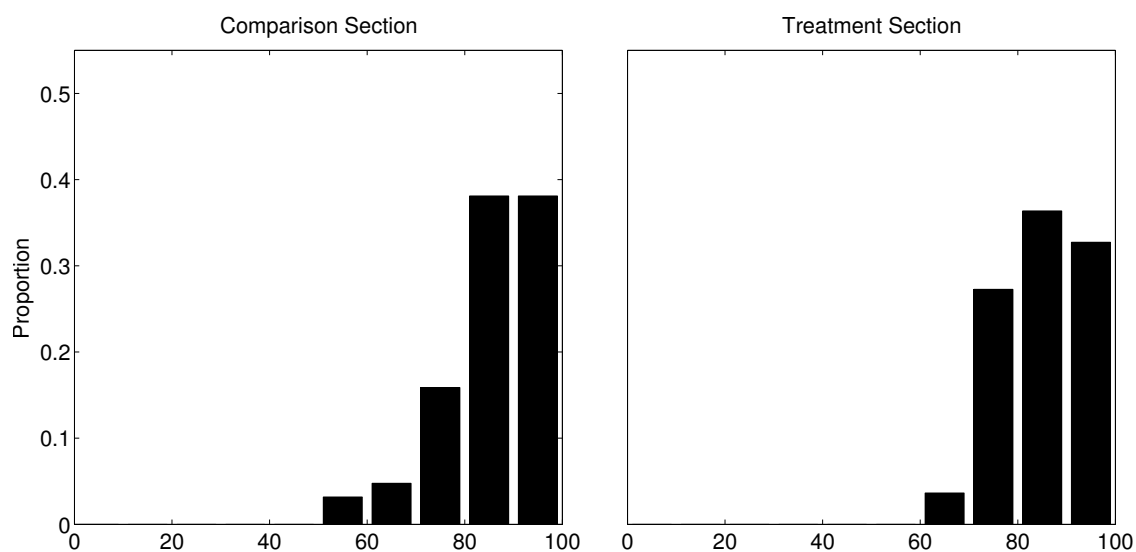


Fig. 4.3: Average exam score distribution by section.

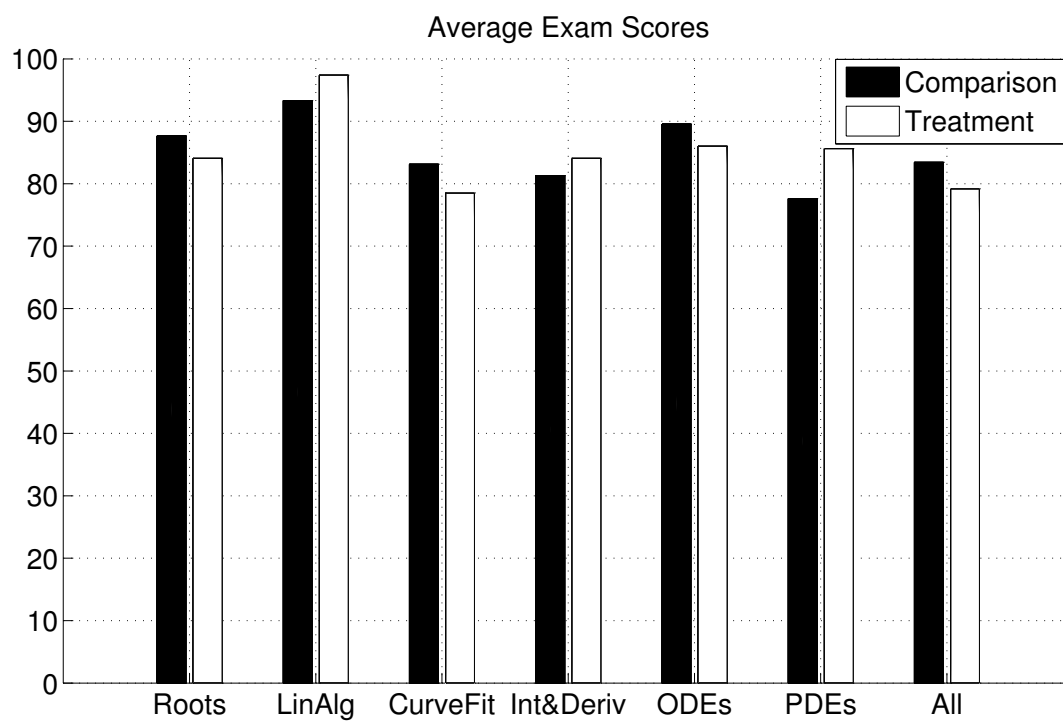


Fig. 4.4: Exam scores by topic.



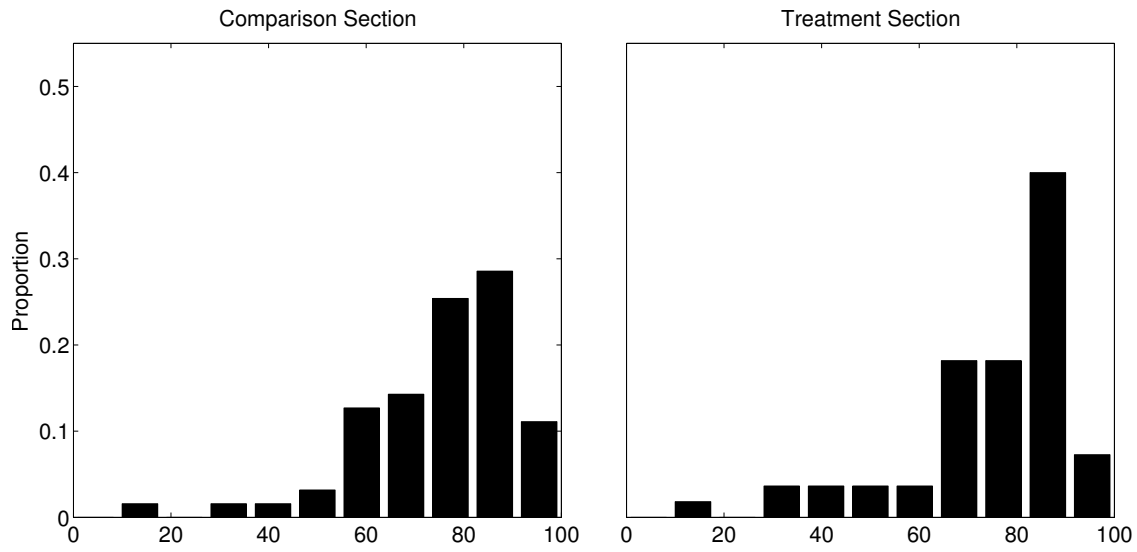


Fig. 4.5: Average quiz score distribution by section.

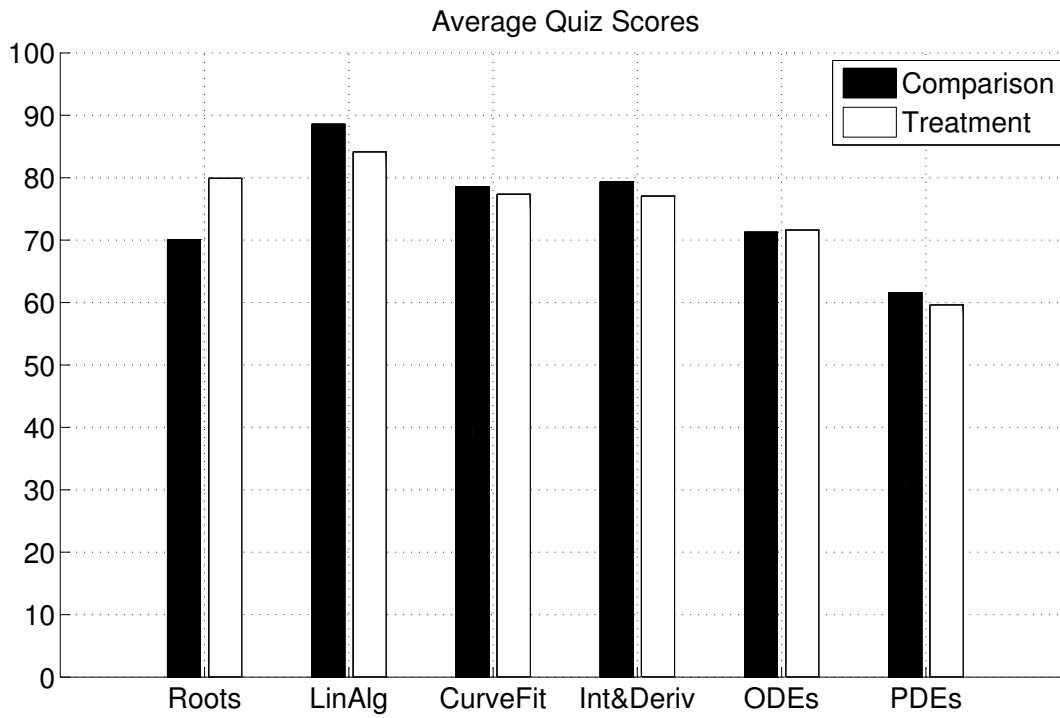


Fig. 4.6: Quiz scores by topic.

#### 4.5 Concept Test

Response proportions for difference scores (CT Post – CT Pre) on the concept test are given in Figure 4.7, while proportions for the concept pre- and posttests are given in Figure 4.8. Statistical tests revealed that sections were not significantly different on pretest scores,  $t(110) = 0.63, p = 0.53, SD = 14.15$ , posttest scores  $t(116) = 0.24, p = 0.81, SD = 11.36$ , nor on difference scores for the conceptual test,  $t(110) = -0.41, p = 0.68, SD = 13.08$ .

Hake’s gain is a normalized gain score used as an effect size for pre-post test measures (Hake, 1998), and is given by

$$g = \frac{\mu_{\text{post}} - \mu_{\text{pre}}}{100 - \mu_{\text{pre}}}. \quad (4.1)$$

The concept test scores, standard deviations, and normalized gain scores for both sections are shown in Table 4.2. Considering the previously described statistical results together with the results from a comparison of average quiz scores between sections confirms the null hypothesis corresponding to the second research question, which was that students taught using the video lecture/MEA format and students taught using the traditional lecture format would attain equal scores on measures of conceptual understanding.

Table 4.2: Mean Scores on Concept Test

Section	<i>N</i>	CT-Pre		CT-Post		CT-Difference		Hake’s Gain
		Mean	<i>SD</i>	Mean	<i>SD</i>	Mean	<i>SD</i>	
Comparison	63	64.76	14.37	78.47	10.75	13.79	14.38	0.396
Treatment	55	63.08	13.91	77.96	12.02	14.82	11.54	0.401

*Note.* Missing concept test scores were excluded. *N* = Sample Size; CT-Pre = Concept Pretest; CT-Post = Concept Posttest; CT-Difference = CT-Post – CT-Pre.

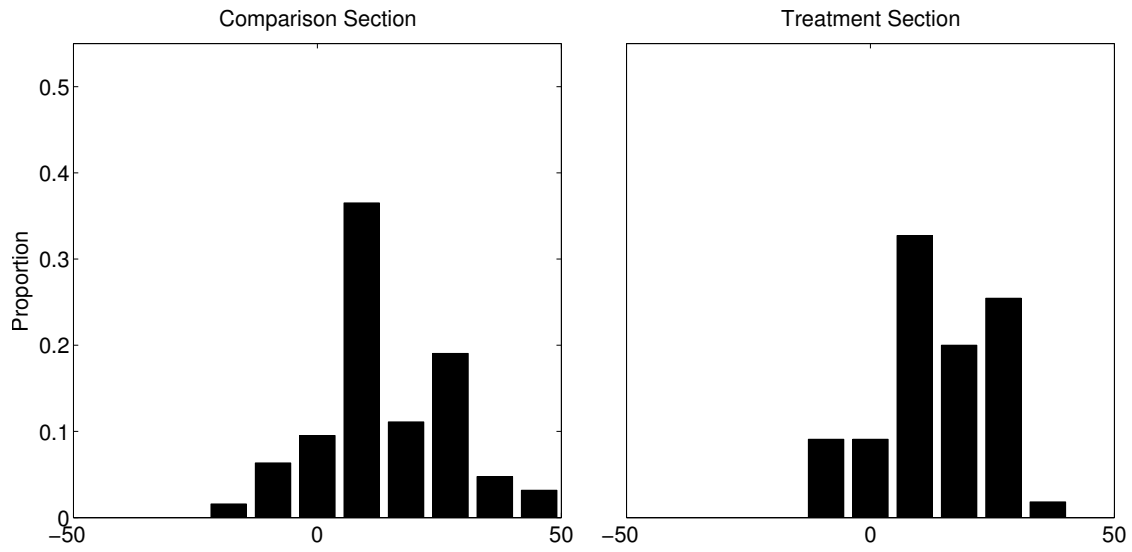


Fig. 4.7: Difference scores for the concept test.

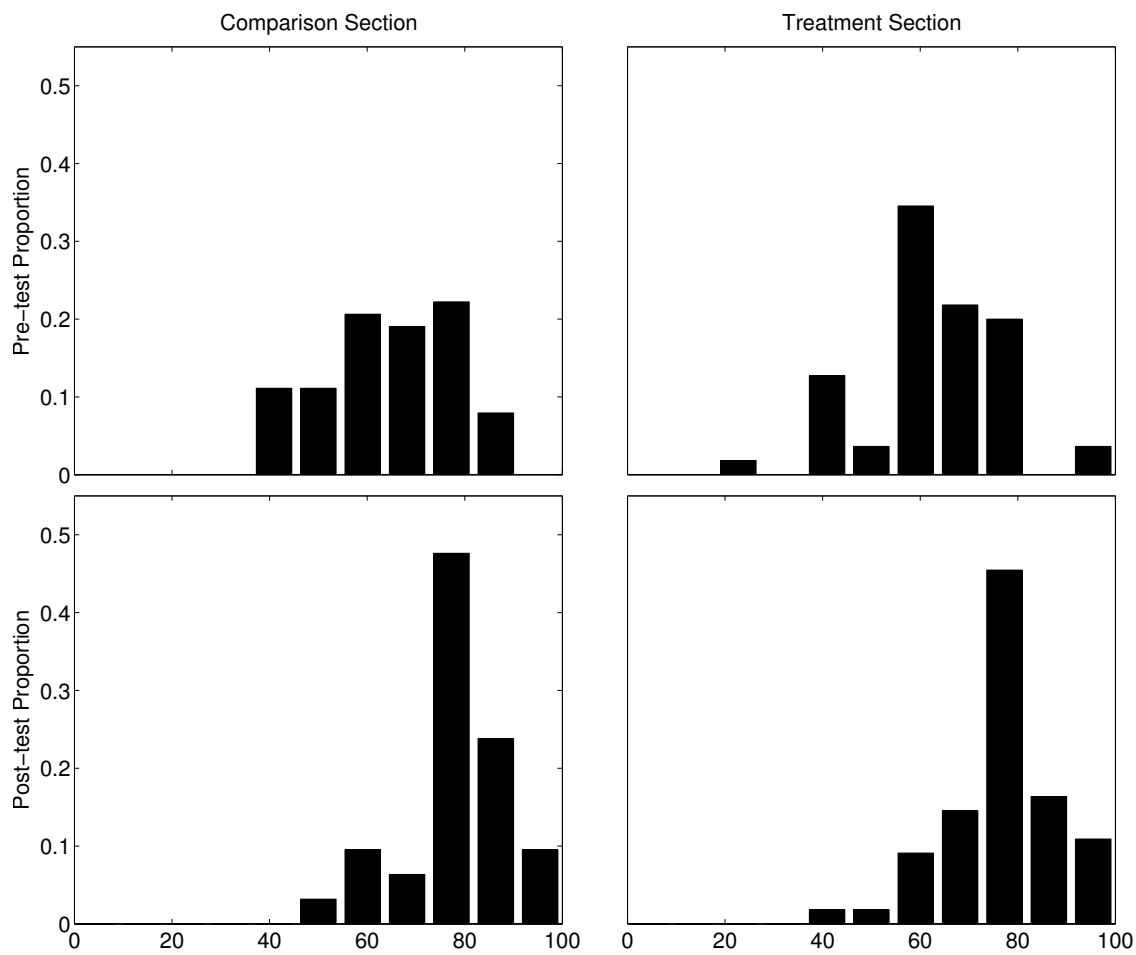


Fig. 4.8: Concept pre- and posttest results.

#### 4.6 Course Evaluations

Results from the course evaluations, which were administered through the IDEA system, are shown in Figure 4.9, broken down by section. These responses were then compared across sections using a two-sample  $t$  test with the Bonferroni correction to control for multiple comparisons, the results of which are shown in Table 4.3. This comparison showed that compared to students in the treatment section, students in the comparison section agreed more strongly that they learned to apply course material (item 3), that the instructor was an excellent teacher (item 17), and that the course was excellent (item 18). In addition to the numerical scores assigned by students in the course evaluation forms, there were also three free-response items. The questions prompts are given as follows

- What changes could be made to improve the teaching or content of this course?
- What aspects of the teaching or content of this course do you feel were especially good?
- Comments: Use the space provided in the text area below for your comments.

Consistent with the numerical items, responses to the open-ended questions were somewhat negative, and also fairly numerous. These reviews provided a significant amount of insight into the student perceptions of the course. After reading all responses, several recurring themes were identified, and the number of times an item was mentioned in a separate response was tallied. Numerical results for themes that were mentioned six times or more are given in Table 4.4.

Table 4.3: Mean Scores on Course Evaluation Items

Item	Wording	Diff	<i>SD</i>	<i>t</i> ( <i>df</i> )	<i>p</i> -value	<i>h</i>	ES
1	Gaining factual knowledge (terminology, classifications, methods, trends)	0.58	1.16	2.65 (111)	0.009	0	
2	Learning fundamental principles, generalizations, or theories	0.68	1.17	3.04 (111)	0.003	0	
3	Learning to apply course material (to improve thinking, problem solving, and decisions)	0.73	1.18	3.25 (111)	0.002	1	0.62
4	Developing specific skills, competencies, and points of view needed by professionals in the field most closely related to this course	0.61	1.28	2.50 (110)	0.014	0	
5	Acquiring skills in working with others as a member of a team	-0.66	1.28	-2.70 (111)	0.008	0	
6	Developing creative capacities (writing, inventing, designing, performing in art, music, drama, etc.)	-0.14	1.20	-0.62 (110)	0.534	0	
7	Gaining a broader understanding and appreciation of intellectual/cultural activity (music, science, literature, etc.)	0.08	1.16	0.36 (109)	0.717	0	
8	Developing skill in expressing myself orally or in writing	-0.32	1.16	-1.46 (110)	0.148	0	
9	Learning how to find and use resources for answering questions or solving problems	0.55	1.17	2.49 (110)	0.014	0	
10	Developing a clearer understanding of, and commitment to, personal values	0.20	1.16	0.89 (108)	0.377	0	
11	Learning to analyze and critically evaluate ideas, arguments, and points of view	-0.02	1.31	-0.06 (109)	0.951	0	
12	Acquiring an interest in learning more by asking my own questions and seeking answers	0.59	1.25	2.48 (109)	0.015	0	
13	As a rule, I put forth more effort than other students on academic work.	0.33	0.79	2.21 (110)	0.029	0	
14	My background prepared me well for this course's requirements.	0.21	1.01	1.11 (110)	0.269	0	
15	I really wanted to take this course regardless of who taught it.	0.24	1.35	0.95 (110)	0.344	0	
16	As a result of taking this course, I have more positive feelings toward this field of study.	0.23	1.33	0.93 (110)	0.354	0	
17	Overall, I rate this instructor an excellent teacher.	1.20	1.14	5.60 (111)	0.000	1	1.06
18	Overall, I rate this course as excellent.	0.68	1.16	3.09 (111)	0.003	1	0.58

*Note.* Diff = comparison – treatment. *t* = *t*-statistic; *h* = hypothesis (1 = significant, 0 = not significant); ES = Effect size. Items 1-13 are rated according to progress on the given objective, while items 14-18 are rated according to the truthfulness of the statement. See Appendix G for evaluation form.

Table 4.4: Frequency and Rank of Themes from Course Evaluation Comments

Theme	Frequency	Section rank	Global rank
Comparison Section			
Disliked switching teachers	23	1	2
Disliked quizzes	18	2	7
Disliked Instructor 1	18	2	1
Liked Instructor 2	13	4	5
Found Excel spreadsheets & study hints useful	8	5	6
Felt the course was disorganized	7	6	14
Liked Instructor 1	6	7	15
Treatment Section			
Disliked MEAs	26	1	3
MEAs consumed too much time	26	1	3
Wanted lecture during class	25	3	6
Disliked Instructor 1	24	4	1
Disliked video lectures	14	5	11
Liked MEAs	14	5	9
Felt MEAs and course material were disconnected	14	5	9
Liked Instructor 2	13	8	5
Disliked switching teachers	12	9	2
Felt they were treated unfairly	12	9	12
Felt the workload was excessive	9	11	13
Found Excel spreadsheets & study hints useful	8	12	7

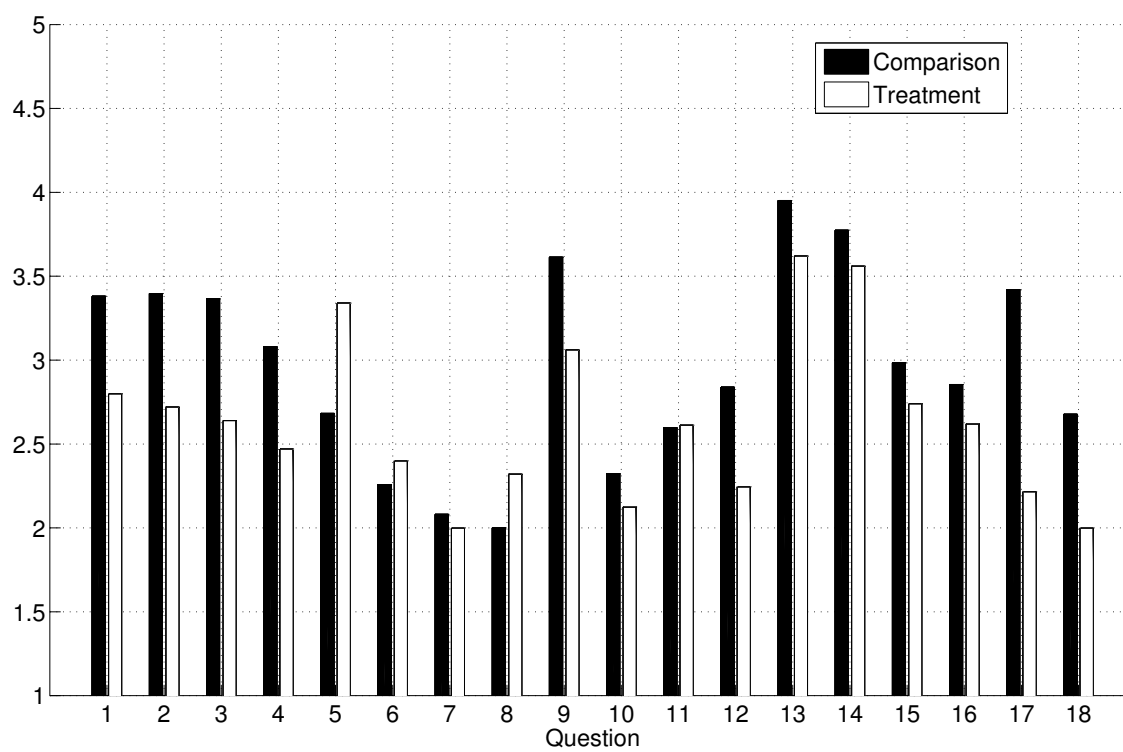


Fig. 4.9: Course evaluation





## Chapter 5

### Discussion

There were three research questions being explored as part of this study. These three questions form the organizational basis for the this chapter, and are given as follows:

1. Will students in a numerical methods course taught using the video lecture/MEA format attain equal or higher conventional problem-solving performance than students in a traditional lecture-based course, as measured by student homework and exam scores?
2. Will students in a numerical methods course taught using the video lecture/MEA format attain equal or higher conceptual understanding than students in a traditional lecture-based course, as measured by conceptual quizzes and a concept test?
3. Will students in a numerical methods course taught using the video lecture/MEA format attain equal or higher opinions of the learning experience as students in a traditional lecture-based course, as measured by students' self-reported perceptions?

Section 5.1 discusses the implications of the results to research question 1, Section 5.2 discusses the implications of the results to research question 2, and Section 5.3 discusses the implications of the results as they relate to research question 3. Because students were relatively well distributed among the two sections, declared major likely had little influence on the outcomes of this study. Nonetheless, an analysis of the results of this study, accounting for declared major, are discussed in Appendix I.

#### 5.1 Research Question 1

The first two research questions are related to whether student learning gains were different according to the treatment used, and were the primary questions of interest for this

study. The main purpose for investigating these questions is that the use of other interactive student-centered teaching methods such as problem-based learning have been shown to have a negative affect on objective knowledge outcomes, but a positive influence on skills-related outcomes. The flipped classroom may be able to overcome the shortcomings observed in studies of other student-centered learning methods such as problem-based learning by combining model-eliciting activities, which meet most of the criteria for problem-based learning, with interactive video lectures. Such a result would potentially remove one of the barriers for adoption of student-centered learning approaches, which is a concern that student-centered learning leads to decreased educational effectiveness (albeit only one aspect of learning).

A comparison of the results on the combined objective learning measures of homework, exams, quizzes, and the conceptual test revealed that there was in fact a difference between the two sections. Further tests revealed that this difference was in homework scores only, with students in the treatment section (using the flipped classroom approach) receiving significantly lower marks. In other words, the hypothesis inherent in research question 1 was not fully supported.

The explanation that seems most plausible is that students prioritized the completion of course activities using an effort-grade optimization framework. The effort-grade optimization hypothesis presented here refers to the idea that students had so many tasks to complete that prioritization was necessary. The criteria for selection was to give top priority to course activities that would maximize the course grade with the minimal amount of effort.

There are several factors that, considered together, support the effort-grade optimization hypothesis. The first factor is the fact that the overall average homework score was worth a mere 5% of the students' final grade. The second factor is the fact that students graded their peer's homework, and when the assignments were completed the students usually got high scores. Finally, it was observed that the students in the treatment section felt that the course workload was excessive, and that that the MEAs consumed too much of their time (both observations are supported by written student feedback to instructors). This same sentiment (of having an excessive workload) was not evident among students in

the comparison section. It therefore seems likely that because students in the treatment section felt the need to drop something from their busy schedules, they chose to simply take a score of zero on one or more homework assignments as a way of cutting back. The rationale behind this is that neglecting one or two assignments would have little impact on a single student's grade. The adequacy of this explanation is further borne out when examining the homework scores across the two sections for each homework assignment (particularly the later ones). Figure 4.2 shows the homework scores for each section, broken down by topic (arranged chronologically in the order the topics were taught). From this bar chart, the distance between homework scores from the two sections appears to be growing as the semester proceeds. The proportions for each of the homework topics are shown in Figure 5.1. From the proportions in Figure 5.1, the relatively high frequency of zeros for the treatment section can be seen.

It is worth noting that homework scores are lower for the treatment section, even if the analysis of section differences is conducted with zero scores removed from both sections. Thus, while the zero scores mentioned provide evidence of reduced effort on homework assignments by students in the treatment section, this reduced effort would have applied to assignments that were (at least partially) complete as well. This is consistent with the effort-grade optimization hypothesis because students in the treatment section might have completed only part of the assigned homework.

Although homework scores were in fact lower on average for students in the treatment section, the difference between average exam scores was less than one fifth of 1%, which was not significant. Since homework scores are a formative assessment, the primary purpose of these was to help students learn the material. Exams are considered a summative evaluation, and in contrast to homework assignments, they are administered in a controlled environment. In the presence of a discrepancy between the two, exams would typically be considered the more valid measure of student learning. This observation further supports the plausibility of the effort-grade optimization hypothesis as an explanation for the discrepancy between scores among the two sections. While it is not clear how exactly the students learned what

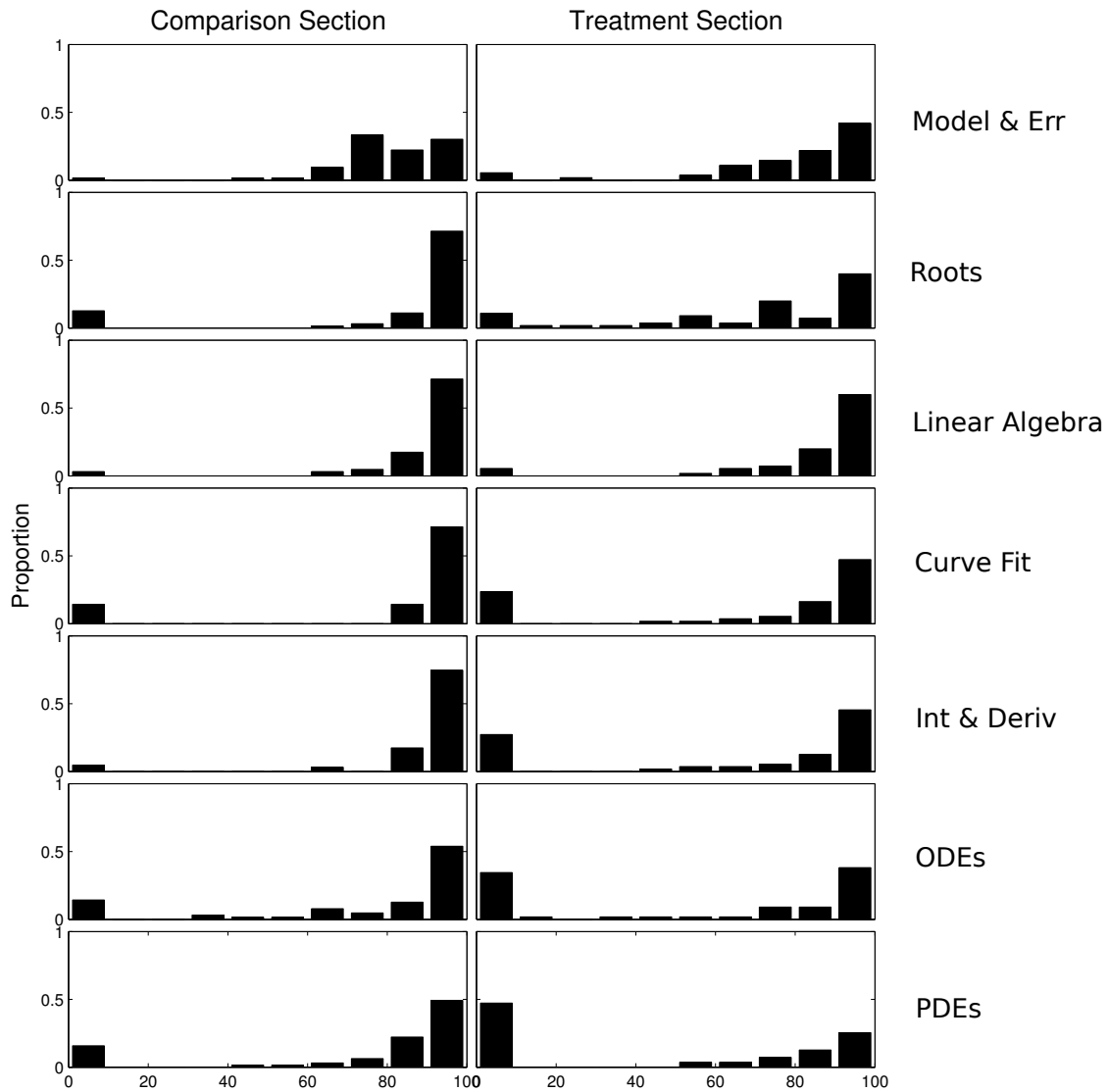


Fig. 5.1: Homework score distribution by assignment and section. This bar chart shows the proportion of students receiving a particular score on a homework assignment. Each row in the array of diagrams corresponds to one homework assignment. Homework assignments are shown in order from top to bottom, with the homework assignment for part 1 on top and the homework assignment for part 8 on the bottom.

they needed to know to pass the exams, it is evident from the scores that they did.

While the effort-grade optimization hypothesis just provided seems to be the most plausible, there are also other possible explanations. Exams may not have been sufficiently reliable to discern a difference in student performance between sections. Another possible explanation is that peers in the treatment section graded homework more stringently than students in the comparison section, and that there were no actual differences in the homework itself.

If, however, it is assumed that the exams were sufficiently reliable and valid, and that peer grading was unbiased, then exam scores can be seen to trump homework scores as a summative measure of student learning, and the results support the conjecture made in the first research question. That is, that students in a numerical methods course taught using the video lecture/MEA format attained equal or higher conventional problem-solving performance than students in a traditional lecture-based course. If, however, both homework and examinations are considered together, then this position is not supported, since the average homework scores for students taught using the video lecture/MEA format were lower than those attained by students in the conventional course. In either case, it is important to consider the fact that (although unintended) the workload for students in the two sections was not equal.

## 5.2 Research Question 2

The second dimension of student learning that was examined in this research study was conceptual understanding. Conceptual understanding was measured with 17 (open-book) quizzes that were administered online after each lecture, and by a 16-question numerical methods concept test. No significant difference was found between the two sections on either quiz scores or on concept test scores from the  $t$  tests used to follow up on the MANOVA results. Thus, the null hypothesis corresponding to the second research question (that students in a numerical methods course taught using the video lecture/MEA format would attain equal or higher conceptual understanding as students in a traditional lecture-based course, as measured by conceptual quizzes and a concept test) was fully supported. This

result is encouraging, although it is necessary to consider the possibility that the instruments were not sufficiently reliable to detect a difference between sections.

Because the same concept test was used as reported previously by Kaw and Yalcin (2011), it is also possible to compare the learning gains observed in this research to their results. The learning gain on pretest posttest measures are often considered in terms of Hake's gain index (Hake, 1998). These scores from the current study are shown side-by-side with those reported by Kaw and Yalcin (2011) in Table 5.1, and graph the corresponding Hake plot in Figure 5.2. The blue line on the main diagonal of Figure 5.2 represents the highest possible gain attainable, so all observed scores will be below and to the left of this line. The Hake gain score for both sections observed in this study were 0.40, which was bounded by those reported by Kaw and Yalcin (2011), who observed a gain of 0.36 to 0.43. Thus, while there is some contradictory evidence with regard to homework scores, the first two research questions were largely confirmed, and the normalized gain scores were similar to those reported elsewhere. Thus, there is both internal and external evidence that indicates that students made the expected level of learning gain in both sections, regardless of the teaching method used.

Table 5.1: Mean Scores on Concept Test

Course (Institution, Section)	CT-Pre		CT-Post		Hake's Gain
	Mean	<i>SD</i>	Mean	<i>SD</i>	
Spring 2008 (USF)	51.25	15.00	68.75	16.88	0.36
Spring 2009 (USF)	51.88	21.88	72.50	24.37	0.43
Spring 2010 (USF)	57.50	16.25	75.00	13.75	0.41
Spring 2013 (USU, Comparison)	64.76	14.37	78.47	10.75	0.40
Spring 2013 (USU, Treatment)	63.08	13.91	77.96	12.02	0.40

*Note.* USF = University of South Florida, as reported in Kaw and Yalcin (2011). USU = Utah State university. CT-Pre = Concept Pretest; CT-Post = Concept Posttest.

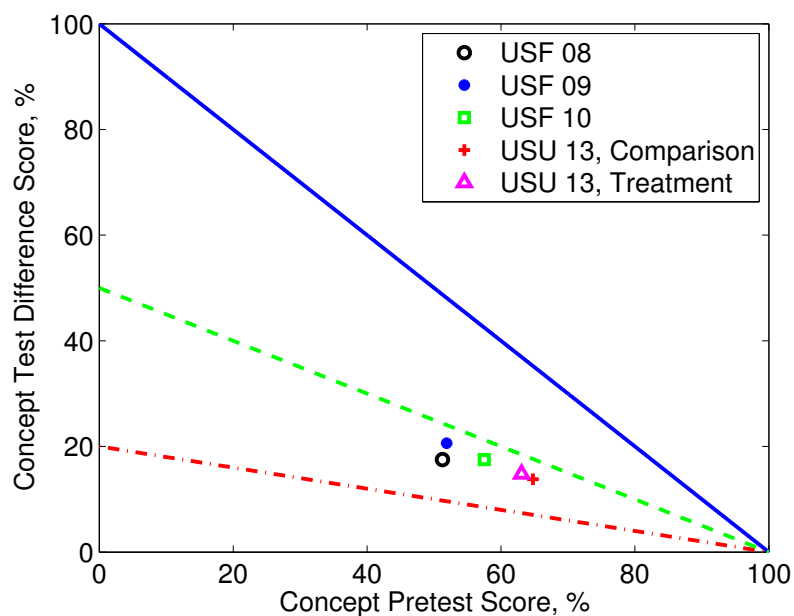


Fig. 5.2: Hake plot of concept test scores for numerical methods. Results from the numerical methods concept test used for this study at Utah State University are shown together with scores from the University of South Florida reported by Kaw and Yalcin (2011).

### 5.3 Research Question 3

Course evaluations showed that students in both sections had relatively low overall perceptions of the course compared to other similar courses at Utah State University. In addition, course evaluations showed that students in the treatment section had significantly lower perceptions of the course overall than students in the comparison section. After controlling the cutoff level for multiple comparison, this difference was significant on three of the 18 questions. The most relevant of these to the third research question was item 3, which asked students to rate progress on “learning to apply course material (to improve thinking, problem solving, and decisions).” The effect size of the difference on this question was 0.62, which is considered large. This result was not aligned with prior expectations based on research question 3, and seems to contradict the claims of other studies of the flipped classroom; however, a closer look suggests that this was at least partially due to several challenges that may be unique to this study.



First, on open response items of the course evaluations, students in both sections frequently mentioned that they disliked having the course taught by two instructors who switched sections every two weeks. Although this was part of the research design used to rule out teacher-level effects, it was not without side-effects. This probably contributed to students feeling the course was disorganized, which was another common complaint (particularly for students in the comparison section). This helps explain why evaluations for this course were lower than for other similar courses.

Another common theme identified in student comments across both sections was a dislike for instructor 1. This can be attributed to several factors. First, it was his first time teaching. Centra (2009) reported that first-year teachers, and also teaching assistants, typically receive ratings significantly lower than teachers who have taught even one or two years, and considerably lower than teachers with more than three years of teaching experience. On the surface, it would appear that this should influence both courses equally, and while it certainly influenced both courses, it is more complicated than that. Although both instructors taught approximately equal time in both sections, students tended to identify the treatment section as belonging to instructor 1, and the comparison section as belonging to instructor 2.<sup>1</sup> Thus, since there was only one evaluation form, it would have been unclear which instructor was being rated. Students may have identified the instructor as described previously. Centra (2009) explained that being an inexperienced teacher has the greatest influence on the response to the evaluation item related to the overall quality of teaching in the course. This was most similar to item 17, which asked students to rate the level to which they agreed with the statement “Overall, I rate this instructor as an excellent teacher.” This may explain a portion of the difference between sections on item 17.

These two factors—having two teachers who alternated teaching across sections, and having an inexperienced instructor—are conditions that will probably not be present in other implementations of the flipped classroom. Additionally, students expressed that they felt they were treated unfairly because they were participating in a study. It is also unlikely

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<sup>1</sup>This is because after students were signed up for the course, a different name needed to be assigned to each course for scheduling purposes. By default, the treatment section was listed under the name of instructor 1, and the comparison section under the name of instructor 2.

that this factor would influence a normal teacher implementing the teaching methods used in the current study. It may, however, be necessary for other researchers choosing to study the flipped classroom to do a controlled study. Because this was not received well, it might be better to implement changes for all sections of a course one semester at a time. Although there are other threats to validity that enter in when groups are not taught concurrently, these may be more minor than those found in this study. Hopefully, as the challenges that have just been described are overcome, future implementations of this course with the same teaching methods used in the present study will obtain results similar to past researchers implementing the flipped classroom (see section 2.4).

There are several aspects of the course implementation that were similar to what other researchers studying the flipped classroom and similar student-centered learning methods have found. First, students resented the fact that they did not receive lectures during class time. In fact, the lack of lectures during class time led many students to feel they were being cheated, and expressed that they were not being taught at all. A similar sentiment was found by Bland (2006), who stated:

During the first semester that this course format was used, there developed a particularly high level of frustration during the final month of the course. Students were first introduced to forced and natural responses of first and second order systems, and they did not grasp the concepts and theory. At the beginning of one class there was a pointed outburst from a group of students, "Are you ever going to lecture prior to assigning problems? We need the lecture because we are lost." This was a rather difficult class period. As the instructor, there was the immediate desire to revert to lecture. It was the easiest way to appease the students and proceed. But it was not the way to reinforce the interventions that they would need later in their career. (p. 8)

Similar moments of frustration and requests to revert to lectures were made during the semester in the flipped section of numerical methods. The bright spot in the comment quoted above by Bland (2006) is the implicit statement that this was not a sustained phenomenon.

Bland (2006) explains that his course was subsequently re-evaluated and re-designed, modified in subtle but strategic ways to both address students concerns, but maintain the integrity of the active learning concepts underlying the structure of the course.

Students observations point to several ways the course could be improved. Students suggested that the MEAs and the course material seemed disconnected. This may be one reason students disliked the MEAs and felt they occupied too much of their time. By seeking tighter integration between MEAs and other course assignments, it may be possible to eliminate some of the homework problems in favor of similar problems but within a more related context. One of the challenges with implementing MEAs faced in the present research study was that since there were several iterations of the procedure (which can be good since it allows time for revision and feedback; Diefes-Dux & Verleger, 2009), a single MEA required about six weeks. This meant that students needed to be able to work through the first MEA with only knowledge they had prior to the course. While numerical methods content is well-suited to realistic problem applications, there are many relatively distinct topics covered throughout the course. Shortening the MEAs into several distinct 1-2 week problems that are closely integrated with homework assignments (ideally, included on the same assignment sheet) and due concurrently should be possible. It might also be necessary to pose some of these problems as model exploration activities (see e.g., Zawojewski et al., 2008).

There are also other ways that the implementation of MEAs in the numerical methods classroom might be improved. The instructors noticed that there was an initial push-back to doing MEAs. This might be minimized by allowing more time to introduce students to the concept of MEAs, and providing more guidance to help them during the early stages of the work as suggested by Bland (2006). For this, it would be useful to have a higher teacher-to-student ratio (TAs in the classroom), and a classroom more conducive to group work. Whether it was documenting the model procedure or testing a new method, computers were inevitably needed to work on MEAs. A room with a table and computer for each group would be ideal. The lecture hall was difficult at times. Rather than spending multiple consecutive

days on MEAs, it might be useful to alternate between solving MEAs and working through group problems. When working on MEAs, a miniature (10-15 minute) lecture/discussion could be implemented to help set the stage with the tasks for that day and address questions that are common across groups.

Finally, the instructors noticed that students did not always get along well with others in their group and resented the possibility that someone might get credit for their work. In a way, it is good to encounter these problems, because it indicates that students are starting to confront some of the challenges associated with working on teams. On the other hand, it was a serious concern for some students. Near the end of the semester, the instructors implemented a team member assessment, which allowed students to rate their peers based on five dimensions of teamwork (Gomez, 2011). This was well-received and helped students feel that their grade proportionately rewarded their efforts.



## Chapter 6

### Conclusions and Recommendations

A controlled study was conducted, which implemented model-eliciting activities (MEAs) and video lectures in a cross-disciplinary sophomore-level undergraduate numerical methods course. This effort was designed to bring together the best parts of both student- and teacher- centered learning approaches to instruction. The past century of theory in student-centered learning methods was surveyed, and it was shown how MEAs are situated within this literature. MEAs were specifically developed to foster students' mathematical modeling abilities, which is also a central focus of numerical methods. Literature in computer-aided instruction, as well as the flipped classroom was also surveyed. The flipped classroom is a term that is being used to describe the result of a fusion of student-centered learning methods with computer-aided instruction, and particularly interactive video lectures. Successful integration of MEAs with video lectures in numerical methods could potentially lead to a new way of teaching that has all the advantages of problem-based learning methods, which are more effective at building important skills, and conventional teaching methods, which typically outperform student-centered methods on knowledge-based outcomes. The primary focus of this research was to compare MEA/video lecture course with a traditional course on knowledge-based outcomes. Conventional problem-solving ability and conceptual understanding were identified as knowledge-based constructs, and were used as instruments to examine student responses on homework, examinations, quizzes, and a conceptual test.

Findings from this study include the following:

- Average scores on examinations, quizzes, and the conceptual test attained by both groups were not significantly different.
- Average homework scores for the section taught with MEAs/video lectures were 15.5 points lower, which was significant and equivalent to an effect size (based on Cohen's

d) of 0.70.

- The hypothesized reason for lower homework scores is that students in the treatment section had more work to complete than students in the comparison section, and consequently prioritized course-related tasks by completing assignments that would have the greatest impact on their grade first, and less grade-critical tasks last (or sometimes not at all). Evidence that supports this hypothesis includes:
  - Higher reported workload among students in the treatment section.
  - Higher frequency of zero scores for homework assignments among students in the treatment section.
  - Homework was worth a relatively small portion of the final grade (5%).
  - No difference was observed in exam, quiz, or concept test scores.
- Regardless of the teaching method used, students attained a learning gain (Hake's  $g$ ) on a conceptual test of 0.40. This was positive, significant, and comparable to the gain attained in similar courses (Kaw & Yalcin, 2011).
- Findings from student evaluations of the course include:
  - Students in both section were confused by the course structure, which involved two instructors who alternated teaching of each section across two-week intervals.
  - Numerical scores were significantly different between the two sections.
    - \* The MEA/video lecture section scored significantly lower on 3 test items, with a large effect size (greater than 0.5) for these items.
    - \* This may have been due to a less than ideal introduction to an entirely new (to them) teaching method (see, e.g., Bland, 2006).
    - \* Students were not particularly excited to be part of a course that used a new educational method.
    - \* Students opinions of MEAs were mixed, but more negative than positive.

Based on the findings from this study, as well as instructor experiences and student feedback, several recommendations for future work are made. Specifically, it is recommended that researchers:

- Avoid alternating instructors (if possible, requires different course times).
- Conduct frequent assessments (the schedule used in this study seemed to work well: units spaced at a 2-week interval with an assessment for each unit).
- Provide students with a careful introduction to the new teaching method during the first week of class (see, e.g., Bland, 2006).
- Maintain equal course workload among different sections.
- Improve the integration between MEAs and core subject material, possibly with shorter MEA problems that are solved more frequently.
- Regularly assess team performance (see, e.g., Gomez, 2011).
- Measure and compare student outcomes based on other ABET objectives (may require development of new instruments).





## References

- Accreditation Board for Engineering and Technology. (2013). *Criteria for accrediting engineering programs*. Baltimore, MD: ABET.
- Adams, G. L., & Engelmann, S. (1996). *Research on direct instruction: 25 years beyond DISTAR*. Seattle, WA: Educational Achievement Systems.
- Adams, W. K., & Wieman, C. E. (2011). Development and validation of instruments to measure learning of expert-like thinking. *International Journal of Science Education*, 33(9), 1289–1312.
- Asimov, N. (2012, May). *Protests as UC regents seek to avoid tuition hike*. San Francisco Chronicle. Retrieved from <http://www.sfgate.com/education/article/Protests-as-UC-regents-seek-to-avoid-tuition-hike-3564366.php>
- Aslan, S., & Reigeluth, C. M. (2011). A trip to the past and future of educational computing: Understanding its evolution. *Contemporary Educational Technology*, 2(1), 1–17.
- Ausubel, D. P. (1960). The use of advance organizers in the learning and retention of meaningful verbal material. *Journal of Educational Psychology*, 51(5), 267–272.
- Ausubel, D. P. (1968). *Educational psychology: A cognitive view*. New York, NY: Holt, Rinehart and Winston.
- Azevedo, R., & Bernard, R. (1995). A meta-analysis of the effects of feedback in computer-based instruction. *Journal of Educational Computing Research*, 13(2), 111–127.
- Barrows, H. (1996). Problem-based learning in medicine and beyond: A brief overview. *New Directions for Teaching and Learning*, 1996(68), 3–12.
- Barry, S., & Webb, T. (2006). Multi-disciplinary approach to teaching numerical methods to engineers using Matlab. *Anziam Journal*, 47(0), 216–230.
- Bayraktar, S. (2002). A meta-analysis of the effectiveness of computer-assisted instruction in science education. *Journal of Research on Technology in Education*, 34(2), 173–188.
- Bland, L. (2006). Applying flip/inverted classroom model in electrical engineering to establish life-long learning. *Proceedings of the ASEE Annual Conference & Exposition*.

- Bonham, S., Deardorff, D., & Beichner, R. (2003). Comparison of student performance using web and paper-based homework in college-level physics. *Journal of Research in Science Teaching*, *40*(10), 1050–1071.
- Bruner, J. S. (1959). Learning and thinking. *Harvard Educational Review*, *29*, 184–192.
- Bruner, J. S. (1961). The act of discovery. *Harvard Educational Review*, *31*, 21–32.
- Carnes, M. T., Cardella, M. E., & Diefes-Dux, H. A. (2010). Progression of Student Solutions over the Course of a Model-Eliciting Activity (MEA). *Proceedings of the IEEE Frontiers in Education Conference (FIE)*, S3G-1–S3G-7. doi: 10.1109/FIE.2010.5673279
- Carroll, J. (1992, April). The role of computer software in numerical analysis teaching. *SIGNUM Newsletter*, *27*(2), 2–31. doi: 10.1145/140682.140686
- Centra, J. A. (2009). *Differences in responses to the student instructional report: Is it bias?* (Tech. Rep.). Princeton, NJ: Educational Testing Service.
- Chapra, S., & Canale, R. (2009). *Numerical methods for engineers*. New York, NY: McGraw-Hill.
- Clayson, D. E. (2009). Student evaluations of teaching: Are they related to what students learn?: A meta-analysis and review of the literature. *Journal of Marketing Education*, *31*(1), 16–30. doi: 10.1177/0273475308324086
- Cohen, P. A., Ebeling, B. J., & Kulik, J. A. (1981). A meta-analysis of outcome studies of visual-based instruction. *Educational Technology Research and Development*, *29*(1), 26–36.
- Coller, B., & Scott, M. (2009). Effectiveness of using a video game to teach a course in mechanical engineering. *Computers & Education*, *53*(3), 900–912.
- Coller, B., & Shernoff, D. (2010). Video game-based education in mechanical engineering: A look at student engagement. *International Journal of Engineering Education*, *25*(2), 308.
- Cook, D. (2005). The research we still are not doing: An agenda for the study of computer-based learning. *Academic Medicine*, *80*(6), 541.

- Corbalan, G., Paas, F., & Cuypers, H. (2010). Computer-based feedback in linear algebra: Effects on transfer performance and motivation. *Computers & Education*, 55(2), 692–703.
- Coursera. (2012). *Courses for everyone*. Retrieved from <https://www.coursera.org/>
- Crouch, C. H., & Mazur, E. (2001). Peer instruction: Ten years of experience and results. *American Journal of Physics*, 69(9), 970-977. doi: 10.1119/1.1374249
- Day, J. A., & Foley, J. D. (2006). Evaluating a web lecture intervention in a human–computer interaction course. *IEEE Transactions on Education*, 49(4), 420–431.
- DeGrazia, J. L., Falconer, J. L., Nicodemus, G., & Medlin, W. (2012). Incorporating screen-casts into chemical engineering courses. *Proceedings of the ASEE Annual Conference & Exposition*.
- Delange, J. (2012, June). *Quebec student protests: Tuition protests planned for Montreal and Quebec city*. Huffington Post. Retrieved from [http://www.huffingtonpost.ca/2012/06/22/quebec-student-protests\\_n\\_1617840.html](http://www.huffingtonpost.ca/2012/06/22/quebec-student-protests_n_1617840.html)
- Demetry, C. (2010). Work in progress-an innovation merging “classroom flip” and team-based learning. *Proceedings of the IEEE Frontiers in Education Conference (FIE)*, T1E-1-T1E-2. doi: 10.1109/FIE.2010.5673617
- Demirci, N. (2010). The effect of web-based homework on university students’ physics achievements. *Turkish Online Journal of Educational Technology*, 9(4), 156–161.
- Deutsch, M. (1949). A theory of cooperation and competition. *Human Relations*, 2(2), 129–152. doi: 10.1177/001872674900200204
- Dewey, J. (1938). *Logic, the theory of inquiry*. New York, NY: Henry Holt and Company.
- Diefes-Dux, H. A., Moore, T. J., Zawojewski, J., Imbrie, P. K., & Follman, D. (2004). A Framework for Posing Open-Ended Engineering Problems: Model-Eliciting Activities. *Proceedings of the 34th Annual Conference Frontiers in Education (FIE)*, F1A-3–F1A-8. doi: 10.1109/FIE.2004.1408556
- Diefes-Dux, H. A., & Verleger, M. A. (2009). Student reflections on peer reviewing solutions to model-eliciting activities. *Proceedings of the 39th IEEE Frontiers in Education*

- Conference (FIE)*, 1–7. doi: 10.1109/FIE.2009.5350403
- Diefes-Dux, H. A., Verleger, M. A., Zawojewski, J., & Hjalmanson, M. (2009). Multi-dimensional tool for assessing student-team solutions to model-eliciting activities. *Proceedings of the ASEE Annual Conference & Exposition*.
- Dochy, F., Segers, M., Van den Bossche, P., & Gijbels, D. (2003). Effects of problem-based learning: a meta-analysis. *Learning and Instruction*, 13(5), 533–568.
- Doolittle, P. (1995, June 2-4). Understanding cooperative learning through Vygotsky. *Lily National Conference on Excellence in College Teaching*.
- Drysdale, M. (2010). *Psychometric properties of postsecondary students' course evaluations*. (Doctoral dissertation), Utah State University, Logan, UT.
- edX. (2012). Retrieved from <http://www.edxonline.org/release.html>
- Ellis, P. (2010). *The essential guide to effect sizes: Statistical power, meta-analysis, and the interpretation of research results*. New York, NY: Cambridge University Press.
- Encyclopaedia Britannica. (2012a). *Encyclopaedia Britannica*. Retrieved from <http://www.britannica.com/EBchecked/topic/186618/Encyclopaedia-Britannica>
- Encyclopaedia Britannica. (2012b). *Encyclopaedia Britannica*. Retrieved from <http://store.britannica.com/>
- Felder, R., & Brent, R. (2003). Designing and Teaching Courses to Satisfy the ABET Engineering Criteria. *Journal of Engineering Education*, 92(1), 7–25.
- Foertsch, J., Moses, G., Strikwerda, J., & Litzkow, M. (2002). Reversing the lecture/homework paradigm using eteach® web-based streaming video software. *Journal of Engineering Education*, 91(3), 267–274.
- Foot, H., & Howe, C. (1998). The psychoeducational basis of peer-assisted learning. In K. Topping & S. Ehly (Eds.), *Peer-assisted learning* (pp. 27–43). Mahwah, NJ: Erlbaum.
- Fynewever, H. (2008). A comparison of the effectiveness of web-based and paper-based homework for general chemistry. *The Chemical Educator*, 13(4), 264–269.

- Gijbels, D., Dochy, F., Van den Bossche, P., & Segers, M. (2005). Effects of problem-based learning: A meta-analysis from the angle of assessment. *Review of Educational Research, 75*(1), 27–61.
- Gijbels, D., Watering, G., Dochy, F., & Bossche, P. (2006). New learning environments and constructivism: The students' perspective. *Instructional Science, 34*(3), 213–226. doi: 10.1007/s11251-005-3347-z
- Gillen, A., Denhart, M., & Robe, J. (2011). *Who subsidizes whom? An analysis of educational costs and revenues* (Policy Paper). Washington, DC: Center for College Affordability and Productivity.
- Gok, T. (2011, August). Comparison of student performance using web and paper-based homework in large introductory physics courses. *International Journal of the Physical Sciences, 6*(15), 3740–3746.
- Gollan, J. (2011, July). *Tuition hyperinflation*. The Bay Citizen. Retrieved from <http://www.baycitizen.org/education/story/tuition-hyperinflation/>
- Gomez, A. G. (2011). *Engineering your future: A project-based introduction to engineering*. Great Lakes Press.
- Goodsell, A. M., Maher, M., & Tinto, V. (Eds.). (1992). *Collaborative learning: A sourcebook for higher education*. University Park, PA: National Center on Postsecondary Teaching, Learning and Assessment.
- Grabinger, R., & Dunlap, J. (1995). Rich environments for active learning: A definition. *Association for Learning Technology Journal, 3*(2), 5–34.
- Hake, R. R. (1998). Interactive-engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses. *American Journal of Physics, 66*(1), 64–74. doi: 10.1119/1.18809
- Hess, M., Kaw, A. K., & Owens, C. (2009). On evaluating and rating online resources for a numerical methods course. *Proceedings of the ASEE Annual Conference & Exposition*.
- Hestenes, D., Wells, M., & Swackhamer, G. (1992). Force concept inventory. *The Physics Teacher, 30*(3), 141–158.

- Heyboer, K. (2012, June). *Long Rutgers board meeting ends with short protest over tuition costs and restructuring plan*. nj.com. Retrieved from [http://www.nj.com/news/index.ssf/2012/06/long\\_rutgers\\_board\\_meeting\\_end.html](http://www.nj.com/news/index.ssf/2012/06/long_rutgers_board_meeting_end.html)
- Hmelo-Silver, C. (2004). Problem-based learning: What and how do students learn? *Educational Psychology Review*, 16(3), 235–266.
- Itmazi, J. A., & Megías, M. G. (2005). *Survey: Comparison and evaluation studies of learning content management systems*. Unpublished manuscript.
- Johnson, D. W. (1970). *The social psychology of education*. New York, NY: Holt, Rinehart and Winston.
- Johnson, D. W. (1984). *Circles of learning: Cooperation in the classroom*. Alexandria, VA: Association for Supervision and Curriculum Development.
- Johnson, D. W., & Johnson, R. T. (2002). Learning together and alone: Overview and meta-analysis. *Asia Pacific Journal of Education*, 22(1), 95–105.
- Johnson, D. W., & Johnson, R. T. (2005). New developments in social interdependence theory. *Genetic, Social, and General Psychology Monographs*, 131(4), 285–358. doi: 10.3200/MONO.131.4.285-358
- Johnson, D. W., & Johnson, R. T. (2009). An educational psychology success story: Social interdependence theory and cooperative learning. *Educational Researcher*, 38(5), 365–379. doi: 10.3102/0013189X09339057
- Johnson, D. W., Johnson, R. T., & Smith, K. A. (1998). Cooperative learning returns to college what evidence is there that it works? *Change: The Magazine of Higher Learning*, 30(4), 26–35.
- Johnson, D. W., Johnson, R. T., & Stanne, M. B. (2000). Cooperative learning methods: A meta-analysis. *Methods*, 1, 1–33.
- Jonassen, D. H., Peck, K. L., & Wilson, B. (1999). *Learning with technology: A constructivist perspective*. Upper Saddle River, NJ: Merrill/Prentice Hall.
- Jung, C. G. (2001). *Modern man in search of a soul* (C. F. Baynes, Trans.). Abingdon, UK: Routledge/Psychology Press. (Original work published 1933)

- Kadel, S., & Keehner, J. (Eds.). (1994). *Collaborative learning: A sourcebook for higher education* (Vol. 2). University Park, PA: National Center on Postsecondary Teaching, Learning, and Assessment.
- Kaw, A. K. (2007). Incorporating a research problem in a numerical methods course for mechanical engineers. *Proceedings of the ASEE Annual Conference & Exposition*.
- Kaw, A. K., & Besterfield, G. H. (2004, November). Novel web-based modules for a course in numerical methods. *ASME International Mechanical Engineering Congress & Exposition*. (2004 ASME Curriculum Innovation Award Winner)
- Kaw, A. K., Besterfield, G. H., & Eison, J. (2005). Assessment of a web-enhanced course in numerical methods. *International Journal of Engineering Education*, 21(4), 712.
- Kaw, A. K., Collier, N., Keteltas, M., Paul, J., & Besterfield, G. H. (2003). Holistic but customized resources for a course in numerical methods. *Computer Applications in Engineering Education*, 11(4), 203–210.
- Kaw, A. K., & Garapati, S. (2010). Development of digital audiovisual lectures for an engineering course: A youtube experience. *Proceedings of the 2010 ASEE Southwest Section Conference*.
- Kaw, A. K., & Garapati, S. (2011). Development and assessment of digital audiovisual youtube lectures for an engineering course in numerical methods. *ASEE Computers in Education Journal*, 2(2), 89–97.
- Kaw, A. K., & Hess, M. (2006). Assessing teaching methods for a course in numerical methods. *Proceedings of the ASEE Annual Conference & Exposition*.
- Kaw, A. K., & Hess, M. (2007). Comparing effectiveness of instructional delivery modalities in an engineering course. *International Journal of Engineering Education*, 23(3), 508–516.
- Kaw, A. K., Keteltas, M., Paul, J., Eison, J., & Besterfield, G. H. (2003). Web-based resources for a course in numerical methods. *Proceedings of the 33rd Annual Frontiers in Education Conference (FIE)*, T3F2. doi: 10.1109/FIE.2003.1263350
- Kaw, A. K., & Yalcin, A. (2008). Problem-centered approach in a numerical methods



- course. *Journal of Professional Issues in Engineering Education and Practice*, 134(4), 359–364.
- Kaw, A. K., & Yalcin, A. (2010). Does collecting homework improve examination performance? *Proceedings of the ASEE Annual Conference & Exposition*.
- Kaw, A. K., & Yalcin, A. (2011). Measuring student learning using initial and final concept test in an STEM course. *International Journal of Mathematical Education in Science and Technology*, 1–14. doi: 10.1080/0020739X.2011.618559
- Kaw, A. K., Yalcin, A., Lee-Thomas, G., Nguyen, D. T., Hess, M., Eison, J., . . . Owens, C. (2012). A holistic view on history, development, assessment, and future of an open courseware in numerical methods. *Proceedings of the ASEE Annual Conference & Exposition*.
- Khan Academy. (2012). *Watch. practice. learn almost anything for free*. Retrieved from <http://www.khanacademy.org/about>
- Kilpatrick, W. (1914). *The Montessori system examined*. Cambridge, MA: Houghton Mifflin.
- Kortemeyer, G., Kashy, E., Benenson, W., & Bauer, W. (2008). Experiences using the open-source learning content management and assessment system LON-CAPA in introductory physics courses. *American Journal of Physics*, 76, 438.
- Kulik, C. L. C., & Kulik, J. A. (1991). Effectiveness of computer-based instruction: An updated analysis. *Computers in Human Behavior*, 7(1-2), 75–94.
- Kulik, J. A., Kulik, C. L. C., & Bangert-Drowns, R. L. (1985). Effectiveness of computer-based education in elementary schools. *Computers in Human Behavior*, 1(1), 59–74.
- Kulik, J. A., Kulik, C. L. C., & Cohen, P. A. (1980). Effectiveness of computer-based college teaching: A meta-analysis of findings. *Review of Educational Research*, 50(4), 525–544.
- Lage, M. J., & Platt, G. (2000). The internet and the inverted classroom. *The Journal of Economic Education*, 31(1), 11.
- Lage, M. J., Platt, G. J., & Treglia, M. (2000). Inverting the classroom: A gateway

- to creating an inclusive learning environment. *The Journal of Economic Education*, 31(1), 30–43.
- Larwin, K., & Larwin, D. (2011). A meta-analysis examining the impact of computer-assisted instruction on postsecondary statistics education: 40 years of research. *Journal of Research on Technology in Education*, 43(3), 26.
- Lee, J. (1999). Effectiveness of computer based instructional simulation: A meta analysis. *International Journal of Instructional Media*, 26(1).
- Lee-Thomas, G., Kaw, A. K., & Yalcin, A. (2011). Using online endless quizzes as graded homework. *Proceedings of the ASEE Annual Conference & Exposition*.
- Lesh, R. A., & Doerr, H. M. (2003). *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Hillsdale, NJ: Erlbaum.
- Lesh, R. A., Hoover, M., Hole, B., Kelly, A. E., & Post, T. (2000). Principles for Developing Thought-Revealing Activities for Students and Teachers. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (p. 591-645). Hillsdale, NJ: Erlbaum.
- Lewin, K. (1935). *Dynamic theory of personality* (D. K. Adams & K. E. Zener, Trans.). New York, NY: McGraw-Hill.
- Lewin, K. (1943). Defining the 'field at a given time'. *Psychological Review*, 50(3), 292–310.
- Lewin, K. (1946). Action research and minority problems. *Journal of Social Issues*, 2(4), 34–46. doi: 10.1111/j.1540-4560.1946.tb02295.x
- Lewin, K. (1947). Frontiers in group dynamics: Concept, method and reality in social science; social equilibria and social change. *Human Relations*, 1(1), 5–41. doi: 10.1177/001872674700100103
- Liao, Y. C. (2007). Effects of computer-assisted instruction on students' achievement in taiwan: A meta-analysis. *Computers & Education*, 48(2), 216–233.
- Mayo, M. J. (2007). Games for science and engineering education. *Communications of the ACM*, 50(7), 30–35.
- Mazur, E. (1997). *Peer instruction: A user's manual*. Upper Saddle River, NJ: Prentice

Hall.

- McNeil, B. J. (1989). *A meta-analysis of interactive video instruction: a 10 year review of achievement effects*. (Doctoral dissertation), University of Idaho, Moscow, ID.
- Means, B., Toyama, Y., Murphy, R., Bakia, M., & Jones, K. (2009). *Evaluation of evidence-based practices in online learning: A meta-analysis and review of online learning studies* (Tech. Rep.). Washington, DC: United States Department of Education.
- Michael, J. (2006). Where's the evidence that active learning works? *Advances in Physiology Education*, 30(4), 159–167.
- MIT. (2012a). *MIT office of the registrar*. Retrieved from <http://web.mit.edu/registrar/reg/costs/>
- MIT. (2012b). *MIT OpenCourseWare*. Retrieved from <http://ocw.mit.edu/about/our-history/>
- Montessori, M. (1912). *The Montessori method: Scientific pedagogy as applied to child education in "the children's houses" : With additions and revisions by the author* (A. E. George, Trans.). New York, NY: Frederick A. Stokes. (Original work published 1909)
- Moore, T. J. (2008). Model-Eliciting Activities: A case-based approach for getting students interested in material science and engineering. *Journal for Materials Education*, 30, 295–310. Retrieved from <http://matdl.org/jme/blog/2008/03/24/model-eliciting-activities/>
- Moore, T. J., & Diefes-Dux, H. A. (2004). Developing Model-Eliciting Activities for Undergraduate Students Based on Advanced Engineering Content. *Proceedings of the 34th Annual Conference Frontiers in Education (FIE)*, F1A-9–F1A-14. doi: 10.1109/FIE.2004.1408557
- Moore, T. J., Diefes-Dux, H. A., & Imbrie, P. K. (2005). Developing first-year students' perceptions of the engineering profession through realistic, client-driven problems. *Proceedings of the 35th Annual Conference Frontiers in Education (FIE)*, S2E-25–S2E-31. doi: 10.1109/FIE.2005.1612228

- Moore, T. J., Diefes-Dux, H. A., & Imbrie, P. K. (2006). Assessment of team effectiveness during complex mathematical modeling tasks. *Proceedings of the 36th Annual Conference Frontiers in Education (FIE)*, T1H-1–T1H-6. doi: 10.1109/FIE.2006.322481
- Moravec, M., Williams, A., Aguilar-Roca, N., & O’Dowd, D. (2010). Learn before lecture: a strategy that improves learning outcomes in a large introductory biology class. *CBE-Life Sciences Education*, 9(4), 473–481.
- Musto, J. C. (2002). A project-based approach to teaching numerical methods. *International Journal of Mechanical Engineering Education*, 30(3), 233–247.
- National Center for Educational Statistics. (2012). *Tuition costs of colleges and universities*. Retrieved from <https://nces.ed.gov/fastfacts/display.asp?id=76>
- Novak, J. D. (1996). Concept mapping: A tool for improving science teaching and learning. In *Improving teaching and learning in science and mathematics* (pp. 32–43). London, UK: Teachers College Press.
- Ormrod, J. E. (1995). *Educational psychology: Principles and applications*. Englewood Cliffs, NJ: Merrill.
- Owens, C., Kaw, A. K., & Hess, M. (2010). Assessing online resources for an engineering course in numerical methods. *Computer Applications in Engineering Education*, 20(3), 426–433. doi: 10.1002/cae.20410
- Papadopoulos, C., & Santiago-Román, A. (2010). Implementing an inverted classroom model in engineering statics: Initial results. *Proceedings of the ASEE Annual Conference & Exposition*.
- Papadopoulos, C., Santiago-Román, A., & Portela, G. (2010). Work in progress—developing and implementing an inverted classroom for engineering statics. *Proceedings of the IEEE Frontiers in Education Conference (FIE)*, F3F-1–F3F-4. doi: 10.1109/FIE.2010.5673198
- Paul, R. (1993). *Critical thinking: What every person needs to survive in a rapidly changing world* (3rd ed.; J. Willson & A. J. A. Binker, Eds.). Rohnert Park, CA: Sonoma State University, Foundation for Critical Thinking.

- Piaget, J. (1967). *Six psychological studies* (D. Elkind, Ed. & A. Tenzer, Trans.). New York, NY: Random House. (Original work published 1964)
- Prince, M. (2004). Does active learning work? A review of the research. *Journal of Engineering Education*, *93*, 223–232.
- Rieber, L. P. (1992). Computer-based microworlds: A bridge between constructivism and direct instruction. *Educational Technology Research and Development*, *40*(1), 93–106.
- Rogers, C. R. (1969). *Freedom to learn*. Columbus, OH: Merrill.
- Rottier, J., & Ogan, B. J. (1991). *Cooperative learning in middle-level schools*. Washington, DC: NEA Professional Library, National Education Association.
- Sappington, J., Kinsey, K., & Munsayac, K. (2002). Two studies of reading compliance among college students. *Teaching of Psychology*, *29*(4), 272–274.
- Sharan, S. (1990). *Cooperative learning: Theory and research*. New York, NY: Praeger.
- Slavin, R. E. (1991). Synthesis of research of cooperative learning. *Educational Leadership*, *48*(5), 71–82.
- Smith, B. L., & MacGregor, J. T. (1992). What is collaborative learning? In A. M. Goodsell, M. Maher, & V. Tinto (Eds.), *Collaborative learning: A sourcebook for higher education* (pp. 10–30). University Park, PA: National Center on Postsecondary Teaching, Learning and Assessment.
- Sosa, G. W., Berger, D. E., Saw, A. T., & Mary, J. C. (2011). Effectiveness of computer-assisted instruction in statistics. *Review of Educational Research*, *81*(1), 97–128.
- Stelzer, T., Brookes, D. T., Gladding, G., & Mestre, J. P. (2010). Impact of multimedia learning modules on an introductory course on electricity and magnetism. *American Journal of Physics*, *78*, 755.
- Streveler, R. A., Miller, R. L., Santiago-Román, A. I., Nelson, M. A., Geist, M. R., & Olds, B. M. (2011). Rigorous methodology for concept inventory development: Using the ‘assessment triangle’ to develop and test the thermal and transport science concept inventory (TTCI). *International Journal of Engineering Education*, *27*(5), 968.
- Sumner, A. (1974). The use of on-line interactive programs for teaching numerical methods.

- International Journal of Mathematical Education in Science and Technology*, 5(3-4), 657–664.
- Tamim, R. M., Bernard, R. M., Borokhovski, E., Abrami, P. C., & Schmid, R. F. (2011). What forty years of research says about the impact of technology on learning a second-order meta-analysis and validation study. *Review of Educational Research*, 81(1), 4–28.
- Thayer-Bacon, B. J. (2012). Maria Montessori, John Dewey, and William H. Kilpatrick. *Education and Culture*, 28(1), 3.
- Thomas, J. S., & Philpot, T. A. (2012). An inverted teaching model for a mechanics of materials course. *Proceedings of the ASEE Annual Conference & Exposition*.
- Timmerman, C. E., & Kruepke, K. A. (2006). Computer-assisted instruction, media richness, and college student performance. *Communication Education*, 55(1), 73–104.
- Topping, K. J., & Ehly, S. W. (Eds.). (1998). *Peer-assisted learning*. Mahwah, NJ: Erlbaum.
- Toto, R., & Nguyen, H. (2009). Flipping the work design in an industrial engineering course. *Proceedings of the 39th IEEE Frontiers in Education Conference (FIE)*, T4F-1–T4F-4. doi: 10.1109/FIE.2009.5350529
- Tudge, J. R. H., & Winterhoff, P. A. (1993). Vygotsky, Piaget, and Bandura: Perspectives on the relations between the social world and cognitive development. *Human Development*, 36, 61.
- Udacity. (2012). *Udacity*. Retrieved from <http://www.udacity.com/udacity>
- VanLehn, K. (2011). The relative effectiveness of human tutoring, intelligent tutoring systems, and other tutoring systems. *Educational Psychologist*, 46(4), 197–221.
- Verleger, M. A. (2009). *Analysis of an informed peer review matching algorithm and its impact on student work on model-eliciting activities*. (Doctoral dissertation), Purdue University, West Lafayette, IN. Retrieved from <http://proquest.umi.com/pqdlink?did=2056269091&Fmt=7&clientId=1652&RQT=309&VName=PQD>
- Verleger, M. A., & Diefes-Dux, H. A. (2008, June). Impact of feedback and revision on student team solutions to model-eliciting activities. *Proceedings of the ASEE Annual*

*Conference & Exposition.*

- Verleger, M. A., Diefes-Dux, H. A., Ohland, M. W., Besterfield-Sacre, M., & Brophy, S. (2010, October). Challenges to informed peer review matching algorithms. *Journal of Engineering Education*, 99(4), 397–408.
- Vygotsky, L. S. (1978). *Mind and society: The development of higher mental processes*. Cambridge, MA: Harvard University Press.
- Warter-Perez, N., & Dong, J. (2012). Flipping the classroom: How to embed inquiry and design projects into a digital engineering lecture. *Proceedings of the 2012 ASEE PSW Section Conference*.
- Wikipedia. (2012). *Wikipedia*. Retrieved from <http://en.wikipedia.org/wiki/Wikipedia>
- Yesilyurt, M. (2011). Meta-analysis of the computer assisted studies in physics: A sample of Turkey. *Energy Educ Sci Technol Part B*, 3, 173–182.
- Yildirim, T., Shuman, L., Besterfield-Sacre, M., & Yildirim, T. (2010). Model eliciting activities: assessing engineering student problem solving and skill integration processes. *International Journal of Engineering Education*, 26(4), 831–845.
- Zappe, S., Liecht, R., Messner, J., Litzinger, T., & Lee, H. W. (2009). “Flipping” the classroom to explore active learning in a large undergraduate course. *Proceedings of the ASEE Annual Conference & Exposition*.
- Zawojewski, J. S., Diefes-Dux, H. A., & Bowman, K. J. (Eds.). (2008). *Models and modeling in engineering education*. Rotterdam, NL: Sense Publishers.
- Zhang, D., Zhou, L., Briggs, R. O., & Nunamaker, J. F. (2006). Instructional video in e-learning: Assessing the impact of interactive video on learning effectiveness. *Information & Management*, 43(1), 15–27.

## Appendices





**Appendix A**  
**Homework Assignments**

**ENGR 2450 – Numerical Methods in Engineering – Assignment 01**

**Solve the following problems using calculators or Excel:**

[1]. [Read Ch. 1] - Newton's law of cooling indicates that the temperature of a body changes at a rate proportional to the difference between its temperature and that of its surrounding medium (the ambient temperature),

$$\frac{dT}{dt} = -k \cdot (T - T_a)$$

where  $T$  = the temperature of the body ( $^{\circ}\text{C}$ ),  $t$  = time ( $\text{min}$ ),  $k$  = the proportionality constant ( $\text{min}^{-1}$ ), and  $T_a$  = the ambient temperature ( $^{\circ}\text{C}$ ). Suppose that a cup of coffee originally has a temperature of  $68^{\circ}\text{C}$ . (a) Find an exact solution for  $T(t)$  and compute the temperature from  $t = 0$  to  $10 \text{ min}$  using a step size of  $1 \text{ min}$  if  $T_a = 21^{\circ}\text{C}$  and  $k = 0.1 \text{ min}^{-1}$ . (b) Use Euler's method to compute the temperatures of part (a). (c) Calculate the true absolute error incurred in calculating the temperatures of part (a). (d) Calculate the true relative error incurred in calculating the temperatures of part (a). Present your results in the form of a table:

t (min)	Exact T( $^{\circ}\text{C}$ )	Euler T( $^{\circ}\text{C}$ )	Abs. Error Et ( $^{\circ}\text{C}$ )	Rel. Error $\epsilon_t$ (%)
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

[2]. [Read Ch. 3] - Evaluate  $e^{-5}$  using two approaches:

$$(a) e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots \quad \text{and} \quad (b) e^{-x} = \frac{1}{e^x} = \frac{1}{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots}$$

and compare with the true value of  $6.737947 \times 10^{-3}$ . Use 20 terms to evaluate each series and compute true and approximate relative errors as terms are added. Present your results in the form of a table:

Term No.	(a) $e^{-x}$	$\epsilon_t(\%)$	$\epsilon_a(\%)$	$e^{-x}$	$1/e^x$	$\epsilon_t(\%)$	$\epsilon_a(\%)$
1							
2							
3							
...	...	...	...	...	...	...	...
18							
19							
20							

[3]. [Read Ch. 3] - The Stefan-Boltzmann law can be employed to estimate the rate of radiation of energy  $H$  from a surface, as in

$$H = A \cdot e \cdot \sigma \cdot T^4$$

where  $H$  is in *watts*,  $A$  = the surface area ( $m^2$ ),  $e$  = the emissivity that characterizes the emitting properties of the surface (dimensionless),  $\sigma$  = a universal constant called the Stefan-Boltzmann constant ( $= 5.67 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ ), and  $T$  = absolute temperature ( $K$ ). Determine the error of  $H$  for a steel plate with  $A = 0.15 \text{ m}^2$ ,  $e = 0.90$ , and  $T = (650 \pm 20) \text{ K}$ . Compare your results with the exact error.

[4]. [Read Ch. 4] - Evaluate and interpret the *condition number* for: (a)  $f(x) = \sin(x)/(1+\cos(x))$ , for  $x = 1.0001 \pi$ .

[5]. **PROGRAMMING ASSIGNMENT.** [Read Chs. 2 and 3] - The “divide and average” method, an old-time method for approximating the square root of any positive number  $a$ , can be formulated as:

$$x = \frac{x+a/x}{2}$$

Write a well-structured function to implement this algorithm based on the algorithm outlined in Figure 3.3 in the Chakra & Canale's textbook.

```

FUNCTION IterMeth(val, es, maxit)
  iter = 1
  sol = val
  ea = 100
  DO
    solold = sol
    sol = ...
    iter = iter + 1
    IF sol ≠ 0 ea=abs((sol - solold)/sol)*100
    IF ea ≤ es OR iter ≥ maxit EXIT
  END DO
  IterMeth = sol
END IterMeth

```

**FIGURE 3.3**

Pseudocode for a generic iterative calculation.

**ENGR 2450 – Numerical Methods in Engineering – Assignment 02**

**Solve the following problems using calculators, or Excel:**

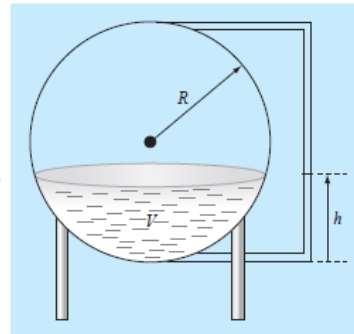
[1]. **[Read Ch. 05]** – (a) Determine the roots of  $f(x) = -13 - 20x + 19x^2 - 3x^3$  graphically. In addition, determine the first root of the function with (b) bisection, and (c) false position. For (b) and (c) use initial guesses of  $x_1 = -1$  and  $x_u = 0$ , and a stopping criterion of 1%. Show your calculations in a table.

[2]. **[Read Chs. 05/06]** – You are designing a spherical tank (see Figure) to hold water for a small village in a developing country. The volume of liquid it can hold can be computed as

$$V = \frac{1}{3} \cdot \pi \cdot h^2 \cdot (3 \cdot R - h) \quad ,$$

where  $V$  = volume [m<sup>3</sup>],  $h$  = depth of water in tank [m], and  $R$  = the tank radius [m].

If  $R = 3$  m, to what depth must the tank be filled so that it holds 30 m<sup>3</sup>? (a) Use three iterations of the false-position method to determine your answer. Determine the approximate relative error after each iteration. Use initial guesses of 0 and  $R$  for  $h$ . (b) Repeat your solution using the Newton-Raphson method with an initial guess of  $R/2$  for  $h$ . Determine the approximate relative error after each iteration. Show your results in a table.



[3]. **[Read Ch. 06]**. A mass balance for a pollutant in a well-mixed lake can be written as:

$$V \cdot \frac{dc}{dt} = W - Q \cdot c - k \cdot V \cdot \sqrt{c}$$

Given the parameter values  $V = 1 \times 10^6$  m<sup>3</sup>,  $Q = 1 \times 10^5$  m<sup>3</sup>/yr,  $W = 1 \times 10^6$  g/yr, and  $k = 0.25$  m<sup>0.5</sup>/g<sup>0.5</sup>/yr, use the modified secant method to solve for the steady-state concentration. Use an initial guess of  $c = 4$  g/m<sup>3</sup> and  $\delta = 0.5$ . Perform three iterations and determine the percent relative error at each iteration. Show your results in the form of a table.

[4]. **[Read Ch. 06]**. For problem [3], the root can be located with fixed-point iteration as

$$c = \left( \frac{W - Q \cdot c}{k \cdot V} \right)^2 \quad \text{or as} \quad c = \frac{W - k \cdot V \cdot \sqrt{c}}{Q} \quad .$$

Only one will converge for initial guesses of  $2 < c < 6$ . To test this statement calculate 6 iterations of the fixed-point method for each of the expressions shown above starting with  $c = 4$ . Calculate the percent relative error at each iteration. Show your results in the form of a table. Based on your results which one of the two expressions is *converging* and which one is *diverging*?

[5]. **[Read Ch. 7].** In control systems analysis, mathematical transfer functions are developed that relate the dynamics of a system's input to its output. A transfer function for a robotic positioning system is given by

$$G(s) = \frac{C(s)}{N(s)} = \frac{s^3 + 12.5 \cdot s^2 + 50.5 \cdot s + 66}{s^4 + 19 \cdot s^3 + 122 \cdot s^2 + 296 \cdot s + 192}$$

where  $G(s)$  = system gain,  $C(s)$  = system output,  $N(s)$  = system input, and  $s$  = Laplace transform complex frequency. Use one of the numerical techniques of Chapter 7 to find the roots of the numerator and denominator and factor these into the form

$$G(s) = \frac{(s+a_1) \cdot (s+a_2) \cdot (s+a_3)}{(s+b_1) \cdot (s+b_2) \cdot (s+b_3) \cdot (s+b_4)}$$

where  $a_i$  and  $b_i$  = the roots of the numerator and denominator, respectively. Report the values of  $a_1$ ,  $a_2$ ,  $a_3$ ,  $b_1$ ,  $b_2$ ,  $b_3$ , and  $b_4$ . Indicate which numerical method (or methods) you used to find those roots.

**ENGR 2450 – Numerical Methods in Engineering – Assignment 03**

**Solve the following problems using calculators, or Excel:**

[1]. (Read Ch. 9) – For the set of equations:

$$\begin{aligned} 2 \cdot x_1 + x_2 + x_3 &= 1 \\ 2 \cdot x_2 + 5 \cdot x_3 &= 1 \\ 3 \cdot x_1 + x_2 &= 2 \end{aligned}$$

- (a) Compute the determinant of the system  
 (b) Use Cramer's rule to solve for the  $x$ 's

[2]. (Read Ch. 10) – (a) Solve the following system of equations by  $LU$  decomposition without pivoting:

$$\begin{aligned} x_1 + 7 \cdot x_2 - 3 \cdot x_3 &= -51 \\ 4 \cdot x_1 - 4 \cdot x_2 + 9 \cdot x_3 &= 62 \\ 12 \cdot x_1 - x_2 + 3 \cdot x_3 &= 8 \end{aligned}$$

[3]. (Read Ch. 11) - Use the Gauss-Seidel method (a) without relaxation and (b) with relaxation ( $\lambda = 1.2$ ) to solve the following system to a tolerance of  $\epsilon_s = 5\%$ .

$$\begin{aligned} 2x_1 - 6x_2 - x_3 &= -38 \\ -3x_1 - x_2 + 7x_3 &= -34 \\ -8x_1 + x_2 - 2x_3 &= -20 \end{aligned}$$

[4]. (Read Ch. 12) - Solve the following system of equations in matrix form:

$$\begin{aligned} 50 &= 5x_3 - 7x_2 \\ 4x_2 + 7x_3 + 30 &= 0 \\ x_1 - 7x_3 &= 40 - 3x_2 + 5x_1 \end{aligned}$$

Use Excel, Calc (OpenOffice.org, or LibreOffice.org), Matlab, Scilab, Mathcad, SMATH Studio, or Maxima, to (a) solve for the unknowns. In addition, compute (b) the transpose and (c) inverse of the coefficient matrix.

**ENGR 2450 – Numerical Methods in Engineering – Homework Assignment For Part 05**

Solve the following problems using calculators, Excel, or SCILAB or MATLAB, as indicated:

[1]. Given the data

0.90	1.42	1.30	1.55	1.63
1.32	1.35	1.47	1.95	1.66
1.96	1.47	1.92	1.35	1.05
1.85	1.74	1.65	1.78	1.71
2.29	1.82	2.06	2.14	1.27

Use Excel, SCILAB, or Matlab to determine (a) the mean, (b) the standard deviation, (c) the variance, (d) the coefficient of variation, and (e) construct a histogram using a range from 0.6 to 2.4 with intervals of 0.2.

[2]. (a) Use your calculator or Excel function “Add Trendline” to find the least-square straight line fitting the following data set:

$x$	1	2	3	4	5	6	7	8	9
$y$	1	1.5	2	3	4	5	8	10	13

Also, determine the coefficient of determination,  $r^2$ .

(b) Use your calculator or Excel function “Add Trendline” to fit a quadratic equation (a parabola) to the data, and determine the coefficient of determination,  $r^2$ .

[3]. A researcher has reported the data tabulated below for an experiment to determine the growth rate of bacteria  $k$  (per day), as function of oxygen concentration  $c$  (mg/L). It is known that such data can be modeled by the following equation:

$$k = \frac{k_{\max} c^2}{c_s + c^2},$$

where  $c_s$  and  $k_{\max}$  are parameters. Use a transformation to linearize this equation. Then use linear regression (in your calculators, or in Excel) to (a) estimate  $c_s$  and  $k_{\max}$ , and predict the grow rate at  $c = 2$  mg/L.

$c$	0.5	0.8	1.5	2.5	4
$k$	1.1	2.4	5.3	7.6	8.9



[4]. The following data are provided:

$x$	1	2	3	4	5
$y$	2.2	2.8	3.6	4.5	5.5

You want to use least-squares regression to fit this data with the following model:

$$y = a + bx + \frac{c}{x}$$

Determine the coefficients by setting up and solving Eq. (17.25) in your textbook.

[5]. Given the data

$x$	1.6	2	2.5	3.2	4	4.5
$f(x)$	2	8	14	15	8	2

- (a) Calculate  $f(2.8)$  using Newton's interpolating polynomials of order 1 through 3. Chose the sequence of the points for your estimates to attain the best possible accuracy.  
 (b) Utilize Eq. (18.18) in your textbook to estimate the error for each prediction.

[6]. The following is the built-in *humps* function that MATLAB uses to demonstrate some of its numerical capabilities:

$$f(x) = \frac{1}{(x-0.3)^2 + 0.01} + \frac{1}{(x-0.9)^2 + 0.04} - 6$$

The *humps* function exhibits both flat and steep regions over a relatively short  $x$  range. Using MATLAB or SCILAB, generate values of this function at intervals of 0.1 over the range from  $x = 0$  to 1. Use functions *splin* and *interp* in SCILAB, or function *spline* in MATLAB, to produce cubic-spline data in the interval  $[0,1]$ . Plot the original *humps* function vs. the spline-interpolated data using SCILAB or MATLAB.

[7]. Use the **Excel Data Analysis Toolpack** to develop a regression polynomial to the following data for the dissolved oxygen concentration of fresh water versus temperature at sea level. Determine the order of the polynomial necessary to match the precision of the data.

$T, ^\circ\text{C}$	0	8	16	24	32	40
$o, \text{mg/L}$	14.62	11.84	9.87	8.42	7.31	6.41

[8]. Use SCILAB or MATLAB to generate 64 points from the function

$$f(t) = \cos(10t) + \sin(3t)$$

from  $t = 0$  to  $2\pi$ . Add a random component to the signal with the function *rand* in SCILAB or with function *randn* in MATLAB. Take an FFT (Fast Fourier Transform) of these values, using the *fft* function provided by SCILAB or MATLAB, and plot the results

**ENGR 2450 – Numerical Methods in Engineering – Homework Assignment For Part 06**

**Solve the following problems using calculators, Excel, or SCILAB or MATLAB (your choice):**

[1]. Evaluate the following integral:

$$\int_1^2 \left(x + \frac{1}{x}\right)^2 dx$$

(a) analytically; (b) single application of the trapezoidal rule; (c) multiple-application trapezoidal rule, with  $n = 2$  and 4; (d) single application of Simpson's 1/3 rule; (e) multiple-application Simpson's 1/3 rule, with  $n = 4$ ; (f) single application of Simpson's 3/8 rule; and (g) multiple-application Simpson's 3/8 rule, with  $n = 5$ . For each of the numerical estimates (b) through (g), determine the percent relative error based on (a). Show the detailed calculations for each item (a) through (g), and then summarize your results in the form of a table:

Result	Integral	Error, $\epsilon_t$
(a) Analytical		
(b) Trap. Single		
(c) Trap. Multiple, $n = 2$		
(c) Trap. Multiple, $n = 4$		
(d) Simpson 1/3 Single		
(e) Simpson 1/3 Mult. $n=4$		
(f) Simpson 3/8 Single		
(g) Simpson 3/8 Mult. $n = 5$		

[2]. Evaluate the integral of the following tabular data (with equally-spaced  $x$  data) using (a) the trapezoidal rule; (b) Simpson's 1/3 rule; and (c) Simpson's 3/8 rule:

$x$	-2	0	2	4	6	8	10
$f(x)$	35	5	-10	2	5	3	20

[3]. The function  $f(x) = 2e^{-1.5x}$  can be used to generate the following table of unequally-spaced data:

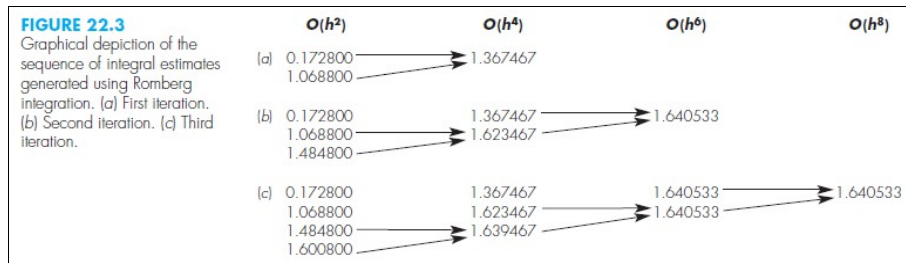
$x$	0	0.05	0.15	0.25	0.35	0.475	0.6
$f(x)$	2	1.8555	1.5970	1.3746	1.1831	0.9808	0.8131

Evaluate the integral from  $a = 0$  to  $b = 0.6$  using (a) analytical means, (b) the trapezoidal rule, and (c) a combination of the trapezoidal and Simpson's rules; employ Simpson's rules wherever possible to obtain the highest accuracy. For (b) and (c), compute the percent relative error ( $\epsilon$ ).

[4]. Use Romberg integration to evaluate the integral

$$I = \int_1^2 \left( 2x + \frac{3}{x} \right)^2 dx$$

to an accuracy of  $\epsilon_s = 0.5\%$ . Your results should be presented in the form of Figure 22.3 in Chapra & Canale's textbook, i.e.,



[5]. Obtain estimates for the integral of problem [4], above, using (a) two-, (b) three-, and (c) four-point Gauss-Legendre formulas. Compute  $\epsilon_i$  for each case on the basis of the analytical solution.

[6]. The following data were collected for the distance traveled,  $y$ , versus time,  $t$ , for a rocket:

$t(s)$	$y(km)$	$v(km/s)$	$a(km/s^2)$
0	0		
25	32		
50	58		
75	78		
100	92		
125	100		

Use numerical differentiation to estimate the rocket's velocity ( $v = dy/dt$ ) and acceleration ( $a = dv/dt$ ) at each time. To present your results, complete the table shown above. Produce plots of  $y$ -vs- $t$ ,  $v$ -vs- $t$ , and  $a$ -vs- $t$ .

**ENGR 2450 – Numerical Methods in Engineering – Homework Assignment For Part 07**

**Solve the following problems:**

[1]. Using Excel, solve the following problem over the interval from  $x = 0$  to 1 using a step size of 0.25 where  $y(0) = 1$ . Display all your results on the same graph:

$$\frac{dy}{dx} = (1 + 2 \cdot x) \cdot \sqrt{y}$$

(a) Analytically; (b) Euler's method; (c) Heun's method without iteration; (d) Ralston's method; (e) Fourth-order Runge-Kutta method.

[2]. Given

$$\frac{dy}{dt} = 30 \cdot (\sin(t) - y) + 3 \cdot \cos(t) \quad ,$$

if  $y(0) = 1$ , use the implicit Euler method in Excel to obtain a solution from  $t = 0$  to 4 using a step size of 0.4.

[3]. A steady-state heat balance for a rod can be represented as

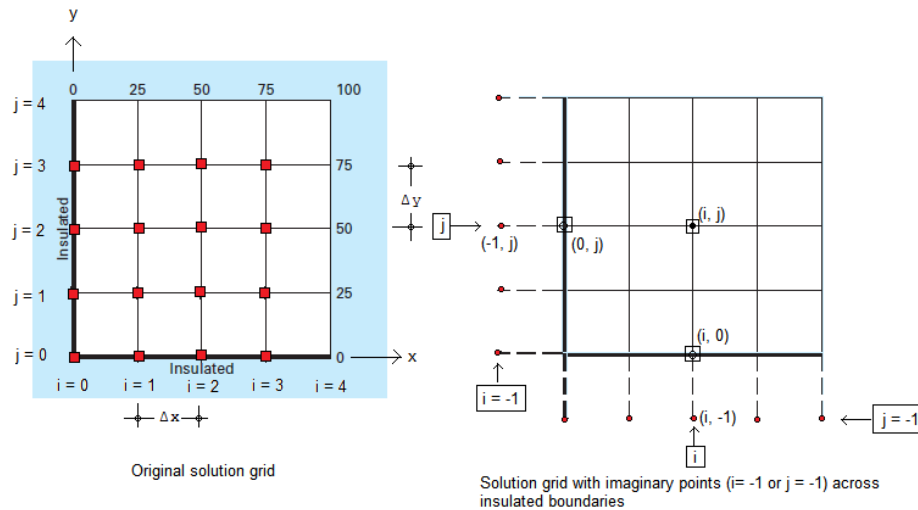
$$\frac{d^2 T}{dx^2} - 0.15 \cdot T = 0 \quad .$$

(a) Obtain an analytical solution for a 10-m rod with  $T(0) = 240$  and  $T(10) = 150$ . (b) Use the shooting method with  $\Delta x = 1.0$  to obtain a solution for this problem. (c) Use the finite difference method with  $\Delta x = 1.0$  to obtain a solution for this problem. You can use Maxima for (a). Use Excel for parts (b) and (c).

**ENGR 2450 – Numerical Methods in Engineering – Homework Assignment For Part 08**

**Solve the following problems:**

[1]. The figure below shows a rectangular domain representing a square metal plate. The solution grid traced on the plate has equal  $x$  and  $y$  increments, namely,  $\Delta x = \Delta y = 10$  cm. The indices  $i$  and  $j$  identify horizontal and vertical lines for the solution grid. The numbers in the upper and right boundaries represent values of the temperature (in  $^{\circ}\text{C}$ ) in those plate boundaries. Since these temperature values at the boundaries are fixed, those two boundaries are examples of *Dirichlet-type boundary conditions*. The other two boundaries (i.e., lines for which  $i=0$  and  $j=0$ ) are insulated. Thus, these two boundary are examples of *Neumann-type boundary conditions*, specifically:  $\frac{\partial T}{\partial x}=0$  for  $i=0$ , and  $\frac{\partial T}{\partial y}=0$  for  $j=0$ .



The temperature distribution in the plate is governed by the Laplace equation:  $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$ . The

Liebmann formula for solving the governing equation for heat transfer in the two-dimensional domain, with  $\Delta x = \Delta y$ , is  $T_{i,j} = \frac{(T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1})}{4}$ . The derivative boundary condition

$\frac{\partial T}{\partial x} = 0$  at  $i=0$ , can be replaced by the finite-difference approximation  $\frac{\partial T}{\partial x} = \frac{T_{1,j} - T_{-1,j}}{2 \cdot \Delta x}$ ,

whereas, the derivative boundary condition  $\frac{\partial T}{\partial y} = 0$  at  $j=0$ , can be replaced by the finite-difference

approximation  $\frac{\partial T}{\partial y} = \frac{T_{i,1} - T_{i,-1}}{2 \cdot \Delta y}$ .

(a) Find expressions for the temperatures  $T_{i,0}$  and  $T_{0,j}$ , by combining Liebmann's formula at points  $(i,0)$  and  $(0,j)$  with the corresponding finite-difference approximations for the derivatives

$$\frac{\partial T}{\partial x}=0, \text{ at } i=0, \text{ and } \frac{\partial T}{\partial y}=0 \text{ at } j=0, \text{ so as to eliminate the terms } T_{i,-1} \text{ and } T_{-1,j}.$$

(b) Using Excel, solve for the temperatures in the points corresponding to the intersection of the lines  $i = 0, 1, 2, 3$  with the lines  $j=0, 1, 2, 3$ , i.e., in the points identified by the red squares in the figure above.

(c) Using Excel, calculate the components,  $q_x$  and  $q_y$ , of the heat fluxes in the points identified by the red squares in the figure above. Recall that these components are given by

$$q_{x(i,j)} = -k \cdot \frac{T_{i+1,j} - T_{i-1,j}}{2 \cdot \Delta x}, \text{ and } q_{y(i,j)} = -k \cdot \frac{T_{i,j+1} - T_{i,j-1}}{2 \cdot \Delta y},$$

respectively. Use  $k = 0.50 \text{ cal}/(\text{s} \cdot \text{cm} \cdot \text{C})$ .

(d) Using Excel calculate the magnitudes  $q$  and directions  $\theta$  of the fluxes at the grid points identified by the red squares in the figure above. Recall that  $q_{(i,j)} = \sqrt{q_{x(i,j)}^2 + q_{y(i,j)}^2}$ , and that  $\theta$  is given by

$$\theta = \tan^{-1} \left( \frac{q_y(i,j)}{q_x(i,j)} \right), \text{ for } q_{x(i,j)} > 0, \text{ and } \theta = \tan^{-1} \left( \frac{q_y(i,j)}{q_x(i,j)} \right) + \pi, \text{ for } q_{x(i,j)} < 0.$$

(e) Produce a plot of the heat fluxes as vectors based on the grid points represented by red rectangles of the solution domain. You can do this plot by hand or using SCILAB or Matlab (Excel doesn't have the ability of producing vector plots).

[2]. Determine the temperatures along a 1-m horizontal rod described by the heat-conduction equation,

$$k \cdot \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}. \text{ Assume that the right boundary (} x = 1 \text{ m) is insulated, i.e., } \frac{\partial T}{\partial x} = 0, \text{ and that the}$$

left boundary ( $x = 0$ ) is represented by  $-k' \cdot \frac{\partial T}{\partial x} = h \cdot (T_a - T_0)$ , where  $k'$  = coefficient of thermal conductivity ( $\text{W}/\text{m} \cdot \text{C}$ ),  $T_a$  = ambient temperature ( $^{\circ}\text{C}$ ), and  $T_0$  = temperature of the rod at  $x = 0$  (in  $^{\circ}\text{C}$ ).

(a) Obtain an explicit scheme to solve for  $T_i^{l+1}$ ,  $i = 1, 2, 3, 4$ , for this case, as well as for the temperatures at the boundaries  $T_0^{l+1}$  and  $T_5^{l+1}$ . The last two expressions will result from combining the formula for  $T_i^{l+1}$ , with  $i = 0$  and  $i = 5$ , respectively, with the values of  $T_{-1}^{l+1}$  and  $T_6^{l+1}$ , obtained from replacing the derivative boundary conditions given above with finite-difference formulas. Notice that the points  $i = -1$  and  $i = 6$  are imaginary points outside of the corresponding boundaries.

(b) Using Excel, and the parameters  $k = 2 \cdot 10^{-5} \text{ m}^2/\text{s}$ ,  $k' = 10 \text{ W}/\text{m} \cdot \text{C}$ ,  $h = 25 \text{ W}/\text{m}^2 \cdot \text{C}$ , and  $T_a = 50 \text{ }^{\circ}\text{C}$ , calculate the temperature as function of time along the rod using a spatial step of  $\Delta x = 20 \text{ cm}$ . Assume that the initial temperature of the rod is zero. Use a value of  $\Delta t = 30 \text{ s}$ . Verify that

$$\lambda = \frac{k \cdot \Delta t}{(\Delta x)^2} < 0.5.$$

(c) Calculate up to  $t = 4800 \text{ s}$  (i.e., 110 time steps of  $\Delta t = 30 \text{ s}$ ), and plot  $T$ -vs- $x$  for  $t = 300, 600, 1200, 2400, \text{ and } 4800 \text{ s}$ .



# Appendix B

## Exams



## Numerical Methods Unit 2 Exam: Root-Finding

**Answer Key****Instructions**

You will have 50 minutes to complete this exam. To get full credit for a problem, you must show all your work. You may use any calculator of your choosing to complete the exam, but use of any pre-programmed notes or other aids will be considered cheating. On root-finding problems, you will be given an interval over which to search for a root. For methods requiring two input points, use the endpoints of the interval provided. For methods requiring only one point, start at the lower-bound of the given interval. Organize your response using a table whenever possible (problems 2-4). If you have questions regarding the meaning of a question, raise your hand and clarification will be provided.

Name: \_\_\_\_\_

A#: \_\_\_\_\_

Section: \_\_\_\_\_

### Problem 1

Given the following equations, label each with the appropriate method name. Newton-Raphson Method, Secant Method, Fixed-Point Iteration, Bisection Method, Modified Secant Method, False-Position Method. (3 pts each = 18 pts)

$$\text{_____ } x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

$$\text{_____ } x_i = \frac{x_{i-1} + x_{i+1}}{2}$$

$$\text{_____ } x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$\text{_____ } x_{i+1} = x_i + f(x_i)$$

$$\text{_____ } x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$$

$$\text{_____ } x_{i+1} = x_i - \frac{\delta x_i f(x_i)}{f(x_i + \delta x_i) - f(x_i)}$$

- a. Fixed-Point Iteration
- b. Secant Method
- c. Newton-Raphson Method
- d. Modified Secant Method
- e. False Position Method
- f. Bisection Method

**Solution** b, f, c, a, e, d

**Problem 2**

Approximate the number of roots of  $f(x) = 1.8 \cdot \cos(x+1) + 1.7 \cdot \sin(7x)$  between -10 and -3 using the incremental search method with 7 equal intervals. (18 pts)

**Solution**

$i$	$x_i$	$f(x_i)$	+/-
1	-10	-2.9556	-
2	-9	-0.54640	-
3	-8	2.2437	+
4	-7	3.3497	+
5	-6	2.0687	+
6	-5	-0.44865	-
7	-4	-2.2425	-
8	-3	-2.1714	-

2 sign changes implies 2 roots.

**Scoring**

8 points for proper method. 5 points for correct evaluation, 5 points for correct interpretation.

### Problem 3

Approximate the root of  $f(x) = 2x^2 - 2x - 180$  between -12 and 6 using 3 iterations of the bisection method. (22 pts)

#### Solution

$i$	$x_l$	$f(x_l)$	$x_u$	$f(x_u)$	$x_r = \frac{x_l + x_u}{2}$	$f(x_l)f(x_r)$
1	-12	84	6	-120	$\frac{-12+6}{2} = -3.0$	$(84)(-120)$
2	-12	84	-3	-156	$\frac{-12-3}{2} = -7.5$	$(84)(-52.5)$
3	-12	84	-7.5		$\frac{-12-7.5}{2} = -9.75$	

#### Scoring

10 points for setting up method. 4 points for sign change check. 4 points for mathematical manipulation. 4 points for correct answer.

### Problem 4

Approximate the root of  $f(x) = 2x^2 - 2x - 180$  between -12 and 12 using 2 iterations of the Newton-Raphson method. (22 pts)

#### Solution

The derivative  $f'(x) = 4x - 2$

$i$	$x_i$	$f(x_i)$	$f'(x_i)$	$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$
1	-12	132	-50	$-12 - \frac{132}{(-50)} = -9.36$
2	-9.36	13.939	-39.440	$-9.36 - \frac{13.939}{-(39.440)} = -9.0066$

#### Scoring

10 points for setting up method properly. 6 points for mathematical manipulation. 6 points for correct answer.

### Problem 5

Distinguish between open methods and bracketing methods. Explain the advantages and disadvantages of each? (20 pts)

**Scoring**

- +5 Open methods may diverge.
  - +5 Bracketing methods never diverge.
  - +5 Bracketing methods slow to converge on some problems.
  - +5 Open methods don't require two start-points, just one guess.
  - +5 Open methods often converge quickly.
- Consider other correct pros/cons for 5 pts each.  
Up to 20 pts.

[1]. Out of the list shown below, select the proper term to fill in the blanks in statements (a) through (e):

**upper triangular - inverse - singular - transpose - symmetric**

(a) If the determinant of matrix A is zero, matrix A is said to be singular.

(b) Matrix A, shown below, is said to be a(n) symmetric matrix.

$$A = \begin{pmatrix} 5 & 1 & 2 \\ 1 & 6 & 3 \\ 2 & 3 & -7 \end{pmatrix}$$

(c) Matrix D, shown below, is said to be a(n) upper triangular matrix.

$$D = \begin{pmatrix} 10 & -2 & 8 \\ 0 & 5 & 5 \\ 0 & 0 & 15 \end{pmatrix}$$

(d) Given matrices B and C, shown below, C is said to be the transpose of B.

$$B = \begin{pmatrix} 5 & 1 & 6 \\ 8 & 6 & 4 \\ 2 & 3 & -7 \end{pmatrix} \quad C = \begin{pmatrix} 5 & 8 & 2 \\ 1 & 6 & 3 \\ 6 & 4 & -7 \end{pmatrix}$$

(e) Let A and B be two square matrices of the same dimensions, say  $n \times n$ . Let I be an  $n \times n$  identity matrix. If  $A \cdot B = I$ , then matrix B is said to be the inverse of matrix A.

**[EACH QUESTION IN PROBLEM [1] IS WORTH 5 POINTS FOR A TOTAL OF 25 POINTS]**

[2]. You are given the set of linear algebraic equations:

$$10 \cdot x_1 + 5 \cdot x_2 - 2 \cdot x_3 = 14$$

$$2 \cdot x_1 + 8 \cdot x_2 + x_3 = 21$$

$$-x_1 + 2 \cdot x_2 + 5 \cdot x_3 = 18$$

which corresponds to the following augmented matrix of coefficients:

$$\begin{pmatrix} 10 & 5 & -2 & 14 \\ 2 & 8 & 1 & 21 \\ -1 & 2 & 5 & 18 \end{pmatrix}$$

The first step in the forward elimination segment, while solving the system of equations using naive Gauss elimination, is given by the following matrix:

$$\begin{pmatrix} 10 & 5 & -2 & 14 \\ 0 & 7 & 1.4 & 18.2 \\ 0 & 2.5 & 4.8 & 19.4 \end{pmatrix}$$

(a) Write out the next step in the forward elimination segment, i.e., write out the missing terms, r and s, in the following matrix:

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$$\begin{pmatrix} 10 & 5 & -2 & 14 \\ 0 & 7 & 1.4 & 18.2 \\ 0 & 0 & r & s \end{pmatrix} \quad \begin{array}{l} r = \text{-----} \\ s = \text{-----} \end{array}$$

**SOLUTION:** Starting from  $\begin{pmatrix} 10 & 5 & -2 & 14 \\ 0 & 7 & 1.4 & 18.2 \\ 0 & 2.5 & 4.8 & 19.4 \end{pmatrix}$ , multiply row 2 by  $\frac{2.5}{7}$  and subtract it

from row 3, then replace row 3 with that result. Thus, we calculate the new row 3 as:

$$(0 \ 2.5 \ 4.8 \ 19.4) - \frac{2.5}{7} \cdot (0 \ 7 \ 1.4 \ 18.2) = (0 \ 0 \ 4.3 \ 12.9)$$

and, replace it into the matrix above to produce:  $\begin{pmatrix} 10 & 5 & -2 & 14 \\ 0 & 7 & 1.4 & 18.2 \\ 0 & 0 & 4.3 & 12.9 \end{pmatrix}$  Thus,  $r=4.3$  5 pts  
 $s=12.9$  5 pts

**NOTE: Some students may have used function "rref" in their calculators, and not show the detailed calculation. GIVE THEM FULL CREDIT IF THAT IS THE CASE AND THE VALUES SHOW ARE CORRECT, OR IF THEY REPORT  $s = 1$  and  $t = 3$ . Otherwise, give them 1/2 points.**

(b) After finding  $r$  and  $s$ , complete the backward substitution segment of the naive Gauss elimination to find the solution to system of equations:

$$x_1 = \text{-----} \quad x_2 = \text{-----} \quad x_3 = \text{-----}$$

**SOLUTION:** The matrix resulting from the forward elimination of the naive Gauss elimination procedure for this problem is:

$$\begin{pmatrix} 10 & 5 & -2 & 14 \\ 0 & 7 & 1.4 & 18.2 \\ 0 & 0 & 4.3 & 12.9 \end{pmatrix} \text{ which is equivalent to the system: } \begin{array}{l} 10 \cdot x_1 + 5 \cdot x_2 - 2 \cdot x_3 = 14 \quad (i) \\ 7 \cdot x_2 + 1.4 \cdot x_3 = 18.2 \quad (ii) \\ 4.3 \cdot x_3 = 12.9 \quad (iii) \end{array}$$

From (iii) we have:  $x_3 = \frac{12.9}{4.3} = 3$

$$x_1 = 1 \quad 5 \text{ pts}$$

From (ii) we have:  $x_2 = \frac{18.2 - 1.4 \cdot 3}{7} = 2 \Rightarrow$

$$x_2 = 2 \quad 5 \text{ pts}$$

TOTAL PROB. [2]:  
25 pts

From (i) we have:  $x_1 = \frac{14 - 5 \cdot 2 + 2 \cdot 3}{10} = 1$

$$x_3 = 3 \quad 5 \text{ pts}$$

**NOTE: Some students may have used the following approach to solve the problem - GIVE THEM FULL CREDIT IF THAT IS THE CASE AND THE RESULTS ARE CORRECT. Otherwise, 1/2 points.**

$$\begin{pmatrix} 10 & 5 & -2 \\ 0 & 7 & 1.4 \\ 0 & 0 & 4.3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 14 \\ 18.2 \\ 12.9 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 10 & 5 & -2 \\ 0 & 7 & 1.4 \\ 0 & 0 & 4.3 \end{pmatrix}^{-1} \begin{pmatrix} 14 \\ 18.2 \\ 12.9 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

[3]. It has been found that the lower triangular matrix,  $L$ , and upper triangular matrix,  $U$ , for the LU decomposition of a matrix of coefficients  $A$  in the matrix equation  $[A]\{X\} = \{B\}$  are:

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & 5 & 1 \end{pmatrix} \quad U = \begin{pmatrix} 2 & 3 & -1 \\ 0 & 5 & 4 \\ 0 & 0 & 2 \end{pmatrix}$$



With the corresponding right-hand-side vector being:

$$B := \begin{pmatrix} 21 \\ -56 \\ 115 \end{pmatrix}$$

(a) Write out the set of equations corresponding to the matrix equation  $[A]\{X\}=\{B\}$  for this case, i.e., write out a set of equations of the form:

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 = b_1$$

$$a_{21} \cdot x_1 + a_{22} \cdot x_2 + a_{23} \cdot x_3 = b_2$$

$$a_{31} \cdot x_1 + a_{32} \cdot x_2 + a_{33} \cdot x_3 = b_3$$

**SOLUTION:** We are given the lower and upper triangular matrices L and U for the system  $A \cdot X = B$ , thus

$$L := \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & 5 & 1 \end{pmatrix} \quad U := \begin{pmatrix} 2 & 3 & -1 \\ 0 & 5 & 4 \\ 0 & 0 & 2 \end{pmatrix} \quad \text{and} \quad A := L \cdot U \quad \text{or} \quad A = \begin{pmatrix} 2 & 3 & -1 \\ -6 & -4 & 7 \\ 8 & 37 & 18 \end{pmatrix} \quad \text{With} \quad B := \begin{pmatrix} 21 \\ -56 \\ 115 \end{pmatrix}$$

the corresponding system of equations is:

$$2 \cdot x_1 + 3 \cdot x_2 - x_3 = 21 \quad \text{5 points}$$

$$-6 \cdot x_1 - 4 \cdot x_2 + 7 \cdot x_3 = -56 \quad \text{5 points}$$

$$8 \cdot x_1 + 37 \cdot x_2 + 18 \cdot x_3 = 115 \quad \text{5 points}$$

**NOTE:** IF THE STUDENTS WROTE THE A MATRIX (SHOWN ABOVE), BUT DID NOT WRITE THE EQUATIONS, GIVE THEN 10 POINTS.

(b) solve for  $x_1$  using Cramer's rule.

**SOLUTION:** In part (a) we found the matrix of coefficients A. To calculate  $x_1$  using Cramer's rule, we create a matrix, call it  $A_1$  resulting from replacing vector B in the matrix A, i.e.,

$$A_1 := \begin{pmatrix} 21 & 3 & -1 \\ -56 & -4 & 7 \\ 115 & 37 & 18 \end{pmatrix}$$

and calculate  $x_1$  dividing the determinant of  $A_1$  by the determinant of A, i.e.,

$$x_1 = \frac{|A_1|}{|A|} = 5 \quad , \quad \text{or} \quad x_1 = \frac{\begin{vmatrix} 21 & 3 & -1 \\ -56 & -4 & 7 \\ 115 & 37 & 18 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & -1 \\ -6 & -4 & 7 \\ 8 & 37 & 18 \end{vmatrix}} = 5 \quad \text{10 points}$$

**TOTAL Problem [3]: 25 points**

[4]. Consider the L and U matrices of problem [3], namely,

$$U := \begin{pmatrix} 2 & 3 & -1 \\ 0 & 5 & 4 \\ 0 & 0 & 2 \end{pmatrix} \quad L := \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & 5 & 1 \end{pmatrix}$$

and right-hand-side vector

$$B := \begin{pmatrix} 21 \\ -56 \\ 115 \end{pmatrix}$$

The solution procedure for a system of equations when using LU decomposition consists in solving the matrix equation  $[L]\{D\} = \{B\}$  for  $\{D\}$ , first, and then solving the matrix equation  $[U]\{X\} = \{D\}$ .

(a) Write out the corresponding equations for the system  $[L]\{D\} = \{B\}$ , and solve for  $\{D\}$ .

$$d_1 = \text{-----} \quad d_2 = \text{-----} \quad d_3 = \text{-----}$$

**SOLUTION:** The matrix equation  $[L]\{D\} = \{B\}$  results in the equations:

$$d_1 = 21$$

$$d_1 = 21 \quad 4 \text{ points}$$

$$-3 \cdot d_1 + d_2 = -56 \Rightarrow d_2 = -56 + 3 \cdot 21 = 7$$

$$d_2 = 7 \quad 4 \text{ points}$$

$$4 \cdot d_1 + 5 \cdot d_2 + d_3 = 115 \Rightarrow d_3 = 115 - 4 \cdot 21 - 5 \cdot 7 = -4$$

$$d_3 = -4 \quad 4 \text{ points}$$

solving problem: 1 point

**NOTE:** Some students may have used the following approach to solve the problem - GIVE THEM FULL CREDIT IF THAT IS THE CASE AND THE RESULTS ARE CORRECT. Otherwise, 1/2 points.

$$\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & 5 & 1 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = \begin{pmatrix} 21 \\ -56 \\ 115 \end{pmatrix} \Rightarrow \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & 5 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 21 \\ -56 \\ 115 \end{pmatrix} = \begin{pmatrix} 21 \\ 7 \\ -4 \end{pmatrix}$$

(b) Write out the corresponding equations for the system  $[U]\{X\} = \{D\}$ , and solve for  $\{X\}$ .

$$x_1 = \text{-----} \quad x_2 = \text{-----} \quad x_3 = \text{-----}$$

**SOLUTION:** The matrix equation  $[U]\{X\} = \{D\}$  results in the equations:

$$2 \cdot x_1 + 3 \cdot x_2 - x_3 = 21 \Rightarrow x_1 = \frac{21 - 3 \cdot 3 + (-2)}{2} = 5$$

$$x_1 = 5 \quad 4 \text{ points}$$

$$5 \cdot x_2 + 4 \cdot x_3 = 7 \Rightarrow x_2 = \frac{7 - 4 \cdot (-2)}{5} = 3$$

$$x_2 = 3 \quad 4 \text{ points}$$

$$2 \cdot x_3 = -4 \Rightarrow x_3 = -\frac{4}{2} = -2$$

$$x_3 = -2 \quad 4 \text{ points}$$

**TOTAL Problem [4]: 25 points**

(see note in next page)

**NOTE: Some students may have used the following approach to solve the problem - GIVE THEM FULL CREDIT IF THAT IS THE CASE AND THE RESULTS ARE CORRECT. Otherwise, 1/2 points.**

$$\begin{pmatrix} 2 & 3 & -1 \\ 0 & 5 & 4 \\ 0 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 21 \\ 7 \\ -4 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 & 3 & -1 \\ 0 & 5 & 4 \\ 0 & 0 & 2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 21 \\ 7 \\ -4 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}$$

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[1]. For each of the following statements, select the proper term to fill in the blank out of this list:

**Fourier – mean – Newton – Lagrange – exponential**

(a) The equation shown below represents a Newton polynomial.

$$f_n(x) = f(x_0) + (x - x_0) f[x_1, x_0] + (x - x_0)(x - x_1) f[x_2, x_1, x_0] \\ + \cdots + (x - x_0)(x - x_1) \cdots (x - x_{n-1}) f[x_n, x_{n-1}, \dots, x_0]$$

(b) The equations shown below represent a Lagrange polynomial.

$$f_n(x) = \sum_{i=0}^n L_i(x) f(x_i) \quad \text{where} \quad L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^n \frac{x - x_j}{x_i - x_j}$$

(c) The equation shown below represents a Fourier series.

$$f(t) = a_0 + \sum_{k=1}^{\infty} [a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)]$$

(d) The expression shown below represents the mean of a numerical sample  $\{x_1, x_2, \dots, x_n\}$  of size  $n$ .

$$\frac{1}{n} \sum_{i=1}^n x_i$$

(e) Suppose you are given a set of ordered pairs  $(x, y)$ . After plotting  $\ln(y)$  versus  $x$ , you find that the data fits a straight line of the form

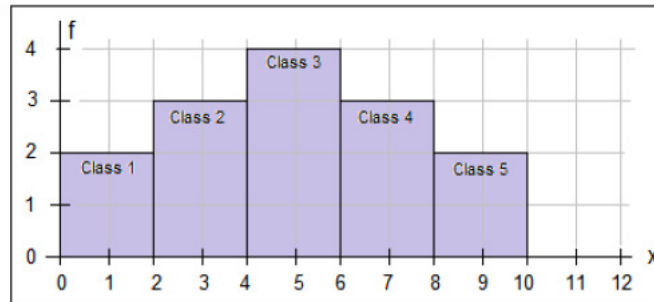
$$\ln(y) = a_0 + a_1 \cdot x$$

If you express  $y$  as a function of  $x$ , i.e.,  $y = f(x)$ , the resulting expression represents a(n)

exponential function.

**GRADING: 4 POINTS EACH, TOTAL = 5X4 = 20 POINTS**

[2]. A histogram shows the frequency count ( $f$ ) for different bins, or *classes*, of data. Each bin or class is bound by a *lower bound* ( $xL_i$ ) and an *upper bound* ( $xU_i$ ). For the histogram shown below, the table shows 5 classes with their upper and lower bounds. (a) Using the information presented in the histogram below, complete the columns for the *class marks* (midpoint of each class) and their corresponding *frequencies* in the table shown:



Class	Lower bound	Upper bound	Class mark	Frequency	
$i$	$xL_i$	$xU_i$	$xM_i$	$f_i$	$xM_i \cdot f_i$
1	0	2	1	2	2
2	2	4	3	3	9
3	4	6	5	4	20
4	6	8	7	3	21
5	8	10	9	2	18
			$\Sigma$	14	70

(a)  
Yellow columns show data needed to complete data table

The mean value ( $\bar{x}$ ) of the data in the frequency table above can be calculated using:

$$\bar{x} = \frac{\sum_{i=1}^n xM_i \cdot f_i}{\sum_{i=1}^n f_i},$$

where  $n$  is the number of classes in the frequency table. (b) Calculate the mean value ( $\bar{x}$ ) of the data in the frequency table above. [If needed, perform calculations in the back of page 1].

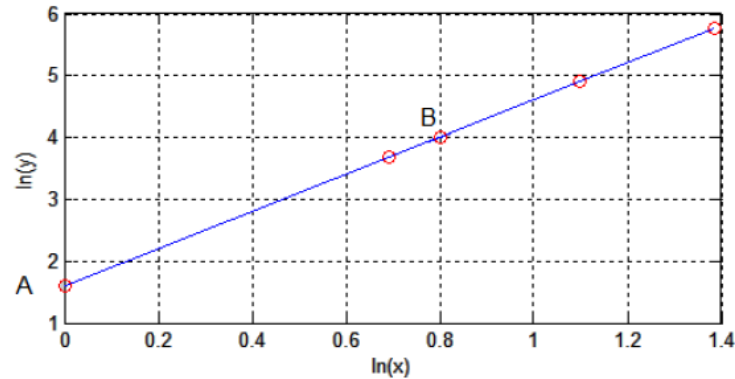
From the table:  $\Sigma xM_i \cdot f_i = 70$ , and  $\Sigma f_i = 14$ , therefore,  $\bar{x} = \frac{\Sigma xM_i \cdot f_i}{\Sigma f_i} = \frac{70}{14} = 5$

GRADING: (a) 15 points; (b) 5 points; TOTAL = 20 POINTS

$\bar{x} = \underline{\quad 5.0 \quad}$

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[3]. You are given a set of ordered pairs  $(x,y)$  as results of some laboratory measurements. When you plot the logarithm of  $y$  versus the logarithm of  $x$ , you find the following graph:



Obviously, the graph above shows a straight line. Therefore, you can write an equation of the form:

$$\ln(y) = a_0 + a_1 \cdot \ln(x)$$

to fit the data. Reading the plot above you find that the coordinates of point A and B, in the graph, are  $A(0.00, 1.61)$  and  $B(0.80, 4.00)$ . Determine (a) the intercept,  $a_0$ , and, (b) the slope,  $a_1$ , of the straight line in the graph shown above. (c) Find the relationship  $y = f(x)$  for the  $(x, y)$  data in this problem.

**SOLUTION:**

Given the coordinates  $A(0.00, 1.61)$  and  $B(0.80, 4.00)$ , the slope is:

$$(b) \quad a_1 = \frac{4.00 - 1.61}{0.80 - 0.00} = 2.9875$$

Thus, the equation is now:  $\ln(y) = a_0 + 2.9875 \cdot \ln(x)$

Replacing the coordinates of  $A(0.00, 1.61)$  into this equation we have:

$$(a) \quad 1.61 = a_0 + 2.9875 \cdot 0 \Rightarrow a_0 = 1.61$$

Thus,  $\ln(y) = 1.61 + 2.9875 \cdot \ln(x) \Rightarrow \ln(y) = 1.61 + \ln(x^{2.9875})$

Applying the exponential function:

$$\begin{aligned} e^{\ln(y)} &= e^{1.61 + \ln(x^{2.9875})} \\ y &= e^{1.61} \cdot e^{\ln(x^{2.9875})} \\ \Rightarrow y &= 5.0028 \cdot x^{2.9875} \quad \text{or} \quad y = 5 \cdot x^3 \end{aligned}$$

$$(a) \quad a_0 = \underline{1.61}$$

$$(b) \quad a_1 = \underline{2.9875}$$

$$(c) \quad \underline{y = 5.0028 x^{2.9875}}$$

**GRADING: (a) 5 points; (b) 5 points; (c) 10 points;  
TOTAL = 20 POINTS**

[4]. Suppose you are given a set of ordered pairs  $(x, y)$  and told that the data fit the relationship:

$$y = \frac{k \cdot x}{r + x},$$

where  $k$  and  $r$  are constant. (a) Produce a linearization of this expression so that you can write an equation of the form:

$$f_1(y) = a_0 + a_1 \cdot f_2(x).$$

Using only two data values, namely  $(x = 2, y = \frac{2}{3})$  and  $(x = 4, y = 1)$ , determine the values of the constants (b)  $k$ , and (c)  $r$  for this fitting.

**SOLUTION:** (a) Linearization: start with  $y = \frac{k \cdot x}{r + x}$ . Divide numerator and denominator of

the right-hand-side of the equation by  $x$ ,  $y = \frac{\frac{k \cdot x}{x}}{\frac{r+x}{x}} = \frac{k}{\frac{r}{x} + 1} \Rightarrow \frac{r}{x} + 1 = \frac{k}{y} \Rightarrow k \cdot \frac{1}{y} = 1 + r \cdot \frac{1}{x}$

$\Rightarrow$  (a)  $\frac{1}{y} = \frac{1}{k} + \frac{r}{k} \cdot \frac{1}{x}$

Compare with  $f_1(y) = a_0 + a_1 \cdot f_2(x)$

Then,  $f_1(y) = \frac{1}{y}$ ,  $f_2(x) = \frac{1}{x}$ ,  $a_0 = \frac{1}{k}$ , and  $a_1 = \frac{r}{k}$

To find  $k$  and  $r$ , you can use the data given as follows:

x	y	1/x	1/y
2	2/3	1/2	3/2
4	1	1/4	1

The slope is:  $a_1 = \frac{\frac{3}{2} - 1}{\frac{1}{2} - \frac{1}{4}} = 2 = \frac{r}{k}$

Replacing the first data point:  $\frac{3}{2} = a_0 + 2 \cdot \frac{1}{2}$

$a_0 = \frac{3}{2} - 1 = \frac{1}{2} = \frac{1}{k} \Rightarrow k = \frac{1}{\frac{1}{2}} = \frac{1}{1/2} = 2$  k=2

Then, from  $a_1 = \frac{r}{k} = \frac{r}{2} = 2 \Rightarrow r = 2 \cdot 2 = 4$  r=4

Alternative solution: start from:  $y = \frac{k \cdot x}{r + x}$  and replace the given data:

(1)  $\frac{2}{3} = \frac{k \cdot 2}{r + 2} \Rightarrow 2 \cdot (r + 2) = 6 \cdot k \Rightarrow 2 \cdot r + 4 = 6 \cdot k \Rightarrow 2 \cdot r - 6 \cdot k = -4$

(2)  $1 = \frac{k \cdot 4}{r + 4} \Rightarrow r + 4 = 4 \cdot k \Rightarrow r - 4 \cdot k = -4$

$\Rightarrow \begin{pmatrix} 2 & -6 \\ 1 & -4 \end{pmatrix} \cdot \begin{pmatrix} r \\ k \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \Rightarrow \begin{pmatrix} r \\ k \end{pmatrix} = \begin{pmatrix} 2 & -6 \\ 1 & -4 \end{pmatrix}^{-1} \cdot \begin{pmatrix} -4 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$  r=4  
k=2

**GRADING: (a) 10 points; (b) 5 points; (c) points;**  
**TOTAL = 20 POINTS**

(a)  $\frac{1}{y} = \frac{1}{k} + \frac{r}{k} \cdot \frac{1}{x}$

(b)  $k = \underline{\quad 2 \quad}$

(c)  $r = \underline{\quad 4 \quad}$

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[5]. The table for this problem shows the variation of the density of liquid water as a function of temperature. Using linear interpolation determine (a) the density of water at +18 °C. (b) Using linear interpolation, determine the temperature, in °C, at which the density of liquid water is 990 kg/m<sup>3</sup>.

**SOLUTION:**

(a) Interpolate density at T = 18 °C

T (°C)	ρ (kg/m <sup>3</sup> )
20	998.2071
18	x
15	999.1026

$$\frac{x - 998.2071}{999.1026 - 998.2071} = \frac{18 - 20}{15 - 20} = \frac{-2}{-5} = \frac{2}{5}$$

$$x = 998.2071 + \frac{2}{5} \cdot (999.1026 - 998.2071) = 998.5653 \frac{\text{kg}}{\text{m}^3}$$

(b) Interpolate temperature for which density = 990 kg/m<sup>3</sup>:

T (°C)	ρ (kg/m <sup>3</sup> )
60	983.2
x	990
40	992.2

$$\frac{x - 60}{40 - 60} = \frac{990 - 983.2}{992.2 - 983.2}$$

$$x = 60 + \frac{(40 - 60) \cdot (990 - 983.2)}{992.2 - 983.2} = 44.8889 \text{ } ^\circ\text{C}$$

**Density of liquid water**

Temp (°C)	Density (kg/m <sup>3</sup> ) <sup>[19][20]</sup>
+100	958.4
+80	971.8
+60	983.2
+40	992.2
+30	995.6502
+25	997.0479
+22	997.7735
+20	998.2071
+15	999.1026
+10	999.7026
+4	999.9720
0	999.8395
-10	998.117
-20	993.547
-30	983.854

The values below 0 °C refer to supercooled water.

(a) For a temperature of + 18 °C, the density is: 998.5653 kg/m<sup>3</sup>(b) For a density of 990 kg/m<sup>3</sup>, the temperature is: 44.8889 °C**GRADING: 10 points each; TOTAL = 20**



**Numerical Methods Unit 6 Exam:  
Numerical Integration and Differentiation - SOLUTION**

**Instructions**

- The exam is closed book and notes, but the use of calculators is allowed.
- You will have 50 minutes to complete this exam.
- To get full credit for a problem, you must show the calculations used in as much detail as possible.
- If you have questions regarding the meaning of a question, raise your hand and clarification will be provided.
- The exam consists of 5 problems: Problem [1] consists of conceptual questions, the other 4 problems require calculations.
- For all problems that involve calculations, use 4 decimals in your solution.
- Problems [2] through [4] require you to calculate approximations to an integral through various methods, while Problem [5] requires you to calculate numerical derivatives, using centered differences.
- Each of problems [2] through [5] requires you to calculate 4 results: (a), (b), (c), and (d). Each one of those results is worth  $\frac{1}{4}$  of the total number of points for each problem.
- Each of the 5 problems included in the exam is worth **20 points**. Thus, the **total points** available for this exam is **100**.

Name: \_\_\_\_\_

A # : \_\_\_\_\_

Section: \_\_\_\_\_

[1]. Conceptual questions on numerical integration and differentiation.

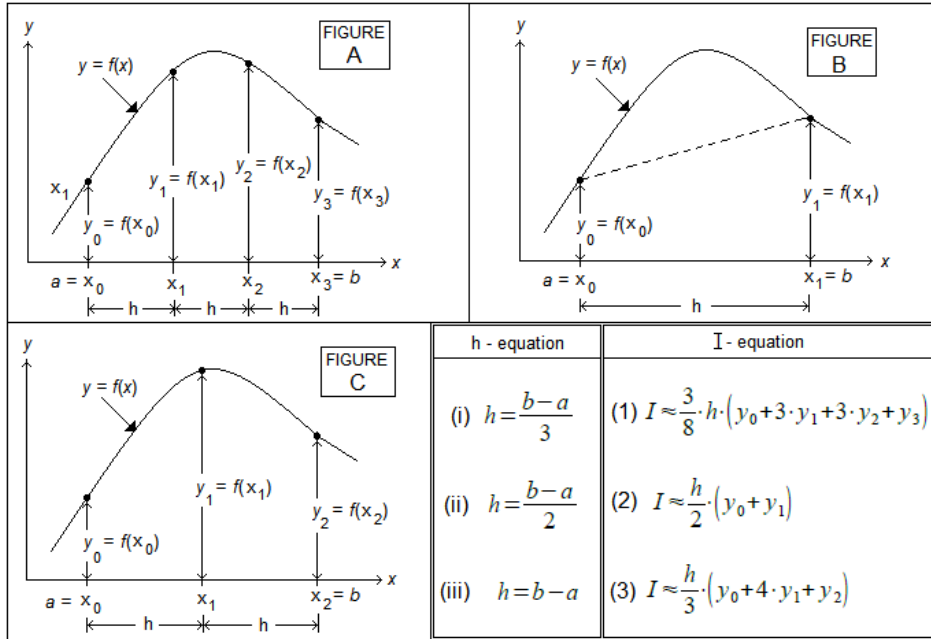
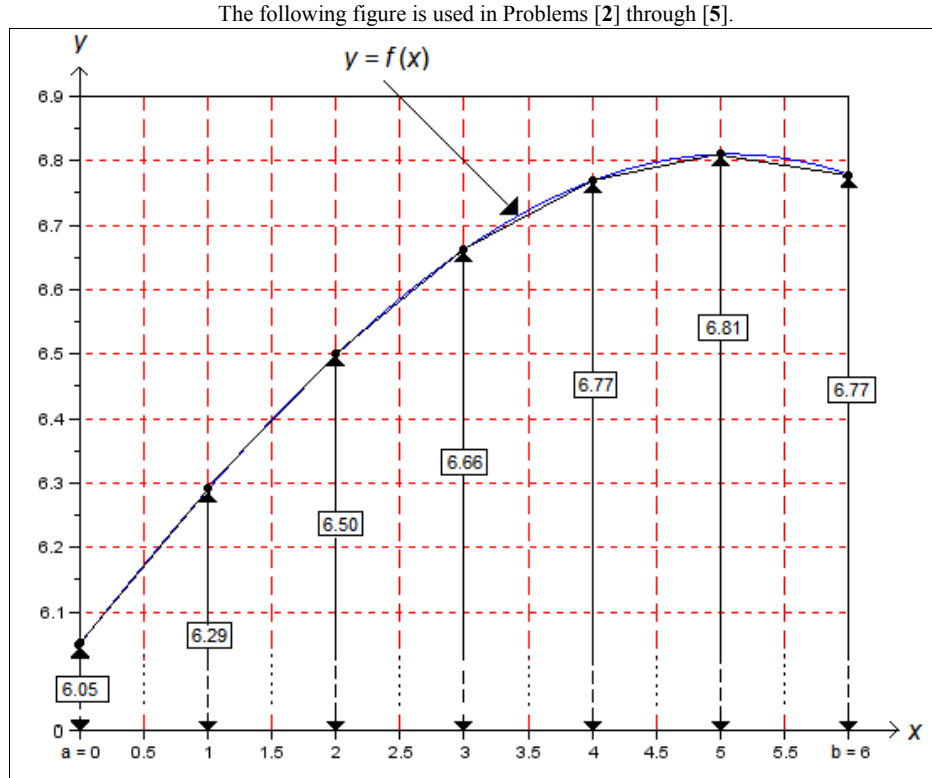


Figure 1

For questions (a), (b), and (c), in the table below, use the information in *Figure 1*, above, to match each one of the **FIGUREs** A, B, and C, with an **h-equation** (i, ii, or iii), and with an **I-equation** (1, 2, or 3) for single applications of the numerical integration methods listed. Also, complete the last column in the table by filling in the proper value of  $n$  = number of intervals in the domain  $[a, b]$  used in each method if  $a = x_0$  and  $b = x_n$ .

Question	Method	FIGURE	h - equation	I - equation	Value of $n$
(a)	Trapezoidal rule	B	(iii)	(2)	1
(b)	Simpson's 1/3 rule	C	(ii)	(3)	2
(c)	Simpson's 3/8 rule	A	(i)	(1)	3

Grading: Each correct cell in the table = 1.5 points each x 12 = 18, then add 2 points to round 20 points



**Figure 2.** Abscissas ( $x$ ) and ordinates ( $y$ ) values for a particular function  $y = f(x)$ .

Use the  $(x,y)$  values listed in this figure to calculate numerical integrals approximating  $I = \int_0^6 f(x) dx$  as required in Problems [2], [3], and [4]. You will also need to use the following:

Richardson's Extrapolation formula : Let  $I(h_1)$  and  $I(h_2)$  be two numerical approximations to the integral  $I = \int_a^b f(x) dx$  based on equally-spaced increments of widths  $h_1$  and  $h_2$ , respectively, and  $h_2 > h_1$ . An improved value of the integral  $I$ , based on  $I(h_1)$  and  $I(h_2)$ , is given by the Richardson's extrapolation formula:

$$I \approx I(h_2) + \frac{I(h_2) - I(h_1)}{\left(\frac{h_1}{h_2}\right)^2 - 1}$$

**[2]. Trapezoidal rule applications and Richardson's interpolation**

(a) Use **two applications of the trapezoidal rule** with equally-spaced data from [Figure 2](#), above, to calculate an estimate of the integral  $I = \int_a^b f(x) dx$  with  $a = 0$ , and  $b = 6$ . Refer to this result as  $I(h_1)$ .

<b>[1] TRAPEZOIDAL RULE</b>						
<b>Problem [2] - (a)</b>						
$x_0 := 0$	$x_1 := 3$	$x_2 := 6$	$y_0 := 6.05$	$y_1 := 6.66$	$y_2 := 6.77$	$h := 3$
$Ih_1 := h \cdot \left( \frac{y_0 + y_1}{2} \right) + h \cdot \left( \frac{y_1 + y_2}{2} \right)$						$Ih_1 = 39.21$

(b) Use **four applications of the trapezoidal rule** with equally-spaced data from [Figure 2](#), above, to calculate an estimate of the integral  $I = \int_a^b f(x) dx$  with  $a = 0$ , and  $b = 6$ . Refer to this result as  $I(h_2)$ .

<b>Problem [2] - (b)</b>						
$x_0 := 0.0$	$x_1 := 1.5$	$x_2 := 3.0$	$x_3 := 4.5$	$x_4 := 6.0$	$h := 1.5$	
$y_0 := 6.05$	$y_1 := 6.40$	$y_2 := 6.66$	$y_3 := 6.80$	$y_4 := 6.77$		
$Ih_2 := h \cdot \left( \frac{y_0 + y_1}{2} \right) + h \cdot \left( \frac{y_1 + y_2}{2} \right) + h \cdot \left( \frac{y_2 + y_3}{2} \right) + h \cdot \left( \frac{y_3 + y_4}{2} \right)$						$Ih_2 = 39.405$

(c) Use the Richardson's extrapolation formula (see page 3, above) to calculate an improved value of the integral,  $I_a$ , based on the values of  $I(h_1)$  and  $I(h_2)$  calculated in (a) and (b), respectively.

**Problem [2] - (c)**

$$h_1 := 3 \quad I_{h_1} = 39.21 \quad + \quad h_2 := 1.5 \quad I_{h_2} = 39.405$$

$$I_a := I_{h_2} + \frac{I_{h_2} - I_{h_1}}{\left(\frac{h_1}{h_2}\right)^2 - 1} \quad I_a = 39.47$$

(d) Calculate the absolute percent error,  $\epsilon_t = \left| \frac{I_{exact} - I_a}{I_{exact}} \right| \cdot 100$  %, for  $I_a$  if the exact value of the integral is  $I_{exact} = 39.4702$ .

**Problem [2] - (d)**

$$I_a = 39.47 \quad I_e := 39.4702$$

$$\epsilon_t = \left| \frac{I_e - I_a}{I_e} \right| \cdot 100 \quad \epsilon_t = 0.0005$$

- (a)  $I(h_1) =$  39.2100 [5 points]  
 (b)  $I(h_2) =$  39.4050 [5 points]  
 (c)  $I_a =$  39.4700 [5 points]  
 (d)  $\epsilon_t =$  0.0005 % [5 points]

**[3]. Simpson's 1/3 rule applications and Richardson's interpolation**

(a) Use a **single** application of the Simpson's 1/3 rule with equally-spaced data from Figure 2, above, to calculate an estimate of the integral  $I = \int_a^b f(x) dx$  with  $a = 0$ , and  $b = 6$ . Refer to this result as  $I(h_1)$ .

**Problem [3] - (a)**

$$x_0 := 0 \quad x_1 := 3 \quad x_2 := 6 \quad y_0 := 6.05 \quad y_1 := 6.66 \quad y_2 := 6.77 \quad h := 3$$

$$Ih_1 := \frac{h}{3} \cdot (y_0 + 4 \cdot y_1 + y_2)$$

$$Ih_1 = 39.46$$

(b) Use **two** applications of the Simpson's 1/3 rule with equally-spaced data from Figure 2, above, to calculate an estimate of the integral  $I = \int_a^b f(x) dx$  with  $a = 0$ , and  $b = 6$ . Refer to this result as  $I(h_2)$ .

**Problem [3] - (b)**

$$x_0 := 0.0 \quad x_1 := 1.5 \quad x_2 := 3.0 \quad x_3 := 4.5 \quad x_4 := 6.0 \quad h := 1.5$$

$$y_0 := 6.05 \quad y_1 := 6.40 \quad y_2 := 6.66 \quad y_3 := 6.80 \quad y_4 := 6.77$$

$$Ih_2 := \frac{h}{3} \cdot (y_0 + 4 \cdot y_1 + y_2) + \frac{h}{3} \cdot (y_2 + 4 \cdot y_3 + y_4)$$

$$Ih_2 = 39.47$$

(c) Use the Richardson's extrapolation formula (see page 3, above) to calculate an improved value of the integral,  $I_a$ , based on the values of  $I(h_1)$  and  $I(h_2)$  calculated in (a) and (b), respectively.

**Problem [3] - (c)**

$$h_1 := 3 \quad I_{h_1} = 39.46 \quad h_2 := 1.5 \quad I_{h_2} = 39.47$$

$$I_a := I_{h_2} + \frac{I_{h_2} - I_{h_1}}{\left(\frac{h_1}{h_2}\right)^2 - 1} \quad I_a = 39.4733$$

(d) Calculate the absolute percent error,  $\epsilon_t = \left| \frac{I_{exact} - I_a}{I_{exact}} \right| \cdot 100$  %, for  $I_a$  if the exact value of the integral is  $I_{exact} = 39.4702$ .

**Problem [3] - (d)**

$$I_a = 39.4733 \quad I_e = 39.4702$$

$$\epsilon_t := \left| \frac{I_e - I_a}{I_e} \right| \cdot 100 \quad \epsilon_t = 0.0079 \%$$

- (a)  $I(h_1) =$  39.4600 [5 points]  
 (b)  $I(h_2) =$  39.4700 [5 points]  
 (c)  $I_a =$  39.4733 [5 points]  
 (d)  $\epsilon_t =$  0.0079 % [5 points]

**[4]. Simpson's 3/8 rule applications and Richardson's interpolation**

(a) Use a **single** application of the Simpson's 3/8 rule with the data from Figure 2, above, to calculate an estimate of the integral  $I = \int_a^b f(x) dx$  with  $a = 0$ , and  $b = 6$ . Refer to this result as  $I(h_1)$ .

**Problem [4] - (a)**

$$x_0 := 0.0 \quad x_1 := 2.0 \quad x_2 := 4.0 \quad x_3 := 6.0 \quad h := 2$$

$$y_0 := 6.05 \quad y_1 := 6.50 \quad y_2 := 6.77 \quad y_3 := 6.77$$

$$Ih_1 := \frac{3}{8} \cdot h \cdot (y_0 + 3 \cdot y_1 + 3 \cdot y_2 + y_3)$$

$$Ih_1 = 39.4725$$

(b) Use **two** applications of the Simpson's 3/8 rule with the data from Figure 2, above, to calculate an estimate of the integral  $I = \int_a^b f(x) dx$  with  $a = 0$ , and  $b = 6$ . Refer to this result as  $I(h_2)$ .

**Problem [4] - (b)**

$$x_0 := 0.0 \quad x_1 := 1.0 \quad x_2 := 2.0 \quad x_3 := 3.0 \quad x_4 := 4.0 \quad x_5 := 5.0 \quad x_6 := 6.0$$

$$y_0 := 6.05 \quad y_1 := 6.29 \quad y_2 := 6.50 \quad y_3 := 6.66 \quad y_4 := 6.77 \quad y_5 := 6.81 \quad y_6 := 6.77$$

$$h := 1$$

$$Ih_2 := \frac{3}{8} \cdot h \cdot (y_0 + 3 \cdot y_1 + 3 \cdot y_2 + y_3) + \frac{3}{8} \cdot h \cdot (y_3 + 3 \cdot y_4 + 3 \cdot y_5 + y_6)$$

$$Ih_2 = 39.4688$$



(c) Use the Richardson's extrapolation formula (see page 3, above) to calculate an improved value of the integral,  $I_a$ , based on the values of  $I(h_1)$  and  $I(h_2)$  calculated in (a) and (b), respectively.

**Problem [4] - (c)**

$$h_1 := 2 \quad I_{h_1} = 39.4725 \quad h_2 := 1 \quad I_{h_2} = 39.4688$$

$$I_a := I_{h_2} + \frac{I_{h_2} - I_{h_1}}{\left(\frac{h_1}{h_2}\right)^2 - 1}$$

$I_a = 39.4675$

(d) Calculate the absolute percent error,  $\epsilon_t = \left| \frac{I_{exact} - I_a}{I_{exact}} \right| \cdot 100$  %, for  $I_a$  if the exact value of the integral is  $I_{exact} = 39.4702$ .

**Problem [4] - (d)**

$$I_a = 39.4675 \quad I_e := 39.4702$$

$$\epsilon_t := \left| \frac{I_e - I_a}{I_e} \right| \cdot 100$$

$\epsilon_t = 0.0068$

- (a)  $I(h_1) =$  39.4725 [5 points]
- (b)  $I(h_2) =$  39.4688 [5 points]
- (c)  $I_a =$  39.4675 [5 points]
- (d)  $\epsilon_t =$  0.0068 % [5 points]

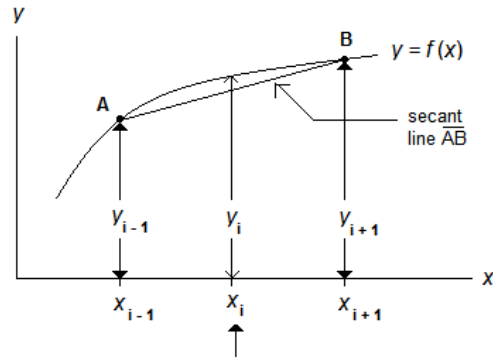
**[5]. Numerical differentiation**

Using a two-point, centered, finite difference formula, calculate the numerical derivatives:

(a)  $f'(2)$ , (b)  $f'(3)$ , and (c)  $f'(4)$ , with increment  $h = 1.0$ , for the data in Figure 2, above. Note:  $f'(x) = dy/dx$ .

(d) Based on the derivatives calculated above, and using centered finite differences again, determine the numerical approximation to the second derivative  $f''(3) = d^2y/dx^2 |_{x=3}$ .

NOTE: The figure to the right shows how to obtain a centered finite difference formula for  $f'(x_i)$ .  $\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow$



$$f'(x_i) = \left. \frac{dy}{dx} \right|_{x=x_i} = \left\{ \begin{array}{l} \text{slope of the} \\ \text{secant line} \\ \overline{AB} \end{array} \right\}$$

**Problem [5]**

$$x_0 := 0.0 \quad x_1 := 1.0 \quad x_2 := 2.0 \quad x_3 := 3.0 \quad x_4 := 4.0 \quad x_5 := 5.0 \quad x_6 := 6.0$$

$$y_0 := 6.05 \quad y_1 := 6.29 \quad y_2 := 6.50 \quad y_3 := 6.66 \quad y_4 := 6.77 \quad y_5 := 6.81 \quad y_6 := 6.77$$

$$h := 1$$

$$(a) \quad f'(2) = \frac{f(3) - f(1)}{3 - 1} = \frac{6.66 - 6.29}{3 - 1} = 0.185$$

$$(b) \quad f'(3) = \frac{f(4) - f(2)}{4 - 2} = \frac{6.77 - 6.50}{4 - 2} = 0.135$$

$$(c) \quad f'(4) = \frac{f(5) - f(3)}{5 - 3} = \frac{6.81 - 6.66}{5 - 3} = 0.075$$

$$(d) \quad f''(3) = \frac{f'(4) - f'(2)}{4 - 2} = \frac{0.075 - 0.185}{4 - 2} = -0.055$$

Alternatively,

$$f''(3) = \frac{f(2) - 2 \cdot f(3) + f(4)}{h^2} = \frac{6.50 - 2 \cdot 6.66 + 6.77}{1^2} = -0.05$$

SEE NEXT PAGE TO SEE COMPARISON WITH TRUE DERIVATIVES:

NOTE:

True derivative - start with:  $f(x) := 5.05 + \sin\left(\frac{x}{4}\right) + \cos\left(\frac{x}{8}\right)$

Calculate derivative:  $\frac{d}{dx} f(x) = \frac{2 \cdot \cos\left(\frac{x}{4}\right) - \sin\left(\frac{x}{8}\right)}{8}$

Define  $f'(x)$ :  $f'(x) := \frac{2 \cdot \cos\left(\frac{x}{4}\right) - \sin\left(\frac{x}{8}\right)}{8}$

First derivatives

True	Numerical
$f'(2) = 0.1885$	0.1850
$f'(3) = 0.1371$	0.1350
$f'(4) = 0.0751$	0.0750

Calculate second derivative:  $\frac{d}{dx} f'(x) = -\frac{4 \cdot \sin\left(\frac{x}{4}\right) + \cos\left(\frac{x}{8}\right)}{64}$

Define  $f''(x)$ :  $f''(x) := -\frac{4 \cdot \sin\left(\frac{x}{4}\right) + \cos\left(\frac{x}{8}\right)}{64}$

Second derivative at x = 3:

True	Numerical
$f''(3) = -0.0571$	-0.05 or -0.055

(a)  $f'(2) =$  0.1850 [5 points]

(b)  $f'(3) =$  0.1350 [5 points]

(c)  $f'(4) =$  0.0750 [5 points]

(d)  $f''(3) =$  -0.0500 or -0.0550 [5 points]

## Numerical Methods Unit 7 Exam: Ordinary Differential Equations

### Instructions

- The exam is closed book and notes, but the use of calculators is allowed.
- You will have 50 minutes to complete this exam.
- To get full credit for a problem, you must show the calculations used in as much detail as possible.
- If you have questions regarding the meaning of a question, raise your hand and clarification will be provided.
- The exam consists of 5 problems: Problem [1] consists of conceptual questions, the other 4 problems require calculations.
- For all problems that involve calculations, use 3 decimals in your solution.
- Each of the 5 problems included in the exam is worth **20 points**. Thus, the **total points** available for this exam is **100**.

Name: \_\_\_\_\_

A # : \_\_\_\_\_

Section: \_\_\_\_\_

[1]. Conceptual questions on ordinary differential equations – To answer the following questions place a check mark in the appropriate solution:

(a) Classify the following ordinary differential equations as linear or non-linear:

$$\frac{d^2x}{dt^2} + 0.5 \cdot \frac{dx}{dt} + 0.2 \cdot x = \sin(x) \quad [x] \text{ Linear} \quad [ ] \text{ Non-linear}$$

$$\frac{d^2x}{dt^2} + 0.5 \cdot x \cdot \frac{dx}{dt} + 0.2 \cdot x^2 = \sin(x) \quad [ ] \text{ Linear} \quad [x] \text{ Non-linear}$$

(b) Classify the following ordinary differential equations as first-order or second-order:

$$\frac{d^2x}{dt^2} + 0.5 \cdot x \cdot \frac{dx}{dt} + 0.2 \cdot x^2 = \sin(x) \quad [ ] \text{ First-order} \quad [x] \text{ Second-order}$$

$$(1+0.5) \cdot \frac{dx}{dt} + 0.2 \cdot x^2 = \sin(x) \quad [x] \text{ First-order} \quad [ ] \text{ Second-order}$$

(c) Classify the following ordinary-differential-equation systems as initial-value or boundary-value problems:

$$\frac{d^2y}{dx^2} + x \cdot y = e^{-t}, y(0) = -2, \frac{dy}{dt} = 2at \text{ at } x=0 \quad [x] \text{ Initial-value problem} \quad [ ] \text{ Boundary-value problem}$$

$$\frac{d^2y}{dx^2} + x \cdot y = e^{-t}, y(0) = -2, y(5) = 2 \quad [ ] \text{ Initial-value problem} \quad [x] \text{ Boundary-value problem}$$

(d) Consider the ordinary differential equation:  $\frac{dy}{dx} = x + y$ . A simple forward finite-difference

formula for  $\frac{dy}{dx}$  is  $\frac{dy}{dx} = \frac{y_{i+1} - y_i}{h}$ , where  $h$  is an increment in  $x$ . Which of the following formulas

represent the Euler method for solving the ordinary differential equation?

$$[ ] y_{i+1} = x_i + y_i \quad [x] y_{i+1} = y_i + (x_i + y_i) \cdot h \quad [ ] y_{i+1} = x_i \cdot h + y_i$$

$$\frac{y_{i+1} - y_i}{h} = x_i + y_i \Rightarrow y_{i+1} = y_i + (x_i + y_i) \cdot h$$

[2]. The Euler method for solving the ordinary-differential equation of the form  $\frac{dy}{dx} = f(x, y)$  follows the formula:  $y_{i+1} = y_i + f(x_i, y_i) \cdot h$ . Consider the ordinary differential equation:

$$2 \cdot \frac{dy}{dx} - 4 \cdot y - 2 \cdot x = 0, \quad y(0) = 1$$

Let  $h$  be an increment on  $x$ . For  $h = 2$ , complete the following table representing the solution to the ordinary differential equation in the range  $0 < x < 6$ .

x	y
0	1.000
2	5.000
4	29.000
6	153.000

$$2 \cdot \frac{dy}{dx} - 4 \cdot y - 2 \cdot x = 0 \quad \Rightarrow \quad \frac{dy}{dx} = 2 \cdot y + x = f(x, y)$$

$$\Rightarrow \quad f(x, y) := 2 \cdot y + x \quad h := 2$$

$$y_{i+1} = y_i + f(x_i, y_i) \cdot h = y_i + (2 \cdot y_i + x_i) \cdot h$$

$$x_0 := 0 \quad y_0 := 1.000$$

$$x_1 := x_0 + h \quad x_1 = 2 \quad y_1 := y_0 + f(x_0, y_0) \cdot h \quad y_1 = 5$$

$$x_2 := x_1 + h \quad x_2 = 4 \quad y_2 := y_1 + f(x_1, y_1) \cdot h \quad y_2 = 29$$

$$x_3 := x_2 + h \quad x_3 = 6 \quad y_3 := y_2 + f(x_2, y_2) \cdot h \quad y_3 = 153$$

[3]. Ralston's method for solving the ordinary-differential equation of the form  $\frac{dy}{dx} = f(x, y)$  requires the following calculations:

$$y_{i+1} = y_i + \left(\frac{1}{3}k_1 + \frac{2}{3}k_2\right)h \quad \text{where } k_1 = f(x_i, y_i), \quad k_2 = f\left(x_i + \frac{3}{4}h, y_i + \frac{3}{4}k_1h\right)$$

Consider the ordinary differential equation:

$$\frac{dy}{dx} - y - x = 0, \quad y(0) = 0$$

Let  $h$  be an increment on  $x$ . For  $h = 3$ , complete the following table representing the solution to the ordinary differential equation in the range  $0 < x < 6$ .

x	$k_1$	$k_2$	y
0	-	-	0.000
2	0.000	1.500	2.000
4	4.000	11.500	20.000

$$\frac{dy}{dx} - y - x = 0 \Rightarrow \frac{dy}{dx} = y + x \Rightarrow f(x, y) = y + x$$

$$\begin{aligned} \boxed{x_0 = 0} & & \boxed{y_0 = 0} \\ x_1 &:= x_0 + h & \boxed{x_1 = 2} & \quad k_1 := f(x_0, y_0) & \quad k_1 = 0 \\ k_2 &:= f\left(x_0 + \frac{3}{4}h, y_0 + \frac{3}{4}k_1h\right) & & \quad k_2 = 1.5 \\ y_1 &:= y_0 + \left(\frac{1}{3}k_1 + \frac{2}{3}k_2\right)h & \boxed{y_1 = 2} \\ \\ x_2 &:= x_1 + h & \boxed{x_2 = 4} & \quad k_1 := f(x_1, y_1) & \quad k_1 = 4 \\ k_2 &:= f\left(x_1 + \frac{3}{4}h, y_1 + \frac{3}{4}k_1h\right) & & \quad k_2 = 11.5 \\ y_2 &:= y_1 + \left(\frac{1}{3}k_1 + \frac{2}{3}k_2\right)h & \boxed{y_2 = 20} \end{aligned}$$

[4]. Consider the ordinary differential equation  $\frac{d^2 T}{dx^2} + \frac{dT}{dx} = x$ ,  $T(0)=1$ ,  $\frac{dT}{dx} = -1$  at  $x=0$ . Transform

this second-order ordinary differential equation into a system of two first-order ordinary differential

equations of the form:  $\frac{dy_1}{dx} = f_1(x, y_1, y_2)$ ,  $\frac{dy_2}{dx} = f_2(x, y_1, y_2)$ ,  $y_1(0) = \text{constant}$ ,

$y_2(0) = \text{constant}$ , using the substitutions  $y_1 = T$  and  $y_2 = \frac{dT}{dx}$ .

$$\frac{d^2 T}{dx^2} + \frac{dT}{dx} = x, \quad T(0)=1, \quad \frac{dT}{dx} = -1 \quad \text{at} \quad x=0$$

$$y_1 = T \quad y_2 = \frac{dT}{dx} \quad \Rightarrow \quad \frac{dy_2}{dx} = \frac{d^2 T}{dx^2}$$

$$y_2 = \frac{dT}{dx} \Rightarrow y_2 = \frac{dy_1}{dx} \Rightarrow$$

$$\boxed{\frac{dy_1}{dx} = y_2}$$

$$\frac{d^2 T}{dx^2} + \frac{dT}{dx} = x \Rightarrow \frac{dy_2}{dx} + y_2 = x \Rightarrow$$

$$\boxed{\frac{dy_2}{dx} = -y_2 + x}$$

$$T(0)=1 \Rightarrow$$

$$\boxed{y_1(0)=1}$$

$$\frac{dT}{dx} = -1 \quad \text{at} \quad x=0 \Rightarrow$$

$$\boxed{y_2(0)=-1}$$



[5]. A stiff ordinary differential equation of the form  $\frac{dy}{dx} = f(x, y)$  can be solved using an implicit

Euler method by writing the formula:  $\frac{y_{i+1} - y_i}{h} = f(x_{i+1}, y_{i+1})$ , and solving for  $y_{i+1}$ , where  $h$  is an

increment in  $x$ . Given the stiff ordinary differential equation:  $\frac{dy}{dx} = -700 \cdot y - 1000 \cdot e^{-x}$ ,  $y(0) = 4$ ,

obtain an expression for  $y_{i+1}$  for this ordinary differential equation.

$$\frac{y_{i+1} - y_i}{h} = f(x_{i+1}, y_{i+1})$$

$$f(x, y) = -700 \cdot y - 1000 \cdot e^{-x}$$

$$\frac{y_{i+1} - y_i}{h} = -700 \cdot y_{i+1} - 1000 \cdot e^{-x_{i+1}}$$

$$y_{i+1} - y_i = -700 \cdot h \cdot y_{i+1} - 1000 \cdot h \cdot e^{-x_{i+1}}$$

$$y_{i+1} + 700 \cdot h \cdot y_{i+1} = y_i - 1000 \cdot h \cdot e^{-x_{i+1}}$$

$$(1 + 700 \cdot h) \cdot y_{i+1} = y_i - 1000 \cdot h \cdot e^{-x_{i+1}}$$

$$y_{i+1} = \frac{y_i - 1000 \cdot h \cdot e^{-x_{i+1}}}{1 + 700 \cdot h}$$

## Numerical Methods Unit 8 Exam: Partial Differential Equations

### Instructions

- The exam is closed book and notes, but the use of calculators is allowed.
- You will have 50 minutes to complete this exam.
- To get full credit for a problem, you must show the derivations and calculations used in as much detail as possible.
- If you have questions regarding the meaning of a question, raise your hand and clarification will be provided.
- The exam consists of 5 problems: Problem [1] consists of conceptual questions, the other 4 problems require derivations of formulas and calculations.
- For all problems that involve calculations, use 2 decimals in your solution.
- Each of the 5 problems included in the exam is worth **20 points**. Thus, the **total points** available for this exam is **100**.

Name: \_\_\_\_\_

A # : \_\_\_\_\_

Section: \_\_\_\_\_

[1]. By analogy with the equations of conic sections:

$$\text{Ellipse: } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad \text{Parabola: } y = k \cdot x^2, \quad \text{Hyperbola: } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$$

determine the type of the following second-order partial differential equations (PDEs). Assume that  $\alpha$ ,  $\beta$ ,  $\kappa$ , and  $k$  are positive constants:

(a) 
$$\frac{\partial^2 \psi}{\partial x^2} - k \cdot \frac{\partial^2 \psi}{\partial y^2} = \alpha$$

Hyperbolic PDE     Parabolic PDE     Elliptic PDE

(b) 
$$\alpha \cdot \frac{\partial^2 \psi}{\partial x^2} + \beta \cdot \frac{\partial^2 \psi}{\partial y^2} = \varphi(x, y)$$

Hyperbolic PDE     Parabolic PDE     Elliptic PDE

(c) 
$$\frac{\partial \psi}{\partial y} = \kappa \cdot \frac{\partial^2 \psi}{\partial x^2}$$

Hyperbolic PDE     Parabolic PDE     Elliptic PDE

[2]. To solve the Laplace equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0,$$

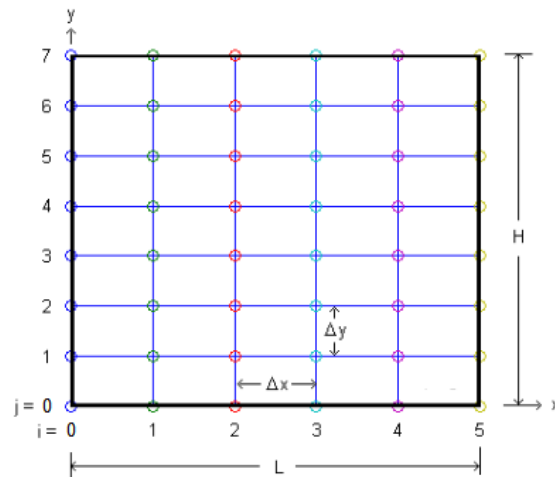
the partial derivatives are replaced by the finite difference formulas:

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2} \quad \text{and} \quad \frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2},$$

and, then, an expression for  $T_{i,j}$  is found. For the case  $\Delta x = \Delta y$ , for example, the resulting equation

is the Liebmann formula, namely,  $T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}}{4}$ .

Consider, on the other hand, a rectangular domain of length  $L = 5$  cm, and height  $H = 3$  cm, with increments  $\Delta x = 1$  cm and  $\Delta y = 0.5$  cm (i.e.,  $\Delta x \neq \Delta y$ ), as illustrated below.



(a) Using  $\beta = \Delta x / \Delta y$ , determine an expression for  $T_{i,j}$  similar to Liebmann's formula, but including  $\beta$ . [Use next page].

Solution part (a):

With  $\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1,j} - 2 \cdot T_{i,j} + T_{i-1,j}}{\Delta x^2}$  and  $\frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j+1} - 2 \cdot T_{i,j} + T_{i,j-1}}{\Delta y^2}$ , in  $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$ , we have:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{T_{i+1,j} - 2 \cdot T_{i,j} + T_{i-1,j}}{\Delta x^2} + \frac{T_{i,j+1} - 2 \cdot T_{i,j} + T_{i,j-1}}{\Delta y^2} = 0, \text{ then, multiply by } \Delta x^2 :$$

$$T_{i+1,j} - 2 \cdot T_{i,j} + T_{i-1,j} + \left(\frac{\Delta x}{\Delta y}\right)^2 \cdot (T_{i,j+1} - 2 \cdot T_{i,j} + T_{i,j-1}) = 0 .$$

Define:  $\beta = \frac{\Delta x}{\Delta y}$ , then  $T_{i+1,j} - 2 \cdot T_{i,j} + T_{i-1,j} + \beta^2 \cdot (T_{i,j+1} - 2 \cdot T_{i,j} + T_{i,j-1}) = 0$  .

Solve for  $T_{i,j}$  :  $T_{i+1,j} - 2 \cdot T_{i,j} + T_{i-1,j} + \beta^2 \cdot (T_{i,j+1} + T_{i,j-1}) - 2 \cdot \beta^2 \cdot T_{i,j} = 0$  , then

$$T_{i+1,j} + T_{i-1,j} + \beta^2 \cdot (T_{i,j+1} + T_{i,j-1}) - 2 \cdot (1 + \beta^2) \cdot T_{i,j} = 0 ,$$

$$2 \cdot (1 + \beta^2) \cdot T_{i,j} = T_{i+1,j} + T_{i-1,j} + \beta^2 \cdot (T_{i,j+1} + T_{i,j-1}) ,$$

$$T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + \beta^2 \cdot (T_{i,j+1} + T_{i,j-1})}{2 \cdot (1 + \beta^2)}$$

(b) The expression for  $T_{i,j}$  that you found in (a) involves  $T_{i+1,j}$ ,  $T_{i-1,j}$ ,  $T_{i,j+1}$ ,  $T_{i,j-1}$  and  $\beta$ . This formula can be used to calculate the values of  $T$  for the points in the interior of the rectangular domain shown in an iterative fashion. Suppose that at a certain step in the iteration you have the following values:  $T_{2,4}=25$ ,  $T_{4,4}=27$ ,  $T_{3,5}=26$ , and  $T_{3,3}=28$ . What would be the value of  $T_{3,4}$  or the next iteration?

Solution to part (b):  $\beta = \frac{\Delta x}{\Delta y} = \frac{1.0 \text{ cm}}{0.5 \text{ cm}} = 2.0$ . Using  $T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + \beta^2 \cdot (T_{i,j+1} + T_{i,j-1})}{2 \cdot (1 + \beta^2)}$ ,

with  $i = 3$  and  $j = 4$ , we have:

$$T_{3,4} = \frac{T_{4,4} + T_{2,4} + 2^2 \cdot (T_{3,5} + T_{3,3})}{2 \cdot (1 + 2^2)} = \frac{27 + 25 + 4 \cdot (26 + 28)}{10} = 26.8$$

[3]. Referring to the figure of problem [2], with  $L = 5$  cm,  $H = 3$  cm,  $\Delta x = 1$  cm, and  $\Delta y = 0.5$  cm:

(a) Suppose that the right boundary of the domain shown above is insulated, so that  $\frac{\partial T}{\partial x} = 0$  at  $i = 5$ .

Replacing this derivative by a finite difference of the form  $\frac{\partial T}{\partial x} = \frac{T_{i+1,j} - T_{i-1,j}}{2 \cdot \Delta x}$ ,  $i = 5$ , and using an imaginary point to the right of the boundary (i.e.,  $i = 6$ ), calculate the value of  $T_{6,j}$ . Then, combine this result with the result you found in (a), using  $i = 5$ , to obtain an expression for calculating the value of  $T_{5,j}$ , i.e., the value of  $T$  at the right boundary.

Solution part (a): For  $i = 5$ ,

$$\frac{\partial T}{\partial x} = \frac{T_{i+1,j} - T_{i-1,j}}{2 \cdot \Delta x} = \frac{T_{6,j} - T_{4,j}}{2 \cdot \Delta x},$$

at which location  $\frac{\partial T}{\partial x} = 0$ , i.e.,

$$\frac{T_{6,j} - T_{4,j}}{2 \cdot \Delta x} = 0, \text{ and } \boxed{T_{6,j} = T_{4,j}}.$$

Next, using

$$T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + \beta^2 \cdot (T_{i,j+1} + T_{i,j-1})}{2 \cdot (1 + \beta^2)},$$

with  $i = 5$  and any  $j$ , we have:

$$T_{5,j} = \frac{T_{6,j} + T_{4,j} + \beta^2 \cdot (T_{5,j+1} + T_{5,j-1})}{2 \cdot (1 + \beta^2)},$$

but, with  $T_{6,j} = T_{4,j}$ ,

$$\boxed{T_{5,j} = \frac{2 \cdot T_{4,j} + \beta^2 \cdot (T_{5,j+1} + T_{5,j-1})}{2 \cdot (1 + \beta^2)}}$$

(b) Suppose that the top boundary of the domain shown above is insulated, so that  $\frac{\partial T}{\partial y} = 0$  at  $j = 7$ .

Replacing this derivative by a finite difference of the form  $\frac{\partial T}{\partial y} = \frac{T_{i,j+1} - T_{i,j-1}}{2 \cdot \Delta y}$ ,  $j = 7$ , and using an imaginary point located above of the boundary (i.e.,  $j = 8$ ), calculate the value of  $T_{i,7}$ . Then, combine this result with the result you found in (a), using  $i = 7$ , to obtain an expression for calculating the value of  $T_{i,7}$ , i.e., the value of  $T$  at the top boundary.

Solution part (a): For  $j = 7$ ,

$$\frac{\partial T}{\partial y} = \frac{T_{i,j+1} - T_{i,j-1}}{2 \cdot \Delta y} = \frac{T_{i,8} - T_{i,6}}{2 \cdot \Delta y},$$

at which location  $\frac{\partial T}{\partial y} = 0$ , i.e.,

$$\frac{T_{i,8} - T_{i,6}}{2 \cdot \Delta y} = 0, \text{ and } \boxed{T_{i,8} = T_{i,6}}.$$

Next, using

$$T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + \beta^2 \cdot (T_{i,j+1} + T_{i,j-1})}{2 \cdot (1 + \beta^2)},$$

with any  $i$  and  $j = 7$ , we have:

$$T_{i,7} = \frac{T_{i+1,7} + T_{i-1,7} + \beta^2 \cdot (T_{i,8} + T_{i,6})}{2 \cdot (1 + \beta^2)},$$

but, with  $T_{i,8} = T_{i,6}$ ,

$$\boxed{T_{i,7} = \frac{T_{i+1,7} + T_{i-1,7} + 2 \cdot \beta^2 \cdot T_{i,6}}{2 \cdot (1 + \beta^2)}}.$$



[4]. Referring to the figure of problem [2], with  $L = 5$  cm,  $H = 3$  cm,  $\Delta x = 1$  cm, and  $\Delta y = 0.5$  cm. If the values of  $T$  in problem [2] represent temperatures, in  $^{\circ}\text{C}$ , in a plate, the heat fluxes in the  $x$  and  $y$  directions, at point  $(i,j)$  in the solution domain, are calculated using

$$q_x = -k' \cdot \frac{\partial T}{\partial x}, \text{ and } q_y = -k' \cdot \frac{\partial T}{\partial y},$$

respectively, where  $k'$  is the heat conductivity given as heat per unit time per unit length per unit temperature increment.

Suppose that, after the iterative calculations for  $T$  converge to a solution, you have the following values of temperatures:  $T_{2,4} = 25$   $^{\circ}\text{C}$ ,  $T_{4,4} = 27$   $^{\circ}\text{C}$ ,  $T_{3,5} = 26$   $^{\circ}\text{C}$ , and  $T_{3,3} = 28$   $^{\circ}\text{C}$ .

(a) Using a value of  $k' = 0.5$  cal/(s m  $^{\circ}\text{C}$ ), and the finite-difference formula  $\frac{\partial T}{\partial x} = \frac{T_{i+1,j} - T_{i-1,j}}{2 \cdot \Delta x}$ , calculate the  $x$  component of the heat flux,  $q_x$ , at point  $(3,4)$ .

Solution to part (a): Combining  $q_x = -k' \cdot \frac{\partial T}{\partial x}$  with  $\frac{\partial T}{\partial x} = \frac{T_{i+1,j} - T_{i-1,j}}{2 \cdot \Delta x}$ , we have

$$q_x = -k' \cdot \frac{T_{i+1,j} - T_{i-1,j}}{2 \cdot \Delta x}.$$

NOTE:  $\Delta x = 1$  cm = 0.01 m

With  $i = 3$  and  $j = 4$ ,

$$q_x = -k' \cdot \frac{T_{4,4} - T_{2,4}}{2 \cdot \Delta x} = -0.5 \frac{\text{cal}}{\text{s} \cdot \text{m} \cdot \text{C}} \cdot \frac{(27 - 25)\text{C}}{2 \cdot 0.01 \text{ m}} = -50 \frac{\text{cal}}{\text{s} \cdot \text{m}^2}$$

(b) Using the value of  $k'$  given in (a), and the finite-difference formula  $\frac{\partial T}{\partial y} = \frac{T_{i,j+1} - T_{i,j-1}}{2 \cdot \Delta y}$ , calculate the  $y$  component of the heat flux,  $q_y$ , at point (3,4).

Solution to part (b): Combining  $q_y = -k' \cdot \frac{\partial T}{\partial y}$  with  $\frac{\partial T}{\partial y} = \frac{T_{i,j+1} - T_{i,j-1}}{2 \cdot \Delta y}$ , we have

$$q_y = -k' \cdot \frac{T_{i,j+1} - T_{i,j-1}}{2 \cdot \Delta y} .$$

NOTE:  $\Delta y = 0.5 \text{ cm} = 0.005 \text{ m}$ ,  $k' = 0.5 \text{ cal/(s m}^\circ\text{C)}$ , and  $T_{3,5} = 26 \text{ }^\circ\text{C}$ , and  $T_{3,3} = 28 \text{ }^\circ\text{C}$ .

With  $i = 3$  and  $j = 4$ ,

$$q_y = -k' \cdot \frac{T_{3,5} - T_{3,3}}{2 \cdot \Delta y} = -0.5 \frac{\text{cal}}{\text{s} \cdot \text{m} \cdot \text{C}} \cdot \frac{(26 - 28)\text{C}}{2 \cdot 0.005 \text{ m}} = 100 \frac{\text{cal}}{\text{s} \cdot \text{m}^2}$$

(c) Using the results found in (a) and (b), calculate the magnitude  $q$  of the vector flux,  $\mathbf{q} = q_x \mathbf{i} + q_y \mathbf{j}$ , at point (3,4). In this expression for vector  $\mathbf{q}$ ,  $\mathbf{i}$  and  $\mathbf{j}$  represent the unit vectors in the  $x$  and  $y$  directions, respectively.

Solution to part (c):

$$q = \sqrt{q_x^2 + q_y^2} = \sqrt{(-50)^2 + 100^2} = 111.8 \frac{\text{cal}}{\text{s} \cdot \text{m}^2}$$

[5] . In fluid mechanics, a classical problem of unsteady flow in two dimensions is known as the

*suddenly-accelerated flat plate* problem (or *Stokes' first problem*). The problem consists in obtaining a solution to the velocity  $u(y,t)$  above an infinitely large flat plate in contact with a fluid, originally at rest, located above the plate. The plate coincides with the  $x$  axis and the fluid is located in the domain  $y > 0$ . At time  $t = 0$  the plate starts moving to the right with a constant velocity  $U$ . It turns out that the governing equation for this problem is the following second-order partial differential equation:

$$\frac{\partial u}{\partial t} = \nu \cdot \frac{\partial^2 u}{\partial y^2}$$

where  $\nu$  (the Greek letter  $\nu$ ) represents a property of the fluid known as the *kinematic viscosity* (for water at 20°C, for example,  $\nu = 10^{-6}$  m<sup>2</sup>/s).

To solve this problem numerically, we would replace the partial derivatives in the governing equation by the finite-difference formulas:

$$\frac{\partial u}{\partial t} = \frac{u_j^{k+1} - u_j^k}{\Delta t}, \text{ and } \frac{\partial^2 u}{\partial y^2} = \frac{u_{j+1}^k - 2 \cdot u_j^k + u_{j-1}^k}{\Delta y^2},$$

where  $u_j^k = u(y_j, t_k)$  is the flow velocity  $u$  at position  $j$  and time level  $k$ .

(a) An explicit scheme will produce a formula in which the velocity  $u_j^{k+1}$ , at position  $j$  and time level  $k+1$ , is expressed as function of  $u_{j+1}^k$ ,  $u_j^k$ , and  $u_{j-1}^k$ , as well as  $\nu$ ,  $\Delta t$ , and  $\Delta y$ . Obtain an explicit scheme for calculating  $u_j^{k+1}$  for this case.

Solution to part (a):

Replacing  $\frac{\partial u}{\partial t} = \frac{u_j^{k+1} - u_j^k}{\Delta t}$ , and  $\frac{\partial^2 u}{\partial y^2} = \frac{u_{j+1}^k - 2 \cdot u_j^k + u_{j-1}^k}{\Delta y^2}$ , into  $\frac{\partial u}{\partial t} = \nu \cdot \frac{\partial^2 u}{\partial y^2}$ , we get:

$$\frac{u_j^{k+1} - u_j^k}{\Delta t} = \nu \cdot \left( \frac{u_{j+1}^k - 2 \cdot u_j^k + u_{j-1}^k}{\Delta y^2} \right).$$

To solve for  $u_j^{k+1}$ , we have  $\boxed{u_j^{k+1} = u_j^k + \frac{\nu \cdot \Delta t}{\Delta y^2} (u_{j+1}^k - 2 \cdot u_j^k + u_{j-1}^k)}$ .

Alternatively, one can define a new parameter  $\lambda = \frac{\nu \cdot \Delta t}{\Delta y^2}$ , and write the result as:

$$\boxed{u_j^{k+1} = u_j^k + \lambda \cdot (u_{j+1}^k - 2 \cdot u_j^k + u_{j-1}^k)}.$$

(b) Suppose that at time level  $k = 5$ , you have the following values of the velocity at given points in the

numerical calculation grid:  $u_3^5 = 15 \text{ m/s}$ ,  $u_4^5 = 12 \text{ m/s}$ , and  $u_5^5 = 10 \text{ m/s}$ . Using the formula you found in (a), and replacing  $j = 4$  and  $k = 5$ , calculate the velocity  $u_4^6$ . Use  $\nu = 10^{-6} \text{ m}^2/\text{s}$ ,  $\Delta t = 5 \text{ s}$ , and  $\Delta y = 1 \text{ cm}$ .

Solution to part (b): Using  $u_j^{k+1} = u_j^k + \frac{\nu \cdot \Delta t}{\Delta y^2} \cdot (u_{j+1}^k - 2 \cdot u_j^k + u_{j-1}^k)$ , and  $j = 4, k = 5$ , we have

$$u_4^6 = u_4^5 + \frac{\nu \cdot \Delta t}{\Delta y^2} \cdot (u_5^5 - 2 \cdot u_4^5 + u_3^5) = 12 \frac{\text{m}}{\text{s}} + \frac{\left(10^{-6} \frac{\text{m}^2}{\text{s}}\right) \cdot (5 \text{ s})}{(0.01 \text{ m})^2} \cdot \left(10 \frac{\text{m}}{\text{s}} - 2 \cdot (12 \frac{\text{m}}{\text{s}}) + 15 \frac{\text{m}}{\text{s}}\right)$$

$$\boxed{u_4^6 = 12.05 \frac{\text{m}}{\text{s}}}$$

Alternative approach for part (c):

First calculate  $\lambda = \frac{\nu \cdot \Delta t}{\Delta y^2} = \frac{\left(10^{-6} \frac{\text{m}^2}{\text{s}}\right) \cdot (5 \text{ s})}{(0.01 \text{ m})^2} = 0.05$ , and, with  $j = 4, k = 5$ , use

$$u_j^{k+1} = u_j^k + \lambda \cdot (u_{j+1}^k - 2 \cdot u_j^k + u_{j-1}^k),$$

i.e.,

$$u_4^6 = u_4^5 + \lambda \cdot (u_5^5 - 2 \cdot u_4^5 + u_3^5) = 12 \frac{\text{m}}{\text{s}} + 0.05 \cdot \left(10 \frac{\text{m}}{\text{s}} - 2 \cdot (12 \frac{\text{m}}{\text{s}}) + 15 \frac{\text{m}}{\text{s}}\right),$$

$$\boxed{u_4^6 = 12.05 \frac{\text{m}}{\text{s}}}$$



## Appendix C

### Quizzes

## Quiz 2.1

<https://usu.instructure.com/courses/220573/quizz...>[Sp13 ENGR-2450-002](#) → [Quizzes](#) → [Quiz 2.1](#)

## Quiz Details

Quiz Instructions:

Complete all questions in this quiz. You will have 15 minutes, and will only be allowed one attempt.

 Show Question Details**Question****1 pts**

The Newton-Raphson method is which of the following types of method?

- Open
- Closed
- Graphical
- Bracketing

**Question****1 pts**

How many starting points are needed for a bracketing method?

- 0
- 1
- 2
- 3

**Question****1 pts**

Which method typically requires the most function evaluations to find a root?

- Open
- Closed
- Graphical
- Bracketing

## Quiz 2.2

<https://usu.instructure.com/courses/220573/quizz...>[Sp13 ENGR-2450-002](#) → [Quizzes](#) → [Quiz 2.2](#)

## Quiz Details

Quiz Instructions:

Complete all questions in this quiz. You will be allowed only one attempt. This quiz is open-book and open-notes.

 Show Question Details**Question****1 pts**

Which root-finding method requires us to know the derivative of the function?

- Fixed Point Iteration
- Bisection Method
- Inverse Quadratic Interpolation
- Newton-Raphson Method
- Secant Method

**Question****1 pts**

Brent's Method is a hybrid method, and employs all of the following methods EXCEPT

- Bisection Method
- False Position Method
- Secant Method
- Inverse Quadratic Interpolation

**Question****1 pts**

Which of the following methods is the most likely to diverge for a given problem?

- Bisection Method
- Brent's Method
- False Position Method
- Secant Method



## Quiz 2.3

<https://usu.instructure.com/courses/220573/quizz...>[Sp13 ENGR-2450-002](#) → [Quizzes](#) → [Quiz 2.3](#)

## Quiz Details

Quiz Instructions:

This quiz is open book, open notes. Answer all of the questions.

 Show Question Details**Question**

1 pts

The **Secant Method** is which of the following (HINT: Select one from each pair for a total of three responses).

- Linear Interpolation Method
- Quadratic Interpolation Method
- Bracketing Method
- Open Method
- Suitable for Finding Complex (imaginary) Roots
- NOT Suitable for Finding Complex (imaginary) Roots

**Question**

1 pts

Müller's Method is which of the following (HINT: Select one from each pair for a total of three responses).

- Linear Interpolation Method
- Quadratic Interpolation Method
- Bracketing Method
- Open Method
- Suitable for Finding Complex (imaginary) Roots
- NOT Suitable for Finding Complex (imaginary) Roots

**Question**

1 pts

Inverse Quadratic Interpolation is which of the following (HINT: Select one from each pair for a total of three responses).

- Linear Interpolation Method
- Quadratic Interpolation Method
- Open Method
- Bracketing Method
- Suitable for Finding Complex (imaginary) Roots
- NOT Suitable for Finding Complex (imaginary) Roots

Quiz 2.3

<https://usu.instructure.com/courses/220573/quizz...>**Question****1 pts**

Polynomial deflation is an important concept underlying the function of which of the following methods?

- Newton-Raphson Method
- Brent's Method
- Bairstow's Method
- Müller's Method

2 of 2

09/26/2013 02:05 PM

## Quiz 3.1

<https://usu.instructure.com/courses/220573/quizz...>[Sp13 ENGR-2450-002](#) → [Quizzes](#) → [Quiz 3.1](#)

## Quiz Details

Quiz Instructions:

You are allowed one attempt to take this quiz. There is no time limit. It is open book, open notes, open calculator, but closed friend.

 Show Question Details

## Question

1 pts

What type of matrix would be used in conjunction with multiplication to change the order of the rows of another matrix?

- A Permutation Matrix
- A Vector
- A Tri-Diagonal Matrix
- A Symmetric Matrix
- The Zero Matrix
- An Upper-Triangular Matrix
- The Matrix Inverse

## Question

1 pts

What type of multiplication would be used in conjunction with the answer to the previous question to produce a change the order of the columns?

- Left Multiplication
- Right Multiplication
- Elementwise Multiplication
- The Cross-Product

## Question

1 pts

What type of matrix would be used in conjunction with multiplication to produce the identity matrix as a product? (i.e.,  $A^{-1}A=I$ )

- The Matrix Inverse
- A Permutation Matrix
- A Vector
- A Lower-Triangular Matrix
- A Banded Matrix
- A Symmetric Matrix
- The Identity Matrix

## Quiz 3.1

<https://usu.instructure.com/courses/220573/quizz...>**Question**

1 pts

Given that:

- $C = A \cdot B$ ,
- $\dim()$  denotes the dimensions of a matrix,
- $\dim(A) = 3 \times 7$ ,
- $\dim(B) = 7 \times 4$ ,

What is  $\dim(C)$ ?

- 3x4
- 7x4
- 4x3
- 3x7
- A and B cannot be multiplied.

**Question**

1 pts

What type of operation would be used to translate an upper triangular matrix into a lower-triangular matrix?

- Inverse
- Transpose
- Product
- Subtraction

## Quiz 3.2

<https://usu.instructure.com/courses/220573/quizz...>
[Sp13 ENGR-2450-002](#) → [Quizzes](#) → [Quiz 3.2](#)

## Quiz Details

Quiz Instructions:

This quiz is open book, open note, open computer/calculator, closed neighbor, open time limit. It closes before class on Monday. I don't know if partial credit will be given on the multiple blank question, but it will not be re-graded.

 Show Question Details

## Question

1 pts

Which method uses determinants to solve a linear system of equations?

- Cramer's Rule
- Naive Gauss Elimination
- Gauss-Jordan Elimination
- Gauss Elimination with Pivoting

## Question

1 pts

Which of the following methods for solving a system of linear equations employs a forward elimination step which results in a triangular matrix?

- Gauss Elimination
- Gauss-Jordan Elimination
- Cramer's Rule
- Elimination of Unknowns

## Question

1 pts

Which of the following methods for solving a system of linear equations employs a forward elimination step which results in an identity matrix?

- Gauss-Jordan Elimination
- Gauss Elimination
- Elimination of Unknowns
- Cramer's Rule

## Question

1 pts

Stability is a property of a(n) [option1] while condition is a property of the [option2]. (HINT: Don't use the same answer twice.)

Show Answers for 

algorithm

## Quiz 3.3

<https://usu.instructure.com/courses/220573/quizz...>

Sp13 ENGR-2450-002 → Quizzes → Quiz 3.3

## Quiz Details

## Quiz Instructions:

This quiz should be easy...I had a hard time coming up with hard questions in m/c format, so you get easy ones. There is no time limit. This quiz is open-book, open-note, open computer/internet, but closed friend.

 Show Question Details

## Question

1 pts

Is LU decomposition related to Gauss Elimination? If so, how?

- Yes. The decomposition step of LU decomposition (step 1) is essentially the same as the elimination step in Gauss Elimination.
- No. It is not related.

## Question

1 pts

Gauss elimination, Modified Gauss Elimination, LU decomposition, and Cramer's method all classify as which of the following types of methods. (HINT: Please think about this first and answer it to yourself, but once you have decided, read p. 300 of Chapra & Canale to make sure.)

- Direct Methods
- Iterative Methods

## Question

1 pts

Matrix decompositions and factorizations (such as LU decomposition) are one of the key concepts in all of linear algebra.

- True
- False

## Question

1 pts

What is LU decomposition useful for?

- Computing the inverse of a matrix.
- Computing the determinant of a matrix
- Solving a linear system of equations.
- ALL of the other answers are correct..

## Question

1 pts

$\text{Cond}(A) = 10000000$ .  $\text{Cond}(M) = 1$ . Which of the following is true.

Quiz 3.3

<https://usu.instructure.com/courses/220573/quizz...>

A is ill-conditioned.

M is ill-conditioned.

## Quiz 5.1

<https://usu.instructure.com/courses/220573/quizz...>

Sp13 ENGR-2450-002 → Quizzes → Quiz 5.1

## Quiz Details

Quiz Instructions:

There is no time limit for this quiz. It is open book, open notes, and open computer, but closed friend.

 Show Question Details

## Question

1 pts

In the method of Least-Squares regression, what criteria is minimized to determine the "best" fit line?

- The sum of the absolute values of the error.
- The maximum error of any point.
- The sum of the squared error.
- The sum of the residuals.

## Question

1 pts

With an appropriate transformation, which of the following can be solved using least squares regression?

- $y = \alpha \cdot \exp(\beta \cdot x)$ .
- $y = \alpha \cdot x^\beta$ .
- $y = \alpha \cdot (x / (\beta + x))$ .
- all of the other answers are correct.

## Question

1 pts

The standard deviation is the most common measure of spread for a sample. Which of the following represents the most direct extension of the concept of standard deviation to regression lines?

- The standard error of the estimate ( $S_{y/x}$ ).
- The coefficient of determination ( $r^2$ ).
- The total sum of squares ( $S_t$ ).
- The residual sum of squares ( $S_r$ ).

## Question

1 pts

Which of the following is a measure of the error reduction due to describing the data in terms of a straight line (fitting the data with  $y = mx + b$ ) rather than as an average value (fitting the data with a constant,  $y = \mu$ )?

- The standard error of the estimate ( $S_{y/x}$ ).
- The coefficient of determination ( $r^2$ ).



## Quiz 5.1

<https://usu.instructure.com/courses/220573/quizz...>

- The total sum of squares ( $S_t$ ).
- The residual sum of squares ( $S_r$ ).

**Question****1 pts**

Given  $n \geq 3$   $(x, y)$  pairs of data points, you are required to determine the line representing a linear fit for the given data. The line chosen may or may not pass through all the data points given. How many unique lines can be drawn or specified that "fit" the data?

- $n-1$
- infinite
- none
- 1

## Quiz 5.2

<https://usu.instructure.com/courses/220573/quizz...>
[Sp13 ENGR-2450-002](#) → [Quizzes](#) → [Quiz 5.2](#)

## Quiz Details

Quiz Instructions:

There is no time limit for this quiz. It is open book, open notes, and open computer, but closed friend.

 Show Question Details

## Question

1 pts

Of the following, which is the WORST method (in terms of likely numerical error) for polynomial interpolation?

- Solving for the coefficients  $a_0, a_1, \dots, a_n$  directly by inverting the Vandermonde matrix, then calculating intermediate values with the resulting polynomial.
- Solving for the intermediate values using Newton's divided difference interpolating polynomials.
- Solving for the intermediate values using Lagrange interpolating polynomials.

## Question

1 pts

For exploratory computations when the order of the polynomial is unknown, one of the following polynomial interpolation methods presented is particularly well-suited. Which one is it?

- Newton's Divided-Difference Interpolating Polynomials.
- Lagrange Interpolating Polynomials.
- Directly Solving for the Polynomial Coefficients  $a_1, a_2, \dots, a_n$ .

## Question

1 pts

The Data points  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$  are given. For conducting quadratic spline interpolation the x-data needs to be:

- Equally spaced
- In ascending or descending order
- Positive
- Integers

## Question

1 pts

In cubic spline interpolation,

- The first derivatives are continuous at the interior knots.
- The second derivatives are continuous at the interior knots.
- The first and second derivatives are continuous at the interior knots.
- The third derivatives are continuous at the interior knots.

## Quiz 5.2

<https://usu.instructure.com/courses/220573/quizz...>

Question	1 pts
For an interpolating polynomial of degree $n$ , the Newton's Divided Difference Polynomial and the Lagrange Interpolating Polynomial are analytically equivalent.	
<input type="radio"/> True	
<input type="radio"/> False	

## Quiz 5.3

<https://usu.instructure.com/courses/220573/quizz...>[Sp13 ENGR-2450-002](#) → [Quizzes](#) → [Quiz 5.3](#)

## Quiz Details

Quiz Instructions:

This quiz is open book, open notes, closed friend, open time limit.

 Show Question Details**Question****1 pts**

Which of the following most closely represents how the discrete Fourier transform works?

- Performs least-squares regression, then computes the Continuous Fourier Transform on the result.
- Performs cubic spline interpolation, then computes the Continuous Fourier Transform on the result.
- Performs the Fourier Transform directly with the discrete data, using discrete Fourier Transform equations analogous to the continuous ones.
- Performs eigenvalue decomposition on the coefficient matrix of the normal equations to identify the dominant frequencies.

**Question****1 pts**

Which of the following algorithms is more efficient for computing the Fourier transform of a discrete signal?

- The Discrete Fourier Transform algorithm in Figure 9.12 of Chapra & Canale.
- The Discrete Fourier Transform algorithm in Figure 9.18 of Chapra & Canale (decimation-in-frequency FFT Algorithm).

**Question****1 pts**

All of the following are closely associated with the Fast-Fourier Transform EXCEPT:

- Butterfly Networks
- Bit Reversal
- Decimation-in-time
- Squared Residual Error
- Decimation-in-frequency

**Question****1 pts**

Quiz 5.3

<https://usu.instructure.com/courses/220573/quizz...>

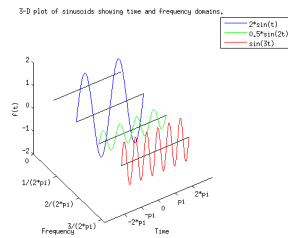
Least squares regression can be performed to fit a continuous function to discrete data, using your choice of basis functions. If the data appears to follow a periodic, regularly repeating pattern, which of the following basis functions would be most appropriate?

- The quadratic polynomial basis  $[1, t, t^2]$
- The trigonometric basis  $[1, \cos(t), \sin(t)]$
- The quadratic Lagrange polynomial basis  $\left[ \left( \frac{t-t_1}{t_0-t_1} \right) \left( \frac{t-t_2}{t_0-t_2} \right), \left( \frac{t-t_0}{t_1-t_0} \right) \left( \frac{t-t_2}{t_1-t_2} \right), \left( \frac{t-t_0}{t_2-t_0} \right) \left( \frac{t-t_1}{t_2-t_1} \right) \right]$
- The quadratic Newton polynomial basis  $[1, (t-t_0), (t-t_0)(t-t_1)]$ .

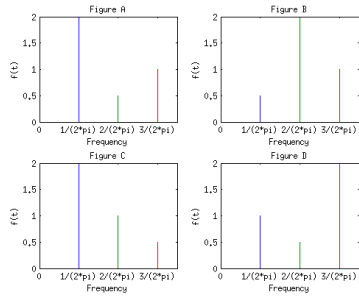
Question

1 pts

The 3-D plot for three functions:  $2 \cdot \sin(t)$ ,  $0.5 \cdot \sin(2 \cdot t)$ ,  $\sin(3t)$  is given by:



Which of the following figures shows the correct frequency-domain representation of these three functions?



- Figure A
- Figure B
- Figure C
- Figure D

## Quiz 6.1

<https://usu.instructure.com/courses/220573/quizz...>

Sp13 ENGR-2450-002 → Quizzes → Quiz 6.1

## Quiz Details

Quiz Instructions:

Open book, open notes, open internet, closed friend. No time limit. One attempt.

 Show Question Details

## Question

1 pts

For which of the following are the Newton-Cotes integration formulas applicable?

- Tabulated Data Values
- A Known Function
- None of the other answers
- Both tabulated data and known functions.

## Question

1 pts

The Newton-Cotes formula in which a cubic polynomial is fit to four data points, and then integrated is known as:

- Trapezoid Rule
- Simpson's 1/3 Rule
- Simpson's 3/8 Rule
- Boole's Rule

## Question

1 pts

Which of the following Newton-Cotes integration methods can always be used, even when data is not evenly spaced?

- Multiple-Application Trapezoid Rule
- Multiple-Application Simpson's 1/3 Rule
- Multiple-Application Simpson's 3/8 Rule
- Multiple-Application Boole's Rule

## Question

1 pts

Which of the following methods is more commonly used for definite integration?

- Newton-Cotes Open Integration Formulas
- Newton-Cotes Closed Integration Formulas

## Quiz 6.1

<https://usu.instructure.com/courses/220573/quizz...>

Question	1 pts
For unevenly spaced data, it is sometimes still possible (and useful) to combine Simpson's 1/3 rule, Simpson's 3/8 rule, and the trapezoid rule.	
<input type="radio"/> True	
<input type="radio"/> False	

Question	1 pts
When should Simpson's 3/8 Rule be used instead of Simpson's 1/3 rule?	
<input type="radio"/> Always. It is more accurate.	
<input type="radio"/> Never. It is less accurate.	
<input type="radio"/> When the number of data points is even. Accuracy is similar to Simpson's 1/3 rule.	

## Quiz 6.2

<https://usu.instructure.com/courses/220573/quizz...>

Sp13 ENGR-2450-002 → Quizzes → Quiz 6.2

## Quiz Details

Quiz Instructions:

Open book, open time limit, closed friend, open computer.

 Show Question Details

## Question

1 pts

Which of the following methods/algorithms automatically detects where a function changes rapidly and adjusts the step-size used in the integration only for that region to improve overall accuracy while minimizing function evaluations?

- Adaptive Quadrature
- Romberg Integration
- Improper Integration
- Gauss Quadrature

## Question

1 pts

Which of the following methods uses successive refinement of the step size over the entire integration interval to improve the estimate of the integral?

- Adaptive Quadrature
- Romberg Integration
- Gauss Quadrature
- Improper Integration

## Question

1 pts

Which of the following methods is used to deal with limits of integration that include infinity?

- Adaptive Quadrature
- Romberg Integration
- Improper Integration
- Gauss Quadrature

## Question

1 pts

Which of the following methods uses the method of undetermined coefficients to derive integration formulas that are highly accurate (e.g., a two-point integration formula that is exact for cubics)?

- Adaptive Quadrature
- Romberg Integration



Quiz 6.2

<https://usu.instructure.com/courses/220573/quizz...>

Improper Integration

Gauss Quadrature

## Quiz 6.3

<https://usu.instructure.com/courses/220573/quizz...>
[Sp13 ENGR-2450-002](#) → [Quizzes](#) → [Quiz 6.3](#)

## Quiz Details

Quiz Instructions:

This quiz is open book, open notes, open computer and internet but closed friend. You will be allowed one attempt.

 Show Question Details

## Question

1 pts

Suppose you have three data points representing the value of a function  $f(x)$  at the points  $x$ : $x$   $f(x)$ 

1 0.5

2 0.8

3 1.2

You want to approximate  $f'(x)$  (the first derivative of the function) at each point. Which of the following methods would be best suited to approximate the derivative at  $x = 1$ ?

- The Forward Finite-Divided-Difference Formula
- The Backward Finite-Divided-Difference Formula
- The Centered Finite-Divided-Difference Formula
- The Infinite-Divided-Difference Formula

## Question

1 pts

Suppose you have three data points representing the value of a function  $f(x)$  at the points  $x$ : $x$   $f(x)$ 

1 0.5

2 0.8

3 1.2

You want to approximate  $f'(x)$  (the first derivative of the function) at each point. Which of the following methods would be best suited to approximate the derivative at  $x = 2$ ?

- The Forward Finite-Divided-Difference Formula
- The Backward Finite-Divided-Difference Formula
- The Centered Finite-Divided-Difference Formula
- The Infinite-Divided-Difference Formula

## Question

1 pts

What tends to happen when data with small errors is differentiated numerically (as compared to what happens when the same data without errors is differentiated numerically)?

- Small errors are attenuated by the differentiation.
- Small errors are amplified by the differentiation.
- Small errors remain constant throughout the differentiation process.
- Small errors tend to cancel each other out.

## Quiz 6.3

<https://usu.instructure.com/courses/220573/quizz...>

Question	1 pts
All of the following methods are suitable for unequally spaced data EXCEPT:	
<input type="radio"/> Using the Centered Finite-Divided-Difference Formula to approximate the derivative.	
<input type="radio"/> Fitting the Lagrange Interpolating Polynomial, then taking the derivative of the result.	
<input type="radio"/> Performing Linear Regression, then taking the derivative of the result.	
<input type="radio"/> Performing Spline Interpolation, then taking the derivative of the result.	

Question	1 pts
Richardson's Extrapolation formulas can be used to refine derivative estimates using successively finer sampling grids, just as they were with integration.	
<input type="radio"/> True	
<input type="radio"/> False	

## Quiz 7.2

<https://usu.instructure.com/courses/220573/quizz...>

Sp13 ENGR-2450-002 → Quizzes → Quiz 7.2

## Quiz Details

Quiz Instructions:

Open book, open notes, open internet and computer, closed friend.

 Show Question Details

## Question

1 pts

How is the error estimated when using adaptive Runge-Kutta ODE solver algorithms?

- I. Using the Taylor Series expansion
- II. Using multiple approximations with different step sizes
- III. Using multiple approximations with different orders of methods
- IV. Using the true value from the analytical solution

 I only I and II II only II and III III only III and IV IV only

## Question

1 pts

Of the following explicit Runge-Kutta methods, which typically provides the most accurate estimates (measured in percent relative error)?

 Euler's Method Heun's Method 3rd-Order Runge-Kutta 4th-Order Runge-Kutta Butcher's 5th-Order RK Method

## Question

1 pts

Explicit Runge-Kutta Methods can be used to solve which of the following?

 First-Order ODEs Systems of coupled first-order ODEs Higher-order diff. eq's (after transformation to a coupled first-order system of ODEs) All of the other answers are correct.

## Quiz 7.2

<https://usu.instructure.com/courses/220573/quizz...>

## Question

1 pts

The evaluation of the definite integral  $\int_a^b f(x) dx$  is equivalent to solving the differential equation  $\frac{dy}{dx} = f(x)$  for  $y(b)$  given the initial condition  $y(a) = 0$ .

- True
- False

## Question

1 pts

What is a stiff ODE (or system of ODEs)?

- An ODE that is nonlinear in its parameters.
- An ODE that has both rapidly and slowly changing components to the solution.
- An ODE that has a steadily changing solution.
- A ballad of praise to a dead man.

## Quiz 7.3

<https://usu.instructure.com/courses/220573/quizz...>

Sp13 ENGR-2450-002 → Quizzes → Quiz 7.3

## Quiz Details

Quiz Instructions:

This quiz is not timed, and is open book, open notes, open computer and internet, but closed friend.

 Show Question Details

## Question

1 pts

Which of the following terms is used for boundary conditions in which only values of the function's derivatives are given?

- Dirichlet Boundary Conditions
- Neumann Boundary Conditions
- Robin Boundary Conditions
- Cauchy Boundary Conditions

## Question

1 pts

Which of the following is an initial value problem?

- $\frac{dy}{dx} = -ky, y(100) = 0.01$ .
- $\frac{d^2T}{dx^2} + h(T_a - T) = 0, T(0) = a, T(L) = b$
- $\frac{dy_1}{dt} = f_1(t, y_1, y_2); \frac{dy_2}{dt} = f_2(t, y_1, y_2); \text{at } t = 0, y_1 = a, \text{at } t = 0, y_2 = b$
- $\frac{d^2y}{dt^2} = f(x, y); \text{at } x = 0, y = a, \text{at } x = L, y = b$

## Question

1 pts

Which of the following problems is most amenable to solving with the finite difference method?

- $\frac{dy}{dx} = xy + 3y - x, y(b) = a$
- $\frac{dy}{dx} = y^2 + 3y - x, y(b) = a$
- $\frac{dy}{dx} = x^2 + 3y - x, y(b) = a$

## Question

1 pts

Which of the following methods approximates the eigenvalues of a system by solving for the roots of the characteristic equation?

- The Power Method
- The Lanczos Method

## Quiz 7.3

<https://usu.instructure.com/courses/220573/quizz...> The Polynomial Method The Arnoldi Method**Question****1 pts**

How does the shooting method deal with the problem of not having initial values?

 Select this answer once you have thought through the question, and understand the answer.

The shooting method guesses an initial value, then compares what it gets at the boundary value to the given point at the boundary. This is repeated. Linear or quadratic interpolation is used to refine the initial value guess based on the results from previous 'shots'.

 Pick this to get the question incorrect. Only do this if you do not know and are unwilling to look up the answer.

The shooting method guesses an initial value, then compares what it gets at the boundary value to the given point at the boundary. This is repeated. Linear or quadratic interpolation is used to refine the initial value guess based on the results from previous 'shots'.

## Quiz 8.1

<https://usu.instructure.com/courses/220573/quizz...>

Sp13 ENGR-2450-002 → Quizzes → Quiz 8.1

## Quiz Details

Quiz Instructions:

Open book, open notes, open computer and internet, closed friend. Select the best answer.

 Show Question Details

## Question

1 pts

What is the main difference between an ordinary differential equation and a partial differential equation?

- An ODE must have at least two independent variables. A PDE may have only one.
- A PDE must have at least two independent variables. An ODE may have only one.
- A PDE must have at least two dependent variables. An ODE may have only one.
- An ODE must have at least two dependent variables. A PDE may have only one.

## Question

1 pts

Which of the following PDEs does not fall into any of the categories: Elliptic, Parabolic, or Hyperbolic? (As an exercise, I suggest that you classify each of them).

- $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = f(x, y)$
- $\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \phi^2} = 0$
- $\frac{\partial u}{\partial x} = 0$
- $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = v \frac{\partial^2 u}{\partial x^2}$

## Question

1 pts

All of the following are true about the finite difference method as applied to elliptic PDEs EXCEPT:

- It employs finite difference approximations to the derivatives.
- Neumann boundary conditions cannot be accommodated for this type of problem.
- Secondary variables may be found after solving the original PDE.
- The Liebman Method (Gauss-Seidel iteration) is the most commonly used approach for solving the resulting system of equations.

## Question

1 pts

Select the appropriate classification for the PDE given by:  $f(x, y) \frac{\partial u}{\partial x} + g(x, y) \frac{\partial u}{\partial y} + h(x, y) u = 0$ 

- Elliptic



## Quiz 8.1

<https://usu.instructure.com/courses/220573/quizz...> Parabolic Hyperbolic Nonlinear**Question****1 pts**

Most important partial differential equations have known analytical solutions.

 True False

## Quiz 8.2

<https://usu.instructure.com/courses/220573/quizz...>
[Sp13 ENGR-2450-002](#) → [Quizzes](#) → [Quiz 8.2](#)

## Quiz Details

Quiz Instructions:

Open book, open notes, open internet and computer, closed friend. Select the best response of those provided.

 Show Question Details**Question****1 pts**

Irregular shaped boundaries and non-constant coefficients can introduce complications in the finite difference method for solving elliptic PDEs. Chapra and Canale mention all of the following as methods used to deal with these issues EXCEPT:

- Decrease the step size and pretend the points lie on the boundary.
- Use weighting coefficients to derive a general finite difference relation.
- Use the control-volume (finite volume method) instead.
- Use the finite difference method with adaptive step sizes.

**Question****1 pts**

According to Chapra and Canale, which of the following methods would be most appropriate for solving the following PDE?

$$\frac{\partial T}{\partial t} = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

- MacCormack's Method
- The Crank-Nicolson Method
- The Alternating-Direction Implicit Method
- The Simple Implicit Method

**Question****1 pts**

What is the main drawback of using EXPLICIT finite divided difference methods for solving parabolic PDEs?

- Restrictions on the step size (related to convergence).
- The need to solve a dense system of linear equations.
- The high number of computations necessary at each step.
- The difficulty in accomodating Neumann (derivative) boundary conditions.

**Question****1 pts**

What is the main drawback of using IMPLICIT finite divided difference methods for solving parabolic PDEs?

- Restrictions on the step size (related to convergence).
- Inability to generate solutions for secondary variables.

## Quiz 8.2

<https://usu.instructure.com/courses/220573/quizz...>

- The high number of computations necessary at each step.
- The difficulty in accommodating Neumann (derivative) boundary conditions.

**Question****1 pts**

Which of the following PDE types is used to characterize time-variable problems?

- Elliptic
- Parabolic

## Quiz 8.3

<https://usu.instructure.com/courses/220573/quizz...>

Sp13 ENGR-2450-002 → Quizzes → Quiz 8.3

## Quiz Details

Quiz Instructions:

Open book, open notes, open internet and computer, closed friend. Select the best answer.

 Show Question Details

## Question

1 pts

The finite element method requires a mesh with equally spaced nodes.

- True
- False

## Question

1 pts

The finite element method is superior to the finite difference method for problems involving complex or irregular geometry.

- True
- False

## Question

1 pts

The method of weighted residuals (used in developing the element equations) is the most conceptually similar to which of the following?

- Root Finding
- Interpolation
- Regression
- Newton-Cotes Integration

## Question

1 pts

A right trapezoid can be represented as the sum of two right triangles.

- True
- False

## Question

1 pts

What were the benefits of performing integration by parts in the Galerkin method?

- I - It reduces the highest-order derivative.

## Quiz 8.3

<https://usu.instructure.com/courses/220573/quizz...> II - It incorporatest he boundary conditions. III - Both I and II None of the other answers is correct.

**Appendix D**  
**Course Schedule**

Table D.1: Course Schedule for ENGR 2450 Spring 2013, Comparison Section

Day/Date	In-class activities	Outside activities			
		Videos	HW	Quiz	Reading
m 7-Jan	Lecture 1.1	-	HW1		Part 1
w 9-Jan	Lecture 1.2	-	HW1	-	Part 1
f 11-Jan	Lecture 1.3	-	HW1	-	Part 1
m 14-Jan	Lecture 1.4	-	HW1	-	Part 1
w 16-Jan					Part 1
f 18-Jan	CT-Pre				
m 21-Jan	-				
w 23-Jan	Lecture 2.1	-	HW2		Part 2
f 25-Jan	Lecture 2.2	-	HW2	2.1	Part 2
m 28-Jan	Lecture 2.3	-	HW2	2.2	Part 2
w 30-Jan	Lecture 2.4	-	HW2	2.3	Part 2
f 1-Feb	P2 Worked Examples				
m 4-Feb	P2 Exam				
w 6-Feb	Lecture 3.1	-	HW3		Part 3
f 8-Feb	Lecture 3.2	-	HW3	3.1	Part 3
m 11-Feb	Lecture 3.3	-	HW3	3.2	Part 3
w 13-Feb	Lecture 3.4	-	HW3	3.3	Part 3
f 15-Feb	Worked Examples				
t 19-Feb	P3 Exam				
w 20-Feb	Lecture 5.1	-	HW5		Part 5
f 22-Feb	Lecture 5.2	-	HW5	5.1	Part 5
m 25-Feb	Lecture 5.3	-	HW5	5.2	Part 5
w 27-Feb	Lecture 5.4	-	HW5	5.3	Part 5
f 1-Mar	Worked Examples				
m 4-Mar	P5 Exam				
w 6-Mar	Lecture 6.1	-	HW6		Part 6
f 8-Mar	Lecture 6.2	-	HW6	6.1	Part 6
m 18-Mar	Lecture 6.3	-	HW6	6.2	Part 6
w 20-Mar	Lecture 6.4	-	HW6	6.3	Part 6
f 22-Mar	Worked Examples				
m 25-Mar	P6 Exam				
w 27-Mar	Lecture 7.1	-	HW7		Part 7
f 29-Mar	Lecture 7.2	-	HW7	7.1	Part 7
m 1-Apr	Lecture 7.3	-	HW7	7.2	Part 7
w 3-Apr	Lecture 7.4	-	HW7	7.3	Part 7
f 5-Apr	Worked Examples				
m 8-Apr	P7 Exam				
w 10-Apr	Lecture 8.1	-	HW8		Part 8
f 12-Apr	Lecture 8.2	-	HW8	8.1	Part 8
m 15-Apr	Lecture 8.3	-	HW8	8.2	Part 8
w 17-Apr	Lecture 8.4	-	HW8	8.3	Part 8
f 19-Apr	Worked Examples				
m 22-Apr	P8 Exam				1-3, 5-8
w 24-Apr	Worked Examples				1-3, 5-8
m 29-Apr	CT-Post, Final Exam				

Table D.2: Course Schedule for ENGR 2450 Spring 2013, Treatment Section

Day/Date	In-class activities	Outside activities			
		Videos	HW	Quiz	Reading
m 7-Jan	Lecture 1.1	-	HW1		Part 1
w 9-Jan	Lecture 1.2	-	HW1	-	Part 1
f 11-Jan	Lecture 1.3	-	HW1	-	Part 1
m 14-Jan	Lecture 1.4	-	HW1	-	Part 1
w 16-Jan					Part 1
f 18-Jan	CT-Pre				
m 21-Jan	-				
w 23-Jan	Video Demo, MEA Overview, Groups	2.1	HW2		Part 2
f 25-Jan	JIT MEA Assignment 1 (INDIV)	2.2	HW2	2.1	Part 2
m 28-Jan	JIT MEA Assignment 2 (TEAM)	2.3	HW2	2.2	Part 2
w 30-Jan	JIT MEA Assignment 2 (TEAM)	2.4	HW2	2.3	Part 2
f 1-Feb	P2 Worked Examples				
m 4-Feb	P2 Exam				
w 6-Feb	JIT MEA Assignment 3 (INDIV)	3.1	HW3		Part 3
f 8-Feb	JIT MEA Assignment 3 (INDIV)	3.2	HW3	3.1	Part 3
m 11-Feb	JIT MEA Assignment 4 (TEAM)	3.3	HW3	3.2	Part 3
w 13-Feb	JIT MEA Assignment 4 (TEAM)	3.4	HW3	3.3	Part 3
f 15-Feb	Worked Examples				
t 19-Feb	P3 Exam				
w 20-Feb	JIT MEA Assignment 5 (INDIV)	5.1	HW5		Part 5
f 22-Feb	JIT MEA Assignment 5 (INDIV)	5.2	HW5	5.1	Part 5
m 25-Feb	JIT MEA Assignment 6 (TEAM)	5.3	HW5	5.2	Part 5
w 27-Feb	JIT MEA Assignment 6 (TEAM)	5.4	HW5	5.3	Part 5
f 1-Mar	Worked Examples				
m 4-Mar	P5 Exam				
w 6-Mar	SD MEA Assignment 1 (INDIV)	6.1	HW6		Part 6
f 8-Mar	SD MEA Assignment 1 (INDIV)	6.2	HW6	6.1	Part 6
m 18-Mar	SD MEA Assignment 2 (TEAM)	6.3	HW6	6.2	Part 6
w 20-Mar	SD MEA Assignment 2 (TEAM)	6.4	HW6	6.3	Part 6
f 22-Mar	Worked Examples				
m 25-Mar	P6 Exam				
w 27-Mar	SD MEA Assignment 3 (INDIV)	7.1	HW7		Part 7
f 29-Mar	SD MEA Assignment 3 (INDIV)	7.2	HW7	7.1	Part 7
m 1-Apr	SD MEA Assignment 4 (TEAM)	7.3	HW7	7.2	Part 7
w 3-Apr	SD MEA Assignment 4 (TEAM)	7.4	HW7	7.3	Part 7
f 5-Apr	Worked Examples				
m 8-Apr	P7 Exam				
w 10-Apr	SD MEA Assignment 5 (INDIV)	8.1	HW8		Part 8
f 12-Apr	SD MEA Assignment 5 (INDIV)	8.2	HW8	8.1	Part 8
m 15-Apr	SD MEA Assignment 6 (TEAM)	8.3	HW8	8.2	Part 8
w 17-Apr	SD MEA Assignment 6 (TEAM)	8.4	HW8	8.3	Part 8
f 19-Apr	Worked Examples				
m 22-Apr	P8 Exam				1-3, 5-8
w 24-Apr	Worked Examples				1-3, 5-8
m 29-Apr	CT-Post, Final Exam				





## **Appendix E**

### **Video Lectures**

Video lectures were recorded for each section of material, corresponding to the parts of Chapra and Canale (2009) covered in this course. This includes parts 1, 2, 3, 5, 7, and 8. The following tables contain the title of each video, organized into units. One unit contains all the videos students were assigned to watch for a single day of lecture, as outlined in the course schedule shown in Appendix D. The title of each video is hyperlinked to the corresponding youtube video URL.

Table E.1: Video Lectures for Part 1, Modeling and Error

Part 1	Index	Video title	Length
Unit 1	1.1.1	Introduction: Numerical vs Analytical Methods	6:05
	1.1.2	Introduction: Chapra & Canale Textbook Overview	2:11
	1.1.3	Introduction: Mathematical Modeling	5:31
	1.1.4	Introduction: Tradeoffs in Mathematical Modeling	10:23
	1.1.5	Introduction: Error Analysis	8:26
Unit 2	1.2.1	Modeling & Error: Formal Error Definitions	6:21
	1.2.2	Modeling & Error: Iterative Refinement	10:17
	1.2.3	Modeling & Error: Integer Number Representation	10:59
	1.2.4	Modeling & Error: Fixed Point Number Representation	2:13
	1.2.5	Modeling & Error: Floating Point Numbers	12:18
	1.2.6	Modeling & Error: Floating Point Numbers	7:55
	1.2.7	Modeling & Error: Normalization	4:10
Unit 3	1.3.2	Modeling & Error: Arithmetic Manipulation	5:04
	1.3.3	Modeling & Error: Arithmetic Manipulation	2:27
	1.3.4	Modeling & Error: Multiplication and Division	2:21
	1.3.5	Modeling & Error: Inner Products	3:29
	1.3.5	Modeling & Error: Examples Interdependent Operations	2:54
Unit 4	1.4.1	Modeling & Error: Subtractive Cancellation	9:52
	1.4.1	Modeling & Error: Stability and Condition	6:23
	1.4.2	Modeling & Error: Taylor Series Intro	4:07
	1.4.3	Modeling & Error: Taylor Series	3:10
	1.4.4	Modeling & Error: Taylor series Example	10:00
	1.4.5	Modeling & Error: Truncation Error	11:40

Table E.2: Video Lectures for Part 2, Roots of Equations

Part 2	Index	Video title	Length
Unit 1	2.1.1	Introduction: and Bisection Method	3:48
	2.1.2	Roots: False Position Method	3:40
	2.1.3	Roots: Fixed Point Iteration	9:32
	2.1.4	Roots: Newton-Raphson Method	2:59
	2.1.5	Roots: Secant Method	3:03
Unit 2	2.2.1	Roots: Graphical Methods	5:28
	2.2.2	Roots: Incremental Search	4:40
	2.2.3	Roots: Inverse Quadratic Interpolation	5:37
	2.2.4	Roots: Comparison of Methods	13:05
	2.2.5	Roots: Brent's (Hybrid) Method	5:39
Unit 3	2.3.0	Roots: Lectures 2.3.X Overview	4:11
	2.3.1	Roots: Solving Systems of Nonlinear Equations	3:12
	2.3.2	Roots: Nonlinear Systems Fixed Point Iteration	13:12
	2.3.3	Roots: Challenges of Equations with Multiple Roots	5:11
	2.3.4	Roots: Modified Newton Raphson Multi-Roots	11:07
	2.3.5	Roots: Eigenvalue Problems Application	5:56
	2.3.6	Roots: Muller's Method	10:26
	2.3.7	Roots: Polynomial Deflation	5:57
Unit 4	2.3.8	Roots: Bairstow's Method	4:12
	2.4.1	Roots: Example 1 Part A	8:33
	2.4.2	Roots: Example 1 Part B	12:44
	2.4.3	Roots: Example 1 Part C	8:37
	2.4.4	Roots: Example 2	7:35
	2.4.5	Roots: Example 3	33:02

Table E.3: Video Lectures for Part 3, Linear Algebra

Part 3	Index	Video title	Length
Unit 1	3.1.01	Linear Algebra: Matrix Notation	3:08
	3.1.02	Linear Algebra: Note on M and N	1:24
	3.1.03	Linear Algebra: Square Matrix	1:59
	3.1.04	Linear Algebra: Vector	3:42
	3.1.05	Linear Algebra: Diagonal and Identity	1:42
	3.1.06	Linear Algebra: Banded Matrix, Tri-Diagonal Matrix	2:03
	3.1.07	Linear Algebra: Triangular Matrix	2:04
	3.1.08	Linear Algebra: Matrix Operations Introduction	0:45
	3.1.09	Linear Algebra: Matrix Addition and Subtraction	3:41
	3.1.10	Linear Algebra: Multiplication Possible Dimensions	2:37
	3.1.11	Linear Algebra: Multiplication	7:48
	3.1.12	Linear Algebra: Matrix Transpose	2:33
	3.1.13	Linear Algebra: Matrix Inverse	2:09
	3.1.14	Linear Algebra: Adjoint Operator	3:06
	3.1.15	Linear Algebra: Self Adjoint and Symmetric Matrices	1:56
	3.1.16	Linear Algebra: Factorization	3:58
	3.1.17	Linear Algebra: Permutation Matrix	3:26
	3.1.18	Linear Algebra: Permutation Matrix Left vs Right Multiplication	0:58
	3.1.19	Linear Algebra: Elementwise Operations	3:32
Unit 2	3.2.01	Linear Algebra: Determinant	6:03
	3.2.02	Linear Algebra: System of Equations to a Matrix	4:04
	3.2.03	Linear Algebra: Solving a Linear System Graphically	5:53
	3.2.04	Linear Algebra: Stability and Condition	4:24
	3.2.05	Linear Algebra: Cramer's Rule	3:59
	3.2.06	Linear Algebra: Cramer's Rule Example	9:42
	3.2.07	Linear Algebra: Elimination and Cramer's Rule	5:52
	3.2.08	Linear Algebra: Cramer's Rule Supplement	8:33
	3.2.09	Linear Algebra: Naïve Gauss Elimination	11:55
	3.2.10	Linear Algebra: Naïve Gauss Elimination Example	26:07
	3.2.11	Linear Algebra: Gauss-Jordan Example	3:24
	3.2.12	Linear Algebra: Shortcomings of Elimination Methods	2:33
	3.2.13	Linear Algebra: Gauss Elimination with Pivoting	5:08
	Unit 3	3.3.1	Linear Algebra: LU Decomposition Overview
3.3.2		Linear Algebra: LU Decomposition Step 1	5:32
3.3.3		Linear Algebra: LU Decomposition Step 2	2:26
3.3.4		Linear Algebra: LU Decomposition Step 3	3:45
3.3.5		Linear Algebra: LU Decompositon-Computing A-Inverse	6:49
3.3.6		Linear Algebra: LU Decompositon-Computing Det(A)	2:23
3.3.7		Linear Algebra: Vector and Matrix Norms	4:01
3.3.8		Linear Algebra: Condition Number	3:24
Unit 4	3.4.1	Linear Algebra: Computational Complexity	10:04
	3.4.2	Linear Algebra: Thomas' Algorithm	6:05
	3.4.3	Linear Algebra: Cholesky Decomposition	8:07
	3.4.4	Linear Algebra: Cholesky Decomposition Example	11:14
	3.4.5	Linear Algebra: Iterative Methods for Linear Systems	1:52
	3.4.6	Linear Algebra: Gauss-Seidel and Jacobi	10:43

Table E.4: Video Lectures for Part 5, Curve Fitting and Regression

Part 5	Index	Video title	Length
Unit 1	5.1.1	Curve Fitting: Regression	6:43
	5.1.2	Curve Fitting: Regression with a Constant	14:29
	5.1.3	Curve Fitting: Regression for a Line	26:30
	5.1.4	Curve Fitting: Comment on $R^2$	1:19
	5.1.5	Curve Fitting: Generalizing Least Squares	5:57
Unit 2	5.2.1	Curve Fitting: Polynomial and Spline Interpolation Overview	5:55
	5.2.2	Curve Fitting: Newton's Divided Difference Polynomials Linear Interpolation	7:08
	5.2.3	Curve Fitting: Newton's Divided Difference Polynomials General Representation	9:31
	5.2.4	Curve Fitting: Lagrange Polynomials, Linear Interpolation	6:06
	5.2.5	Curve Fitting: Lagrange Polynomials, Quadratic Interpolation	5:33
	5.2.6	Curve Fitting: Solving for Coefficients	5:47
	5.2.7	Curve Fitting: Spline Interpolation	14:38
Unit 3	5.3.1	Curve Fitting: Least Squares Regression with Sinusoids	5:11
	5.3.2	Curve Fitting: Continuous Fourier Approximation	3:49
	5.3.3	Curve Fitting: Time and Frequency Domain	3:10
	5.3.4	Curve Fitting: Discrete Fourier Transform	2:40
	5.3.5	Curve Fitting: Fast Fourier Transform	13:44
Unit 4	5.4.1	Curve Fitting: Worked Example 1	12:18
	5.4.2	Curve Fitting: Worked Example 2	6:12
	5.4.3	Curve Fitting: Worked Example 3 with Matlab	3:05
	5.4.4	Curve Fitting: Worked Example 3 by Hand	10:48
	5.4.5	Curve Fitting: Worked Example 4 Regression with both Trigonometric and Polynomial Functions	7:30
	5.4.6	Curve Fitting: Worked Example 5: FFT with Sunspot Data	5:41
	5.4.7	Curve Fitting: Note on Polynomial Interpolation	2:43
	5.4.8	Curve Fitting: Splines with Matlab Demonstration	1:50

Table E.5: Video Lectures for Part 6, Numerical Integration and Differentiation

Part 6	Index	Video title	Length
Unit 1	6.1.1	Overview of Remaining Material	1:39
	6.1.2	Numerical Integration: Overview	3:59
	6.1.3	Numerical Integration: Newton-Cotes Integration Formulas Overview	5:07
	6.1.4	Numerical Integration: Trapezoid Rule Formulation	10:11
	6.1.5	Numerical Integration: Newton-Cotes Integration Formulas	5:21
	6.1.6	Numerical Integration: Composite Newton-Cotes	5:50
	6.1.7	Numerical Integration: Multi-Dimensional Newton-Cotes	6:27
	6.1.8	Numerical Integration: Error and Newton-Cotes Formulas	4:25
Unit 2	6.2.1	Numerical Integration: Newton-Cotes Formulas Summary	2:50
	6.2.2	Numerical Integration: Romberg Integration and Richardson's Extrapolation	7:28
	6.2.3	Numerical Integration: Richardson Extrapolation Derivation	5:49
	6.2.4	Numerical Integration: Error for Multiple-Application Trapezoid Rule	4:57
	6.2.5	Numerical Integration: Adaptive Integration	6:29
	6.2.6	Numerical Integration: Gauss Quadrature Part 1	10:36
	6.2.7	Numerical Integration: Gauss Quadrature Part 2	9:32
	6.2.8	Numerical Integration: Improper Integrals	5:20
Unit 3	6.3.1	Numerical Differentiation: Introduction	2:31
	6.3.2	Numerical Differentiation: Derivation of Forward and Backward Difference	7:46
	6.3.3	Numerical Differentiation: Derivation of Centered Difference	3:41
	6.3.4	Numerical Differentiation: Higher-Order Finite Divided Difference Tables	2:39
	6.3.5	Numerical Differentiation: Richardson Extrapolation	1:09
	6.3.6	Numerical Differentiation: Unequally Spaced Data	2:48
	6.3.7	Numerical Differentiation: Noisy Data	2:51
	6.3.8	Numerical Differentiation: Finite Difference with Partial Derivatives	2:54
Unit 4	6.4.1	Numerical Integration & Differentiation: Worked Example 1	22:03
	6.4.2	Numerical Integration & Differentiation: Worked Example 2	6:31
	6.4.3	Numerical Integration & Differentiation: Worked Example 3	12:14

Table E.6: Video Lectures for Part 7, Ordinary Differential Equations

Part 7	Index	Video title	Length
Unit 1	7.1.1	ODEs: Introduction to Ordinary Differential Equations	11:59
	7.1.2	ODEs: Introduction to Runge-Kutta Methods	5:56
	7.1.3	ODEs: Euler's Method	2:18
	7.1.4	ODEs: Heun's Method	3:07
	7.1.5	ODEs: General Runge-Kutta Framework	4:07
	7.1.6	ODEs: Second-Order Runge-Kutta	6:03
	7.1.7	ODEs: Third-Order Runge-Kutta	3:43
	7.1.8	ODEs: Classical Fourth-Order Runge-Kutta	4:35
	7.1.9	ODEs: Butcher's Fifth-Order Runge-Kutta	3:00
Unit 2	7.2.1	ODEs: Adaptive Runge-Kutta	9:16
	7.2.2	ODEs: Stiff Systems	6:55
	7.2.3	ODEs: Recap of Explicit Runge-Kutta Methods	3:45
	7.2.4	ODEs: Explicit versus Implicit Solution Methods	6:10
	7.2.5	ODEs: Implicit Euler's Method	2:18
	7.2.6	ODEs: Non Self-Starting Heun Method	5:02
	7.2.7	ODEs: Milne's Method (3rd order) and Adam's Method (4th order)	7:16
	7.2.8	ODEs: Recap of Explicit and Implicit Methods	4:43
Unit 3	7.3.1	ODEs: Boundary Conditions	5:52
	7.3.2	ODEs: Shooting Method	8:58
	7.3.3	ODEs: Finite Difference Method	14:12
	7.3.4	ODEs: Eigenvalue Problem Motivation	4:13
	7.3.5	ODEs: Polynomial Method for Eigenvalues	2:27
	7.3.6	ODEs: Power Iteration for Eigenvalues	7:15
	7.3.7	ODEs: Other Methods for Eigenvalues	1:12
Unit 4	7.4.1	ODEs: Worked Example–Problem Statement	2:37
	7.4.2	ODEs: Worked Example–Analytical Solution	5:30
	7.4.3	ODEs: Worked Example–Euler's Method	7:10
	7.4.4	ODEs: Worked Example–Heun's Method	8:56
	7.4.5	ODEs: Worked Example–Midpoint Method	10:54
	7.4.6	ODEs: Worked Example–Ralston's Method	8:45
	7.4.7	ODEs: Worked Example–Final Plot	3:59



Table E.7: Video Lectures for Part 8, Partial Differential Equations

Part 7	Index	Video title	Length
Unit 1	8.1.1	PDEs: Ordinary versus Partial Differential Equations	6:00
	8.1.2	PDEs: Classification of Partial Differential Equations	10:54
	8.1.3	PDEs: Classification Continued—Classifying Second-Order PDEs	8:20
	8.1.4	PDEs: Boundary Conditions and Solution Methods Overview	2:35
	8.1.5	PDEs: Derivation of the Laplace Equation	6:12
	8.1.6	PDEs: Finite-Difference Method for Laplace Equation	11:31
	8.1.7	PDEs: Secondary Variables	5:56
Unit 2	8.2.1	PDEs: Finite Divided Difference for Elliptic PDEs with Irregular Boundaries	8:42
	8.2.2	PDEs: Finite Volume Method (Control Volume Approach)	15:18
	8.2.3	PDEs: Explicit Finite Difference Method for Parabolic PDEs	12:10
	8.2.4	PDEs: Convergence and Stability	4:52
	8.2.5	PDEs: Implicit Finite Divided Difference for Parabolic PDEs	5:57
	8.2.6	PDEs: Crank-Nicolson Implicit Finite Divided Difference Method	8:00
	8.2.7	PDEs: Parabolic PDEs In Two Spatial Dimensions (ADI Method)	10:26
Unit 3	8.3.1	PDEs: Introduction to Finite Element Method	4:50
	8.3.2	PDEs: Finite Element Method: Domain Discretization	4:28
	8.3.3	PDEs: Finite Element Method: Element Equations Part 1	14:06
	8.3.4	PDEs: Finite Element Method: Element Equations Part 2	13:00
	8.3.5	PDEs: Finite Element Method: Assembly	4:41
	8.3.6	PDEs: Finite Element Method: Boundary Conditions	3:00
	8.3.7	PDEs: Finite Element Method: Solution and Post-Processing	2:26

**Appendix F**  
**Model-Eliciting Activities**

**ENGR 2450 – Spring 2013  
JIT MEA – Assignment 1****MEA 1-A1****Model-Eliciting Activity (MEA) 1 - Just-In-Time Manufacturing**  
*This is to be completed individually.*

1. Individually read the company profile and CEO request below.

**Company Profile – D. Dalton Technologies**

*The Bowman and Ceramica Divisions of D. Dalton Technologies develop advanced piezoceramics and custom-made ultrasonic transducers.*

D. Dalton Technologies (DDT) was founded in 2000 to advance the ultrasound field using piezoelectric materials. DDT acquired Bowman Transducers, a company that is well known for producing innovative transducers, and Ceramica, a manufacturer of piezoelectric ceramic materials located in Lincoln, Nebraska. These companies were purchased in the summer of 2001 and have each been strengthened by the addition of high-tech equipment and an increase in the number of amply qualified engineers. DDT is the up-and-coming premier supplier of top-quality transducers in the United States.

Through the improved vision of D. Dalton Technologies, Ceramica is broadening its line of piezoelectric materials. Piezoelectric materials convert electrical signals into a mechanical response (as in a speaker) or mechanical signals into an electrical response (as in a microphone). Most piezoelectric materials are compounds of metals and oxides that require precise processing conditions to produce optimum properties. Presently, Ceramica's primary focus is on the expansion of the composite materials product line. Several applications are now under active development with current and potential Ceramica customers. Ceramica is developing a new line of materials that will withstand high temperatures and still retain their polarization. This is an example of one of the exciting new projects in the works for Ceramica.

Bowman, located in Noblesville, Indiana, is also benefiting from the direction and the investment of D. Dalton Technologies. Bowman has added additional transducer design professionals to its engineering staff. Transducers are devices that convert one form of energy into another. The diverse customers of Bowman want custom transducers for products such as sensors that monitor oil well drilling tips, medical instruments such as diagnostic imaging ultrasound, discs for accelerated bone healing and devices for intravascular procedures, and in-home electronic gadgets such as telephones and stereo equipment. Bowman takes extra care to create custom transducers that fit the needs of each of its clients.

D. Dalton Technologies, together with Ceramica and Bowman Transducers, is ready to design and manufacture transducers to fit your specific needs. D. Dalton Technologies understands that progress is made not only by taking advantage of the latest in technical innovations, but also adhering to the best of the "tried and true". Therefore, it is the goal of the company to provide clients with the highest quality customer service and products. Because of the dedication and insight of the president and founder, Devon Dalton, D. Dalton Technologies can honestly say, "Your ideas plus our commitment equals perfect solutions."

ENGR 2450 – Spring 2013  
JIT MEA – Assignment 1

MEA 1-A1

## Interoffice Memo

**To:** Applications Engineering Team  
**From:** Devon Dalton, CEO  
**RE:** Shipping Issues  
**Priority:**[Urgent]

---

Our company operates a just-in-time manufacturing system. After several years of shipping with Pathways Transit (PT), it has come to my attention that PT has not been meeting our shipping needs. We are having problems with late arrival times. The fact that PT is not consistently arriving at the time they have promised is causing D. Dalton Technologies (DDT) production problems. This means that our Logistics Manager needs a method to identify a new shipping company.

I want to make use of your team's analytical expertise. DDT is small; therefore, we need your team to serve in an engineering project management function on this project. Your team's task is to design a procedure to rank potential shipping companies. My assistant has collected historical data on several potential companies for you. Eight shipping companies have been identified as able to transport materials directly from Ceramica to Bowman. As you know, arrival time of materials is a big issue for DDT. Since the piezoelectric materials are designed specifically for each custom order, it is imperative that the delivery of materials occur just-in-time for Bowman to begin the manufacturing process that uses all of the shipped materials. Because we operate with a small workforce and only one shift, minutes to a few hours can make a difference in our ability to complete devices for our custom applications by our contracted delivery date. This makes arrival time of materials of great importance. We have in excess of 250 data points for each shipping company. At this time, the data for only four companies is available. This data is stored in a file called **jit\_data\_partial.txt**. The four shipping companies are Iron Horse Expeditors (IHE), Delphi Shipping (DS), ShipCorp (SC), and United Express (UE). The data is in hours late for shipping runs from Lincoln, Nebraska to Noblesville, Indiana.

Your team should brainstorm different ways in which to analyze the shipping data. Then, your engineering team will use the sampling of data provided for the four shipping companies to develop a procedure to rank the shipping companies in order of most likely to least likely able to meet our timing needs.

In a memo to my attention, please include your team's procedure and the rank order of the shipping companies generated by applying your procedure to the sample data. Be sure to include additional quantitative results as appropriate to demonstrate the functionality of your procedure. Please be sure to include your team's reasoning for the each step, heuristic (i.e. rule), or consideration in your team's procedure.

Please send your complete memo to me by next week.

DD

**ENGR 2450 – Spring 2013  
JIT MEA – Assignment 1**

**MEA 1-A1**

2. Access the MEA portal and answer the Individual Questions at:  
<https://www.mealearning.com>
3. **Just-in-Time** (JIT) is a management philosophy that originated in Japan and was put into practice in the 1970s. Its practice was developed and perfected by Toyota.

Use and document (with proper citations) at least **two** external resources to learn **three** things about JIT that are relevant to this problem context.

In the Context Setting box on the MEA 1 interface, list, **in your own words**, the **three** things you learned and **explain how each is relevant to this problem context**. In the same box, below the three things learned, list your citations using APA format. (For help with APA reference formats, see <http://owl.english.purdue.edu/owl/resource/560/01/>)

4. Individually, answer the following questions.  
**NOTE:** *Before proceeding, you should review the lecture materials from ENGR 2450 concerning expectations for good responses to the individual questions posted on CANVAS in the MEA FOLDER.*

**Problem Formulation:** Parts a and b ask you to take a big-picture view of the problem.

- a. List as many stakeholders as you can think of who may be impacted by the deliverable your team has been asked to create. For each stakeholder, explain the relationship between the stakeholder, the problem, and the deliverable.
- b. Your solution will be implemented in the context described here and potentially in other contexts. Describe issues (minimum five) that might arise for stakeholders when your generalizable solution is implemented.

Here your team has only been asked to consider late shipping times in the development of your solution. What other issues may need to be considered when committing to a shipping company and attempting to operate in a just-in-time fashion.

**Problem Identification:** Parts c to e ask you to take a task-level view of the problem.

- c. Consider your list of stakeholders. Who is the direct user of the deliverable your team is being asked to create?
- d. In a few sentences and in your own words, what does the direct user need? (Remember to describe the deliverable, its function, the criteria for success, and the constraints.)
- e. Consider the immediate problem as described and the sample data provided. Describe at least two ideas you have for why this problem might be complex to solve.

**Deliverables:** *Electronic submission of your responses to steps 3 and 4a-e via the MEA portal.*

**ENGR 2450 – Spring 2013  
JIT MEA – Assignment 2**

**MEA 1-A2**

**Model-Eliciting Activity (MEA) 1 - Just-In-Time Manufacturing  
Team Activity**

**You should work with the team assigned by your TA. Before you start this task, select one team member to be the Timekeeper, another to be the Recorder, another to be the Meeting Coordinator, and one to be to be the Encourager/Gatekeeper. Once you have decided on the role of each member for this lab session, begin working on this task with all members of your team working at one computer.**

**The team Timekeeper should monitor the time spent on tasks. The team Recorder should serve as the initial keyboard operator. The Encourager/Gatekeeper should ensure equal team member participation. The Meeting Coordinator should facilitate arrangements for the team to complete this task outside of class.**

1. Gather around one computer and access one team member's MEA Team Draft 1 interface. This will enable you to see each team member's individual responses to the Problem Formulation and Problem Identification questions. You will also have access a space to enter your team's Draft 1 response. **Within your team, compare your answers to the individual questions from Assignment 1. Your team must come to consensus on these two questions:**
  - c. Who is the direct user of the deliverable your team is being asked to create?
  - d. In a few sentences and in your own words, what does the direct user need? (Remember to describe the deliverable, its function, the criteria for success, and the constraints.)

***NOTE:** Before proceeding, your team should review the lecture materials from ENGR 2450 concerning expectations for good responses to the individual questions posted on CANVAS in the MEA FOLDER.*

**Write your team responses in a Word file. Be sure to email the completed file to all team members.**

When you are sure that your team has come to consensus, submit your responses MEA Team Consensus on Problem Identification interface at <http://mealearning.com>.

2. In your team, formulate a plan to use the historical data to develop a procedure to rank shipping companies in terms of most likely to meet DDT's timing needs to least likely to meet the timing needs.

***NOTE:** Before proceeding, your team should review the lecture materials from ENGR 2450 concerning expectations for a high quality team response posted on CANVAS in the MEA FOLDER.*

**Your team will type the body of the memo to Devon Dalton in a Microsoft Word document. Be sure to email the completed memo to all team members.**

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JIT MEA – Assignment 2**

**MEA 1-A2**

When your response is complete, copy your memo to the MEA Team Draft 1 interface at <http://mealearning.com>.

Use the following outline to help organize your team's response and ensure that your team has not forgotten necessary items.

**CAUTION: The memo that your team submits should not contain outline formatting when it is complete. The outline below is just a content guide.**

Items I A-C are typically all covered in the first paragraph and item II is typically in easy-to-follow numbered steps. Item III could be in a combination of paragraph and tabular form, depending on the nature and quantity of the results generated by your team's solution. (NOTE: Your team cannot receive a grade higher than a D if you do not present results (Item III). Why? A supervisor and direct user would want to see results. Without results, your team has only attempted part of the task (provided the supervisor and direct user with a solution); your team would not have provided evidence that it actually works.). Item IV includes any other requested information.

TO: Name, Title  
FROM: Team #  
RE: Subject

**I. Introduction (*Re-usability*)**

**A. In your own words, describe the problem. (~2-3 sentences)**

This should include your team's consensus on who the direct user is and what the direct user needs in terms of the deliverable, criteria for success, and constraints.

**B. Provide an overarching description of what the procedure is designed to do or find – be specific (~1- 2 sentences)**

**C. State your assumptions about the conditions under which it is appropriate to use your procedure. Another way to think about this is to describe the limitations of your procedure.**

**II. List the steps of your procedure (*Mathematical Model*). Provide clear rationales for the critical steps, assumptions associated with individual procedural steps (*Modifiability*), and clarifying explanations (e.g. sample computations) for steps that may be more difficult for the direct user to understand or replicate (*Shareability*).**

**III. Present results of applying the procedure to the specified data in the form requested. (*Shareability*)**

**IV. Other requested information**

**Hint 1:** Spelling and grammar are important (*Shareability*). Let Word check for some of these errors. Do realize that Word will not find all of your errors and it will identify some that are not really errors at all. Your team needs to proof-read carefully.

**Hint 2:** Be sure to give your team's reasoning for the each step, heuristic (i.e. rule), or consideration in your team's procedures. (*Modifiability*)

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**MEA 1-A2**

**JIT MEA – Assignment 2**

4. **Finish your memo and upload it to the MEA portal as part of Homework 3.** Your TA will provide feedback and a grade for your responses to the individual questions. Your Peers will provide feedback on your MEA Team Draft 1.



**ENGR 2450 – Spring 2013**  
**JIT MEA – Assignment 3**

**MEA 1 – A3**

**MEA 1: Just-in-Time Manufacturing – Peer Calibration**

*This is to be completed individually*

*This problem is part of your MEA grade*

The purpose of the Peer Review is to provide a critical evaluation of another team's work such that the team can use your feedback to improve their solution. Your feedback should enable and encourage the team receiving your critique to make substantive (e.g. major, practical, fundamental) changes to improve their MEA solution.

An ability to critically evaluate other people's work and provide constructive feedback that encourages improvement is a necessary professional skill that takes practice. Through this assignment, you will practice giving feedback on the work of two sample student teams through a Calibrated Peer Review. Just as your Teaching Assistant will evaluate your team's MEA work in the future, you will evaluate two teams' MEA Draft 1 work along four dimensions: Mathematical Model, Re-Usability, Modifiability, and Share-ability. After you have provided feedback on the sample team's work, you will have access to an example of how an expert evaluated that same sample team. A review of the expert evaluation will enable you to better gauge the depth and specificity for the critique you will do of one of your peer's work. Once you have reviewed the expert evaluation, you will be asked to reflect on three ways that you can improve your ability to provide a MEA Peer Review.

You will access **Calibration 1** through the MEA portal at: <http://mealearning.com>

The instructions below walk you through the items that must be completed in a MEA Peer Review.

***Keep in mind that a "good" reviewer:***

- *Accepts the responsibility for helping others improve their work*
- ***Is Always Respectful!!***
- *Focuses on content rather than the team of individuals that wrote the MEA solution*
- *Reviews and understands the evaluation criteria (e.g. rubric items)*
- *Provides evidence that the work has been read, observed, examined... carefully*
- *Conducts a thorough and complete evaluation*
- *Shares their expertise by providing specific examples and suggestions for improvement*
- *Is clear and organized*
- *Is specific - avoid general comments and clarify pronouns such as "it", "that", etc.*

1. Read the team's response. This appears on the left side of the screen. The Peer Review Feedback and Assessment Form you will complete appears on the right side of the screen.
2. You will now critically evaluate the **Mathematical Model** the team has designed.
  - a. Under the Mathematical Model heading in the textbox provided, briefly summarize in your own words the mathematics used in the procedure.

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**JIT MEA – Assignment 3**

**MEA 1 – A3**

- b. To provide a critical review, you must actually try to apply the team's procedure. Use the student survey data stored as **jit\_data\_partial.txt** on CANVAS. Be sure to use the procedure as written. In the textbox provided, **state the results found by applying the procedure as written to the historical data; provide a final ranking for the shipping companies with quantitative results.** Also, describe any problem(s) you experienced when trying to apply the procedure.
  - c. Based on your written comments to parts 2 a-b above:
    - Select the level to which you believe the team addressed the complexity of the problem.
    - Select whether the team used all the data provided or they reasonably explained why they chose to not use all of the data provided.
  - d. In the last Mathematical Model section textbox, provide constructive recommendations on how to better address the complexity of the problem or eliminate errors. **You worked this problem too; so you have some insights into solving it. Share what you understand about the problem and its complexity.**
3. You will also critically evaluate the Re-Usability of the team's work. Recall that **Re-usability means that the procedure can be used by the direct user for new but similar situation because the direct user can understand for whom and what the procedure was designed.** A re-usable procedure: (1) identifies who the direct user is and what the direct user needs in terms of the deliverable, its function, criteria for success, and constraints, (2) provides an overarching description of the procedure, and (3) clarifies assumptions and limitations concerning the use of procedure. If there are no limitations to the use of the procedure, this should be stated.
- a. Under the Re-Usability heading in the textbox provided, briefly summarize in your own words the information provided in the procedure that contributes to its re-usability.
  - b. Select the level to which you feel the team's procedure is re-usable.
  - c. In the last Re-usability section textbox, provide constructive recommendations on how to make the procedure more re-usable.
4. You will now critically evaluate the Modifiability of the team's work. Recall that **Modifiability means that the procedure can be modified easily by the client for slightly different situations because design decisions are clearly explained and justified using evidence or context based rationales.** A modifiable procedure (1) contains acceptable rationales for critical steps in the procedure and (2) clearly states assumptions associated with individual procedural steps.

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JIT MEA – Assignment 3**

**MEA 1 – A3**

- a. Under the Modifiability heading in the textbox provided, briefly summarize in your own words the rationales and assumptions provided.
  - b. Select the level to which you feel the team's procedure is modifiable.
  - c. In the last Modifiability section textbox, provide constructive recommendations on how make the procedure more easily modifiable.
5. Finally, you will critically evaluate how well the team attended to Share-ability. Recall that Share-ability means that the direct user can apply the procedure and replicate results.
- a. Check whether the team presented the results of applying their procedure to **jit\_data\_partial.txt** – remember that both a ranking of the shipping companies and quantitative results are expected.
  - b. Select the level to which you feel the procedure is easy for the direct user to apply and replicate results.
  - c. Select whether the team included extraneous information in their memo.
  - d. In the last Share-ability section textbox, provide constructive recommendations to make the procedure easier for the direct user to use and replicate.
6. Save your **Calibration 1**.
7. You will now review an expert's critique of the same piece of student work you looked at in the Calibration 1 above. Access the Calibration 1 Comparison to Expert through the MEA portal at: <http://mealearning.com>

For each of the four dimensions (Mathematical Model, Re-Usability, Modifiability, and Share-ability) compare your marks and comments to those of the expert. In the textbox provided, briefly state three things that you will do differently to improve the quality of MEA Peer Reviews that you will provide in the future.

8. Be sure to save your **Calibration 1 Comparison to Expert**.
9. Repeat steps 1-8 above to complete **Calibration 2** and **Calibration 2 Comparison to Expert**.

**Deliverables:** *Electronic submission of your Calibration 1, Calibration 2, Calibration 1 Comparison to Expert, and Calibration 2 Comparison to Expert. No hardcopy printout is required. The TAs will review the completion of this assignment through the online interface.*

**ENGR 2450 – Spring 2013**  
**JIT MEA – Assignment 3**

**MEA 1 – A3**

**X. MEA 1 Peer Review**

*This is to be completed individually*

*This problem is part of your MEA grade*

Through this assignment, you will provide a MEA Peer Review on the work of a student team currently in ENGR 2450. Just as you did in Calibration 1 and 2 (above), you will evaluate the team's MEA Draft 1 solution along four dimensions: Mathematical Model, Re-Usability, Modifiability, and Share-ability.

The MEA 1 **Peer Review** can be accessed through the MEA portal at: <http://mealearning.com>

Follow steps 1-5 in MEA Peer Review Calibration and Comparison to Expert (above). Then save your MEA 1 **Peer Review**.

***Deliverables: Electronic submission of your MEA Peer Review. No hardcopy printout is required. The TAs will review the completion of this assignment through the online interface.***

**ENGR 13200 – Spring 2012**  
**MEA1-Assignment 4**

**MEA 1-A4**

**X. MEA 1: Just-in-Time Manufacturing - Team Draft 2**

*This problem is part of your MEA grade.*

**Your team will type the body of the revised memo Devon Dalton and your revision documentation (as described below) in a Microsoft Word document first. Be sure to email the completed Word file to all team members. When all documentation is complete, copy and paste your responses to [mealearning.com](http://mealearning.com).**

Your team will complete a revision of your solution to MEA 1 – this was started in Assignment 2. You will have access to your MEA 1 – Team Draft 2 link on [www.mealearning.com](http://www.mealearning.com)

*When you open the Team Draft 2 link, you should see 3-4 Peer Critiques.*

ENGR 13200 – Spring 2012  
 MEA1-Assignment 4

MEA 1-A4

## Interoffice Memo

**To:** Applications Engineering Team  
**From:** Devon Dalton, CEO  
**RE:** Shipping Issues (2)  
**Priority:** [Urgent]

Thank you for your prompt attention to my first memo. The information I have received begins to help DDT and our Logistics Manager think about a process for selecting a new shipping company. The next part of the task is detailed in this memo.

Now that your team has an initial procedure in place to rank potential shipping companies, I need your team to refine that procedure using additional historical data. My assistant has gathered data on four more companies that we are currently considering. Data for all eight companies is stored in a file called `jit_data_full.txt`. The data is arranged in columns, with each column representing a different shipping company. The columns are ordered as in the table below. The data is in hours late for shipping runs from Lincoln, Nebraska to Noblesville, Indiana.

Column Number	Shipping Company
1	BF = Blue Freight
2	DFC = Direct Freight Shipping
3	NPS = National Package Service
4	FPS = Federal Parcel Service
5	IHE = Iron Horse Expeditors
6	DS = Delphi Shipping
7	SC = ShipCorp
8	UE = United Express

In a memo to my attention, please include your revised procedure and the rank order of the shipping companies generated by applying your procedure to the historical data for all eight companies above. Be sure to include additional quantitative results as appropriate to demonstrate the functioning of your procedure. Please be sure to include your team's reasoning for the each step, heuristic (i.e. rule), or consideration in your team's procedure. Remember that DDT will need to use your procedure in the future, when other shipping company data or other information about any of these shipping companies becomes available. So, DDT must not only be able to apply your procedure to this and future data but also understand your procure well enough to modify it as needed.

Thank you for your team's dedicated work. I look forward to seeing your results.

DD

**ENGR 13200 – Spring 2012**  
**MEA1-Assignment 4**

**MEA 1-A4**

At this time, Devon Dalton would like your team to revise your MEA 1 - Team Draft 1 procedure(s) and provide documentation on your revisions. The data file (**jit\_data\_full.txt** referred to in the memo) is available on CANVAS.

Recall that the direct user is looking for a share-able, re-usable, and modifiable procedure that is easy to use, replicate results, use on similar data sets, and modify. The mathematical model must also address the complexity of the problem. To prepare to make revisions, your team should:

- Read the latest memo from Devon Dalton (below)
- Review the feedback from your peers (will appear in MEA 1 – Team Draft 2), paying close attention to (1) the results presented by your peers for the test case used in their critique and (2) the level to which your team has addressed the Mathematical Model, Re-Usable, Modifiable, and Share-able criteria..
- Review the original memo from Devon Dalton (class 2b).
- Review your team’s first response to Devon Dalton (will appear in MEA 1 – Team Draft 2).
- Review the feedback from your TA on your responses to the individual and team consensus questions (MEA 1 - Grader Feedback on Context and Individual & Grader Feedback on Team Consensus)
- Attempt to use your MEA solution, *as written by your team*, to rank the shipping companies using the data provided (**jit\_data\_full.txt** is available on CANVAS). Avoid making assumptions or reading between the lines of your own procedure – execute it as written. Your team might also consider having someone not on your team try to use your procedure (even someone not in the class) while you observe. This would help you identify difficult or unclear steps in your procedure.
- Be sure to give your team’s reasoning for the each step, heuristic (i.e. rule), or consideration in your team’s procedures.
- Use the following outline to help organize your team’s response and ensure that your team has not forgotten necessary items.

***CAUTION: The memo that your team submits should not contain outline formatting when it is complete. The outline below is just a content guide.***

Items I A-C are typically all covered in the first paragraph and item II is typically in easy-to-follow numbered steps. Item III could be in a combination of paragraph and tabular form, depending on the nature and quantity of the results generated by your team’s solution.

*(NOTE: Your team cannot receive a grade higher than a D if you do not present results (Item III). Why? A supervisor and direct user would want to see results. Without results, your team has only attempted part of the task (provided the supervisor and direct user with a solution); your team would not have provided evidence that it actually works.).* Item IV includes any other requested information.

TO: Name, Title  
 FROM: Team #  
 RE: Subject

**I. Introduction (*Re-usability*)**

A. In your own words, describe the problem. (~2-3 sentences)

This should include your team’s consensus on who the direct user is and what

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**MEA1-Assignment 4**

**MEA 1-A4**

- the direct user needs in terms of the deliverable, criteria for success, and constraints.
- B. Provide an overarching description of what the procedure is designed to do or find – be specific (~1- 2 sentences)
  - C. State your assumptions about the conditions under which it is appropriate to use your procedure. Another way to think about this is to describe the limitations of your procedure.
  - II. List the steps of your procedure (*Mathematical Model*). Provide clear rationales for the critical steps, assumptions associated with individual procedural steps (*Modifiability*), and clarifying explanations (e.g. sample computations) for steps that may be more difficult for the direct user to understand or replicate (*Shareability*).
  - III. Present results of applying the procedure to the specified data in the form requested. (*Shareability*)
  - IV. Other requested information

When responding to feedback (say from your supervisor), it is common practice to document what feedback you are responding to and how you are addressing the feedback in the revision. One would also document the reason for other changes to a work product that are not explicitly requested. You will create this documentation as your team revises your MEA solution. On the mealearning.com MEA 1 - Team Draft 2 interface, you will see six text boxes with instructions for documenting your revisions with regards to the four dimensions of a high quality solution - Mathematical Model, Re-usability, Modifiability, and Share-ability:

- Identify 1 or more things for the Mathematical Model dimension that your team needs to address in order to improve your work. Write out how you can / will address these things.
- Identify 1 or more things for the Re-Usability dimension that your team needs to address in order to improve your work. Write out how you can / will address these things.
- Identify 1 or more things for the Modifiability dimension that your team needs to address in order to improve your work. Write out how you can /will address these things.
- Identify 1 or more things for the Shareability dimension that your team needs to address in order to improve your work. Write out how you can / will address these things.
- Are there comments in your feedback that your team does not agree with? Copy-and-paste what you don't agree with and explain why you do not agree.
- Are there comments in your feedback that your team does not understand? Copy-and-paste the comments you don't understand and explain what specifically needs to be clarified.

Each textbox should be completed as requested.

Your team will submit your final memo to Devon Dalton and your revision documentation as MEA 1 – Team Draft 2 through the mealearning.com interface.

***Deliverable: Electronic submission of your MEA 1 –Team Draft 2 with revision documentation. No hardcopy printout is required. The TAs will review the completion of this assignment through their online interface.***



**0. MEA1: Individual Created Data Sets**

*(This problem is to be completed individually and contributes to your MEA 1 grade.)*

The shipping company data sets for the JIT Manufacturing MEA are test cases. They do *not* represent the extent of all possible data sets one might expect to encounter in potential shipping companies. So you need additional ways of testing your solution. Consider the 8 shipping companies that were provided by Ollie Fiji. Create two additional shipping company data sets (*yourloginA* and *yourloginB*) each with 100 data points. These data sets must, in one or more ways, be different from the ones you have been provided and thus enable further testing of your team's solution to the JIT Manufacturing MEA. Explain how your two data sets are different from the existing 8 data sets and therefore how they further test your solution in ways the other data sets do not.

Create your data sets using any tools you like, but save the data in an Excel spreadsheet file called **jit\_datasets\_yourlogin.xls** in two column format WITH these labels at the top of the column (column 1 - data set *yourloginA*; column 2 – data set *yourloginB*).

Access [www.mealarning.com](http://www.mealarning.com) and the MEA 1 Individual Created Data Sets assignment.

1. Enter your explanation for your two data sets. You must respond to these statements for *each* data set:
  - a. Describe your data set *quantitatively*.
  - b. Describe *quantitatively* how your data set is different from those given.
  - c. Describe how your data set will test your mathematical model differently than those given. This might include thinking about how you want your data set to be ranked amongst those provided by your model.
2. **Below** your explanation, copy-paste your two data sets arranged in two in column format WITH these labels at the top of the column (column 1 - data set *yourloginA*; column 2 – data set *yourloginB*).

**Deliverable:**

- On-line Submission of MEA 1 Individual Created Data Sets on mealarning.com.
- Upload your Excel file, **jit\_datasets\_yourlogin.xls**, to Canvas under the assignment entitled **MEA 1 Assignment 5 - Individual Datasets**. **Only the Excel file containing Individual Created Data Sets belongs in this location.**

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**MEA 1 Assignment 6**

**MEA 1-A6**

**X. MEA 1: Just-in-Time Manufacturing - Team Final Response**

*This problem is part of your MEA grade.*

**Your team will type the body of the revised memo Devon Dalton and your revision documentation (as described below) in a Microsoft Word document first. Be sure to email the completed Word file to all team members. When all documentation is complete, copy and paste your responses to [mealearning.com](http://mealearning.com).**

Your team will finalize your solution to MEA 1 – this was started in Assignment 2. You will have access to your MEA 1 – Team Final Response link on [www.mealearning.com](http://www.mealearning.com)

*When you open the **Team Final Response** link, you should see feedback from a TA.*

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ME A 1 Assignment 6

ME A 1-A6

## Interoffice Memo

**To:** Applications Engineering Team  
**From:** Devon Dalton, CEO  
**RE:** Shipping Issues (3)  
**Priority:** [Urgent]

The Logistics Manager is anxiously awaiting your final revisions on a process for ranking potential shipping companies. Data for all eight companies is stored in a file called `jit_data_full.txt` **PLUS 4 data sets your team created for testing your procedure. Explain how these four data sets are different from the existing 8 data sets and therefore how they further test your solution in ways the other data sets do not.** The data in the file is arranged in columns, with each column representing a different shipping company. The columns are ordered as in the table below. The data is in hours late for shipping runs from Lincoln, Nebraska to Noblesville, Indiana.

Column Number	Shipping Company
1	BF = Blue Freight
2	DFC = Direct Freight Shipping
3	NPS = National Package Service
4	FPS = Federal Parcel Service
5	IHE = Iron Horse Expeditors
6	DS = Delphi Shipping
7	SC = ShipCorp
8	UE = United Express

In a memo to my attention, please include your finalized procedure and the rank order of the shipping companies generated by applying your procedure to the historical data for the eight companies above. Be sure to include additional quantitative results as appropriate to demonstrate the functioning of your procedure. Please be sure to include your team's reasoning for the each step, heuristic (i.e. rule), or consideration in your team's procedure. Remember that the Logistics Manager will need to use your procedure in the future, when other shipping company data or other information about any of these shipping companies becomes available. So, the Logistics Manager must not only be able to apply your procedure to this and future data but also understand your procedure well enough to modify it as needed.

Thank you for your team's dedicated work. I look forward to seeing your results.  
DD

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**MEA 1-A6**

At this time, Devon Dalton would like your team to revise your MEA 1 - Team Draft 2 procedure(s) and provide documentation on your revisions (see below). You will continue to work with the same data file (**jit\_data\_full.txt** referred to in the memo); this is available on Blackboard.

At this time, Devon Dalton would like your team to revise your MEA 1 - Team Draft 2 procedure(s) and provide documentation on your revisions (see below). You will continue to work with the same data file (**jit\_data\_full.txt** referred to in the memo) **PLUS 4 of the data sets (1 or more per team member) that you individually created for Homework 6**; this is available on Blackboard.

Recall that the direct user is looking for a share-able, re-usable, and modifiable procedure that is easy to use, replicate results, use on similar data sets, and modify. The mathematical model must also address the complexity of the problem. To prepare to make revisions, your team should:

- Read the original memo from Devon Dalton (class 2b) and subsequent documentation (Homework 5 and below)
- Review your team's first and second response to the Devon Dalton (Team Draft 1 and 2)
- Review the feedback from your TA (will appear in MEA 1 – Team Final Response), paying close attention to (1) the results presented by your TA for the test case used in their feedback and (2) the level to which your team has addressed the Mathematical Model, Re-Usable, Modifiable, and Share-able criteria..
- Review the feedback from your TA on your responses to the individual and team consensus questions (MEA 1 - Grader Feedback on Context and Individual & Grader Feedback on Team Consensus)
- Attempt to use your MEA solution, *as written by your team*, to rank the shipping companies using the data provided (**jit\_data\_full.txt** is available on Blackboard) **PLUS 4 of the data sets (1 or more per team member) that you individually created for Homework 6**. Avoid making assumptions or reading between the lines of your own procedure – execute it as written. Your team might also consider having someone not on your team try to use your procedure (even someone not in the class) while you observe. This would help you identify difficult or unclear steps in your procedure.
- Be sure to give your team's reasoning for the each step, heuristic (i.e. rule), or consideration in your team's procedures.
- Use the following outline to help organize your team's response and ensure that your team has not forgotten necessary items.

***CAUTION: The memo that your team submits should not contain outline formatting when it is complete. The outline below is just a content guide.***

Items I A-C are typically all covered in the first paragraph and item II is typically in easy-to-follow numbered steps. Item III could be in a combination of paragraph and tabular form, depending on the nature and quantity of the results generated by your team's solution.

(NOTE: Your team cannot receive a grade higher than a D if you do not present results (Item III). Why? A supervisor and direct user would want to see results. Without results, your team has only attempted part of the task (provided the supervisor and direct user with a solution); your team would not have provided evidence that it actually works.). Item IV includes any other requested information.

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**MEA 1 Assignment 6**

**MEA 1-A6**

TO: Name, Title  
 FROM: Team #  
 RE: Subject

- I. Introduction (*Re-usability*)
  - A. In your own words, describe the problem. (~2-3 sentences)  
 This should include your team's consensus on who the direct user is and what the direct user needs in terms of the deliverable, criteria for success, and constraints.
  - B. Provide an overarching description of what the procedure is designed to do or find – be specific (~1- 2 sentences)
  - C. State your assumptions about the conditions under which it is appropriate to use your procedure. Another way to think about this is to describe the limitations of your procedure.
- II. List the steps of your procedure (*Mathematical Model*). Provide clear rationales for the critical steps, assumptions associated with individual procedural steps (*Modifiability*), and clarifying explanations (e.g. sample computations) for steps that may be more difficult for the direct user to understand or replicate (*Shareability*).
- III. Present results of applying the procedure to the specified data in the form requested. (*Shareability*)
- IV. Other requested information

When responding to feedback (say from your supervisor), it is common practice to document what feedback you are responding to and how you are addressing the feedback in the revision. One would also document the reason for other changes to a work product that are not explicitly requested. You will create this documentation as your team revises your MEA solution. On the mealearning.com MEA 1 - Team Final Response interface, you will see six text boxes with instructions for documenting your revisions with regards to the four dimensions of a high quality solution - Mathematical Model, Re-usability, Modifiability, and Share-ability:

- Identify 1 or more things for the Mathematical Model dimension that your team needs to address in order to improve your work. Write out how you can / will address these things.
- Identify 1 or more things for the Re-Usability dimension that your team needs to address in order to improve your work. Write out how you can / will address these things.
- Identify 1 or more things for the Modifiability dimension that your team needs to address in order to improve your work. Write out how you can /will address these things.
- Identify 1 or more things for the Shareability dimension that your team needs to address in order to improve your work. Write out how you can / will address these things.
- Are there comments in your feedback that your team does not agree with? Copy-and-paste what you don't agree with and explain why you do not agree.
- Are there comments in your feedback that your team does not understand? Copy-and-paste the comments you don't understand and explain what specifically needs to be clarified.

Each textbox should be completed as requested.

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**MEA 1 Assignment 6**

**MEA 1-A6**

Your team will submit your final memo to Devon Dalton and your revision documentation as MEA 1 - Team Final Response through the mealearning.com interface.

***Deliverable: Electronic submission of your MEA 1 –Team Final Response with revision documentation. No hardcopy printout is required. The TAs will review the completion of this assignment through their online interface.***

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SD MEA – Assignment 1

MEA 2-A1

## 1. MEA 2 – Shredded Document: Problem Formulation & Identification

*This is to be completed individually.*

*This problem is part of your MEA grade.*

1. Individually read the two memos from Dorothy Belding.

### MEMORANDUM

**TO:** Engineering Team  
**FROM:** Dorothy Belding, Evidence Documents Manager, Federal Security Services  
**RE:** Shredded Document Recovery Process

Federal Security Services (FSS) is a freelance security consulting firm with numerous governmental contracts. Due to the recent increase in illegal business practices, state and federal government prosecuting teams have been contacting us to aid in their investigations. We need to develop a structured method for the recovery of shredded documents that meets a target level accuracy appropriate for our clients. The high profile nature of many of their cases has resulted in more intense scrutiny of how evidence of this nature is obtained and recovered. Currently, few formally documented processes exist for the recovery of shredded documents, causing key documents to be excluded as evidence due to the potential for tampering. FSS would like to develop a computer-based tool to handle document recovery.

The picture shown in Figure 1 is a digitized, grayscale image. This image happens to be a view seen in an optical microscope. Similar images are currently being created of scanned document strips. An image is stored as a data file which contains an array of numbers. Each number refers to the grayscale value for each pixel of the image. A sub-sample of the complete Figure 1 file is shown in Figure 2 and Table 1. This sub-sample shows the grayscale values for the square superimposed on Figure 1 and exploded in Figure 2.

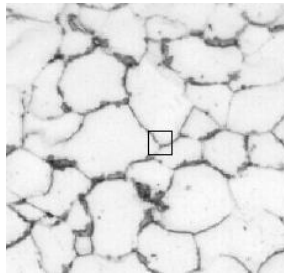


Figure 1. Micrograph of crystals with sub-sample shown.



Figure 2. Sub-sample of micrograph shown in Figure 1.

Grayscale values range from 0 for black to 255 for white. In a true black and white document, such as a text-only document, the grayscale values present will be limited to 0 (black) and 255 (white). A perfectly black and white image will contain ONLY 0's and 255's.

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**MEA 2-A1**

Table 1. Digital file for sub-sample from micrograph of crystals shown in Figure 2.

128	160	221	221	221	241	241	234	241	255	241	241	241	241	255	255	234	234
178	153	178	178	227	234	234	234	255	241	234	234	234	241	241	241	241	241
234	192	192	178	221	227	234	234	255	241	234	241	234	241	234	234	255	241
241	204	178	134	192	221	234	227	234	234	234	241	234	255	234	234	241	234
234	234	215	153	160	215	234	227	227	234	234	241	234	255	234	234	234	227
227	241	221	153	150	192	234	234	234	234	234	234	241	234	241	234	241	227
234	241	221	192	153	178	204	221	234	234	227	234	234	241	227	241	221	221
241	234	227	227	160	153	153	192	227	227	221	234	234	227	221	234	204	192
241	234	234	227	215	160	178	192	234	221	227	227	227	227	227	221	215	178
241	234	234	234	227	215	215	192	204	204	215	192	204	204	192	178	153	150
241	241	241	234	227	241	234	204	160	160	160	153	134	134	150	134	178	192
241	241	241	234	227	234	227	221	153	134	134	153	119	128	153	153	192	221
241	241	241	234	234	227	221	221	160	150	153	192	204	215	234	221	227	234
241	241	241	234	241	234	227	215	178	178	192	221	227	227	234	221	227	234
241	241	234	234	234	234	234	204	192	204	227	227	241	234	234	234	241	241
241	234	234	234	234	227	234	221	227	221	234	227	227	234	234	241	227	234
234	241	227	241	221	215	221	227	227	227	234	241	234	234	255	227	234	241
234	234	241	204	215	204	204	215	227	234	227	227	255	227	241	255	241	221

The grayscale value of each pixel is accessible information that can be used to make decisions about whether or not document strips fit together.

Last week, a government raid on a major automotive company resulted in the seizing of 8 bags of shredded documents. Though the majority of the documents are most likely legitimate confidential business memos, there is evidence to suggest that some documents were shredded to cover up indications that a high ranking vice-president was aware of poor product performance which resulted in a number of deaths.

I would like your engineering team to develop an algorithm that the FSS Software Development Team will use to reassemble documents that have been shredded into full length strips. The FSS Software Development Team will take your written description and translate it into a computer-based tool. Because the FSS Software Development Team is still evaluating several potential computer-based means of implementation, *you should avoid using any specific computer tool language to describe your procedure.*

Dorothy Belding



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SD MEA – Assignment 1

MEA 2-A1

MEMORANDUM

**TO:** Engineering Team  
**FROM:** Dorothy Belding, Evidence Documents Manager, Federal Security Services  
**RE:** Shredded Document Recovery Process

To help your team develop an algorithm to reassemble shredded documents, I asked the FSS Software Development Team to create a test case for you. Here is the information they sent to me:

The complete set of pixel information for an entire document is very large. So, the test case we have provided is only a couple of characters from a set of scanned grayscale document shreds. The original document image is provided in **G6.png**. The scanned shreds are provided in two formats for your convenience: (1) 8 individual shreds saved as eight .dat files called **im0shredA.dat** to **im0shredH.dat**, and (2) an Excel file called **image0\_shreds\_f11.xlsx** into which each .dat file was loaded into a separate tab. Note that all scanned shreds are oriented in the same direction.

In the Excel file, conditional settings are used to grayscale shade the cells containing data according to the pixel value. This allows you to have visual cues about the test data. If you set the View>Zoom to 10%, you can see a relatively sharp image of the scan. As an alternative, the data in the .dat files can

viewed in MATLAB using the following code:

```
>> shredA = load('im0shredA.dat') % loads shredA data
>> shredA_image_matrix(:,1)=shredA/255 % these three lines
>> shredA_image_matrix(:,2)=shredA/255 % create a viewable
>> shredA_image_matrix(:,3)=shredA/255 % image matrix
>> image(shredA_image_matrix) % creates an image of shredA in a figure window
```

Using this information, please develop an algorithm to reassemble grayscale documents. Your final procedure should describe the entire process for reconstructing a document from a set of scanned strips. Further, The FSS Software Development Team has specifically requested that you pay special attention to describing how to decide whether two strips should be joined together.

Your team must set a target level accuracy for reassembly of documents. As your team develops your model, you will need to analyze the accuracy of your model using the original document image as your guide. That is, your team must *quantify* the accuracy with which your model re-assembles documents. Your team should use this quantification and your target to improve the level of accuracy of your model.

In a memo to my attention, please include:

- Your team's procedure for re-assembling a grayscale document. Be sure to include your team's reasoning for the each step, heuristic (i.e. rule), or consideration in your team's procedure.
- The results of applying your general procedure to the data provided.
  - The order of the scanned strips (A to H) generated by applying your procedure to the test case and key quantitative results generated by your model used to determine the order of the strips.
  - A quantitative assessment of your model. You must be able to quantitatively answer the question, "how accurate is your team's model?" Further, you should use your measure(s) of accuracy to identify weaknesses in your model (e.g. how is your model failing? under what conditions does your model fail?)

Please send your complete memo to me.

Dorothy Belding

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SD MEA – Assignment 1**

**MEA 2-A1**

2. Access the MEA portal at <http://www.mealarning.com> and answer the Context Setting and Individual Questions for MEA 2.
3. Document shredders are a standard item in a busy office. What kinds of shredding can one expect from such devices? What challenges do these shreds present for re-assembly? What are the issues around using re-assembled documents as evidence of a crime?

Use and document (with proper citations) at least **two** external resources to learn **three** things about document shredding and re-assembly that are relevant to this problem context.

In the Context Setting box on the MEA 2 interface, list, *in your own words*, the **three** things you learned and *explain how each is relevant to this problem context*. In the same box, below the three things learned, list your citations using APA format. (For help with APA reference formats, see <http://owl.english.purdue.edu/owl/resource/560/01/>)

4. Individually, answer the following questions.

NOTE: You may wish to review the video lectures:

- Mathematical Models and Development:  
[https://engineering.purdue.edu/I2IApps/Diefes-Dux/MEA/MEA\\_01/MEA\\_MathModelsAndDevelopment\\_08-16-12.htm](https://engineering.purdue.edu/I2IApps/Diefes-Dux/MEA/MEA_01/MEA_MathModelsAndDevelopment_08-16-12.htm)
- Problem Formulation and Identification:  
[https://engineering.purdue.edu/I2IApps/Diefes-Dux/MEA/MEA\\_02/MEA\\_ProblemFormulationIdentification.htm](https://engineering.purdue.edu/I2IApps/Diefes-Dux/MEA/MEA_02/MEA_ProblemFormulationIdentification.htm)
- High Quality Solutions: [https://engineering.purdue.edu/I2IApps/Diefes-Dux/MEA/MEA\\_03/MEA\\_HighQualityModels\\_08-22-12.htm](https://engineering.purdue.edu/I2IApps/Diefes-Dux/MEA/MEA_03/MEA_HighQualityModels_08-22-12.htm)

and your feedback from MEA 1 Individual Questions before proceeding.

**Problem Formulation:** Parts a and b ask you to take a big-picture view of the problem.

- a. List as many stakeholders as you can think of who may be impacted by the deliverable your team has been asked to create. For each stakeholder, explain the relationship between the stakeholder, the problem, and the deliverable.
- b. Your solution will be implemented in the context described here and potentially in other contexts. Describe issues (minimum five) that might arise for stakeholders when your generalizable solution is implemented.

Here your team has only been asked to consider scans of strips from single grayscale images in the development of your solution. What other issues may need to be considered when the computer-based tool that will eventually result from your team's work is implemented.

**Problem Identification:** Parts c to e ask you to take a task-level view of the problem.

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**MEA 2-A1**

- c. Consider your list of stakeholders. Who is the direct user of the deliverable your team is being asked to create?
- d. In a few sentences and in your own words, what does the direct user need? (Remember to describe the deliverable, its function, the criteria for success, and the constraints.)
- e. Consider the immediate problem as described and the sample data provided. Describe at least two ideas you have for why this problem might be complex to solve. You may wish to consider the small problem below as you think about the complexity of the larger problem.

Given the following values for the edges of two strips, develop a method for quantitatively rating how well these two pieces align.

Left Strip, Right Edge - 1	Left Strip, Right Edge	Right Strip, Left Edge	Right Strip, Left Edge + 1
254	255	255	255
255	255	255	255
255	255	255	255
64	255	62	255
63	63	64	63
50	64	255	65
64	62	64	64
67	255	59	58
255	255	255	58
255	255	255	255

**Deliverables:**

- *Electronic submission of your responses to steps 3 and 4a-e via the MEA portal.*

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SD MEA – Assignment 2**

**MEA 2-A2**

**Model-Eliciting Activity (MEA) 2 – Shredded Document  
Team Activity**

**You should work with the team assigned by your TA. Before you start this task, select one team member to be the Timekeeper, another to be the Recorder, another to be the Meeting Coordinator, and one to be to be the Encourager/Gatekeeper. Once you have decided on the role of each member for this lab session, begin working on this task with all members of your team working at one computer.**

**The team Timekeeper should monitor the time spent on tasks. The team Recorder should serve as the initial keyboard operator. The Encourager/Gatekeeper should ensure equal team member participation. The Meeting Coordinator should facilitate arrangements for the team to complete this task outside of class.**

1. Gather around one computer and access one team member's MEA 2 Team Draft 1 interface. This will enable you to see each team member's individual responses to the Problem Formulation and Problem Identification questions. You will also have access to a space to enter your team's Draft 1 response. **Within your team, compare your answers to the individual questions from MEA 2 Assignment 1. Your team must come to consensus on these two questions:**
  - c. Who is the direct user of the deliverable your team is being asked to create?
  - d. In a few sentences and in your own words, what does the direct user need? (Remember to describe the deliverable, its function, the criteria for success, and the constraints.)

**Write your team responses in a Word file. Be sure to email the completed file to all team members.**

When you are sure that your team has come to consensus, submit your responses MEA Team Consensus on Problem Identification interface at <http://mealearning.com>.

2. In your team, formulate a plan to create your algorithm as requested by Dorothy Belding.

***NOTE:** Before proceeding, your team may wish to review the video lectures:*

- Mathematical Models and Development: [https://engineering.purdue.edu/I2IApps/Diefes-Dux/MEA/MEA\\_01/MEA\\_MathModelsAndDevelopment\\_08-16-12.htm](https://engineering.purdue.edu/I2IApps/Diefes-Dux/MEA/MEA_01/MEA_MathModelsAndDevelopment_08-16-12.htm)
- Problem Formulation and Identification: [https://engineering.purdue.edu/I2IApps/Diefes-Dux/MEA/MEA\\_02/MEA\\_ProblemFormulationIdentification.htm](https://engineering.purdue.edu/I2IApps/Diefes-Dux/MEA/MEA_02/MEA_ProblemFormulationIdentification.htm)
- High Quality Solutions: [https://engineering.purdue.edu/I2IApps/Diefes-Dux/MEA/MEA\\_03/MEA\\_HighQualityModels\\_08-22-12.htm](https://engineering.purdue.edu/I2IApps/Diefes-Dux/MEA/MEA_03/MEA_HighQualityModels_08-22-12.htm)

**Your team will type the body of the memo to Dorothy Belding in a Microsoft Word document. Be sure to email the completed memo to all team members.**

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**SD MEA – Assignment 2**

**MEA 2-A2**

When your response is complete, copy your memo to the MEA 2 Team Draft 1 interface at <http://mealearning.com>.

Use the following outline to help organize your team's response and ensure that your team has not forgotten necessary items.

**CAUTION: The memo that your team submits should not contain outline formatting when it is complete. The outline below is just a content guide.**

Items I A-C are typically all covered in the first paragraph and item II is typically in easy-to-follow numbered steps. Item III could be in a combination of paragraph and tabular form, depending on the nature and quantity of the results generated by your team's solution.

(NOTE: Your team cannot receive a grade higher than a D if you do not present results (Item III). Why? A supervisor and direct user would want to see results. Without results, your team has only attempted part of the task (provided the supervisor and direct user with a solution); your team would not have provided evidence that it actually works.). Item IV includes any other requested information.

TO: Name, Title  
 FROM: Team #  
 RE: Subject

**I. Introduction (*Re-usability*)**

**A. In your own words, describe the problem. (~2-3 sentences)**

This should include your team's consensus on who the direct user is and what the direct user needs in terms of the deliverable, criteria for success, and constraints.

**B. Provide an overarching description of what the procedure is designed to do or find – be specific (~1- 2 sentences)**

**C. State your assumptions about the conditions under which it is appropriate to use your procedure. Another way to think about this is to describe the limitations of your procedure.**

**II. List the steps of your procedure (*Mathematical Model*). Provide clear rationales for the critical steps, assumptions associated with individual procedural steps (*Modifiability*), and clarifying explanations (e.g. sample computations) for steps that may be more difficult for the direct user to understand or replicate (*Shareability*).**

**III. Present results of applying the procedure to the specified data in the form requested. (*Shareability*)**

**IV. Other requested information**

**Hint 1:** Spelling and grammar are important (*Shareability*). Let Word check for some of these errors. Do realize that Word will not find all of your errors and it will identify some that are not really errors at all. Your team needs to proof-read carefully.

**Hint 2:** Be sure to give your team's reasoning for the each step, heuristic (i.e. rule), or consideration in your team's procedures. (*Modifiability*)

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**SD MEA – Assignment 2**

**MEA 2-A2**

4. **Finish your memo and upload it to the MEA portal.** Your TA will provide feedback and a grade for your responses to the individual questions. Your Peers will provide feedback on your MEA 2 Team Draft 1.

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**SD MEA – Assignment 3**

**MEA 2-A3**

**X. MEA 2: Shredded Document – Peer Calibration**

*This is to be completed individually*

*This problem is part of your MEA grade*

The purpose of the Peer Review is to provide a critical evaluation of another team's work such that the team can use your feedback to improve their solution. Your feedback should enable and encourage the team receiving your critique to make substantive (e.g. major, practical, fundamental) changes to improve their MEA solution.

An ability to critically evaluate other people's work and provide constructive feedback that encourages improvement is a necessary professional skill that takes practice. Through this assignment, you will practice giving feedback on the work of two sample student teams through a Calibrated Peer Review. Just as your Teaching Assistant will evaluate your team's MEA work in the future, you will evaluate two teams' MEA 2 Draft 1 work along four dimensions: Mathematical Model, Re-Usability, Modifiability, and Share-ability. After you have provided feedback on the sample team's work, you will have access to an example of how an expert evaluated that same sample team. A review of the expert evaluation will enable you to better gauge the depth and specificity for the critique you will do of one of your peer's work next week. Once you have reviewed the expert evaluation, you will be asked to reflect on three ways that you can improve your ability to provide a MEA Peer Review.

You will access MEA 2 **Calibration 1** through the MEA portal at: <http://mealearning.com>

The instructions below walk you through the items that must be completed in a MEA Peer Review.

***Keep in mind that a "good" reviewer:***

- *Accepts the responsibility for helping others improve their work*
- ***Is Always Respectful!!***
- *Focuses on content rather than the team of individuals that wrote the MEA solution*
- *Reviews and understands the evaluation criteria (e.g. rubric items)*
- *Provides evidence that the work has been read, observed, examined... carefully*
- *Conducts a thorough and complete evaluation*
- *Shares their expertise by providing specific examples and suggestions for improvement*
- *Is clear and organized*
- *Is specific - avoid general comments and clarify pronouns such as "it", "that", etc.*

1. Read the team's response. This appears on the left side of the screen. The Peer Review Feedback and Assessment Form you will complete appears on the right side of the screen.
2. You will now critically evaluate the **Mathematical Model** the team has designed.
  - a. Under the Mathematical Model heading in the textbox provided, briefly summarize in your own words the mathematics used in the procedure.

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**MEA 2-A3**

- b. To provide a critical review, you must actually try to apply the team's procedure. As appropriate for the team's procedure that you are reviewing, use the shredded document image data stored as .dat files called **im0shredA.dat** to **im0shredH.dat** or as an Excel file called **image0\_shreds\_f11.xlsx** on Blackboard. Be sure to use the procedure as written. In the textbox provided, **state the results found by applying the procedure as written to the scanning shredded document image data; provide a final order of the document shreds with quantitative results for determining the order of shreds. and (if a procedure is given) provide quantitative results concerning the document re-assembly accuracy of the model.** Also, describe any problem(s) you experienced when trying to apply the procedure.
- c. Based on your written comments to parts 2 a-b above:
- Select the level to which you believe the team addressed the complexity of the problem.
  - Select whether the team used all the data provided or they reasonably explained why they chose to not use all of the data provided.
- d. In the last Mathematical Model section textbox, provide constructive recommendations on how to better address the complexity of the problem or eliminate errors. **You worked this problem too; so you have some insights into solving it. Share what you understand about the problem and its complexity.**
3. You will also critically evaluate the Re-Usability of the team's work. Recall that **Re-usability means that the procedure can be used by the direct user for new but similar situation because the direct user can understand for whom and what the procedure was designed.** A re-usable procedure: (1) identifies who the direct user is and what the direct user needs in terms of the deliverable, its function, criteria for success, and constraints, (2) provides an overarching description of the procedure, and (3) clarifies assumptions and limitations concerning the use of procedure. If there are no limitations to the use of the procedure, this should be stated.
- a. Under the Re-Usability heading in the textbox provided, briefly summarize in your own words the information provided in the procedure that contributes to its re-usability.
- b. Select the level to which you feel the team's procedure is re-usable.
- c. In the last Re-usability section textbox, provide constructive recommendations on how to make the procedure more re-usable.
4. You will now critically evaluate the Modifiability of the team's work. Recall that **Modifiability means that the procedure can be modified easily by the direct user for slightly different situations because design decisions are clearly explained and justified using evidence or context based rationales.** A modifiable procedure (1) contains



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**MEA 2-A3**

acceptable rationales for critical steps in the procedure and (2) clearly states assumptions associated with individual procedural steps.

- a. Under the Modifiability heading in the textbox provided, briefly summarize in your own words the rationales and assumptions provided.
  - b. Select the level to which you feel the team's procedure is modifiable.
  - c. In the last Modifiability section textbox, provide constructive recommendations on how make the procedure more easily modifiable.
5. Finally, you will critically evaluate how well the team attended to Share-ability. Recall that Share-ability means that the direct user can apply the procedure and replicate results.
- a. Check whether the team presented the results of applying their procedure to the set of shredded document images – **remember that all of the following are expected:**
    - o The order of the scanned strips (A to H) generated by applying the procedure to the test case and key quantitative results generated by the model and used to determine the order of the strips.
    - o A quantitative assessment of the model.
  - b. Select the level to which you feel the procedure is easy for the direct user to apply and replicate results.
  - c. Select whether the team included extraneous information in their memo.
  - d. In the last Share-ability section textbox, provide constructive recommendations to make the procedure easier for the direct user to use and replicate.
6. Save your **Calibration 1**.
7. You will now review an expert's critique of the same piece of student work you looked at in the Calibration 1 above. Access the Calibration 1 Comparison to Expert through the MEA portal at: <http://mealearning.com>

For each of the four dimensions (Mathematical Model, Re-Usability, Modifiability, and Share-ability) compare your marks and comments to those of the expert. In the textbox provided, briefly state three things that you will do differently to improve the quality of MEA Peer Reviews that you will provide in the future.

8. Be sure to save your **Calibration 1 Comparison to Expert**.
9. Repeat steps 1-8 above to complete **Calibration 2** and **Calibration 2 Comparison to Expert**.

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**MEA 2-A3**

*Deliverables: Electronic submission of your MEA 2 Calibration 1, Calibration 2, Calibration 1 Comparison to Expert, and Calibration 2 Comparison to Expert. No hardcopy printout is required. The TAs will review the completion of this assignment through the online interface.*

**X. MEA 2: Shredded Document - Peer Review**

*This is to be completed individually  
This problem is part of your MEA grade*

Through this assignment, you will provide a MEA 2 Peer Review on the work of a student team currently in ENGR 13200. Just as you did in Calibration 1 and 2 (above), you will evaluate the team's MEA 2 Draft 1 solution along four dimensions: Mathematical Model, Re-Usability, Modifiability, and Share-ability.

The MEA 2 **Peer Review** can be accessed through the MEA portal at: <http://mealearning.com>

Follow steps 1-5 in MEA Peer Review Calibration and Comparison to Expert (above). Then save your MEA 2 **Peer Review**.

*Deliverables: Electronic submission of your MEA 2 Peer Review. No hardcopy printout is required. The TAs will review the completion of this assignment through the online interface.*

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SD MEA — Assignment 4

MEA2-A4

### MEA 2: Shredded Document – Draft 2

*(This problem is to be completed by your entire team and contribute to your MEA grade.)*

Your team will complete a revision of your solution to MEA 2 – this was started in class 9b. You will have access to your MEA 2 – Team Draft 2 link on [www.mealarning.com](http://www.mealarning.com)

*When you open the Team Draft 2 link, you should see 3-4 Peer Critiques.*

At this time, Dorothy Belding would like your team to revise your MEA 2 - Team Draft 1 procedure(s) and provide documentation on your revisions (see the new memo below). The data files referred to in the memo below are available on CANVAS.

#### MEMORANDUM

**TO:** ENGINEERING TEAM  
**FROM:** DOROTHY BELDING, EVIDENCE DOCUMENTS MANAGER, FEDERAL SECURITY SERVICES  
**RE:** SHREDDED DOCUMENT RECOVERY PROCESS (UPDATE)

I see your model to reassemble shredded documents is coming along.

To help your team continue in its development work, I have asked the FSS Software Development Team to create another test case for you. Here is what they have sent to me:

This second test case is a bit larger image than the first. The original document image is provided in **gear1.jpg**. The scanned shreds are again provided in two formats for your convenience: (1) 11 individual shreds saved as eleven .dat files called **im1shredA.dat** to **im1shredK.dat**, and (2) an Excel file called **image1\_shreds\_Sp2012.xlsx** into which each .dat file was loaded into a separate tab.

In your next memo to my attention, please include:

- Your team's *revised* procedure for re-assembling a grayscale document. Please review my last memo for all of the details that must be included concerning your procedure.
- The results of applying your *revised* general procedure to *both* sets of data provided.
  - The order of the scanned strips (A to H for image0 and A to K for image1) generated by applying your procedure to the test case and key quantitative results generated by your model used to determine the order of the strips.
  - A quantitative assessment of your model. You must be able to quantitatively answer the question, "how accurate is your team's model?" Further, you should use your measure(s) of accuracy to identify weaknesses in your model (e.g. how is your model failing? under what conditions does your model fail?)

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**SD MEA — Assignment 4**

**MEA2-A4**

Recall that the direct user is looking for a share-able, re-usable, and modifiable procedure that is easy to use, replicate results, use on similar data sets, and modify. The mathematical model must also address the complexity of the problem. To prepare to make revisions, your team should:

- Read the latest memo from Dorothy Belding (above)
- Review the feedback from your peers (will appear in MEA 2 – Team Draft 2), paying close attention to (1) the results presented by your peers for the test case used in their critique and (2) the level to which your team has addressed the Mathematical Model, Re-Usable, Modifiable, and Share-able criteria..
- Review the original memo from Dorothy Belding (from previously; see the “MEAs – SD” folder in CANVAS).
- Review your team’s first response to Dorothy Belding (will appear in MEA 2 – Team Draft 2).
- Review the feedback from your TA on your responses to the individual and team consensus questions (MEA 2 - Grader Feedback on Context and Individual & Grader Feedback on Team Consensus).
- Attempt to use your MEA solution, *as written by your team*, to re-assemble the new shredded document sample provided (available on CANVAS). Avoid making assumptions or reading between the lines of your own procedure – execute it as written. Your team might also consider having someone not on your team try to use your procedure (even someone not in the class) while you observe. This would help you identify difficult or unclear steps in your procedure.
- Be sure to give your team’s reasoning for the each step, heuristic (i.e. rule), or consideration in your team’s procedures.
- Use the following outline to help organize your team’s response and ensure that your team has not forgotten necessary items.

TO: Name, Title  
 FROM: Team #  
 RE: Subject

- I. Introduction (***Re-usability***)
  - A. In your own words, describe the problem. (~2-3 sentences)  
 This should include your team’s consensus on who the direct user is and what the direct user needs in terms of the deliverable, criteria for success, and constraints.
  - B. Provide an overarching description of what the procedure is designed to do or find – be specific (~1- 2 sentences)
  - C. State your assumptions about the conditions under which it is appropriate to use your procedure. Another way to think about this is to describe the limitations of your procedure.
- II. List the steps of your procedure (***Mathematical Model***). Provide clear rationales for the critical steps, assumptions associated with individual procedural steps (***Modifiability***), and clarifying explanations (e.g. sample computations) for steps that may be more difficult for the direct user to understand or replicate (***Shareability***).

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**MEA2-A4**

- III. Present results of applying the procedure to the specified data in the form requested.  
*(Shareability)*
- IV. Other requested information

***CAUTION: The memo that your team submits should not contain outline formatting when it is complete. The outline above is just a content guide.***

Items I A-C are typically all covered in the first paragraph and item II is typically in easy-to-follow numbered steps. Item III could be in a combination of paragraph and tabular form, depending on the nature and quantity of the results generated by your team's solution. (*NOTE: Your team cannot receive a grade higher than a D if you do not present results (Item III). Why? A supervisor and direct user would want to see results. Without results, your team has only attempted part of the task (provided the supervisor and direct user with a solution); your team would not have provided evidence that it actually works.*). Item IV includes any other requested information.

When responding to feedback (say from your supervisor), it is common practice to document what feedback you are responding to and how you are addressing the feedback in the revision. One would also document the reason for other changes to a work product that are not explicitly requested. You will create this documentation as your team revises your MEA solution. On the mealearning.com MEA 2 - Team Draft 2 interface, you will see six text boxes with instructions for documenting your revisions with regards to the four dimensions of a high quality solution - Mathematical Model, Re-usability, Modifiability, and Share-ability:

- Identify 1 or more things for the Mathematical Model dimension that your team needs to address in order to improve your work. Write out how you can / will address these things.
- Identify 1 or more things for the Re-Usability dimension that your team needs to address in order to improve your work. Write out how you can / will address these things.
- Identify 1 or more things for the Modifiability dimension that your team needs to address in order to improve your work. Write out how you can /will address these things.
- Identify 1 or more things for the Share-ability dimension that your team needs to address in order to improve your work. Write out how you can / will address these things.
- Are there comments in your feedback that your team does not agree with? Copy-and-paste what you don't agree with and explain why you do not agree.
- Are there comments in your feedback that your team does not understand? Copy-and-paste the comments you don't understand and explain what specifically needs to be clarified.

Each textbox should be completed as requested.

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**MEA2-A4**

Your team will submit your final memo to Dorothy Belding and your revision documentation as MEA 2 – Team Draft 2 through the mealearning.com interface.

***Deliverable:***

- Electronic submission of your MEA 2 - Team Draft 2 with revision documentation through the MEA portal. No hardcopy printout is required. The TAs will review the completion of this assignment through the online interface.
- Electronic submission of any MEA 2 files that your team created. If your team created MATLAB m-files or Excel .xlsx files to test your mathematical model on the data files, your team must upload them into the CANVAS assignment entitled, MEA2\_Draft2. Please note that your TA is not required to review these files, but s/he may elect to review them to better understand your team's mathematical model.

## 1. MEA 2 Assignment 5: Individual Created Data Sets

*This problem is to be completed individually.*

*This is part of your MEA 2 grade.*

The shredded documents (or images of document shreds) your team has worked with so far are test cases. They do *not* represent the extent of all possible single-document gray-scale shred sets. So you need additional ways of testing your solution. Consider the shredded documents that you have worked with while creating your recovery procedure. Create two additional sets of document shreds. These two new document image sets must, in one or more ways, be different from the ones you have been provided and thus enable further testing of your team's solution to the Shredded Document MEA. You will need to explain how your two shredded documents are different from the existing ones and therefore how they further test your solution in ways the other data sets do not. Include a statement about the correct order of the shreds for future reference.

To shred your image, you will use the function `ShredImage` which is available on CANVAS in the MEAs->SD folder. To use `ShredImage`, do the following:

1. Create a folder called **MEA2\_SHREDDER** on your hard drive.
2. Download the folder encrypted file: **ShredImage.p** from Canvas to **MEA2\_SHREDDER**.
3. Move a copy of the image file you wish to shred to **MEA2\_SHREDDER**. `ShredImage` supports common image file types (.bmp, .jpg, .gif and others). It is recommended that the image you select be not too much larger than the **G6.bmp** and the **gear1.jpg** image files, else the files will be too large to work with in Excel. So aim for image files between 5 and 20 KB in size. Your images can be in color; they will be converted to gray-scale automatically in the steps that follow.
4. Launch MATLAB and set the Current Directory to **MEA2\_SHREDDER**.
5. Call the shredding function at the MATLAB prompt:

```
>> ShredImage
```

You will be asked for three pieces of information.

- The name of the image file you wish to shred - type the full filename of your image file with extension.
- The approximate number of shreds – enter 12. Depending on the size of your image, you might get one more or less than you ask for.
- The prefix you would like to prepend to the output filenames – type **im1yourlogin** for your first image and **im2yourlogin** for your second image.

This step should create a folder called **im1yourlogin** (or **im2yourlogin** if you are working with your second image) that contains 12 or so numbered files named with a .bmp and a .dat extension.

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MEA2-A5

6. Work with your .dat or .bmp files to determine the order that the shreds need to be in to have a correctly re-assembled image.

For example, you might order your shreds (if your login is A01234567):

im1A01234567: 12-9-3-1-2-8-7-5-10-4-6-11

7. Access [www.melearning.com](http://www.melearning.com) and the MEA 2 Individual Created Data Sets assignment.
- Upload two zipped folders (.zip, not .bz or .tgz), each containing the shredded images.
  - Provide the names of your two image files and an explanation for the selection of these two data sets. Also provide with the correct order of the image shreds.
  - Enter your explanation for your two data sets. You must respond to these statements for *each* data set:
    - Describe your data set *quantitatively*.
    - Describe *quantitatively* how your data set is different from those given.
    - Describe how your data set will test your mathematical model differently than those given.

***Deliverable:***

- ***Online submission of your two original images and their shred sets on melearning.com.***
- ***Online submission of your responses to all questions in Parts 7b and 7c on melearning.com.***



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SD MEA - Assignment 6**

**MEA 2-A6**

**X. MEA 2: Shredded Document - Team Final Response**

*This problem is part of your MEA grade.*

Your team will finalize your solution to MEA 2. You will have access to your MEA 2 – Team Final Response link on [www.mealearning.com](http://www.mealearning.com)

*When you open the Team Final Response link, you should see feedback from a TA.*

At this time, Dorothy Belding would like your team to revise your MEA 2 - Team Draft 2 procedure(s) and provide documentation on your revisions (see the new memo below). For your Team Final Response, your team will need to generate results for the **G6.jpg** and **gear1.jpg**; these data files are available on Canvas. Your team will need to also generate results for **2 of the shredded documents that your team members individually created for Homework 13.**

**Memorandum**

To: Engineering Team  
From: Dorothy Belding, Evidence Documents Manager, Federal Security Services  
Re: Shredded Document Recovery Process (Update)

I would like your team to finalize your procedure for re-assembling shredded documents. I understand that your team has generated a number of shredded documents of your own to further test your procedure. Please include results for these in your final memo as described below.

In your final memo to my attention, please include:

- Your team's revised and final procedure for re-assembling a grayscale document. Please review my previous memos for all of the details that must be included concerning your procedure.
- The results of applying your *final* general procedure to **both** sets of data previously provided (**G6.jpg** and **gear1.jpg**) **plus 2 of your own shredded documents**.
  - **Please describe the images contained in your 2 shredded documents (include the original filenames). Also explain how these documents are different from the ones I provided and therefore how they further test your solution in ways the other two data sets do not.**
  - The order of the scanned strips generated by applying your procedure to the test cases and key quantitative results generated by your model used to determine the order of the strips. **Please supply the correct order of your ALL shredded documents for comparison with the results of your procedure.**
  - A quantitative assessment of your model. You must be able to quantitatively answer the question, "how accurate is your team's model?" Further, you should use your measure(s) of accuracy to identify weaknesses in your model (e.g. how is your model failing? under what conditions does your model fail?)

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**MEA 2-A6**

Recall that the direct user is looking for a share-able, re-usable, and modifiable procedure that is easy to use, replicate results, use on similar data sets, and modify. The mathematical model must also address the complexity of the problem. To prepare to make revisions, your team should:

- Read the original memo and subsequent documentation from Dorothy Belding.
- Review your team's first and second response to the Dorothy Belding (Team Draft 1 and 2).
- Review the feedback from your TA (will appear in MEA 2 – Team Final Response), paying close attention to (1) the results presented by your TA for the test case used in their feedback and (2) the level to which your team has addressed the Mathematical Model, Re-Usable, Modifiable, and Share-able criteria.
- Review the feedback from your TA on your responses to the individual and team consensus questions (MEA 2 - Grader Feedback on Context and Individual & Grader Feedback on Team Consensus)
- Attempt to use your MEA solution, *as written by your team* using all of the data provided (G6.jpg and gear1.jpg are available on Canvas) **plus 2 of the shredded documents that your team members individually created for Homework 14**. Avoid making assumptions or reading between the lines of your own procedure – execute it as written. Your team might also consider having someone not on your team try to use your procedure (even someone not in the class) while you observe. This would help you identify difficult or unclear steps in your procedure.
- Be sure to give your team's reasoning for the each step, heuristic (i.e. rule), or consideration in your team's procedures.
- Use the following outline to help organize your team's response and ensure that your team has not forgotten necessary items.

**CAUTION: The memo that your team submits should not contain outline formatting when it is complete. The outline below is just a content guide.**

Items I A-C are typically all covered in the first paragraph and item II is typically in easy-to-follow numbered steps. Item III could be in a combination of paragraph and tabular form, depending on the nature and quantity of the results generated by your team's solution.

(NOTE: Your team cannot receive a grade higher than a D if you do not present results (Item III). Why? A supervisor and direct user would want to see results. Without results, your team has only attempted part of the task (provided the supervisor and direct user with a solution); your team would not have provided evidence that it actually works.). Item IV includes any other requested information.

TO: Name, Title  
 FROM: Team #  
 RE: Subject

**I. Introduction (*Re-usability*)**

A. In your own words, describe the problem. (~2-3 sentences)

This should include your team's consensus on who the direct user is and what

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**MEA 2-A6**

- the direct user needs in terms of the deliverable, criteria for success, and constraints.
- B. Provide an overarching description of what the procedure is designed to do or find – be specific (~1- 2 sentences)
  - C. State your assumptions about the conditions under which it is appropriate to use your procedure. Another way to think about this is to describe the limitations of your procedure.
- II. List the steps of your procedure (**Mathematical Model**). Provide clear rationales for the critical steps, assumptions associated with individual procedural steps (**Modifiability**), and clarifying explanations (e.g. sample computations) for steps that may be more difficult for the direct user to understand or replicate (**Shareability**).
  - III. Present results of applying the procedure to the specified data in the form requested. (**Shareability**)
  - IV. Other requested information

When responding to feedback (say from your supervisor), it is common practice to document what feedback you are responding to and how you are addressing the feedback in the revision. One would also document the reason for other changes to a work product that are not explicitly requested. You will create this documentation as your team revises your MEA solution. On the mealearning.com MEA 2 - Team Final Response interface, you will see six text boxes with instructions for documenting your revisions with regards to the four dimensions of a high quality solution - Mathematical Model, Re-usability, Modifiability, and Share-ability:

- Identify 1 or more things for the Mathematical Model dimension that your team needs to address in order to improve your work. Write out how you can / will address these things.
- Identify 1 or more things for the Re-Usability dimension that your team needs to address in order to improve your work. Write out how you can / will address these things.
- Identify 1 or more things for the Modifiability dimension that your team needs to address in order to improve your work. Write out how you can /will address these things.
- Identify 1 or more things for the Shareability dimension that your team needs to address in order to improve your work. Write out how you can / will address these things.
- Are there comments in your feedback that your team does not agree with? Copy-and-paste what you don't agree with and explain why you do not agree.
- Are there comments in your feedback that your team does not understand? Copy-and-paste the comments you don't understand and explain what specifically needs to be clarified.

Each textbox should be completed as requested.

Your team will submit your final memo to Dorothy Belding and your revision documentation as MEA 2 - Team Final Response through the mealearning.com interface.

**Deliverable:**

- **Electronic submission of your MEA 2 –Team Final Response with revision documentation. No hardcopy printout is required. The TAs will review the completion of this assignment through their online interface.**

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**MEA 2-A6**

- *If your team created any new shredded documents (beyond those submitted for Homework 13) that are referred to in your MEA 2 – Team Final Response, upload the original image(s) and shred set(s) with your final draft.*
- Electronic submission of any MEA 2 files that your team created. Please note that your TA is not required to review these files, but s/he may elect to review them to better understand your team's mathematical model.



**Appendix G**  
**Course Evaluations**

## SHORT FORM - STUDENT REACTIONS TO INSTRUCTION AND COURSES



Institution:	Instructor:
Course Number:	Time and Days Class Meets:

### IMPORTANT!



Twelve possible learning objectives are listed below, not all of which will be relevant in this class. Describe the amount of progress you made on each (even those not pursued in this class) by using the following scale:

- 1-No apparent progress
- 2-Slight progress; I made small gains on this objective.
- 3-Moderate progress; I made some gains on this objective.
- 4-Substantial progress; I made large gains on this objective.
- 5-Exceptional progress; I made outstanding gains on this objective.

#### Progress on:

- 1. (1) (2) (3) (4) (5) Gaining factual knowledge (terminology, classifications, methods, trends)
- 2. (1) (2) (3) (4) (5) Learning fundamental principles, generalizations, or theories
- 3. (1) (2) (3) (4) (5) Learning to *apply* course material (to improve thinking, problem solving, and decisions)
- 4. (1) (2) (3) (4) (5) Developing specific skills, competencies, and points of view needed by professionals in the field most closely related to this course
- 5. (1) (2) (3) (4) (5) Acquiring skills in working with others as a member of a team
- 6. (1) (2) (3) (4) (5) Developing creative capacities (writing, inventing, designing, performing in art, music, drama, etc.)
- 7. (1) (2) (3) (4) (5) Gaining a broader understanding and appreciation of intellectual/cultural activity (music, science, literature, etc.)
- 8. (1) (2) (3) (4) (5) Developing skill in expressing myself orally or in writing
- 9. (1) (2) (3) (4) (5) Learning how to find and use resources for answering questions or solving problems
- 10. (1) (2) (3) (4) (5) Developing a clearer understanding of, and commitment to, personal values
- 11. (1) (2) (3) (4) (5) Learning to *analyze* and *critically evaluate* ideas, arguments, and points of view
- 12. (1) (2) (3) (4) (5) Acquiring an interest in learning more by asking my own questions and seeking answers

For the remaining questions, use the following code:

- 1=Definitely False
- 2=More False Than True
- 3=In Between
- 4=More True Than False
- 5=Definitely True

- 13. (1) (2) (3) (4) (5) As a rule, I put forth more effort than other students on academic work.
- 14. (1) (2) (3) (4) (5) My background prepared me well for this course's requirements.
- 15. (1) (2) (3) (4) (5) I really wanted to take this course regardless of who taught it.
- 16. (1) (2) (3) (4) (5) As a result of taking this course, I have more positive feelings toward this field of study.
- 17. (1) (2) (3) (4) (5) Overall, I rate this instructor an excellent teacher.
- 18. (1) (2) (3) (4) (5) Overall, I rate this course as excellent.

#### EXTRA QUESTIONS

If your instructor has extra questions, answer them in the space designated below (questions 19-38).

- 19. (1) (2) (3) (4) (5)    24. (1) (2) (3) (4) (5)    29. (1) (2) (3) (4) (5)    34. (1) (2) (3) (4) (5)
- 20. (1) (2) (3) (4) (5)    25. (1) (2) (3) (4) (5)    30. (1) (2) (3) (4) (5)    35. (1) (2) (3) (4) (5)
- 21. (1) (2) (3) (4) (5)    26. (1) (2) (3) (4) (5)    31. (1) (2) (3) (4) (5)    36. (1) (2) (3) (4) (5)
- 22. (1) (2) (3) (4) (5)    27. (1) (2) (3) (4) (5)    32. (1) (2) (3) (4) (5)    37. (1) (2) (3) (4) (5)
- 23. (1) (2) (3) (4) (5)    28. (1) (2) (3) (4) (5)    33. (1) (2) (3) (4) (5)    38. (1) (2) (3) (4) (5)

Comments:

**DO NOT  
WRITE  
IN THE  
SHADED  
AREA**



**Urroz, Gilberto**  
**Utah State University**

ENGR 2450, Section: 001  
 Numerical Methods for Engineers  
 Spring 2013  
 Local code: 11858000



IDEA Short Form Report

To learn more, see the Interpretive Guide: [www.theideacenter.org/shortguide.pdf](http://www.theideacenter.org/shortguide.pdf)

Of the 91 students enrolled, 63 responded (69%). Feedback from individual classes is always useful to guide improvement efforts. Typically, multiple classes should be used for evaluation, using more classes when they are small (fewer than 10) or when they have low response rates (less than 60%) (see [www.theideacenter.org/AdminDecisions](http://www.theideacenter.org/AdminDecisions)).

**Summary Evaluation of Teaching Effectiveness**

Teaching effectiveness is assessed in two ways: **A. Progress on Relevant Objectives**, a weighted average of student ratings of the progress they reported on objectives selected as "Important" or "Essential" (double weighted) and **B. Overall Ratings**, the average student agreement with statements that the teacher and the course were excellent. The **SUMMARY EVALUATION** is the average of these two measures. Individual institutions may prefer to combine these measures in some other manner to arrive at a summary judgment.

**Converted Averages** are standardized scores that take into account the fact that the average ratings for items on the IDEA form are not equal; students report more progress on some objectives than on others. Converted scores all have the same average (50) and the same variability (a standard deviation of 10); about 40% of them will be between 45 and 55. Because measures are not perfectly reliable, it is best to regard the "true score" as lying within plus or minus 3 of the reported score.

**For comparative purposes, use converted averages.** Your converted averages are compared with those from all classes in the IDEA database. If enough classes are available, comparisons are also made with classes in the same broad *discipline* as this class and/or with all classes that used IDEA at your *institution*. The *Interpretive Guide* offers some suggestions for using comparative results; **some institutions may prefer to establish their own "standards" based on raw or adjusted scores rather than on comparative standing.**

Both unadjusted (raw) and adjusted averages are reported. The latter makes classes more comparable by considering factors that influence student ratings, yet are beyond the instructor's control. Scores are adjusted to take into account student work habits (item 13), student desire to take the course regardless of who taught it (item 15), and instructor reported class size.

**Your Average Scores**

	Your Average (5-point scale)	
	Raw	Adj.
<b>A. Progress on Relevant Objectives</b> <sup>1</sup> Seven objectives were selected as relevant (Important or Essential – see page 2)	3.2	3.3
<b>Overall Ratings</b>		
B. Excellent Teacher	3.4	3.5
C. Excellent Course	2.7	2.6
<b>D. Average of B &amp; C</b>	3.1	3.1
<b>Summary Evaluation (Average of A &amp; D)</b> <sup>1</sup>	3.2	3.2

<sup>1</sup> If you are comparing Progress on Relevant Objectives from one instructor to another, use the converted average.

<sup>2</sup> The process for computing Progress on Relevant Objectives for the Discipline and Institution was modified on May 1, 2006. Do not compare these results with reports generated prior to this date.

**Your Converted Average When Compared to All Classes in the IDEA Database**

Comparison Category	A. Progress on Relevant Objectives	Overall Ratings						Summary Evaluation (Average of A & D)	
		B. Excellent Teacher		C. Excellent Course		D. Average of B & C			
		Raw	Adj.	Raw	Adj.	Raw	Adj.	Raw	Adj.
<b>Much Higher</b> Highest 10% (63 or higher)									
<b>Higher</b> Next 20% (56–62)									
<b>Similar</b> Middle 40% (45–55)									
<b>Lower</b> Next 20% (38–44)									
<b>Much Lower</b> Lowest 10% (37 or lower)	38	39	38	39	30	28	34	34	36 37

**Your Converted Average When Compared to Your:**<sup>2</sup>

Discipline (IDEA Data)	37	41	41	44	30	32	36	38	37	40
Institution	33	39	34	38	23	30	29	34	31	37

**IDEA Discipline used for comparison:**  
 Engineering

### Student Ratings of Learning on Relevant (Important and Essential) Objectives

Average unadjusted (raw) and adjusted progress ratings are shown below for those objectives you identified as "Important" or "Essential." **Progress on Relevant Objectives** (also shown on page 1) is a weighted average of student ratings of the progress they reported on objectives selected as "Important" or "Essential" (double weighted). The percent of students rating each as "1" or "2" (either "no" or "slight" progress) and as "4" or "5" ("substantial" or "exceptional" progress) is also reported. These results should help you identify objectives where improvement efforts might best be focused. For resources on improving learning and teaching, please refer to the **POD-IDEA Center Notes** ([www.theideacenter.org/podidea](http://www.theideacenter.org/podidea)), and **POD-IDEA Center Learning Notes** ([www.theideacenter.org/podidea/PODNotesLearning.html](http://www.theideacenter.org/podidea/PODNotesLearning.html)).

	Importance Rating	Your Average (5-point scale)		Percent of Students Rating	
		Raw	Adj.	1 or 2	4 or 5
		1. Gaining factual knowledge (terminology, classifications, methods, trends)	Essential	3.4	3.2
2. Learning fundamental principles, generalizations, or theories	Essential	3.4	3.2	21%	46%
3. Learning to <i>apply</i> course material (to improve thinking, problem solving, and decisions)	Essential	3.4	3.4	21%	48%
4. Developing specific skills, competencies, and points of view needed by professionals in the field most closely related to this course	Important	3.1	3.1	27%	38%
5. Acquiring skills in working with others as a member of a team	Minor/None				
6. Developing creative capacities (writing, inventing, designing, performing in art, music, drama, etc.)	Minor/None				
7. Gaining a broader understanding and appreciation of intellectual/cultural activity (music, science, literature, etc.)	Minor/None				
8. Developing skill in expressing myself orally or in writing	Minor/None				
9. Learning how to find and use resources for answering questions or solving problems	Essential	3.6	3.9	16%	61%
10. Developing a clearer understanding of, and commitment to, personal values	Minor/None				
11. Learning to <i>analyze</i> and <i>critically evaluate</i> ideas, arguments, and points of view	Important	2.6	2.7	50%	26%
12. Acquiring an interest in learning more by asking my own questions and seeking answers	Essential	2.8	3.1	47%	32%
<b>Progress on Relevant Objectives</b>		<b>3.2</b>	<b>3.3</b>		

<sup>1</sup> The process for computing Progress on Relevant Objectives for the Discipline and Institution was modified on May 1, 2006. Do not compare these results with reports generated prior to this date.

Your Converted Average When Compared to Group Averages					
IDEA Database		IDEA Discipline		Your Institution <sup>1</sup>	
Raw	Adj.	Raw	Adj.	Raw	Adj.
37	34	34	35	32	36
Much Lower	Much Lower	Much Lower	Much Lower	Much Lower	Much Lower
39	36	35	36	34	36
Lower	Much Lower	Much Lower	Much Lower	Much Lower	Much Lower
38	39	37	41	34	41
Lower	Lower	Much Lower	Lower	Much Lower	Lower
32	32	31	35	29	36
Much Lower	Much Lower	Much Lower	Much Lower	Much Lower	Much Lower
48	53	48	56	43	51
Similar	Similar	Similar	Higher	Lower	Similar
29	31	35	38	26	31
Much Lower	Much Lower	Much Lower	Lower	Much Lower	Much Lower
33	38	36	43	29	39
Much Lower	Lower	Much Lower	Lower	Much Lower	Lower
38	39	37	41	33	39

Much Higher = Highest 10% of classes (63 or higher)  
 Higher = Next 20% (56-62)  
 Similar = Middle 40% (45-55)  
 Lower = Next 20% (38-44)  
 Much Lower = Lowest 10% (37 or lower)

### Description of Students

The two items describing your students relate to their academic motivation and work habits and are key factors in developing adjusted ratings.

Student Description	Your Average (5-point scale)
13. As a rule, I put forth more effort than other students on academic work.	4.0
15. I really wanted to take this course regardless of who taught it.	3.0

Your Converted Average When Compared to Group Averages					
IDEA Database		IDEA Discipline		Your Institution	
60	Higher	54	Similar	53	Similar
44	Lower	38	Lower	34	Much Lower

Much Higher = Highest 10% of classes (63 or higher)  
 Higher = Next 20% (56-62)  
 Similar = Middle 40% (45-55)  
 Lower = Next 20% (38-44)  
 Much Lower = Lowest 10% (37 or lower)

**Statistical Detail**

The details on this page are of interest primarily to those who want to confirm scores reported on pages 1 and 2 or who want to determine if responses to some items were distributed in an unusual manner.

Converted Averages are reported only for relevant learning objectives (Important or Essential – see page 2) and other items for which comparisons were provided.

	Number Responding								Converted Avg.		Comparison Group Average		
	1	2	3	4	5	Omit	Avg.	s.d.	Raw	Adj.	IDEA	Discipline	Institution
	<b>1. Gaining factual knowledge (terminology,...</b>	2	12	19	20	10	0	3.4	1.1	37	34	4.0	4.1
<b>2. Learning fundamental principles, generalizations, or...</b>	3	10	21	17	12	0	3.4	1.1	39	36	3.9	4.1	4.2
<b>3. Learning to apply course material (to improve...</b>	4	9	20	20	10	0	3.4	1.1	38	39	4.0	4.0	4.2
<b>4. Developing specific skills, competencies, and points...</b>	9	8	22	17	7	0	3.1	1.2	32	32	4.0	4.0	4.2
5. Acquiring skills in working with others as a member of a...	13	17	16	11	6	0	2.7	1.3	NA	NA	3.9	4.0	4.0
6. Developing creative capacities (writing, inventing,...	20	18	14	8	2	1	2.3	1.1	NA	NA	3.9	3.5	4.0
7. Gaining a broader understanding and appreciation of...	26	13	17	4	2	1	2.1	1.1	NA	NA	3.7	3.2	3.9
8. Developing skill in expressing myself orally or in writing	28	16	10	6	2	1	2.0	1.1	NA	NA	3.8	3.4	3.9
<b>9. Learning how to find and use resources for...</b>	2	8	14	26	12	1	3.6	1.0	48	53	3.7	3.7	4.0
10. Developing a clearer understanding of, and...	18	17	18	7	2	1	2.3	1.1	NA	NA	3.8	3.5	3.9
<b>11. Learning to analyze and critically evaluate ideas,...</b>	15	16	15	11	5	1	2.6	1.3	29	31	3.8	3.5	4.0
<b>12. Acquiring an interest in learning more by asking...</b>	9	20	13	12	8	1	2.8	1.3	33	38	3.8	3.6	4.0

Key: 1=No apparent progress 2=Slight progress 3=Moderate progress 4=Substantial progress 5=Exceptional progress      Bold=Selected as Important or Essential

13. As a rule, I put forth more effort than other students on...	0	2	13	33	14	1	4.0	0.8	60	NA	3.6	3.8	3.9
14. My background prepared me well for this course's...	1	2	20	26	13	1	3.8	0.9	NA	NA	NA	NA	NA
15. I really wanted to take this course regardless of who...	8	18	14	11	11	1	3.0	1.3	44	NA	3.3	3.5	3.9
16. As a result of taking this course, I have more positive...	11	16	16	9	10	1	2.9	1.3	33	31	3.9	3.9	4.1
17. Overall, I rate this instructor an excellent teacher.	4	7	22	17	12	1	3.4	1.1	38	39	4.2	4.0	4.3
18. Overall, I rate this course as excellent.	17	7	21	13	4	1	2.7	1.3	30	28	3.9	3.9	4.2

Key: 1 = Definitely False 2 = More False than True 3 = In Between 4 = More True than False 5 = Definitely True      Item 14 is an experimental item. Therefore, no comparative information is available.

No Additional Questions.

Notes
Discipline code selected on FIF: 1400 Discipline code used for comparison: 1400

**Urroz, Gilberto**  
**Utah State University**

ENGR 2450, Section: 002  
 Numerical Methods for Engineers  
 Spring 2013  
 Local code: 20260000



IDEA Short Form Report

To learn more, see the Interpretive Guide: [www.theideacenter.org/shortguide.pdf](http://www.theideacenter.org/shortguide.pdf)

Of the 67 students enrolled, 51 responded (76%). Feedback from individual classes is always useful to guide improvement efforts. Typically, multiple classes should be used for evaluation, using more classes when they are small (fewer than 10) or when they have low response rates (less than 60%) (see [www.theideacenter.org/AdminDecisions](http://www.theideacenter.org/AdminDecisions)).

**Summary Evaluation of Teaching Effectiveness**

Teaching effectiveness is assessed in two ways: **A. Progress on Relevant Objectives**, a weighted average of student ratings of the progress they reported on objectives selected as "Important" or "Essential" (double weighted) and **B. Overall Ratings**, the average student agreement with statements that the teacher and the course were excellent. The **SUMMARY EVALUATION** is the average of these two measures. Individual institutions may prefer to combine these measures in some other manner to arrive at a summary judgment.

**Converted Averages** are standardized scores that take into account the fact that the average ratings for items on the IDEA form are not equal; students report more progress on some objectives than on others. Converted scores all have the same average (50) and the same variability (a standard deviation of 10); about 40% of them will be between 45 and 55. Because measures are not perfectly reliable, it is best to regard the "true score" as lying within plus or minus 3 of the reported score.

**For comparative purposes, use converted averages.** Your converted averages are compared with those from all classes in the IDEA database. If enough classes are available, comparisons are also made with classes in the same broad *discipline* as this class and/or with all classes that used IDEA at your *institution*. The *Interpretive Guide* offers some suggestions for using comparative results; **some institutions may prefer to establish their own "standards" based on raw or adjusted scores rather than on comparative standing.**

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**Your Average Scores**

	Your Average (5-point scale)	
	Raw	Adj.
<b>A. Progress on Relevant Objectives</b> <sup>1</sup> Nine objectives were selected as relevant (Important or Essential – see page 2)	2.7	2.8
<b>Overall Ratings</b>		
B. Excellent Teacher	2.2	2.3
C. Excellent Course	2.0	1.9
<b>D. Average of B &amp; C</b>	2.1	2.1
<b>Summary Evaluation (Average of A &amp; D)</b> <sup>1</sup>	2.4	2.5

<sup>1</sup> If you are comparing Progress on Relevant Objectives from one instructor to another, use the converted average.

<sup>2</sup> The process for computing Progress on Relevant Objectives for the Discipline and Institution was modified on May 1, 2006. Do not compare these results with reports generated prior to this date.

**Your Converted Average When Compared to All Classes in the IDEA Database**

Comparison Category	A. Progress on Relevant Objectives	Overall Ratings						Summary Evaluation (Average of A & D)		
		B. Excellent Teacher		C. Excellent Course		D. Average of B & C				
		Raw	Adj.	Raw	Adj.	Raw	Adj.	Raw	Adj.	
<b>Much Higher</b> Highest 10% (63 or higher)										
<b>Higher</b> Next 20% (56–62)										
<b>Similar</b> Middle 40% (45–55)										
<b>Lower</b> Next 20% (38–44)										
<b>Much Lower</b> Lowest 10% (37 or lower)	29	30	19	21	18	17	19	19	24	25

**Your Converted Average When Compared to Your:**<sup>2</sup>

<b>Discipline (IDEA Data)</b>	28	33	24	27	18	21	21	24	25	29
<b>Institution</b>	25	32	14	18	11	19	13	19	19	26

**IDEA Discipline used for comparison:**  
 Engineering

### Student Ratings of Learning on Relevant (Important and Essential) Objectives

Average unadjusted (raw) and adjusted progress ratings are shown below for those objectives you identified as "Important" or "Essential." **Progress on Relevant Objectives** (also shown on page 1) is a weighted average of student ratings of the progress they reported on objectives selected as "Important" or "Essential" (double weighted). The percent of students rating each as "1" or "2" (either "no" or "slight" progress) and as "4" or "5" ("substantial" or "exceptional" progress) is also reported. These results should help you identify objectives where improvement efforts might best be focused. For resources on improving learning and teaching, please refer to the **POD-IDEA Center Notes** ([www.theideacenter.org/podidea](http://www.theideacenter.org/podidea)), and **POD-IDEA Center Learning Notes** ([www.theideacenter.org/podidea/PODNotesLearning.html](http://www.theideacenter.org/podidea/PODNotesLearning.html)).

	Importance Rating	Your Average (5-point scale)		Percent of Students Rating	
		Raw	Adj.	1 or 2	4 or 5
		1. Gaining factual knowledge (terminology, classifications, methods, trends)	Essential	2.8	2.8
2. Learning fundamental principles, generalizations, or theories	Essential	2.7	2.7	48%	28%
3. Learning to <i>apply</i> course material (to improve thinking, problem solving, and decisions)	Essential	2.6	2.7	48%	24%
4. Developing specific skills, competencies, and points of view needed by professionals in the field most closely related to this course	Important	2.5	2.5	55%	27%
5. Acquiring skills in working with others as a member of a team	Essential	3.3	3.4	28%	46%
6. Developing creative capacities (writing, inventing, designing, performing in art, music, drama, etc.)	Minor/None				
7. Gaining a broader understanding and appreciation of intellectual/cultural activity (music, science, literature, etc.)	Minor/None				
8. Developing skill in expressing myself orally or in writing	Essential	2.3	2.6	56%	16%
9. Learning how to find and use resources for answering questions or solving problems	Essential	3.1	3.3	36%	40%
10. Developing a clearer understanding of, and commitment to, personal values	Minor/None				
11. Learning to <i>analyze</i> and <i>critically evaluate</i> ideas, arguments, and points of view	Important	2.6	2.8	53%	29%
12. Acquiring an interest in learning more by asking my own questions and seeking answers	Essential	2.2	2.4	59%	18%
<b>Progress on Relevant Objectives</b>		<b>2.7</b>	<b>2.8</b>		

Your Converted Average When Compared to Group Averages					
IDEA Database		IDEA Discipline		Your Institution <sup>1</sup>	
Raw	Adj.	Raw	Adj.	Raw	Adj.
26	25	22	26	20	26
Much Lower	Much Lower	Much Lower	Much Lower	Much Lower	Much Lower
25	24	21	24	19	25
Much Lower	Much Lower	Much Lower	Much Lower	Much Lower	Much Lower
24	25	22	27	20	28
Much Lower	Much Lower	Much Lower	Much Lower	Much Lower	Much Lower
20	21	19	24	17	26
Much Lower	Much Lower	Much Lower	Much Lower	Much Lower	Much Lower
41	41	39	42	40	45
Lower	Lower	Lower	Lower	Lower	Similar
28	33	33	39	25	32
Much Lower	Much Lower	Much Lower	Lower	Much Lower	Much Lower
38	43	38	44	33	41
Lower	Lower	Lower	Lower	Much Lower	Lower
29	32	35	39	26	32
Much Lower	Much Lower	Much Lower	Lower	Much Lower	Much Lower
22	26	25	31	19	28
Much Lower	Much Lower	Much Lower	Much Lower	Much Lower	Much Lower
<b>29</b>	<b>30</b>	<b>28</b>	<b>33</b>	<b>25</b>	<b>32</b>

Much Higher = Highest 10% of classes (63 or higher)  
 Higher = Next 20% (56-62)  
 Similar = Middle 40% (45-55)  
 Lower = Next 20% (38-44)  
 Much Lower = Lowest 10% (37 or lower)

<sup>1</sup> The process for computing Progress on Relevant Objectives for the Discipline and Institution was modified on May 1, 2006. Do not compare these results with reports generated prior to this date.

### Description of Students

The two items describing your students relate to their academic motivation and work habits and are key factors in developing adjusted ratings.

Student Description	Your Average (5-point scale)
13. As a rule, I put forth more effort than other students on academic work.	3.6
15. I really wanted to take this course regardless of who taught it.	2.7

Your Converted Average When Compared to Group Averages					
IDEA Database		IDEA Discipline		Your Institution	
49	Similar	42	Lower	43	Lower
39	Lower	32	Much Lower	30	Much Lower

Much Higher = Highest 10% of classes (63 or higher)  
 Higher = Next 20% (56-62)  
 Similar = Middle 40% (45-55)  
 Lower = Next 20% (38-44)  
 Much Lower = Lowest 10% (37 or lower)

### Statistical Detail

The details on this page are of interest primarily to those who want to confirm scores reported on pages 1 and 2 or who want to determine if responses to some items were distributed in an unusual manner.

Converted Averages are reported only for relevant learning objectives (Important or Essential – see page 2) and other items for which comparisons were provided.

	Number Responding							Converted Avg.		Comparison Group Average			
	1	2	3	4	5	Omit	Avg.	s.d.	Raw	Adj.	IDEA	Discipline	Institution
	<b>1. Gaining factual knowledge (terminology,...</b>	<b>9</b>	<b>14</b>	<b>9</b>	<b>14</b>	<b>4</b>	<b>1</b>	<b>2.8</b>	<b>1.3</b>	<b>26</b>	<b>25</b>	<b>4.0</b>	<b>4.1</b>
<b>2. Learning fundamental principles, generalizations, or...</b>	<b>9</b>	<b>15</b>	<b>12</b>	<b>9</b>	<b>5</b>	<b>1</b>	<b>2.7</b>	<b>1.2</b>	<b>25</b>	<b>24</b>	<b>3.9</b>	<b>4.1</b>	<b>4.2</b>
<b>3. Learning to apply course material (to improve...</b>	<b>11</b>	<b>13</b>	<b>14</b>	<b>7</b>	<b>5</b>	<b>1</b>	<b>2.6</b>	<b>1.3</b>	<b>24</b>	<b>25</b>	<b>4.0</b>	<b>4.0</b>	<b>4.2</b>
<b>4. Developing specific skills, competencies, and points...</b>	<b>17</b>	<b>10</b>	<b>9</b>	<b>8</b>	<b>5</b>	<b>2</b>	<b>2.5</b>	<b>1.4</b>	<b>20</b>	<b>21</b>	<b>4.0</b>	<b>4.0</b>	<b>4.2</b>
<b>5. Acquiring skills in working with others as a member...</b>	<b>5</b>	<b>9</b>	<b>13</b>	<b>10</b>	<b>13</b>	<b>1</b>	<b>3.3</b>	<b>1.3</b>	<b>41</b>	<b>41</b>	<b>3.9</b>	<b>4.0</b>	<b>4.0</b>
6. Developing creative capacities (writing, inventing,...	17	10	11	10	2	1	2.4	1.3	NA	NA	3.9	3.5	4.0
7. Gaining a broader understanding and appreciation of...	25	7	11	4	2	2	2.0	1.2	NA	NA	3.7	3.2	3.9
<b>8. Developing skill in expressing myself orally or in...</b>	<b>16</b>	<b>12</b>	<b>14</b>	<b>6</b>	<b>2</b>	<b>1</b>	<b>2.3</b>	<b>1.2</b>	<b>28</b>	<b>33</b>	<b>3.8</b>	<b>3.4</b>	<b>3.9</b>
<b>9. Learning how to find and use resources for...</b>	<b>7</b>	<b>11</b>	<b>12</b>	<b>12</b>	<b>8</b>	<b>1</b>	<b>3.1</b>	<b>1.3</b>	<b>38</b>	<b>43</b>	<b>3.7</b>	<b>3.7</b>	<b>4.0</b>
10. Developing a clearer understanding of, and...	21	9	11	5	2	3	2.1	1.2	NA	NA	3.8	3.5	3.9
<b>11. Learning to analyze and critically evaluate ideas,...</b>	<b>13</b>	<b>13</b>	<b>9</b>	<b>8</b>	<b>6</b>	<b>2</b>	<b>2.6</b>	<b>1.4</b>	<b>29</b>	<b>32</b>	<b>3.8</b>	<b>3.5</b>	<b>4.0</b>
<b>12. Acquiring an interest in learning more by asking...</b>	<b>19</b>	<b>10</b>	<b>11</b>	<b>7</b>	<b>2</b>	<b>2</b>	<b>2.2</b>	<b>1.2</b>	<b>22</b>	<b>26</b>	<b>3.8</b>	<b>3.6</b>	<b>4.0</b>

Key: 1=No apparent progress 2=Slight progress 3=Moderate progress 4=Substantial progress 5=Exceptional progress

Bold=Selected as Important or Essential

13. As a rule, I put forth more effort than other students on...	2	0	18	25	5	1	3.6	0.8	49	NA	3.6	3.8	3.9
14. My background prepared me well for this course's...	3	5	16	13	13	1	3.6	1.2	NA	NA	NA	NA	NA
15. I really wanted to take this course regardless of who...	15	4	17	7	7	1	2.7	1.4	39	NA	3.3	3.5	3.9
16. As a result of taking this course, I have more positive...	15	7	14	10	4	1	2.6	1.3	29	30	3.9	3.9	4.1
17. Overall, I rate this instructor an excellent teacher.	19	11	13	7	1	0	2.2	1.2	19	21	4.2	4.0	4.3
18. Overall, I rate this course as excellent.	21	13	14	2	1	0	2.0	1.0	18	17	3.9	3.9	4.2

Key: 1 = Definitely False 2 = More False than True 3 = In Between 4 = More True than False 5 = Definitely True

Item 14 is an experimental item. Therefore, no comparative information is available.

No Additional Questions.

#### Notes

Consider selecting fewer objectives as "Important" or "Essential."  
Discipline code selected on FIF: 1400  
Discipline code used for comparison: 1400



**Appendix H**  
**Informed Consent Form**





Department of Engineering Education  
4160 Old Main Hill  
Logan, UT 84322-4160  
(435) 797-7020



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### INFORMED CONSENT

#### Testing the Flipped Classroom with Numerical Methods for Engineers

**Purpose** In an effort to improve the learning experience of students at Utah State University, we—Dr. Gilberto Urroz and Jacob Bishop—are introducing new instructional strategies and supporting course materials. These methods and materials have been designed to provide an enhanced learning experience for students. To support this effort, we will also be analyzing student performance on homework and tests. We are asking students enrolled ENGR 2450 – Numerical Methods for Engineers, Spring 2013 to provide consent for us to analyze data from their course assignments, including data from homework and tests.

**Procedures** If you provide consent, copies of your homework and tests—after personally identifiable information has been removed—will be used in an effort to study and improve on current teaching methods & materials. If you do not provide consent, you will still be required to participate in course activities, but none of your data will be analyzed and used for research purposes. On this form, “participation in this research” constitutes permission to use your data for analysis as outlined above.

**Risks** There is a small risk that the information on homework, exams, or other course materials could be accidentally disclosed. Several measures will be used to minimize this risk; see “Confidentiality” below.

**Benefits** Consent to have your data included in this study may not lead to any direct immediate benefits. However, the results may provide evidence and important information regarding the nature of effective instruction. This study may fill an important gap in many literature bases. Thus, the implications might generalize to classroom instructional practice as well as other instruction such as employee training.

**Explanation & offer to answer questions** Dr. Gilberto Urroz or Jacob Bishop has explained this research study to you and answered your questions. If you have other questions or research-related problems, you may reach Dr. Urroz at (435) 797-3379 or by e-mail at gilberto.urroz@usu.edu.

**Compensation:** You will not be paid or compensated directly for your participation.

**Voluntary nature of participation and right to withdraw without consequence** Participation in this research is entirely voluntary. You may withdraw your consent at any time. If you choose to do so, your data will still be protected according to federal, state, and university guidelines, but will not be copied or included with other data as part of this research study.

**Confidentiality** Research records will be kept confidential, consistent with federal and state regulations. Only the investigator and research staff listed on the bottom of this form will have access to the data which will be kept in a locked file cabinet in a locked room to maintain confidentiality. Before records are used for research, they will be irreversibly anonymized—all personally identifiable information will be removed and replaced with an anonymous identifier. No link of the identifier to a specific individual will be maintained. All consent forms will be stored in a locked drawer in a locked office. Electronic information will be kept in password protected computer files.



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Page 2 of 2

### INFORMED CONSENT

Testing the Flipped Classroom with Numerical Methods for Engineers

**IRB Approval Statement** The Institutional Review Board for the protection of human participants at USU has approved this research study. If you have any pertinent questions or concerns about your rights or a research-related injury, you may contact the IRB Administrator at (435) 797-0567 or email [irb@usu.edu](mailto:irb@usu.edu). If you have a concern or complaint about the research and you would like to contact someone other than the research team, you may contact the IRB Administrator to obtain information or to offer input.

**Copy of consent** You have been given two copies of this Informed Consent. Please sign both copies and retain one copy for your files.

**Investigator Statement** “I certify that the research study has been explained to the individual, by me or my research staff, and that the individual understands the nature and purpose, the possible risks and benefits associated with taking part in this research study. Any questions that have been raised have been answered.”

\_\_\_\_\_  
Gilberto E. Urroz, PhD, Principal Investigator  
(435) 797-3379  
[gilberto.urroz@usu.edu](mailto:gilberto.urroz@usu.edu)

\_\_\_\_\_  
Jacob L. Bishop, Student Researcher  
(801) 796- 2529  
[jacob.bishop@usu.edu](mailto:jacob.bishop@usu.edu)

**Signature of Participant** By signing below, I agree to participate.

\_\_\_\_\_  
Participant's signature

\_\_\_\_\_  
Date



## Appendix I

### Analysis of Scores by Declared Major

The purpose of this section is to ascertain whether average scores attained by students on the measures used in this study differ by major, and whether there is an interaction between this and group membership (treatment or comparison). Average scores attained by students, grouped by major only, are shown in Table I.1. Since a multivariate analysis of variance (MANOVA) considering group (treatment or comparison) was already conducted, it is not necessary to conduct this analysis again using major as a second independent variable. Rather, it is sufficient to note from the previous MANOVA results that a significant difference does in fact exist. The next step is to perform four separate  $2 \times 4$  factorial ANOVAs for each dependent variable (homework, exams, quizzes, concept test).

For each of these ANOVAs, section and major are the independent variables. Section has two levels (treatment and control), whereas major has four (biological engineering [BIEN], civil engineering [CIEN], computer engineering [CMPE], and electrical engineering [ELEN]). There were too few individuals with declared majors other than those just listed to warrant analysis. The  $p$ -values corresponding to the main effects of major and section that resulted

Table I.1: Mean Differences by Major

Major	N	Homework	Exams	Quizzes	CT Pre	CT Post	CT Diff
BIEN	23	93.275	85.605	85.780	66.964	77.989	12.202
CIEN	41	86.994	84.889	70.542	63.322	79.573	15.789
CMPE	13	90.623	88.096	77.315	69.231	80.769	11.538
ELEN	29	87.821	84.478	74.857	62.284	76.509	14.224
ENVE	5	82.346	88.759	75.080	58.750	80.000	21.250
OTHER	7	79.636	78.502	60.692	58.333	72.321	12.500

*Note.* BIEN = biological engineering; CIEN = civil engineering; CMPE = computer engineering; ELEN = electrical engineering; ENVE = environmental engineering. CT Pre = concept pretest; CT Post = concept posttest; CT Diff = concept test difference score (posttest - pretest).

from this analysis are shown in Table I.2. No interaction effects were significant, but two significant main effects were discovered. First, the main effect of section on homework score. This has been discussed in detail in Section 5.1 of the main document. The second main effect that was present was a main effect of major on quiz scores. As a follow-up to the  $2 \times 4$  ANOVA with quiz scores as the dependent variable,  $t$ -tests were conducted to discover which majors were different from each other. The results of these pairwise comparisons are shown in Table I.3, and indicated that biological engineering majors scored significantly higher on quizzes than both civil and electrical engineering majors.

The observed differences between groups with different declared majors on quizzes shown in Table I.1 may be due to actual differences in competency between students of these majors, with biological engineering students being the most capable. If this is the case, however, it is curious that biological engineering students only scored higher on non-proctored assignments, including homework and quizzes (although differences in homework scores were not statistically significant) and did not score higher on proctored evaluations, including section exams and the concept test. Quizzes were administered online, and were to be completed individually. A single attempt was allowed for each student, after which the quiz was scored, and the correct were shown. This administration method allows for the possibility that biological engineering students were collaborating to complete quizzes, whereas students with other majors tended to complete them individually.

In summary, the analysis of scores by major revealed significant differences in quiz

Table I.2: ANOVA Results, Showing  $p$ -values for the Main Effects of Major and Section.

Measure	$p$ -value	
	Major	Section
Homework	0.15	<0.01
Examinations	0.34	0.98
Quizzes	<0.01	0.26
Concept Test	0.67	0.87

*Note.*  $p$  = probability of getting a test statistic at least as extreme as observed, given the null.

Table I.3: Pairwise Comparison Results for Quiz Scores by Major

Major	h	$p$	$t$	df	$SD$	99.17% CI	
						Lower	Upper
BIEN-CIEN	1	0.000	3.71	62	15.78	4.04	26.44
BIEN-CMPE	0	0.034	2.21	34	11.02	-2.24	19.18
BIEN-ELEN	1	0.003	3.18	50	12.29	1.49	20.35
CIEN-CMPE	0	0.232	-1.21	52	17.59	-22.13	8.59
CIEN-ELEN	0	0.298	-1.05	68	16.95	-15.49	6.86
CMPE-ELEN	0	0.610	0.51	40	14.33	-10.82	15.74

*Note.* BIEN = biological engineering; CIEN = civil engineering; CMPE = computer engineering; ELEN = electrical engineering; ENVE = environmental engineering. h = null hypothesis rejected? (1 = true, 0 = false);  $p$  = probability of getting a test statistic at least as extreme as  $t$  given the null;  $t$  = value of  $t$ -statistic with df degrees of freedom; CI = confidence interval.

scores, but not in scores on homework, exams, or the concept test. These differences in quiz scores could be due to greater ability or preparation for these students or due to more widespread cheating among biological engineering students (perhaps the most plausible explanation). Further study (with more careful safeguards against cheating) is necessary before the importance of major on the outcomes of homework and quizzes can be determined. The results of this analysis suggest that comparisons among engineering students on numerical methods problem solving and conceptual understanding abilities using the proctored examinations from this study were appropriate, without regard to declared major. Comparisons of students on homework assignments also appears to be justified. However, it is recommended that future studies carefully consider students' declared major, since differences according to group membership may arise, as was observed for this study on quiz scores.



## Curriculum Vitae



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## Curriculum Vitae

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### Personal Information

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Name Jacob Lowell Bishop  
 Email jacob.bishop@usu.edu, bishop.jacob@gmail.com  
 Phone (801) 796-2529  
 Citizenship United States  
 Languages English, Spanish

### Education

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Dec. 2013 **PhD-Engineering Education**  
 Utah State University - Logan, UT  
 Dissertation: *A Controlled Study of the Flipped Classroom with Numerical Methods for Engineers*  
 Advisors: Dr. Gilberto Urroz and Dr. Matthew Verleger  
 Dec. 2010 GPA: 3.88  
**MS-Mechanical Engineering**  
 Brigham Young University - Provo, UT  
 Thesis: *Search Pattern Generation and Path Management for Search over Rough Terrain with a Small UAV*  
 Advisor: Dr. Timothy McLain  
 Apr. 2007 GPA: 3.39  
**BS-Mechanical Engineering**  
 Brigham Young University-Idaho - Rexburg, ID  
 GPA: 3.61

### Research and Teaching Appointments

---

**Graduate Research Assistant**  
 Utah State University Department of Psychology Aug. 2012 Present  
**Graduate Instructor**  
 Utah State University Department of Engineering Education Jan. 2013 May 2013  
**Research/Teaching Assistant**  
 Utah State University Department of Engineering Education Jan. 2011 Aug. 2012  
**Graduate Research Assistant**  
 Brigham Young University Department of Mechanical Eng. Sep. 2007 Dec. 2010  
**Summer Researcher**  
 Air Force Research Laboratory, Wright-Patterson AFB,  
 Air Vehicles Directorate May 2009 July 2009  
**Mech. Engineering Intern; Engineering Geophysics Research**  
 Idaho National Laboratory May 2006 Aug. 2007  
**Mathematics Tutor**

**Jacob Bishop** **Curriculum Vitae (page 2)**

Brigham Young University—Idaho Department of Mathematics Jan. 2005 Apr. 2006

**Research Interests**

- |  |  |
|--|--|
| <ul style="list-style-type: none"> <li>• Open Online Education</li> <li>• Flipped Classroom</li> <li>• Intelligent Tutoring Systems</li> <li>• Educational Data Mining</li> <li>• Psychometrics</li> </ul> | <ul style="list-style-type: none"> <li>• Structural Equation Modeling</li> <li>• Latent Variable Analysis</li> <li>• Control of Unmanned Aerial Vehicles</li> <li>• Robot Path Planning</li> </ul> |
|--|--|

**Leadership and Volunteer Service****Chair**

School Community Council, Bridger Elementary School      Sept. 2013    Present

**President**

ASEE Student Section at Utah State University      Apr. 2013    Present

**Information Chair**

ASEE Student Division (national-level)      June 2013    Present

**Co-Chair**

School Community Council, Bridger Elementary School      Sept. 2012    Sept. 2013

**Webmaster/VCS Administrator**

MAGICC Lab at Brigham Young University      Apr. 2008    Dec. 2010

**Vice-Chair**

ASME Student Section at BYU-Idaho      Sept. 2006    Sept. 2007

**Secretary**

ASME Student Section at BYU-Idaho      Sept. 2005    Sept. 2006

**Publications****Journal Articles (Peer-Reviewed)**

Geiser, C., Bishop, J., Lockhart, G., Shiffman, S., & Grenard, J. (in press). Analyzing latent state-trait and multiple-indicator latent growth curve models as multilevel structural equation models. *Frontiers in Quantitative Psychology and Measurement*.

Bishop, J., & Geiser, C. (2013). Modeling latent growth with multiple indicators: A comparison of three approaches. *Psychological Methods*.

**Conference Papers (Peer-Reviewed)**

Bishop, J. (2013, October). Testing the flipped classroom with model-eliciting activities and video lectures in a mid-level undergraduate engineering course. In *Frontiers in education conference (FIE), 2013*. Oklahoma City, Oklahoma.

Last Updated October 16, 2013

**Jacob Bishop****Curriculum Vitae (page 3)**

- Bishop, J., & Verleger, M. (2013, June). The flipped classroom: A survey of the research. In *American society for engineering education annual conference*. Atlanta, Georgia. Retrieved from <http://www.asee.org/public/conferences/20/papers/6219/view>
- Bishop, J., & Verleger, M. (2011). Work in progress—using the Levenshtein distance to examine changes to teams' model-eliciting activity solutions throughout a semester. In *Frontiers in education conference (FIE), 2011* (Vol. 0, p. S4D-1-S4D-3). Los Alamitos, CA, USA: IEEE Computer Society. doi:10.1109/FIE.2011.6142809
- Bishop, J., & Verleger, M. (2012, October). Feedback effects: Comparing the change resulting from peer and TA feedback to student solutions of model-eliciting activities. In *Frontiers in education conference (FIE), 2012* (Vol. 0, pp. 1–6). Los Alamitos, CA, USA: IEEE Computer Society. doi:10.1109/FIE.2012.6462434
- Millet, P. T., Casbeer, D. W., Mercker, T., & Bishop, J. (2010, August). Multi-agent decentralized search of a probability map with communication constraints. In *AIAA guidance, navigation and control conference*. Toronto, Ontario Canada: American Institute of Aeronautics and Astronautics. doi:10.2514/6.2010-8424

**Conference Posters/Abstracts (Peer-Reviewed)**

- Bishop, J. (2013, June). Proposing the exploration of stochastic models for research in engineering education. In *American society for engineering education annual conference*. Atlanta, Georgia.
- Bishop, J. & Geiser, C. (2013, May). Multiple-indicator latent growth curve models: An analysis of the second-order growth model and two less restrictive alternatives. In *Modern modeling methods (M<sup>3</sup>) conference*. Storrs, Connecticut.

**Teaching Experience**

Spring 2013

**Numerical Methods for Engineers**

Numerical analyses applied to engineering problems, including simultaneous solutions of linear and nonlinear equations, optimization, curve fitting, numerical differentiation and integration, and numerical solutions of differential equations.

**Ad Hoc Journal Reviewer**

Frontiers in Psychology (Quantitative Psychology and Measurement)

Computers in Education

Jacob Bishop

Curriculum Vitae (page 4)

Journal of Field Robotics

Last Updated October 16, 2013