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An Investigation of Methods for Estimating Marginal Values of Irrigation Water

Richard L. Johnson
Utah State University

AN INVESTIGATION OF METHODS FOR ESTIMATING

MARGINAL VALUES OF IRRIGATION WATER

by

Richard L Johnson

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Richard L Johnson

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ABSTRACT

An Investigation of Methods for Estimating

Marginal Values of Irrigation Water

by

Richard L Johnson, Master of Science

Utah State University, 1967

Major Professor: Lynn H. Davis
Department: Agricultural Economics

Marginal values of water used in irrigation are needed if
water is to be optimally allocated among alternative uses. Cobb-
Douglas production function analysis and linear programming
methods were studied in this investigation to find their fruit-
fulness in predicting these marginal values. The theoretical
properties of both methods indicate that they are conceptually
capable of yielding valid marginal value estimates for irrigation
water.

Further investigation of the two methods was carried out as
an empirical test in the Milford area of Utah. Marginal values
of water used for irrigation in that area were estimated by both
procedures. Although inviolable criteria for testing the validity
of the estimates are not available, imperfect standards of measure
imply that they are sound. Linear programming and Cobb-Douglas
production function analysis are therefore concluded to be fruitful
methods of estimating marginal values of water used for irrigation.

INTRODUCTION AND JUSTIFICATION

This thesis is about marginal values of water used for irrigation. Its purpose is to investigate methods by which marginal values may be estimated. A need exists for reliable marginal value estimates in allocating water among alternative uses. If resources are to be optimally allocated, the general allocative model of economic theory must hold for all resources and all products.

$$\frac{MVPx_1\ (y_1)}{MCFx_1} = \ .\ \ .\ \ .\ = \frac{MVPx_1\ (y_n)}{MCFx_1} = \frac{MVPx_n\ (y_n)}{MCFx_n} = \ .\ \ .\ \ .\ = \frac{MVPx_n\ (y_n)}{MCFx_n}$$

Where MVP is marginal value product, MCF is marginal cost of the factor, $x_1\ .\ \ .\ \ .\ _n$ are any number of production inputs, and $y_1\ .\ \ .\ \ .\ _n$ are any number of products.

In the first two components of the model let the production input x_1 be water and $y_1\ .\ \ .\ \ .\ _n$ be the different products that may be produced by water in any of its uses. Water will be optimally allocated where these two components equal each other. It is combined with other factors in such a way that shifting one unit of water from one use to another would reduce the total net benefits to society. Since irrigation is one of the major uses of water, marginal values for irrigation water are needed. The work of this study is to investigate methods of estimating these marginal values.

OBJECTIVES

The objective of this investigation is to yield valid and meaningful information about marginal values of irrigation water by investigating methods by which these marginal values may be estimated and testing the validity of the methods empirically.

Design of the Investigation

The specific purposes and procedures of the study were: (a) To discuss some of the problems of applying marginal value estimation techniques to irrigation water; (b) to investigate two methods of marginal value analysis and appraise conceptually the fruitfulness of these methods in estimating marginal values of irrigation water; and (c) to test the validity of these two methods by estimating marginal values for irrigation water in an empirical test area.

REVIEW OF LITERATURE

Heady and Dillon[1] have explained several types of production functions, told briefly of the history of them, and described in detail the characteristics of each one. Certain concepts and methods relating to the prediction and use of production functions in agriculture are summarized and methods of data collection are considered. Production surfaces are illustrated and problems of choice concerning alternative models are explained.

Heady, Johnson, and Hardin[2] edited a book of conference proceedings. The objective of those who contributed to this book was to review some of the thinking and research in the measurement of resource productivity in farm production. In chapter 8, Heady discusses the relationship of scale analysis to productivity analysis. The discussion was useful to this study because of its treatment of elasticity and marginal product relationships. In Chapter 9, Johnson describes some classification and accounting problems in fitting production functions to farm record and survey data. A few general considerations concerning sampling problems and the effect they have on regression coefficients in production function analysis are presented. In Chapter 11, Beringer discusses

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[1]Earl O. Heady and John L. Dillon, Agricultural Production Functions (Ames, Iowa: Iowa State University Press, 1961).

[2]Earl O. Heady, Glenn L. Johnson and Lowell S. Hardin (EDS.) Resource Productivity, Returns to Scale, and Farm Size (Ames, Iowa: Iowa State College Press, 1956).

problems in finding a method to estimate marginal value productiv-
ities for input and investment categories on multiple-enterprise
farms. Many of these problems are common to this investigation.

Tintner has written Chapter 14 concerning significance tests
in production function research. He lists the important applications
of tests of significance to agricultural production studies, and
sets forth the conditions necessary for testing the significance
of a given marginal productivity. An example using the Cobb-Douglas
function is presented, which is of special significance to this
investigation.

Some of the criticisms of the Cobb-Douglas function for
marginal value estimation are considered by Haver in Chapter 18
and by McAlexander in Chapter 17.

Beringer[3] discusses some of the conceptual problems in
determining production functions for water. He suggests that
agronomists no longer try to determine a production function for
water simply by applying various quantities of it on a number of
plots and measuring the resulting production response.[4] Instead,
they have concentrated on finding plant-water relationships which
are independent of soil types and water quantity. Three terms
are used extensively in describing these relationships: (a) Field

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[3]Christoph Beringer, "Some Conceptual Problems in Determining
the Production Function for Water," The West in a Growing Economy
(Logan, Utah: Proceedings of the Western Farm Economics Association,
1959), pp. 58-70.

[4]Earlier studies have, however, employed this method. See for
example, John A. Widtsoe and L. A. Merrill, "The Yields of Crops with
Different Quantities of Irrigation Water" (Utah Agricultural Experi-
ment Station Bulletin 117, 1912).

capacity is defined as the amount of water a soil will hold against gravity when allowed to drain freely; (b) the wilting point designates a soil moisture content at which plants growing in that soil become permanently wilted. Once this point is reached, no further growth will occur; and (c) moisture tension or moisture stress is a measure of the force with which water particles are held by a particular soil. The plant must overcome this moisture tension if it is to take water from the soil. As the water content of the soil decreases, the water remaining is held more tightly by the soil and it is increasingly difficult for plants to maintain themselves. At some point of water depletion, the plant reaches the wilting point and finally dies. Beringer refers to irrigation studies which have concluded that between the wilting percentage and field capacity, plants extract the soil moisture necessary for their continued growth equally well, as illustrated in Figure 1.
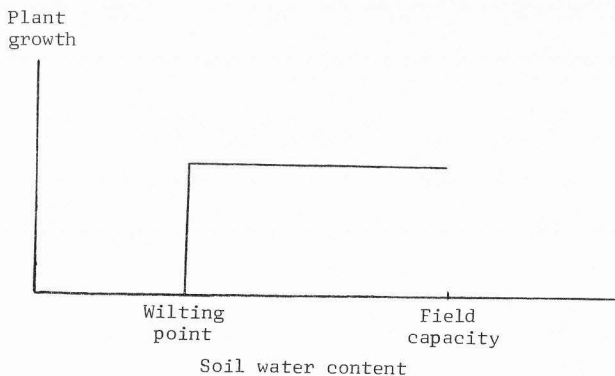
Figure 1. Theoretical production function illustrating zero marginal value product of water.

No plant growth can occur when the level of water application is below the wilting point level. Above that point, the marginal product of increasing amounts of water inputs is zero. These conclusions seem to be in sharp contrast to the laws of diminishing returns. The economic implications are obvious: The marginal value product of water will be zero and profits will be maximized when water application is kept just above the wilting point.

Beringer questions whether or not it is possible to maintain a soil moisture content just above the wilting point. He cites references indicating that it is impossible to wet any soil mass to less than its field capacity. If a small quantity of water is applied to a mass of dry soil, the uppermost layer is filled to field capacity while the rest of the soil remains uneffected. As more moisture is added the soil is wetted to greater depths, but only after the soil above it has already reached field capacity. The depth to which the soil must be wetted will depend largely upon the root depth of the planted crop.

With respect to the shape of the crop-response curve, Mr. Beringer sees these considerations to be of considerable importance:

> If only a very small amount of water is applied to a soil planted to a given crop, only the uppermost part of the soil will be wetted. Germination, root development, and plant growth being restricted to this layer of soil will be retarded: and the resulting yield response will be zero or, at best, a very small amount. As more water is applied, a second layer of soil will be wetted; germination and root development will be improved, and so will the resulting production.[5]

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[5] Beringer, pp. 63-64.

As this process is carried on, it should result in a production
response curve which approximates the usual concept of the law
of diminishing returns illustrated in Figure 2.
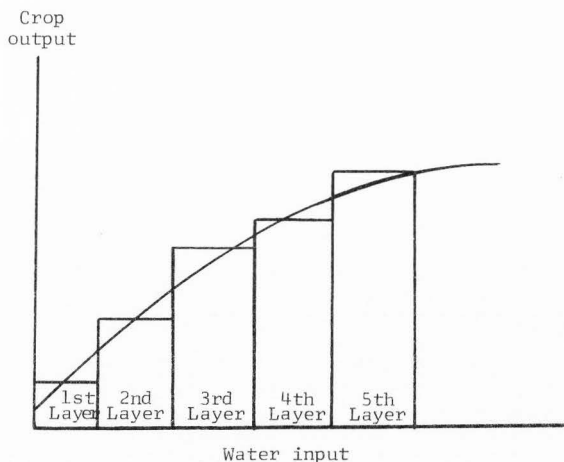
Figure 2. Water required to wet a soil to increasing depth.

Fullerton[6] had an objective to determine the relative
efficiency of different allocative schemes for irrigation water.
In the persuit of this objective institutional factors affecting
water transfers were examined to determine how critical they are
in misallocation. Considerable attention was given to describing

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[6]Herbert H. Fullerton, "Transfer Restrictions and Misal-
location of Irrigation Water" (Unpublished MS thesis, Utah State
University Library, Logan, Utah, 1965).

a water market and determining the values of water in the market. A perfectly competitive model was assumed; thus the market value of water was declared to approximate the marginal value product derived from the use of water for irrigation. The reliability of many of the measures used in the analysis rested upon the hypothesis that a market for irrigation water does in fact exist, and that a perfectly competitive model may be used to describe it.

Fullerton hypothesized that institutional changes which eliminate barriers to free transfer of water will result in an increase in economic benefits. Results of the analysis indicated that a significant differential does exist between rental prices occurring under different transfer policies. This conclusion takes on added significance with the assumption that the rental price approximates the marginal value product of water. The average rental price under the policy which permitted intercompany transfer of water was $9.60 per acre foot. When policy restricted this intercompany transfer the rental price was $3.21 per acre foot.

Hartman and Whittelsey[7], of Colorado State University have conducted a study entitled "Marginal Values of Irrigation Water." The intent of this study was not the estimation of a single value for an increment of water, but rather to indicate a range of values that would apply under different conditions. The specific

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[7]L. M. Hartman and Norman Whittelsey, "Marginal Values of Irrigation Water" (Colorado Agricultural Experiment Station Research Bulletin 70, 1960).

problem considered involves estimating the value to an individual
farm of an increased supply of water. Linear programming pro-
cedures were used to estimate how water supply changes affect income.
Enterprise alternatives were varied by using three models to
reflect differing ambitions of farm operators, and risk preferences.
Three levels of crop yields and three levels of water application
were used. Resource levels for land, labor, and operating capital
were held constant.

Three separate models were defined according to variances in
activity organization. The coefficients for input requirements,
resource quantities available, and net revenues from each enterprise
were largely obtained from previously published data sources.

Results of the analysis estimated marginal values of water
for the most extensive model as being $14.49, $38.49, and $14.40,
per acre foot for July, August, and September respectively. A
conclusion was drawn that the timing of water's availability is
an important factor in determining its worth to a particular farm.
These monthly results were forthcoming by changing the water supply
in increments in each month when water was a limiting resource.

The linear programming models of the study demonstrate the
effect of certain factors upon the marginal value of water and
indicate the type of adjustments that would be economic in response
to a change in water supply. It was found that the kind of
adjustments farmers make to changes in water supply has an effect
upon the value of the additional water. Market conditions for
products and for resource inputs, and native land characteristics

also help determine value of presently used water and of marginal increments.

Miller[8] completed a Ph.D dissertation entitled, "An Investigation of Alternative Methods of Valuing Irrigation Water," in June of 1965. His principal objective was to compare and evaluate alternative procedures for estimating the marginal values of irrigation water. The three methods he evaluated are budgeting, linear programming, and production function analysis.

The conceptual and statistical problems associated with the use of each method are discussed.

The data for the study are derived from two basic sources: (a) Physical response experiments conducted on a plot control basis; and (b) a survey of farms in four counties of Oregon. Marginal value productivities are estimated for each of the two crops using both data sources. These values are predicted for a range of from 2 to 14 inches of water in 2 inch increments. Marginal value products of water for sweet corn as estimated by a Cobb-Douglas function range from $2.38 per acre inch when 2 inches of water are applied per acre to .38 cents when 14 inches are applied per acre. Corresponding values for bush beans are $9.70 per acre inch and $1.58 per acre inch, respectively.

The equation and the variables used in fitting the Cobb-Douglas function for corn from survey data were as follows:

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[8]Stanley F. Miller, "An Investigation of Alternative Methods of Valuing Irrigation Water" (Unpublished PhD dissertation, Oregon State University Library, Corvallis, Oregon, 1965).

$$Y = aX_1^{b1} X_2^{b2} X_3^{b3} X_4^{b4} U$$

where Y = gross income arising from the sale of dry shelled corn

per acre;

X_1 = dollar value of purchased inputs per acre;

X_2 = hours of machinery use per acre;

X_3 = water use in acre-inches per acre;

X_4 = drainage in feet per acre; and

U = the stochastic error term.

Tests for multicollinearity were made by comparing the highest correlation coefficients between independent variables with the over-all multiple correlation coefficients. For the field corn survey the R value was .67 and the highest correlation between independent variables was .49. The respective figures for bush beans were .61 and .55. It was concluded that the intercorrelation between independent variables was not high enough relative to the respective R values to indicate multicollinearity.

Four separate linear programming models were developed to obtain marginal value products of water from the survey data.

The resulting marginal value productivities for each source of data collection and for each method of analysis are compared graphically and some general conclusions are stated. It is concluded that at the average level of water use, both the survey and the experimental functions gave almost identical estimates of yield. This does not hold for other levels of water input, however.

CONCEPTUAL PROBLEMS

The theory of marginality is a powerful tool in economics.
Strictly speaking, an optimum allocation of resources cannot be
determined without it.[9] Marginal analysis has therefore been
highly refined and its methodologies have been studied extensively.
Problems in applying these methods to resources used in agriculture
still remain however, especially where water for irrigation is
concerned.

The purpose of this section is to describe some of the problems
involved in applying methods of marginal analysis to water for
irrigation.

Previous economic studies of irrigation in Utah have been
unable to find a significant relationship between water applied
per acre and yields per acre.[10,11] It is not supposed that such a
lack of correlation exists if all other variable factors are held
constant. Economic theory would predict that as small increments
of water are added to a constant unit of land, other factors
remaining constant, yields per unit of land will increase to a

[9]Heady and Dillon, p. 228.

[10]Clyde E. Stewart, "Profitable Farm Adjustments in the Use of
Irrigation Water in Ashley Valley, Utah" (Utah Agricultural Experi-
ment Station Ag. Ec. Series 65-2, 1965), p. iv.

[11]Gordon L. Langford, "An Economic Study of Adjustment
Possibilities in Farm Organization and Resource Allocation in the
Sevier River Valley in Piute County, Utah, 1961" (Unpublished MS
thesis, Utah State University Library, Logan, Utah, 1965), p. 19.

point and then decrease. Such a relationship has in fact been shown when controlled experiments are conducted and all factors other than water application are held constant.[12] In correlation of cross sectional data, however, there may be factors influencing yield per acre which are not held constant as quantities of water applied vary. For example, soil types may differ between the sample units of the survey; fertilizer application may be different among farms; present cropping patterns are different; the historical cropping patterns and farming practices influence the present productive ability of farms. Inputs other than water, such as capital, labor, and management vary between farms and are also important contributors to production. The efficiency of water use and the timeliness of harvesting methods are not constant; and estimates of the exact quantity of inputs used and yields achieved are not always accurate. These many differences provide an intuitive explanation for the lack of simple correlation between water applied and yields attained in cross sectional analysis. An illustration may indicate possible economic consequences of these observations.

Suppose that I, II, and III in Figure 3 represent production functions for three different soil types. The points a, b, and c represent the location of current production on each of the three soil types. The slope of each of these functions become the marginal products of the respective function. Since the rate of

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[12]Widtsoe and Merrill, p. 119.

Output

Input
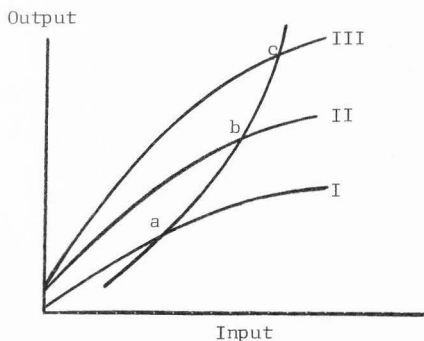
Figure 3.   Theoretical production functions for three different
            soil types.

increase in each of the functions is decreasing, the marginal

product of each will be declining. If data from the three soil

types are aggregated, a function traced out by line a, b, c may

be the resulting prediction. This curve is convex to the origin

and its rate of increase is increasing. This implies a possitively

sloped marginal curve and increasing marginal productivity of the

factor inputs. Thus the nature of the marginal productivity

estimation is greatly changed by aggregation of various soil

types into a single production investigation.

Another deterent to meaningful production response to inputs

in agriculture is apparent. Consider the general hypothetical

production function in Figure 4. It is assumed that inputs can begin at 0 and be added in increments throughout the range of the function. Marginal product is both increasing, constant, and decreasing, depending upon the quantity of factor used. In cross sectional analysis of farm data, however, it is unlikely that such a wide range of input application is real. If farmers are operating in the rational stage of production they will not apply less input than that represented by point a. To do so would be to sacrifice a greater average product per unit of input.
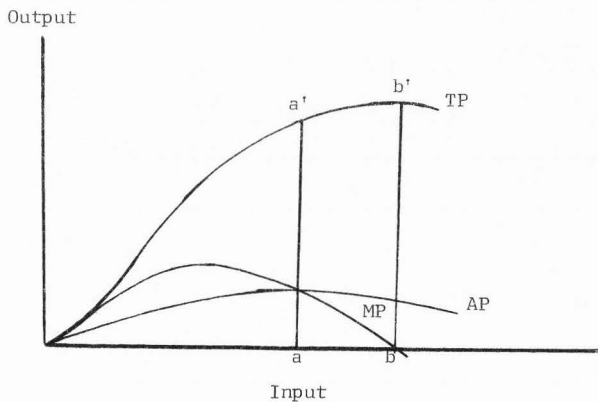
Figure 4. General hypothetical production function.

Neither will they use more factor inputs than that represented by point b; for each unit of input used beyond this stage would effect a decrease in total product. Thus the producer seeks rationally to operate in the relatively small area on the production function between a' and b'. This obviously reduces the range over which the predicting function is relevant and diminishes the variance in the quantities of inputs applied. It is more difficult to establish a causal relationship between inputs and output because of this shorter range in the magnitude of inputs. This reduction in explained variation increases the standard error of the regression coefficient and decreases the reliability of the marginal value product estimates.

In addition to these theoretical problems, difficulties in data collection and empirical procedures make it hard to apply methods of marginal analysis to water for irrigation. Controlled experiments are lacking and data must come from ex post decisions made by farm operators rather than from planned production experiments conducted by the researcher. It is unlikely that information will be recorded on all of the variables which may be relevant to the problem, and problems of interview bias may retard accuracy of the survey data. Also, there is a myriad of input and output factors relevant to any real-world response phenomena. Account cannot be taken of all of them because they are too numerous or because no satisfactory scale of measurement exists for them. These problems may be solved in part through aggregating inputs and outputs into categories, but this can lead to meaningless specification of the production function and results that are not useful.

CONCEPTUAL CONCLUSIONS

Some of the problems and considerations of estimating marginal values of irrigation water have been discussed. This section describes two methods of marginal analysis and assess their fruitfulness in estimating marginal values for irrigation water.

The two methods to be analyzed are Cobb-Douglas production function analysis and linear programming. They were chosen because of their popularity with agricultural economists in similar studies and because of the conceptual properties described in this section. A brief history of the development of each method will be given and the basic theoretical properties and assumptions will be described. After both methods have been discussed some general comparisons will be made between them.

The Cobb-Douglas Production Function

The Cobb-Douglas function has been the most popular algebraic form used in farm-firm production function analysis.[13]

Brief history

Paul H. Douglas[14] credits T. R. Malthus and Edward West,

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[13]Heady and Dillon, p. 228.

[14]Paul H. Douglas, "Are There Laws of Production?" The American Economic Review, Vol. XXXVIII (1948).

in 1815, with pointing out that if successive combined doses of
labor and capital were applied to a given piece of land, the
amount of the product would increase by diminishing increments.
Two years later this principle was adopted by Richardo in his
Principles of Political Economy as the basis for his theory of
distribution. He thought that the quantities of labor and capital
would not vary in relation to each other but were bound together
in fixed and unvarying proportions. There was therefore, no way
of isolating the specific contributions of these two factors as
a means of determining the rate of wages and interest.

Several years later (1840) in Germany, Von Thunen theoretically
separated labor and capital and pointed out that when each of the
factors were separately increased and the others held constant, the
product increased by diminishing increments. He stated that the
rates of wages and of interest were equal to the amount of the
product added by the last increments of each. Marginal productivity
was thus probably discovered by Von Thunen. It did not receive
the influence which it deserved, however, until some forty-eight
years later (1888), when it was "rediscovered" by John Bates Clark.
He said that,

> an increasing amount of labor applied to a fixed amount of
> pure capital goods yields a smaller and smaller rate of
> return . . . General wages tend to be equal to the actual
> product created by the last laborer that is added to the
> social working force. The earnings of capital are subject
> to identically the same law as those of labor; they are
> fixed by the product of the last increment that is brought
> into the field.[15]

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[15]John Bates Clark, "The Possibility of a Scientific Law of
Wages," Publications, American Economic Association, IV (March,
1889), pp. 39-63.

A next important step in marginal productivity was made by
Philip Wicksteed.[16] Wicksteed wrote in 1894 that if production
were characterized by a homogeneous linear function of the first
degree (that is, if when each of the factors of production were
increased, product would increase in the same proportion), then
with each factor receiving its marginal product, the total product
would be absorbed in payments to the factors without either
surplus or deficit. Wicksell[17] later detailed it further when
he proposed that only under perfect competition would each firm
tend to carry its scale of output to the point where the rate of
return was constant.

The theoretical discussion of marginal productivity became
largely inactive at this point, and it is 34 years later (1928)
that Douglas' work makes its contribution. One of the main
objectives in his now famous work was the measurement of the
marginal productivities of capital and labor. He was at the time
working with indexes for American manufacturing of the labor
employed, capital used, and physical output produced for the
years 1899-1922. He was lecturing at Amherst College, and
suggested to his friend Charles W. Cobb, that they seek to develop
a formula which would measure the relative effect of labor and
capital upon product during this period. They originally proposed

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[16]Philip H. Wicksteed, The Common Sense of Political Economy
and Selected Papers and Reviews on Economic Theory (London, England:
Routledge and Kegan Paul Limited, 1933), pp. 358-398.

[17]Knut Wicksell, Lectures on Political Economy (London, England:
George Routledge and Sons, Limited, 1934, Vol. I), pp. 101-133.

the formula:

$$P = bL^k C^{1-k}$$

Where P represented total value productivity of industry deflated
for price changes, L was total labor employed in production and
C total fixed capital available for production. The parameters
b and k were found by the method of least squares and the value
of k was found to be .75. The value of the capital exponent was
then .25 or 1-k. These results largely coincided with what Cobb
and Douglas had expected, and were later verified by other similar
studies conducted by them using time series data.[18]

The original equation, by requiring the exponents to sum to
unity, assumed constant returns to scale and perfect competition.
In 1937 this restriction was relaxed largely upon the urgings of
David Durand.[19] He pointed out that the use of the original
function assumed the existance of an economic law, namely constant
returns. He was not convinced that such a law existed and did
not accept the assumption that the product will be exactly dis-
tributed in accordance with the productivity principle. Cobb and
Douglas accepted this criticism and left it as a task of science
to test whether an economic law may or may not require constant
returns to scale. They adopted the less restrictive equation:

$$P = bL^k C^j ;$$

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[18] Heady and Dillon, pp. 18-20.

[19] David Durand, "Some Thoughts on Marginal Productivity with
Special Reference to Professor Douglas' Analysis," Journal of
Political Economy, XLV (December, 1937), pp. 740-758.

where j was estimated independently. This allowed the sum of the
exponents to be either greater than, equal to, or less than unity,
and hence to show increasing, constant, or decreasing returns to
scale. This means the sacrifice of the productivity principle,
for the new formula does not suggest that the total product will
be exactly distributed in accordance with the productivity principle.

Theoretical properties and assumptions

The Cobb-Douglas function may be generalized as $Y = aX^b$,
where Y is output, a is a constant, X is a variable input, and b
defines the transformation rate when the magnitude of X changes.
This production function merely states symbolically that the out-
put of a productive effort depends upon the inputs used. In
this case, only one input is used, and output is a function of
the quantity of X applied.

The marginal product of X can be estimated as the first
derivative with respect to X of the production function.

$$MP = \frac{dY}{dX} = baX^{b-1} \text{ or } \frac{baX^b}{X}$$

The elasticity of production can be found directly from this
marginal as follows:

$$\mathcal{E}p = \frac{\Delta Y}{Y} \Big/ \frac{\Delta X}{X} = \frac{\Delta Y}{\Delta X} \cdot \frac{X}{Y}$$

Substituting $\frac{baX^b}{X}$ in for $\frac{\Delta Y}{\Delta X}$, $\mathcal{E}p = \frac{baX^b}{X} \cdot \frac{X}{Y}$ but, from
the original function, $Y = aX^b$. Therefore, $\frac{bY}{X} \cdot \frac{X}{Y} = \mathcal{E}p$;
the X's and Y's cancel, and $\mathcal{E}p = b$; hence the elasticity of
production may be estimated directly from the fitted Cobb-Douglas

function as the b values of the equation. From the above compu-
tations it is also evident that the function assumes a constant
elasticity of production, or that successive equal increments of
input add the same percentage to total output.[20]

The function allows either constant, increasing or decreasing
marginal productivity, depending upon the magnitude of b. If $b = 1$,
constant returns to scale hold. If $b < 1$ decreasing returns to scale
exist, and if $b > 1$, increasing returns to scale are indicated.
Since b cannot at the same time be less than and greater than one,
both increasing and decreasing marginal product cannot hold. The
product curve flattens out as input increases but never reaches a
maximum. The rate of decrease in the marginal product declines
but never becomes zero.

Given these properties, the Cobb-Douglas function cannot be
used satisfactorily for data where there are ranges of both
increasing and decreasing marginal productivity. Neither can it
yield satisfactory estimates for data which might have both
positive and negative marginal products. Since a maximum product
is never defined, the Cobb-Douglas function may overestimate the
quantity of inputs which will equate marginal revenue and marginal
cost.

A Cobb-Douglas function which uses more than one variable
input retains the same properties as that of the simplified
equation. In the equation $Y = aX_1^{b1} X_2^{b2} \ldots X_n^{bn}$ the individual

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[20]The b values are the elasticities of production and they
do not change as the magnitude of X changes.

Neither will they use more factor inputs than that represented by point b; for each unit of input used beyond this stage would effect a decrease in total product. Thus the producer seeks rationally to operate in the relatively small area on the production function between a' and b'. This obviously reduces the range over which the predicting function is relevant and diminishes the variance in the quantities of inputs applied. It is more difficult to establish a causal relationship between inputs and output because of this shorter range in the magnitude of inputs. This reduction in explained variation increases the standard error of the regression coefficient and decreases the reliability of the marginal value product estimates.

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b values are the elasticities of production for each respective
input when all other inputs are held constant. The assumptions
of constant elasticity and marginal products with only a plus or
minus sign regardless of input or output magnitudes are retained.
The sum of the elasticity coefficients (b values) predicts the
elasticity of production for the entire equation. Returns to
scale are decreasing, constant or increasing depending upon whether
the b values sum to less than one, equal to one, or greater than
one respectively. The function also implies that at least some
quantity of each input must be used if output is to be nonzero.
Since the equation is multiplicative, a zero magnitude of any
one of the inputs would set the whole equation to zero. More
restrictive, none of the observations may contain zero units of
an input; for as the raw data is converted to logarithmic form,
the logarithm of zero would be minus infinity.

The b values are commonly estimated rather simply by multiple
regression. The equation is linear when it is estimated in its
natural logarithmic form. It then becomes:

$\log Y = \log a + b_1 \log X_1 + b_2 \log X_2 + \ldots + b_3 \log X_3.$

The Cobb-Douglas function is a relatively efficient user of
degrees of freedom, containing only one parameter for each variable.

The merits of the Cobb-Douglas production function as a
means of estimating marginal values may now be summarized. (a) It
permits the phenomena of diminishing marginal returns without
using as many degrees of freedom as would be required by other
quadratic functions. This is an aid in obtaining significant

results from survey data.   (b) The function is rather simply
estimated through multiple regression techniques and the regression
coefficients are the elasticities of production.   (c) The marginal
products of the factors may be estimated at their means from the
elasticities or regression coefficients.   (d) The function
becomes linear when transformed into its logarithmic form.   This
simplifies the interpretation of results and permits the graphing
of functions by calculating only one point in addition to the
intercept value.   (e) The residuals are normally distributed, or
at least their distribution does not deviate too much from the
normal.   This assumption permits the use of the t distribution for
testing the significance of the marginal productivities of each
of the inputs.   The significance of the correlation coefficients
may also be investigated.

Linear Programming Analysis

Brief history

The rudiments of linear programming are thought of by some
as lying with Elements of Political Economy, by the Frenchman
Leon Walras.[21]   This acknowledgement seems cogent if only the
most fundamental concepts are ascribed to him.   He showed that
the price of any number of commodities at a single time can be
determined by solving simultaneously the correct number of

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[21]As cited in Robert O. Ferguson and Lauren F. Sargent,
Linear Programming: Fundamentals and Applications (New York:
McGraw-Hill Book Company, Inc., 1958), p. 6.

equations in terms of the unknowns for which a solution is sought. It was this first attempt to solve problems of scarcity by stating problem conditions in equation form that gives Walras a claim to the development of linear programming.

Walras is credited as being one of the three men who independently pioneered the marginalist doctrine.[22] He was highly abstract in his approach and relied heavily upon mathematical notation and reasoning. His work was continued by other marginalists, noteably Vilfredo Pareto, Knut Wicksell, John Bates Clark, and Philip Wicksteed, and admired by such eminent economists as Irving Fisher, and Joseph Shumpeter. Of contemporary economists, J. R. Hicks is the leading exponent of Walrasian economics. Thus the marginal productivity principle is a common stem of development for linear programming and for the Cobb-Douglas production function. Additionally, the system of equations first used by Walras became the forerunner of the linear programming equation system, although methods of solving the equations are completely different.

A more definitive contribution to linear programming was by Wassily W. Leontief in the 1920's. He was working with a broader scope of activity analysis referred to as input-output analysis. Much that is basic to linear programming can be found in his study.

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[22] The other two were W. S. Jevons of England and Carl Menger of Austria. Even the innovations of these men were anticipated earlier by such men as Dupuit and Cournot of France and Von Thunen and Gossen of Germany, who wrote in the first half of the 19th Century.

Linear programming was refined during World War II. Groups of scientists were charged with finding solutions to several critical war problems. Linear programming was used as a method of minimizing travel distances, and of allocating scarce manpower, tools, weapons, and plant facilities among alternative uses.

George B. Dantzig is credited with developing the Simplex Method of linear programming in 1947.[23] His method was essentially a means of solving simultaneous equations for an optimum solution. Since that time linear programming has become an important tool of private firms and research organizations in their decision making processes. It is used extensively by agricultural economists as they seek to optimize the organization of resources and enterprises on farms, to suggest desirable farm adjustments, to indicate optimum interregional patterns of resource use and product specialization in agriculture, and to solve other related problems.

Theoretical properties and assumptions

Three quantitative components of a problem must exist if linear programming is to be used in seeking solutions in agriculture. First, there must be an objective function which a farm manager chooses to optimize. This objective is often to maximize income or to minimize costs. It may be modified to provide for other individual choices of farmers, such as risk aversion, enterprise preferences, fertility conservation, or leisure time. Second, there must be alternative methods of attaining the chosen objectives.

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[23]Ferguson and Sargent, p. 6.

Otherwise a decision opportunity does not exist. Third, there is no problem unless resources are limited. Deductively from these three components the intent of linear programming is to optimize a preconditioned objective subject to germane restrictions. Its method is computational of the following algebraic form:

(a) Maximize $Z_o = C_1 X_1 + C_2 X_2 + \ldots + C_n X_n$

subject to (b)

$$P_{11} X_1 + P_{12} X_2 + \ldots + P_{1n} X_n \leqq b_1$$

$$P_{21} X_1 + P_{22} X_2 + \ldots + P_{23} X_n \leqq b_2$$

$$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

$$P_{m1} X_1 + P_{m2} X_2 + \ldots + P_{mn} X_n \leqq b_m$$

Where for (a) Z_o is the function to be maximized, (profit), X_i are productive activities and C_i are net prices for the respective activity production. In (b) P_{ij} are the requirement coefficients indicating the amount of the ith resource (in rows) required to produce one unit of the jth activity (in columns). The X_i are the productive activities, and the b_m values are the amounts of each ith resource that are available for use in the productive process. Thus, the general problem is to maximize (a), subject to the restrictions of a set of linear inequalities, (b).

The restriction equations are mathematically solvable when they are changed from inequalities to equalities. This is accomplished through the use of slack variables or disposal activities. These slack variables provide for resources to go unused. If a term is added to each m relationship in (b), re-

presenting the amount of resource going into nonuse or disposal, the inequality sign may be replaced with an equal sign.

To leave the general form and consider a model having four activities and three resources, the problem can be put in the following matrix form:

maximize

(c) $Z_o = \begin{bmatrix} c_1 c_2 c_3 c_4 \end{bmatrix}$ $\overset{X}{\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}}$

where the first bracket is labeled c^1 and the column is labeled X.

subject to the programming restrictions

(d) $\overset{P}{\begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & 1 & 0 & 0 \\ P_{21} & P_{22} & P_{23} & P_{24} & 0 & 1 & 0 \\ P_{31} & P_{32} & P_{33} & P_{34} & 0 & 0 & 1 \end{bmatrix}}$ $\overset{X^1}{\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{bmatrix}} = \overset{B}{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix}}$

and (e) $X \gtreqless 0$: or more concisely, maximize $f(X) = c^1 X$ subject to the programming restrictions $PX = B$ with $X \gtreqless 0$.

The second restriction, that X must not be less than zero, simply states that the quantity of each activity level contained in X cannot be less than zero, or any value assigned to activities must not be negative. It has relevance in defining the maximum level to which an activity can be increased. One variable cannot be increased to a level causing the magnitude of another variable to become less than zero. It is this restriction which limits

the maximum level of an activity to that defined by the most limiting resource.

Marginal value products for each limiting factor of production are determined simultaneously as the systems of equations are solved. They appear in the solution as shadow prices and represent the reduction that would occur in the total returns if the availability of a resource is reduced by one unit and all other conditions are constant.

The simplex method is often used as a means of solving the matrix. It is a simple computational table for determining feasible and optimum programs. Its procedures are usually explained in the form of examples, and will not be included in this study. Detailed description of the practical application and of the mathematical properties of the simplex method is found in Heady and Chandler's book.[24]

There are four basic assumptions of linear programming which must be considered. First, the activities must be linear. This suggests that the rate of return to resource inputs is constant. Each increment of output requires the same amount of inputs as every other equal unit requires. The reality of this assumption is increased by omitting from the accounting procedure and productive costs which are fixed. Such expenses as machinery and building depreciation and taxes remain constant for a farm regardless of the enterprise combinations or the varying levels

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[24]Earl O. Heady and Wilfred Chandler, _Linear Programming Methods_ (Ames, Iowa: Iowa State College Press, 1958), Chapters 3, 4, 11, and 12.

of gross returns. Their inclusion in the program would effect a
tendency toward increasing rather than constant returns to scale,
and the assumption of linearity would be invalid. Therefore,
they usually are not included in the program but may be deducted
from returns after an optimum solution has been reached.

The second assumption is that the activities must be additive--
that is, that the total value product of any number of activities
carried on simultaneously must be the sum of their individual value
products. Furthermore, the total amount of resources used by
several activities must be equal to the sum of the resources used
by each individual enterprise. No interaction is possible in the
amount of resources required per unit of output or in the amount
of product produced.

Divisibility is the third assumption. It requires that
factors can be used and commodities can be produced in quantities
which are fractional units. Resources and products are considered
to be continuous, or infinitely divisible. Slight departure from
this assumption does not cause serious decision error, and divisions
can usually be rounded to the nearest whole unit.

Fourth, it is assumed that there is a limit to the number of
alternative activities and to the resource restrictions which need
to be considered. If the number of alternatives available were
unlimited, the task of describing additional activities could not
be finished nor the optimum solution selected.

The merits of linear programming as a method of estimating
marginal values can be summarized as follows: (a) Precise problem

formulation is required. The objectives and restrictions must be expressed in equation form, assuring an understanding of the main components of the problem by the research worker. (b) The computational procedures are well defined and are easily used. Solutions by the simplex method can be reached with accuracy by following simple computational instructions. (c) Large quantities of data can be processed. The burden of clerical operations is minimized, and highly complex problems involving many activities and restrictions can be analyzed, and (d) marginal value products of each limiting resource in a problem are given directly in the solution. No additional computations are needed.

Comparison of Methods

An a priori comparison of linear programming and Cobb-Douglas production function analysis is now in order. No attempt will be made to decide whether one method is better than the other for the purpose at hand, but differences between them will be indicated.

Linear programming is principally a normative procedure which works to explain how phenomena ought to be. It prescribes resource organization and commodity combinations which will optimize a goal previously decided upon. It has predictive value, in that it is not tied to procedures as they are, but is free to propose solutions contingent upon how they ought to be. Linear programming does not provide physical production functions and can be used for estimating value productivity only when the input-output coefficients are already known. It requires constant returns to

resources and a linear relationship between factor inputs and products. Thus marginal returns to any one resource do not change as the use of the resource varies, unless quantities of other resources used are also permitted to change.

The computational facilities of linear programming permit the use of many inputs and products in the analysis, and problems of aggregation are minimized.

The Cobb-Douglas production function is a more positive method of estimating marginal values. It seeks to explain phenomenon as they exist rather than as they ought to be. The function need not be linear, thus allowing either increasing or decreasing returns to factors. It does, however, require a constant elasticity of production.

Productivity coeficients are estimated directly by multiple regression and a prior knowledge of the relevant input-output relationships is not needed.

Problems of aggregation of outputs and inputs are important in the Cobb-Douglas analysis. This is especially true if estimates are to be made when several commodities are considered from multiple enterprise farms.

Conclusions

The methods of marginal analysis described in this section have intuitive appeal as methods of estimating marginal values of irrigation water. Theoretically, they seem capable of yielding fruitful results if assumptions peculiar to each one are not

forgotten. This theoretical aptness might be concluded as a necessary condition for use of the models in estimating marginal values of water for irrigation. It is not sufficient evidence, however. Nor can much be said concerning the comparitive usefulness of the two models in relation to each other. The argument about which of several unrealistic assumptions is most realistic soon loses interest. More positive answers and more conclusive evidence must be sought through empirical procedures.

EMPIRICAL TEST

Two methods of marginal value estimation have been examined. The fruitfulness of the models has been tested in part by cursorily examining the assumptions pertinent to the models. In a larger sense, theory must not rely for its validity upon the reality of its assumptions. Indeed, complete "realism" is clearly unattainable, and whether a model is realistic enough rests with whether it yields predictions that are good enough for the purpose in hand; or, in lieu of this, that are better than predictions from alternative models.

If these abstract models of marginal value estimation are to be tested effectively, criteria for estimating the reality of their predictions are needed. Inviolable criteria are obviously lacking; for if the marginal values of irrigation water were known, a study concerning methods for estimating them would not be important. Moreover, if feasible methods of estimation other than the two suggested by this study were thought to yield more accurate results, they would have been investigated in lieu of the two which were chosen. Thus, a dilemma exists. The validity of the models rests with the reality of their predictions--but since the true marginal value is not known, how can the predictions be tested? The vindicable reply to that question is the objective of this section. An empirical test area will first be described and data collection techniques explained. The data for the area

will then be used in each of the two models, and the resulting
marginal value estimates will be presented. Finally, some
criteria for appraisal will be suggested and the predictions will
be scrutnized for validity.

The Test Area

The farming area of Milford, Utah was chosen as the empirical
test area. It is located in Beaver County in southwestern Utah.
It is bordered on the north and south by slight rolling hills, and
on the east and west with higher mountains. The valley floor is
about 5,000 feet in elevation, and is a rather flat plain which
slopes gently to the north. It contains approximately 9,000
acres of irrigated land.[25]

The climate of the Milford valley is semiarid with an average
rainfall of 8.44 inches. Water for irrigation is pumped from
wells. The average duration of the frost free period is from
May 3 to October 3 and winters are generally quite cold.[26]

There are several reasons why the Milford area was chosen as
a test site for this study.

Soil types and topography

Soils are quite homogeneous among farms in the Milford valley.
They are mostly of a sandy loam nature and do not differ significantly

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[25]Wayne D. Criddle, Karl Harris, and Lyman S. Willardson,
"Consumptive Use and Water Requirements for Utah" (Office of Utah
State Engineer, Technical Publication No. 8 (revised), 1962), p. 10.

[26]Ibid.

in erosion factors. The effects of weather upon crops do not vary
much among farms and consumptive use requirements of water for
various crops are uniform among farms.

Availability of hydrological data

All of the irrigation water used in the Milford pump area is
pumped from underground and is applied by surface flow methods.
This fact alone implies significant advantages over areas which
may use surface water for irrigation or may have various means of
applying irrigation water. Problems which involve the cooperative
use of canals, reservoirs, and ditches are largely avoided and the
institutions which aportion water and enforce rights are simplified.
Yearly and monthly fluctuations in water supply are also minimized.
Moreover, because of a gradually decreasing water table during
the past several years, explicit attention has been given the
Milford pump area by the office of the State Engineer. Precise
measurements of the flow of wells have been made and recorded and
pumping limits have been set. In June of 1960, the District
Court for Iron County, State of Utah, concluded the following, in
part

> that withdrawals of water from said underground water
> basin have substantially exceeded the recharge during each
> of the years for at least twelve years past. That the
> underground water level has thereby been substantially
> lowered . . . (5) That, with reasonable care and efficiency
> in the use of water, four acre feet per acre of land
> irrigated is adequate for production of crops ordinarily
> grown on average land in this area . . .

> Now, therefore, pursuant to the foregoing findings and
> conclusions it is ordered:

      1.   That during the year 1960 the use of water from the underground basin involved herein shall be limited to four acre feet of water per acre of land awarded a water right under the Proposed Determination herein.

    . . . 3.   That the State Engineer and the water commissioner appointed by him are charged with the duty of enforcing obedience to this order by shutting off wells or by instituting contempt proceedings against persons violating this order.[27]

This restriction of four acre feet per acre has been renewed each of the years following 1960, as was provided for by the original decree. In order to enforce the restriction as charged, the State Engineer and his appointed commissioner have kept accurate records of each of the wells over these years. In addition to this, studies have been made of the area to determine the effects of this water restriction and the adequecy of the four acre foot per acre limit.[28,29] Such careful attention to the supply of irrigation water in the Milford valley complements this study.

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[27]Lee Strong, "Annual Report of Water Distribution in Escalante Valley, Utah; Milford Area" (Office of Utah State Engineer, unpublished report, 1964).

[28]Antonio H. Giles Saez, "Economics of Allocating Limited Water Supplies Within the Farm With Special Reference to Escalante Valley, Utah" (Unpublished MS thesis, Utah State University Library, Logan, Utah, 1959).

[29]Duane R. Price, "Some Economic Effects of Irrigation Water Pumping From a Declining Water Table in the Milford Area of Utah" (Unpublished MS thesis, Utah State University Library, Logan, Utah, 1966).

Cropping patterns

Cropping patterns for farms in the study tend to be quite
uniform. Six field crops were produced during the summer of
1964: alfalfa, potatoes, wheat, corn for silage, barley, and oats.
All farms grew some alfalfa, and 73 percent of the cropped acreage
was planted to alfalfa. None of the remaining five crops accounted
for more than 10 percent of the total acreage. This extensive
production of alfalfa permitted the use of a single crop in the
Cobb-Douglas estimation model.

Data Collection Procedures

The same data are used for the analysis of both methods.
They come mainly from two studies conducted during the growing
season of 1964. Duane R. Price[30] was the principal enumerator
of a study conducted by the Agricultural Economics Department of
Utah State University. Cooperation was established with 26
farmers in the Milford pumping area. Schedules were given the
cooperators at the beginning of the summer and were filled out
during the cropping year. Periodic visits were made to each
cooperator to aid him in keeping the records up to date. The
schedules were designed to acquire cost and return information
needed for the preparation of budgets.

The other main source of data was from an irrigation efficiency
study conducted in the Milford pumping area by the Agricultural

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[30]Ibid.

Research Service.[31] Pumping efficiencies were calculated for approximately 140 wells in 1964. Water use efficiencies were estimated on a farm basis in the following manner. The acreage of each crop on each farm was measured from maps using a planimeter. The consumptive use requirement for each crop for that particular year was estimated and a total consumptive use requirement was found for each farm. The amount of water pumped on each farm was divided by the estimated consumptive use of the crops on that farm, then multiplied by 100 to calculate water use efficiency as a percentage.

Cobb-Douglas Model

Some of the most serious problems in production function analysis involve classification and accounting problems. It is here that judgment may have to be exercised, sometimes arbitrarily.

Output category

The output in the production function was measured in terms of gross returns per acre of alfalfa. This eliminates multiple enterprise accounting problems by reducing the number of enterprises to one crop. Data collected on other enterprises were used in linear programming analysis, but not in the Cobb-Douglas function. This accounting procedure also permitted a direct

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[31]Lyman S. Willardson, "Water Use Efficiencies, Well Efficiencies and Physical Data for Milford, Utah, Summer, 1964" (Unpublished report, Department of Irrigation Engineering, Agricultural Research Service, Utah State University, 1964).

estimation of marginal value, since the output was already in terms of value rather than some physical unit.

Gross returns to alfalfa result from the sale of the alfalfa produced, or its sale value if used on the farm. It is the product of the price received per ton of alfalfa and the quantity sold. There was some cross-sectional diversity in prices received by farmers for their hay. There was no indication that these price variations resulted from differences in quality of the hay. They probably represent differences in management effectiveness and do not reflect the productivity of the water resources in the producing year. Therefore, the average price received for hay during the study year was used as a single price in determining the gross returns per acre to each farm.

Input categories

One reasonable rule for grouping inputs into categories is to group good substitutes and good complements.[32] Following this suggestion, these input categories were defined in the study:

X_1 - Water applied per acre in acre inches. The greatest possible accuracy is needed in measuring the amount of water applied per acre because it is from this input that the marginal values of water must be estimated. This was done as follows:

$$Wa = \frac{CU}{WUE}$$

where Wa is water applied to alfalfa, CU is consumptive use

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[32]Heady, Johnson, and Hardin, p. 90.

requirements of water for a farm, and WUE is the water use efficiency for that particular farm. Reflection upon the origin of the water use efficiency for each farm implies that: Since water use efficiency is total consumptive use requirements for a farm divided by total water applied; and water applied to alfalfa is the consumptive use requirement for alfalfa on a farm divided by the water use efficiency for the farm; then water applied to alfalfa is to consumptive use of alfalfa as water applied to the total farm is to the consumptive use requirements for the total farm.

Symbolically,

$$\frac{Wa}{CUa} = \frac{Wt}{CUt} \quad , \quad \text{and } Wa = \frac{Wt \cdot CUa}{CUt}$$

where Wa is water applied to alfalfa, Wt is water applied to the total farm, CUa is consumptive use requirement for alfalfa, and CUt is the consumptive use requirements for the total farm.

Obviously, such a method assumes that water is used on alfalfa just as efficiently as it is on the entire farm. Since 73 percent of the crop acres in the study were planted to alfalfa, the consumptive use for the farm is largely a function of the consumptive use for alfalfa, and the assumption is strengthened.

The total water applied to each farm is known quite precisely from the records kept on all wells by the water commissioner, and the water applied to alfalfa can be accurately estimated with the proposed procedure.

X_2 - Material and energy costs per acre. This input includes the cost of materials used in the productive process and of energy (usually electricity) of pumping water from wells. A cost of energy per acre foot of water pumped was obtained for each well.[33] This was multiplied by the number of acre inches per acre of water used for alfalfa on the particular farm, yielding an energy cost per acre for alfalfa grown on each farm.

X_3 - Machine and irrigation equipment value per acre. This input category includes several individual factors. (a) Machinery value charged to alfalfa is the sum of the values of each piece of equipment used in the production of alfalfa. When the equipment was used in the production of other crops as well as alfalfa, the value attributed to alfalfa was based on the approximate fraction alfalfa use was of the total use of the particular implement. This was an estimation made by the farmer during a personal interview. This means of valuing machinery used on alfalfa land was not needed for much of the equipment used in the production of alfalfa. The primary investment in equipment for alfalfa is a swather (or a mower and side delivery rake) and a baler. These two items of machinery receive little use on any crop other than alfalfa.[34] Additionally, much of the plowing, tilling, leveling, and planting equipment used extensively in the production of other crops does not find frequent use in the pro-

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[33]Willardson.

[34]The baler may also be used to bale straw, and the swather to cut grain.

duction of alfalfa. Hauling equipment may be of more mutual use between crops, but even here versatile trucks and wagon and tractor combinations are not always used. Much of the hay is hauled and stacked with self propelled bale wagons operated by one man, which pick up, haul, and stack the hay automatically.

Because of the rather clear cut differences in the type of equipment used in producing alfalfa and in producing other crops, the farmer's estimate of the value of machinery used in alfalfa production should be comparatively accurate.

(b) Pump and well investment were also calculated for the production of alfalfa only. The total investment of the pump and well was prorated to various crops depending upon the number of acre feet of water pumped which was used in the production of each crop in 1964.

(c) Concrete ditches or pipelines for irrigation on parts of their farms were installed by 13 of the 26 cooperating farmers. The value attributed to alfalfa of these facilities was in the same proportion to their total value as water applied to alfalfa was to the total water applied to the farm. Since zero magnitudes of inputs cannot be used in Cobb-Douglas analysis, the farms which had no cement ditches were assumed to have an investment of 10 cents in lined ditches.

X_4 - Machine use and labor is the fourth variable input category. Both are measured in hours, and intuitively are complementary inputs. They include both farm owned and hired labor and machine useage.

X_5 - Length of life of the alfalfa stand was the final input variable. Life of the alfalfa stand was ascertained on individual farms. The average for all farms was 6.9 years, and the range was from 4 to 14 years.

Other combinations and variations of these variable inputs were also considered and tested in the model. Some gain in the R^2 value can be obtained when the inputs are not aggregated into the above categories, but are each considered as separate inputs. However, the simplicity of the model is sacrificed and the degrees of freedom are lessened. More important, problems of inter-correlation between independent variables become serious when these inputs are not aggregated.

Multiple correlation analysis

Multiple correlation techniques were used to estimate the parameters of the function and to test the significance of the independent variables in explaining the variation in gross returns per acre. Two of the five variables were found to have little explanatory power. Labor and machinery use in hours (X_4), and length of life of the alfalfa stand (X_5) did not add significantly to the over-all R^2 of the model. When these two variables were deleted from the model the multiple coefficient of determination decreased by less than one percentage point.

Other studies have failed to show significance between labor input per acre and yields per acre.[35] A lack of correlation

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[35]G. Tintner and O. H. Brownlee, "Production Functions Derived from Farm Records," Journal of Farm Economics, 26:55 (1944).

between the per acre use of machinery and yields per acre is no
less surprising, because machinery use in hours and labor used
are obviously so closely related. A priori reasoning suggests
explanations for this lack of correlation between hours of
machine and labor useage and yields per acre. Notably, the
capacity of a given machine used on a farm may determine in large
measure the number of hours needed to perform a given operation.
Since the capacity of machinery on different farms may vary
greatly, the number of hours needed to produce a crop may also
vary without having any effect on the per acre yields produced.

The variable concerning the life of the alfalfa stand was
originally included in the model in an attempt to measure variation
due to the quality of the alfalfa stand. The variable did not
prove significant in explaining yields per acre, however, and was
dropped from the model.

The results of the regression analysis on the remaining three
variables are summarized in Table 1, where X_1 is water applied
per acre, X_2 is material and energy costs per acre and X_3 is
machinery and irrigation equipment value per acre.

The mean square for the model is .2279 and the mean square
error (residual error) is .0248. This gives a calculated F value
of 9.177. With 3 and 22 degrees of freedom this value is significant
at the .01 level. The null hypothesis that all the partial re-
gression coefficients are equal to 0 is thus rejected and the over-
all model has a significant effect upon the dependent variable.

Calculated F values for each of the X_i values in Table 1 exceeds the tabular F at the .10 level. All three variables are therefore said to be significant at the .10 level of confidence. That is to say, the probability that the variables are significant because of pure chance is less than 10 percent; each of the three variables probably have a significant influence on gross returns per acre.

Table 1. Calculated and tabular F values, and standard partial, and multiple correlation coefficients for three independent variables

Independent variable	Calculated F value	(b) Partial correlation coefficients	(b') Standard partial coefficients
X_1	3.89	.38	.32 (2)[a]
X_2	4.23	.24	.36 (1)
X_3	4.00	.23	.31 (3)

Multiple correlation coefficient (R) = .75

Tabular F at .10 level and 1 and 22 D.F. = 2.95

[a]The numbers in parentheses are relative rankings.

The partial correlation coefficients (b values) indicate how gross returns vary with each of the independent X_i values. Since the X values are not all in the same unit of measure, the b's are not comparable unless put in standard form. The standard partial correlation coefficients (b' values) are the b values in standard deviations form. A comparison of the b' values indicates the

number of standard deviations by which estimated gross returns would vary if each of the X_i values considered separately were changed by one standard deviation. The relative influence of the independent variables can thus be observed in Table 1.

The fitted equation is $\hat{Y} = 1.32 \, X_1^{.38} X_2^{.24} X_3^{.23} \, e$; where

$\hat{Y}$ is estimated gross returns per acre,

X_1 is water applied per acre in acre inches,

X_2 is material and energy costs per acre,

X_3 is machinery and irrigation value per acre, and

e is the error due to the fact that the independent variables do not completely explain Y.

The correlation between the observed values of gross returns and the corresponding estimated gross returns (Y) is given by the multiple correlation coefficient, $R = .75$. The coefficient of multiple determination, $R^2 = .56$, indicates the percentage of the variation in the observed values that is explained by the fitted regression equation.

The simple correlation coefficients are given in Table 2. They show the relationship or intercorrelation between independent variables.

The degree of intercorrelation between X_1 and X_2 (.47) and between X_2 and X_3 (.40) raise questions of multicollinearity. Multicollinearity is in general terms the tendency of economic phenomenon to move together. It denotes excessive correlation between the independent variables which introduces indeterminacy of the function. It is of especial importance if something is to

Table 2. Simple correlation coefficients for three independent
          variables

	X_1	X_2	X_3
X_1	1.00	.47	.16
X_2		1.00	.40
X_3			1.00

be said about individual independent variables rather than merely
the over-all function. The marginal value products of this study
must be estimated from one variable under the assumption that
other inputs remain constant. This clearly is not feasible if the
inputs are tied together in causal relationships. If the correlation
among independent variables is high relative to the multiple cor-
relation coefficient of the model, multicollinearity is suspected.
The highest correlation between independent variables in this
analysis is .47 (Table 2), which is not high relative to the .75
multiple R value. It is of sufficient magnitude to imply possible
relationships between the independent variables, but excessive
correlation is not indicated.

The fitted function

The b values estimated in the correlation analysis are the
parameters of the Cobb-Douglas function. In natural logarithmic
form, the function is:

$$\log Y = 1.32 + .38 \log X_1 + .24 \log X_2 + .23 \log X_3$$

Marginal values of water can be found for any level of water input as the partial derivative of Y with respect to X_1.

Symbolically,

$$MVP = \frac{\partial Y}{\partial X_1} .$$

Since $MVP = \frac{\partial Y}{\partial X}$ and $\mathcal{E} p$ or $b = \frac{\partial Y}{\partial X} \cdot \frac{X}{Y}$,

MVP can be calculated as $b \frac{Y}{X}$

At the mean levels of water (59.26 acre inches) and gross returns ($104.97), the marginal value product of water is

$$.38 \ \frac{\$104.97}{59.26} = \$.68 \text{ per acre inch or } \$8.16 \text{ per acre foot.}$$

This marginal value can be interpreted as follows: Ceteris paribus, the addition of one acre inch of water at the mean level of application will increase gross returns by .68 cents. Marginal values for other water levels are given in Table 3.

Table 3. Total value products and marginal value products of water at various levels of water input, Milford area, Utah, 1964

Water level per acre		Total value product per acre	Marginal value	
acre inch	acre feet	dollars	acre inch	acre feet
12	1	57	$1.82	$21.79
24	2	75	1.20	14.35
36	3	87	.92	11.10
48	4	97	.77	9.24
60	5	106	.67	8.04
72	6	114	.61	7.87

Assuming the function to be continuous, the total value product and marginal value product of water can be graphed over the relevant range of water application as in Figure 5.

Linear Programming Model

All six of the crops grown in the Milford valley in 1964 were used in the linear programming model. Budgets of average costs and returns were established for each crop from the survey data collected. Prices, yields, and costs are per acre averages of all farms in the survey for the year 1964. The only exception to this is the price used for seed potatoes. Prices received for seed potatoes in 1964 were much higher than had been received during any other year. Therefore the price used in the potato budget was a 10 year weighted average for seed potato prices.[36] The prices received for all other crops in 1964 were comparable with the prices received during the previous five years and were used directly.[37]

The objective function

Input-output coefficients from the crop budgets were used in maximizing the following linear function:

$$Z_O = 74.80X_1 + 142.66X_2 + 44.31X_3 + 35.44X_4 + 25.48X_5 + 1.67X_6$$

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[36]Facts and Figures, Prices of Selected Crops, Utah, 1917-1964 (United States Department of Agriculture Statistical Reporting Service).
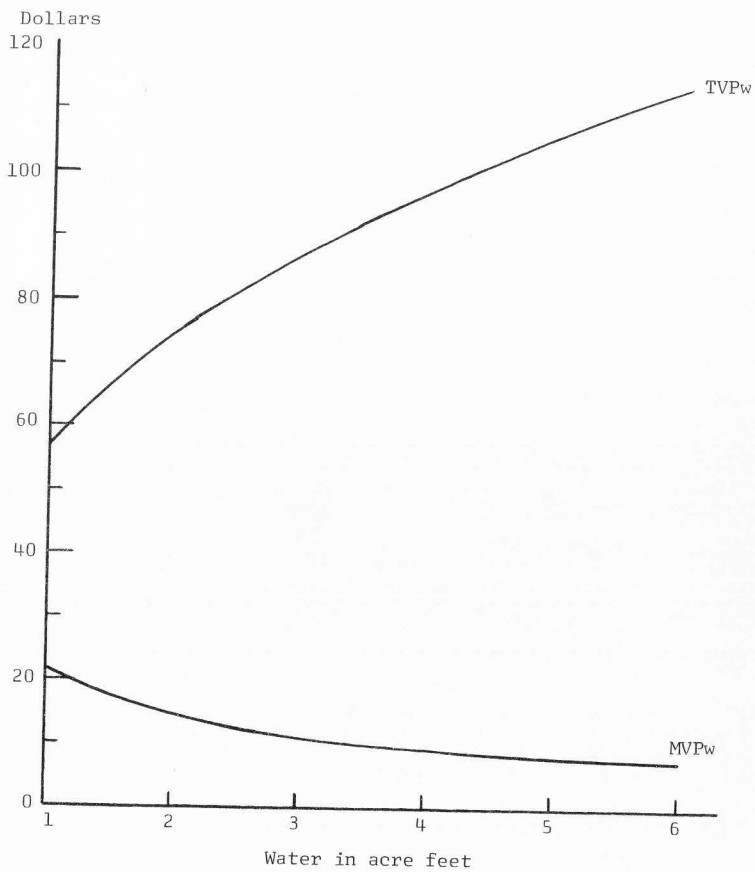
[37]Price, p. 28.

Figure 5. Cobb-Douglas production function and marginal value curve for irrigation water in the Milford area of Utah, 1964

where Z_o is returns to fixed factors,

X_1 is acres of alfalfa,

X_2 is acres of seed potatoes,

X_3 is acres of wheat,

X_4 is acres of corn silage,

X_5 is acres of barley, and

X_6 is acres of oats.

The X_i coefficients are the respective returns above variable costs received per acre from each crop.

The returns used in the calculations are returns to fixed factors. They are gross returns[38] less variable costs of power, materials, and interest on the money invested in the crop. Fixed costs such as interest on capital investment, building and machinery depreciation and repair, and taxes have not been deducted.

Resource requirements and restrictions equations

Resource requirements were taken from the cost and returns budgets. They represent the average quantities of resources used per acre in the production of each crop in the survey during 1964. The amount of total production is limited by the quantities of each resource available:

(land) $\quad 1X_1 + 1X_2 + 1X_3 + 1X_4 + 1X_5 + 1X_6 \leqq 160$ acre

_____

[38]Gross returns are the product of the prices received per unit of the commodity produced and the number of units produced per acre.

(capital) $9.82X_1 + 20.60X_2 + 17.47X_3 + 181.74X_4 + 15.74X_5 +$
$15.08X_6 \leqq \$4,000$ [39]

(water) $59.3X_1 + 49.8X_2 + 38.5X_3 + 40.6X_4 + 53.5X_5 + 55.4X_6 \leqq$
7680 ac. in. [40]

(labor) $7.79X_1 + 5.77X_2 + 3.39X_3 + 16.55X_4 + 21.7X_5 + 8.55X_6 \leqq$
1885 hrs.

where X_1 is acres of alfalfa,

X_2 is acres of potatoes,

X_3 is acres of wheat,

X_4 is acres of corn silage,

X_5 is acres of barley,

X_6 is acres of oats, and where the productive resources
are as follows:

Land. The X_i coefficients for land are all ones. This
merely indicates that it takes one acre of land to produce one
acre of any crop. The quantity of land available is assumed to
be 160 acres of irrigated land. This is near the 166 acre per
farm average as found in the survey.

Capital. Capital requirement coefficients for each crop are
taken from the cost and return budgets. They represent the amount
of capital needed to meet the variable expenses incurred in the

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[39]Three levels of capital were used in the equations: $3,000,
$4,000, and $5,000.

[40]Five water levels were used in the equations.   They ranged
from three to five acre feet per acre in increments of .5 acre feet.

production of each crop. The costs of power, fertilizer, wire, spray, seed, machine and labor hire, and interest on the money invested in the crop make up the capital requirements per acre.

Capital restrictions were set at $4,000 per farm. This was the amount of money assumed available to the farmer to cover his costs of production during the 1964 growing season. It may either be owned by the farm operator or borrowed by him.

Two additional levels of capital are also used in the analysis. Three thousand dollars and $5,000 capital availability provide results which show how returns change as the amount of capital available varies.

Water. Water requirements per acre for each crop were computed from consumptive use requirements in the manner already explained on page 41.

The water restriction of four acre feet per acre of land (7680 acre inches for the entire farm) is the limit set by the office of the state engineer. This limit was established in 1960 in an effort to stabilize a gradually declining water table.

Four other water levels (3.0, 3.5, 4.5, and 5.0) were also used in the analysis.

Labor. The hours of labor required to produce an acre of each crop was estimated from the cost and returns budgets.

The average supply of farm labor was assumed to be 1,885 hours for April through August. This consists of the operator supplying 250 hours per month and a 16 year old boy supplying 250 hours per month during the off-school months and 50 hours per month while attending school. Some hired labor is used by most operators, and was assumed to be available if needed.

Procedure and results

A solution to the object function and the resource restriction
equations may be found by solving the system of equations. This
was done through use of the simplex method. Four slack variables
were added to the equations, one for each resource used in production.
This provided for the non-use of any part of any of the resources
and converted the original equation to equality form. The require-
ment coefficients, resource restrictions, and returns per acre
for each crop were entered in a simplex table. An I.B.M. 1620
computer was used to find optimum solutions to the programming
problems. The results, using five levels of water and three
levels of capital are given in Table 4.

Table 4. Optimum combinations, marginal values of restricting
resources, and returns to fixed factors for varying
levels of water and capital

Capital levels	Crops, marginal values, and returns	Water levels in acre feet				
		3.0	3.5	4.0	4.5	5.0
$3,000	Percent alfalfa	56	65	77	87	95
	Percent potatoes	7	7	6	6	5
	MVP - water ($)	14.04	14.04	14.04	14.04	0
	MVP - capital ($)	.52	.52	.52	.52	.39
	MVP - land ($)	-	-	-	-	70.92
	Returns ($)	8,334	9,345	10,590	11,718	12,530
$4,000	Percent alfalfa	53	63	74	85	91
	Percent potatoes	11	10	10	9	9
	MVP - water ($)	14.04	14.04	14.04	14.04	0
	MVP - capital ($)	.52	.52	.52	.52	.39
	MVP - land ($)	-	-	-	-	70.92
	Returns ($)	8,857	9,868	11,113	12,240	12,926
$5,000	Percent alfalfa	51	60	72	82	88
	Percent potatoes	14	14	13	13	12
	MVP - water ($)	14.04	14.04	14.04	14.04	0
	MVP - capital ($)	.52	.52	.52	.52	.39
	MVP - land ($)	-	-	-	-	70.92
	Returns ($)	9,379	10,390	11,635	12,760	13,332

Only two crops, alfalfa and seed potatoes, were a part of any optimum plan. The acreages of crops were converted to percentage figures for presentation. The percent of the land to be planted to alfalfa ranges from 51 to 95 percent and the percent to be planted to potatoes ranges from 5 to 14 percent. Of special interest is the optimum combination of activities when capital is varied and water is held constant at four acre feet per acre. This water level is the actual level established by decree in the pump area. The optimum organization of activities for these conditions requires that from 72 to 77 percent of the land be planted to alfalfa and from 6 to 13 percent be planted to potatoes, depending upon which capital level is assumed. The actual cropping pattern in 1964 fell within those narrow ranges, as 73 percent of the cropped acreage was planted to alfalfa and 6 percent was planted to potatoes.

The linear programming model places all land other than that used in the production of alfalfa and potatoes in non-use activity. For four acre feet of water, this amounts to from 15 to 17 percent of the land. The remainder of the land in the actual survey cases for 1964 was not left unused but was divided among other crops as follows. Corn for silage 4 percent, wheat 6 percent, barley 10 percent, and oats 1 percent. Water for the production of these crops was obtained by using less than optimal amounts in the production of alfalfa and potatoes. Thus, the amount of land used in the production of alfalfa and potatoes in 1964 was very near that suggested by linear programming, but the use of the remaining land differed from the linear programming solution.

Three of the four resources were limiting at some combination of activities. (1) The resource which was most often a limiting resource was capital, which was restricting at all levels of water and capital. (2) Water was a restricting input at all combinations of resources except when five acre feet per acre were available. (3) Some land was in non-use for all combinations of resources except when five acre feet per acre of water were available. This water level was sufficiently high to bring all of the 160 acres of land into production. (4) Labor was in excess supply in all cases of resource availability.

The marginal value of water given by this linear programming model was constant whenever water was a limiting resource. It remained at $1.17 per acre inch ($14.04 per acre foot) for all water levels up to five acre feet per acre. When this level was reached, land replaced water as the limiting variable and excess water entered the water disposal activity. The rigidity of the marginal values of water at different levels resulted from the nature of the model. The requirements for each crop were established independently of the linear programming model. They did not change as resources were varied in the program. Therefore, any increase in total returns to fixed factors following an increase in the quantity of water available was not a consequence of increased productivity per unit; instead, it reflected an increase in the quantity of other inputs used. As more water was assumed available, additional units of land and labor were shifted from non-use or disposal activities to the production of real activities

and total product increased. The value of the increase must be attributed to labor and land as well as to water, because additional units of all three inputs were used.

The marginal value of capital was also constant for all resource combinations except when five acre feet of water were available per acre. At water levels between three and 4.5 acre feet per acre the marginal value of capital is $.52, and at five acre feet per acre its marginal value is $.39.

This optimizing model was inadequate for arriving at several different marginal value estimates as water levels vary. The marginal value does not change as long as only two crops are used in the optimum combinations. Potatoes, which require large amounts of capital, were planted in the greatest amounts possible given the capital restrictions. Alfalfa came in as the next most profitable crop, and any additions to water after that point merely allows for the production of additional acres of alfalfa. Since the amount of water required to produce an acre of alfalfa was constant, the marginal value of water did not change as water levels changed.

An altered model. Alterations in the restriction equations of the model allowed the estimation of several marginal values rather than only one as in the original optimizing plan. The procedure was to restrict the number of acres of each crop that could be planted. The survey results were used to determine the maximum number of acres of each crop that could be produced. That is, the percent of the total acres planted to each crop was that

found to exist in the Milford area in 1964. The percentages and
results of the model are shown in Table 5.

Table 5. Marginal values of water for six crops and for various
levels of water availability, the Milford area of Utah,
1964

Crop	Percent of cropland	Acres	Total water in acre inches, accumulative	Water per acre in acre feet, accumulative	MVP of water	
					acre inch	acre feet
Potatoes	6	9	365	.19	$3.51	$42.12
Alfalfa	73	117	7,303	3.80	1.26	15.12
Wheat	7	11	7,726	4.02	1.15	13.80
Corn	3	5	8,047	4.19	.66	7.92
Barley	10	16	8,844	4.61	.51	6.12
Oats	1	2	8,899	4.63	.03	.36

The marginal value products of water used in the production
of each of the six crops were given by this model. For example,
the total amount of water that could be used in the production of
potatoes was 365 acre inches. At this level of water usage, the
marginal value of water was $3.51 per acre inch ($42.12 per acre
foot). At this level of production, the acreage constraint on
potatoes restricted further use of irrigation water by potatoes,
and alfalfa entered the program as a user of the water resource.
The alfalfa maximum restriction permitted the use of additional
water, up to a total of 7,303 acre inches. At this level of
useage the marginal value of water used on the farm was $1.26 per
acre inch ($15.12 per acre foot). This process was continued until

each of the crops had entered the program to the maximum limit set by the acreage restrictions.

To make the marginal values obtained from the crop restriction model more obviously comparable to the results of the other models, water application was put in terms of water per acre in acre feet and entered in Table 5. This was done by dividing the total acre feet of water used by 160 acres, the total number of acres in the representative farm. Thus, for example, the 7,303 acre inches of water that can be used in producing potatoes and alfalfa represents 3.80 acre feet of water per acre for the entire 160 acres.

Empirical Conclusions

Linear programming techniques and Cobb-Douglas analysis have been used to estimate marginal values of irrigation water in the Milford area. No infallible criteria are known for measuring how realistic these marginal value estimates are. Lacking this, the following four indicators will be used as imperfect standards of measure.

Cobb-Douglass--linear programming comparison

Marginal value estimates from each of the models are generally near each other in magnitude in the relevent range of two to four acre feet of water (Table 6). This lends mutual support to the validity of the estimates from each model.

Table 6. Marginal value estimates of three models for various
          water levels, Milford area, Utah, 1964

Water level (acre feet)	Cobb-Douglas	Linear program original	Linear program altered
2	$14.35	$14.04	$15.12
3	11.10	14.04	15.12
4	9.24	14.04	13.80
5	8.24	0	6.12

Comparisons with water costs

Under conditions of perfect competition, farmers will seek to
operate where the marginal value of water is equal to its price.
An estimate of marginal value is thus provided if the price of the
factor is known. A market for water is not defined in the Milford
area, however. Some transfer of pumping rights does take place,
but this is often on a yearly trade basis and no market price is
established. The costs of obtaining water through pumping and of
applying it to the land can be interpreted as being a price for
water, and therefore an approximation of marginal value under
competitive and profit maximization conditions.

Price found that the cost per acre foot of obtaining and
applying water to farm land in the Milford area in 1964 was $4.26.
The estimates for marginal value (Table 6) found in this investigation
are greater than these estimated costs of irrigation, suggesting one
or more of the following phenomena.

(a)   The estimated marginal values may not be realistic.
Because of its assumptions of linearity, linear programming holds
the marginal value of water at one level for the production of any
particular crop if other resources are held constant. It stays at
that level until water is no longer a limiting resource, and at
that point marginal value of water is zero. This may bias the
marginal estimate upward.

The Cobb-Douglas function assumes constant elasticities of
production, and a maximum total product is not defined. This
effects an over estimation of the level of water input which
equates marginal value and marginal cost.[41]

(b)   Water costs may be invalid. These costs are averages for
all farms in the survey. They represent the annual operating
costs of pumping water, and are calculated directly from cost
records.[42]

(c)   Farmers may not be operating at points of profit maximi-
zation or the market for irrigation water may not approach a perfect-
ly competitive market. The institutional restriction of four acre
feet of water per acre retards the increase of water application
rates toward an optimum level. A farmer can apply more than four
acre feet of water per acre only by letting some of his land lie
idle or by borrowing or renting additional pumping rights from
other farmers or from his own water supply of the coming years.

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[41]Heady and Dillon, p. 76.

[42]Price, p. 31.

Comparisons with results of other investigations

Fullerton[43] found average rental prices of irrigation water
in the Delta area of Utah to be $3.21 per acre foot where inter-
company transfer was restricted and $9.60 per acre foot when inter-
company transfer was permitted. Under conditions of competition,
these values would approximate marginal values.

The $9.60 per acre foot value which Fullerton found to exist
is in general terms, near the values found by this investigation
for average levels of water application.

In a study of farm organization and resource allocation in
Piute County, Utah in 1961, Langford[44] found marginal values of
water to vary between $19.08 and $20.40 per acre foot when two
feet of water were available per acre. These values were obtained
through linear programming techniques.

Intuitive assessments

Water application levels in the study ranged from less than
two acre feet per acre to more than six acre feet per acre. How-
ever, 23 of the 26 farms surveyed had application rates between
three and five acre feet per acre. The marginal values estimated
by the methods used in this section seem intuitively reasonable
for the application range of from three to five acre feet per
acre. Values for application rates of less than two acre feet

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[43]Fullerton, p. 106.

[44]Langford, p. 54.

seem overestimated. It is not likely that any returns would be forthcoming if so little water were used. This observation does not apply to the linear programming estimates, since a constant amount of water per acre is applied to the various crops regardless of the amount of water available. The number of acres planted is greatly restricted by low water availability, however, and much land must be left idle.

The marginal values estimated by the Cobb-Douglas function at water levels greater than five acre feet are also probably over-estimated.

SUMMARY

If water is to be optimally allocated among its alternative
uses, it must be used in such a manner as to satisfy the general
allocative model of economic theory. Specifically, the quotient
$\left(\dfrac{\text{marginal value of water}}{\text{marginal cost of water}}\right)$ must be equal for all uses of water.
The theory of marginality is essential to this model. It is a
powerful tool in economic analysis. Considerable progress has
been made in methods of finding marginal values, and the paths of
these procedures can be traced through carefully written literature.
Problems in applying these methods to resources used in agriculture
still remain, however, especially where water used for irrigation
is concerned.

Many problems come about because of a lack of controlled
experimental data. Information must come from ex post decisions
made by farm operators who vary greatly in age, goals, preferences,
and management ability, and in the amounts and quality of resources
used in production. Reliable knowledge concerning yields per acre
and input-output coefficients are also difficult to obtain. In
addition to these problems of heterogeneity, it is also difficult
to specify the production process. The number of input-output
relationships are too numerous to work with and are not always
measurable; and aggregation of these may lead to meaningless
production function specification.

Another deterent to meaningful marginal value analysis for irrigation water results from the narrow range over which the predicting production function is relevant. This makes it difficult to establish a causal relationship between inputs and output.

Two methods of marginal value estimation which are often used in agriculture are the Cobb-Douglas production function and linear programming analysis. A survey of the analytical properties of these methods gives reasons for their favored use.

The Cobb-Douglas function has been the most popular algebraic form used in farm-firm production function analysis. It was originally developed from marginal productivity theory by Paul H. Douglas and Charles W. Cobb in 1928. As presently used by agricultural economists, the function permits diminishing marginal returns with a minimum usage of degrees of freedom. It is rather simply estimated through multiple regression techniques, and the estimated coefficients are the elasticities of production. The marginal product of the factors may be estimated at their means from the elasticities or regression coefficients, and the function is linear in its logarithmic form. The residuals are assumed to be normally distributed, which permits the use of the t distribution for testing the significance of the marginal productivities.

Linear programming is a form of activity analysis which was largely pioneered by Wassily W. Leontief in the 1920's. It facilitates precise problem formulation and its computational procedures are well defined and easily used. Large quantities of data can be processed, thus minimizing problems of aggregation.

The marginal value product of each limiting resource used in the production process are given directly in the solution.

Both linear programming and Cobb-Douglas analyses seem theoretically capable of yielding fruitful marginal value estimates for irrigation water. A more conclusive test of their validity can be made by applying the two models to an empirical study area and assessing the reality of the resultant estimates. Such an empirical test was conducted by this study in the Milford area of Utah. In 1964 cooperation was established with 26 farmers, and schedules were completed. The data thus obtained was used in both of the models studies.

The Cobb-Douglas function fitted to the data was in natural log form;

$$\log \hat{Y} = 1.32 + .38 \log X_1 + .24 \log X_2 + .23 \log X_3,$$

where $\hat{Y}$ is the estimated gross returns to alfalfa per acre,

X_1 is water applied per acre,

X_2 is material and energy costs, and

X_3 is machinery and irrigation value per acre.

All of the parameters were found to be significant at the .10 level, and the multiple correlation coefficient was .75. Marginal values of water were estimated as the partial derivative with respect to water. They were $11.10, $9.24, and $8.04, for 3, 4, and 5 acre feet of water respectively.

The linear programming model included as activities, all six of the crops grown in the Milford area. However, only two crops, namely alfalfa and potatoes, entered the optimum solution. Marginal

values of water were constant at $14.04 whenever water was a
limiting resource.

Alterations in the restriction equations of the model allowed
the estimation of several marginal values rather than only one as
in the original optimizing plan. The number of acres of each crop
which could be planted was restricted, thus requiring a marginal
value estimate for each of the six activities. These predicted
marginal values per acre foot of water were $15.12 for 3.80 acre
feet, and $13.80 for 4.02 acre feet.

Estimates of marginal values of irrigation water were thus
obtained through empirical application of the models. The remain-
ing task was to assess the reality of the estimates, and by that,
the validity of the models themselves. Lacking infallible criteria
for measuring how realistic these marginal value estimates are,
some imperfect indicators were used as standards of measure.

First, the estimates obtained by each method were near enough
to each other over the relevent range of water application to lend
mutual support to validity of the models.

Second, marginal values as estimated by both methods are near
the estimates of prices of water found by Fullerton in a nearby
study area.

Third, the marginal values estimated by both methods seem
intuitively reasonable for the water application range of from
three to five acre feet per acre. Twenty three of the 26 farms
surveyed fell within this range of application. There is good
reason to doubt the validity of the marginal value estimate for
water levels both higher and lower than this range.

CONCLUSIONS

The results of this study indicate that Cobb-Douglas production function analysis and linear programming methods are both conceptually capable of yielding estimates of marginal values of irrigation water.

The estimates do not share equivalent interpretations, however. The marginal value as estimated by the Cobb-Douglas function is forthcoming from an increment of water, with all other inputs held constant. Alternatively, the marginal value attributed to water through linear programming methods results from an increment of water and the additional use of other resources. Input-output relationships are constant, and a changing marginal product requires a change in the mix of inputs used in production.

Empirical tests of the methods resulted in reasonable estimates of marginal values of irrigation water. Inviolable criteria for testing the reality of these predictions are lacking, but imperfect standards of measure imply that they are sound.

It is therefore concluded that linear programming and Cobb-Douglas production function analysis can be used to yield meaningful estimates of marginal values of irrigation water.

A more positive assessment of the fruitfulness of the two models awaits the development of more precise criteria for measuring the reality of the predictions.

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VITA

Richard L Johnson

Candidate for the Degree of

Master of Science

Thesis: An Investigation of Methods for Estimating Marginal
           Values of Irrigation Water

Major Field: Agricultural Economics

Biographical Information:

    Personal Data: Born at Salina, Utah (hometown Aurora, Utah),
        February 16, 1940, son of Vernon R. and Golda Lindquist
        Johnson; married Nancy Ellen Rasmussen March 9, 1963.

    Education: Attended elementary school in Aurora, Utah;
        graduated from North Sevier High School in 1958; received
        the Bachelor of Science degree from Utah State University,
        with a major in Agricultural Economics, in 1965; received
        Master of Science degree from Utah State University in
        1967.