

Utah State University

DigitalCommons@USU

All Graduate Theses and Dissertations

Graduate Studies

5-1966

The Dynamics of Duopoly

Gordon E. Johnson

Utah State University

Follow this and additional works at: <https://digitalcommons.usu.edu/etd>



Part of the [Economics Commons](#)

Recommended Citation

Johnson, Gordon E., "The Dynamics of Duopoly" (1966). *All Graduate Theses and Dissertations*. 2805.
<https://digitalcommons.usu.edu/etd/2805>

This Thesis is brought to you for free and open access by the Graduate Studies at DigitalCommons@USU. It has been accepted for inclusion in All Graduate Theses and Dissertations by an authorized administrator of DigitalCommons@USU. For more information, please contact digitalcommons@usu.edu.



THE DYNAMICS OF DUOPOLY

by

Gordon E. Johnson

A thesis submitted in partial fulfillment
of the requirements for the degree

of

MASTER OF SCIENCE

in

Economics

UTAH STATE UNIVERSITY
Logan, Utah

1966

TABLE OF CONTENTS

STATEMENT OF PROBLEM	1
METHODS OF ANALYSIS	9
Theoretical Derivation of Dynamic Reaction Curves	9
Measurement of Conjectural Variation	15
RESULTS AND DISCUSSION	35
Identical Cost Curves, Limited Information	35
Identical Cost Curves, Complete Information	36
Similar Cost Curves, Limited Information	37
Dissimilar Cost Curves, Limited Information	38
Dissimilar Cost Curves, Complete Information	40
Discussion	41
BIBLIOGRAPHY	43
APPENDIXES	44
Appendix I	44
Appendix II	50
Appendix III	52
Appendix IV	54
Appendix V	56
Appendix VI	58
Appendix VII	60
Appendix VIII	62
Appendix IX	64
Appendix X	66
Appendix XI	68

LIST OF TABLES

Table	Page
1A. Conjectural variations for duopolists 1 with identical cost curves, limited information	21
1B. Conjectural variations for duopolists 2 with identical cost curves, limited information	23
2A. Conjectural variations for duopolists 1 with identical cost curves, complete information	25
2B. Conjectural variations for duopolists 2 with identical cost curves, complete information	26
3A. Conjectural variations for duopolists 1 with similar cost curves, limited information	27
3B. Conjectural variations for duopolists 2 with similar cost curves, limited information	28
4A. Conjectural variations for duopolists 1 with dissimilar cost curves, limited information	29
4B. Conjectural variations for duopolists 2 with dissimilar cost curves, limited information	31
5A. Conjectural variations for duopolists 1 with dissimilar cost curves, complete information	33
5B. Conjectural variations for duopolists 2 with dissimilar cost curves, complete information	34

STATEMENT OF THE PROBLEM

A monopolist, knowing the demand curve for his product, can in a given period produce the quantity of this product which will maximize his profit. Any larger or smaller quantity will result in less profit. When another manufacturer starts producing the same or similar product, a duopoly results. The new manufacturer, in order to maximize his profit, according to Cournot (2), will choose a quantity that is derived on the assumption that the original manufacturer's quantity will remain fixed.

Cournot's treatment of competition between two producers, originally published in 1838, was translated into English in 1897. As an abstract example Cournot considers two proprietors who each own springs and who sell springwater to the same market with negligible costs of production. The profits of proprietors 1 and 2 are respectively expressed as $D_1 f(D_1 + D_2)$ and $D_2 f(D_1 + D_2)$ where $D_1 + D_2$ is total production and where price is a function of total production.

Proprietor 1 can have no direct influence on the determination of D_2 . But he can adjust his price for the value of D_1 which is best for him according to the condition:

$$\frac{\partial [D_1 f(D_1 + D_2)]}{\partial D_1} = 0$$

and proprietor 2 can determine D_2 in terms of D_1 by the analogous condition:

$$\frac{\partial [D_2 f(D_1 + D_2)]}{\partial D_2} = 0$$

After the new manufacturer enters the market, with a quantity based on the assumption that his competitor's quantity will remain fixed, the original manufacturer must change his quantity of production in order to again maximize his profit since both duopolists share the same linear demand curve. This quantity is derived using the assumption that the new manufacturer won't change his quantity. One sets an output; this induces the first to readjust his, and so on. Each quantity adjustment by a manufacturer is smaller than his previous adjustment and in the opposite direction. Theoretically, an equilibrium will be reached only after an infinite number of adjustments. Practically, an equilibrium is reached with a finite number of adjustments because it is impossible for a manufacturer to produce and market fractions of a unit.

Only with production quantities at the Cournot equilibrium are the conditions on the previous page simultaneously satisfied, and with the conditions simultaneously satisfied neither duopolist has an incentive to change his production quantity. However, at the Cournot equilibrium the total income is less than the income received if either proprietor were a monopolist. With a collusive agreement to share the market and produce the monopoly output, each duopolist would receive greater income than if production were at the Cournot equilibrium. Without a collusive agreement the monopoly output is unstable because either duopolist could fix his

production at a higher or lower rate with a temporary benefit. The proprietor who adopts this course of action is soon punished because the other proprietor will then adopt a new scale of production. These successive reactions, instead of bringing both duopolists nearer to the condition of monopoly, separate them farther and farther from it.

Cournot further describes the path to equilibrium and shows mathematically why the monopoly quantity is unstable with two proprietors. This is also done for more than two proprietors and for proprietors that have limitations of productive capacity and varying production costs.

What Cournot explains mathematically is showed graphically by Chamberlin (1). Chamberlin also shows that with a linear demand curve and zero or identical constant costs of production a duopolist maximizes his profit by selecting a production quantity that is one-half the competitive output minus one-half the production quantity of the other duopolist. The competitive output is defined as the aggregate output that yields zero profits to each of the duopolists if the duopolists had no production costs. Therefore, if the current production of duopolist 1 is zero, the monopoly output for duopolist 2 is one-half the competitive output. If the current production of duopolist 2 is one-half the competitive output, duopolist 1 will enter the market with a production of one-quarter the competitive output. The next quantity for duopolist 2 will be three-eighths the competitive output, and the following quantity for duopolist 1 will be five-sixteenths of the competitive output. Both geometric series converge to one-third the competitive output so the aggregate output at the Cournot

equilibrium is two-thirds of the competitive output.

In real life it is doubtful that each duopolist readjusts his output with absolutely no expectation of retaliation. Thus Hicks (6) adds the concept of conjectural variation to the Cournot model in an effort to anticipate these expectations. The characteristic feature of the Hicksian model is that when there are only a few proprietors, the fear of retaliation is great. A duopolist, when changing his production quantity to maximize his profits, may attempt to predict the resulting quantity change of his competitor. Under these conditions marginal revenue to duopolist 1 becomes:

$$\frac{\partial [D_1 f(D_1 + D_2)]}{\partial D_1} = f(D_1 + D_2) + D_1 f'(D_1 + D_2) + D_1 f'(D_1 + D_2) \frac{\partial D_2}{\partial D_1}$$

where $\frac{\partial D_2}{\partial D_1}$, hereafter referred to as conjectural variation, is the degree to which duopolist 1 expects duopolist 2 to expand or contract output, if he himself expands output by an increment ΔD_1 .

A reaction curve can be constructed giving the profit maximizing output of duopolist 1 corresponding to each possible output of duopolist 2 when it is assumed that $\frac{\partial D_2}{\partial D_1}$ is invariant over time for each output of duopolist 2. Likewise a reaction curve giving the profit maximizing output of duopolist 2 corresponding to each possible output of duopolist 1 can be constructed. The intersection of these two reaction curves establishes a stable equilibrium. Movement away from this intersection

causes the output of one duopolist to rise and that of the other to fall. However, as pointed out by Fellner (3), equilibrium may not be at the intersection of these two reaction curves since the reaction curves may shift before equilibrium is achieved. This shifting occurs whenever original predictions, by both duopolists concerning their rival's behavior, are modified. An equilibrium is stable only as long as nobody realizes his notions are incorrect. It is extremely likely that these notions will be tested--particularly if previous quantity decisions by a duopolist had resulted in less than the maximum possible profit.

According to Stackleberg's duopoly analysis, if equilibrium is to be the outcome, it will not occur at the Cournot equilibrium. Rather the equilibrium will be at quantities near the monopoly output as achieved by a tacit or collusive agreement or else the equilibrium will be at the quantities where one duopolist succumbs to the leadership of the other. A follower adjusts his output level given the quantity decisions of the leader, and the leader knows it. A leader does not observe his own reaction curve. He assumes that the rival acts as a follower, and the leader proceeds to maximize his profit. Conjectural variation of the leader is the slope of the follower's reaction curve. This reaction function gives the profit maximizing output of the leader corresponding to each possible output of the follower. In this case the leader's conjectural variation is not necessarily zero, since the leader knows what the follower will do. It is possible that the follower will not react in

attempting to stop the leader short of complete leadership. A Cournot time path to equilibrium occurs when both duopolists act as followers. This equilibrium occurs at the intersection of the two reaction curves.

A Stackleberg profit-indifference map shows the combination of outputs by one proprietor on one axis and outputs by the other proprietor on the other axis which results in identical profits to the proprietor in question. The reaction function of the proprietor is then defined as the locus of tangency points of the family of curves to lines perpendicular to his rival's axis. The leadership equilibrium point of the proprietor is that point of the rival's (follower's) reaction curve where it becomes tangent to a profit-indifference curve of the leader. The followership point of a proprietor is the same as his rival's leadership point.

Microeconomic theory (5) shows with numerical examples profits to each duopolist at the Cournot equilibrium and profits to each duopolist when one has succumbed to the leadership of the other. Clearly the duopolist who becomes a leader receives the most profit. For this reason Stackleberg predicts both duopolists will strive to become leaders and that disequilibrium will be the outcome since both reaction curves will be constantly shifting.

Conjectural variation is the degree to which one duopolist expects the other to expand or contract output in retaliation for a quantity change; so if both duopolists exhibit a Cournot dynamic time path to equilibrium, the conjectural variation associated with each quantity

change is zero. A dynamic time path, that differs from the Cournot time path, to the Cournot equilibrium causes conjectural variations that originally are non-zero to approach zero. Stackleberg disequilibrium causes conjectural variations to vary randomly without a tendency to approach any fixed values. A leader-follower relationship causes conjectural variation of the follower to approach zero and the conjectural variations of the leader to approach a fixed negative value. Collusive agreements cause the conjectural variations of both duopolists to approach fixed positive values.

Duopoly bargaining situations are simulated in the laboratory to determine under what conditions one of the above outcomes is to be expected. Students are given actual cash payments according to production quantities selected. Following each selection the conjectural variation exhibited by the student in selecting that quantity is calculated. The empirical data from these simulations is used in developing models to predict conjectural variation for future time periods. Once these models are obtained and if Stackleberg disequilibrium is not the outcome, the complete equations for both reaction curves can be derived. Students play these games having either identical, similar, or dissimilar cost curves. Fixed and variable costs are such that with identical cost curves the Cournot equilibrium for both duopolists is 63 units of production. With similar cost curves the Cournot equilibrium is 66 and 56 respectively for duopolists 1 and 2. With dissimilar cost curves the

Cournot equilibrium is 72 and 42. On some games the players know the profit level of their opponent in addition to their own.

Empirical data obtained in the same manner by Lawrence E. Fouraker and Sidney Siegel (4) support the following conclusions:

1. Bargainers under incomplete information (knowing only their own profit levels but not the profit levels of their opponents) tend to negotiate transactions at the Cournot equilibrium.
2. Increasing the amount of relevant information available to bargainers decreases the tendency to the Cournot point.
3. Bargaining groups under complete information (knowing the profit level of their opponents in addition to their own), show no strong tendency or typical solution in their negotiated transactions, but rather show a multi-modal distribution of results. Under complete information some oligopolies show a tendency to the Paretian optima (outputs that result in the largest aggregate profit), some to the competitive point, and some to mixed solutions.

However, in the study by Fouraker and Siegel when students exhibited a tendency to negotiate to equilibriums, conjectural variation was not measured and no attempt was made to derive equations of the reaction curves. Only payoff matrices were used that had the same Cournot equilibrium quantity for both duopolists, and zero cost curves were assumed.

METHODS OF ANALYSIS

Theoretical Derivation of Dynamic Reaction Curves

Equation (1) is the demand function for the combined output of two duopolists; equations (2) and (3) are cost functions for duopolists 1 and 2 respectively.

$$p(t) = a - b [q_1(t) + q_2(t)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$T_1[q_1(t)] = k_1 + c_1 q_1(t) + d_1 q_1^2(t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$T_2[q_2(t)] = k_2 + c_2 q_2(t) + d_2 q_2^2(t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Given that each duopolist has a two period planning horizon, the profit to duopolist 1 is :

$$\begin{aligned} \pi_1 = & [a - bq_1(t) - bq_2(t)] q_1(t) - [k_1 + c_1 q_1(t) + d_1 q_1^2(t)] \\ & + [a - bq_1(t+1) - bq_2(t+1)] q_1(t+1) - [k_1 + c_1 q_1(t+1) \\ & + d_1 q_1^2(t+1)] \end{aligned}$$

The profit to duopolist 1 can then be maximized with respect to $q_1(t)$ as follows :

$$\begin{aligned}
\frac{\partial \pi_1}{\partial q_1(t)} &= [a - bq_1(t) - bq_2(t)] + q_1(t) \left[-b - b \frac{\partial q_2(t)}{\partial q_1(t)} \right] - c_1 \\
&\quad - 2d_1 q_1(t) + [a - bq_1(t+1) - bq_2(t+1)] \frac{\partial q_1(t+1)}{\partial q_1(t)} \\
&\quad + q_1(t+1) \left[-b \frac{\partial q_1(t+1)}{\partial q_1(t)} - b \frac{\partial q_2(t+1)}{\partial q_1(t)} - c_1 \frac{\partial q_1(t+1)}{\partial q_1(t)} \right] \\
&\quad - 2d_1 q_1(t+1) \frac{\partial q_1(t+1)}{\partial q_1(t)} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)
\end{aligned}$$

Since duopolist 1 does not expect duopolist 2 to react instantaneously to changes in $q_1(t)$, $\frac{\partial q_2(t)}{\partial q_1(t)} = 0$. However, $\frac{\partial q_2(t+1)}{\partial q_1(t)}$ is not necessarily zero;

$$\frac{\partial q_2(t+1)}{\partial q_1(t)},$$

the conjectural variation of duopolist 1 and designated by $\ell_1(t)$, is the degree to which duopolist 1 expects duopolist 2 to expand or contract output, if he himself changes output by an increment Δq_1 . Duopolist 1 will not alter output in the next period solely as a result of changes in output occurring during the present period; therefore

$$\frac{\partial q_1(t+1)}{\partial q_1(t)} = 0,$$

and equation (5) becomes :

Solving for $q_1(t+1)$:

$$q_1(t+1) = \frac{a - c_1 - b q'_2(t+1)}{2(b + d_1)}$$

The anticipated output by duopolist 1 of duopolist 2 in period $t+1$ is $q'_2(t+1)$. (A primed quantity indicates the quantity is an anticipated output by the other duopolist rather than the actual output. This distinction is necessary in the following computations.)

By definition of conjectural variation $q_1(t+1)$ can be arrived at as follows:

$$q'_2(t) = q_2(t-1) + [q_1(t-1) - q_1(t-2)] \frac{\partial q_2(t)}{\partial q_1(t-1)} \quad (10)$$

$$q'_2(t+1) = q_2(t) + [q_1(t) - q_1(t-1)] \frac{\partial q_2(t+1)}{\partial q_1(t)} \quad (11)$$

where $\frac{\partial q_2(t)}{\partial q_1(t-1)}$ and $\frac{\partial q_2(t+1)}{\partial q_1(t)}$ are conjectural variations for two successive period. Thus $q_1(t+1)$ becomes:

$$q_1(t+1) = \frac{a - c_1 - b [q'_2(t) + \{q_1(t) - q_1(t-1)\} \ell_1(t)]}{2(b + d_1)} \quad (12)$$

where $\ell_1(t) = \frac{\partial q_2(t+1)}{\partial q_1(t)}$. Substituting equation (12) into (4) :

$$\begin{aligned}
\frac{\partial \pi_1}{\partial q_1(t)} &= a - bq_1(t) - bq'_2(t) - bq_1(t) - c_1 - 2d_1q_1(t) \\
&\quad - \frac{b}{2(b+d_1)} \left(a - c_1 - b [q'_2(t) + \{q_1(t) - q_1(t-1)\}] \right. \\
&\quad \left. \ell_1(t) \right) \ell_1(t) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)
\end{aligned}$$

Solving for $q_1(t)$:

$$\begin{aligned}
q_1(t) &= \frac{2[b+d_1][a-c_1-bq'_2(t)] - b[a-c_1-bq'_2(t)+bq_1(t-1)\ell_1(t)]\ell_1(t)}{b^2[4-\ell_1^2(t)] + 4d_1(2b+d_1)} \\
&\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)
\end{aligned}$$

By differentiating equations (1) and (2) with respect to $q_1(t)$ after multiplying equation (1) by $q_1(t)$, marginal revenue and marginal costs are obtained. Equating marginal revenue to marginal cost and solving for $q_1(t)$ yields:

$$q_1(t) = \frac{a - c_1 - bq'_2(t)}{2(b+d_1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Equation (15) shows $q_1(t)$ that maximizes profit to duopolist 1 for any value $q'_2(t)$ selected by duopolist 2.

Let conjectural variation be defined as follows:

$$\ell_1(t) = \alpha_1 + \beta_1 \left(\frac{a - c_1 - bq_2(t-1)}{2(b+d_1)} - q_1(t-1) \right) + \gamma_1 \ell_1(t-1) \quad (16)$$

The expression in parenthesis in equation (16) is the quantity by which duopolist 1 missed his profit-maximizing quantity on the last production decision; the expression satisfies the following conditions: (a) If duopolist 2 is not in the market and duopolist 1 is producing a profit maximizing quantity, the conjectural variation is zero; this is expected in real life because duopolist 1 does not expect any reaction from duopolist 2 until duopolist 2 is known to be in the market. (b) When both duopolists are at their respective Cournot equilibria, conjectural variation defined as $\ell_1(t)$ is zero; this is so because duopolist 1 expects duopolist 2 to eventually produce a quantity specified by the Cournot equilibrium and maintain that quantity.

The constant β exists only when both duopolists fail to reach the Cournot equilibrium. If both duopolists reach the Cournot equilibrium α will be zero. If duopolist 2 becomes a follower, α_2 will be zero and α_1 for duopolist 1 will be negative; the degree of leadership by duopolist 1 is indicated by how negative α_1 is. With Stackleberg disequilibrium, both α_1 and α_2 are negative. If duopolist 1 is attempting to reach a tacit agreement or has obtained one α_1 is positive.

The constant γ exists only when a duopolist considers previous conjectural variations in making current decisions of production quantities.

Using identical procedures, the reaction curve for duopolist 2 is obtained.

$$q_2(t) = \frac{2[b+d_2][a-c_2-bq_1'(t)] - b[a-c_2-bq_1'(t) + bq_2(t-1)\ell_2(t)]\ell_2(t)}{b^2[4-\ell_2^2(t)] + 4d_2(2b+d_2)} \quad (17)$$

$$q_1'(t) = q_1(t-1) + [q_2(t-1) - q_2(t-2)]\ell_2(t-1)\ell_2(t-1) \quad (18)$$

$$\ell_2(t) = d_2 + \beta_2 \left(\frac{a-c_2-bq_1(t-1)}{2(b+d_2)} - q_2(t-1) \right) + \gamma_2 \ell_2(t-1) \quad (19)$$

Measurement of Conjectural Variation

Solving equation (14) for $\ell_1(t)$:

$$\ell_1(t) = \frac{-b\{a-c_1-bq_2'(t)\} \pm \sqrt{b^2[a-c_1-bq_2'(t)]^2 - 4b^2\{q_1(t-1) - q_1(t)\}\{q_1(t)[4b^2+8d_1b+4d_1^2] - 2[b+d_1]\}}}{2b^2[q_1(t-1) - q_1(t)]} \quad (20)$$

where

$$q_2'(t) = q_2(t-1) + \{q_1(t-1) - q_1(t-2)\}\ell_1(t-1)$$

To show that the desired root exists only when the square root in the above equation is (+), it is necessary to evaluate limit $\ell_1(t)$ as $q_1(t) \rightarrow q_1(t-1)$ using a (+) sign. Multiplying both numerator and denominator of equation (20) by

$$-b \{a - c_1 - bq'_2(t)\} - \sqrt{b^2[a - c_1 - bq'_2(t)]^2 - 4b^2\{q_1(t-1) - q_1(t)\} \{q_1(t) - q_1(t-1)\}}$$

and letting $q_1(t) \rightarrow q_1(t-1)$:

$$\lim_{q_1(t) \rightarrow q_1(t-1)} \ell_1(t) = \frac{-[4b^2 + 8d_1b + 4d_1^2]q_1(t) + 2[b + d_1][a - c_1 - bq'_2(t)]}{b[a - c_1 - bq'_2(t)]} \quad (21)$$

Using a (-) sign, $\lim_{q_1(t) \rightarrow q_1(t-1)} \ell_1(t) = \infty$

Equation (21) provides a method of evaluating conjectural variation when the production quantity of a producer remains constant from one period to the next. Of course equation (20) can only be used to evaluate conjectural variation when the production quantity changes.

Using identical calculations, the equations for measuring conjectural variation of duopolist 2 are obtained:

$$\ell_2(t) = \frac{-b \{a - c_2 - bq'_1(t)\} + \sqrt{b^2 [a - c_2 - bq'_1(t)]^2 - 4b^2 \{q_2(t-1) - q_2(t)\}}}{2b^2 [q_2(t-1) - q_2(t)]}$$

$$\frac{q_2(t) \{q_2(t) [4b^2 + 8d_1 b + 4d_1^2] - 2[b + d_1] [a - c_1 - bq'_1(t)]\}}{\dots \dots \dots (22)}$$

where

$$q'_1(t) = q_1(t-1) + \{q_2(t-1) - q_2(t-2)\} \ell_2(t-1)$$

$$\lim_{q_2(t) \rightarrow q_2(t-1)} \ell_2(t) = \frac{-[4b^2 + 8d_1 b + 4d_1^2] q_2(t) + 2[b + d_1] [a - c_1 - bq'_1(t)]}{b[a - c_2 - bq_1(t)]} \dots \dots \dots (23)$$

The conjectural variation for each decision of every game was computed with the aid of an IBM 1620 computer using equations (20) through (23).

Description of Games

Five empirical games were played; and in each game there were 10 to 21 players representing duopolist 1 and 10 to 21 players representing duopolist 2 for a total of approximately 160 players. The players were undergraduate students at Utah State University who volunteered and had completed Freshman Economics. As each student volunteered he was arbitrarily assigned a future game time that was not in conflict with a previously scheduled game or the student's class schedule.

Physical arrangements were such that a player did not know the identity of his opponent until after the game. It was felt that asking each player not to discuss the experiment with anyone would cause more discussion than if they were not asked. Also discussion probably could not bias the results since a player and any preceding player had only one chance in eight of being assigned matrices with the same payoff levels.

When a bargaining pair first met with the administrator, each player was furnished with an instruction sheet (see Appendix I), a copy of one of various compatible payoff matrices (see Appendixes II through XI) and the starting production quantity of both duopolists. The starting point on all matrices was the quantity that results in maximum profit to a duopolist if his opponent is not in business. In other words the conjectural variation is zero for both duopolists when they first start production; theoretically, neither duopolist knows the other exists, and neither duopolist expects retaliation until he is aware of the competition.

Next each duopolist examined his payoff matrix and secretly marked the quantity of production on a slip of paper. The administrator examined both slips of paper, recorded $q_1(1)$ and $q_2(1)$ along with the payoff, and advised each player of his payoff and opponents move by returning the paper. After 9 more such moves the payoffs were totaled and paid. No conversation was permitted between participants during the session.

Calculation of Payoff Matrices

All payoff matrices contain 962 cells. (See Appendixes II through XI) In each cell the lower number is the payoff to duopolist 1 and the upper number is the payoff to duopolist 2. A cell is selected by duopolist 1 choosing a q_1 on the abscissa and duopolist 2 simultaneously choosing a q_2 on the ordinate. If duopolist 1 holds his q_1 constant, duopolist 2 can select 31 cells by varying his q_2 and visa versa. If a game is classified as limited information, as contrasted to full information, each player knows only his own payoff so there is just one payoff number in each cell.

Payoffs to duopolists 1 and 2 with identical cost curves (Cournot equilibrium of 63 for both duopolists) are calculated from the following equation:

$$\pi_1 = \pi_2 = \{ 117 - 0.5 [q_1(t) + q_2(t)] \} q_1(t) - 810 - 9.0 q_1(t) - 0.1 q_1^2(t) \quad (24)$$

Payoffs to duopolist 1 with similar cost curves (Cournot equilibrium of 66) are calculated from the following equation:

$$\pi_1 = \{ 117 - 0.5 [q_1(t) + q_2(t)] \} q_1(t) - 1260 - 9.0 q_1(t) - 0.1 q_1^2(t) \quad (25)$$

Payoffs to duopolist 2 with similar cost curves (Cournot equilibrium of 56) are calculated from the following equation:

$$\pi_2 = \{117 - 0.5 [q_2(t) + q_1(t)]\} q_2(t) - 880 - 5.0 q_2(t) - 0.2 q_2^2(t) \quad (26)$$

Payoffs to duopolist 1 with dissimilar cost curves (Cournot equilibrium of 72) are calculated from the following equation :

$$\pi_1 = \{117 - 0.5 [q_1(t) + q_2(t)]\} q_1(t) - 1890 - 9.0 q_1(t) - 0.1 q_1^2(t) \quad (27)$$

Payoffs to duopolist 2 with dissimilar cost curves (Cournot equilibrium of 42) are calculated from the following equation :

$$\pi_2 = \{117 - 0.5 [q_2(t) + q_1(t)]\} q_2(t) - 297 - 18.0 q_2(t) - 0.25 q_2^2(t) \quad (28)$$

Outcomes of Games

Table 1 lists $q_1(1) \dots q_1(10)$ for all players representing duopolist 2 with identical cost curves and limited information; below each quantity is listed the conjectural variation exhibited by the player in selecting that quantity. Table 2 contains the same information for all players with identical cost curves and complete information. Table 3 contains the same information for all players with similar cost curves and limited information. Table 4 contains the same information for all players with dissimilar cost curves and limited information. Table 5 contains the same information for all players with dissimilar cost curves and complete information.

Table 1A. Conjectural variations for duopolists 1 with identical cost curves, limited information.

62.	64.	62.	62.	64.	62.	60.	60.	62.	66.
-.4870	-.2633	.1470	.0239	-.0576	-.0111	-.1419	-.1465	-.1148	-.1987
54.	60.	70.	68.	66.	62.	62.	68.	64.	62.
-.0700	.1191	-.0993	-.3801	-.0489	-.0477	.0476	-.2094	-.4138	.0558
64.	44.	48.	66.	70.	70.	60.	50.	56.	64.
-.6000	.5957	.8445	-.2572	-.1735	-.1407	.0954	.5461	.5285	.1370
60.	54.	52.	54.	68.	62.	56.	66.	60.	62.
-.3766	.2479	.4708	.4456	-.0311	.1729	.2335	-.1432	.0869	-.0045
58.	68.	80.	68.	48.	70.	80.	58.	66.	64.
-.2699	.0314	-.5375	-.2068	.4213	-.3366	-.2537	.2038	-.4165	-.1368
64.	72.	60.	66.	74.	68.	64.	68.	66.	66.
-.6000	-.3866	.1694	-.1331	-.3819	-.1945	-.0120	-.2109	-.0552	-.0078
58.	54.	58.	60.	64.	62.	58.	56.	64.	70.
-.2699	.4433	.3884	.3249	.1713	.2391	.2898	.4209	.0497	-.1257
70.	56.	48.	70.	70.	56.	68.	64.	42.	72.
-.9404	-.3181	.6252	.0143	-.0335	-.0438	-.0881	-.0733	.7100	-.0732
58.	64.	62.	50.	58.	62.	62.	62.	64.	66.
-.2699	-.2427	.0105	.3878	.1843	-.0708	-.0084	.0192	-.0253	-.1001
64.	90.	66.	68.	64.	64.	64.	40.	70.	62.
-.6000	-1.1490	.2980	.0568	.0045	.0372	-.0253	.6957	-.1371	-.1136
60.	60.	70.	80.	62.	64.	58.	60.	66.	64.
-.3766	-.0563	-.1505	-.5800	-.0497	-.1402	.2325	.1174	-.0097	.0071
50.	64.	54.	60.	70.	82.	52.	56.	62.	64.
.1104	-.0186	.3993	.0649	-.2552	-.9335	.4551	.4816	.1123	-.0043
64.	60.	62.	58.	58.	60.	62.	66.	62.	62.
-.6000	-.4296	-.1098	-.0893	.0076	.0649	.0485	-.1024	.0568	.0845
46.	42.	56.	52.	70.	70.	62.	60.	66.	62.
.2714	.6346	.3222	.0867	-.5593	.1100	-.0130	-.0017	-.1007	.0287

Table 1A. Continued

78.	66.	58.	58.	56.	58.	80.	58.	58.	58.
-1.3356	-.2079	.1034	.0933	.1902	.1488	-.6557	.3048	.3960	.1427
66.	56.	54.	60.	70.	64.	60.	64.	66.	70.
-.7143	.0081	.3794	.0770	-.2907	-.0109	.1249	-.1160	-.1951	-.3883
66.	58.	70.	72.	76.	68.	72.	66.	66.	70.
-.7143	-.2371	.0015	-.1288	-.8862	.0339	.0997	-.0752	-.1085	-.0000
66.	66.	68.	60.	64.	64.	52.	52.	52.	64.
-.7143	-.3378	-.0789	.1278	-.0091	.0068	.4158	.4921	.4551	-.0252
52.	64.	58.	66.	56.	64.	64.	54.	64.	66.
.0227	.1379	.3128	-.3501	.2837	-.0439	-.0197	.2926	-.0106	-.1324
60.	60.	64.	60.	60.	60.	56.	54.	54.	58.
-.3766	.0116	.1801	.0211	.1276	.1558	.2480	.3385	.3079	.1121
68.	64.	66.	64.	66.	64.	62.	64.	66.	62.
-.8284	-.2779	-.0864	-.0225	-.0068	-.0251	.0802	.0087	-.1012	.0536

Table 1B. Conjectural variations for duopolists 2 with identical cost curves, limited information.

64.	58.	66.	66.	68.	80.	80.	74.	70.	66.
-.6000	-.0139	-.1019	-.0556	-.1431	-.5977	-.4590	-.3367	-.2198	-.0829
62.	54.	76.	60.	70.	64.	66.	86.	62.	60.
-.4870	.2987	-.3471	.1482	-.2763	-.0132	-.0698	-.7422	.2010	.1958
56.	52.	70.	64.	58.	66.	60.	50.	50.	68.
-.1676	.3494	.0753	.1866	.1879	-.1821	.0561	.5392	.7196	-.0477
60.	62.	58.	54.	56.	68.	68.	68.	68.	68.
-.3766	-.0679	.3367	.5125	.4361	-.2568	-.0935	-.0480	-.2112	-.1108
42.	60.	72.	72.	78.	50.	66.	86.	74.	78.
.4145	.4807	-.5501	-.5083	-.6201	.6533	.0868	-1.1001	.0320	-.5787
52.	64.	68.	64.	68.	62.	66.	62.	58.	62.
.0227	-.0116	-.3146	.0558	-.2062	-.1008	-.1783	.0614	.1459	.0284
48.	52.	48.	50.	50.	58.	54.	60.	56.	54.
.1933	.6058	.6566	.6117	.5393	.2043	.3553	.2342	.3628	.3712
78.	54.	52.	50.	84.	58.	68.	66.	64.	72.
-1.3356	.0069	.5265	.6908	-.7523	.4255	.1110	-.1885	-.0313	.0157
68.	68.	72.	70.	70.	68.	66.	64.	64.	62.
-.8284	-.4022	-.3256	-.1967	-.0347	-.0791	-.0712	.0040	.0064	.0505
64.	64.	56.	62.	60.	64.	74.	74.	78.	74.
-.6000	-.3026	-.1617	-.0015	.0650	-.0232	-.3924	-.3336	-.1522	-.5145
64.	58.	64.	76.	70.	62.	66.	58.	62.	64.
-.6000	.0205	.0388	-.5759	-.4291	.0415	-.0954	.2865	.1444	-.0670
68.	60.	72.	64.	80.	70.	62.	58.	62.	64.
-.8284	.0611	-.3120	.1744	-.5020	-.2208	-.3158	.3284	.1870	-.0054
78.	72.	82.	76.	68.	64.	64.	64.	62.	64.
-1.3356	-.6698	-.6775	-.3268	-.1108	.0536	.0402	.0062	.0192	.0068
90.	64.	82.	82.	50.	68.	72.	64.	66.	74.
-1.7143	.2239	-.2217	-.4802	.5786	.0192	-.4419	.0333	-.0327	-.4305

Table 1B. Continued.

54.	68.	72.	70.	68.	68.	70.	56.	68.	66.
-.0700	-.4684	-.2474	-.1363	-.0834	-.0506	-.1516	.0334	-.0713	.0069
64.	62.	68.	66.	66.	64.	70.	70.	72.	76.
-.6000	-.2581	-.0558	.0581	-.0351	-.1251	-.1596	-.1596	-.3270	-.4991
72.	46.	52.	84.	54.	36.	62.	64.	48.	52.
-1.0485	.3545	.6227	-.8417	.5309	.2498	-.2798	-.2798	.5334	.5175
60.	58.	64.	62.	64.	64.	64.	62.	64.	64.
-.3766	-.0086	-.0577	-.0850	.1106	-.0217	-.0246	.2218	.1586	.1478
54.	54.	80.	66.	68.	64.	68.	66.	66.	64.
-.0700	.4738	-.5750	.1987	-.1633	.1004	-.1693	-.0901	.0507	-.0253
60.	50.	70.	64.	62.	66.	66.	68.	70.	72.
-.3766	.3987	-.1185	.0119	.1117	-.0334	-.0348	-.0480	-.0873	-.1569
60.	62.	64.	58.	64.	62.	62.	64.	64.	62.
-.3766	-.2118	-.0185	.1722	-.0089	.0200	.0511	.0062	-.0255	.0192

Table 2A. Conjectural variations for duopolists 1 with identical cost curves, complete information.

58.	58.	62.	68.	86.	64.	60.	62.	64.	62.
-.2699	-.1825	-.0460	-.2063	-.9839	.0017	-.0580	-.1924	-.1529	-.3424
78.	62.	64.	60.	64.	82.	70.	66.	62.	76.
-1.3356	-.3640	-.2291	.3213	.1429	-.8481	-.2713	-.2652	-.0640	-.3966
70.	80.	74.	82.	76.	86.	84.	84.	40.	50.
-.9404	-1.1236	-.2458	-1.2364	-.6162	-1.2298	-.7850	-1.1464	.8553	1.0543
50.	68.	82.	86.	78.	74.	76.	80.	84.	60.
.1104	-.2664	-.6169	-.6001	-.5648	-.5801	-.4830	-.5285	-.6494	.2140
54.	70.	66.	78.	78.	78.	70.	70.	70.	78.
-.0700	-.1578	.0317	-.7974	-.6445	-.6773	-.1864	-.1762	.2323	-.8170
30.	66.	70.	66.	68.	62.	66.	64.	80.	74.
.7562	.5473	-.5080	-.5323	-.1786	.0248	-.2357	.0511	-.7374	-.1719
88.	52.	54.	64.	60.	62.	60.	60.	62.	62.
-1.6670	.0350	.6091	-.1447	-.0092	-.0806	.0027	.0330	-.0130	.0196
50.	80.	86.	88.	76.	70.	64.	56.	80.	88.
.1104	-.4066	-.6035	-.8892	-.5906	-.3115	-.0556	.2694	-.4788	-.6086
58.	74.	54.	60.	80.	74.	76.	58.	58.	86.
-.2699	-.9405	.2472	.0156	-.8393	-.3960	-.9132	-.0240	-.1392	-1.1337
74.	78.	70.	82.	72.	86.	80.	76.	72.	70.
-1.1511	-.8448	-.2673	-.9899	-.2302	-1.3407	-.4746	-.7159	-.5045	-.2191
60.	60.	58.	62.	58.	60.	60.	60.	50.	60.
-.3766	-.0218	.2576	-.0046	.2570	.0495	.0313	.0960	.6404	-.0269
80.	86.	86.	86.	86.	88.	88.	78.	80.	72.
-1.4164	-1.4922	-.5671	-.7352	-1.1383	-1.4660	-1.4713	-1.2278	-.9810	-.2281
78.	78.	78.	78.	78.	78.	78.	78.	78.	78.
-1.3356	-1.0567	-.6773	-.5174	-.4800	-.3065	-.4435	-.4435	-.4435	-.5174

Table 2B. Conjectural variations for duopolist 2 with identical cost curves, complete information.

78.	70.	66.	76.	80.	76.	78.	72.	86.	76.
-1.3356	-.4515	-.0625	-.4349	-.6147	-.9311	-.6248	-.2395	-.7897	-.2899
70.	70.	50.	54	74.	80.	70.	70.	60.	60.
-.9404	-.9828	.4977	.4853	-.3508	-.4888	-.5681	-.4734	.0954	.1697
70.	66.	88.	80.	80.	76.	78.	46.	48.	90.
-.9404	-.6002	-1.1690	-.3534	-1.1129	-.7426	-1.1172	.3899	.4896	-.5016
74.	66.	60.	66.	68.	62.	60.	60.	60.	60.
-1.1511	-.1776	.0422	-.4237	-.5506	-.1686	-.0513	-.0704	-.1412	-.2182
56.	58.	78.	78.	70.	60.	58.	30.	78.	62.
-.1676	.2635	-.6333	-.3620	-.5386	-.1877	-.0543	1.0170	-.1673	.0808
62.	58.	88.	62.	66.	72.	60.	72.	62.	60.
-.4870	.4569	-.8092	.2899	-.0132	-.3948	.1872	-.3161	.1070	-.1215
86.	44.	70.	74.	72.	72.	70.	68.	66.	68.
-1.6141	.2865	.1113	-.2770	-.3099	-.2691	-.2188	-.1180	-.0406	-.1444
58.	64.	66.	70.	60.	64.	64.	60.	60.	60.
-.2699	.0575	-.4000	-.6689	-.2495	-.2787	-.1058	.1259	.2468	-.1412
84.	86.	76.	78.	80.	82.	80.	84.	86.	90.
-1.5552	-.9206	-.6750	-.4908	-.5318	-1.0305	-.8077	-1.0636	-.6478	-.8367
60.	68.	78.	70.	88.	74.	72.	70.	60.	50.
-.3766	-.5764	-.7146	-.2357	-1.2304	-.1678	-.8605	-.6082	-.0908	.3774
62.	60.	68.	60.	70.	70.	66.	50.	80.	70.
-.4870	-.0276	-.1110	.2224	-.1861	-.1222	-.0370	.5239	-.3772	.1252
78.	58.	58.	76.	86.	88.	88.	68.	60.	56.
-1.3356	-.4131	-.3442	-.8640	-.9423	-1.2144	-1.4863	-.7472	-.4200	-.0319
70.	70.	62.	60.	50.	58.	58.	58.	62.	60.
-.9404	-.9828	-.1899	-.1325	.3024	.0311	-.0252	-.0209	-.1868	-.0910

Table 3A. Conjectural variations for duopolists 1 with similar cost curves, limited information.

60.	70.	68.	48.	66.	72.	72.	70.	74.	60.
-.1459	-.2214	-.0757	.6595	.2615	-.2345	-.2016	-.0289	-.1990	.1925
60.	64.	56.	70.	80.	60.	68.	64.	64.	66.
-.1459	.0908	.2959	-.1131	-.4735	.1631	-.3652	-.3356	-.0148	-.0369
70.	60.	62.	58.	58.	54.	66.	62.	58.	66.
-.6215	.1189	.0380	.2825	.2739	.4020	-.1730	.1403	.3168	-.1797
64.	64.	66.	64.	66.	66.	58.	64.	66.	66.
-.3316	-.1013	.0240	.0362	.2415	-.0135	.1416	.0837	.0461	-.0384
60.	62.	64.	60.	58.	52.	80.	64.	66.	68.
-.1459	-.0864	-.1566	-.0285	.1098	.5204	-.4281	.1761	.0361	-.1118
62.	60.	54.	54.	70.	58.	52.	68.	56.	66.
-.2375	.1164	.5407	.3703	-.3483	.3245	.5610	-.0574	.4528	.0452
80.	80.	62.	70.	66.	66.	66.	68.	68.	66.
-1.0729	-.5677	.2173	-.0013	-.0059	.0236	.0240	-.0178	-.0475	-.0369
52.	52.	56.	56.	60.	66.	66.	60.	68.	68.
.1878	.6700	.4620	.3075	.1565	.2342	-.0277	.3145	.0672	-.0258
60.	64.	56.	50.	64.	62.	64.	60.	58.	64.
-.1459	-.0313	.1906	.5598	.0874	.1505	.2102	.1778	.1227	.0407
58.	62.	60.	58.	80.	62.	64.	70.	68.	70.
-.0572	.1427	.4054	.2132	-.3829	.4246	.2276	-.0065	.0964	.0854
64.	54.	56.	62.	64.	68.	68.	74.	64.	64.
-.3316	.4820	.2622	.0109	.0052	-.0790	-.0131	-.2944	.0631	.1051
70.	66.	68.	68.	66.	68.	68.	68.	68.	68.
-.6215	-.4523	-.0759	-.0766	.0240	-.0472	-.0466	-.0480	-.0480	-.0480
62.	40.	62.	62.	66.	66.	64.	62.	64.	66.
-.2375	.7857	.3557	-.0770	-.0369	.0555	.0959	.1134	.0702	-.0082

Table 3B. Conjectural variations for duopolists 2 with similar cost curves, limited information

58.	60.	52.	52	54.	58.	50.	52.	60.	52.
-.6663	-.3482	.1633	.2084	.3963	-.0923	.2246	.1495	-.2570	.1182
52.	62.	58.	58	66.	78.	84.	62.	60.	64.
-.2561	-.2939	.0093	.0939	-.5446	-1.2046	-.8652	-.2223	-.2334	-.3422
52.	66.	58.	60.	60.	70.	60.	56.	70.	68.
-.2561	-.6800	.1780	-.0783	-.0311	-.4915	.1100	.0404	-.5574	-.2682
60.	56.	60.	40.	58.	68.	58.	54.	60.	62.
-.8079	-.2540	-.1582	.7518	.2065	-.6298	.0323	.2530	-.1210	-.2615
68.	72.	74.	70.	56.	58.	62.	58.	60.	54.
-1.3391	-.9894	-.6901	-.6099	.0819	.0796	-.0286	-.3588	-.1663	.1255
58.	48.	66.	70.	60.	56.	60	50.	60.	56.
-.6663	.2419	-.2995	-.3266	.0539	-.0410	-.0365	.5079	-.1194	.2041
50.	52.	52.	58.	56.	56.	54.	56.	60.	54.
-.1291	-.1092	-.0268	-.0069	-.0502	.0195	.1203	.0255	-.2152	.1007
50.	50.	60.	62.	40.	58.	50.	50.	54.	56.
-.1291	.4684	.0654	-.1045	.8428	.3190	.2228	.3467	.2197	-.0303
60.	70.	60.	60.	56.	48.	60.	70.	60.	60.
-.8079	-.8618	.0103	.0017	.2750	.4538	-.0399	-.5974	.0325	-.0282
56.	42.	64.	52.	42.	54.	48.	46.	42.	42.
-.5260	.6182	-.1377	.3504	.8254	.0194	.4657	.5805	.6664	.7247
44.	70.	66.	62.	58.	54.	58.	60.	56.	60.
.2134	-.4425	-.0509	-.0971	-.0137	.1525	-.1032	-.2069	-.1193	-.1482
70.	56.	58.	56.	56.	56.	56.	56.	56.	56.
-1.4531	-.3610	-.1727	-.0081	-.0147	.0213	-.0144	-.0144	-.0144	-.0144
58.	60.	64.	60.	54.	56.	60.	58.	58.	58.
-.6663	-.3905	.0847	-.1101	.1784	.0400	-.1779	.0294	-.0079	-.0420

Table 4A. Conjectural variations for duopolists 1 with dissimilar cost curves, limited information.

60.	66.	62.	64.	68.	66.	70.	68.	66.	72.
.0942	.1757	.2843	.2717	.0611	.1064	.0030	.0957	.1660	-.0636
62.	68.	74.	68.	82.	68.	70.	72.	74.	64.
.0191	-.0704	-.1007	-.0388	-.2809	-.0492	.0736	-.0132	-.3656	.1336
60.	68.	64.	66.	66.	68.	68.	70.	70.	70.
.0942	.1345	.0802	.1943	.1045	-.0480	-.0465	.0558	.0543	.0828
60.	66.	62.	40.	80.	90.	70.	74.	76.	78.
.0942	.0654	.3403	.6950	-.0596	-.6873	.1912	.0814	-.1497	-.1773
76.	64.	64.	68.	70.	64.	62.	68.	78.	72.
-.5462	.1816	.1270	.0405	.0260	-.0586	.0136	.0409	-.2132	.0453
64.	74.	76.	78.	66.	66.	70.	66.	78.	78.
-.0582	-.0151	.0228	-.1531	.0285	.0291	-.0000	-.0061	-.0158	-.0130
60.	62.	62.	64.	64.	66.	66.	68.	70.	70.
.0942	.1799	.1906	.2575	.2249	.1902	.1849	.1492	.0788	.0537
66.	64.	74.	68.	68.	68.	68.	74.	68.	70.
-.1374	-.0141	.1087	.0533	.0450	.1745	-.0178	.1088	.0594	.1647
68.	74.	70.	76.	78.	74.	72.	78.	68.	62.
-.2182	-.3419	-.3263	-.2556	-.2808	-.1291	-.0184	-.1525	-.1980	.4527
64.	78.	66.	54.	60.	54.	64.	70.	74.	68.
-.0582	-.4867	.0648	.4036	.2537	.3809	.0714	.0461	-.0535	.0986
72.	74.	76.	68.	54.	58.	58.	52.	50.	50.
-.3823	-.0621	-.225	.1215	.4723	.5572	.4993	.5834	.7148	.6591
58.	64.	58.	66.	62.	60.	70.	62.	70.	64.
.1669	.2759	.4095	.0009	.2724	.4462	-.0159	.1688	-.2504	.1250
66.	62.	66.	68.	70.	70.	68.	72.	72.	72.
-.1374	-.0344	.0802	.0070	-.0002	.0282	.0685	.0184	.0155	.0166
70.	72.	60.	62.	62.	64.	66.	66.	66.	64.
-.3000	-.2576	.1326	.0749	.1935	.0369	.0523	.1084	.0533	-.0576

Table 4A. Continued

54.	40.	80.	72.	70.	68.	68.	70.	68.	70.
.3042	1.0162	-.0991	-.1301	.0136	.0408	.1756	.0282	.1219	.0861
<hr/>									
64.	66.	60.	66.	64.	68.	70.	70.	78.	82.
-.0582	.0604	.1816	.0692	.1462	.0447	-.0315	.0009	-.2100	-.2867
<hr/>									
50.	58.	80.	78.	62.	70.	78.	84.	84.	88.
.4306	.4102	-.7435	-.2379	.0430	.0923	-.3138	-.4311	-1.1950	-.6435
<hr/>									
74.	68.	68.	70.	78.	78.	68.	68.	74.	70.
-.4646	-.1684	-.0636	-.1198	-.3960	-.3535	-.0481	-.0554	-.0218	.0575
<hr/>									
58.	66.	66.	58.	50.	70.	80.	64.	74.	66.
.1669	.0728	.1293	.2279	.7193	.1337	-.2849	.1345	.0074	.2836
<hr/>									
38.	74.	66.	74.	74.	50.	64.	90.	70.	60.
.7472	.3847	-.0038	-.2290	-.2338	.5133	.4357	-.7505	.2434	.2984

Table 4B. Conjectural variations for duopolists 2 with dissimilar cost curves, limited information.

48.	46.	44.	48.	50.	48.	46.	46.	48.	44.
-1.2679	-.6007	-.0276	-.1768	-.3389	-.3073	-.1514	-.2420	-.3215	.0145
58.	46.	56.	40.	60.	42.	44.	62.	54.	54.
-2.1879	-.5213	-.9932	.3012	-.8883	.1703	.0240	-1.1688	-.3105	-.9995
46.	56.	44.	50.	56.	56.	44.	44.	42.	42.
-1.0297	-1.1752	.2029	-.2945	-.7430	-.6934	-.0464	-.0592	.0468	.0490
56.	42.	76.	44.	52.	40.	36.	44.	42.	40.
-2.0593	-.2257	-1.6180	.8154	.4673	-.1531	-.0520	-.0982	-.0290	.0477
42.	56.	50.	46.	66.	66.	50.	44.	42.	50.
-.5729	-1.3018	.0706	-.0800	-1.3142	-.8501	-.3750	-.0463	.0860	-.7110
38.	32.	40.	56.	56.	48.	58.	30.	30.	46.
-.1746	.7437	.2001	-1.0418	-.6877	-.2774	-.9523	.8793	1.2761	-.4258
58.	54.	44.	46.	44.	44.	42.	42.	44.	42.
-2.1879	-1.0714	-.0065	-.0455	.0468	.0468	.1361	.1423	-.0461	.0490
60.	30.	48.	50.	40.	54.	30.	48.	36.	44.
-2.2918	.6194	.1722	-.7137	.2559	-.6250	.9106	.1581	.3158	.0402
56.	70.	50.	50.	48.	44.	40.	66.	30.	30.
-2.0593	-2.1608	.0818	-.4712	-.5459	-.3341	.0650	-1.3367	1.0154	1.3789
60.	60.	60.	60.	60.	60.	44.	42.	46.	48.
-2.2918	-1.4907	-1.5000	-1.0909	-.7500	-.9130	.2434	.2578	-.2199	-.4560
36.	32.	44.	54.	42.	38.	50.	48.	52.	56.
-.0000	.6988	-.1217	-.8745	.2693	.6651	-.1524	-.0460	-.2063	-.3826
48.	46.	60.	48.	40.	50.	56.	66.	56.	46.
-1.2679	-.5381	-.9645	.1831	.3116	-.2449	-.5744	-1.3751	-.3750	-.3414
66.	52.	52.	48.	46.	48.	42.	42.	42.	42.
-2.5000	-.5813	-.6602	-.2750	-.2125	-.3839	.0643	.1009	0.0000	0.0000
54.	64.	64.	54.	60.	54.	50.	54.	66.	44.
-1.9019	-2.0693	-.9266	-.5431	-1.0820	-.4181	-.4278	-.7145	-1.2627	.2958

Table 4B. Continued

50.	46.	56.	46.	50.	40.	46.	44.	42.	50.
-1.5000	-.4570	-.2240	-.4608	-.6912	.2425	-.1256	-.0286	.0455	-.4468
52.	60.	54.	52.	50.	50.	48.	44.	42.	40.
-1.7143	-1.6991	-.3429	-.4456	-.4352	-.3802	-.3247	-.1099	.0367	.0018
66.	70.	62.	64.	42.	50.	48.	84.	50.	46.
-2.5000	-1.2167	-.8962	-2.1163	-.0430	-.3257	-.3091	-2.0569	.5519	-.1187
56.	56.	56.	56.	56.	56.	56.	40.	44.	40.
-2.0593	-1.8740	-.8769	-.8769	-.9375	-1.2000	-1.2000	.2246	.0358	.0931
58.	48.	62.	44.	44.	42.	56.	40.	36.	44.
-2.1879	-.5515	-1.2411	.3354	.2884	.4433	-.8347	.2182	.6319	-.1287
46.	44.	54.	56.	60.	42.	50.	48.	56.	44.
-1.0297	.1590	-.8511	-.5821	-1.2633	.0703	-.0148	-.2218	-.5152	.1714

Table 5A Conjectural variations for duopolists 1 with dissimilar cost curves, complete information.

62.	72.	78.	82.	86.	72.	70.	70.	72.	74.
.0191	-.1502	-.3111	-.3143	-.3911	.0377	.0629	.0300	-.0112	-.0493
58.	62.	52.	66.	62.	64.	66.	66.	70.	64.
.1669	.3181	.6229	.1141	.0869	.1843	.0185	.0234	.1352	.0287
70.	60.	64.	64.	64.	66.	64.	64.	64.	64.
-.3006	.3822	.3294	.2652	.2567	.1902	.2759	.2636	.2814	.2567
44.	60.	60.	68.	66.	70.	82.	70.	70.	74.
.6000	.5655	.1105	-.1752	-.0783	-.0312	-.4298	.0974	-.2674	-.1070
74.	78.	90.	72.	90.	90.	90.	70.	74.	74.
-.4646	-.5373	-.7549	.1328	-.5122	.4294	-.5793	-.1219	-.0005	.0320
80.	80.	84.	80.	78.	90.	78.	80.	84.	84.
-.7051	-.1750	-.4746	-.6867	-.1349	-.9845	.0575	-.9106	-.4445	-.3216
80.	56.	70.	80.	66.	66.	68.	60.	66.	70.
-.7051	.4606	-.1026	-.5812	.0788	.0692	-.0178	.2277	.1634	-.0433
64.	78.	80.	62.	58.	90.	70.	70.	68.	72.
-.0582	-.3864	-.1399	.0236	.1782	-.7522	.2495	.0141	.0117	-.1907
76.	74.	80.	76.	88.	86.	90.	80.	88.	90.
-.5462	-.2260	-.4450	-.43520	-.9464	-.5637	-1.0524	-.6125	-1.0605	-.8616
72.	90.	86.	90.	80.	80.	68.	90.	78.	70.
-.3823	-1.0385	-.1447	-.6852	-.4436	-.5621	.1475	-.9334	.0170	.1112

Table 5B. Conjectural variations for duopolists 2 with dissimilar cost curves, complete information.

54.	52.	44.	40.	42.	44.	46.	44.	42.	44.
-1.9019	-1.1589	-.2045	-.0415	-.2621	-.5145	-.2580	-.0814	.0431	-.1404
50.	44.	56.	60.	50.	56.	56.	38.	60.	46.
-1.5000	-.4231	-.7286	-.4836	-.3701	-.7818	-.6340	.3885	-.7870	.1489
38.	42.	42.	44.	44.	42.	44.	42.	44.	42.
-.1746	-.0702	.2664	.0448	.0428	.1786	.0081	.1783	.0527	.1764
58.	56.	64.	64.	50.	50.	46.	66.	46.	32.
-2.1879	-.6996	-1.1405	-.9151	-.4871	-.5948	-.2361	-1.6750	.4152	.8183
56.	56.	42.	42.	42.	42.	56.	36.	36.	36.
-2.0593	-1.8740	-.1527	-.5707	0.0000	-.5000	-1.4092	.4339	.6295	.3871
30.	48.	68.	36.	66.	36.	78.	48.	40.	64.
.4365		1.5970	.6789	-1.0255	.7415	-1.5824	.6211	.3577	-1.4794
40.	66.	64.	58.	54.	54.	56.	48.	50.	48.
-.3655	-1.7451	-.0742	-1.1443	-1.4486	-.8509	-.8083	-.2873	-.3146	-.2582
54.	40.	66.	66.	58.	52.	52.	52.	56.	54.
-1.9019	-.2457	-1.5560	-.7487	-.8842	-.5336	-1.4657	-.6563	-.9115	-.6427
46.	54.	64.	66.	60.	66.	66.	66.	66.	66.
-1.0297	-1.5925	-1.1205	-1.5597	-1.4114	-2.4854	-1.6804	-2.5000	-2.0339	-2.4000
64.	40.	48.	56.	56.	42.	68.	48.	40.	32.
-2.4439	.0274	-.9143	-1.1374	-1.3040	-.2086	-1.6802	.4856	-.0586	.5689

RESULTS AND DISCUSSION

Regression techniques were used to fit the conjectural variation models described by equations (16) and (19) to the measured conjectural variations of each game. The constants β and γ were evaluated and, when significant at the 1% level or better, included in the model for conjectural variation. The constant α can be considered the value of conjectural variation approached as the number of time periods become large. For each game α_1 was substituted for $\ell_1(t)$ in equation (14) and α_2 was substituted for $\ell_2(t)$ in equation (17), and the equations were solved by iterations to determine $\lim_{t \rightarrow \infty} q_1(t)$ and $\lim_{t \rightarrow \infty} q_2(t)$. The models for conjectural variation can be used to predict the dynamic time path of subsequent games with the same demand curve, cost curve, and knowledge about the rival's profit.

Identical Cost Curves, Limited Information

$$\alpha_1 = 0.0082$$

$$\beta_1 = 0.0122 \text{ at a significance level of } 0.1\%$$

$$\gamma_1 = -0.0149 \text{ with no significance}$$

Therefore, the model for conjectural variation becomes :

$$\ell_1(t) = 0.0082 + 0.0122 \left(\frac{117 - 9 - 0.5 q_2(t-1)}{2(0.5 + 0.1)} - q_1(t-1) \right)$$

at a significance level of 0.1% (29)

$$\alpha_2 = -0.0163$$

$$\beta_2 = 0.0155 \text{ at a significance level of } 0.1\%$$

$$\gamma_2 = -0.1134 \text{ at a significance level of } 25\%$$

Therefore, the model for conjectural variation becomes:

$$\ell_2(t) = -0.0163 + 0.0155 \left(\frac{117 - 9 - 0.5 q_1(t-1)}{2(0.5 + 0.1)} - q_2(t-1) \right) \quad (30)$$

at a significance level of 0.1%

$$\lim_{t \rightarrow \infty} q_1(t) = \lim_{t \rightarrow \infty} q_2(t) = 63$$

which is the Cournot equilibrium of both duopolists.

Identical Cost Curves, Complete Information

$$\alpha_1 = -0.1558$$

$$\beta_1 = 0.01594 \text{ at a significance level of } 0.1\%$$

$$\gamma_1 = 0.1991 \text{ with no significance}$$

Therefore the model for conjectural variation becomes :

$$\ell_1(t) = -0.1558 + 0.01594 \left(\frac{117 - 9 - 0.5 q_2(t-1)}{2(0.5 + 0.1)} - q_1(t-1) \right) \quad (31)$$

at a significance level of 0.1%

$$\alpha_2 = -0.1685$$

$$\beta_2 = 0.0195 \text{ at a significance level of } 0.1\%$$

$$\gamma_2 = -0.0670 \text{ with no significance}$$

Therefore, the model for conjectural variation becomes:

$$q_2(t) = -0.1685 + 0.0195 \left(\frac{117 - 9 - 0.5 q_1(t-1)}{2(0.5 + 0.1)} - q_2(t-1) \right) \quad (32)$$

at a significance level of 0.1%

$$\lim_{t \rightarrow \infty} q_1(t) = \lim_{t \rightarrow \infty} q_2(t) = 67$$

as compared with the Cournot equilibrium of 63. Since quantities of production of 67 are unstable, Stackleberg disequilibriums are centered about 67. With production at 67 both duopolists could fix their production at a lower rate with temporary benefits to both.

Similar Cost Curves, Limited Information

$$\alpha_1 = 0.0467$$

$$\beta_1 = 0.01112 \text{ at a significance level of } 0.1\%$$

$$\gamma_1 = -0.0308 \text{ with no significance}$$

Therefore, the model for conjectural variation becomes :

$$\ell_1(t) = 0.0467 + 0.01112 \left(\frac{117 - 9 - 0.5 q_2(t-1)}{2(0.5 + 0.1)} - q_1(t-1) \right) \quad (33)$$

at a significance level of 0.1%

$$\alpha_2 = -0.01665$$

$$\beta_2 = 0.01701 \text{ at a significance level of 0.1\%}$$

$$\gamma_2 = 0.08512 \text{ with no significance}$$

Therefore, the model for conjectural variation becomes :

$$\ell_2(t) = -0.01665 + 0.0171 \left(\frac{117 - 5 - 0.5 q_1(t-1)}{2(0.5 + 0.2)} - q_2(t-1) \right) \quad (34)$$

at a significance level of 0.1%

$$\lim_{t \rightarrow \infty} q_1(t) = 66, \quad \lim_{t \rightarrow \infty} q_2(t) = 56$$

which is the Cournot equilibrium.

Dissimilar Cost Curves, Limited Information

$$\alpha_1 = 0.03016$$

$$\beta_1 = 0.00472 \text{ at a significance level of 1\%}$$

$$\gamma_1 = 0.2687 \text{ at a significance level of 1\%}$$

Therefore, the model for conjectural variation becomes :

$$\begin{aligned} \ell_1(t) = & 0.03016 + 0.00472 \left(\frac{117 - 9 - 0.5q_2(t-1)}{2(0.5 + 0.1)} - q_1(t-1) \right) \\ & + 0.2687 \ell_1(t-1) \end{aligned} \quad (35)$$

at a significance level of 0.1%

$$\alpha_2 = -0.1983$$

$$\beta_2 = 0.0309 \text{ at a significance level of } 0.1\%$$

$$\gamma_2 = 0.00000642 \text{ with no significance}$$

Therefore, the model for conjectural variation becomes :

$$\ell_2(t) = -0.1983 + 0.0309 \left(\frac{117 - 18 - 0.5 q_1(t-1)}{2(0.5 + 0.25)} - q_2(t-1) \right) \quad (36)$$

at a significance level of 0.1%

$$\lim_{t \rightarrow \infty} q_1(t) = 71, \quad \lim_{t \rightarrow \infty} q_2(t) = 46 \quad \text{as compared with the Cournot}$$

equilibrium of 72 and 42. Since quantities of 71 and 46 are stable,

duopolist 2 has become a price leader. With production at 71 and 46

duopolist 1 can not fix his production at a higher or lower rate with a temporary benefit.

Dissimilar Cost Curves, Complete Information

$$\alpha_1 = -0.0534$$

$$\beta_1 = 0.01445 \text{ at a significance level of } 0.1\%$$

$$\gamma_1 = -0.00834 \text{ with no significance}$$

Therefore, the model for conjectural variation becomes :

$$\ell_1(t) = -0.0534 + 0.01445 \left(\frac{117 - 9 - 0.5q_2(t-1)}{2(0.5 + 0.1)} - q_1(t-1) \right) \quad (37)$$

at a significance level of 0.1%

$$\alpha_2 = -0.391$$

$$\beta_2 = 0.0221 \text{ at a significance level of } 1\%$$

$$\gamma_2 = 0.0863 \text{ with no significance}$$

Therefore, the model for conjectural variation becomes :

$$\ell_2(t) = -0.391 + 0.0221 \left(\frac{117 - 18 - 0.5q_1(t-1)}{2(0.5 + 0.25)} - q_2(t-1) \right) \quad (38)$$

at a significance level of 5%.

$$\lim_{t \rightarrow \infty} q_1(t) = 70, \quad \lim_{t \rightarrow \infty} q_2(t) = 50 \quad \text{as compared with the Cournot}$$

equilibrium of 72 and 42. Since quantities of 70 and 50 are stable,

duopolist 2 has become a price leader.

Discussion

With the dissimilar cost curves used in this study, the Cournot equilibrium is 72 and 42 for duopolists 1 and 2 respectively. Limited information games resulted in equilibrium of 71 and 46. Complete information games resulted in equilibrium of 70 and 50. Studies of Fouraker and Siegel (4) also showed that increasing the amount of relevant information available to bargainers decreases their tendency to the Cournot equilibrium. With the identical cost curves used in this study, the Cournot equilibrium is 63 for both duopolists. Limited information games resulted in equilibrium of 63. Complete information games resulted in disequilibrium centered about 67.

Although a few bargaining pairs achieved a tacit agreement, more bargaining pairs were competitive. As a group only duopolists 1 with dissimilar cost curve, limited information considered previous conjectural variations in making current decisions of production as indicated by the only significant γ .

At the beginning of most games it appears the theoretical reaction curves did not coincide with the actual reaction curves because the participants used about three moves trying to maneuver or trick their opponents. After the next four moves most games that eventually ended at equilibrium were at equilibrium. A few players used moves 8 and 9 in an attempt to

"set their opponent up for a kill" on move 10. Therefore, the theoretical data fits the empirical data best during the middle moves of the game.

BIBLIOGRAPHY

1. Chamberlin, E. H. 1936. The theory of monopolistic competition. Harvard University Press, Cambridge, Mass. p. 30-55.
2. Cournot, Augustine. 1963. The mathematical principles of the theory of wealth. Richard D. Irwin, Inc. Homewood, Illinois. p. 65-74.
3. Fellner, William. 1949. Competition among the few. Alfred A. Knopf. New York. p. 55-116.
4. Fouraker, Lawrence E. and Sidney Siegel. 1961. Bargaining behavior : II. The Pennsylvania State University. University Park, Pennsylvania. p. 76.
5. Henderson, James M. and Richard E. Quandt. 1958. Micro-economic theory. McGraw-Hill Book Company, Inc. New York. p. 164-200.
6. Hicks, J. R. 1935. Annual survey of economic theory: the theory of monopoly. *Econometrica* 3:1-20. (Jan. 1935.)

APPENDIXES

Appendix I

Instruction Sheets

Instructions to Duopolist 1

The Ford Foundation has provided funds for research by the Utah State University Department of Economics regarding economic decisions. If you follow instructions carefully and make appropriate decisions, you may earn an appreciable amount of money. You may keep all the money that you earn. You cannot lose your own money but poor choices will result in small profit for you.

You will be paired at random with one other person hereafter called duopolist 2. You will not see this person or speak with him at any time. You will never know the identity of your competitor nor will he be aware of yours.

Imagine that you and duopolist 2 are the sole producers of some standardized commodity. You and duopolist 2 will engage in a series of transactions by written bids, the bids on each transaction representing the quantity of the commodity produced for that transaction. You must bid on each transaction.

You will be furnished with a payoff matrix which shows in red the various levels of profit or loss you can attain. You may or may not be given the level of profit of your competitor (duopolist 2). If you are given his profit levels, they will appear in green directly above your profit levels. The quantities you may produce from 30 to 90 are listed in red down the left hand margin; the quantities duopolist 2 may produce from

30 to 90 are listed in green across the top of the matrix. For any transaction, your profit, and maybe the profit of duopolist 2, is at the intersection of the row selected by you, and column selected by duopolist 2. The actual cash payoff is scaled at 1 cent per 50¹ points, so the objective of the game is to win for yourself the maximum number of points.

On your payoff matrix is encircled the profit resulting from your first quantity of production and the first quantity of production by duopolist 2. The process is continued by each player selecting and recording a quantity on a sheet provided for that purpose. You will select any quantity listed down the left hand margin, record it, and then give the sheet to the administrator. When the administrator returns your sheet, it will show for that transaction your quantity bid, your profit or loss from that transaction, and duopolist 2's quantity bid.

The process is repeated 10 times. You may select the same quantity each time; however you do not have to do so. Decisions will ordinarily be made every few minutes, but extra time will be granted when necessary.

For each production quantity of duopolist 2, you have one and only one corresponding quantity that maximizes your profit.

At the end of the session we will add up your profit and loss column and give you the resulting amount of money.

Are there any questions?

¹With dissimilar cost curves, the actual cash payoff was scaled at 1 cent per 33-1/3 points.

Instructions to Duopolist 2

The Ford Foundation has provided funds for research by the Utah State University Department of Economics regarding economic decisions. If you follow instructions carefully and make appropriate decisions, you may earn an appreciable amount of money. You may keep all the money that you earn. You cannot lose your own money, but poor choices will result in small profit to you.

You will be paired at random with one other person hereafter called duopolist 1. You will not see this person or speak with him at any time. You will never know the identity of your competitor nor will he be aware of yours.

Imagine that you and duopolist 1 are the sole producers of some standardized commodity. You and duopolist 1 will engage in a series of transactions by means of written bids, the bids on each transaction representing the quantity of the commodity produced for that transaction. You must bid on each transaction.

You will be furnished with a payoff matrix which shows in green the various levels of profit or loss you can attain. You may or may not be given the level of profit of your competitor (duopolist 1). If you are given his profit levels, they will appear in red directly below your profit levels. The quantities you may produce from 30 to 90 are listed in green across the top; the quantities duopolist 1 may produce from 30 to 90 are listed in red down the left hand margin of the matrix. For any transaction,

your profit and maybe the profit of duopolist 1, is at the intersection of the column selected by you and the row selected by duopolist 1. The actual cash payoff is scaled at 1 cent per 50^2 points, so the objective of the game is to win for yourself the maximum number of points.

On your payoff matrix is encircled the profit resulting from your first quantity of production and the first quantity of production by duopolist 1. The process is continued by each player selecting and recording a quantity on a sheet provided for that purpose. You will select any quantity listed across the top, record it, and then give the sheet to the administrator. When the administrator returns your sheet it will show for that transaction your quantity bid, your profit or loss from that transaction, and duopolist 1's quantity bid.

The process is repeated 10 times. You may select the same quantity each time; however you do not have to do so. Decisions will ordinarily be made every few minutes, but extra time will be granted when necessary.

For each quantity of production by duopolist 1, you have one and only one corresponding quantity that maximizes your profit.

At the end of the session, we will add up your profit and loss column and give you the resulting amount of money.

Are there any questions?

²With dissimilar cost curves games, the actual cash payoff was scaled at 1 cent per $33\frac{1}{3}$ points.

Appendix II

Payoff Matrix for Duopolist 1

Identical Cost Curves

Limited Information

DUOPOLIST 1

	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90
30	1440	1410	1380	1350	1320	1290	1260	1230	1200	1170	1140	1110	1080	1050	1020	990	960	930	900	870	840	810	780	750	720	690	660	630	600	570	540
32	1552	1520	1488	1456	1424	1392	1360	1328	1296	1264	1232	1200	1168	1136	1104	1072	1040	1008	976	944	912	880	848	816	784	752	720	688	656	624	592
34	1658	1624	1590	1556	1522	1488	1454	1420	1386	1352	1318	1284	1250	1216	1182	1148	1114	1080	1046	1012	978	944	910	876	842	808	774	740	706	672	638
36	1760	1724	1688	1652	1616	1580	1544	1508	1472	1436	1400	1364	1328	1292	1256	1220	1184	1148	1112	1076	1040	1004	968	932	896	860	824	788	752	716	680
38	1858	1820	1782	1744	1706	1668	1630	1592	1554	1516	1478	1440	1402	1364	1326	1288	1250	1212	1174	1136	1098	1060	1022	984	946	908	870	832	794	756	718
40	1950	1910	1870	1830	1790	1750	1710	1670	1630	1590	1550	1510	1470	1430	1390	1350	1310	1270	1230	1190	1150	1110	1070	1030	990	950	910	870	830	790	750
42	2038	1996	1954	1912	1870	1828	1786	1744	1702	1660	1618	1576	1534	1492	1450	1408	1366	1324	1282	1240	1198	1156	1114	1072	1030	988	946	904	862	820	778
44	2120	2076	2032	1988	1944	1900	1856	1812	1768	1724	1680	1636	1592	1548	1504	1460	1416	1372	1328	1284	1240	1196	1152	1108	1064	1020	976	932	888	844	800
46	2198	2152	2106	2060	2014	1968	1922	1876	1830	1784	1738	1692	1646	1600	1554	1508	1462	1416	1370	1324	1278	1232	1186	1140	1094	1048	1002	956	910	864	818
48	2272	2224	2176	2128	2080	2032	1984	1936	1888	1840	1792	1744	1696	1648	1600	1552	1504	1456	1408	1360	1312	1264	1216	1168	1120	1072	1024	976	928	880	832
50	2340	2290	2240	2190	2140	2090	2040	1990	1940	1890	1840	1790	1740	1690	1640	1590	1540	1490	1440	1390	1340	1290	1240	1190	1140	1090	1040	990	940	890	840
52	2402	2350	2300	2248	2196	2144	2092	2040	1988	1936	1884	1832	1780	1728	1676	1624	1572	1520	1468	1416	1364	1312	1260	1208	1156	1104	1052	1000	948	896	844
54	2462	2408	2354	2300	2246	2192	2138	2084	2030	1976	1922	1868	1814	1760	1706	1652	1598	1544	1490	1436	1382	1328	1274	1220	1166	1112	1058	1004	950	896	842
56	2516	2460	2404	2348	2292	2236	2180	2124	2068	2012	1956	1900	1844	1788	1732	1676	1620	1564	1508	1452	1396	1340	1284	1228	1172	1116	1060	1004	948	892	836
58	2566	2508	2450	2392	2334	2276	2218	2160	2102	2044	1986	1928	1870	1812	1754	1696	1638	1580	1522	1464	1406	1348	1290	1232	1174	1116	1058	1000	942	884	826
60	2610	2550	2490	2430	2370	2310	2250	2190	2130	2070	2010	1950	1890	1830	1770	1710	1650	1590	1530	1470	1410	1350	1290	1230	1170	1110	1050	990	930	870	810
62	2650	2588	2526	2464	2402	2340	2278	2216	2154	2092	2030	1968	1906	1844	1782	1720	1658	1596	1534	1472	1410	1348	1286	1224	1162	1100	1038	976	914	852	790
64	2688	2624	2560	2496	2432	2368	2304	2240	2176	2112	2048	1984	1920	1856	1792	1728	1664	1600	1536	1472	1408	1344	1280	1216	1152	1088	1024	960	896	832	768
66	2714	2648	2582	2516	2450	2384	2318	2252	2186	2120	2054	1988	1922	1856	1790	1724	1658	1592	1526	1460	1394	1328	1262	1196	1130	1064	998	932	866	800	734
68	2740	2672	2604	2536	2468	2400	2332	2264	2196	2128	2060	1992	1924	1856	1788	1720	1652	1584	1516	1448	1380	1312	1244	1176	1108	1040	972	904	836	768	700
70	2760	2690	2620	2550	2480	2410	2340	2270	2200	2130	2060	1990	1920	1850	1780	1710	1640	1570	1500	1430	1360	1290	1220	1150	1080	1010	940	870	800	730	660
72	2778	2708	2638	2568	2498	2428	2358	2288	2218	2148	2078	2008	1938	1868	1798	1728	1658	1588	1518	1448	1378	1308	1238	1168	1098	1028	958	888	818	748	678
74	2788	2718	2648	2578	2508	2438	2368	2298	2228	2158	2088	2018	1948	1878	1808	1738	1668	1598	1528	1458	1388	1318	1248	1178	1108	1038	968	898	828	758	688
76	2794	2724	2654	2584	2514	2444	2374	2304	2234	2164	2094	2024	1954	1884	1814	1744	1674	1604	1534	1464	1394	1324	1254	1184	1114	1044	974	904	834	764	694
78	2802	2732	2662	2592	2522	2452	2382	2312	2242	2172	2102	2032	1962	1892	1822	1752	1682	1612	1542	1472	1402	1332	1262	1192	1122	1052	982	912	842	772	702
80	2810	2740	2670	2600	2530	2460	2390	2320	2250	2180	2110	2040	1970	1900	1830	1760	1690	1620	1550	1480	1410	1340	1270	1200	1130	1060	990	920	850	780	710
82	2818	2748	2678	2608	2538	2468	2398	2328	2258	2188	2118	2048	1978	1908	1838	1768	1698	1628	1558	1488	1418	1348	1278	1208	1138	1068	998	928	858	788	718
84	2826	2756	2686	2616	2546	2476	2406	2336	2266	2196	2126	2056	1986	1916	1846	1776	1706	1636	1566	1496	1426	1356	1286	1216	1146	1076	1006	936	866	796	726
86	2830	2760	2690	2620	2550	2480	2410	2340	2270	2200	2130	2060	1990	1920	1850	1780	1710	1640	1570	1500	1430	1360	1290	1220	1150	1080	1010	940	870	800	730
88	2832	2762	2692	2622	2552	2482	2412	2342	2272	2202	2132	2062	1992	1922	1852	1782	1712	1642	1572	1502	1432	1362	1292	1222	1152	1082	1012	942	872	802	732
90	2830	2760	2690	2620	2550	2480	2410	2340	2270	2200	2130	2060	1990	1920	1850	1780	1710	1640	1570	1500	1430	1360	1290	1220	1150	1080	1010	940	870	800	730

Appendix III

Payoff Matrix for Duopolist 2

Identical Cost Curves

Limited Information

Appendix IV

Payoff Matrix for Duopolist 1

Identical Cost Curves

Complete Information

Appendix V

Payoff Matrix for Duopolist 2

Identical Cost Curves

Complete Information

Appendix VI

Payoff Matrix for Duopolist 1

Similar Cost Curves

Limited Information

Appendix VII

Payoff Matrix for Duopolist 2

Similar Cost Curves

Limited Information

DUOPOLIST 2'

	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90
30	1400	1507	1609	1705	1785	1880	1957	2033	2101	2163	2220	2271	2317	2357	2391	2420	2443	2461	2473	2479	2480	2478	2465	2449	2426	2400	2368	2329	2285	2236	2180
32	1376	1475	1578	1669	1757	1834	1917	1989	2055	2115	2170	2219	2263	2301	2333	2366	2391	2410	2424	2431	2437	2440	2438	2426	2409	2384	2352	2314	2270	2221	2168
34	1346	1443	1541	1633	1719	1800	1875	1946	2009	2067	2120	2167	2209	2246	2276	2300	2319	2333	2341	2343	2346	2343	2331	2317	2297	2270	2236	2198	2151	2103	2050
36	1310	1411	1507	1597	1681	1760	1833	1901	1963	2019	2070	2115	2155	2189	2217	2240	2257	2269	2275	2278	2278	2273	2263	2248	2228	2199	2161	2113	2066	2010	
38	1280	1379	1473	1561	1643	1720	1791	1857	1917	1971	2020	2063	2101	2133	2159	2180	2195	2205	2209	2207	2200	2187	2169	2145	2114	2078	2038	1990	1941	1884	1830
40	1255	1347	1437	1525	1605	1680	1749	1813	1871	1923	1970	2011	2049	2077	2101	2120	2133	2141	2143	2139	2130	2115	2095	2069	2036	2004	1956	1909	1855	1796	1730
42	1230	1315	1405	1494	1577	1654	1727	1789	1845	1895	1942	1987	2028	2063	2092	2112	2123	2124	2119	2109	2094	2074	2049	2016	1982	1936	1890	1836	1783	1724	1650
44	1190	1281	1371	1458	1539	1616	1685	1745	1797	1843	1884	1927	1965	1998	2026	2049	2067	2080	2087	2088	2074	2054	2028	1994	1958	1912	1866	1812	1750	1689	1620
46	1160	1251	1337	1421	1499	1576	1643	1703	1755	1800	1839	1873	1902	1927	1948	1965	1978	1987	1991	1989	1974	1949	1914	1878	1831	1785	1730	1676	1614	1553	1484
48	1130	1219	1303	1381	1453	1520	1581	1637	1687	1731	1770	1803	1831	1853	1869	1880	1885	1885	1879	1867	1850	1827	1799	1765	1724	1680	1628	1573	1511	1444	1370
50	1100	1187	1269	1345	1415	1480	1537	1587	1634	1678	1720	1751	1777	1797	1811	1820	1823	1821	1813	1799	1780	1755	1725	1689	1646	1600	1546	1489	1425	1356	1280
52	1070	1155	1235	1309	1377	1440	1497	1549	1595	1635	1676	1709	1733	1750	1760	1761	1757	1747	1731	1710	1683	1651	1613	1568	1526	1474	1415	1350	1283	1216	1140
54	1040	1123	1201	1273	1339	1400	1458	1505	1549	1587	1620	1647	1669	1685	1695	1700	1699	1693	1681	1663	1640	1611	1577	1537	1490	1440	1382	1324	1255	1180	1100
56	1010	1091	1167	1237	1301	1360	1413	1461	1503	1539	1570	1595	1615	1629	1637	1640	1637	1629	1615	1595	1570	1539	1500	1461	1412	1360	1300	1237	1167	1092	1010
58	980	1059	1133	1201	1263	1320	1371	1417	1457	1491	1520	1543	1561	1573	1579	1580	1575	1565	1549	1527	1498	1460	1423	1385	1338	1280	1221	1150	1081	1004	920
60	950	1027	1099	1165	1225	1280	1329	1373	1411	1443	1470	1491	1507	1517	1521	1520	1513	1501	1483	1457	1430	1395	1355	1309	1256	1200	1136	1069	995	916	830
62	920	998	1065	1129	1187	1240	1287	1329	1365	1395	1420	1439	1453	1461	1463	1460	1451	1437	1417	1391	1366	1323	1281	1233	1178	1120	1054	985	909	828	740
64	890	963	1031	1093	1149	1200	1245	1285	1319	1347	1370	1387	1399	1405	1405	1400	1389	1373	1351	1323	1299	1251	1207	1157	1100	1040	970	901	823	740	650
66	860	931	997	1057	1111	1160	1203	1241	1273	1297	1320	1335	1345	1349	1347	1340	1327	1309	1285	1255	1230	1179	1133	1081	1022	960	890	817	737	652	560
68	830	901	966	1024	1073	1120	1161	1197	1227	1251	1276	1283	1291	1293	1297	1290	1276	1255	1229	1197	1170	1117	1065	1005	944	880	808	733	651	564	470
70	800	871	936	995	1045	1092	1139	1175	1203	1223	1240	1246	1253	1251	1240	1223	1203	1181	1153	1119	1080	1035	985	929	866	800	726	649	565	476	380
72	770	835	895	949	997	1045	1089	1135	1165	1197	1223	1238	1243	1243	1230	1210	1183	1153	1117	1087	1051	1010	963	911	853	788	720	644	565	479	388
74	740	803	861	913	959	1000	1035	1065	1089	1107	1126	1137	1140	1135	1115	1090	1079	1053	1021	983	940	891	833	777	710	640	562	481	393	300	200
76	710	771	827	877	921	960	993	1020	1039	1059	1070	1075	1075	1069	1057	1040	1021	998	955	915	870	819	763	701	632	560	480	397	307	212	110
78	680	739	793	841	883	920	951	977	997	1011	1020	1023	1021	1013	999	980	955	925	889	847	800	747	689	625	559	480	398	313	221	124	20
80	650	707	759	805	845	880	909	933	951	963	970	971	967	957	941	920	893	861	823	779	720	655	585	519	446	360	276	189	95	3	-70
82	620	678	725	769	807	840	867	889	905	915	920	919	913	901	883	860	831	797	757	711	660	603	540	473	400	324	237	145	49	-52	-160
84	590	640	691	733	769	800	825	845	859	867	870	867	859	845	825	800	769	733	691	643	590	531	467	397	320	240	150	61	-73	-140	-250
86	560	611	657	697	731	760	783	801	813	819	820	815	805	789	767	740	707	669	625	575	520	457	393	321	242	160	70	-23	-122	-232	-346
88	530	579	623	661	693	720	741	757	767	771	770	763	751	733	709	680	645	605	559	507	450	387	319	245	169	80	-12	-107	-207	-316	-430
90	500	547	589	625	655	680	699	713	721	723	720	711	697	677	651	620	583	541	493	439	380	318	245	169	86	0	-94	-191	-295	-404	-520

Appendix VIII

Payoff Matrix for Duopolist 1

Dissimilar Cost Curves

Limited Information

DUOPOLIST 1"

	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90		
30	360	330	300	270	240	210	180	150	120	90	60	30	0	-30	-60	-90	-120	-150	-180	-210	-240	-270	-300	-330	-360	-390	-420	-450	-480	-510	-540		
32	472	440	408	376	344	312	280	248	216	184	152	120	88	56	24	-8	-40	-72	-104	-136	-168	-200	-232	-264	-296	-328	-360	-392	-424	-456	-488		
34	578	544	510	476	442	408	374	340	306	272	238	204	170	136	102	68	34	0	-34	-68	-102	-136	-170	-204	-238	-272	-306	-340	-374	-408	-442		
36	680	644	608	572	536	500	464	428	392	356	320	284	248	212	176	140	104	68	32	-4	-76	-112	-148	-184	-220	-256	-292	-328	-364	-400	-436		
38	778	740	702	664	626	588	550	512	474	436	398	360	322	284	246	208	170	132	94	56	18	-20	-58	-96	-134	-172	-210	-248	-286	-324	-362		
40	870	830	790	750	710	670	630	590	550	510	470	430	390	350	310	270	230	190	150	110	70	-30	-70	-110	-150	-190	-230	-270	-310	-350	-390		
42	958	916	874	832	790	748	706	664	622	580	538	496	454	412	370	328	286	244	202	160	118	76	-34	-8	-50	-92	-134	-176	-218	-260	-302		
44	1040	996	952	908	864	820	776	732	688	644	600	556	512	468	424	380	336	292	248	204	160	116	72	-28	-16	-60	-104	-148	-192	-236	-280		
46	1118	1072	1026	980	934	888	842	796	750	704	658	612	566	520	474	428	382	336	290	244	198	152	106	60	14	-32	-78	-124	-170	-216	-262		
48	1192	1144	1096	1048	1000	952	904	856	808	760	712	664	616	568	520	472	424	376	328	280	232	184	136	88	40	-8	-56	-104	-152	-200	-248		
50	1260	1210	1160	1110	1060	1010	960	910	860	810	760	710	660	610	560	510	460	410	360	310	260	210	160	110	60	10	-40	-90	-140	-190	-240		
52	1324	1272	1220	1168	1116	1064	1012	960	908	856	804	752	700	648	596	544	492	440	388	336	284	232	180	128	76	24	-28	-80	-132	-184	-236		
54	1382	1328	1274	1220	1166	1112	1058	1004	950	896	842	788	734	680	626	572	518	464	410	356	302	248	194	140	86	32	-22	-76	-130	-184	-238		
56	1436	1380	1324	1268	1212	1156	1100	1044	988	932	876	820	764	708	652	596	540	484	428	372	316	260	204	148	92	36	-20	-76	-132	-188	-244		
58	1486	1428	1370	1312	1254	1196	1138	1080	1022	964	906	848	790	732	674	616	558	500	442	384	326	268	210	152	94	36	-22	-76	-132	-188	-244		
60	1530	1470	1410	1350	1290	1230	1170	1110	1050	990	930	870	810	750	690	630	570	510	450	390	330	270	210	150	90	30	-30	-80	-130	-180	-230		
62	1570	1508	1446	1384	1322	1260	1198	1136	1074	1012	950	888	826	764	702	640	578	516	454	392	330	268	206	144	82	-20	-42	-94	-146	-198	-250		
64	1604	1540	1476	1412	1348	1284	1220	1156	1092	1028	964	900	836	772	708	644	580	516	452	388	324	260	196	132	68	4	-60	-124	-188	-252	-316		
66	1634	1568	1502	1436	1370	1304	1238	1172	1106	1040	974	908	842	776	710	644	578	512	446	380	314	248	182	116	50	-16	-82	-148	-214	-280	-346		
68	1660	1592	1524	1456	1388	1320	1252	1184	1116	1048	980	912	844	776	708	640	572	504	436	368	300	232	164	96	28	-40	-108	-176	-244	-312	-380		
70	1680	1610	1540	1470	1400	1330	1260	1190	1120	1050	980	910	840	770	700	630	560	490	420	350	280	210	140	70	0	-70	-140	-210	-280	-350	-420		
72	1695	1624	1551	1479	1407	1335	1263	1191	1119	1047	975	903	831	759	687	615	543	471	399	327	255	183	111	39	-33	-105	-177	-249	-321	-393	-465		
74	1704	1632	1558	1484	1410	1336	1262	1188	1114	1040	966	892	818	744	670	596	522	448	374	300	226	152	78	4	-70	-144	-218	-292	-366	-440	-514		
76	1712	1636	1560	1484	1408	1332	1256	1180	1104	1028	952	876	800	724	648	572	496	420	344	268	192	116	40	-36	-72	-112	-184	-256	-328	-400	-472		
78	1713	1634	1557	1479	1401	1323	1245	1167	1089	1011	933	855	777	699	621	543	465	387	309	231	153	75	-3	-81	-129	-207	-285	-363	-441	-519	-597		
80	1710	1630	1553	1475	1397	1319	1241	1163	1085	1007	929	851	773	695	617	539	461	383	305	227	149	110	30	-50	-79	-120	-200	-280	-360	-440	-520		
82	1702	1620	1543	1465	1387	1309	1231	1153	1075	997	919	841	763	685	607	529	451	373	295	217	139	110	30	-50	-79	-120	-200	-280	-360	-440	-520		
84	1688	1604	1526	1448	1370	1292	1214	1136	1058	980	902	824	746	668	590	512	434	356	278	200	122	110	30	-50	-79	-120	-200	-280	-360	-440	-520		
86	1670	1584	1506	1428	1350	1272	1194	1116	1038	960	882	804	726	648	570	492	414	336	258	180	102	110	30	-50	-79	-120	-200	-280	-360	-440	-520		
88	1648	1560	1482	1404	1326	1248	1170	1092	1014	936	858	780	702	624	546	468	390	312	234	156	74	110	30	-50	-79	-120	-200	-280	-360	-440	-520		
90	1620	1530	1450	1370	1290	1210	1130	1050	970	890	810	730	650	570	490	410	330	250	170	90	0	-70	-180	-270	-360	-450	-540	-630	-720	-810	-900	-990	-1080

Appendix IX

Payoff Matrix for Duopolist 2

Dissimilar Cost Curves

Limited Information

Appendix X

Payoff Matrix for Duopolist 1

Dissimilar Cost Curves

Complete Information

DUOPOLIST 2"

DUOPOLIST 1"

30	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90
30	180	182	184	186	188	190	192	194	196	198	200	202	204	206	208	210	212	214	216	218	220	222	224	226	228	230	232	234	236	238	240
32	340	342	344	346	348	350	352	354	356	358	360	362	364	366	368	370	372	374	376	378	380	382	384	386	388	390	392	394	396	398	400
34	400	402	404	406	408	410	412	414	416	418	420	422	424	426	428	430	432	434	436	438	440	442	444	446	448	450	452	454	456	458	460
36	460	462	464	466	468	470	472	474	476	478	480	482	484	486	488	490	492	494	496	498	500	502	504	506	508	510	512	514	516	518	520
38	520	522	524	526	528	530	532	534	536	538	540	542	544	546	548	550	552	554	556	558	560	562	564	566	568	570	572	574	576	578	580
40	580	582	584	586	588	590	592	594	596	598	600	602	604	606	608	610	612	614	616	618	620	622	624	626	628	630	632	634	636	638	640
42	640	642	644	646	648	650	652	654	656	658	660	662	664	666	668	670	672	674	676	678	680	682	684	686	688	690	692	694	696	698	700
44	700	702	704	706	708	710	712	714	716	718	720	722	724	726	728	730	732	734	736	738	740	742	744	746	748	750	752	754	756	758	760
46	760	762	764	766	768	770	772	774	776	778	780	782	784	786	788	790	792	794	796	798	800	802	804	806	808	810	812	814	816	818	820
48	820	822	824	826	828	830	832	834	836	838	840	842	844	846	848	850	852	854	856	858	860	862	864	866	868	870	872	874	876	878	880
50	880	882	884	886	888	890	892	894	896	898	900	902	904	906	908	910	912	914	916	918	920	922	924	926	928	930	932	934	936	938	940
52	940	942	944	946	948	950	952	954	956	958	960	962	964	966	968	970	972	974	976	978	980	982	984	986	988	990	992	994	996	998	1000
54	1000	1002	1004	1006	1008	1010	1012	1014	1016	1018	1020	1022	1024	1026	1028	1030	1032	1034	1036	1038	1040	1042	1044	1046	1048	1050	1052	1054	1056	1058	1060
56	1060	1062	1064	1066	1068	1070	1072	1074	1076	1078	1080	1082	1084	1086	1088	1090	1092	1094	1096	1098	1100	1102	1104	1106	1108	1110	1112	1114	1116	1118	1120
58	1120	1122	1124	1126	1128	1130	1132	1134	1136	1138	1140	1142	1144	1146	1148	1150	1152	1154	1156	1158	1160	1162	1164	1166	1168	1170	1172	1174	1176	1178	1180
60	1180	1182	1184	1186	1188	1190	1192	1194	1196	1198	1200	1202	1204	1206	1208	1210	1212	1214	1216	1218	1220	1222	1224	1226	1228	1230	1232	1234	1236	1238	1240
62	1240	1242	1244	1246	1248	1250	1252	1254	1256	1258	1260	1262	1264	1266	1268	1270	1272	1274	1276	1278	1280	1282	1284	1286	1288	1290	1292	1294	1296	1298	1300
64	1300	1302	1304	1306	1308	1310	1312	1314	1316	1318	1320	1322	1324	1326	1328	1330	1332	1334	1336	1338	1340	1342	1344	1346	1348	1350	1352	1354	1356	1358	1360
66	1360	1362	1364	1366	1368	1370	1372	1374	1376	1378	1380	1382	1384	1386	1388	1390	1392	1394	1396	1398	1400	1402	1404	1406	1408	1410	1412	1414	1416	1418	1420
68	1420	1422	1424	1426	1428	1430	1432	1434	1436	1438	1440	1442	1444	1446	1448	1450	1452	1454	1456	1458	1460	1462	1464	1466	1468	1470	1472	1474	1476	1478	1480
70	1480	1482	1484	1486	1488	1490	1492	1494	1496	1498	1500	1502	1504	1506	1508	1510	1512	1514	1516	1518	1520	1522	1524	1526	1528	1530	1532	1534	1536	1538	1540
72	1540	1542	1544	1546	1548	1550	1552	1554	1556	1558	1560	1562	1564	1566	1568	1570	1572	1574	1576	1578	1580	1582	1584	1586	1588	1590	1592	1594	1596	1598	1600
74	1600	1602	1604	1606	1608	1610	1612	1614	1616	1618	1620	1622	1624	1626	1628	1630	1632	1634	1636	1638	1640	1642	1644	1646	1648	1650	1652	1654	1656	1658	1660
76	1660	1662	1664	1666	1668	1670	1672	1674	1676	1678	1680	1682	1684	1686	1688	1690	1692	1694	1696	1698	1700	1702	1704	1706	1708	1710	1712	1714	1716	1718	1720
78	1720	1722	1724	1726	1728	1730	1732	1734	1736	1738	1740	1742	1744	1746	1748	1750	1752	1754	1756	1758	1760	1762	1764	1766	1768	1770	1772	1774	1776	1778	1780
80	1780	1782	1784	1786	1788	1790	1792	1794	1796	1798	1800	1802	1804	1806	1808	1810	1812	1814	1816	1818	1820	1822	1824	1826	1828	1830	1832	1834	1836	1838	1840
82	1840	1842	1844	1846	1848	1850	1852	1854	1856	1858	1860	1862	1864	1866	1868	1870	1872	1874	1876	1878	1880	1882	1884	1886	1888	1890	1892	1894	1896	1898	1900
84	1900	1902	1904	1906	1908	1910	1912	1914	1916	1918	1920	1922	1924	1926	1928	1930	1932	1934	1936	1938	1940	1942	1944	1946	1948	1950	1952	1954	1956	1958	1960
86	1960	1962	1964	1966	1968	1970	1972	1974	1976	1978	1980	1982	1984	1986	1988	1990	1992	1994	1996	1998	2000	2002	2004	2006	2008	2010	2012	2014	2016	2018	2020
88	2020	2022	2024	2026	2028	2030	2032	2034	2036	2038	2040	2042	2044	2046	2048	2050	2052	2054	2056	2058	2060	2062	2064	2066	2068	2070	2072	2074	2076	2078	2080
90	2080	2082	2084	2086	2088	2090	2092	2094	2096	2098	2100	2102	2104	2106	2108	2110	2112	2114	2116	2118	2120	2122	2124	2126	2128	2130	2132	2134	2136	2138	2140

Appendix XI

Payoff Matrix for Duopolist 2

Dissimilar Cost Curves

Complete Information

