Design of CubeSat Solar Panels Considering Acoustic Pressure

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Premise

During launch the rocket produces a tremendous amount of sound, averaging around **160 dB** in the local area. This acoustic pressure can crack solar cells if the array wall deflects too much.

Figure 1: Falcon 9 launch. Image provided by SpaceX.

The University of Illinois at Urbana-Champaign has designed thin carbon fiber solar panels which simultaneously act as the outer skin of the CubeSat bus. The solar cell arrays run down the panel, connected by an innovative ultrathin flexible power and data harness. The cells are very delicate and will crack if their center deflects approximately 1.8 mm relative to its edges.

An analysis is conducted to find the minimum thickness for the carbon fiber panel of the system under acoustic pressure. For worst case analysis, a 6U bus is chosen because it has the most surface area for acoustic pressure to act upon out of the buses the University makes. On board electronics will fail if they are subjected to an acoustic pressure greater than 130 dB. Therefore, this pressure level will serve as the upper bound to which the panels need to be subjected. The deflection is solved numerically using Matlab to find the minimum thickness before the solar cells crack under deflection.

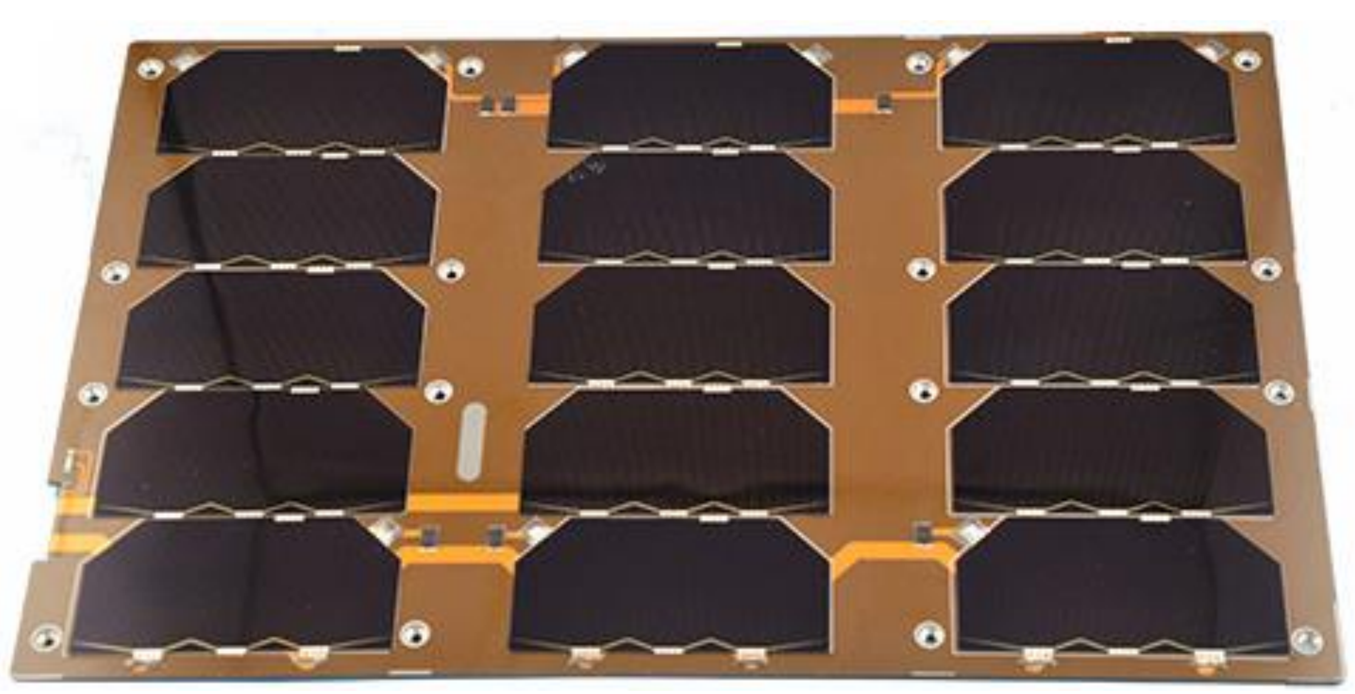

Figure 2: 6U solar panel array. Image provided by Innovative Solutions In Space.

Assumptions

1. The satellite is subject to 130 dB of sound. This is a worst case approach because at greater sound intensities major electronic components begin to fail.
2. The sound pressure is uniform across the face of the CubeSat.
3. Analysis on the largest drum face will ensure that smaller drum faces also do not fail.
4. The edges of the drum face are firmly clamped such that it can neither rotate nor deflect. The corners of the drum face are screwed to the frame of the CubeSat.
5. The panel is a symmetrically supported thin plate.
6. The material properties are for standard carbon fiber for any thickness.

Schematic

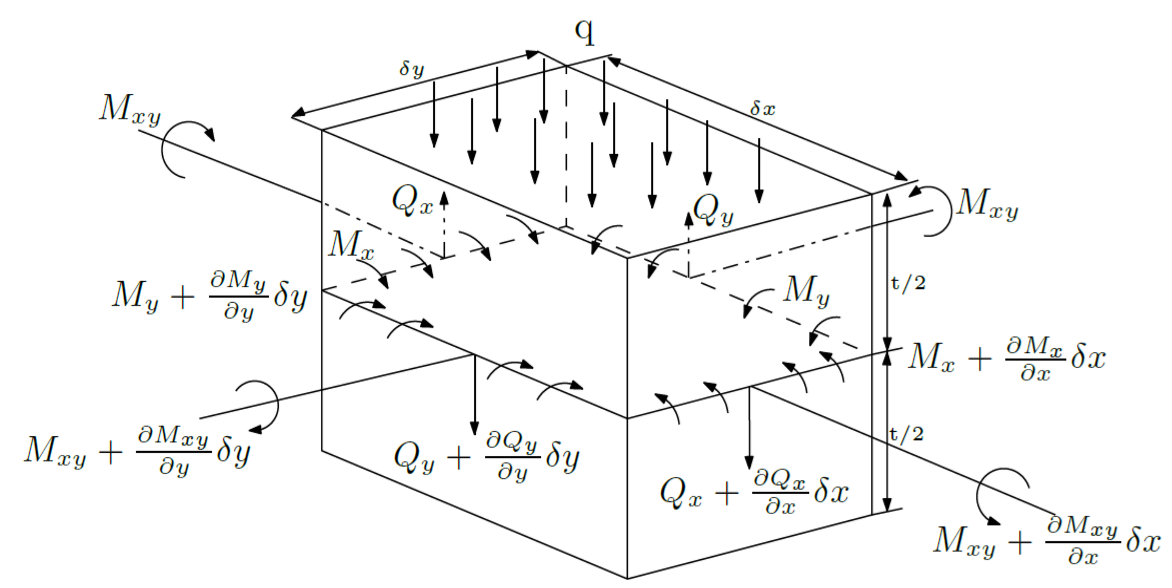

Figure 3: Plate Element Subject to Bending, Twisting, and Transverse Load[1]

The differential equation for deflection can be expressed as the following:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

Using the Fourier method of separation of variables, with the origin at the center of the plate of length 2a and width 2b, the general form of the solution is:

$$w = w_0 + \frac{q}{D}\sum_{n=0}^{\infty}(-1)^n\left(A_n\frac{\cosh\alpha_n y}{\cosh\alpha_n b} + B_n\frac{y\sinh\alpha_n y}{b\cosh\alpha_n b}\right)\cos\alpha_n x$$
$$+\frac{q}{D}\sum_{n=0}^{\infty}(-1)^n\left(C_n\frac{\cosh\beta_n x}{\cosh\beta_n a} + D_n\frac{x\sinh\beta_n x}{a\cosh\beta_n a}\right)\cos\beta_n y$$

Boundary Conditions

The largest drum face across the entire side of a 6U CubeSat which has symmetric boundaries of two opposite sides clamped and two opposite sides simply supported. The two opposite simply supported edges have the following boundary conditions[2]:

$$(w)_{x=-a,x=a} = 0, \qquad \left(\frac{\partial^2 w}{\partial x^2}\right)_{x=-a,x=a} = 0$$

Which provides the particular solution:

$$w_0 = \frac{q}{24D}(x^4 - 6a^2x^2 + 5a^4)$$

It also determines that coefficients C_n and D_n are equal to zero.
Applying the clamped boundary conditions to determine the other coefficients. For the clamped edges:

$$(w)_{y=-b,y=b} = 0 \;\&\; \left(\frac{\partial w}{\partial y}\right)_{y=-b,y=b} = 0$$

Which yield the expressions for the coefficients A_n and B_n:

$$A_n = -B_n\left(\coth\alpha_n b + \frac{1}{\alpha_n b}\right) \;\&\; B_n = \frac{2b\tanh\alpha_n b}{a\,\alpha_n^4(\tanh\alpha_n b + \alpha_n b\cosh^{-2}\alpha_n b)}$$

Failure Conditions

Condition 1:

The solar cells the University uses have been found to crack at a deflection of 1.8 mm relative to their corner. For a marginal factor of safety that tolerance is set to 1 mm. Therefore, the simulation has a constraint that the difference in deflection from the center of the panel and a point at (40 mm, 20 mm) cannot exceed 1 mm. The smallest thickness that meets this condition would be the minimum thickness.

Condition 2:

A conservative method of making sure the solar cell does not break is to use the law of similar triangles. This assumes the deflection is linear which, for small acoustic pressures, is accurate under small angle approximation. Therefore the angle of deflection for the plate cannot exceed the angle of deflection of the solar array at fracture. The distance from the corner of the solar cell to the center is 44.7 mm and over that distance it can deflect at most 1mm, which is a deflection angle of 1.281 degrees. The distance from the corner to the center of the plate is 203.4 mm which allows a maximum deflection of 4.55 mm.

Results

The simulation iterates over decreasing thicknesses until either condition is met. As a result the minimum thickness is found to be 0.1579 mm which generates the following surface plot over the entire plate:

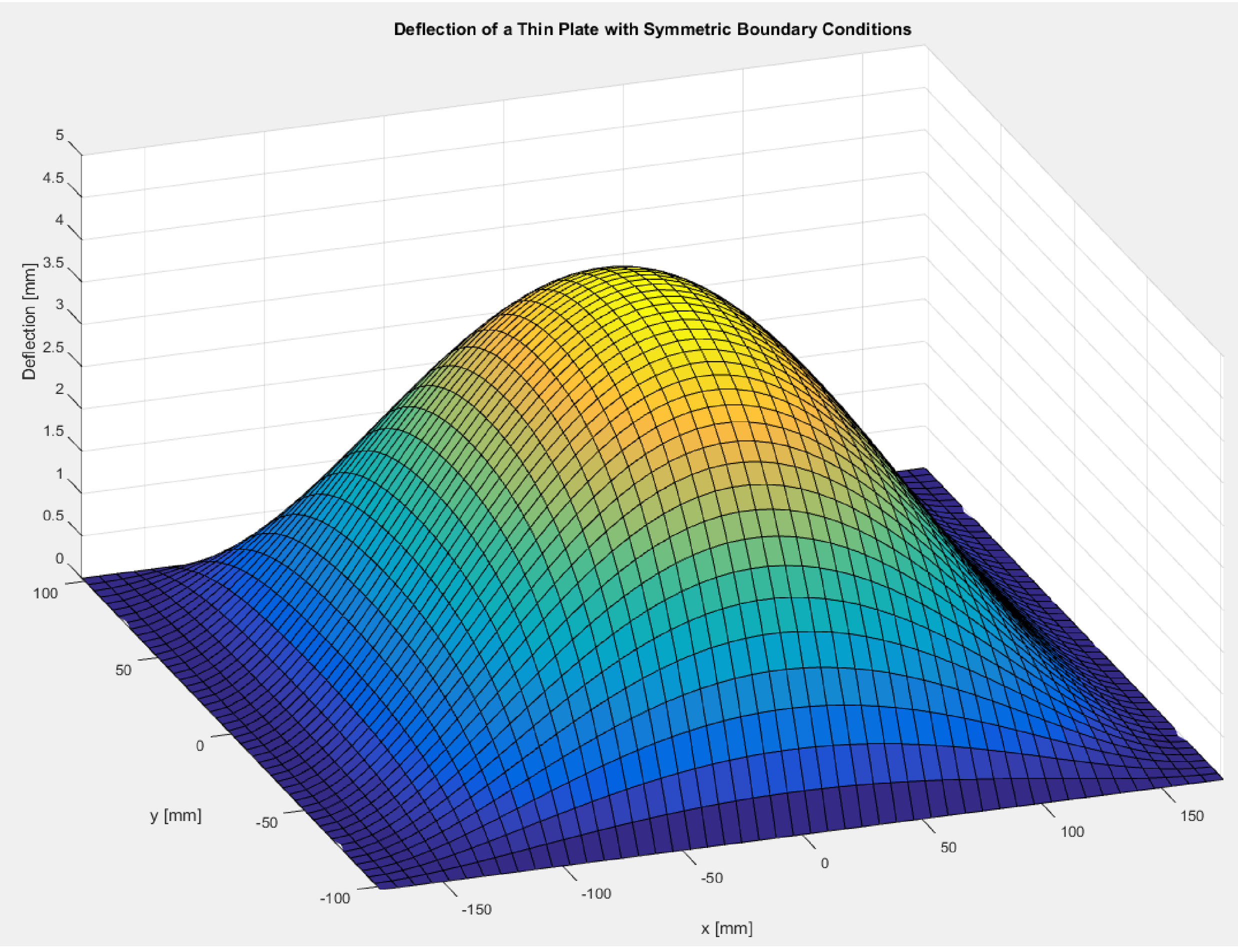

Figure 4: Deflection of plate over entire 6U CubeSat face.

Ultimately the analytical solutions were able to model the plate system used on 6U CubeSats. While the minimum thickness was numerically derived there were several assumptions required that can be further explored. In particular, the elastic modulus of the carbon fiber is of particular interest because the panels are made at the University and might not align well with commercial samples. In addition, the material properties used in the simulation do not shift for varying thicknesses. This is important for carbon fiber because the final calculated thickness is 0.1579 mm which is about the thickness of a single carbon fiber layer. Therefore, the ARES lab at the University currently makes six layer carbon fiber panels for smaller CubeSats, which is more than sufficient to handle the loads induced by acoustic pressure.

References

1. Megson T. H. G., *Aircraft Structures for Engineering Students*, 5th Edition, Elsevier Ltd., Waltham, MA, 2013.
2. Batista M., "Uniformly Loaded Rectangular Thin Plates with Symmetrical Boundary Conditions," University of Ljubljana, 2010. Retrieved from Cornell University Library.