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An Application of Statistical Decision Theory to Farm Management in Sevier County, Utah

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AN APPLICATION OF STATISTICAL DECISION THEORY TO

FARM MANAGEMENT IN SEVIER COUNTY, UTAH

by

Suwaphot Lakawathana

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Suwaphot Lakawathana

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An Application of Statistical Decision Theory to

Farm Management in Sevier County, Utah

by

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The major purpose of this study is to present selected empirical results of a study employing decision-making theory as a framework for considering decision making under risk. The particular problem involves choices between alternative crop rotations for Sevier County farmers. The study demonstrates the usefulness of the Bayesian theory that gives more than a point estimation.

A multiple regression model using two linear terms was employed to determine the influence of snow pack and reservoir storage on water availability for irrigation purposes during July, August, and September.

The Bayesian approach was employed. The optimal action or decision was first determined where only the knowledge of the *a priori* probabilities of the states of nature was available. Optimal strategies were then determined where run-off observation was available and the *a posteriori* probabilities of the states of nature were determined.

Study results indicate that the expected value of the additional information is substantial and come out very close to the expected value of a perfect predictor and higher than the expected value of the "no data" problems. It means that the Bayesian approach gives more than a point estimation and is useful for farm management decision making under risk.

(89 pages)

INTRODUCTION

The importance of risk and uncertainty to decision making in agricultural production has long been recognized by agricultural economists. While important conceptual and empirical contributions to the understanding of risk and uncertainty have been made by many writers, this particular aspect of production economics has lacked any generally accepted unifying theory. The recent development of statistical decision theory perhaps comes closest to providing an acceptable theoretical framework for the study of decision making under uncertainty.

During the past few years, Bayes' theorem has been increasingly employed by agricultural economists to conduct research in utilization and development of resources. The Bayesian approach is useful when dealing with risk of agricultural production where probabilities can be assigned to the recurrence of a state of nature in the world. It is a method of systematically incorporating available information about the frequency distribution of these factors directly into the decision process. As Robert Schlaifer (27) mentions, the main idea of Bayes' approach is that probability is orderly opinion, and influence from data is nothing more than revision of probability in light of additional information.

This study provides an empirical application of Bayesian decision theory to farm management decision under risk. The empirical problem is the choice between alternative crop rotations. A major random variable affecting crop production is irrigation water supply that is dependent upon the snow pack and water stored in major reservoirs. The optimal action or decision is first determined where only the knowledge of the *a priori* probabilities of the state of nature is available. Optimal strategies are then determined where run-off observation is available and the *a posteriori* probabilities of the states of nature are determined. The value of the additional information provided by the run-off observation is substantial.

STATEMENT OF THESIS PROBLEM

Justification for the study

Decision-making in the realm of certainty poses no particular problems since each action has a single-valued or known outcome. However, decision problems under risk and uncertainty have several possible outcomes associated with each action. A set of decision rules, consistent with the farmer's objective (utility) functions, is needed to select the course of action that maximizes utility.

There are numerous variables that affect both the total acres cultivated by the farmer and the acres planted to specific crops. In the Mountain States where farming is dependent on irrigation, the following variables, in addition to farmers' habits, are important in this decision-making process:

1. Physical variability, such as water supplies from snow pack measurement, climate, insects, diseases, biological pests, and unpredicted freezes and scattered soil, which are the determinants of yield or technical variability.

2. Variability in product prices that depend upon (a) the fluctuations in national income and prosperity, (b) the recurring commodity cycles for farm products generated by discontinuous production cycles, and (c) random disturbances growing out of weather fluctuation.

3. Variability of input prices.

4. Need for enterprise combination in crop rotations.

In general, farmers attempt to predict or consider all of the preceding variables that may influence farm income, but each farmer's habits also have a direct influence on his decision. However, crop price trends and relative crop price trends tend to be somewhat stable or more easily predicted than forecasts of water supplies and crop risk.

In the Sevier County area, water for irrigation is dependent primarily on snow pack deposited in surrounding mountains during the winter and on the amount of water carried over in the reservoirs (Otter Creek and Piute Reservoirs) from previous years. Springs, streams and rivers carry melting snow to irrigated areas. The principal run-off occurs during the spring and early summer. Stream flow during July, August and September is more stable but at much lower levels. Therefore, not only does stream flow fluctuate from year to year because of variation in the snow pack, but also from month to month as a result of the melting process. The latter problem is perhaps eliminated for some farmers where adequate storage facilities from two major reservoirs are available. The annual fluctuations, however, can only be dealt with by providing long-term carryover storage from year to year, but the storage is not enough to eliminate this problem. Thus, farmers are required to make decisions concerning crop rotations, acres planted and other input needs based on an uncertain supply of water which fluctuates from year to year and also during the irrigation season. Any farmer, then, must choose among several alternative farm enterprise combinations best suited to the anticipated water supply for a given year.

The hypothesis of this thesis is that, due to these risks, a farmer should base this decision on a Bayesian approach where the method of incorporating additional information is provided and can be substantial, rather than select crops by a "no data" method.

Objectives of study

1. To demonstrate the usefulness of the Bayesian theory that gives more than a point estimation. It measures the magnitude of the difference between alternative actions and provides a variety of estimates for consideration.

2. To present selected empirical results of a study employing decision-making theory as a framework for considering decision making under uncertainty.

3. To evaluate the question of what is the optimum crop rotation as an isolated annual decision.

Method of study

To estimate the influence of snow pack and reservoir storage on water availability for irrigation purposes during July, August and September, the period of frequent shortage, a multiple regression (8) model was used. The mathematical model is of the form:

$$Y = b_o + b_1X_1 + b_2X_2 + e$$

where:

Y = available irrigation water during July, August and September in acre-feet.

X_1 = water content of snow pack as measured jointly on April 1 of each year by USDA-SCS and State Engineer of Utah on the watershed of Upper Sevier River (south of Richfield, Utah) (1937-1968).

X_2 = acre-feet of water stored in Otter Creek and Piute Reservoirs as measured by U.S. Geological Survey on April 1 of each year (1937-1968).

b_i = coefficient to be determined.

e = error on amount of deviation in the estimated $\hat{Y}$ from the true Y.

Using the above model, the multiple correlation coefficient (R), a measurement of the degree of correlation between run-off and all the factors included in the regression equation, is 0.909 (R^2 = 0.826). This means the variables tended to move together; $\hat{Y}$ is very close to Y and the fitted model is a good predictor of stream flow; but the fluctuation around the mean of annual snow pack and run-off during July, August and September was very large in many years (Appendixes A-C).

The problem is further exemplified by observing the average stream flows by month for the 32-year period and the consumptive use rate of the major irrigated crops using the methods of Criddle, Harris, and Willardson (5) (Appendixes D-F).

A number of decision rules (for example, maximum gain, minimum regret, the principle of insufficient reason and the pessimism-optimism index) have been suggested for the cases where the probability distribution of the state of nature is unknown. All of these criteria have severe defects, as shown by Luce and Raiffa (19). Furthermore, it is difficult to conceive of decision problems in which the decision-maker has no information, either objective or subjective, regarding the probabilities of the states of nature (θ_j). Thus, recent emphasis in decision theory has shifted toward the so-called Bayesian strategies which employ relevant probability distributions.

Bayesian analysis is concerned with the basic problem of assessing some underlying "state of nature" that is in some way uncertain. The Bayesian decision model provides a framework for developing decision criteria for problems characterizied by uncertain outcomes. The model incorporates the available objective and/or subjective information into a decision process to select the optimum action.

Given: (1) a set of A_i possible actions (crop rotations, A_1, $A_2, \ldots A_i$); (2) the set θ_j of alternative states of nature (actual stream flows during July, August and September, θ_1, $\theta_2, \ldots \theta_j$, the values of one or more exogenous factors that directly affect the outcome of a particular action but cannot be controlled with certainty by the decision-maker); (3) the utility index, U_{ij}, associated with the selection of A_i and occurrence of θ_j; (4) each outcome (λ_{ij}, loss or gain from each crop rotation and this matrix formulation of decision problem is obtained by replacing λ_{ij} with U_{ij} in Table 1); (5) vector of the <u>a priori</u> information about the relative frequency of actual stream flows in Sevier River (θ_j), called a probability distribution,

$$P(\theta_j) = \begin{bmatrix} P(\theta_1) \\ P(\theta_2) \\ \cdot \\ \cdot \\ \cdot \\ P(\theta_j) \end{bmatrix}$$

Table 1. Matrix representation of outcome plan

Action (A_i)	State of nature (θ_j)				
	θ_1	θ_2	θ_3	..	θ_j
A_1	λ_{11}	λ_{12}	.	.	λ_{ij}
A_2	λ_{21}	λ_{22}	.	.	λ_{2j}
A_3	λ_{31}	λ_{32}	.	.	λ_{3j}
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
A_1	λ_{i1}	λ_{i2}	.	.	λ_{ij}

Then the action can be selected, (A_i (crop rotations A_i)), for which expected utility, $\hat{U}_i = \Sigma_j U_{ij} P(\theta_j)$, is a maximum.

Where $P(\theta_j)$ is the a priori probability that states of nature (θ_j) will occur, this becomes the information that the decision maker has about the relative frequency of state of nature, θ_j (actual stream flows during July, August and September) upon which to make decisions. This information is expressed in the form of a probability distribution, $P(\theta_j)$, that provides some distribution of the likelihood of a particular value of states of nature, (θ_j), occurring ($\Sigma P(\theta_j) = 1$). It is derived from the histrogram showing the relative frequencies of stream flows of Sevier River in the past 32 years (1937-68).

$\hat{U}_i$ is the expected value of the utility that results from taking action, A_i, in the states of nature, θ_j, and is equal to the summation of U_{ij} (replaced λ_{ij}) multiplied by $P(\theta_j)$.

U_{ij} is the utility function ($U_{ij} = U(\lambda_{ij})$, and is assumed to be linear.

λ_{ij} is each outcome (net farm income) and is represented as a point in an action (crop rotations, A_i) - state of nature, (θ_j), $\lambda_{ij} = (A_i, \theta_j)$ as shown in Table 1.

In addition to the prior knowledge of the probability distribution, $P(\theta_j)$, it may be possible for the decision maker to gain additional information about the likelihood of a particular state of nature, θ_j, by making an observation, Z_k ($K = 1, 2, \ldots K$), on the water content of snow pack (X_1) and acre-feet of water in Otter Creek and Piute Reservoirs (X_2) as measured on April 1 each year. The results of the observation (Z_k) will serve as a predictor of the states of nature, θ_j (run-off in Sevier River). That is, the decision maker can construct a conditional probability distribution, $P(Z_k/\theta_j)$, which incorporates the a priori information, $P(\theta_j)$, with information about the past performance of Z_k as a predictor of run-off in Sevier River, θ_j.

The a posteriori probability distribution, $P(\theta_j/Z_k)$, is calculated using Bayes' formula, shown in Table 2.

$$P(\theta_j/Z_k) = \frac{P(\theta_j)\ P(Z_k/\theta_j)}{P(Z_k)}$$

where:

$P(Z_k/\theta_j)$ = conditional probability of Z_k observations on water content of snow pack (X_1) and acre-feet of water in

Table 2. Derivation of posteriori probabilities, $P(\theta_j/Z_k)$

States of nature	Conditional probabilities $P(Z/\theta)$				A priori probabilities	Joint probabilities $P(\theta)$ $P(Z/\theta)$			
	Observations (Z_k)					Observations (Z_k)			
(θ_j)	Z_1	Z_2		Z_k	$P(\theta_j)$	Z_1	Z_2		Z_k
θ_1	$P(Z_1/\theta_1)$	$P(Z_2/\theta_1)$	...	$P(Z_k/\theta_1)$	$P(\theta_1)$	$P(\theta_1)P(Z_1/\theta_1)$	$P(\theta_1)P(Z_2/\theta_1)$	...	$P(\theta_1)P(Z_k/\theta 1)$
θ_2	$P(Z_1/\theta_2)$	$P(Z_2/\theta_2)$	...	$P(Z_k/\theta_2)$	$P(\theta_2)$	$P(\theta_2)P(Z_1/\theta_2)$	$P(\theta_2)P(Z_2/\theta_2)$	...	$P(\theta_2)P(Z_k/\theta_2)$
.	.	.	...	.	.	.	.	...	.
.	.	.	...	.	.	.	.	...	.
.	.	.	...	.	.	.	.	...	.
θ_j	$P(Z_1/\theta_j)$	$P(Z_2/\theta_j)$	...	$P(Z_k/\theta_n)$	$P(\theta_j)$	$P(\theta_j)P(Z_1/\theta_j)$	$P(\theta_j)P(Z_2/\theta_j)$	...	$P(\theta_j)P(Z_k/\theta_j)$
					$P(Z_k)$	$\Sigma P(\theta_j)P(Z_1/\theta_j)$	$\Sigma P(\theta_j)P(Z_2/\theta_j)$		$P(\theta_j)P(Z_k/\theta_j)$

States of nature	Posteriori probabilities $P(\theta_j/Z_k) = \frac{P(\theta_j)P(Z_k/\theta_j)}{P(Z_k)}$			
	Observations (Z_k)			
(θ_j)				
θ_1	$P(\theta_1/Z_1)$	$P(\theta_1/Z_2)$	. . . .	$P(\theta_1/Z_k)$
θ_2	$P(\theta_1/Z_1)$	$P(\theta_2/Z_2)$	. . . .	$P(\theta_2/Z_k)$
.	.	.	. . . .	.
.	.	.	. . . .	.
.	.	.	. . . .	.
θ_j	$P(\theta_j/Z_1)$	$P(\theta_j/Z_2)$	. . . .	$P(\theta_j/Z_k)$

Otter Creek and Piute Reservoirs (X_2) on April 1 each year when, in fact, there will be θ_j,

$P(Z_k)$ = the probability of observing a particular observation result, $P(Z_k) = \Sigma P(\theta_j)\ P(Z_k/\theta_j)$, and

$P(\theta_j)\ P(Z_k/\theta_j)$ = the joint probability of the two distributions that resulted from the probability of state of nature in a given year multiplied by the probability of observation when giving the state of nature.

The observation information expands our knowledge about the likelihood of θ_j from the $P(\theta_j)$ vector to a (jxk) matrix of conditional probabilities in the lower right-hand of Table 2. $P(\theta_j/Z_k)$ is the probability of θ_j occurring, given Z_k as the observation result (prediction of θ_j). If the observation Z_k is a perfect predictor of θ_j, the lower right-hand of Table 2 will consist of ones along the diagonal and zeros elsewhere.

The optimal Bayesian strategy is generally defined as one which maximizes expected utility. If the utility function is linear over the relevant range, maximizing expected profit is equivalent to maximizing expected utility. With data provided by the observation, the Bayesian strategy becomes: Given a projection of θ_j (for example, Z_k), select the action, A_i, for which the expected utility

$$\hat{U}_i^k = \Sigma_j U_{ij} P(\theta_j/Z_k)$$

is a maximum. Thus the Bayesian strategy consists of a set of optimal actions--at least one for each observation result.

Decision problems which involve the use of prior probabilities are often called "no data" problems, and those involving <u>a posteriori</u>

probabilities are called "data" problems. The increase in expected income which results from using data rather than no-data probabilities is variously called "value of the data," "value of added information," or "value of the observation" (4).

$$V = \Sigma_k \hat{U}_i^k P(Z_k) - \hat{U}_i$$

where:

V = value of the data that can be compared with the cost of making the observation to evaluate the net contribution of the observation information to expected income

$\hat{U}_i^k P(Z_k)$ = the expected value of the data strategy and is calculated by multiplying the expected value of optimum action for each observation result by the probability of observing the appropriate observation result, $P(Z_k)$, and summing over all possible results:

$$\left[\Sigma_k \left\{\Sigma_j \; U_{ij} P\,(\Theta_j/Z_k)\right\} P(Z_k)\right]$$

U_i^k = the expected value of the utility that results from taking action, A_i, in the status of nature, Θ_j, is equal to the summation of U_{ij} multiplied by $P(\Theta_j/Z_k)$

The Bayesian decision model presented above provides a framework for developing decision criteria for problems characterized by uncertain outcomes and appears to be useful in farm management. The model incorporates the available objective and/or subjective information into the decision process. Data requirements are modest; *a priori* information is available from past stream flow records. Additional information is obtained from observations on the snow pack, water storage in the reservoirs, and the Bayesian theory which decrease the uncertainty. As the process is repeated each year, the input-output will be improved.

CHARACTERISTICS OF THE STUDY AREA

Sevier County is located in south central Utah. The study area extends from the town of Sevier on the south to Sigurd on the north. The major cities in the area are Richfield, Monroe and Sevier. The area contains about 644,200 acres. Approximately 44,360 acres of land are irrigated within the area (34).

The principal crops grown in the cultivated and irrigated lands are alfalfa, permanent pasture, meadow hay, barley, corn silage, rotation pasture, wheat and sugar beets. The proportions of crops are alfalfa 53.0 percent, permanent pasture 18.7 percent, meadow hay 1.2 percent, barley 17.1 percent, corn silage 5.0 percent, rotation pasture 1.0 percent, wheat 1.1 percent and sugar beets 2.9 percent (34).

The valley floor formed by the flood plain of the Sevier River is very flat laterally with land sloping from both sides of the valley to the Sevier River which runs from south to the north along the floor of the valley. The Sevier River, which drains the valley, rises in the high plateaus of Southern Utah above an altitude of 10,000 feet and flows northward through the trough of the Sevier Valley for about 175 miles before turning westward into the Sevier Desert. The river is fed along its course by numerous tributaries which drain into it from the surrounding mountains and plateaus (36).

The climate in the Sevier County ranges from semi-arid on the valley floor to humid on the mountains and plateaus bordering the valley. Average annual precipitation ranges from less than 10 inches on the valley floor to 30 inches or more at the higher altitudes. Because of sparse precipitation on the valley floor, most crop production is dependent upon irrigation (36).

Soils are relatively homogeneous and generally range from medium to moderately fine in texture. Soils of any one texture tend to be located in blocks and soils on individual farms are usually of one type (31).

Irrigation water comes from the Sevier River, tributary streams, springs, and storage in Otter Creek and Piute Reservoirs. The average annual water resource of the area has been estimated to be 148,160 acre-feet of which 104,970 acre-feet are consumptively used by irrigated crops and 26,230 acre-feet consumptively used on non-irrigated meadows and saltgrass area (34). Irrigation efficiency is 40 percent in the area (6). Irrigation water supplies are short during the months of July, August and September.

The average size of farms in the area was 246 acres in 1962. Crop and forage were harvested from 87 percent of the acreage while 13 percent of the acreage was idle. Farmers owned 61.7 percent of the land they operated and rented the remaining 38.3 percent (34).

REVIEW OF LITERATURE

Prior to this study, no results have been published of attempts to determine optimum enterprise by using the Bayesian approach for Sevier County farmers.

Bayesian statistics, other than the initial contribution of Bayes in 1762, were begun in 1955 with the publication of Probability and Statistics for Business Decisions by Robert Schlaifer (27). This book introduced the key ideas of Bayesian statistics, namely, that probability is orderly opinion, and that inference from data is nothing more than the revision of such opinion in the light of relevant new information.

Herman Chernoff and Lincoln E. Moses (4) present the general decision-making formulation which somewhat parallels Howard Raiffa and Robert Schlaifer (25) for a basic problem.

In the Chernoff, Moses, and Raiffa formulations of the decision-making problem, the state of nature is unknown and partial insight into this unknown can be obtained from gathering data. The data requirement is a probability distribution of the states of nature, $P(\theta)$. This is referred to as the *a priori* distribution which is either known or assigned before choice of an experiment is made. The second requirement is for the *a posteriori* distribution $P(\theta/Z,e)$ which is the conditional probability of a given observation, Z, occurring when given a certain θ and a particular experiment, e. The *a priori* and *a posteriori* distributions are convertible into the two distributions used in selecting a terminal action by the Bayesian theorem. The joint distribution of Θ and Z is given by $P(\Theta \text{ and } Z) = P(\Theta)P(Z/\Theta,e)$. The conditional distribution $P(\Theta/Z,e)$ is given by $\frac{P(\Theta)P(Z/\Theta,e)}{P(Z/e)}$ where $P(Z/e)$ is the marginal distribution of Z given the particular experiment selected. Then, the minimizing loss or maximizing gain can be selected from the losses or gain matrix multiplied by a column of the $P(\Theta/Z,e)$.

McConnen (21) considered a problem of stocking rates by the Bayesian theory which was determined by the five levels of range productivity in terms of animal unit days. There are three actions: A_1 = heavy stocking, A_2 = medium stocking and A_3 = light stocking. He presented a table of gross range profits for each action given a particular state of nature. He then presented a table of the a posteriori probability distributions $P\left\{\Theta/Z,e\right\}$ (the probabilities from the result of the experiment, e_i, as in the Raiffa formulation, $P\left\{\Theta/Z,e\right\}$) rather than the frequency response table of $P(Z/\Theta,e)$. McConnen's procedure assumes that he knows the "best" a priori distribution (the probabilities of the level of range productivity in terms of animal days) which he obtained from the results of the experiment at the U.S.D.A. Range Livestock Experiment Station at Miles City, Montana, from 1933 to 1959. His states of nature (Θ) were specified as the level of range productivity, and Z is the observation on the different rates of precipitation.

In the same year that McConnen was using Bayes' approach to determine the stocking rates, T. A. Walther used statistical decision theory applied to western range problems and ranch management. He clarifies some of the concepts that have prevailed in trying to apply decision-making theory and points out that the use of a choice criterion such as minimax makes sense only if one thinks nature is trying to do the worst she can. The minimax regret criterion defines strategy with the minimum-maximum regret as the "best." At any rate, the various possible criteria for selecting alternatives do not quite fit the situation. According to T. A. Walther (29), the crux of the problem is that these would fit in a war game situation or perhaps for rival store owners in a community where the opponent is intelligent and realizes that his gain is the other's loss and acts accordingly. However, to say that nature realizes that her gain is the decision maker's loss is going somewhat far afield and means that this type of model is not readily applicable to most range management decisions. Then he shows how solutions can be obtained from statistical decision models which can utilize any relevant information which is available to the decision maker.

Gerald W. Dean (7) employed the Bayesian theorem to evaluate the alternative stocking rates of cattle ranches in the foothill range area of Northern California, where stocker cattle are purchased in fall or early winter and sold in late spring or early summer. He used two sources of uncertainty--the range feed supply and cattle prices. He succeeded in obtaining reasonable appearing estimates of the _a priori_ and _a posteriori_ probabilities of various range conditions. From the calculated _a posteriori_ probabilities and pay-off matrix for stocking rates under various conditions, he was able to obtain expected net returns for alternative actions, given the observed January 1 range condition. His treatment of the other major aspect of uncertainty affecting ranchers (cattle prices) seemed somewhat less impressive than his treatment of stocking rates since his price prediction model does not utilize physical or economic variables such as cattle numbers.

Vernon R. Eidman, Gerald W. Dean and Harold O. Carter (9) used the Bayesian theory to solve the particular problems involving choices between contract producer and independent producer for California turkey producers under the uncertainty of prices and mortality. The study demonstrates that several well-known quantitative tools used previously in dealing with risk and uncertainty probability distributions, prices, forecasting equations, and simulation are used in developing the components of the decision problem.

Harold H. Hiskey and Darwin B. Nielsen (18) employed the Bayesian theory to select the optimum rotation crop for the farmers in Cache County as an isolated annual decision under the risk of run-off in the Logan River, where run-off is dependent upon snow pack in the surrounding area. This is referred to as the state of nature and can be described as an a priori probability distribution. Then they constructed a conditional probability distribution by making the observation of snow pack as measured on April first each year. They showed the value of the application of Bayes' approach as a tool for farm management. Although the Bayesian approach is no panacea, we should be alert and profit from this work. By more comprehensive study the very nature of agriculture could change; therefore, it is important to recognize this tool.

STUDY ASSUMPTIONS AND ANALYTICAL FRAMEWORK

The purpose of this section is to give the assumptions, reasoning and the relationship between crop production and resource use made in this study.

Production function

A production function is the technical relationship telling the maximum amount of output that can be produced by each combination of specified factors of production. It simply means the relationship between the physical inputs and the physical outputs of a firm. The term input-output relationship is also used at times by economists as a counterpart of the production function (10).

A production function for a crop tells the relationship between all inputs and the resulting yield of the crop. The production of a crop is the result of many factors such as land, seed, water, labor, fertilizers, machinery and management. A crop can never be produced by a single factor alone. The variation in the crop production due to one variable input can be determined if all the inputs required for the production of a crop are held constant, except one variable input. This procedure is commonly used by physical scientists and economists when determining the short-run production function.

Short-run production function

The theoretical short-run production functions for a crop and irrigation water are shown in Figure 1. The curve Y_p shows the yield of crop on an acre of land with varying quantities of irrigation water. The increment of crop production with addition of more units of water is represented by curve M_p, computed by dividing the addition to total product by the corresponding addition to total input. It is the "average" marginal product of additional input, rather than the marginal product of each lost unit of input. The curve A_p shows the average yield per unit of irrigation water that can be computed by dividing each total product by the corresponding total unit of irrigation water input.

There are three stages of production function. The area of rational use of inputs is in stage 2. The optimum point within the stage can be determined only after prices of inputs and outputs are known. Any level of resource use falling in stages 1 and 3 is irrational. Stage 1 is uneconomical because the use of one additional unit of the variable input will increase the average return for all inputs, and a reduction of the fixed inputs while the variable inputs are maintained constant will increase the total production. Stage 3 is uneconomical because the use of one additional unit of water input will decrease total production. So, stage 2 is the only stage where the marginal productivity of both variable and fixed inputs is greater than zero. In other words, if production is in stage 1 or 3, total product can be increased by decreasing either the variable input or the fixed input until stage 2 is reached; this is why they are called the stage of irrational production.

Given prices of inputs and output, problems of efficiency and

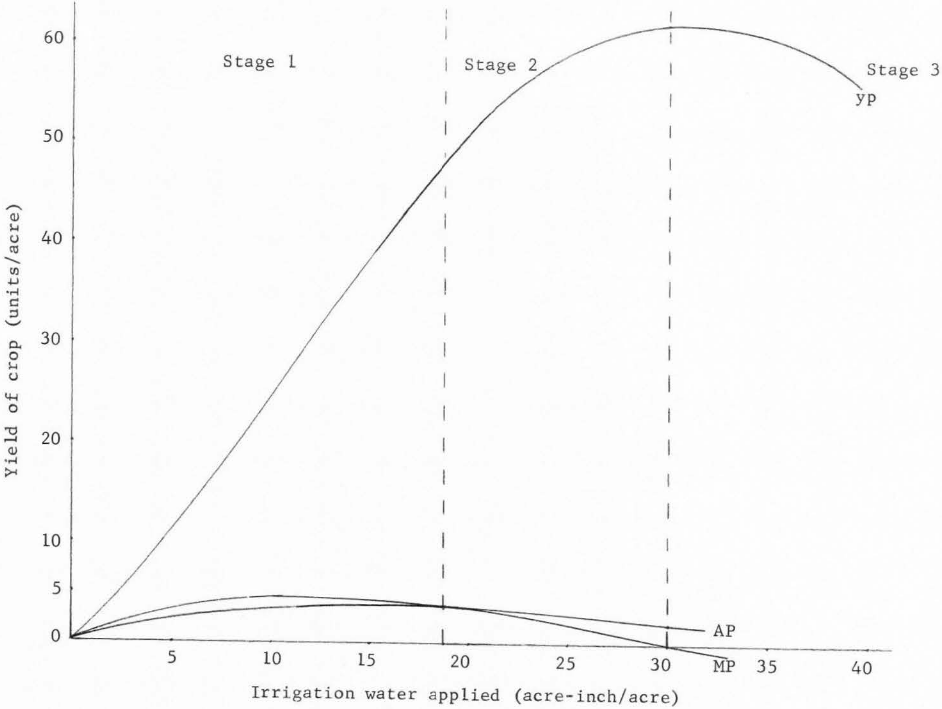

Figure 1. Illustrative short-run physical production function showing the relationship between crop yield and irrigation water applied.

allocation of resources can be solved. An input is used efficiently if the marginal cost of the input is equal with the marginal value product of the input. At this point, another unit of water would cost more than the additional income. If a resource such as water is limited, however, a farmer cannot maximize profits for each use. His problem, then, is to allocate the available water among alternative uses so as to maximize total profits. He must allocate this inadequate supply of water among alternative crops. An alternative is to leave some land idle and water a smaller acreage more heavily. The efficient allocation of water for several crops can be determined by equating the marginal value product of water on all crops.

Many production processes do not conform to the smooth curves shown in Figure 1. In production of livestock and livestock products, for instance, production is actually not achieved at all until a substantial amount of resource is utilized. Some feed is necessary for body maintenance before production occurs. In crop production, also, a substantial amount of irrigation water may be necessary before any production occurs.

Most of the forage crops have a linear relationship between water input and yield in the relevant portion of their production function (18) where harvest is periodic or continuous, since alfalfa growth continues as long as soil moisture is available to the plant in sufficient amounts. When water is no longer available or is available in less than biological optimum amounts, production is stopped or retarded. Thus, even though the rate of growth is influenced by many factors, the production function of forage crops tends to approach a linear relationship. Other crops which are usually harvested only at maturity do not have a linear production function. Regardless of water applied during the early part of the growing season, discontinuance later results in little or no production. This means the last one or two irrigations may add more production to the total product than all previous irrigations, because without the late irrigation water, the crop would not mature.

We assume, therefore, in this study that whenever late season irrigation water shortages exist, available water supplies will be allocated to mature the row crops where the marginal value product of water is higher. Forage crops with lower marginal value products will be the first to be shorted unless there is some minimal level of forage crop production necessary to support a livestock enterprise.

Crop rotations

Three representative farms have been studied: range beef farms, feeder farms and small dairy farms. It is assumed that these farms are located such that their irrigation water is obtained from the Sevier River. In addition to crops which support the main enterprise, some cash crops are grown to supplement farm income. A normal rotation is usually followed but can be altered by varying the acreage planted to small grains, cash crops and alfalfa.

Cropping pattern and farm types

Two significant adjustments that farmers may make within a given water year are changes in cropping pattern and livestock numbers. For example, farmers with a poor water supply cannot successfully grow corn for silage. Restrictions of range land and markets for grade A milk and feed crops limit the adjustments that can be made to more livestock enterprises and cash crop farms. Only cropping pattern has been studied in this thesis. The following rotations, as shown in Tables 3, 4 and 5, are considered as being feasible under differing circumstances and are in general practice for each type of farm where applied in Sevier County area. When a greater amount of late season water is expected, a farmer could plow up more acreage of alfalfa than usual and grow more corn silage because corn silage produces more feed nutrients per acre than alfalfa; or he could reduce acreage of alfalfa and increase acreage of sugar beets if he expects higher level of water. On the contrary, the farmer will retain more acreage of alfalfa in the field and grow

Table 3. Percentage of rotation in range beef farms in Sevier County, Utah, 1968

	Percentage of cropland			
	Rotation			
Crop	A_{B1}	A_{B2}	A_{B3}	A_{B4}
Alfalfa (short rotation)	58	--	52	--
Alfalfa (long rotation)	--	66	--	61
Permanent pasture	18	18	18	18
Barley	12	8	10	7
Barley (nurse crop)	12	8	10	7
Corn silage	--	--	10	7
Total	100	100	100	100

Table 4. Percentage of rotation in feeder farm in Sevier County, Utah, 1968

	Percentage of cropland			
	Rotation			
Crop	A_{F1}	A_{F2}	A_{F3}	A_{F4}
Alfalfa (short rotation)	52	--	46	--
Alfalfa (long rotation)	--	61	--	54
Permanent pasture	18	18	18	18
Barley	10	7	9	7
Barley (nurse crop)	10	7	9	7
Corn silage	10	7	9	7
Sugar beets	--	--	9	7
Total	100	100	100	100

Table 5. Percentage of rotation in small dairy farms in Sevier County, Utah, 1968

	Percentage of cropland			
	Rotation			
Crop	A_{D1}	A_{D2}	A_{D3}	A_{D4}
Alfalfa (short rotation)	58	--	52	46
Alfalfa (long rotation)	--	66	--	--
Permanent pasture	18	18	18	18
Barley	12	8	10	9
Barley (nurse crop)	12	8	10	9
Corn silage	--	--	10	9
Sugar beets	--	--	--	9
Total	100	100	100	100

small grains rather than row crops if he expects to have lower water supply.

Range beef farm

Alternative crop rotations for the range beef farm are as follows: A_{B1} represents a rotation where forage crops are grown to maintain the main livestock enterprise. Small grain is included but no row crops. Alfalfa fields are plowed up after 5 years and followed by 1 year barley and 1 year barley (nurse crop). A_{B2} has a pattern similar to A_{B1}, but alfalfa is retained in the field 8 years and followed by 1 year barley, then barley (nurse crop). There are no row crops in these two crop rotations. A_{B3} retains alfalfa for a 5-year period and is then plowed up. Barley follows for 1 year. After barley the crop following is corn silage. This is followed by barley (nurse crop). Alfalfa is retained in the field for 8 years for crop rotation A_{B4} and followed by barley, corn silage and barley (nurse crop). In this crop rotation small grain and corn silage are retained in the field for 1 year but have smaller proportional acreage than crop rotation A_{B3}.

Feeder farm

Crop rotation A_{F1} of feeder farm retains alfalfa for a short rotation (5 years) and then it is plowed up. The following crops are barley, corn silage and barley (nurse crop) which are retained 1 year in the field (about 10 percent of cropland). Crop pattern in A_{F2} is similar to A_{F1} but alfalfa is retained for a longer rotation (8 years and about 61 percent of cropland) and includes a smaller acreage of small grain and row crops (1 year and about 7 percent each of cropland for barley, corn silage and barley (nurse crop)). A_{F3} and A_{F4} are more intensive crop rotations and have the same crop pattern. But A_{F3} retains alfalfa shorter period (5 years) and higher percent of cropland for small grain and row crops than A_{F4} (8 years alfalfa, 1 year barley, 1 year corn silage and sugar beets and 1 year barley (nurse crop)).

Small dairy farm

Crop rotations (action) A_{D1} and A_{D2} are similar. Crop pattern A_{D1} retains alfalfa for short rotation (5 years) and includes a larger acreage of small grain than A_{D2} (8 years alfalfa and 1 year in small grains). There are no row crops following small grain in these crop patterns. Crop rotations (action) A_{D3} and A_{D4} represent the small dairy farms where forage crops are grown to maintain the dairy enterprise. Alfalfa is retained only 5 years in A_{D3} and 8 years in A_{D4}. After plowing up alfalfa in action A_{D3}, there is a larger acreage of small grains and corn silage than in action A_{D4}. Sugar beets are grown to supplement farm income and follow small grains.

Water requirements of crop rotations

Figure 2 shows the potential consumptive use of water for the major crops in Sevier County, Utah. Each crop requires different amounts of water in the different periods of time. Usually the less water that is required by a crop during the shortage period of water supply, the smaller is the net income per acre. This is similar to

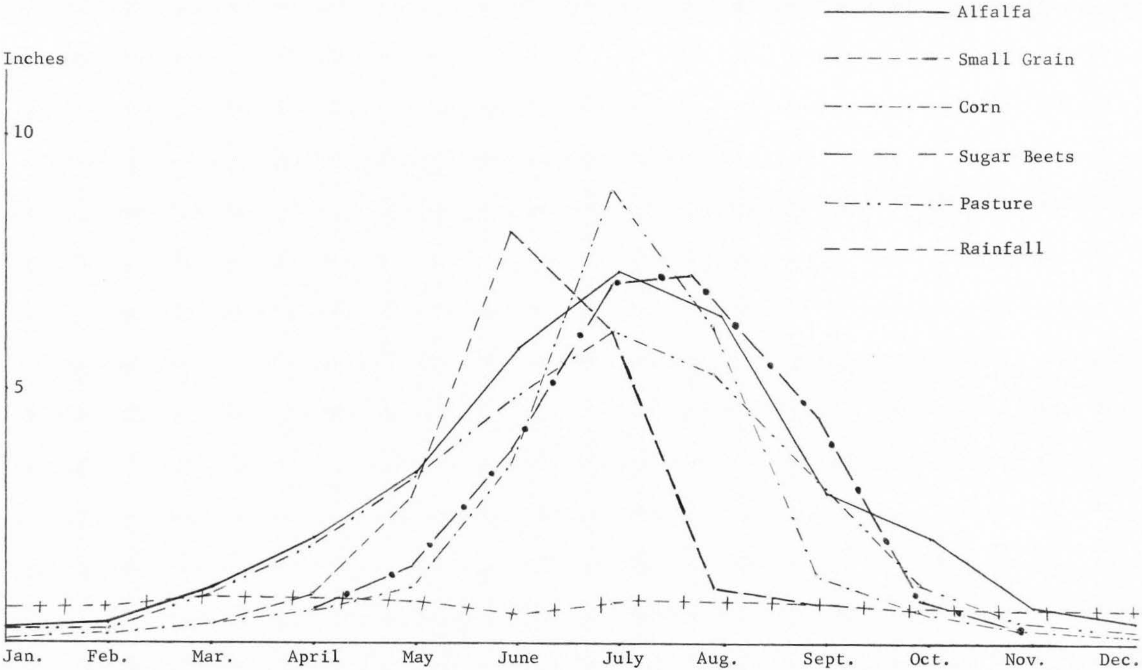

Figure 2. Potential consumptive use of water for major crops in Sevier County, Utah.

Source: U.S. Department of Agriculture. Unpublished data compiled by U.S.D.A. Sevier Basin Field Party, 1966.

the pattern for net income per acre of a set of crop rotations. Because small grains which have lower return per acre than row crops are harvested mostly in the latter part of July, they are not greatly affected by late season shortages. But row crops (higher return per acre) are harvested up to late fall and need late season irrigation. In general, we can say that the net farm income of crops is positively related to water requirements in the shortage period of water supply.

Capital

It is assumed in this study that the farm has a line of machinery to perform most farm operations. Seasonal labor and operating capital are also available.

Irrigation water

Rainfall in Sevier County is unpredictable. Water for irrigation is available from the natural flow of Sevier River on the basis of shares of stock owned in an irrigation company by the irrigator. As stream flows decline during the middle or latter part of the irrigation season, water deliveries to the farm diminish. As stream flow diminishes, available irrigation water will be used economically (husbanded) to ensure that those acres chosen for irrigation will continue to receive an adequate supply. Water shortages are most critical in July, August and September. When water is critically short, the first 1.84 acre feet per acre of water are allocated to corn and sugar beets which have higher marginal value products than the other crops.

In general, irrigators in Sevier County will receive a larger percentage of the total stream flow during the periods of high run-off and lower percentage during the periods of low run-off because of preferential "first right" of irrigators above them on the stream. Water deliveries (states of nature) to the farm assumed in this study are 1.84, 2.67, 2.95 and 3.25 acre-feet per acre during the period of July, August and September. These quantities of water are dependent upon the run-off in Sevier River during spring and early summer. In other words, the water deliveries to the farm are proportional to water supply in Sevier River.

The Sevier River water supply is dependent primarily on snow pack deposited in mountain areas surrounding the basin. Run-off occurs in the spring and early summer as snow pack melts. Water available annually for irrigation fluctuates with greater magnitude than fluctuation in annual precipitation because of the rather constant consumptive use requirement by vegetation on the watershed. Thus, run-off tends to be relatively small in years when snow pack is below normal and relatively large in years when snow pack is above normal (Appendixes A-C).

<u>The profit maximization</u>

It is assumed that the farmer is a profit maximizer, or at least struggles toward maximizing net farm income over time and his profit function is:

$$P_t = \sum_{i=1}^{n} L_i R_i$$

where: L_i = acres in production of the i crop and: R_i = return to farm labor and management per acre for the i crop

where: $R_i = Y_i P_i - C_i$

and: Y_i = yield per acre of the i crop

P_i = price per unit of field for the i crop

C_i = costs of production per acre for the i crop

ANALYSIS AND APPLICATION OF THE MODEL

The purpose of this section is to present the optimal crop rotation in each type of farm for which decisions are to be made under the uncertainty of amount of water to be available for irrigation. Optimal crop rotations selected by "data" methods, a perfect predictor, and "no data" methods are shown later in this report in Tables 13, 14, 15, 16, 17, 18, 19, 20, 21, 27, 28 and 29 respectively. The discussion here is limited to evaluation of alternative crop rotations only. They all present a problem in which only variability in states of nature (run-off) is considered.

This means the decision maker is faced with choosing the optimal course of action, A_i, from a set of possible actions. The outcomes of these various actions are dependent on the occurrence of alternative states of nature, θ_j.

The states of nature

The states of nature can be defined as the values of one or more exogenous factors that directly affect the outcome of a particular action but cannot be controlled with certainty by the decision maker (2). This is known as a random variable. For this study, the stream flow in Sevier River during July, August and September is regarded as a random variable, referred to as the state of nature (θ_j). From the 32-year (1937-1968) stream flow records, as shown in Appendixes A and B, four states of nature are considered:

Poor (θ_1) = less than 40 1,000 acre/feet

Fair (θ_2) = 40 - 55 1,000 acre/feet

Good (θ_3) = 56 - 85 1,000 acre/feet

Excellent (θ_4) = 86 or over 1,000 acre/feet

These correspond to 1.84, 2.67, 2.95 and 3.25 acre-feet per acre of water delivered to the farm. The class interval of the states of nature was classified after inspection of the distribution of actual run-off data of Sevier River.

Run-off distribution

In the past, the four states of nature have been observed. The result of the observed frequencies in Table 6 are 6, 8, 11 and 7 years and the distribution of the occurrence of θ_j[P (θ_j)] during the 32-year period analyzed is as follows:

θ_1 = 18.8 percent;

θ_2 = 25.0 percent;

θ_3 = 34.4 percent;

and θ_4 = 21.8 percent.

These are the *a priori* probabilities of the states of nature, as shown in Table 6.

Conditional probabilities

Since θ_j is a random variable which the farmer needs to know more about in order to make the correct decision, one scheme is to employ annual April first snow pack measurement on the watershed of Upper Sevier River (south of Richfield, Utah) and water stored in Otter Creek and Piute Reservoirs to arrive at an estimate of expected

Table 6. Frequencies of various run-off conditions of Sevier River, Sevier County, Utah, 1937-1968, and calculation of the *a priori* probability

Average run-off condition 1937-1968			Number of years observed	Probabilities $P(\theta_j)$
Description		Run-off index interval (ac. ft.)		
Poor	θ_1	less than 40	6	0.188
Fair	θ_2	41-55	8	0.250
Good	θ_3	56-85	11	0.344
Excellent	θ_4	86 or over	7	0.218
Total			32	1.000

value of θ_j. (As was shown previously, there is a relationship between April first snow pack, water stored in Otter Creek and Piute Reservoirs and run-off in Sevier River during July, August and September.) The results of the observed frequencies which served as a predictor of the states of nature, θ_j, are shown in Table 7. If θ_1 (poor water year, less than 40,000 acre-feet) is the state of nature, 4 of the 6 years will be expected to be poor water years; 2 of the 6 years will be expected to be fair water years; no year is expected to be a good year or an excellent year. The rest of the table is interpreted in this way.

From the *a priori* experience, the conditional probability distribution of such observation can now be computed:

$$P\ (Z_k/\theta_j)$$

where: Z_k = the observation on snowpack and water stored

Table 7. Frequencies of four observations on April 1 snow pack and water storage in Otter Creek and Piute Reservoirs given the state of nature (actual stream flows)

States of nature (1,000 acre-feet)			Observation on April 1 snow pack and storage				
			Z_1 Poor	Z_2 Fair	Z_3 Good	Z_4 Excellent	Total (θ_j)
					Years		
Poor	θ_1	less than 40	4	2	0	0	6
Fair	θ_2	41-55	3	5	0	0	8
Good	θ_3	56-85	0	2	8	1	11
Excellent	θ_4	86 or over	0	0	3	4	7
Total (Z_k)			7	9	11	5	32

θ_j = the states of nature (actual run-off in Sevier River)

Table 8 shows the conditional probabilities of four observations of snow pack and water stored given the state of nature, θ_j.

The 0.667 in the third column of Table 8 means that the farmers will expect a poor water year (Z_1) of stream flow on April first 66.70 percent of the time when in fact there will be 40,000 acre feet or less (poor water year, θ_1). Likewise, they will expect a fair water year (Z_2) of stream flow on April first 33.30 percent of the time when in fact there will be 40,000 acre-feet or less (poor water year, θ_1). The other conditional probabilities in Table 8 are derived similarly (it is Z_k which is observed and not θ_j). The states of nature are unknown at the time of decision in April; only

Table 8. Conditional probability of four observations on April 1 snow pack and water storage in Otter Creek and Piute Reservoirs (denoted $P(Z_k/\theta_j)$ given the state of nature (actual stream flows)

States of nature (1,000 acre feet)			Observations on April 1 snow pack and storage Z_1 Poor	Z_2 Fair	Z_3 Good	Z_4 Excellent	Total
Poor	θ_1	less than 40	0.667	0.333	0	0	1.000
Fair	θ_2	41-55	0.375	0.625	0	0	1.000
Good	θ_3	56-85	0	0.182	0.727	0.091	1.000
Excellent	θ_4	86 or over	0	0	0.429	0.571	1.000

Z_k is known.

A posteriori probabilities

By utilizing the conditional probabilities of Table 8 and the a priori probabilities, the joint probability $P(\theta_j)P(Z_k/\theta_j)$ (the product of two distributions) can be calculated. From column 8 in the right part of Table 9, 0.125 (0.188 x 0.667) is the probability that the water year which occurs is both an actual poor water year and a poor observed water year; only 0.063 (0.188 x 0.333) which occurred is both a poor actual water year and fair observed water year; nothing which occurred is both a poor actual water year and good and excellent observed water year; 0.094 (0.250 x 0.375), 0.156 (0.250 x 0.625) and 0 (zero) (0.250 x 0) which occur is both a fair actual water year and poor, fair, good and excellent observed water year; 0 (zero) (0.344 x 0), 0.063 (0.344 x 0.182), 0.250 (0.344 x

Table 9. Derivation of posteriori probabilities $P(\theta_j/Z_k)$ and calculations

States of nature θ_j		Conditional probabilities $P(Z_k/\theta_j)$ observations (Z_k): Z_1 Poor	Z_2 Fair	Z_3 Good	Z_4 Excellent	Priori probabilities $P(\theta_j)$	Joint probabilities $P(\theta_j)P(Z_k/\theta_j)$ observations (Z_k): Z_1 Poor	Z_2 Fair	Z_3 Good	Z_4 Excellent
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Poor	θ_1	0.667	0.333	0	0	0.188	0.125	0.063	0	0
Fair	θ_2	0.375	0.625	0	0	0.250	0.094	0.156	0	0
Good	θ_3	0	0.182	0.727	0.091	0.344	0	0.063	0.250	0.031
Excellent	θ_4	0	0	0.429	0.571	0.218	0	0	0.094	0.124
						$P(Z_k)$	0.219	0.282	0.344	0.155

States of nature (θ_j)		Posteriori probabilities $P(\theta_j/Z_k) = \frac{P(\theta_j)P(Z_k/\theta_j)}{P(Z_k)}$ Observations (Z_k): Z_1 Poor	Z_2 Fair	Z_3 Good	Z_4 Excellent
Poor	θ_1	0.571	0.223	0	0
Fair	θ_2	0.429	0.554	0	0
Good	θ_3	0	0.223	0.727	0.200
Excellent	θ_4	0	0	0.273	0.800
Total		1.000	1.000	1.000	1.000

0.727) and 0.031 (0.344 x 0.091) which occur is both a good actual water year and poor, fair, good and excellent observed water year; 0 (zero) (0.218 x 0), 0.094 (0.218 x 0.429) and 0.124 (0.218 x 0.571) occur is both an excellent water year and poor, fair, good and excellent observed water year, respectively. These figures show that the predictor device is close to a perfect predictor.

The probability of observing (Z_k) is given by summing the $P(Z_k/\theta_j)$ over all θ_j for a particular Z_k; Z_1, Z_2, Z_3 and Z_4 are 21.90, 28.20, 34.40 and 15.50 percent, respectively. These are shown in Table 9.

The a posteriori probabilities, $P(\theta_j/Z_k)$, are determined by Bayes' formula as shown in the lower right-hand part of Table 9. These probabilities are called a posteriori distribution of θ_j because it is the distribution after having observed Z_k and tell the decision maker what state of nature he can expect given an observation on snow pack and water stored, i.e., the probabilities of the true run-off condition (θ_j), given the observed April first run-off condition (Z_k) (7).

After weighting the a priori probability by the conditional probability, 57.1 percent of the time θ_1 (poor water year) will occur and 42.9 percent of the time θ_2 (fair water year) will occur when observing Z_1; 22.3, 55.4, 22.3 and zero or 0 percent for θ_1, θ_2, θ_3 and θ_4 when Z_2 (fair observed water year) is observed; zero, 72.7, 27.3 percent for θ_1, θ_2, θ_3 and θ_4 when Z_3 (good observed water year) is observed; and zero, 20.0 and 80.0 percent for θ_1, θ_2, θ_3 and θ_4 when Z_4 is observed.

Pay-off matrix

Net farm income (gain matrix) can be calculated for each of the crop rotations (action A_i) for each type of farm given the state of nature (Θ_j). The difference in the state of nature is the difference in water input delivered to the farms. An increased acreage of high valued crops results in a higher degree of variability of net income over a wider range of possibilities. A significantly higher net income per acre will be obtained if water is plentiful and a loss in net income may result if a serious water shortage occurs, because the level of production will be seriously damaged. Thus, a poor water year will reduce the size of the total return obtained and for some rotation will not cover the high total costs of the inputs used for the production of the high valued crops (labor, machinery, fertilizer, water, etc.).

A set of decision rules in this study consistent with the farmer's objective (utility) function is needed to select the course of action that maximizes utility. However, the derivation of such a utility function is no small undertaking. Thus, as a matter of practical application, it has been assumed in this study that the utility function is linear with respect to money over the relevant range. Consequently, maximization of monetary gain is equivalent to maximizing utility.

The actions (crop rotations), as shown in Tables 3, 4 and 5, have been assumed to represent the possible farm practices in Sevier County. Crops were valued at the time of harvest, and net farm income does not include income from the livestock enterprise as crops used on the farm were valued at the market price. The gains or losses for range beef farms, feeder farms and small dairy farms are shown in Tables 10, 11 and 12, respectively. These data are derived from each physical crop-water production function. It is measured in dollars of net farm income per acre.

The physical productivity of irrigation water as shown in Appendix G was used to compute the economic productivity of irrigation water applied to alfalfa, barley, corn silage and sugar beets. Once the physical productivity of irrigation water has been established for all alternative uses, the economic productivity in different uses can be determined by attaching monetary values to output and resources inputs. The price of output and inputs is the average price reported by U.S.D.A. (35). The average production cost is shown in Table 26, Appendix G. The net return shown is the net cash income (cash receipts minus cash expenses).

Table 10. Pay-off table of range beef farms in Sevier County, Utah, 1968

States of nature (run-off in Sevier River) 1,000 ac. ft.			Actions (crop rotations)			
			A_{B1}	A_{B2}	A_{B3}	A_{B4}
			---------dollars per acre----------			
Poor	θ_1	less than 40	9.34	3.54	4.88	1.08
Fair	θ_2	41 - 55	35.98	31.54	32.56	29.45
Good	θ_3	56 - 85	40.49	44.88	45.85	45.19
Excellent	θ_4	86 or over	51.77	49.81	54.62	52.07

Table 11. Pay-off table of feeder farms in Sevier County, Utah, 1968

States of nature (run-off in Sevier River) 1,000 ac. ft.			Actions (crop rotations)			
			A_{B1}	A_{B2}	A_{B3}	A_{B4}
			---------dollars per acre---------			
Poor	θ_1	less than 40	7.55	7.85	0.60	0.12
Fair	θ_2	41 - 55	36.36	36.20	30.32	26.79
Good	θ_3	56 - 85	52.54	51.89	63.43	60.29
Excellent	θ_4	86 or over	59.85	58.73	72.32	67.59

Table 12. Pay-off table of small dairy farms in Sevier County, Utah, 1968

States of nature (run-off in Sevier River) 1,000 ac. ft.			Actions (crop rotations)			
			A_{D1}	A_{D2}	A_{D3}	A_{D4}
			---------dollars per acre---------			
Poor	θ_1	less than 40	0.95	3.54	4.88	0.60
Fair	θ_2	41 - 55	27.60	23.10	32.56	30.32
Good	θ_3	56 - 85	37.18	36.45	45.85	63.43
Excellent	θ_4	86 or over	43.39	41.39	54.62	72.32

From Table 10, we found that when the "poor" water state of nature occurred, net farm income per acre (\$9.34) from range beef farm from crop rotation A_{B1} was higher than net farm income (\$1.08) from crop rotation A_{B4} which required late water irrigation higher than crop rotations A_{B1}. The farm income per acre from crop rotation A_{B4} was \$1.08, because the income on this crop rotation is not sufficient to cover the costs of the input used for producing the high value crops. On the other hand, when the water supply is adequate for all late irrigation water requirements (good year, θ_3, and excellent year, θ_4), net farm income per acre from crop rotation A_{B4} (\$45.19 and \$52.07) is higher than net farm income per acre from crop rotation A_{B1} (\$40.49 and \$51.77). These same relationships occur for farm income of the feeder farm and small dairy farm, as shown in Tables 11 and 12.

"No data" decision making

The following discussion will illustrate what Chernoff and Moses (4) have called the "no data" problem utilizing only the _a priori_ probabilities. Table 6 presents the relevant frequency distribution of run-off condition in the Sevier River (states of nature, θ_j); column $P(\theta_j)$ gives the _a priori_ distribution over the four states of nature. Tables 10, 11, and 12 present the pay-off matrix (net farm incomes) for each type of farm in which alternative actions (crop rotations) are considered. Each set of crop rotations of individual farm (range beef, feeder, and small dairy farm) is selected to approximately utilize the water supplies for each of the four alternative run-off conditions.

The expected value of actions using "no data" methods can be found by multiplying each possible net farm income for the states of nature by the *a priori* probability distribution of its occurrence and then taking the sum of these products. Utilizing the criterion of maximizing expected monetary value, action A_{B3} of range beef farm is chosen, giving an expected net income of \$36.736 per acre; action A_{F3} of feeder farm is chosen, giving an expected net income of \$45.278 per acre. Action A_{D4} of dairy farm with an expected value of \$45.278 per acre is then optimal. These expected values are shown in Appendixes H-J.

"Data" decision making

The above discussion illustrates the "no data" problem. However, by April first, when the farmer starts cultivation, he has some notion of whether run-off conditions during July, August and September will be poor, fair, good or excellent--an opinion based on snow pack and water stored conditions up to that date. In fact, the observed run-off conditions on April first were used as an indicator of the true run-off conditions during the subsequent spring cultivation period. Once the probabilities of the true run-off condition (Θ_j) are known, a Bayes strategy is found by maximizing the estimated income for each action, given the observation on snow pack and water stored. The expected income can be computed--the estimated income for each action is multiplied by the *a posteriori* probabilities for each observation (Z_k). The strategy bundle resulting from use of the *a posteriori* probabilities represents the "data" problem of decision theory (4).

Applying the a posteriori probabilities of Table 9 to original gain table (Table 10), the range beef farmer finds that run-off condition Z_1 (poor) action A_{B1} provides the maximum expected value of \$20.769 per acre. Likewise, when Z_2 (fair water year) is observed, action A_{B1} provides the maximum expected value of \$31.045 per acre; when Z_3 (good water year) is observed, action A_{B3} provides the maximum expected value of \$48.244 per acre; and when Z_4 (excellent water year) is observed, action A_{B3} provides the maximum expected value of \$52.866 per acre. These expected values are presented in Table 13. The underlined income figures indicate the maximum expected value for each observed Z_k. Thus, the optimum Bayes strategy bundle is defined as (A_{B1}, A_{B1}, A_{B3} and A_{B3}) meaning that action A_{B1} is taken in response to observation Z_1, A_{B1} to Z_2, A_{B3} to Z_3 and A_{B3} to Z_4. By determining the probability distribution of the predicted run-off,

Table 13. Pay-off table of expected net returns for alternative actions (crop rotations) given the observed April 1 run-off in Sevier River using posterior probabilities or "data" problems of range beef farms in Sevier County, Utah, 1968

Observed April 1 run-off condition		Actions (crop rotations)			
		dollars/acre			
Poor	Z_1	<u>20.769</u>	15.552	16.755	13.251
Fair	Z_2	<u>31.045</u>	28.271	29.351	26.634
Good	Z_3	43.569	46.226	<u>48.244</u>	47.068
Excellent	Z_4	49.514	48.824	<u>52.866</u>	50.694

Note: The underlined income figures indicate the maximum expected value of each observed Z_k (1,2,3 and 4).

$P(Z_k)$, the range beef farmer can calculate his expected income if he follows the strategy bundle (A_{B1}, A_{B1}, A_{B3} and A_{B3}). Given the $P(Z_k)$ of Table 9, the expected value of the optimal strategy bundle in Table 13 is $20.769 (0.219) + $31.045 (0.282) + $48.244 (0.344) + $52.866 (0.155) = $38.093 per acre as shown in Table 14.

By the same manner, the optimum Bayes strategy bundle for feeder farmer is (A_{F2}, A_{F1}, A_{F3}, and A_{F3}) as shown in Table 15 and the expected value of optimal strategy bundle in Table 15 is $20.012 (0.219) + $33.544 (0.282) + $65.857 (0.344) + $70.542 (0.155) = $47.454 per acre as shown in Table 16. The optimum Bayes strategy bundle for a small dairy farmer is (A_{D3}, A_{D4}, A_{D4} and A_{D4}), and the expected value of optimal strategy bundle is $46.021 per acre as shown in Tables 17 and 18, respectively.

Table 14. Expected value of optimal strategies for "data" problems of range beef farm in Sevier County, Utah, 1968

Rotation	Net return dollars/acre	Probabilities of observing Z $P(Z_k)$	Expected value of optimum strategies dollars/acre
A_{B1}	20.769	0.219	4.548
A_{B1}	31.045	0.282	8.755
A_{B3}	48.244	0.344	16.596
A_{B3}	52.866	0.155	8.194
Total		1.000	38.093

Table 15. Pay-off table of expected net return for alternative actions (crop rotations) given the observed April 1 run-off in the Sevier River using posterior probabilities or "data" problems of feeder farms in Sevier County, Utah, 1968

Observed April 1 run-off condition		Actions (crop rotations)			
		A_{F1}	A_{F2}	A_{F3}	A_{F4}
		dollars/acre			
Poor	Z_1	19.909	<u>20.012</u>	13.350	11.561
Fair	Z_2	<u>33.544</u>	33.377	31.076	28.313
Good	Z_3	54.536	53.757	<u>65.857</u>	62.283
Excellent	Z_4	58.388	57.362	<u>70.542</u>	66.130

Note: The underlined income figures indicate the maximum expected value of each observed Z_k (1,2,3 and 4).

Table 16. Expected value of optimal strategies for "data" problems of feeder farms in Sevier County, Utah, 1968

Optimal strategies		Probabilities of observing Z $P(Z_k)$	Expected value of optimal strategies dollars/acre
Rotation	Net return dollars/acre		
A_{F2}	20.012	0.219	4.406
A_{F1}	33.544	0.282	9.459
A_{F3}	65.857	0.344	22.655
A_{F3}	70.542	0.155	10.934
Total		1.000	<u>47.454</u>

Table 17. Pay-off table of expected net return for alternative actions (crop rotations) given the observed April 1 run-off in the Sevier River using posterior probabilities or "data" problems of small dairy farms in Sevier County, Utah, 1968

Observed April 1 run-off condition		Actions (crop rotations)			
		A_{D1}	A_{D2}	A_{D3}	A_{D4}
		dollars/acre			
Poor	Z_1	12.383	11.931	<u>16.755</u>	13.350
Fair	Z_2	23.793	21.715	29.351	<u>31.076</u>
Good	Z_3	38.875	37.799	48.244	<u>65.857</u>
Excellent	Z_4	42.148	40.402	52.866	<u>70.542</u>

Note: The underlined income figures indicate the maximum expected value of each observed Z_k (1,2,3 and 4).

Table 18. Expected value of optimal strategies for "data" problems of small dairy farms in Sevier County, Utah, 1968

Optimal strategies		Probabilities of observing Z	Expected value of optimal strategies
Rotation	Net return dollars/acre	$P(Z_k)$	dollars/acre
A_{D3}	16.755	0.219	3.669
A_{D4}	31.076	0.282	8.763
A_{D4}	65.857	0.344	22.655
A_{D4}	70.542	0.155	10.934
Total		1.000	<u>46.021</u>

"Perfect knowledge" decision making

If the state of nature is known prior to the decision-making period, it would be a simple matter to choose a crop rotation which would maximize net farm income. But since the state of nature is not known in advance, the farmer must make a decision. A method is needed to allow him to make judicious use of all the information available.

If the states of nature could be predicted with certainty in the spring, the a posteriori probability distribution, $P(\Theta_j/Z_k)$, portion of Table 9 would show value of 1.0 down the diagonal (with zeros elsewhere). Thus, the optimum perfect knowledge strategy bundle of a range beef farm is (A_{B1}, A_{B1}, A_{B3}, and A_{B3}) with expected value of optimum perfect strategy \$9.34 (0.188) + \$35.98 (0.250) + \$45.85 (0.344) + \$54.62 (0.218) = \$38.430 per acre as shown in Table 19.

The optimum perfect knowledge strategy bundle of feeder farm and small dairy farm is derived similarly. Optimum perfect knowledge strategy bundle of feeder farm is (A_{F2}, A_{F1}, A_{F3} and A_{F3}) and the expected value of optimum strategy is \$48.152 per acre as shown in Table 20. Table 21 presents the optimum perfect knowledge strategy bundle (A_{D1}, A_{D3}, A_{D4} and A_{D4}) of the small dairy farm and their expected value of optimum strategy, \$46.642 per acre.

Value of the data

The derivation of Bayesian decisions by using only the a priori probability distribution, $P(\Theta_j)$, is referred to as the "no data" problem. The decision problem using the a posteriori distribution

Table 19. Expected value of a perfect predictor of range beef farms in Sevier County, Utah, 1968

Optimal strategies		Probabilities of observing Z_p $P(Z_{kp})$	Expected value of optimum strategies dollars/acre
Rotation	Net return dollars/acre		
A_{B1}	9.34	0.188	1.756
A_{B1}	35.98	0.250	8.995
A_{B3}	45.85	0.344	15.772
A_{B3}	54.62	0.218	11.907
Total		1.000	38.430

Table 20. Expected value of a perfect predictor of feeder farms in Sevier County, Utah, 1968

Optimal strategies		Probabilities of observing Z_p $P(Z_{kp})$	Expected value of optimal strategies dollars/acre
Rotation	Net return dollars/acre		
A_{F2}	7.85	0.188	1.476
A_{F1}	36.36	0.250	9.090
A_{F3}	63.43	0.344	21.820
A_{F3}	72.32	0.218	15.766
Total		1.000	48.152

Table 21. Expected value of a perfect predictor of small dairy farms in Sevier County, Utah, 1968

Optimal strategies		Probabilities of observing Z_p $P(Z_{kp})$	Expected value of optimal strategies dollars/acre
Rotation	Net return dollars/acre		
A_{D1}	4.88	0.188	0.917
A_{D3}	32.56	0.250	8.140
A_{D4}	63.43	0.344	21.819
A_{D4}	72.32	0.218	15.766
Total		1.000	46.642

is called "data" problems. The difference in expected incomes resulting from using the "data" strategy bundle relative to the "no data" strategy can be interpreted as the value of the data or the value of the added information provided by the observation (Z_k). The value of data of the range beef farm is \$1.357 per acre; the feeder farm is \$2.167 per acre; and the small dairy farm is \$0.743 per acre as shown in Table 22.

Value of a perfect predictor

The difference in expected value of a perfect predictor relative to the expected value of the *a priori* distribution of the random variable, $P(\Theta_j)$, is the value of a perfect predictor. It is the value of a set of run-off forecasting. This value is usually higher than the value of the "data." As presented in Table 22, the value

Table 22. Value of the perfect predictor and value of the data of range beef farm, feeder farm and small dairy farm, Sevier County, Utah, 1968

Value	Range beef	Feeder	Dairy
	dollars per acre		
Expected value of a perfect predictor	38.430	48.152	46.642
Expected value of the "no data" problems	36.736	45.278	45.278
Value of the perfect predictor	1.694	2.874	1.364
Expected value of the "data" problems	38.093	47.454	46.021
Expected value of the "no data" problems	36.736	45.278	45.278
Value of the data	1.357	2.176	0.743

of a perfect predictor of range beef farm is $1.694 per acre; the feeder farm is $2.874 per acre; and the dairy farm is $1.364 per acre.

The expected incomes from the "data" method in our problem are $38.093, $47.454 and $46.021 per acre for range beef farm, feeder farm and small dairy farm, respectively, an increase of only $1.357, $2.176 and $0.743 over the expected income for the "no data" methods. Thus, the "value of the data" is slight. However, the value of a "perfect" run-off predictor would be only $1.694, $2.874 and $1.364 in this case.

From the standpoint of this study, the strategies derived from statistical decision theory have allowed only relatively slight improvements in expected net incomes over strategies already used by farmers. However, even if devices for perfectly predicting run-off conditions were available, the possibilities of increasing expected income would be slight within the scope of production possibilities presented here.

SUMMARY

If decision making is made in the realm of certainty, it would be a simple matter to select a crop rotation which would maximize net farm income. But since the decision problems under risk and uncertainty have several possible outcomes corresponding to each crop rotation, a set of decision rules, consistent with the farmer's objective (utility) function is needed to select the course of action that maximizes utility.

The major purpose of this study is to present selected empirical results of a study employing decision making theory as a framework for considering decision making under risk. The particular problem involves choices between alternative crop rotations for Sevier County farmers. The study demonstrates the usefulness of the Bayesian theory that gives more than a point estimation. It describes the magnitude of the difference between alternative actions, and provides a variety of estimates for consideration.

Several alternative crop rotations are available to Sevier County farmers in each year. Thus, a major problem facing a Sevier County farmer each year is this: Given the uncertainties of irrigation water supply, what combination of small grain, forage crops and row crops should be grown? The analysis used evaluates this question, as an isolated annual decision.

By employing a multiple regression ($y = b_o + b_1 \, x \, b_2 x_2 + e$), it was found that influence of snow pack and reservoir storage on water availability for irrigation purposes during July, August and September is very high ($R = 0.909$, $R^2 = 0.826$).

A number of decision rules have been suggested for cases where the probability distribution of the states of nature is unknown. All of these criteria have severe defects. Furthermore, it is difficult to conceive of decision problems in which the decision maker has no information, either objective or subjective, regarding the probabilities of the states of nature, θ_j. Thus, recent emphasis in decision theory has shifted toward the so-called Bayes strategies, which employ relevant probability distributions. The optimal Bayes strategy is generally defined as one which maximizes expected utility. If the utility function is linear over the relevant range, maximizing expected profits is equivalent to maximizing expected utility.

The decision problem in this study involves four crop rotations (A_{B1}, A_{B2}, A_{B3} and A_{B4}) for range beef farms; four crop rotations (A_{F1}, A_{F2}, A_{F3} and A_{F4}) for feeder farms; and four crop rotations (A_{D1}, A_{D2}, A_{D3} and A_{D4}) for small dairy farms--and four states of nature: poor water year (θ_1), fair water year (θ_2), good water year (θ_3) and excellent water year (θ_4). These four states of nature are correspondent to 1.84, 2.67, 2.95 and 3.25 acre-feet of water delivered to the farm. They are equivalent to an input for producing a crop.

Outcomes of each action-state pair are derived from crop-water production function. It is measured in dollars of profit of net farm income per acre. This is called pay-off matrix of outcome plan.

By making the observation on the actual run-off from the past 32-year period (1937-1968) of the Sevier River, the a priori probabilities (the probability distribution of states of nature $P(\Theta_j)$) can be calculated: $P(\Theta) = 0.188$; $P(\Theta_2) = 0.250$; $P(\Theta_3) = 0.344$; $P(\Theta_4) = 0.218$. To gain additional information about the likelihood of a particular state of nature (Θ_j), the decision makers will make the observation on snow pack and water stored (Z_k) that serves as a predictor of states of nature (Θ_j). That is, he can construct a conditional probability distribution, $P(Z_k/\Theta_j)$. For example, in the past he has observed poor water year (Θ_1) in 6 years. In 4 of those 6 years, the run-off observation was for poor water year (Z_1), while in the other 2 years, it was for fair water year (Z_2). Therefore, the conditional probabilities of obtaining particular observations, given the underlying state of nature, Θ_1, are $P(Z_1/\Theta_1) = 0.667$, $P(Z_2/\Theta_1) = 0.333$, $P(Z_3/\Theta_1 = 0$ and $P(Z_4/\Theta_1) = 0$. The other conditional probabilities are derived similarly. The a priori probability distribution, whether objective or subjective, is given by $P(\Theta_j)$. The joint probability $P(\Theta_j)\ P(Z_k/\Theta_j)$ is simply the product of the two distributions. The $P(Z_k)$ is given by summing the $P(Z_k/\Theta_j)$ over all Θ_j for a particular Z_k. By utilizing data on the run-off in the past (Z_k) and actual run-off which occurred (Θ_j), the a posteriori probability distribution, $P(\Theta_k/Z_k)$, is then determined by Bayes' theorem.

$$P(\Theta_j/Z_k) = \frac{P(\Theta_j)\ P(Z_k/\Theta_j)}{P(Z_k)} \text{ and } P(\Theta_1/Z_1) = 0.571,\ P(\Theta_2/Z_1) = 0.429$$

$P(\theta_1/Z_2) = 0.223$, $P(\theta_2/Z_2) = 0.554$, $P(\theta_3/Z_2) = 0.223$, $P(\theta_3/Z_3) = 0.727$, $P(\theta_4/Z_3) = 0.273$, $P(\theta_3/Z_4) = 0.200$, $P(\theta_4/Z_4) = 0.800$,

and the remainders are zeros.

In this study, the $P(Z_k)$s are: $P(Z_1) = 0.219$, $P(Z_2) = 0.282$, $P(Z_3) = 0.344$ and $P(Z_4) = 0.155$.

Applying objective or subjective a priori probabilities, $P(\theta_j)$, to the states of nature and original pay-off table, action (crop rotation) A_{B3}, A_{F3}, and A_{D4} of the range beef farm, feeder farm and small dairy farm, with expected values \$36.736, \$45.278 and \$45.278 per acre are then the optimal respective action. These are called the "no data" problem optimum strategies.

Utilizing those a posteriori probabilities over the states of nature and the original pay-off table, it is possible to calculate the expected income for each action, given the observed April first run-off condition. Thus, the optimal strategy bundle of range beef farms is (A_{B1}, A_{B1}, A_{B3} and A_{B3}), meaning that A_{B1} is taken in response to observation Z_1, A_{B1} to Z_2, A_{B3} to Z_3, and A_{B3} to Z_4. Similarly, the optimal strategy bundle of feeder farm and small dairy farm is (A_{F2}, A_{F1}, A_{F3} and A_{F3}) and (A_{D3}, A_{D4}, A_{D4} and A_{D4}), respectively.

The strategy bundle resulting from use of the a posteriori probability distributions shows the "data" problem of the decision theory. The expected value of the strategy bundle can be computed by multiplying the expected values of the optimum action for each observed Z_k by the probability of run-off observing Z_k, $P(Z_k)$, and summing. The expected net income per acre of range beef farm, feeder farm and small dairy farm from the "data" problem is \$38.093, \$47.454 and \$46.021, respectively.

If a perfect run-off observation device were available, the <u>a posteriori</u> probability distribution, $P(\Theta_j/Z_k)$, would be value of 1.0 down the diagonal and zeros elsewhere and lead to the optimal strategy bundle of (A_{B1}, A_{B1}, A_{B3}, and A_{B3}) with expected value \$38.430 per acre for range beef farm; (A_{F2}, A_{F1}, A_{F3} and A_{F3}) with expected value \$48.152 per acre for feeder farm; and (A_{D1}, A_{D3}, A_{D4} and A_{D4}) with expected value of \$46.642 per acre for small dairy farm.

The difference in expected incomes per acre resulting from using the "data" problem and the "no data" problem is \$1.357, \$2.176 and \$0.743 for range beef farm, feeder farm, and small dairy farm respectively. These are represented for the value of the data or the value of added information.

The value of a perfect predictor, the difference between expected incomes per acre of a perfect predictor and the expected incomes per acre from the "no data" problem, for range beef farm, feeder farm and small dairy farm is \$1.694, \$2.874 and \$1.364 respectively.

CONCLUSION

This study involves the derivation of the a posteriori probability function by a weighting of the a priori probabilities, $P(\Theta_j)$, by conditional probabilities, $P(Z_k/\Theta_j)$. These conditional probabilities, in turn, are the probabilities of observing the particular additional information given the possible values of the random variable (states of nature, Θ_j).

The observation was made only on two variables (snow pack and water stored), but the degree of multiple correlation coefficient between those two variables and coming late season run-off shows a high degree of correlation in this study.

The figures from this study show that the expected value of the "data" problems for each type of farm came out very close to the expected value of a perfect predictor and higher than the expected value of the "no data" problems. This means that the Bayes approach gives more than a point estimation and is useful for farm management decision making under risk. Even though the value of data and the value of a perfect predictor are only slight, the cost for making these observations is not high. It appears that this process can improve farm enterprises gradually, for each year the answers will become "better and better" (to paraphrase Gompers), even though McConnen's (21, p. 65) words are true and we get ". . . bad answers to problems to which otherwise worse answers are given."

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APPENDIXES

Appendix A

Table 23. Hydraulic records of Sevier County, Utah, 1937-1968

Year	Rainfall records (inches)	Snow pack (water contain) X_1 (inches)	Water stored (1,000 ac-ft) X_2	Sevier River run-off (July-Sept.) 1,000 ac-ft Y	Annual run-off (1,000 ac-ft)	Run-off predictions (1,000 ac-ft) Y
1937	10.36	215.6	68.97	69.86	151.0	107.41
1938	7.56	169.8	118.84	92.68	200.0	111.59
1939	7.59	97.1	105.20	56.13	152.7	61.61
1940	9.77	82.2	82.39	47.20	123.3	39.16
1941	11.29	157.0	74.46	77.89	189.4	77.18
1942	6.10	145.9	119.73	89.34	260.0	98.43
1943	7.24	131.7	116.09	72.44	160.9	88.08
1944	9.22	174.6	104.22	96.78	185.3	105.42
1945	10.68	178.1	108.81	78.21	134.6	110.23
1946	11.09	83.0	124.92	75.22	154.1	65.56
1947	10.73	101.8	100.40	88.32	121.9	61.38
1948	8.19	146.8	122.74	92.81	182.9	100.79

Table 23. Continued

Year	Rainfall records (inches)	Snow pack (water contain) X_1 (inches)	Water stored (1,000 ac-ft) X_2	Sevier River run-off (July-Sept.) 1,000 ac-ft Y	Annual run-off (1,000 ac-ft)	Run-off predictions (1,000 ac-ft) Y
1949	7.41	193.7	85.92	95.11	183.2	105.20
1950	6.60	97.1	120.98	76.80	157.6	71.24
1951	7.70	54.0	77.13	48.52	105.4	19.79
1952	6.73	252.0	68.48	86.14	143.9	127.97
1953	6.45	73.3	121.26	72.83	141.4	57.77
1954	5.68	121.8	76.66	46.39	112.3	58.35
1955	5.50	107.3	60.60	38.45	85.48	40.25
1956	4.53	74.8	40.86	18.91	72.64	9.59
1957	11.15	108.0	32.70	51.45	76.53	28.63
1958	4.69	171.4	92.33	80.62	166.30	96.33
1959	7.14	58.9	95.41	42.84	120.30	33.75
1960	7.70	87.9	58.79	26.59	85.04	28.03
1961	10.42	81.1	42.62	19.60	74.95	14.27

Table 23. Continued

Year	Rainfall records (inches)	Snow pack (water contain) X_1 (inches)	Water stored (1,000 ac-ft) X_2	Sevier River run-off (July-Sept.) 1,000 ac-ft Y	Annual run-off (1,000 ac-ft)	Run-off predictions (1,000 ac-ft) Y
1962	7.89	165.0	70.08	65.29	122.50	72.09
1963	7.48	41.3	47.03	24.67	57.11	-5.85
1964	8.31	76.6	45.73	38.97	75.35	13.59
1965	9.78	117.7	51.08	41.98	92.23	40.40
1966	7.32	75.6	95.29	42.86	106.65	43.24
1967	9.39	50.2	70.57	50.30	106.07	13.61
1968	8.49	133.4	92.07	66.07	125.23	74.40
Total	260.32	3824.7	2692.36	1971.27	4206.28	1961.49
Average	8.135	119.52	84.14	61.60	131.45	61.30

Sources: U. S. Geological Survey, Surface water supply of the United States, 1931-60, Paper 1314-1734. The Great Basin. 1960, 1968.

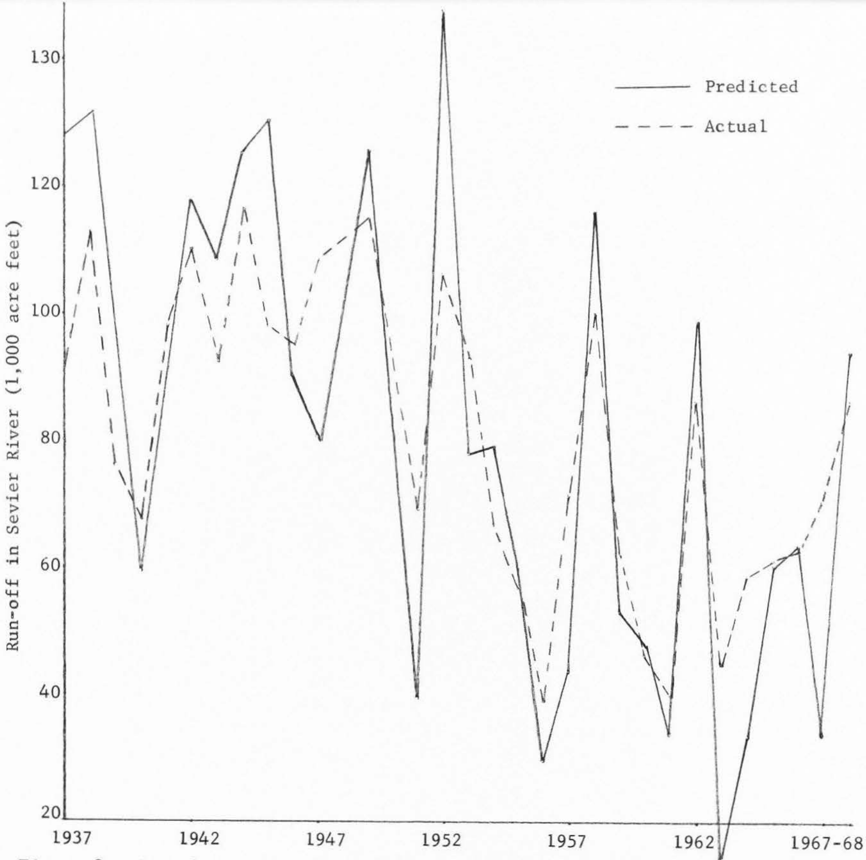

Figure 3. Actual run-off and predicted run-off of Sevier River, Sevier County, Utah, 1937-1968.

Appendix C

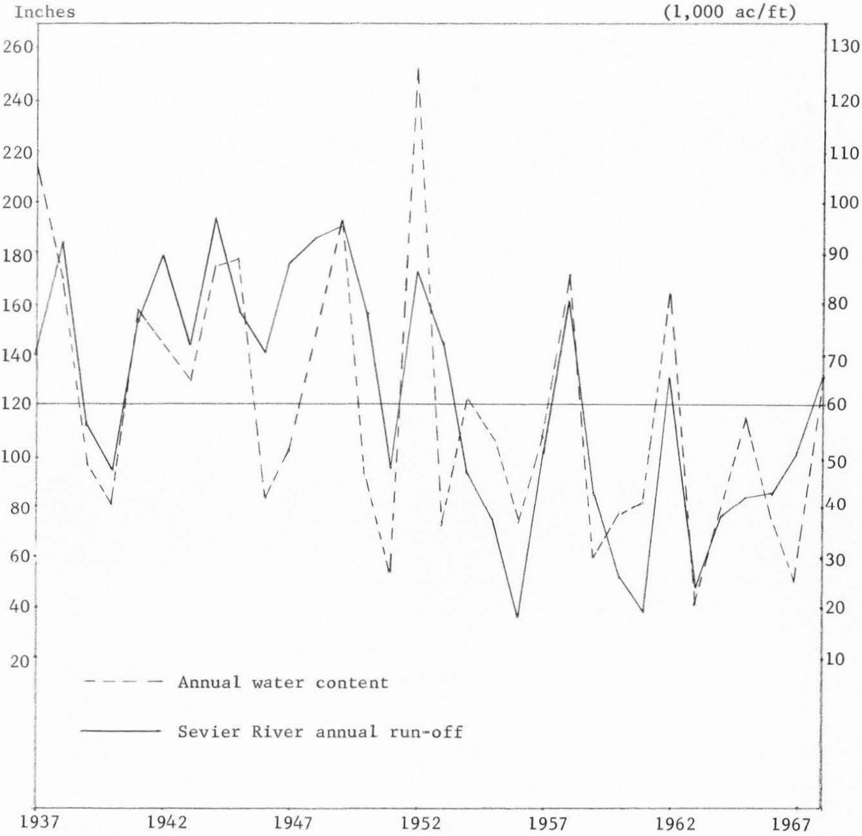

Figure 4. Fluctuation of annual snow pack on Upper Sevier Watershed and stream run-off of Sevier River, Sevier County, Utah, 1937-1968.

Appendix D

Table 24. Monthly average run-off and rainfall distribution of Sevier County, Utah, 1937-1968

Run-off period	Run-off 1,000 ac/ft	Average run-off	Percent	Rainfall inches	Percent
April	368.05	11.50	8.75	0.752	9.24
May	724.70	22.65	17.22	0.724	8.90
June	573.89	17.93	13.64	0.536	6.56
July	839.82	26.24	19.96	0.755	9.28
August	651.69	20.36	15.49	0.755	9.28
September	479.76	15.00	11.41	0.751	9.23
Annually	4206.28	131.45		8.135	

Appendix E

Table 25. Potential consumptive use of water for major crops of Sevier County, Utah, 1966

	Alfalfa		Small grain		Corn silage		Sugar beets		Pasture	
Month	Monthly use rate inches	Percent	Monthly use rate inches	Percent	Monthly use rate inches	Percent	Monthly use rate inches	Percent	Monthly use rate inches	Percent
April	2.19	6.34	0.53	4.32	0.60	2.58	0.60	2.27	1.89	6.53
May	3.99	11.55	2.81	13.04	1.12	4.81	1.49	5.64	3.32	11.46
June	5.75	16.65	8.14	37.77	3.49	15.00	3.75	14.18	4.78	16.50
July	7.32	21.25	6.25	29.00	8.72	37.46	7.05	26.66	6.78	23.41
August	6.44	18.65	1.02	4.73	6.31	27.10	7.24	27.38	5.44	18.78
Sept.	3.94	11.41	0.70	3.25	1.34	5.76	4.45	16.83	3.36	11.60

Appendix F

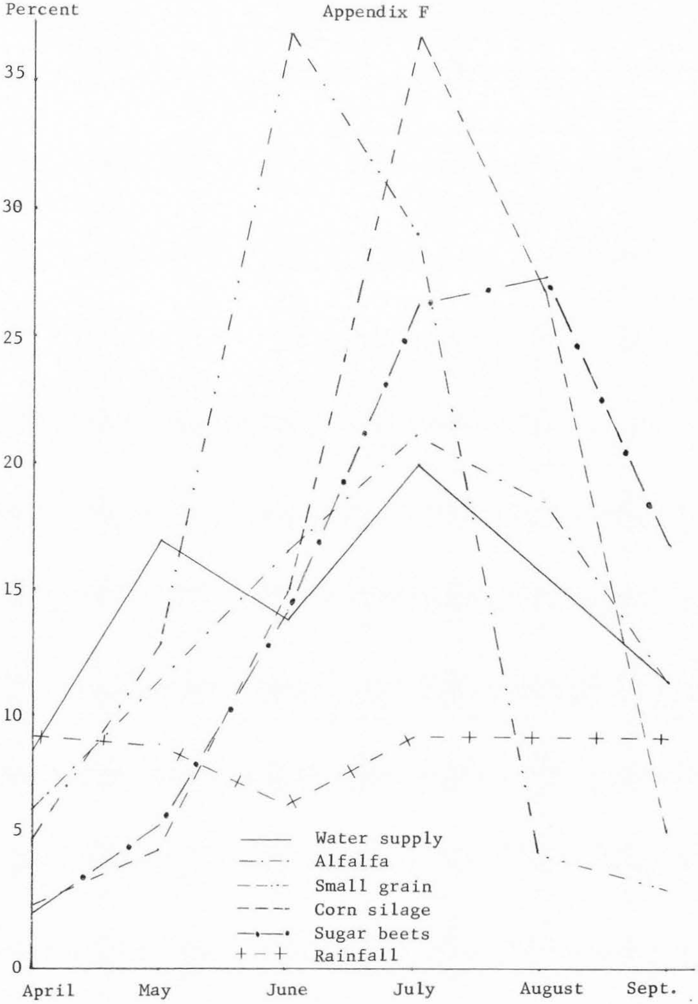

Figure 5. Potential consumptive use of water of major crops and water supply distribution of Sevier River, Sevier County, Utah, 1966.

Source: U. S. Department of Agriculture. Unpublished data compiled by U.S.D.A. Sevier Basin Field Party 1966 and U.S. Geological Survey, Surface Water Supply of the United States. The Great Basin, paper 1314-1736.

Appendix G

Table 26. Crop yield and cost for four water supply situations of Sevier County, Utah, 1968

Water supply situation	Average rotation use of water	Alfalfa				Barley				Corn silage		Sugar beets	
		5-year rotation		8-year rotation		Seed crop		Nurse crop					
water year	ac/ft per acre	Yield ton/ac	Cost $/ac	Yield ton/ac	Cost $/ac	Yield bu/ac	Cost $/ac	Yield bu/ac	Cost $/ac	Yield ton/ac	Cost $/ac	Yield ton/ac	Cost $/ac
Poor	1.84	1.80	38.95	1.65	38.95	75.15	44.81	45	44.37	12	78.93	16	119.34
Fair	2.67	3.10	39.56	3.20	39.40	83.50	44.96	50	44.37	12	79.88	16	119.34
Good	2.95	4.05	40.27	4.00	39.98	83.50	45.11	50	44.37	17.3	81.93	17.5	131.11
Excellent	3.25	4.30	40.78	4.20	40.56	88.50	45.26	60	44.37	20.8	82.88	19.8	131.11

Sources: 1. David L. Wilson. Agricultural economy of Sevier River Basin, Utah. USDA. March 1969.
2. Clyde E. Stewart. Profitable farm adjustments in the use of irrigation water in Ashley Valley, Utah. Utah Agricultural Experiment Station, Ag. Econ. Series 65-2. March 1965.
3. Jay L. Haddock. Yield, quality and nutrient content of sugar beets as affected by irrigation regime and fertilizers. Amer. Soc. Sugar Beet Tech. Proc. 10(4):290-355. January 1959.

Appendix H

Table 27. Optimal strategy for the "no data" problems of range beef farm in Sevier County, Utah, 1968

States of nature (run-off in Sevier River) θ_j			Actions (crop rotations)				Priori probabilities $P(\theta_j)$
			A_{B1}	A_{B2}	A_{B3}	A_{B4}	
		1,000 sq. ft.	dollars/acre				
Poor	θ_1	less than 40	1.756	0.666	0.917	0.203	0.188
Fair	θ_2	41 - 55	8.995	7.885	8.140	7.362	0.250
Good	θ_3	56 - 85	13.929	15.438	15.772	15.545	0.344
Excellent	θ_4	86 or over	11.285	10.859	11.907	11.351	0.218
Expected value of actions using priori probabilities $P(\theta_j)$			35.965	34.848	<u>36.736</u>	34.461	1.00

Appendix I

Table 28. Optimal strategies for the "no data" problems of feeder farms in Sevier County, Utah, 1968

States of nature (run-off in Sevier River) θ_j			Actions (crop rotations) A_{F1}	A_{F2}	A_{F3}	A_{F4}	Priori probabilities $P(\theta_j)$
		1,000 ac./ft.		dollars/acre			
Poor	θ_1	less than 40	1.419	1.476	0.113	0.023	0.188
Fair	θ_2	41 - 55	9.090	9.050	7.580	6.698	0.250
Good	θ_3	56 - 85	18.074	17.850	21.819	20.740	0.344
Excellent	θ_4	86 or over	13.047	12.803	15.766	14.734	0.218
Expected value of action using priori probabilities $P(\theta_j)$			41.630	41.179	45.278	42.195	1.000

Appendix J

Table 29. Optimal strategies for the "no data" problems of small dairy farms in Sevier County, Utah, 1968

States of nature (run-off in Sevier River) Θ_j			Actions (crop rotations) A_{D1}	A_{D2}	A_{D3}	A_{D4}	Priori probabilities
		1,000 ac./ft.	dollars/acre				
Poor	Θ_1	less than 40	0.179	0.666	0.917	0.113	0.188
Fair	Θ_2	41 - 55	6.900	5.775	8.140	7.580	0.250
Good	Θ_3	56 - 85	12.790	14.540	15.772	21.820	0.344
Excellent	Θ_4	86 or over	9.459	9.022	11.907	15.766	0.218
Expected value of actions using priori probabilities $P(\Theta_j)$			29.328	30.003	36.736	45.278	1.000

Appendix K

Table 30. Frequencies of actual run-off condition of Sevier River, snow pack observation, water storage observation of Otter Creek and Piute Reservoirs, and observed run-off of Sevier River, Sevier County, Utah, 1937-1968

Description of condition	Actual run-off: run-off interval Θ_i (1,2,3and4) (ac-ft)	Actual run-off: Observed frequency (No. of years)	Snow pack: Water content interval (inches)	Snow pack: Observed frequency (No. of years)	Water stored: Storage interval (ac-ft)	Water stored: Observed frequency (No. of years)	Observed run-off Z_k (1,2,3 and 4) (No. of years)
Poor	less than 40	6	less than 80	8	less than 60	8	7
Fair	41 - 55	8	81 - 110	9	61 - 80	7	9
Good	56 - 85	11	111 - 170	9	81 - 110	10	11
Excellent	86 or over	7	171 or over	6	111 or above	7	5
Total		32		32		32	32

Appendix L

Table 31. Irrigation water applied to an acre of crop and crop yield, Sevier County, Utah, 1968

Water supply situation	Alfalfa		Barley				Corn silage		Sugar beets	
			Seed crop		Nurse crop					
water year	Water applied inches	Yield ton/ac	Water applied inches	Yield bu/ac	Water applied inches	Yield bu/ac	Water applied inches	Yield ton/ac	Water applied inches	Yield ton/ac
Poor	30.0	1.80	25.00	75.15	50.00	45	29.13	12	40	16.0
Fair	42.5	3.10	36.75	83.50	52.08	50	29.13	12	40	16.0
Good	52.5	4.05	41.75	83.50	52.08	50	42.00	17.3	50	17.5
Excellent	65.0	4.30	46.75	88.50	62.5	60	50.5	20.8	64.5	19.8

Note: Assumes a 40 percent water application efficiency.

Sources: 1. David L. Wilson. Agricultural economy of Sevier River Basin, Utah. USDA. March 1969.
2. Jay L. Haddock. Yield, quality and nutrient content of sugar beets as affected by irrigation regime and fertilizers. Amer. Soc. Sugar Beet Tech. Proc. 10(4):290-355. January 1955.

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