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Temperature Forecasting at Logan, Utah, for the Snowmelt-Runoff Season

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TEMPERATURE FORECASTING AT LOGAN, UTAH, FOR THE SNOWMELT-RUNOFF SEASON

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INTRODUCTION

The purpose of this thesis is to investigate the practicability of forecasting temperatures for the spring and summer snowmelt-runoff season. A successful temperature forecast for Logan, Utah, would be useful in studies of Logan River streamflow, as well as other related applications.

Logan, Utah, is situated in a region where the snow accumulates during the winter and early spring at high altitudes and remains until the proper sequence of meteorological events provides the thermodynamic conditions essential for melting. Snow surveys provide an index to the water available on a watershed. Temperature conditions during the melt period largely determine the available water yield. Therefore, a temperature forecast is needed, along with results from the snow surveys, to predict the time, rate, and volume of streamflow from snowmelt.

From the results of a temperature forecast, snow surveys, precipitation records, and streamflow data, the probable shape of the streamflow hydrograph for a watershed can be predicted. A predicted hydrograph would allow time for planning to obtain more efficient use of water for power, irrigation, and other uses. Also, in some areas the information on expected streamflow may be helpful in planning the operation of multiple-purpose reservoirs or for the prediction of flood or drought conditions.

Since temperatures during the snowmelt season are an excellent index of snowmelt conditions, this thesis concentrates on the derivation of a satisfactory temperature forecasting procedure for April, May, and June. Also, a forecast is made of monthly summer temperatures.

Objectives

The major objectives of this study are:

1. To derive an economical, fast, and efficient temperature forecasting procedure for the snowmelt-runoff season.

2. To estimate the reliability of the forecast.

REVIEW OF LITERATURE

Snowmelt index

Much has been written concerning the many factors which contribute to snowmelt, but from a practical point of view, the following quotation shows the utility of temperature as a snowmelt index (17, p. 193):

> Temperature was used because it was generally thought to be the best index of the heat transfer processes associated with snowmelt and because it was (and in many cases will continue to be) the only reliable and regularly available meteorological variable.

Likewise, Riesbol (13, p. 596) states: "For practical application, however, the temperature of the air seems to be the most significant single index of melting conditions." In view of such statements, air temperature is assumed to be a satisfactory index of snowmelt.

The choice remains as to the most representative snowmelt index among maximum, minimum, and average daily temperatures. Gay (7) suggests preference for maximum daily temperature over minimum or mean daily temperatures. There may be some benefit from using a combination of maximum and minimum temperatures, but the computations are more difficult and of doubtful value (17). Maximum daily temperatures have been selected as the snowmelt index for this study.

Weather forecasting

Temperature forecasting may be considered a part of long-range weather forecasting; therefore, the following discussion will deal mainly with the more broad field of long-range weather forecasting.

As indicated by Ezekiel (5), weather forecasting, like any other forecast, is hazardous for the future can never be perfectly known.

In everyday life, estimates of future conditions are usually based on hunches, waves of opinion, recent happenings, rule-of-thumb analyses, or even blind guess-work. But, to be useful, all that is required is that a forecast method be more reliable, on the average, than past methods.

Accurate long-range weather forecasts have considerable value; therefore, many forecasting methods have been attempted. According to the observations of Lester (10), numerous of these forecasts have been based on plant, animal, bird, or insect traits. Also, Hewson (8) indicates that much consideration has been given to the position of the moon as an indicator of future weather conditions, even though such considerations lack substantiation by fact.

Turning to more scientific weather forecasting, Byers (3) suggests that 3 main approaches are used: (1) the statistical approach, (2) the geophysical or cosmic approach, and (3) the meteorological approach. This classification seems to agree with the writing of McNish (11). Each approach is briefly discussed in this order.

The statistical approach is applied in various ways, but is generally based on so called "persistence forecasts" or the tendency of weather to continue along certain patterns. This approach depends upon the better than chance correlation of weather conditions during one period with weather conditions during the following period (2). The procedure generally employed to discover such correlations is to select elements which suggest a relationship and then to measure the success by statistical means (11). Gay (7) has studied the relationship between temperature conditions before and during the snowmelt season. Koelzer and Ford (9) have used temperature conditions preceding snowmelt as an index to improve multiple correlation results in snowmelt-runoff studies. Correlation studies, conducted for a large number of meteorological series, show that variations in some series are followed by corresponding variations in other series. From results of these studies, forecasts were made for summer rainfall in India, Australia, and South Africa (2). A similar procedure was used by Schell (15) to predict the winter precipitation in Montana.

The geophysical or cosmic approach is used to relate temperature and other weather elements with phenomena known to influence the weather. Some investigators claim that a correlation exists between energy output of the sun and the world weather. At present, however, this method has not proven to be of great value (8). Studies of other phenomena, such as ocean currents, extent of the polar ice cap, and sea-water temperatures produce successful results in some regions (3).

The meteorological approach is probably more extensive and more widely accepted than any other approach. Generally, it is highly scientific and requires an understanding of large-scale air movement, as well as data on world-wide meteorological conditions. As early as 1920, a Russian bureau called "The Institute of Long-Range Weather Forecasts" issued forecasts for 10 days to 3 months in advance, and after 1932 forecasts were issued as much as 5 months in advance. The general basis for this system of forecasting is explained by Schell (14, p. 142):

> In sub-polar regions are found strong accumulations of heavy cold air. For reasons as yet not completely understood, lumps or masses of air separate from these regions every now

> and then. If one can determine the places from which these air-masses or "tongues" of air start and the paths along which they travel, one can naturally draw corresponding conclusions regarding the weather in regions over which these air-masses travel. One can also draw conclusions regarding the weather in adjoining regions, since the weather in the latter is determined to a large extent by that in the region occupied by the air-masses.

Similarly, Van Ornum (16, p. 5) states, "Long range weather forecasting (for the Northern Hemisphere) is basically a study of these two air currents, the westerlies and the polar outbreaks." Also he says (p. 4), "It turns out that changes in airflow half way around the earth can very definitely affect our weather within a week, and may continue to do so for several months."

With due respect for the advancements made in the field of long-range weather forecasting, further research is necessary to arrive at an acceptable procedure which can be easily understood and applied to all regions.

A. E. Chard* says of a weather forecast used in 1956, "The over-all forecast for the 3-months period was fairly accurate but weather variation for days and weeks during the period were often inaccurately forecast." The present use of long-range weather forecasts as an aid in runoff forecasting is further explained by Chard (4).

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* A. E. Chard, Development Engineer, Powell River Company Limited, Powell River, British Columbia, Canada. The use of a 3 months weather forecast has been explained in correspondence with Mr. Chard.

PROCEDURE

Selection of forecasting method

As previously discussed, long-range temperature forecasting is approached by statistical, geophysical, or meteorological methods. Some of the more important factors which have determined the approach to be used in this study are:

(1) The geophysical or cosmic approach has not proven of great value in long-range forecasting.

(2) The effective use of the meteorological approach requires an understanding of the mechanics and thermodynamics of large-scale air-mass movement beyond the scope of this particular study.

(3) The meteorological approach is not fully developed for all regions and all seasons of the year.

(4) Improvement of an existing statistical method, utilizing readily available data, shows promise.

The more simple statistical approach is used in this study. The apparent success of a statistical forecasting procedure, based on the persistence tendency of the weather, developed by Robert W. Gay* further supports this method (7). Although a statistical approach may not predict the day-to-day temperature sequence accurately, it does offer the opportunity to tell whether the temperatures are going to be above or below normal.

Source of data

The daily maximum temperature data for this study were taken

* Robert W. Gay was Hydrologist for the Bureau of Reclamation at Boise, Idaho, until his death in May, 1953. Correspondence from the Boise office, dated August 31, 1956, is written, "We use some of his procedures in present operations. No further research on this problem has been done."

from Weather Bureau publications under the title "Climatological Data for Utah." The Logan Weather Station temperature data were tabulated for the years of 1923 to 1956 (snow survey records begin in 1924 for the area) and checked for errors. Missing data were provided by comparing temperature trends at Logan with temperature trends at nearby stations. After the tabulations of the temperature data were checked and missing data filled in (table 1), the entire record was transferred into punch cards to facilitate computation by IBM accounting machines.

General multiple linear regression

Multiple linear regression methods are used to relate antecedent temperatures to temperatures for the following snowmelt-runoff period. The general form of the regression equation is (12, 6):

$$Y = b_0 + b_1 X_1 + b_2 X_2 + \dots + b_n X_n \quad (1)$$

in which

Y = An average daily maximum temperature for a designated period of the snowmelt-runoff season.

X's = Average daily maximum temperatures, or some function of them, for designated periods preceding the snowmelt-runoff season.

b_0 = A constant (Y intercept).

$b_1, b_2, \dots b_n$ = The regression coefficients relating X and Y terms.

To develop systems of independent (X) variables in equation (1), to represent the temperatures during the period preceding snowmelt, 4 methods are used (figure 1). The antecedent time interval is arbitrarily set to include the 180-day period ending March 16 to allow time for the preparation of temperature forecasts prior to the preparation of runoff forecasts on April 1. The 4 methods of selecting the independent variables are designated Methods I, II, III, and IV. Each of these methods is briefly discussed.

Method I is based on an orthogonal polynomial procedure. Daily temperature measurements are equally spaced in time; therefore, a convenient method of fitting the time-temperature curve is by the application of orthogonal polynomials.* In this regression procedure the orthogonal polynomials are so constructed that any term of the polynomial is independent of all other terms. The form of the orthogonal polynomial equation is:

$$Y = A_0 + A_1\xi_1' + A_2\xi_2' + \ldots + A_k\xi_k' \qquad (2)$$

where

Y = Dependent variable(2-day sums of maximum daily temperatures)

A's are constants (orthogonal polynomial coefficients)

ξ''s are orthogonal polynomial values

The coefficients (A's) are evaluated by fitting the orthogonal polynomials to the temperature data. These coefficients are used to define the time-temperature curves. For a more complete discussion of the derivation and application of orthogonal polynomials, see Anderson and Houseman (1).

The independent variables for Methods II, III, and IV are average daily maximum temperatures for the numbers of days shown in column 2 of tables 2, 3, and 4. For example: Method II variable (X_1) is the average daily maximum temperature for the 5-day period from March 12-16 (inclusive), and variable (X_2) is the average temperature for the

* Polynomials which are independent (non-correlated)

40-day period the previous fall. Thus, for Method II, temperatures are averaged for gradually decreasing numbers of days nearest the snowmelt season. Method III variables represent shorter day-periods nearest the snowmelt season and much longer periods during the previous fall. Method IV allows only one variable for the fall and winter average temperatures and is similar to a system suggested by Gay (7).

The dependent (Y) variables are the same for all 4 methods and represent average daily maximum temperatures for 5-day, 15-day, and monthly day-periods as shown in table 5.

After the dependent and independent variables are determined for each year of record, the sums, sums of squares, and sums of products are computed. The system of normal equations are developed as follows:

$$n\,b_0 + \Sigma X_1\,b_1 + \Sigma X_2\,b_2 + \ldots + \Sigma X_n\,b_n = \Sigma Y$$

$$\Sigma X_1\,b_0 + \Sigma X_1^2\,b_1 + \Sigma X_1 X_2\,b_2 + \ldots + \Sigma X_1 X_n\,b_n = \Sigma X_1 Y$$

$$\vdots$$

$$\Sigma X_n\,b_0 + \Sigma X_1 X_n\,b_1 + \Sigma X_2 X_n\,b_2 + \ldots + \Sigma X_n^2\,b_n = \Sigma X_n Y$$

The solution of this system of equations yields the multiple regression coefficients b_0, b_1, b_2, . . . b_n. After the regression coefficients are determined, an estimated temperature ($\hat{Y}$) is computed for any appropriate set of independent (X) variables by solving the forecast equation:

$$\hat{Y} = b_0 + b_1 X_1 + b_2 X_2 + \ldots + b_n X_n \qquad (3)$$

PRESENTATION AND ANALYSIS OF DATA

Computations and presentation

Practically all of the computations for this study were made on IBM accounting machines, and no attempt is made to discuss each step of the machine operations. However, a typical example of the computations shows the steps required in obtaining the final solutions. Also, much of the basic data, solutions, and results are tabulated.

Orthogonal polynomials (Method I)

Temperature data for the period from September 19, 1923, to March 16, 1924, is chosen to illustrate the method of fitting orthogonal polynomials. Figure 2 shows the 2-day sums of maximum daily temperatures plotted against time for the 180-day period from September 19, 1923, to March 16, 1924. These sums are also given in the first two columns of table 6. In table 6 the series of temperature sums are read in sequence beginning September 19-20 to the following March 15-16 by starting at the bottom of column 1, reading upward to the top of column, and then down column 2. The (s) values in column 3 are the sums of values in columns 1 and 2 for the same line. The (d) values in column 4 are found by subtracting the values in column 2 from those in column 1. As a check, the sum of column 3 should equal the sum of columns 1 and 2, while column 4 should equal the difference between columns 1 and 2. Columns 5, 6, 7, 8, and 9 are orthogonal polynomial values taken from the tables for N = 90 (1, p. 658). Only the lower half of the table of polynomial values (ξ''s) is given in table 6, since the upper half of the table is symmetrical with the lower half, except for a change of sign in columns 5, 7, and 9. Also, values of $(\Sigma \xi'^2)$, as shown in the computations to follow, are copied from the tables of orthogonal polynomial values for N = 90.

From the data in table 6 the polynomial regression coefficients A_0, A_1, A_2, A_3, A_4, and A_5 are determined as follows:

$$A_0 = \frac{s}{N} = \frac{7528}{90} = 83.6444$$

$$A_1 = \frac{\Sigma(d\,\xi_1')}{\Sigma(\xi_1')^2} = \frac{79{,}796}{242{,}970} = 0.3284$$

$$A_2 = \frac{\Sigma(s\,\xi_2')}{\Sigma(\xi_2')^2} = \frac{1{,}113{,}323}{73{,}765{,}692} = 0.015093$$

$$A_3 = \frac{\Sigma(d\,\xi_3')}{\Sigma(\xi_3')^2} = \frac{-7{,}350{,}777}{47{,}368{,}112{,}220} = -0.00015518$$

$$A_4 = \frac{\Sigma(s\,\xi_4')}{\Sigma(\xi_4')^2} = \frac{-243{,}704{,}537}{74{,}457{,}409{,}286{,}260} = -0.000003273$$

$$A_5 = \frac{\Sigma(d\,\xi_5')}{\Sigma(\xi_5')^2} = \frac{9{,}413{,}640{,}819}{19{,}677{,}062{,}617{,}741{,}620} = 0.0000004784$$

The result of the fitting process for a fifth degree polynomial is illustrated by the curve in figure 2. The ordinates to the regression curve are determined by substitution of the ξ' values in the following equation:

$$Y = 83.6444 + .3284\,\xi_1' + .01509\,\xi_2' - .0001552\,\xi_3' - .000003273\,\xi_4' + .0000004784\,\xi_5'$$

The computed ordinate values for the curve in figure 2 are given in table 7. Estimated 2-day temperature sums in columns 6 and 7 of table 7 are for the same periods as the values in columns 1 and 2 of table 6. Values in column 7 of table 7 are determined by changing the sign of terms in columns 1, 3, and 5.

At this point the tests of significance are made. The results in table 8 show the reduction in sums of squares attributable to each stage of regression. Also included in table 8 are the significance tests for the 1924-25 data which show that the fitting of a fifth degree polynomial is not always significant. Nevertheless, regression is carried to the fifth degree in all polynomial computations.

Table 9 contains the polynomial coefficients for the 33 periods from 1923-24 to 1955-56. These coefficients are used in the multiple regression procedure as a function of temperatures during the 180-day periods preceding March 17.

The volume of 2-day sums of maximum daily temperature comprising the basic data of this portion of the study is too great to be included in this thesis. However, the data are placed in the research project (Project 459) files for reference.

Multiple linear regression

The values of the independent (X) and dependent (Y) variables used in the multiple regression computations are given in tables 10 to 14. The values in table 10 are obtained by coding the orthogonal polynomial coefficients in table 9. The system of coding the polynomial coefficients to obtain the independent variables in Method I is:

$$X_1 = A_0/2 \qquad X_4 = A_3/2 + .0001800$$

$$X_2 = A_1/2 \qquad X_5 = A_4/2 + .00000550$$

$$X_3 = A_2/2 \qquad X_6 = A_5/2 + .0000002500$$

Coding makes all X's positive and simplifies the computations. The values shown in tables 11, 12, 13, and 14 are average daily maximum temperatures for the periods specified in tables 2, 3, 4, and 5, respectively.

The great bulk of computations required in solving the multiple regression equations prevents the inclusion of all data in this thesis. However, a single computation is given to illustrate how the multiple regression coefficients (b_0, b_1, . . . b_n) are determined. For reasons of simplicity in coding and presentation, Method III is chosen for this computation example. Also, the example is restricted to the relation between the independent (X) variables and a single dependent variable (Y_1).

The general system of linear normal equations for Method III and Y_1 is shown in table 15. The sums, sums of squares, and sums of products were computed by IBM machines from the data in tables 12 and 14 (for example: X_2 X_1 in column 2, line 3 was computed by adding the products of X_2 and X_1 in columns 2 and 3 of table 12; and X_1 Y_1 in column 8, line 2 was computed by adding the products of column 2 of table 12 and column 2 of table 14). The results of these computations are shown in table 16, which is designated "the original information matrix."

To facilitate the inversion of the matrix, the original information matrix is coded. Table 17 shows this matrix in coded form (the portion below the main diagonal is deleted because of symmetry). The coding factor notation is K_r (for rows) and K_c (for columns), as shown in table 16 for Method III.

The inverse matrix and regression coefficients were obtained by a "Machine Method of Matrix Inversion," developed by Dr. Rex L. Hurst.* The coded inverse matrix and coded regression coefficients are shown in table 18. Column 8 contains the coded values of the multiple regression coefficients (b'_0, B'_1, B'_2, B'_3, B'_4, B'_5, and b'_6) for Y_1. The coefficients are decoded by the formula:

$$b = b' \; (k_r/k_c)$$

where

b = Decoded regression coefficient

b' = Coded regression coefficient

k_r = Coding factor for the row containing b'

k_c = Coding factor for the column containing b'

The decoded multiple regression coefficients for all Y's and methods are given in tables 19-22.** (See column Y_1 in table 21 for decoded coefficients of Method III and Y_1.)

It is now possible to express the relationship between temperatures during the 180 days prior to March 16 and the following 5-day period (Y_1), for Method III. The expression is:

$$\hat{Y}_1 = 46.356 + .7028 X_1 + .3591 X_2 + .3673 X_3 - .3478 X_4 + .2604 X_5 - 1.0426 X_6$$

The substitution of X values from table 12 (for 1923-24) in the foregoing expression yields the predicted 5-day average maximum daily temperature for the period of March 17-21, 1924. The predicted temperature is:

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* Dr. Rex L. Hurst, Head of the Applied Statistics Department, Utah State Agricultural College, Logan, Utah.

** The accuracy of solutions shown in these tables is much less than was carried in the machine computations and exact duplication of results from presented values is not possible.

$\hat{Y}_1 = 42.885$ (actual temperature was 37.6° F.)

Coefficients of multiple correlation

At this point, the quantity R is introduced. R is designated the "coefficient of multiple correlation." R^2 is the proportion of the sum of squares of the dependent variable which is explained by multiple regression and is expressed as a ratio. This ratio is computed as follows:

$$R^2 = \frac{\text{Sum of squares attributable to regression}}{\text{Corrected total sums of squares of Y}}$$

Thus, R shows what part of the variability in the independent (Y) variables is accounted for by regression with all of the independent (X) variables. A more complete explanation of the coefficient of multiple correlation is given by Ostle (12, pp. 202-227).

Table 23 contains the total sums of squares, corrections, corrected sums of squares, and sums of squares attributable to regression for the 4 methods. From this table the value of R^2 for Method I, Y_1 is:

$$R^2 = \frac{1121.88}{2047.52} = 0.5479$$

Therefore, nearly 55 per cent of the variance in Y is explained by regression with the X's. Table 24 contains the complete tabulation of R^2 values for all Y's by the 4 methods. However, the R^2 values cannot be directly compared for each method, since the degrees of freedom associated with the 4 methods are not the same.

To obtain a valid comparison among methods, the F-test is used. The appropriate F-test (12, pp. 217-219) for Method I, Y_1 is presented by means of table 25, which leads to $F = 186.25 / 35.60 = 5.252$ with $\nu_1 = 6$ and $\nu_2 = 26$ degrees of freedom. The F value at the 0.5 per cent probability level is 4.1027 (from statistical F tables). Since the computed value is greater than 4.1027, it is significant at the 0.5 per cent probability level. The computed probability levels are shown (in parenthesis) in table 25 for all Y's and methods. In this table the lower probability levels indicate a high significance, and these probabilities are a valid comparison for each line of the table (Y periods). Thus, for period Y_1, all of the methods show high significance. However, for period Y_3, Method I is much better. The rank of methods from best to poorest, on a basis of probability levels, is: (1) Method I, (2) Method IV, (3) Method III, and (4) Method II. Also, it appears that Method I is much better during the spring, while during the summer, Method IV is best.

Comparison of trial forecasts

Trial forecasts are made by each of the 4 methods. Results are compared with actual temperatures for the years of 1923 and 1956 as a test of the reliability of each method. The basic data for 1923 are not used in the regression analysis; therefore, this year is independent. Data for 1956 are included, and the forecast for this year is historical (probably biased).

Table 26 contains a list of the independent (X) variables for 1922-23 and 1955-56. These values were computed by the procedures explained earlier. The forecasted temperatures for all Y periods were obtained by the solution of the forecasting equations, using X variables from table 26 and regression coefficients from tables 19-22. The actual temperatures for 1923, the 33-year average temperatures, and forecasted temperatures for 1923 are listed in table 27. Also included in this table are average, maximum, and standard deviations from actual temperatures for the periods Y_1 to Y_{21} (5-day periods). Similar information for 1956 is given in table 28.

Any forecast must show an improvement over the 33-year average temperature to be of value; therefore, the standard deviation from actual temperatures is used as an index of the success of the forecast methods. The per cent improvement over the 33-year average values is computed as follows:

$$\text{Improvement } (\%) = \frac{S_a - S_m}{S_a} \times 100$$

where

S_a = Standard deviation from actual of 33-year average temperatures

S_m = Standard deviation from actual of the method being studied

The per cent improvement for the 4 methods is given as follows:

Year	Method I	Method II	Method III	Method IV
1923	11.1%	0.6%	11.2%	19.0%
1956	33.1%	31.0%	20.4%	None

The results shown are in agreement with results from the F-tests and show Method I is preferable to the other methods.

Figure 3 shows actual 5-day average maximum temperatures plotted against forecasted temperatures for the periods Y_1 to Y_{21}, inclusive. The 45° diagonal line indicates the location of points having perfect agreement between forecasted and actual temperatures. Figure 4 shows actual temperatures for the same Y periods plotted against 33-year average 5-day temperatures. As shown in figure 4, 1923 temperatures were generally colder than normal, since most of the points lie above the diagonal line. Some improvement in favor of the forecasted temperatures is evident by a comparison of figures 3 and 4 and is due to a more even distribution of points around the diagonal line. Figures 5 and 6 are similar to figures 3 and 4 and illustrate temperature relationships for 1956. Figure 6 shows that 1956 temperatures were warmer than normal, since most of the points fall below the diagonal line. Also, the 1956 forecasted temperatures show much better agreement with actual temperatures and the points are again well distributed around the diagonal line.

THE 1957 TEMPERATURE FORECAST

The 1957 temperature forecast is presented to illustrate the complete forecasting procedure and to allow future evaluation of the forecasting methods.

Maximum daily temperature data, used to compute the independent (X) variables, were obtained from Weather Bureau publications and from the Logan, Utah, weather station on the Utah State Agricultural College campus. Missing data in the Logan record were estimated from data available at the experiment farm in North Logan, Utah. Table 29 contains the temperature data used in computing the 1957 forecast and underlined values indicate estimated temperatures missing from the Logan weather station record.

The 2-day sums of maximum daily temperatures and computations of the orthogonal polynomial coefficients are given in table 30. The orthogonal polynomial values for these computations are the same as shown in table 6 and are not repeated in table 30. The system of coding the polynomial coefficients for the independent variables in Method I are shown in table 31. The independent variables for Methods II, III, and IV, which are average temperatures for the intervals shown in tables 2, 3, and 4, are shown in table 31.

The solution of equation (3) for $\hat{Y}$ is the final step in the forecasting procedure. For substitution in this equation, the appropriate regression coefficients (b's) are chosen from tables 19, 20, 21, or 22 and the independent variables (X's) from table 31. Thus, to predict the 5-day average maximum daily temperature for the period from March 17 to 21, inclusive, the b's are taken from table 19 and X's from table 31 for Method I. The forecast equation, substituted values for Method I, and the forecasted temperature are given as follows:

$$\hat{Y} = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_5 X_5 + b_6 X_6$$

$$\hat{Y} = 56.2710 + (.1586)(45.38) + (-162.1186)(.1666)$$
$$+ (2008.3162)(.012849) + (-41168.871)(.0001763)$$
$$+ (1918832.8)(.000004968) + (-21506714)(.0000002851)$$

$\hat{Y} = 58.3^{\circ}$ F. (actual temperature for the period = 55.2° F.)

The forecasted average daily maximum temperatures for the 28 prediction periods (Y_1 to Y_{28}) are given in table 32 for the 4 methods studied. Actual temperatures for the March 17 to March 31 interval are included since the forecast was made on April 1, 1957.

CONCLUSIONS

1. As shown by the trial forecasts and higher significance level of the correlation coefficients, Method I probably gives the best overall temperature forecast among methods used in this study.
2. Due to the generally poor correlations found in the regression analysis, the statistical F-tests and trial forecasts are not conclusive in the comparison among methods.
3. Although Method I appears to give a more reliable forecast than other methods, it requires considerably more computation. Therefore, Method IV should be given consideration as a possible method for use.
4. Temperatures during the periods from April 1 to April 15, as well as other short intervals, do not correlate well under any method, and this is probably due to the extreme temperature fluctuations during these periods.
5. Temperature forecasting by the statistical approach used in this study is quick and simple after the relationships have been determined by multiple regression. However, the initial computations are laborious, especially without computing machines.
6. Fore more precise long-range temperature forecasting, the meteorological approach apparently offers more promise of ultimate success than any other method. This approach allows the physical factors causing temperature changes to be considered and evaluated.

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APPENDIX

Table 1. Maximum daily temperature data (°F.) for 1923

Day	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	33	15	42	54	60	60	88	81	84	66	52	32
2	31	18	43	52	64	66	90	83	85	64	54	33
3	38	19	34	43	67	70	92	84	85	68	52	34
4	34	18	27	46	70	70	84	84	85	62	49	31
5	43	19	29	57	70	69	83	82	85	58	51	40
6	44	21	37	41	72	73	85	82	85	57	54	44
7	51	29	49	46	75	75	85	83	84	56	55	52
8	46	32	38	50	78	73	87	85	86	49	51	35
9	42	24	31	48	81	79	86	87	87	48	54	25
10	48	28	35	59	73	79	86	87	86	48	53	21
11	41	30	37	63	60	87	90	89	88	54	52	24
12	37	31	36	60	61	84	91	90	82	52	52	36
13	30	30	39	59	49	66	89	86	82	53	51	36
14	33	32	27	54	52	76	88	88	81	54	49	35
15	32	30	39	61	61	76	85	85	80	57	48	34
16	47	31	43	69	62	59	89	83	79	58	53	30
17	48	30	28	70	69	66	87	87	59	56	49	36
18	37	32	32	63	67	71	90	83	65	52	46	32
19	34	33	45	46	75	72	88	84	70	55	45	24
20	39	39	37	48	76	73	93	85	80	60	49	32
21	38	37	35	43	64	64	92	82	74	64	48	35
22	42	34	38	36	66	71	89	85	75	57	51	34
23	42	33	45	48	72	74	91	86	74	50	53	29
24	38	44	35	53	81	67	85	87	62	46	61	33
25	32	37	44	58	75	78	85	86	68	49	52	40
26	36	32	43	54	68	75	81	80	61	50	41	33
27	33	32	48	60	62	75	87	81	41	54	36	34
28	32	36	53	67	68	81	88	85	48	52	38	32
29	27		59	64	50	87	87	81	60	46	40	31
30	25		61	51	65	89	91	77	65	51	38	34
31	22		64		64		82	78		50		29

Table 2. System of independent variables for Method II

Designation of variables	Number of days represented	Dates (inclusive)	
		Leap year	Other years
X_1	5	Mar. 12-16	Mar. 12-16
X_2	10	Mar. 2-11	Mar. 2-11
X_3	15	Feb. 16-Mar. 1	Feb. 15-Mar. 1
X_4	20	Jan. 27-Feb. 15	Jan. 26-Feb. 16
X_5	25	Jan. 2-26	Jan. 1-25
X_6	30	Dec. 3-Jan. 1	Dec. 2-31
X_7	35	Oct. 29-Dec. 2	Oct. 28-Dec. 1
X_8	40	Sept. 19-Oct. 28	Sept. 18-Oct. 27
	Total 180		

Table 3. System of independent variables for Method III

Designation of variables	Number of days represented	Dates (inclusive)	
		Leap year	Other years
X_1	3	Mar. 14-16	Mar. 14-16
X_2	6	Mar. 8-13	Mar. 8-13
X_3	12	Feb. 25-Mar. 7	Feb. 24-Mar. 7
X_4	24	Feb. 1-24	Jan. 31-Feb. 23
X_5	48	Dec. 15-Jan. 31	Dec. 14-Jan. 30
X_6	87	Sept. 19-Dec. 14	Sept. 18-Dec. 13
	Total 180		

Table 4. System of independent variables for Method IV

Designation of variables	Number of days represented	Dates (inclusive) Leap year	Other years
X_1	5	Mar. 12-16	Mar. 12-16
X_2	5	Mar. 7-11	Mar. 7-11
X_3	5	Mar. 2-6	Mar. 2-6
X_4	5	Feb. 26-Mar. 1	Feb. 25-Mar. 1
X_5	10	Feb. 16-Feb. 25	Feb. 15-Feb. 24
X_6	10	Feb. 6-Feb. 15	Feb. 5-Feb. 14
X_7	140	Sept. 19-Feb. 5	Sept. 18-Feb. 4
	Total 180		

Table 5. System of dependent variables for all methods

Designation of variables	Number of days represented	Dates (inclusive)
Y_1	5	March 17-21
Y_2	5	March 22-26
Y_3	5	March 27-31
Y_4	5	April 1-5
Y_5	5	April 6-10
Y_6	5	April 11-15
Y_7	5	April 16-20
Y_8	5	April 21-25
Y_9	5	April 26-30
Y_{10}	5	May 1-5
Y_{11}	5	May 6-10
Y_{12}	5	May 11-15
Y_{13}	5	May 16-20
Y_{14}	5	May 21-25
Y_{15}	5	May 26-30
Y_{16}	5	May 31-June 4
Y_{17}	5	June 5-9
Y_{18}	5	June 10-14
Y_{19}	5	June 15-19
Y_{20}	5	June 20-24
Y_{21}	5	June 25-29

Table 5. System of dependent variables for all methods (continued)

Designation of variables	Number of days represented	Dates (inclusive)
Y_{22}	15	March 17-31
Y_{23}	30	April 1-30
Y_{24}	31	May 1-30
Y_{25}	30	June 1-30
Y_{26}	31	July 1-31
Y_{27}	31	August 1-31
Y_{28}	30	September 1-30

Table 6. Two-day temperature sums for 1923-24 and orthogonal polynomial values for N = 90

Sum of 2-day temperatures		Sum (s)	Diff. (d)	ξ_1'	ξ_2'	ξ_3'	ξ_4'	ξ_5'
66	56	122	10	1	-1012	- 1012	1022626	1022626
69	67	136	2	3	-1009	- 3031	1012521	3053731
72	63	135	9	5	-1003	- 5035	992381	5042521
45	73	118	-28	7	- 994	- 7014	962346	6961206
60	67	127	- 7	9	- 982	- 8958	922626	8782626
96	63	159	33	11	- 967	-10857	873501	10480503
71	63	134	8	13	- 949	-12701	815321	12029693
67	40	107	27	15	- 928	-14480	748506	13406438
70	60	130	10	17	- 904	-16184	673546	14588618
78	18	96	60	19	- 877	-17803	591001	15556003
77	51	128	26	21	- 847	-19327	501501	16290505
113	61	174	52	23	- 814	-20746	405746	16776430
104	63	167	41	25	- 778	-22050	304506	17000730
97	41	138	56	27	- 739	-23229	198621	16953255
91	55	146	36	29	- 697	-24273	89001	16627005
102	68	170	34	31	- 652	-25172	- 23374	16018382
97	37	134	60	33	- 604	-25916	- 137454	15127442
103	44	147	59	35	- 553	-26495	- 252119	13958147
105	58	163	47	37	- 499	-26899	- 366179	12518617
105	60	165	45	39	- 442	-27118	- 478374	10821382
109	62	171	47	41	- 382	-27142	- 587374	8883634
100	64	164	36	43	- 319	-26961	- 691779	6727479
106	60	166	46	45	- 253	-26565	- 790119	4380189
102	57	159	45	47	- 184	-25944	- 880854	1874454
97	63	160	34	49	- 112	-25088	- 962374	- 751366
106	76	182	30	51	- 37	-23987	-1032999	- 3452989
99	81	180	18	53	41	-22631	-1090979	- 6179959
96	75	171	21	55	122	-21010	-1134494	- 8875394
121	71	192	50	57	206	-19114	-1161654	-11475734
115	82	197	33	59	293	-16933	-1170499	-13910489
108	87	195	21	61	383	-14457	-1158999	-16101987
115	85	200	30	63	476	-11676	-1125054	-17965122
107	78	185	29	65	572	- 8580	-1066494	-19407102
106	74	180	32	67	671	- 5159	- 981079	-20327197
96	76	172	20	69	773	- 1403	- 866499	-20616487
105	85	190	20	71	878	2698	- 720374	-20157610
115	93	208	22	73	986	7154	- 540254	-18824510
130	83	213	47	75	1097	11975	- 323619	-16482185
130	80	210	50	77	1211	17171	- 67879	-12986435
125	78	203	47	79	1328	22752	229626	- 8183610
89	82	171	7	81	1448	28728	571626	- 1910358
129	78	207	51	83	1571	35109	960921	6006627
136	73	209	63	85	1697	41905	1400381	15750857
149	72	221	77	87	1826	49126	1892946	27516302
150*	76	226	74	89	1958	56782	2441626	41507642
4529	2999	7528	1530					

* Sum of maximum daily temperatures for September 19 and 20, 1923.

Table 7. Calculations of the ordinate values for the fifth degree polynomial curve in figure 2

A_0 = 83.6444

$A_1 \xi_1'$	$A_2 \xi_2'$	$A_3 \xi_3'$	$A_4 \xi_4'$	$A_5 \xi_5'$	Polynomial ordinates	Polynomial ordinates
0.33	-16.12	0.16	-3.35	0.49	65.15	63.19
0.99	-16.07	0.47	-3.31	1.46	67.18	61.34
1.64	-15.98	0.78	-3.25	2.41	69.24	59.58
2.30	-15.83	1.09	-3.15	3.33	71.38	57.94
2.96	-15.64	1.39	-3.02	4.20	73.53	56.43
3.61	-15.40	1.69	-2.86	5.01	75.69	55.07
4.27	-15.12	1.97	-2.67	5.76	77.85	53.85
4.93	-14.78	2.25	-2.45	6.41	80.00	52.82
5.58	-14.41	2.51	-2.20	6.98	82.11	51.97
6.24	-13.97	2.76	-1.93	7.44	84.18	51.30
6.90	-13.49	3.00	-1.64	7.79	86.20	50.82
7.55	-12.97	3.22	-1.33	8.03	88.14	50.54
8.21	-12.39	3.42	-1.00	8.13	90.01	50.49
8.87	-11.77	3.61	-0.65	8.11	91.81	50.63
9.52	-11.10	3.77	-0.29	7.95	93.49	51.01
10.18	-10.39	3.91	0.08	7.66	95.08	51.58
10.84	- 9.62	4.02	0.45	7.24	96.57	52.37
11.49	- 8.81	4.11	0.83	6.68	97.94	53.38
12.15	- 7.95	4.17	1.20	5.99	99.20	54.58
12.81	- 7.04	4.21	1.57	5.18	100.37	55.97
13.46	- 6.09	4.21	1.92	4.25	101.39	57.55
14.12	- 5.08	4.18	2.26	3.22	102.34	59.30
14.78	- 4.03	4.12	2.59	2.10	103.20	61.20
15.43	- 2.93	4.03	2.88	0.90	103.95	63.23
16.09	- 1.78	3.89	3.15	-0.36	104.63	65.39
16.75	- 0.59	3.72	3.38	-1.65	105.25	67.61
17.41	0.65	3.51	3.57	-2.96	105.82	69.90
18.06	1.94	3.26	3.71	-4.25	106.36	72.22
18.72	3.28	2.97	3.80	-5.49	106.92	74.52
19.38	4.67	2.63	3.83	-6.66	107.49	76.79
20.03	6.10	2.24	3.79	-7.70	108.10	78.96
20.69	7.58	1.81	3.68	-8.59	108.81	80.99
21.35	9.11	1.33	3.49	-9.28	109.64	82.84
22.00	10.69	0.80	3.21	-9.72	110.62	84.46
22.66	12.31	0.22	2.84	-9.86	111.81	85.77
23.32	13.99	-0.42	2.36	-9.64	113.25	86.73
23.97	15.71	-1.11	1.77	-9.01	114.97	87.27
24.63	17.48	-1.86	1.06	-7.89	117.06	87.30
25.29	19.29	-2.66	0.22	-6.21	119.57	86.73
25.94	21.16	-3.53	-0.75	-3.92	122.54	85.56
26.60	23.07	-4.46	-1.87	-0.91	126.07	83.61
27.26	25.03	-5.45	-3.15	2.87	130.20	80.84
27.91	27.03	-6.50	-4.58	7.54	135.04	77.14
28.57	29.09	-7.62	-6.20	13.16	140.64	72.42
29.23	31.19	-8.81	-7.99	19.86	147.12	66.56

Table 8. Tests of significance

1923-24

Source of variation	Degrees of freedom	Sum of squares	Mean square	F-test
Total Y^2	90	688,328	--	
Correction for mean	1	629,675	--	
Deviations from mean	89	58,653	--	
Linear regression	1	26,205	26,205.00	71.97 (.005)
Deviations from linear regression	88	32,448	368.73	
Second degree term	1	17,731	17,731.00	104.82 (.005)
Deviation from regression	87	14,717	169.16	
Third degree term	1	1,140	1,140.00	7.22 (.01)
Deviation from regression	86	13,577	157.87	
Fourth degree term	1	797	797.00	5.30 (.025)
Deviation from regression	85	12,780	150.25	
Fifth degree term	1	4,530	4,530.00	46.03 (.005)
Deviation from regression	84	8,267	98.42	

1924-25

Source of variation	Degrees of freedom	Sum of squares	Mean square	F-test
Total Y^2	90	771,924	--	
Correction for mean	1	706,496	--	
Deviation from mean	89	65,428	--	
Linear regression	1	18,044	18,044.00	33.51 (.005)
Deviation from regression	88	47,384	538.45	
Second degree term	1	29,186	29,186.00	139.53 (.005)
Deviation from regression	87	18,198	209.17	
Third degree term	1	1,005	1,005.00	5.03 (.05)
Deviation from regression	86	17,193	199.92	
Fourth degree term	1	7,825	7,825.00	71.00 (.005)
Deviation from regression	85	9,368	110.21	
Fifth degree term	1	450	450.00	4.24 (.05)
Deviation from regression	84	8,918	106.17	

Table 9. Orthogonal polynomial coefficients, 1923-24 to 1955-56

Year	A_0	A_1	A_2	A_3	A_4	A_5
1923-24	83.64	.3284	.01509	-.0001552	-.00000327	+.0000004784
1924-25	88.60	.2727	.01989	-.0001457	-.00001025	+.0000001512
1925-26	91.34	.2711	.01712	-.0001629	+.00000096	-.0000000834
1926-27	93.29	.3668	.01530	-.0002269	-.00000467	+.0000001401
1927-28	90.99	.3810	.01872	-.0002586	-.00000496	+.0000001225
1928-29	84.58	.4440	.02300	-.0001493	-.00000061	+.0000000021
1929-30	91.68	.3404	.01948	-.0002456	-.00000015	+.0000001588
1930-31	81.11	.3326	.02469	-.0001624	-.00000608	-.0000000257
1931-32	80.26	.4177	.02004	-.0000938	-.00000860	-.0000003251
1932-33	85.20	.4340	.01812	-.0001431	-.00000008	-.0000000781
1933-34	103.64	.2688	.01747	-.0001128	-.00000122	-.0000001651
1934-35	90.67	.3433	.01436	-.0001587	-.00000451	-.0000001116
1935-36	90.14	.3491	.02127	+.0000514	+.00000309	+.0000000411
1936-37	86.58	.4517	.02124	-.0002025	+.00000190	+.0000002823
1937-38	98.41	.3242	.01563	-.0000588	-.00000125	-.0000000209
1938-39	89.23	.4469	.01929	+.0000936	+.00000534	-.0000002257
1939-40	98.57	.3562	.01429	-.0002158	+.00000148	+.0000003468
1940-41	90.04	.3217	.01780	-.0000562	-.00000360	-.0000002109
1941-42	83.02	.4035	.01252	-.0001815	-.00000158	-.0000000080
1942-43	90.48	.3797	.01835	-.0000022	-.00000226	-.0000002106
1943-44	89.71	.4482	.01915	-.0000233	-.00000135	+.0000001553
1944-45	89.64	.3846	.01889	+.0000265	-.00000638	-.0000003941
1945-46	87.14	.3362	.01820	-.0001531	-.00000542	-.0000003508
1946-47	88.67	.2599	.01625	-.0000857	+.00000169	+.0000002494
1947-48	87.77	.3966	.01960	+.0001378	-.00000332	-.0000002749
1948-49	82.83	.4475	.02512	-.0003516	-.00000210	+.0000000220
1949-50	92.49	.3438	.01655	-.0000857	+.00000023	+.0000005306
1950-51	93.37	.4020	.01335	-.0001545	-.00000592	+.0000000194
1951-52	82.86	.4250	.01501	+.0001044	-.00000360	-.0000001066
1952-53	99.92	.3997	.02253	+.0001430	-.00000507	-.0000004798
1953-54	95.94	.3761	.01912	-.0000646	-.00000255	+.0000000299
1954-55	87.79	.5064	.01566	-.0003399	-.00000104	+.0000001753
1955-56	89.18	.3888	.01137	-.0000154	+.00000012	-.0000003773

Table 10. Independent variables for Method I (coded values from table 9)

Year	X_1	X_2	X_3	X_4	X_5	X_6
1923-24	41.82	.1642	.00755	.0001024	.00000386	.0000004892
1924-25	44.30	.1364	.00995	.0001071	.00000037	.0000003256
1925-26	45.67	.1356	.00856	.0000985	.00000598	.0000002083
1926-27	46.65	.1834	.00765	.0000665	.00000316	.0000003201
1927-28	45.50	.1905	.00936	.0000507	.00000302	.0000003113
1928-29	42.29	.2220	.01150	.0001053	.00000519	.0000002511
1929-30	45.84	.1702	.00974	.0000572	.00000542	.0000003294
1930-31	40.56	.1663	.01235	.0000988	.00000246	.0000002371
1931-32	40.13	.2089	.01002	.0001331	.00000120	.0000000874
1932-33	42.60	.2170	.00906	.0001084	.00000546	.0000002109
1933-34	51.82	.1344	.00874	.0001236	.00000489	.0000001674
1934-35	45.34	.1717	.00718	.0001006	.00000324	.0000001942
1935-36	45.07	.1746	.01064	.0002057	.00000705	.0000002706
1936-37	43.29	.2259	.01062	.0000787	.00000645	.0000003912
1937-38	49.21	.1621	.00782	.0001506	.00000487	.0000002395
1938-39	44.62	.2235	.00965	.0002268	.00000817	.0000001371
1939-40	49.29	.1781	.00715	.0000721	.00000624	.0000004234
1940-41	45.02	.1609	.00890	.0001519	.00000370	.0000001445
1941-42	41.51	.2018	.00626	.0000892	.00000471	.0000002460
1942-43	45.24	.1899	.00918	.0001789	.00000437	.0000001447
1943-44	44.86	.2241	.00958	.0001683	.00000482	.0000003277
1944-45	44.82	.1923	.00945	.0001933	.00000231	.0000000529
1945-46	43.57	.1681	.00910	.0001034	.00000279	.0000000746
1946-47	44.34	.1300	.00813	.0001371	.00000635	.0000003747
1947-48	43.89	.1983	.00980	.0002489	.00000384	.0000001125
1948-49	41.42	.2238	.01256	.0000042	.00000445	.0000002610
1949-50	46.25	.1719	.00828	.0001371	.00000562	.0000005153
1950-51	46.69	.2010	.00668	.0001027	.00000254	.0000002597
1951-52	41.43	.2125	.00751	.0002322	.00000370	.0000001967
1952-53	49.96	.1999	.01127	.0002515	.00000296	.0000000101
1953-54	47.97	.1881	.00956	.0001477	.00000422	.0000002650
1954-55	43.90	.2532	.00783	.0000100	.00000498	.0000003377
1955-56	44.59	.1944	.00569	.0001723	.00000556	.0000000613

Table 11. Independent variables for Method II

Year	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
1923-24	36.6	39.7	41.3	34.6	25.3	32.9	48.3	58.2
1924-25	42.6	49.8	44.7	41.9	31.3	26.9	46.9	63.1
1925-26	58.0	47.3	40.6	38.4	32.4	37.9	46.6	62.6
1926-27	45.4	45.7	43.2	35.2	36.3	32.9	51.2	66.9
1927-28	45.8	48.4	39.0	36.1	33.0	29.1	50.5	67.6
1928-29	40.8	50.0	34.1	28.9	26.2	28.4	46.6	67.0
1929-30	56.4	45.3	45.1	38.1	22.7	42.3	44.6	67.1
1930-31	51.2	43.3	38.4	34.0	27.9	23.5	41.7	62.4
1931-32	37.8	35.2	34.8	33.8	24.1	30.7	39.8	64.2
1932-33	42.2	41.3	35.6	25.1	35.1	26.1	50.5	64.6
1933-34	63.2	53.2	45.1	47.1	38.9	44.0	50.6	70.0
1934-35	51.2	37.4	41.7	35.7	34.4	35.4	49.4	63.5
1935-36	47.6	46.5	42.7	32.5	33.8	36.7	42.1	67.5
1936-37	45.8	41.6	38.0	33.2	17.5	38.1	46.0	68.1
1937-38	49.6	48.2	40.7	42.8	38.9	40.9	50.7	67.2
1938-39	46.8	42.3	30.5	32.0	34.8	38.3	42.4	69.4
1939-40	51.0	45.4	44.5	40.1	33.4	43.7	54.2	66.2
1940-41	44.8	45.3	41.7	34.5	34.0	39.0	40.8	66.6
1941-42	38.4	38.7	25.9	36.3	25.6	36.3	46.1	60.9
1942-43	46.2	42.0	37.9	35.7	34.0	35.7	45.3	67.6
1943-44	36.0	43.3	37.1	35.3	27.1	37.4	46.5	69.3
1944-45	44.8	38.9	36.5	39.4	35.8	33.0	42.5	68.7
1945-46	43.4	46.2	39.9	32.1	32.6	33.0	43.9	64.6
1946-47	52.8	43.9	41.7	39.5	27.7	40.4	44.0	60.4
1947-48	43.6	35.0	43.2	31.2	37.7	31.2	41.1	68.6
1948-49	51.8	43.2	39.7	22.9	21.4	31.8	42.3	68.5
1949-50	38.0	47.3	46.8	31.2	34.9	35.5	54.4	62.3
1950-51	40.0	38.3	39.9	37.8	34.8	36.9	50.5	68.0
1951-52	32.4	34.5	29.3	35.4	30.9	30.8	45.1	63.3
1952-53	48.4	47.9	40.7	43.7	43.5	36.0	45.3	75.9
1953-54	39.2	49.7	47.2	33.7	38.3	34.7	52.3	68.3
1954-55	43.0	38.8	33.1	27.5	27.0	34.7	53.7	66.4
1955-56	33.0	43.1	36.1	25.7	41.0	39.6	43.9	65.6

Table 12. Independent variables for Method III

Year	X_1	X_2	X_3	X_4	X_5	X_6
1923-24	37.3	37.5	42.1	37.2	28.4	50.9
1924-25	46.0	39.0	51.0	43.0	29.1	52.4
1925-26	60.7	49.8	45.1	40.3	34.4	52.6
1926-27	42.3	43.7	45.7	38.3	32.4	57.3
1927-28	46.0	49.3	42.6	37.1	30.2	56.4
1928-29	45.3	45.5	40.5	29.6	28.0	53.6
1929-30	54.3	50.5	41.3	44.3	29.0	55.6
1930-31	53.7	50.0	38.1	35.9	25.5	49.4
1931-32	44.3	30.0	40.0	33.3	27.7	49.4
1932-33	41.7	47.0	34.9	28.2	31.4	53.5
1933-34	60.3	61.3	47.3	48.8	39.7	59.1
1934-35	47.0	42.3	40.4	37.9	36.1	53.3
1935-36	46.3	49.0	44.7	37.0	33.0	53.7
1936-37	47.7	44.2	39.8	35.3	26.5	55.0
1937-38	51.7	49.8	47.0	41.0	38.8	57.4
1938-39	41.3	53.0	33.2	30.6	35.4	54.7
1939-40	57.0	43.8	48.9	40.0	35.8	59.4
1940-41	48.7	42.8	45.3	37.1	35.0	52.7
1941-42	36.7	42.5	33.7	30.3	31.0	51.6
1942-43	39.7	48.8	40.0	35.8	34.9	54.2
1943-44	37.0	42.0	40.8	36.7	31.0	55.8
1944-45	38.7	50.5	35.8	40.1	34.0	53.2
1945-46	41.0	51.2	42.1	35.0	32.8	51.6
1946-47	57.3	46.3	39.3	42.8	31.7	51.9
1947-48	40.3	39.7	37.3	37.3	33.8	52.6
1948-49	54.7	44.3	41.7	31.6	22.5	53.9
1949-50	44.0	38.5	51.3	36.1	33.5	56.0
1950-51	46.3	40.2	36.3	39.7	34.4	57.3
1951-52	31.0	36.7	30.5	33.0	32.3	51.0
1952-53	42.7	58.0	44.6	40.3	40.1	58.5
1953-54	44.0	50.0	46.8	36.9	37.0	57.2
1954-55	37.0	44.7	38.6	28.0	28.5	57.7
1955-56	35.0	38.3	40.8	27.8	40.9	52.5

Table 13. Independent variables for Method IV

Year	X_1	X_2	X_3	X_4	X_5	X_6	X_7
1923-24	36.6	39.6	39.8	44.0	40.0	38.5	42.5
1924-25	42.6	42.2	57.4	45.8	44.2	39.2	44.3
1925-26	58.0	47.8	46.8	44.4	38.7	42.2	45.9
1926-27	45.4	42.8	48.6	41.2	44.2	29.7	48.3
1927-28	45.8	51.4	45.4	40.0	38.5	33.8	46.8
1928-29	40.8	52.2	47.8	31.0	35.7	19.0	44.3
1929-30	56.4	45.6	45.0	35.6	49.9	42.6	45.8
1930-31	51.2	48.0	38.6	39.0	38.1	34.8	40.6
1931-32	37.8	32.8	37.6	41.8	31.3	36.5	41.4
1932-33	42.2	48.4	34.2	34.2	36.3	21.1	45.0
1933-34	63.2	56.0	50.4	45.6	44.8	50.8	52.1
1934-35	51.2	36.4	38.4	43.8	40.7	35.6	46.8
1935-36	47.6	49.6	43.4	45.0	41.5	36.9	45.7
1936-37	45.8	44.0	39.2	41.0	36.5	34.4	44.5
1937-38	49.6	51.2	45.2	48.6	36.7	45.4	50.5
1938-39	46.8	49.4	35.2	30.8	30.3	33.0	47.1
1939-40	51.0	45.8	45.0	52.2	40.7	40.2	50.7
1940-41	44.8	45.6	45.0	46.2	39.5	39.0	45.8
1941-42	38.4	41.6	35.8	32.8	22.4	33.7	44.1
1942-43	46.2	45.2	38.8	41.2	36.2	38.9	46.7
1943-44	36.0	44.2	42.4	40.8	35.2	36.1	46.7
1944-45	44.8	47.0	30.8	39.0	35.3	43.1	46.3
1945-46	43.4	50.8	41.6	41.2	39.3	30.6	44.7
1946-47	52.8	45.6	42.2	34.2	45.4	42.1	44.5
1947-48	43.6	36.0	34.0	39.2	45.2	31.7	45.5
1948-49	51.8	42.2	44.2	39.0	40.1	29.5	42.0
1949-50	38.0	41.2	53.4	53.8	43.3	39.0	46.9
1950-51	40.0	44.0	32.6	37.8	40.9	47.4	48.2
1951-52	32.4	36.4	32.6	30.0	28.9	31.6	44.3
1952-53	48.4	57.2	38.6	50.2	35.9	38.9	52.0
1953-54	39.2	58.6	40.8	49.4	46.1	31.1	49.5
1954-55	43.0	39.4	38.2	41.4	29.0	26.8	46.7
1955-56	33.0	40.0	46.2	38.0	35.2	30.2	47.0

Table 14. Dependent variables for all Y's and methods

Year	X_1	X_2	X_3	X_4	X_5	X_6	X_7
1922-23	35.4	41.0	57.0	50.4	48.8	59.4	59.2
1923-24	37.6	39.0	37.8	49.2	59.2	55.6	50.4
1924-25	54.0	60.4	61.2	60.2	62.2	70.4	56.6
1925-26	57.2	57.8	45.0	48.2	57.0	64.8	69.4
1926-27	37.6	50.8	53.8	51.6	53.6	49.0	49.2
1927-28	62.2	57.6	49.2	52.6	45.4	53.4	55.6
1928-29	51.0	38.4	50.6	61.6	41.2	55.2	64.2
1929-30	52.8	50.0	50.2	64.6	75.8	63.8	62.4
1930-31	56.6	44.6	43.8	53.2	65.8	64.0	63.6
1931-32	45.8	42.8	51.8	59.4	53.2	66.8	65.0
1932-33	42.4	45.0	54.0	55.4	41.8	50.8	57.8
1933-34	60.8	57.2	59.8	49.2	69.8	72.4	67.8
1934-35	44.6	47.0	50.4	54.8	49.2	64.6	64.4
1935-36	52.8	37.0	40.2	37.2	52.2	75.4	77.4
1936-37	46.0	43.2	48.2	45.8	51.2	63.2	53.8
1937-38	45.6	46.4	40.0	47.6	52.6	59.6	65.0
1938-39	54.8	60.6	54.6	62.8	54.8	60.4	64.2
1939-40	57.8	65.0	55.4	56.0	55.0	63.8	64.2
1940-41	56.0	49.4	60.6	52.0	56.2	52.2	45.0
1941-42	39.2	40.6	46.4	62.2	64.6	67.4	66.6
1942-43	37.8	54.8	63.0	71.0	57.8	67.4	71.2
1943-44	43.2	41.6	46.2	63.8	57.0	52.0	46.6
1944-45	48.6	48.4	51.8	41.6	53.6	42.2	63.8
1945-46	52.0	49.8	61.6	54.8	51.2	67.0	75.8
1946-47	63.2	55.4	57.6	50.8	49.4	63.2	60.8
1947-48	40.0	50.0	48.8	54.0	50.6	55.8	63.8
1948-49	58.6	42.8	41.8	54.2	61.2	64.0	70.4
1949-50	48.8	48.2	44.2	52.0	58.4	58.2	61.0
1950-51	40.2	52.4	46.2	58.6	63.6	61.4	64.0
1951-52	40.4	35.8	47.6	45.4	54.4	55.4	59.2
1952-53	49.2	51.2	65.0	55.2	43.4	43.6	55.0
1953-54	47.6	44.4	45.6	60.2	58.2	65.6	68.8
1954-55	35.0	39.6	46.6	40.8	53.2	54.8	56.6
1955-56	56.0	68.0	55.8	46.2	59.6	62.0	65.2

Table 14. Dependent variables for all Y's and methods (continued)

Year	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}
1922-23	47.6	59.2	66.2	75.8	56.6	69.8	71.6
1923-24	61.2	59.6	65.4	65.8	72.8	72.0	73.4
1924-25	51.0	67.4	71.2	69.8	64.6	72.8	69.4
1925-26	65.8	73.4	73.8	53.2	66.8	72.8	77.8
1926-27	67.0	75.6	66.6	60.0	75.0	77.2	62.8
1927-28	62.8	72.4	66.8	78.2	64.0	72.2	78.4
1928-29	50.0	60.0	61.4	61.6	72.0	75.2	77.2
1929-30	75.6	62.2	67.4	53.6	67.8	68.6	70.4
1930-31	53.0	65.4	66.2	60.0	77.8	63.2	79.4
1931-32	52.0	54.2	66.0	67.6	74.4	77.4	64.4
1932-33	62.8	62.2	52.8	48.8	59.8	67.6	65.6
1933-34	75.0	71.4	71.2	78.0	73.8	82.2	84.2
1934-35	55.4	56.6	59.4	64.8	60.2	59.4	73.0
1935-36	74.8	64.6	73.6	66.4	83.4	72.4	72.6
1936-37	57.0	54.8	71.2	70.4	70.6	73.8	77.8
1937-38	70.8	68.8	51.4	62.0	69.8	57.2	71.2
1938-39	65.6	74.2	77.2	72.4	69.8	71.2	63.4
1939-40	64.0	56.8	71.8	74.0	77.6	72.6	78.4
1940-41	65.0	65.8	65.4	69.2	77.4	67.2	76.2
1941-42	61.6	47.8	52.4	65.8	52.0	61.6	79.6
1942-43	69.0	65.4	65.4	60.2	57.6	60.2	72.8
1943-44	54.8	59.0	62.4	70.4	80.2	63.4	64.6
1944-45	58.2	65.4	77.2	73.2	64.8	58.0	69.2
1945-46	70.8	70.4	70.0	61.0	63.8	69.8	66.2
1946-47	54.2	65.4	83.0	76.2	62.0	71.2	74.0
1947-48	57.0	63.2	54.6	57.8	65.8	76.6	74.6
1948-49	68.4	69.2	63.0	65.6	73.8	63.4	63.2
1949-50	63.4	51.6	49.0	47.2	68.4	67.0	73.0
1950-51	59.6	57.0	56.4	67.0	61.4	68.2	71.6
1951-52	62.0	72.4	74.6	68.2	74.8	63.0	62.8
1952-53	71.2	64.6	54.4	62.6	58.4	65.0	56.4
1953-54	70.4	65.4	58.0	60.0	76.6	83.8	73.2
1954-55	59.2	54.6	65.4	73.6	74.6	67.2	74.0
1955-56	70.4	61.2	69.4	70.6	53.2	77.2	81.2

Table 14. Dependent variables for all Y's and methods (continued)

Year	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	X_{20}	X_{21}
1922-23	62.6	66.0	73.8	78.4	68.8	69.8	79.2
1923-24	64.2	78.8	70.4	82.8	78.2	83.6	87.0
1924-25	77.8	58.2	65.0	70.4	75.0	82.4	88.4
1925-26	73.6	79.6	85.4	78.8	76.0	79.4	95.0
1926-27	64.8	66.0	79.6	78.4	81.4	86.4	80.6
1927-28	81.4	73.8	76.2	68.0	67.8	79.0	87.4
1928-29	62.8	72.2	76.6	74.8	71.0	79.4	89.0
1929-30	85.0	64.6	81.4	77.0	84.8	79.2	81.4
1930-31	68.4	82.2	82.0	84.6	85.8	87.8	90.0
1931-32	66.6	66.8	62.4	76.0	73.6	88.0	89.8
1932-33	75.2	80.0	75.6	89.0	89.6	89.0	85.0
1933-34	80.4	73.8	66.6	81.6	79.4	80.2	82.8
1934-35	63.6	65.8	81.8	83.6	76.0	84.0	82.6
1935-36	81.4	63.0	68.4	86.4	83.8	92.6	86.2
1936-37	70.2	63.8	68.8	74.4	79.0	87.6	84.4
1937-38	76.2	81.0	80.8	74.8	76.6	78.8	84.8
1938-39	81.6	75.0	68.0	81.0	62.2	77.4	86.2
1939-40	75.4	77.6	70.0	87.2	95.6	88.0	88.8
1940-41	70.8	71.8	58.6	77.0	82.6	88.4	74.2
1941-42	62.8	74.0	80.6	71.4	75.0	79.8	74.6
1942-43	80.0	51.8	71.2	69.4	80.8	81.2	86.0
1943-44	76.0	61.0	69.2	69.0	73.6	78.8	77.0
1944-45	73.4	64.2	62.0	64.2	72.6	82.0	70.8
1945-46	57.0	76.0	79.8	78.4	75.4	80.0	82.2
1946-47	72.4	67.0	71.2	66.0	79.8	70.6	74.2
1947-48	79.2	73.6	82.0	82.6	79.6	63.8	79.0
1948-49	80.2	63.4	68.0	80.8	77.8	77.6	79.0
1949-50	69.0	69.0	68.6	74.8	79.2	80.4	78.0
1950-51	76.6	61.0	69.8	73.4	82.4	75.6	79.4
1951-52	75.6	78.8	84.2	79.6	79.8	75.4	67.4
1952-53	66.2	68.4	64.6	82.4	81.8	79.0	77.4
1953-54	66.6	64.2	63.4	69.4	72.0	87.6	83.2
1954-55	67.8	57.0	76.0	79.0	71.6	88.4	80.6
1955-56	73.0	82.2	80.6	67.8	75.6	78.4	84.6

Table 14. Dependent variables for all Y's and methods (continued)

Year	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}	X_{28}
1922-23	44.5	54.1	67.0	73.5	87.5	84.1	74.9
1923-24	38.1	55.9	69.0	80.8	86.5	85.8	74.7
1924-25	58.5	61.3	70.6	74.1	87.8	82.6	72.4
1925-26	53.3	63.1	69.8	83.0	86.9	85.5	72.8
1926-27	47.4	57.7	67.6	79.0	87.1	82.8	73.7
1927-28	56.3	57.0	73.5	75.7	87.5	85.2	78.7
1928-29	46.7	55.4	68.7	77.6	88.7	88.8	70.3
1929-30	51.0	67.4	68.5	78.9	88.3	83.2	74.3
1930-31	48.3	60.8	69.6	85.3	91.3	87.5	75.8
1931-32	46.8	58.4	69.1	77.1	85.9	86.7	78.0
1932-33	47.1	55.1	62.4	84.6	91.1	83.6	76.9
1933-34	59.3	67.6	78.3	77.8	90.5	88.5	72.8
1934-35	47.3	57.5	63.2	79.4	89.8	87.7	80.3
1935-36	43.3	63.6	74.7	80.6	89.9	87.3	76.5
1936-37	45.8	54.3	71.8	77.3	87.6	87.4	80.2
1937-38	44.0	60.7	64.9	79.4	84.4	85.5	81.7
1938-39	56.7	63.7	72.5	75.7	88.1	87.9	75.0
1939-40	59.4	60.0	75.3	84.2	89.2	90.1	74.3
1940-41	55.3	56.0	70.9	75.6	86.3	82.4	69.9
1941-42	42.1	61.7	62.5	76.3	90.4	87.2	76.8
1942-43	51.9	67.0	65.7	74.4	87.9	86.2	80.3
1943-44	43.7	55.5	69.6	72.1	86.9	86.1	75.9
1944-45	49.6	54.1	69.0	69.6	87.8	84.1	70.7
1945-46	54.5	65.0	64.7	79.2	88.1	87.5	75.3
1946-47	58.7	57.3	72.8	71.9	89.0	84.5	77.0
1947-48	46.3	57.4	68.5	77.1	86.6	86.8	79.2
1948-49	47.7	64.6	68.3	74.5	85.5	87.6	79.0
1949-50	47.1	57.4	62.7	75.4	84.6	85.2	74.7
1950-51	46.3	60.7	67.2	73.6	87.0	83.3	76.6
1951-52	41.3	58.1	69.9	77.9	84.7	86.7	81.5
1952-53	55.1	55.5	60.6	76.4	89.4	86.1	80.0
1953-54	45.9	64.8	72.6	73.9	89.1	84.1	76.1
1954-55	40.4	53.2	70.7	75.0	86.3	87.6	77.2
1955-56	59.9	60.8	71.1	81.8	88.4	85.6	80.4

Table 15. The general system of normal equations for Method III and Y_1

X_0	X_1	X_2	X_3	X_4	X_5	X_6		Y_1
$n\ b_0$	$+ \Sigma X_1\ b_1$	$+ \Sigma X_2\ b_2$	$+ \Sigma X_3\ b_3$	$+ \Sigma X_4\ b_4$	$+ \Sigma X_5\ b_5$	$+ \Sigma X_6\ b_6$	$=$	ΣY_1
$\Sigma X_1\ b_0$	$+ \Sigma X_1^2\ b_1$	$+ \Sigma X_1 X_2\ b_2$	$+ \Sigma X_1 X_3\ b_3$	$+ \Sigma X_1 X_4\ b_4$	$+ \Sigma X_1 X_5\ b_5$	$+ \Sigma X_1 X_6\ b_6$	$=$	$\Sigma X_1 Y_1$
$\Sigma X_2\ b_0$	$+ \Sigma X_2 X_1\ b_1$	$+ \Sigma X_2^2\ b_2$	$+ \Sigma X_2 X_3\ b_3$	$+ \Sigma X_2 X_4\ b_4$	$+ \Sigma X_2 X_5\ b_5$	$+ \Sigma X_2 X_6\ b_6$	$=$	$\Sigma X_2 Y_1$
$\Sigma X_3\ b_0$	$+ \Sigma X_3 X_1\ b_1$	$+ \Sigma X_3 X_2\ b_2$	$+ \Sigma X_3^2\ b_3$	$+ \Sigma X_3 X_4\ b_4$	$+ \Sigma X_3 X_5\ b_5$	$+ \Sigma X_3 X_6\ b_6$	$=$	$\Sigma X_3 Y_1$
$\Sigma X_4\ b_0$	$+ \Sigma X_4 X_1\ b_1$	$+ \Sigma X_4 X_2\ b_2$	$+ \Sigma X_4 X_3\ b_3$	$+ \Sigma X_4^2\ b_4$	$+ \Sigma X_4 X_5\ b_5$	$+ \Sigma X_4 X_6\ b_6$	$=$	$\Sigma X_4 Y_1$
$\Sigma X_5\ b_0$	$+ \Sigma X_5 X_1\ b_1$	$+ \Sigma X_5 X_2\ b_2$	$+ \Sigma X_5 X_3\ b_3$	$+ \Sigma X_5 X_4\ b_4$	$+ \Sigma X_5^2\ b_5$	$+ \Sigma X_5 X_6\ b_6$	$=$	$\Sigma X_5 Y_1$
$\Sigma X_6\ b_0$	$+ \Sigma X_6 X_1\ b_1$	$+ \Sigma X_6 X_2\ b_2$	$+ \Sigma X_6 X_3\ b_3$	$+ \Sigma X_6 X_4\ b_4$	$+ \Sigma X_6 X_5\ b_5$	$+ \Sigma X_6^2\ b_6$	$=$	$\Sigma X_6 Y_1$

Table 16. Original information matrix for Method III and Y_1 (uncorrected sums of squares and products)

	$K_{c1} = 10^{-1}$	$K_{c2} = 10^{-2}$	$K_{c3} = 10^{-2}$	$K_{c4} = 10^{-2}$	$K_{c5} = 10^{-2}$	$K_{c6} = 10^{-2}$	$K_{c7} = 10^{-2}$	$K_{c8} = 10^{-2}$
$K_{r1} = 10^{-1}$	33.00	1497.00	1500.20	1367.50	1206.30	1074.80	1791.40	1615.40
$K_{r2} = 10^{-2}$	1497.00	69710.18	68659.90	62595.91	55420.00	48724.72	81408.53	74568.22
$K_{r3} = 10^{-2}$	1500.20	68659.90	69497.72	62300.93	55207.58	49161.04	81687.22	74067.88
$K_{r4} = 10^{-2}$	1367.50	62595.91	62300.93	57475.03	50387.05	44696.22	74418.45	67390.26
$K_{r5} = 10^{-2}$	1206.30	55420.00	55207.58	50387.05	44870.69	39482.42	65621.55	59456.68
$K_{r6} = 10^{-2}$	1074.80	48724.72	49161.04	44696.22	39482.42	35599.22	58501.83	52679.96
$K_{r7} = 10^{-2}$	1791.40	81408.53	81687.22	74418.45	65621.55	58501.83	97481.90	87696.76

Table 17. Coded information matrix (See original matrix and coding factors in table 16.)

X_0	X_1	X_2	X_3	X_4	X_5	X_6	Y_1
.3300	1.497000	1.500200	1.367500	1.206300	1.074800	1.791400	1.615400
	6.971018	6.865990	6.259591	5.542000	4.872472	8.140853	7.456822
		6.949772	6.230093	5.520758	4.916104	8.168722	7.406788
			5.747503	5.038705	4.469622	7.441845	6.739026
				4.487069	3.948242	6.562155	5.945668
					3.559922	5.850183	5.267996
						9.748190	8.769676

Table 18. Inverse coded matrix

X_0	X_1	X_2	X_3	X_4	X_5	X_6	b'
1297.4881	-14.2684	20.5083	16.2319	- 7.5142	- 1.9835	-249.8486	4.6356
	11.5737	- 4.5863	- 5.3113	- 7.4414	6.2179	2.1324	.7028
		12.9498	5.2427	- 1.5983	- 4.8060	- 10.8324	.3591
			21.8069	- 5.6655	- 3.2144	- 13.8452	.3673
				24.7867	- 6.2708	.3375	.3478
					24.9124	- 9.0763	.2604
						69.1024	1.0426

Table 19. Decoded regression coefficients for Method I

	Y_1	Y_2	Y_3	Y_4
b_0	+ 56.2710	+ 35.1071	+ 36.8505	+ 40.4099
b_1	+ .1586	+ .9792	+ .6534	+ .1454
b_2	- 162.6686	- 87.5085	- 32.4245	+ 26.5478
b_3	+ 2008.3162	- 885.9600	- 15.1837	+ 526.2648
b_4	- 41168.8710	- 25124.6340	- 7003.2750	- 1966.9050
b_5	+ 1918832.8000	+ 732674.9000	- 322670.5000	-648385.0000
b_6	-21506714.0000	-23544682.0000	-28475903.0000	+608355.0000

	Y_5	Y_6	Y_7	Y_8
b_0	+ 77.1483	+ 110.1461	+ 89.6519	+ 13.4445
b_1	- .0276	- .5987	- .2815	+ 1.2438
b_2	- 76.1377	- 133.8886	- 78.0981	- 27.3174
b_3	- 336.4172	+ 228.2090	+ 422.1336	+ 14.6621
b_4	- 25313.2450	- 33255.5400	- 29535.4650	- 23947.4540
b_5	- 73050.3000	+1488461.9000	+ 1831175.6000	+ 1735872.3000
b_6	+1840373.0000	-9882350.0000	-35044034.0000	-23742154.0000

	Y_9	Y_{10}	Y_{11}	Y_{12}
b_0	+ 57.1229	+ 119.7649	+ 31.2105	+ 22.7738
b_1	+ .3394	- .8587	+ .7290	+ .3172
b_2	- 83.7381	- 116.3636	+ 45.3287	+ 20.5039
b_3	+ 1161.0484	+ 325.3505	- 437.0799	+ 2076.2206
b_4	- 5088.4530	- 8179.8640	- 2834.1340	+ 16015.5680
b_5	+ 453990.0000	+ 1643571.6000	-498117.8000	- 17242.8000
b_6	-20212484.0000	-13558710.0000	+ 47344.0000	+29484807.0000

Table 19. Decoded regression coefficients for Method I (continued)

	Y_{13}	Y_{14}	Y_{15}	Y_{16}
b_0	+ 54.4650	+ 102.9712	+ 34.1543	+ 134.1702
b_1	+ .3402	- .1954	+ .5466	- .8730
b_2	- 25.8812	- 84.2351	- 7.3617	- 94.7048
b_3	+ 365.1320	- 723.5171	+ 939.8034	- 1048.7913
b_4	- 7834.7500	- 33379.9050	+ 13342.3450	- 1315.0100
b_5	+ 451582.7000	+1218506.7000	+1024179.8000	+ 1542638.6000
b_6	+1148995.0000	-4558464.0000	+2725519.0000	-18845740.0000

	Y_{17}	Y_{18}	Y_{19}	Y_{20}
b_0	+ 137.3996	+ 88.1649	+ 72.5941	+ 71.3829
b_1	- .9036	- .1835	+ .2368	+ .1532
b_2	- 38.7629	- 8.5137	- 43.1901	+ 14.5833
b_3	- 1523.6618	- 156.1663	+ 82.9538	+ 440.0016
b_4	- 30277.7270	- 13479.5270	- 1451.3910	- 28589.2300
b_5	+ 1082596.1000	+ 1367859.1000	+ 63417.8000	- 83416.1000
b_6	-15589199.0000	-15682066.0000	+8578046.0000	+2407793.0000

	Y_{21}	Y_{22}	Y_{23}	Y_{24}
b_0	+ 88.2110	+ 42.5667	+ 64.5940	+ 60.9939
b_1	- .0310	+ .5993	+ .1379	+ .1436
b_2	- 54.3693	- 93.7787	- 62.0565	- 25.9128
b_3	+ 967.8115	+ 366.2834	+ 335.5130	+ 388.4525
b_4	- 37050.0720	- 24295.7180	- 19947.5640	- 4160.5450
b_5	+ 747111.6000	+ 775244.2000	+ 802790.3000	+ 649982.2000
b_6	-6435700.0000	-24473896.0000	-14428912.0000	+2446428.0000

Table 19. Decoded regression coefficients for Method I (continued)

	Y_{25}	Y_{26}	Y_{27}	Y_{28}
b_0	+ 99.6189	+ 89.4723	+ 82.8363	+ 72.9470
b_1	- .2869	+ .0154	- .0257	- .0194
b_2	- 40.0677	- 15.3954	+ 11.3298	+ 36.4598
b_3	- 149.0296	+ 163.2162	+ 157.1917	- 366.3973
b_4	- 16423.1030	- 6930.7070	- 4054.0500	+ 8369.3390
b_5	+ 777271.3000	+ 300725.6000	+ 447324.9000	- 22341.0000
b_6	-7353308.0000	-5556347.0000	-2811526.0000	-985030.0000

Table 20. Decoded regression coefficients for Method II

	Y_1	Y_2	Y_3	Y_4	Y_5
b_0	+55.9730	+30.7745	+29.7698	+36.4400	+68.1289
b_1	+ .3761	+ .2211	+ .0166	- .1901	- .0126
b_2	+ 1.0688	+ .2533	+ .2780	+ .2034	- .0031
b_3	- .0388	+ .0077	- .1799	- .0216	+ .3024
b_4	- .2036	- .0374	+ .2569	+ .2443	+ .2332
b_5	+ .0862	+ .6365	+ .3503	- .2935	- .3029
b_6	+ .1855	+ .4156	+ .0059	- .1371	+ .4596
b_7	- .8115	- .2010	- .4385	- .0438	- .2325
b_8	- .5032	- .4126	+ .2318	+ .3859	- .4159

	Y_6	Y_7	Y_8	Y_9	Y_{10}
b_0	+102.6472	+88.4939	+11.2835	+55.3205	+111.6309
b_1	+ .1994	+ .4617	+ .1597	+ .3631	+ .2219
b_2	+ .3641	+ .1173	+ .3858	+ .4778	+ .2683
b_3	+ .1339	- .2116	- .0070	- .1868	- .2361
b_4	- .1231	- .4464	- .4448	+ .0130	+ .1864
b_5	- .1616	+ .4073	+ .4201	+ .4532	- .1482
b_6	+ .4059	+ .1563	+ .7008	- .3691	+ .3224
b_7	- .3619	- .3555	- .1225	- .4763	- .7907
b_8	- .9146	- .4605	+ .1715	- .0205	- .5285

	Y_{11}	Y_{12}	Y_{13}	Y_{14}	Y_{15}
b_0	+26.0439	+33.4592	+51.4079	+96.0425	+31.7018
b_1	- .2904	- .1206	- .2011	+ .1886	+ .2553
b_2	+ .1822	+ .1876	+ .4507	+ .3062	- .1613
b_3	- .2553	+ .5245	+ .4392	- .1123	+ .1188
b_4	+ .4554	- .0464	- .2339	- .1471	- .0502
b_5	- .2292	- .5662	- .0387	+ .0491	- .0123
b_6	+ .3378	- .0513	+ .1351	+ .3456	+ .1682
b_7	+ .0113	- .0047	- .1102	+ .1283	- .1719
b_8	+ .5167	+ .5047	+ .0000	- .8488	+ .5449

Table 20. Decoded regression coefficients for Method II (continued)

	Y_{16}	Y_{17}	Y_{18}	Y_{19}	Y_{20}
b_0	+145.2779	+126.9837	+93.7041	+71.5291	+70.7749
b_1	+ .2368	+ .4823	+ .4069	+ .1931	+ .0056
b_2	+ .4026	- .2245	- .2754	- .4452	+ .2286
b_3	- .6221	- .5154	+ .0194	+ .5490	+ .0259
b_4	- .2238	- .2999	- .7002	- .0205	- .1357
b_5	+ .6918	+ .3328	+ .4631	+ .0317	- .1831
b_6	+ .0720	- .1168	+ .0906	+ .0400	- .1253
b_7	- .2345	+ .3542	+ .0230	+ .1269	+ .3082
b_8	- 1.2863	- .8822	- .2726	- .1816	+ .0013

	Y_{21}	Y_{22}	Y_{23}	Y_{24}	Y_{25}
b_0	+85.2178	+38.6698	+60.3131	+58.9091	+99.8420
b_1	+ .2705	+ .2031	+ .1637	+ .0183	+ .2427
b_2	+ .4416	+ .5331	+ .2584	+ .2036	+ .0393
b_3	- .0036	- .0713	+ .0018	+ .0624	- .0763
b_4	- .2164	+ .0062	- .0892	+ .0041	- .2379
b_5	- .0346	+ .3571	+ .0871	- .1357	+ .1966
b_6	- .2615	+ .2035	+ .2043	+ .1998	- .0460
b_7	+ .1073	- .4822	- .2651	- .1303	+ .0692
b_8	- .3233	- .2255	- .2088	+ .0182	- .4790

	Y_{26}	Y_{27}	Y_{28}
b_0	+88.2023	+83.5821	+73.1750
b_1	+ .1151	+ .0968	+ .0120
b_2	+ .0731	- .0052	- .3251
b_3	- .1007	- .1616	- .0343
b_4	- .0088	- .0712	- .0566
b_5	+ .0667	- .0356	+ .0565
b_6	- .0637	+ .0481	+ .0398
b_7	- .0129	+ .0634	+ .1089
b_8	- .0571	+ .0541	+ .1793

Table 21. Decoded regression coefficients for Method III

	Y_1	Y_2	Y_3	Y_4	Y_5
b_0	+46.3560	+3.7619	+42.9584	+50.8121	+48.7152
b_1	+ .7028	+ .4241	- .1309	- .0173	+ .0786
b_2	+ .3591	- .1454	+ .2895	- .0327	- .1361
b_3	+ .3673	+ .0694	+ .1334	- .1084	- .1257
b_4	- .3478	- .0699	+ .1340	+ .0692	+ .6537
b_5	+ .2604	+1.0484	+ .5185	- .1690	- .2155
b_6	- 1.0426	- .0352	- .4926	+ .2325	- .0411

	Y_6	Y_7	Y_8	Y_9	Y_{10}
b_0	+80.3272	+82.6631	+10.0908	+64.7039	+100.5490
b_1	+ .4987	+ .4130	- .0458	- .0665	+ .1273
b_2	- .0692	+ .3402	+ .5172	+ .6016	+ .2313
b_3	+ .2730	- .0165	+ .3261	+ .1109	- .2890
b_4	- .1819	- .5244	- .2424	+ .2265	+ .4673
b_5	+ .1888	+ .5429	+ .5105	+ .0807	- .1211
b_6	- .9294	- .9713	+ .1878	- .7545	- .9713

	Y_{11}	Y_{12}	Y_{13}	Y_{14}	Y_{15}
b_0	+31.8153	+46.2756	+50.3227	+68.2235	+32.2793
b_1	- .0792	+ .0317	+ .1440	+ .4394	+ .2340
b_2	- .0482	- .0287	- .1219	- .1055	- .0957
b_3	- .2873	+ .3969	+ .3965	+ .0628	- .5345
b_4	+ .2998	+ .0466	- .1933	- .2626	+ .3809
b_5	+ .0402	- .7745	+ .1368	+ .3533	- .0341
b_6	+ .7280	+ .5398	+ .0787	- .2945	+ .7990

Table 21. Decoded regression coefficients for Method III (continued)

	Y_{16}	Y_{17}	Y_{18}	Y_{19}	Y_{20}
b_0	+110.4507	+101.0960	+81.6873	+56.4026	+78.0714
b_1	+ .4687	+ .2164	+ .3740	+ .3238	- .0343
b_2	+ .0683	- .0253	- .0363	- .2808	+ .1758
b_3	- .0684	- .4313	+ .0655	- .1143	+ .6050
b_4	- .6539	- .2385	- .7739	+ .1624	- .4716
b_5	+ .9410	+ .2376	+ .5313	+ .1215	- .3000
b_6	- 1.2686	- .3301	- .2089	+ .2681	- .0194

	Y_{21}	Y_{22}	Y_{23}	Y_{24}	Y_{25}
b_0	+85.3466	+30.8973	+56.1260	+54.5468	+88.0356
b_1	+ .3575	+ .3307	+ .1437	+ .1627	+ .2598
b_2	+ .1198	+ .1674	+ .2037	- .0351	+ .0173
b_3	+ .5550	+ .1882	+ .0773	- .0480	+ .1198
b_4	- .5969	- .0936	- .0016	+ .0872	- .3935
b_5	- .0302	+ .6092	+ .1567	- .0504	+ .2401
b_6	- .4581	- .5192	- .3777	+ .1675	- .3975

	Y_{26}	Y_{27}	Y_{28}
b_0	+89.7842	+85.3375	+75.5630
b_1	+ .0412	+ .0867	+ .0176
b_2	+ .1571	+ .0439	- .1178
b_3	- .0373	- .0391	- .2128
b_4	- .0523	- .1925	- .0923
b_5	+ .0475	+ .0100	+ .1597
b_6	- .1667	+ .0559	+ .2269

Table 22. Decoded regression coefficients for Method IV

	Y_1	Y_2	Y_3	Y_4	Y_5
b_0	+29.8306	-26.2719	+7.6924	+52.7135	+55.2836
b_1	+ .3415	+ .1285	+ .0844	- .1738	- .0563
b_2	+ .5127	- .2204	+ .0114	+ .1520	- .1050
b_3	+ .5453	+ .4433	+ .1876	+ .1249	+ .3128
b_4	- .2021	- .3918	- .2725	- .3560	- .3274
b_5	- .0147	+ .1005	- .0245	+ .1518	+ .1011
b_6	+ .2096	+ .2227	- .0382	+ .0586	+ .7140
b_7	- .8863	+ 1.4134	+ .9629	+ .0747	+ .4656

	Y_6	Y_7	Y_8	Y_9	Y_{10}
b_0	+70.8790	+66.6023	-1.4208	+34.4722	+79.4078
b_1	+ .2412	+ .3114	+ .1176	+ .1837	+ .2536
b_2	- .0091	+ .2401	+ .2512	+ .3534	+ .0496
b_3	+ .3663	- .1837	- .0005	+ .1603	+ .2493
b_4	+ .0012	+ .0513	+ .0164	- .4137	- .5664
b_5	- .0763	- .0541	+ .0646	+ .1995	- .0519
b_6	+ .2699	- .0091	- .0045	- .0045	+ .3665
b_7	- .9369	- .4688	+ .9695	+ .1653	- .5644

	Y_{11}	Y_{12}	Y_{13}	Y_{14}	Y_{15}
b_0	+44.6808	+78.4586	+40.3897	+63.0240	+41.2124
b_1	- .1994	+ .0764	- .1703	+ .1353	+ .1472
b_2	+ .2296	+ .1466	+ .0798	+ .0088	- .1287
b_3	+ .0259	+ .0080	+ .2585	+ .2904	+ .1055
b_4	- .1926	+ .4704	- .1444	- .0569	- .5490
b_5	- .2072	+ .1048	+ .5009	- .1034	+ .2350
b_6	+ .3095	- .1427	- .2210	- .1486	+ .3228
b_7	+ .5090	- .8345	+ .3674	- .1899	+ .6107

Table 22. Decoded regression coefficients for Method IV (continued)

	Y_{16}	Y_{17}	Y_{18}	Y_{19}	Y_{20}
b_0	+78.2927	+74.3170	+69.2928	+66.2329	+95.7672
b_1	+ .0656	+ .3528	+ .4272	+ .1938	+ .1237
b_2	+ .2173	- .4548	- .3694	- .1979	+ .1951
b_3	+ .0791	+ .1231	- .1782	- .2807	- .0442
b_4	- .0640	- .4742	+ .3197	+ .2917	+ .6884
b_5	- .2702	- .0193	- .0045	+ .3718	- .2966
b_6	- .0622	- .3377	- .5551	- .0680	- .2573
b_7	- .2019	+ .6640	+ .4323	- .0017	- .7493

	Y_{21}	Y_{22}	Y_{23}	Y_{24}	Y_{25}
b_0	+85.9433	+3.6554	+46.3046	+57.6968	+79.6712
b_1	+ .2867	+ .1831	+ .1044	+ .0412	+ .2272
b_2	+ .1528	+ .1014	+ .1467	+ .0596	- .0627
b_3	+ .2434	+ .3919	+ .1305	+ .1562	- .0008
b_4	+ .3173	- .2906	- .1721	- .1708	+ .1719
b_5	- .1479	+ .0205	+ .0653	+ .0729	- .0560
b_6	- .2136	+ .1320	+ .1693	+ .1151	- .2288
b_7	- .7221	+ .5013	- .1072	+ .0068	- .1370

	Y_{26}	Y_{27}	Y_{28}
b_0	+85.5197	+87.7888	+78.1836
b_1	+ .0804	+ .1526	+ .0330
b_2	+ .1189	- .0403	- .1786
b_3	- .0568	- .0164	- .1800
b_4	- .0291	+ .0813	+ .0707
b_5	+ .0171	- .1812	- .0545
b_6	- .0546	- .1007	- .0796
b_7	- .0399	+ .0224	+ .3099

Table 23. Total sums of squares, corrections, corrected sums of squares, and sums of squares attributable to regression for the 4 methods

Period	Total Y^2	Corrections	y^2	Method I y^2	Method II y^2	Method III y^2	Method IV y^2
Y_1	81123.80	79076.28	2047.52	1121.88	1253.18	1167.37	1090.66
Y_2	81176.88	79056.70	2120.18	843.97	799.44	790.26	755.32
Y_3	86614.64	84998.64	1616.00	533.12	426.23	316.17	226.70
Y_4	96964.92	95172.51	1792.41	74.92	136.35	21.47	119.13
Y_5	103570.64	101748.17	1822.47	284.37	455.08	299.89	711.29
Y_6	121386.36	119448.88	1937.48	539.20	601.97	499.64	466.78
Y_7	128571.52	126703.24	1868.28	518.40	413.27	486.62	229.67
Y_8	132682.12	130977.00	1705.12	788.07	890.22	755.10	573.86
Y_9	134931.36	133381.94	1549.42	467.20	686.75	456.72	476.53
Y_{10}	142827.92	140597.45	2230.47	432.57	670.00	438.78	564.48
Y_{11}	145492.32	143378.64	2113.68	140.14	402.55	156.27	269.06
Y_{12}	157428.76	155461.36	1967.40	600.45	420.16	438.50	271.58
Y_{13}	160436.52	158995.40	1441.12	86.73	320.46	176.97	419.64
Y_{14}	171826.16	170496.48	1329.68	413.69	375.03	231.31	152.27

Table 23. Total sums of squares, corrections, corrected sums of squares, and sums of squares attributable to regression for the 4 methods (continued)

Period	Total Y^2	Corrections	y^2	Method I y^2	Method II y^2	Method III y^2	Method IV y^2
Y_{15}	175370.96	173847.97	1522.90	267.12	322.99	323.28	455.08
Y_{16}	163119.76	161084.59	2035.17	468.84	727.53	642.57	122.93
Y_{17}	177585.60	175827.80	1757.80	416.89	565.48	216.45	502.16
Y_{18}	199032.56	197664.12	1368.44	164.81	453.17	299.07	369.68
Y_{19}	202251.32	200990.46	1260.86	131.79	300.65	208.49	290.00
Y_{20}	220347.24	219243.15	1104.09	134.64	136.58	259.06	390.58
Y_{21}	224901.84	223732.60	1169.24	254.81	317.31	412.93	343.89
Y_{22}	82219.85	81016.73	1203.12	538.84	562.06	476.94	439.72
Y_{23}	117985.36	117435.94	549.42	175.21	165.97	133.94	152.00
Y_{24}	157540.63	157016.41	524.22	78.43	93.31	68.08	77.94
Y_{25}	198294.74	197849.91	444.83	80.25	128.21	114.06	82.94
Y_{26}	254708.12	254602.48	105.64	17.17	22.51	33.21	33.91
Y_{27}	244034.09	243913.22	120.87	25.59	28.50	21.91	74.82
Y_{28}	192614.98	192283.67	331.31	54.03	77.37	58.11	78.77

Table 24. R^2 values and probability levels for all Y periods and methods

Period	Method I (R^2)	Method II (R^2)	Method III (R^2)	Method IV (R^2)
Y_1	.54792 (.005)*	.61205 (.005)	.57014 (.005)	.53267 (.005)
Y_2	.39807 (.05)	.37706 (.25)	.37273 (.05)	.35625 (.10)
Y_3	.32990 (.10)	.26376 (.50)	.19565 (.50)	.14029 (n.s.)
Y_4	.04180 (n.s.)**	.07607 (n.s.)	.01198 (n.s.)	.06647 (n.s.)
Y_5	.15604 (n.s.)	.24971 (.50)	.16455 (n.s.)	.39029 (.10)
Y_6	.27830 (.25)	.31070 (.50)	.25788 (.25)	.24092 (.50)
Y_7	.27748 (.25)	.22120 (n.s.)	.26046 (.25)	.12293 (n.s.)
Y_8	.46218 (.01)	.52209 (.025)	.44285 (.025)	.33655 (.25)
Y_9	.30153 (.25)	.44323 (.05)	.29477 (.25)	.30755 (.25)
Y_{10}	.19394 (.50)	.30038 (.50)	.19672 (.50)	.25308 (.50)
Y_{11}	.06630 (n.s.)	.19045 (n.s.)	.07393 (n.s.)	.12729 (n.s.)
Y_{12}	.30520 (.25)	.21356 (n.s.)	.22288 (.50)	.13804 (n.s.)
Y_{13}	.06019 (n.s.)	.22237 (n.s.)	.12280 (n.s.)	.29119 (.25)
Y_{14}	.31112 (.25)	.28205 (.50)	.17396 (n.s.)	.11452 (n.s.)
Y_{15}	.17539 (.50)	.21208 (n.s.)	.21227 (.50)	.29881 (.25)
Y_{16}	.23037 (.50)	.35748 (.25)	.31574 (.25)	.06040 (n.s.)
Y_{17}	.23717 (.50)	.32170 (.25)	.12314 (n.s.)	.28567 (.25)
Y_{18}	.12044 (n.s.)	.33091 (.25)	.21855 (.50)	.27015 (.50)
Y_{19}	.10452 (n.s.)	.23845 (n.s.)	.16536 (n.s.)	.23000 (.50)
Y_{20}	.12194 (n.s.)	.12371 (n.s.)	.23464 (.50)	.35376 (.25)
Y_{21}	.21793 (.50)	.27138 (.50)	.35316 (.10)	.29411 (.25)

Table 24. R^2 values and probability levels for all Y periods and methods (continued)

Period	Method I (R^2)	Method II (R^2)	Method III (R^2)	Method IV (R^2)
Y_{22}	.44787 (.025)	.46717 (.05)	.39642 (.05)	.36548 (.10)
Y_{23}	.31889 (.10)	.30207 (.50)	.24379 (.50)	.27665 (.50)
Y_{24}	.14961 (n.s.)	.17801 (n.s.)	.12987 (n.s.)	.14868 (n.s.)
Y_{25}	.18042 (.50)	.28821 (.50)	.25641 (.25)	.18645 (n.s.)
Y_{26}	.16254 (n.s.)	.21310 (n.s.)	.31438 (.25)	.32103 (.25)
Y_{27}	.21174 (.50)	.23580 (n.s.)	.18129 (.50)	.61905 (.005)
Y_{28}	.16310 (n.s.)	.23352 (n.s.)	.17540 (.50)	.23775 (.50)

* Values in parenthesis are the approximate probability levels associated with tabled F values.
** Not significant at 50% level of probability or less.

Table 25. Analysis of variance for the multiple linear regression of Method I, Y_1

Source of variation	Degrees of freedom	Sum of squares	Mean squares
Due to regression	6	1121.88	186.98
Deviations about regression	26	925.64	35.60
Total	32	2047.52	

Table 26. Independent (X) variables for 1922-23 and 1955-56

1922-23

Method I	Method II	Method III	Method IV
X_1 = 44.03	X_1 = 36.8	X_1 = 36.3	X_1 = 36.8
X_2 = 0.2375	X_2 = 36.0	X_2 = 36.0	X_2 = 38.0
X_3 = 0.00845	X_3 = 34.8	X_3 = 36.8	X_3 = 34.0
X_4 = 0.0002577	X_4 = 26.1	X_4 = 27.8	X_4 = 35.8
X_5 = 0.000006698	X_5 = 39.2	X_5 = 36.2	X_5 = 34.3
X_6 = 0.0000001395	X_6 = 35.0	X_6 = 54.7	X_6 = 27.6
	X_7 = 42.3		X_7 = 47.0
	X_8 = 70.8		

1955-56

Method I	Method II	Method III	Method IV
X_1 = 44.59	X_1 = 33.0	X_1 = 35.0	X_1 = 33.0
X_2 = 0.1944	X_2 = 43.1	X_2 = 38.3	X_2 = 40.0
X_3 = 0.005690	X_3 = 36.1	X_3 = 40.8	X_3 = 46.2
X_4 = 0.0001723	X_4 = 25.7	X_4 = 27.8	X_4 = 38.0
X_5 = 0.000005560	X_5 = 41.0	X_5 = 40.9	X_5 = 35.2
X_6 = 0.0000000613	X_6 = 39.6	X_6 = 52.5	X_6 = 30.2
	X_7 = 43.9		X_7 = 47.0
	X_8 = 65.6		

Table 27. Actual, computed, and deviations from actual temperatures for 1923

Period	Actual temp.	33-year average temperatures (Comp.)	(Dev.)	Method I (Comp.)	(Dev.)	Method II (Comp.)	(Dev.)	Method III (Comp.)	(Dev.)	Method IV (Comp.)	(Dev.)
Y_1	35.4	49.0	+13.6	40.8	+ 5.4	41.5	+ 6.1	41.0	+ 5.6	36.8	+ 1.4
Y_2	41.0	49.0	+ 8.0	45.1	+ 4.1	49.1	+ 8.1	50.6	+ 9.6	47.2	+ 6.2
Y_3	57.0	50.8	- 6.2	49.9	- 7.1	52.6	- 4.4	49.1	- 7.9	51.2	- 5.8
Y_4	50.4	53.7	+ 3.3	52.8	+ 2.4	51.6	+ 1.2	53.5	+ 3.1	53.9	+ 3.5
Y_5	48.8	55.5	+ 6.7	48.3	- 0.5	49.1	+ 0.3	50.2	+ 1.4	49.4	+ 0.6
Y_6	59.4	60.2	+ 0.8	54.9	- 4.5	52.3	- 7.1	56.9	- 2.5	52.7	- 6.7
Y_7	59.2	62.0	+ 2.8	62.0	+ 2.8	64.5	+ 5.3	61.2	+ 2.0	58.6	- 0.6
Y_8	47.6	63.0	+15.4	64.0	+16.4	67.2	+19.6	61.1	+13.5	60.7	+13.1
Y_9	59.2	63.6	+ 4.4	54.8	- 4.4	63.0	+ 3.8	56.0	- 3.2	59.8	+ 0.6
Y_{10}	66.2	65.3	- 0.9	64.1	- 2.1	60.7	- 5.5	58.3	- 7.9	60.6	- 5.6
Y_{11}	75.8	65.9	- 9.9	66.3	- 9.5	64.8	-11.0	66.2	- 9.6	65.4	-10.4
Y_{12}	56.6	68.6	+12.0	67.3	+10.7	64.5	+ 7.9	63.8	+ 7.2	64.4	+ 7.8
Y_{13}	69.8	69.4	- 0.4	67.6	- 2.2	68.0	- 1.8	69.6	- 0.2	69.1	- 0.7
Y_{14}	71.6	71.9	+ 0.3	67.2	- 4.4	65.6	- 6.0	72.1	+ 0.5	67.8	- 3.8
Y_{15}	62.6	72.6	+10.0	75.1	+12.5	74.8	+12.2	70.7	+ 8.1	71.3	+ 8.7

Table 27. Actual, computed, and deviations from actual temperatures for 1923 (continued)

Period	Actual temp.	33-year average temperatures (Comp.)	(Dev.)	Method I (Comp.)	(Dev.)	Method II (Comp.)	(Dev.)	Method III (Comp.)	(Dev.)	Method IV (Comp.)	(Dev.)
Y_{16}	66.0	70.0	+ 4.0	71.7	+ 5.7	69.6	+ 3.6	73.9	+ 7.9	68.9	+ 2.9
Y_{17}	73.8	73.0	- 0.8	72.8	- 1.0	72.4	- 1.4	76.1	+ 2.3	78.5	+ 4.7
Y_{18}	78.4	77.4	- 1.0	80.2	+ 1.8	84.2	+ 5.8	82.7	+ 4.3	81.2	+ 2.8
Y_{19}	68.8	78.0	+ 9.2	74.7	+ 5.9	76.3	+ 7.5	77.4	+ 8.6	77.5	+ 8.7
Y_{20}	69.8	81.5	+11.7	77.7	+ 7.9	78.1	+ 8.3	80.4	+10.6	78.4	+ 8.6
Y_{21}	79.2	82.3	+ 3.1	76.7	- 2.5	76.4	- 2.8	80.3	+ 1.1	77.0	- 2.2
Y_{22}	44.5	49.6	+ 5.1	45.3	+ 0.8	47.8	+ 3.3	46.9	+ 2.4	45.1	+ 0.6
Y_{23}	54.1	59.7	+ 5.6	57.0	+ 2.9	57.9	+ 3.8	56.5	+ 2.4	55.9	+ 1.8
Y_{24}	67.0	69.0	+ 2.0	68.1	+ 1.1	66.6	- 0.4	67.2	+ 0.2	66.7	- 0.3
Y_{25}	73.5	77.4	+ 3.9	76.2	+ 2.7	76.4	+ 2.9	78.5	+ 5.0	77.1	+ 3.6
Y_{26}	87.5	87.8	+ 0.3	87.3	- 0.2	87.1	- 0.4	86.7	- 0.8	87.2	- 0.3
Y_{27}	84.1	86.0	+ 1.9	87.3	+ 3.2	86.3	+ 2.2	86.7	+ 2.6	86.3	+ 2.2
Y_{28}	74.9	76.3	+ 1.4	79.5	+ 4.6	80.1	+ 5.2	79.8	+ 4.9	79.5	+ 4.6
Maximum dev. from actual			15.400		16.400		19.600		13.500		13.100
Average dev. (Y_1 to Y_{21})			5.929		5.419		6.176		5.576		5.019
Standard dev. (Y_1 to Y_{21})			7.744		6.888		7.695		6.879		6.276

Table 28. Actual, computed, and deviations from actual temperatures for 1956

Period	Actual temp.	33-year average temperatures (Comp.)	(Dev.)	Method I (Comp.)	(Dev.)	Method II (Comp.)	(Dev.)	Method III (Comp.)	(Dev.)	Method IV (Comp.)	(Dev.)
Y_1	56.0	49.0	- 7.0	45.4	-10.6	50.1	- 5.9	45.9	-10.1	43.3	-12.7
Y_2	68.0	49.0	-19.0	55.0	-13.0	55.0	-13.0	55.0	-13.0	51.4	-16.6
Y_3	55.8	50.8	- 5.0	54.9	- 0.9	53.0	- 2.8	54.0	- 1.8	52.5	- 3.3
Y_4	46.2	53.7	+ 7.5	51.1	+ 4.9	50.4	+ 4.2	51.8	+ 5.6	55.9	+ 9.7
Y_5	59.6	55.5	- 4.1	54.6	- 5.0	52.8	- 6.8	48.3	-11.3	54.5	- 5.1
Y_6	62.0	60.2	- 1.8	60.7	- 1.3	60.2	- 1.8	60.1	- 1.9	56.9	- 5.1
Y_7	65.2	62.0	- 3.2	67.3	+ 2.1	66.8	+ 1.6	66.1	+ 0.9	55.7	- 9.5
Y_8	70.4	63.0	- 7.4	67.8	- 2.6	72.4	+ 2.0	65.6	- 4.8	60.8	- 9.6
Y_9	61.2	63.6	+ 2.4	63.0	+ 1.8	63.2	+ 2.0	59.9	- 1.3	61.0	- 0.2
Y_{10}	69.4	65.3	- 4.1	67.6	- 1.8	64.1	- 5.3	59.1	-10.3	62.5	- 6.9
Y_{11}	70.6	65.9	- 4.7	66.8	- 3.8	65.2	- 5.4	63.7	- 6.9	67.1	- 3.5
Y_{12}	53.2	68.6	+15.4	57.2	+ 4.0	63.1	+ 9.9	60.4	+ 7.2	65.3	+12.1
Y_{13}	77.2	69.4	- 7.8	67.9	- 9.3	73.0	- 4.2	71.2	- 6.0	72.6	- 4.6
Y_{14}	81.2	71.9	- 9.3	74.5	- 6.7	73.3	- 7.9	73.8	- 7.4	71.0	-10.2
Y_{15}	73.0	72.6	- 0.4	70.6	- 2.4	70.5	- 2.5	66.1	- 6.9	71.7	- 1.3

Table 28. Actual, computed, and deviations from actual temperatures for 1956 (continued)

Period	Actual temp.	33-year average temperatures (Comp.)	(Dev.)	Method I (Comp.)	(Dev.)	Method II (Comp.)	(Dev.)	Method III (Comp.)	(Dev.)	Method IV (Comp.)	(Dev.)
Y_{16}	82.2	69.9	-12.3	78.1	- 4.1	78.8	- 3.4	80.4	- 1.8	69.5	-12.7
Y_{17}	80.6	73.0	- 7.6	80.8	+ 0.2	73.6	- 7.0	75.9	- 4.7	75.8	- 4.8
Y_{18}	87.8	77.4	-10.4	81.8	- 6.0	83.7	- 4.1	85.3	- 2.5	75.9	-11.9
Y_{19}	75.6	78.0	+ 2.4	75.9	+ 0.3	74.6	- 1.0	75.9	+ 0.3	73.8	- 1.8
Y_{20}	78.4	81.5	+ 3.1	78.3	- 0.1	79.4	+ 1.0	81.9	+ 3.5	78.3	- 0.1
Y_{21}	84.6	82.3	- 2.3	79.1	- 5.5	79.2	- 5.4	83.2	- 1.4	79.2	- 5.4
Y_{22}	59.9	49.6	-10.3	51.8	- 8.1	52.7	- 7.1	51.6	- 8.3	49.1	-10.8
Y_{23}	60.8	59.7	- 1.1	60.7	- 0.1	61.0	+ 0.2	58.7	- 2.1	57.5	- 3.3
Y_{24}	71.1	69.0	- 2.1	67.6	- 3.5	68.5	- 2.6	66.1	- 5.0	68.5	- 2.6
Y_{25}	81.8	77.4	- 4.4	79.2	- 2.6	78.5	- 3.3	80.7	- 1.1	75.8	- 6.0
Y_{26}	88.4	87.8	- 0.6	88.2	- 0.2	87.2	- 1.2	87.5	- 0.9	86.3	- 2.1
Y_{27}	85.6	86.0	+ 0.4	86.4	+ 0.8	85.7	+ 0.1	86.5	- 0.9	85.2	- 0.4
Y_{28}	80.4	76.3	- 4.1	78.4	- 2.0	77.3	- 3.1	78.9	- 1.5	76.7	- 3.7
Maximum dev. from actual			19.000		13.000		13.000		13.000		16.600
Average dev. (Y_1 to Y_{21})			6.533		4.114		4.629		5.219		7.005
Standard dev. (Y_1 to Y_{21})			8.196		5.484		5.655		6.528		8.567

Table 29. Maximum daily temperature data for Logan, Utah, 1956-57

	1956				1957		
Day	September	October	November	December	January	February	March
1	74	70	34	42	34	33	45
2	82	71	32	38	31	41	50
3	80	74	30	39	40	40	48
4	77	77	31	42	25	35	54
5	79	75	34	49	31	32	41
6	83	74	42	32	16	32	47
7	84	74	47	31	23	34	44
8	85	77	46	15	26	39	54
9	84	73	50	14	34	48	64
10	87	66	53	18	28	46	57
11	80	67	58	32	26	36	45
12	86	74	58	45	41	53	50
13	85	51	57	45	44	44	57
14	82	59	56	44	39	37	39
15	84	62	35	39	41	39	44
16	84	65	43	44	39	35	56
17	85	69	45	42	26	37	57
18	87	70	45	41	20	37	49
19	86	65	32	34	23	37	52
20	89	65	24	41	41	52	58
21	82	62	28	37	36	50	60
22	65	65	33	34	32	47	39
23	72	62	37	25	22	54	37
24	72	59	40	32	29	52	46
25	77	35	41	31	31	56	44
26	80	46	40	32	30	51	45
27	82	57	41	22	29	52	48
28	77	62	39	22	29	50	54
29	73	34	40	29	21		59
30	70	41	41	30	15		56
31		55		30	21		50

Table 30. Computation of orthogonal polynomial coefficients for 1956-57

Sum of 2-day temperatures		Sum (s)	Difference (d)
83	83	166	0
89	75	164	14
77	71	148	6
32	57	89	-25
46	63	109	-17
81	44	125	37
81	59	140	22
80	64	144	16
81	71	152	10
80	56	136	24
81	39	120	42
77	60	137	17
61	54	115	7
56	85	141	-29
90	80	170	10
78	65	143	13
113	43	156	70
116	77	193	39
103	54	157	49
93	60	153	33
76	59	135	17
61	50	111	11
66	36	102	30
96	74	170	22
96	75	171	21
103	64	167	39
94	73	167	21
127	94	221	33
127	89	216	38
135	81	216	54
134	74	208	60
121	74	195	47
125	89	214	36
133	97	230	36
150	106	256	44
148	107	255	41
152	102	254	50
145	95	240	50
140	102	242	38
150	88	238	62
162	98	260	64
149	121	270	28
137	95	232	42
171	96	267	75
173	100	273	73
	Sum	8168	

Computation of coefficients

$$A_0 = \frac{\Sigma s}{N} = \frac{8168}{90} = 90.7556$$

$$A_1 = \frac{\Sigma (d\,\xi_1')}{\Sigma \xi_1'^2} = \frac{80,932}{242,970} = 0.3331$$

$$A_2 = \frac{\Sigma (s\,\xi_2')}{\Sigma \xi_2'} = \frac{1,895,539}{73,765,692} = 0.025697$$

$$A_3 = \frac{\Sigma (d\,\xi_3')}{\Sigma \xi_3'^2} = \frac{-350,879}{47,368,112,220}$$

$$= -0.000007408$$

$$A_4 = \frac{\Sigma (s\,\xi_4')}{\Sigma \xi_4'^2} = \frac{-79,176,627}{74,457,409,286,260}$$

$$= -0.000001063$$

$$A_5 = \frac{\Sigma (d\,\xi_5')}{\Sigma \xi_5'^2} = \frac{1,378,759,355}{19,677,062,617,741,620}$$

$$= 0.00000007007$$

Table 31. Independent (X) variables for the 1957 forecasts

Method I

$$X_1 = \frac{A_0}{2} = \frac{90.7556}{2} = 45.38$$

$$X_2 = \frac{A_1}{2} = \frac{0.3331}{2} = 0.1666$$

$$X_3 = \frac{A_2}{2} = \frac{0.025697}{2} = 0.012849$$

$$X_4 = \frac{A_3}{2} + 0.0001800 = \frac{-0.000007408}{2} + 0.0001800 = 0.0001763$$

$$X_5 = \frac{A_4}{2} + 0.00000550 = \frac{-0.000001063}{2} + 0.000000550 = 0.000004968$$

$$X_6 = \frac{A_5}{2} + 0.00000025 = 0.00000007007 + 0.000000250 = 0.0000002851$$

Method II	Method III	Method IV
X_1 = 49.2	X_1 = 46.3	X_1 = 49.2
X_2 = 50.4	X_2 = 54.5	X_2 = 52.8
X_3 = 46.3	X_3 = 49.2	X_3 = 48.0
X_4 = 34.8	X_4 = 40.0	X_4 = 50.8
X_5 = 31.1	X_5 = 31.5	X_5 = 44.0
X_6 = 33.6	X_6 = 53.4	X_6 = 40.1
X_7 = 41.9		X_7 = 45.2
X_8 = 69.4		

Table 32. The 1957 temperature forecast

Period	33-year average	Method I	Method II	Method III	Method IV	Actual
Y_1	49.0	58.3	59.5	55.2	57.3	55.2
Y_2	49.0	46.1	49.5	47.2	47.0	42.2
Y_3	50.8	49.9	54.0	54.6	48.5	53.4
Y_4	53.7	54.8	56.1	52.8	52.5	
Y_5	55.5	54.6	56.9	55.9	57.4	
Y_6	60.2	62.3	62.7	62.1	65.0	
Y_7	62.0	63.2	62.9	63.8	64.5	
Y_8	63.0	63.2	66.2	68.6	64.9	
Y_9	63.6	69.1	69.4	71.2	64.9	
Y_{10}	65.3	68.4	68.0	67.8	64.6	
Y_{11}	65.9	63.3	65.5	63.5	64.8	
Y_{12}	68.6	78.4	75.3	72.0	75.4	
Y_{13}	69.4	71.5	75.1	70.6	71.1	
Y_{14}	71.9	69.7	70.0	70.8	74.0	
Y_{15}	72.6	78.0	75.8	68.4	69.7	
Y_{16}	69.9	67.2	65.5	68.3	70.0	
Y_{17}	73.0	66.0	65.1	68.8	65.1	
Y_{18}	77.4	76.4	75.9	74.9	75.6	
Y_{19}	78.0	79.7	78.3	75.0	80.2	
Y_{20}	81.5	81.7	82.1	86.5	87.8	
Y_{21}	82.3	85.5	85.3	86.4	88.2	
Y_{22}	49.6	51.4	54.5	52.5	50.9	50.3

Table 32. The 1957 temperature forecast (continued)

Period	33-year average	Method I	Method II	Method III	Method IV	Actual
Y_{23}	59.7	61.2	62.3	62.4	61.5	
Y_{24}	69.0	71.4	71.4	68.6	69.8	
Y_{25}	77.4	76.9	76.2	77.5	78.4	
Y_{26}	87.8	88.4	88.0	88.9	88.3	
Y_{27}	86.0	86.3	85.0	85.4	85.5	
Y_{28}	76.3	74.5	73.9	72.9	73.7	

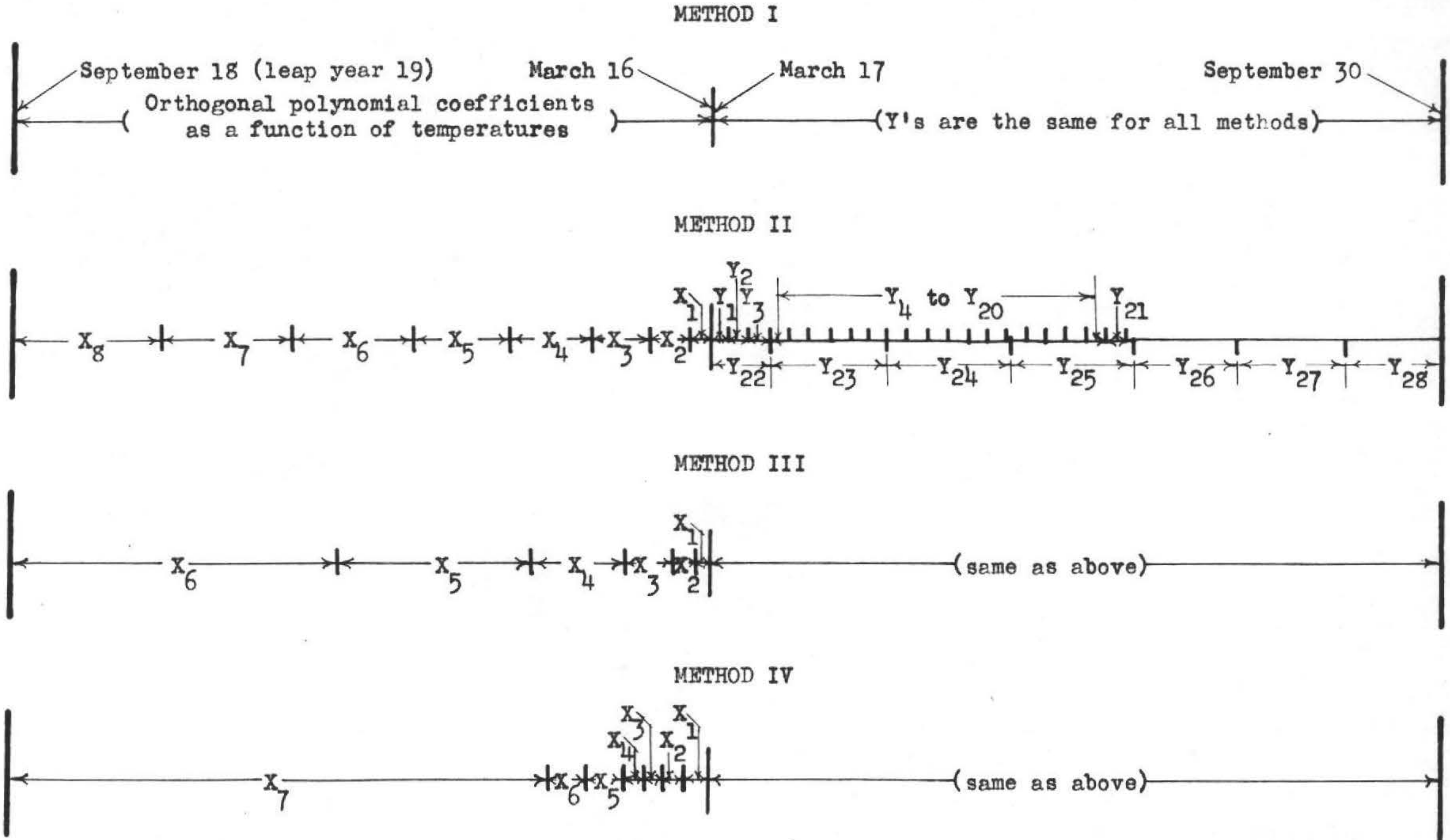

Figure 1. Graphic representation of the dependent (Y) variables and the 4 methods of independent (X) variables (See tables 2, 3, 4, and 5 for the days in each variable)

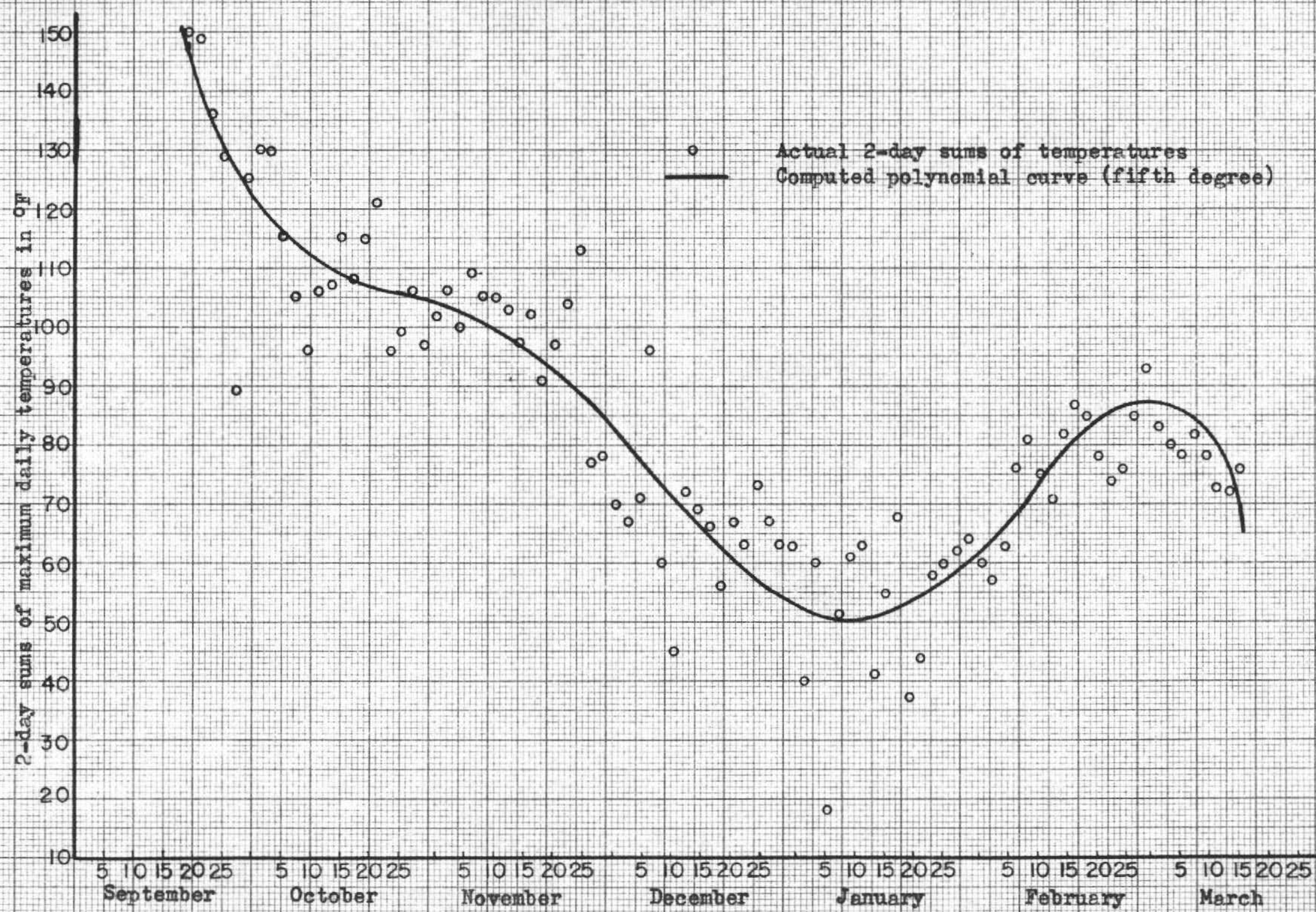

Figure 2. 2-day sums of maximum daily temperatures and the fifth degree polynomial curve for the period from September 19, 1923 to March 16, 1924

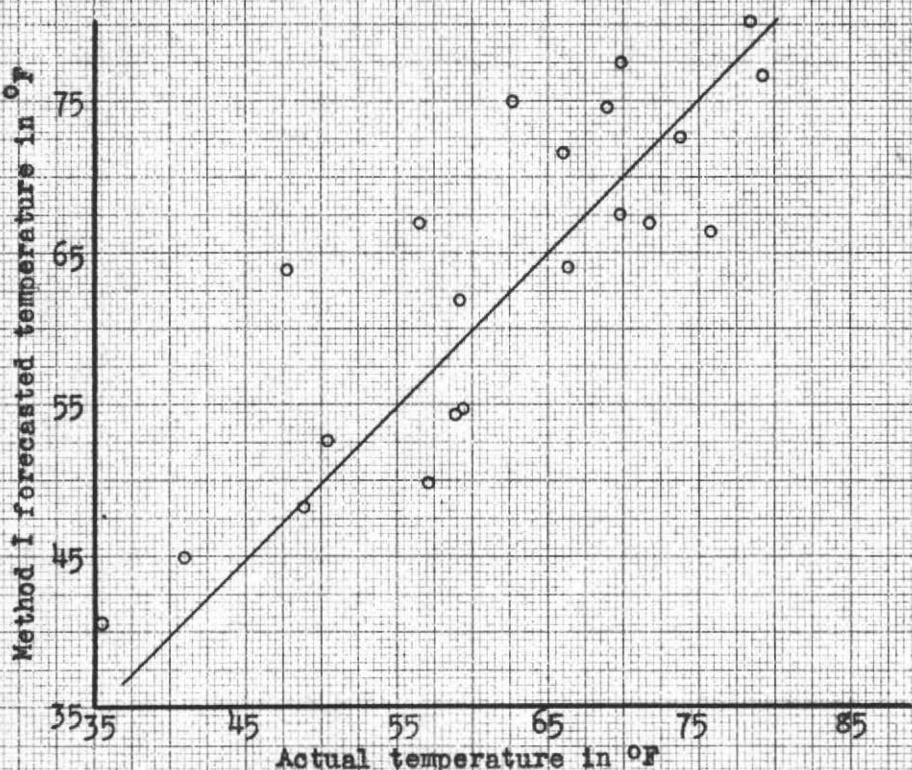

Figure 3. Actual and forecasted temperatures by method I from March 17 to June 29, 1923

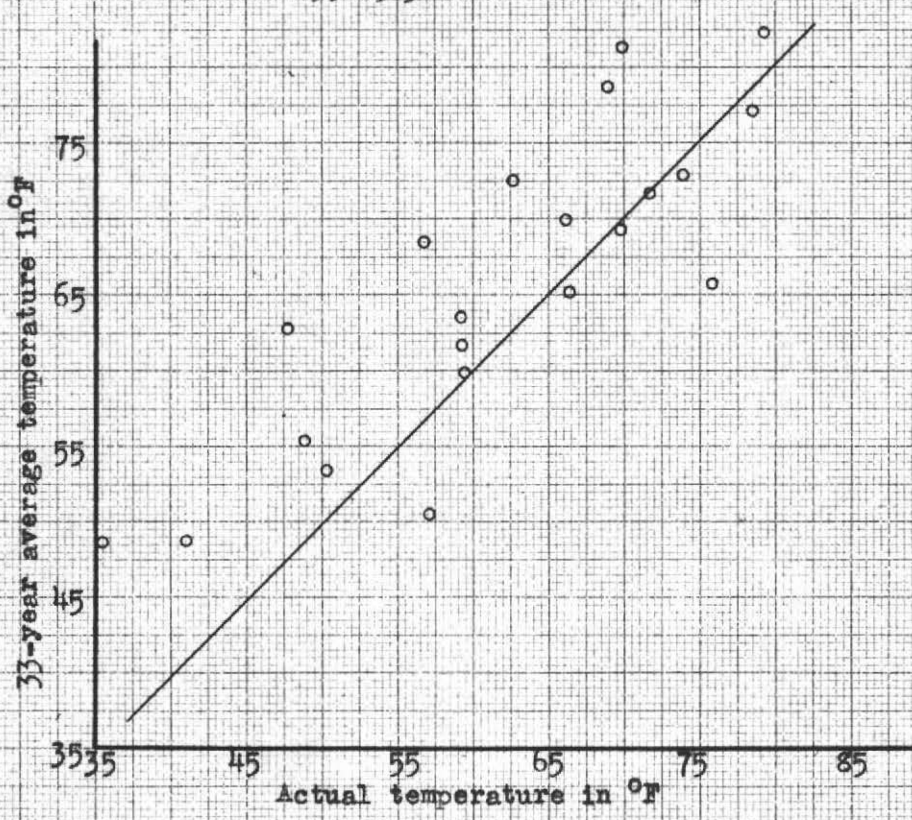

Figure 4. 33-year average temperatures and actual temperatures from March 17 to June 29, 1923

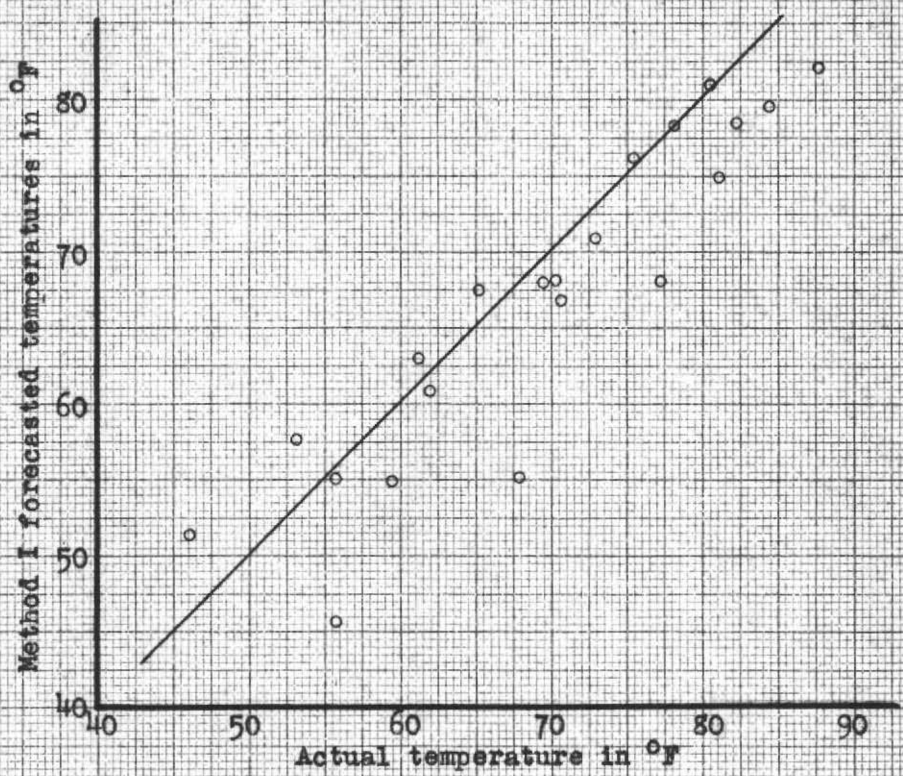

Figure 5. Actual and forecasted temperatures by method I from March 17 to June 29, 1956

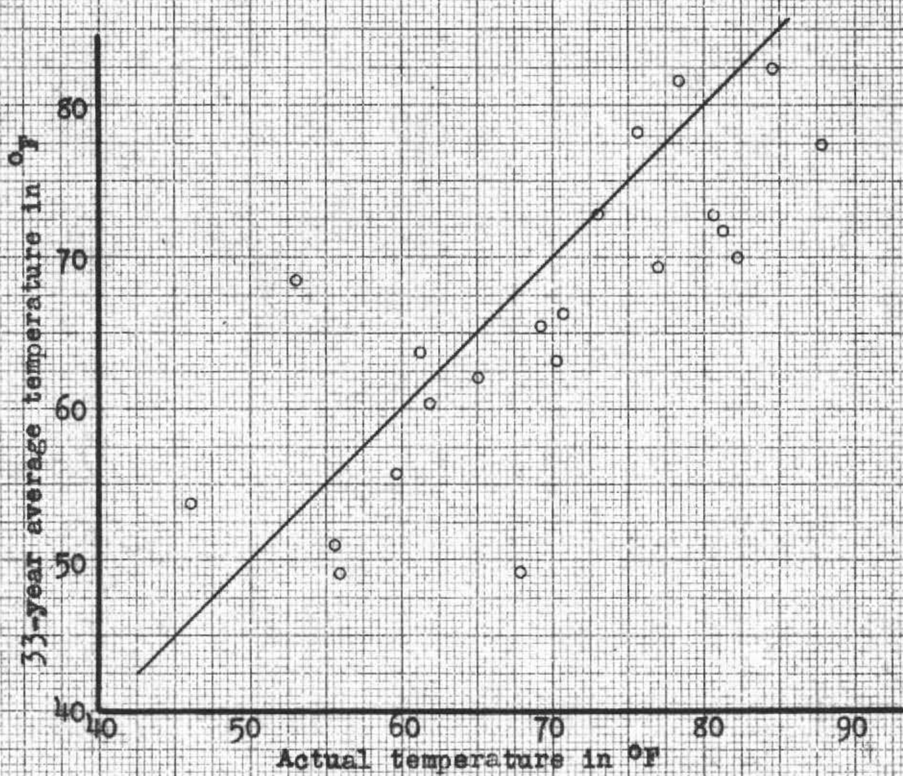

Figure 6. 33-year average temperatures and actual temperatures from March 17, 1956 to June 29, 1956