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Estimating Agricultural Production Functions from Experimental Data for Different Crops in Relation to Irrigation, Fertilization and Soil Management in Northern Utah

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ESTIMATING AGRICULTURAL PRODUCTION FUNCTIONS FROM EXPERIMENTAL DATA FOR DIFFERENT CROPS IN RELATION TO IRRIGATION, FERTILIZATION AND SOIL MANAGEMENT IN NORTHERN UTAH

by

Subramaniam Swami Nathan

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Estimating Agricultural Production Functions from Experimental Data for Different Crops in Relation to Irrigation, Fertilization and Soil Management in Northern Utah

by

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Major Professor: Dr. Herbert H. Fullerton
Department: Agricultural Economics

Estimates of agricultural production functions from experimental data for four different crops in relation to six variable inputs are calculated by this study. There are four basic sections in the study. The first section covers the review of production function concepts and the procedures and problems that specifically pertain to this study. Also the importance of joint economic-agronomic research efforts, methodologies and applications of agricultural production functions are cited.

The second section includes the presentation data and postulated functional relationships in estimating production functions. Model building programs are used in developing three dimensional figures, which aid in the selection of the appropriate model. A multiple regression model using linear, non-linear and interaction terms is employed in deriving three production function for each crop. The problem of selecting a "best" model from the above three models is solved on the basis of economic theory, observed biologic physical production process, projected three dimensional production surfaces and statistical analyses. The polynomial form was selected as the "best" model for each crop.

The third section of this study analyzes the results and the economic implications. Optimal rates of input use are determined. Qualification of these results are required because of the non significant statistical relationships including the F values of the regression coefficients and relatively low coefficient of determination (R^2), and, also, because some optimal inputs values did not seem reasonable relative to observed rates. Further statistical analyses are carried out to determine the confidence interval for each input's marginal productivity and this results in unbounded solutions. As an alternative, the above confidence interval problem is rephrased as a system of equalities and solved simultaneously to obtain optimal input levels at the marginal productivities maximum and minimum values and these estimates are shown not to be confidence intervals.

Finally, in the fourth section of this study, summary and conclusions are given. Also, limitation and recommendations to the study are discussed.

(63 pages)

INTRODUCTION

The estimation of an agricultural production function provides a basic tool for economic analysis of the relationship between inputs and outputs. Knowledge of the production function is essential for making sound farm management decisions. Basically, the production function can be used to determine to what extent output of a product can be increased by altering resource use levels and combinations. In development applications, the magnitude of the production coefficients serves as the basis for determining comparative advantage and specifying an optimal pattern for regional or international trade. If the goal is to maximize output from the available resource supplies, a production function derived for a region, firm or crop, etc., with an associated estimate of the marginal product schedule, can provide a basis or guide for attaining that goal (assuming price competition in the resource market).

Recently, agronomic field and laboratory studies, in correlation with output performance studies, were conducted by the Utah State Experiment Station and the United States Department of Agriculture. These studies provide estimates of output responses for alfalfa, canning peas, potatoes, and sugar beets using varying levels and combinations of water and fertilizers. Estimates of variation in yield response associated with the sequence of crop rotation and the mode of water applications were also obtained.

Fertilizer and water applications play an important part in crop production. Since crop production economics is of great importance today, establishing more exact estimates of crop response to fertilizer and water applications on a given soil should be a useful research area. In addition, there is a need to know the rate at which inputs substitute for one another in the production of a given yield, so as to have a basis for determining least cost input combinations.

It should be possible to estimate production functions for the above-mentioned crops. Such production functions should provide information which will contribute to the optimization decision of input use.

OBJECTIVES OF THE STUDY

The main objectives of this study are to:

(a) Estimate the basic production functions from the experimental data of the four crops (alfalfa, canning peas, potatoes, and sugar beets).

(b) Apply output and input prices to translate physical outputs and inputs into monetary units.

(c) Calculate the value of the marginal product for each input.

(d) Determine the optimal levels and allocation of inputs.

REVIEW OF THE LITERATURE

This section, of the review of literature, will be devoted to the summarizing certain concepts of the agricultural production functions, empirical methods, and research which relates specifically to this type of study. The subject area covered in this section includes discrepancies in estimating and interpreting controlled experimental results in contrast to farm production, along with the technical considerations of estimating the various types of production functions and using regression analysis in selecting a production function.

Discrepancies in Estimating and Interpreting Controlled Field Experimental Results

Davidson, Martin and Mauldin (1) suggest that field experiments are the scientists' chief means of assessing animal and plant productivity potential. The evidence assembled in this article indicates that farm yields are less than experimental yields for important classes of farms and experiments. These variations are the results of differences in the circumstances under which experiments are conducted and those under which the farms are normally operated. Scientists are able to perform the cultural operations at a precise time and take maximum advantage of the environmental conditions because the experiments are conducted on a small area, while the farmer works with a larger acreage and smaller amounts of labor and capital per unit area. This prevents him from completing his cultural operations at a precise time, as the scientist does, and, thus, there is a tendency to perform operations at marginally less favorable times, accounting for the reduced yields on farms. Logically the extent of the reduction increases with the rise in crop acreage. Also, experiments are designed to highlight differences between varieties and treatments. Because experimenters are interested in isolating particular effects, they commonly attempt to supply all other resources in luxurious abundance compared to the farm's normal supply of these resources. Therefore, farm yields can be expected to be less than experimental yields for these reasons, and these reasons should be borne in mind by those who plan experiments and interpret experimental results.

Estimation of Different Types of Production Functions

Heady and Dillon (4) illustrate several types of production functions: Cobb-Douglas, quadratic, and square root. They consider certain concepts and methods relating to the production and use of production functions in agriculture and methods of data collection. They explain the illustration of production surfaces to the above-mentioned functions, as well as others, and the problems choosing of alternative models. The authors suggest that in formulating an economic model of the productive process, the logic of economic, biologic, or physical processes of production have to be considered. Also, they discuss the general type of recommendations from fitted production functions including the economic analyses of marginal productivity theory, optimum combination of resource input required for a specified output, and the maximization of net revenues.

In the book Resources, Productivity Return to Scale and Scale and Farm Size, Heady, Johnson and Hardin (5) discuss the technical problems involved in estimating production functions. This discussion is useful in the present study, as they explain that a conventional procedure is to predict the total output or output surface with the use of regression analysis. From the regression equation, the marginal product of individual resources can be estimated from production function the first derivative of that particular resource. Also, the marginal production relationships can be used to determine an optimum resource input allocation through a system of simultaneous equations. This optimum allocation is determined by equating the resource to product price ratio and the respective marginal product equations, equating the value of the marginal product to the price of the resource.

Fox (6) utilized experimental data in an agricultural production function to demonstrate the uses of multiple regression analysis. He analyzes several different functional forms (linear, quadratic, and square root) and indicates that a particular functional form might appear to give a better fit to the data. Also, he notes that increases and decreases in total variance from one functional form to another can be expected from the same basic population of both the dependent and independent variables. Furthermore, he points out the interesting feature of a controlled experimental design, that it is possible and appropriate that the inter-correlation problem can be "designed-out" and inter-correlations reduced to zero.

Stritzel (9) develops an analysis similar to the present study. However, in contrast to the data used in this study, he uses data derived from a controlled experiment run over a four-year period. A unique feature of Stritzel's study is the close cooperation between agronomist and economist in giving treatment to both agronomic and economic questions. A variety of rates of variable inputs are included in the experiment to provide an adequate basis for economic analysis. This facilitates statistical analysis by eliminating such problems as intercorrelation. A procedure for determining the best fitting equations to characterize yield data was investigated. The procedure involved the selection of significant variables by analysis of variance, subdividing the sum of squares of the significant treatment variables into their linear, quadratic, square root, etc., components on the basis of agronomic logic.

Stritzel (9) concludes that no one algebraic form of equation will adequately characterize the response function for any one crop under all soil and climatic conditions. However, he also concludes that it is possible to establish a generalized function under a given climatic condition and on a given soil for a specific crop.

Pesek and Heady (8) discuss the procedures used in determining the highest net return per dollar invested in fertilizer application in the field. The fertilizer application rate, thus determined, represents both the economic minimum rate and the lower limit that can be utilized in making agronomic fertilizer recommendations. Calling the yield increase, Y_1, this output can be expressed in the quadratic form,

$$Y_1 = sx + tx^2,$$

where x is the rate of fertilization, and s and t are constants. The cost of the applied fertilizer can be expressed as

$$Y_2 = m + rx ,$$

where m is the fixed application unit area and r is the price ratio of the unit of fertilizer to a unit of yield increase.

METHODS AND PROCEDURES

Experimental Procedures

The data used in the present study was derived from an experiment, initiated in the spring of 1949 and continued over a period of eight years. It was conducted on a calcareous Millville silt loam near Logan, Utah. Alfalfa, canning peas, potatoes, and sugar beets were the crops used in rotation during this period.

Soil tests were made with the following results: Millville silt loam used in the study has a 2 percent surface slope in each of two directions (west and south); the loam is derived from the dolomitic limestone; the profile is uniform in texture to a depth of more than 20 feet. The pH varies from 7.9 to 8.2 and contains from 45 to 75 percent $CaCO_3$ equivalent, increasing with profile depth; the average moisture percentage at one-third atmosphere tension is 21.0 and at 15 atmospheres is 8.7; the electrical conductivity (EC 10^3 @ 25 C) of saturated extract varies from 0.35 to 0.52 millimlos per cm.; and the cation exchange capacity is 13.3, with calcium constituting 12.4, sodium 0.4, and potassium 0.5 milliequivalents per 100 grams of soil.

The irrigation water used in the experiment contains 1, 10, 85, and 240 pounds of potassium (K), sodium (Na), magnesium (Mg), and calcium (Ca) respectively, per 24 acre inches of water. Land preparation, seeding, harvesting, and experimental field plot design are described in detail by Haddock, Taylor, and Milligan (2) in their manuscript Irrigation, Fertilization and Soil Management of Crops in Rotation. All peas yield data was adjusted to tenderometer reading of 105.0. For alfalfa, two cuttings of the first year and three of the second year were obtained as yield data.

For the present study, the year 1954 was chosen out of the eight-year experiment because the experiment was designed solely by agronomists, with the object of agronomic evaluation studies. Therefore, only two rates of fertilizer application were utilized. Also, the amount of residual nitrogen and phosphorous in the soil was determined only for the years 1953 and 1956 after the harvesting of crops. Because two rates of fertilizer application and the residual fertilizer data is not available throughout the experiment, it is not possible to establish a consistent production function for all eight years of the experiment, except for 1954. Therefore, for that year (1954) the amount of the residual fertilizer in the soil, the amount of water applied, and the methods of irrigation.

Statistical Analysis

Model building, analysis of variance, and multiple regression equations were computed and selected using the Utah State University computer write-up programs, Model Building (MODEL), Multivariate Data Collection Revised (MDCR) and Stepwise Multiple Regression Revised (SMRR) for crop yields. The regressions, together with the standard errors, inverse matrix, mean squares, and coefficient of variation (R^2) were computed at the Department of Applied Statistics and Computer Science of Utah State University.

Theoretical and Analytical Framework

Model estimation

The term production function is applied to the physical relation between resource inputs and a firm's product output. Product output is determined partly by the quantities of resource inputs and partly by the farmer's production techniques. This can be expressed in mathematical terminology as

$$Y = f\ (a,\ b,\ c)$$

where output of goods is represented by Y and resource input is represented by a, b, and c. The equation can be expanded readily to include as many different resources as are used in the production of a given commodity.

Alternate Forms and Derivation of the Production Function

Consider the classical production function in Figure 1. It is assumed that input per unit time can begin at 0 and be added in increments throughout the range of the function. Marginal product is shown to be increasing, constant and decreasing, depending upon the quantity of factor used (relative to the magnitude of other factor inputs). If a farmer is operating in the rational stage of production, he will not apply less input than that represented by point d (stage 1). To do so would sacrifice a greater average product per unit of input. Neither will the farmer use more factor inputs than represented by point e, since each unit of input used beyond this stage would effect a decrease in total product (stage 2). Thus the rational farmer seeks

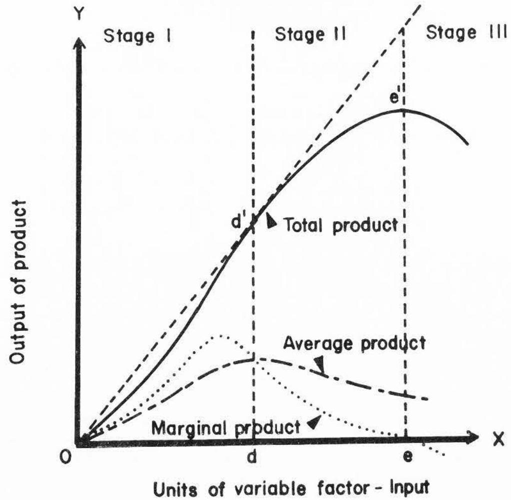

Figure 1. Classical production function.

to operate in relatively small area on the production function between d' and e'. This obviously reduces the range over which the predicting function is relevant and diminishes the variance in the quantities of inputs applied. It is difficult to establish a causal relationship between inputs and outputs within this shorter range of the inputs. This small range becomes relevant when variation increases the standard error of regression coefficient and decreases the reliability of the marginal product estimates (7).

Heady (3) discusses the analytical framework and methods for selecting a production function. He suggests that the knowledge of biological and economic factors aids in the selection, and also, that the algebraic form of the function, as well as the magnitude of its coefficient, will vary due to environmental conditions, type of crop, variable resources, magnitude of inputs, etc. Hence, to select the algebraic form of the function should be consistent with the above-mentioned factors. By way of illustration, Heady discusses a few general types of production functions. First, the Cobb-Douglas function, the most popular algebraic form used in farm-firm production function analysis, may be generalized as $Y = ax^b$, where Y is output, a is a constant, x is a variable input and b defines the transformation rate when the magnitude of input x changes. The production function merely states symbolically that the productive effort output depends upon the input used. In this case, only one input is used and output is a function of the quantity of x applied.

The marginal product of x(MP) can be estimated as the first derivative with respect to x of the production function.

$$MP = \frac{dy}{dx} = bax^{b-1} \text{ or } \frac{bax^b}{x}$$

The elasticity of production (EP) can be found directly from these marginal values as follows:

$$Ep = \frac{\frac{\Delta y}{y}}{\frac{\Delta x}{x}} = \frac{\Delta y}{\Delta x} \cdot \frac{x}{y} = \frac{bax^b}{x} \cdot \frac{x}{ax^b} = b$$

Hence, production elasticity may be estimated directly from estimated Cobb-Douglas function as the b values of the equation. From the above computation, it is also evident that the Cobb-Douglas function assumes a constant production elasticity, or that successive equal input increments add the same percentage to output. The function allows either constant, increasing, or decreasing marginal productivity depending upon the magnitude of b. If b equals one, constant return to scale exists; if b is less than one, decreasing return to scale exists; and if b is greater than one, increasing returns to scale are indicated. Since b cannot at the same time be less than and greater than one, both increasing and decreasing marginal product cannot hold for the same function. The rate of decrease in the marginal product declines, but never becomes zero. Given these properties, the Cobb-Douglas function cannot be used satisfactorily for data where there are ranges of both increasing and decreasing marginal productivity. Neither can it yield satisfactory estimates for data which might exhibit both positive and negative marginal products (stage 3 of production). Since a maximum product is never defined, the Cobb-Douglas function may over-estimate the quantity of inputs which will equate marginal revenue and marginal cost.

Besides the Cobb-Douglas, Heady (3) also indicates some other possible combination of linear and non-linear terms, as well as cross product terms in the equations as follows:

(1) $Y = a + b_1x_1 + b_2x_2 - b_3x_1^2 - b_4x_2^2$

(2) $Y = a + b_1x_1 + b_2x_2 + b_3x_1^{.5} + b_4x_2^{.5}$

(3) $Y = a + b_1x_1 + b_2x_2 + b_3x_1^2 + b_4x_2^2 + b_5x_1x_2$

In these estimates where Y is the total output, a is a constant, b's are the coefficients, and x_1 and x_2 are the variable inputs.

The polynomial equation (1) above with linear and squared terms has a greater flexibility than the Cobb-Douglas function because it assumes no constant elasticities of response, allowing the elasticity to change with greater inputs. The function can be applied to all observations and allows diminishing product, following a negative marginal product or declining total yield.

Heady continues to explain that another alternative is the equation (2) with linear and square root terms where one expects extremely large marginal products at lower input rates, followed by a long range of small and fairly constant marginal products. This square-root function may provide a useful form of the production relationship, but for marginal products of medium magnitude for low rates of input, followed by an early maximum in total product, it may be advisable to select the squared terms as in equation (1).

In addition to the terms used in Heady's equations (1) and (2), Fox (6) discusses the uses of an additional cross product term to these respective equations. In equations (1) and (2), the properties of second degree parabola in both the variables are to show that the effects of inputs x_1 and x_2 are strictly additive. But to test the hypothesis that a unit of input x_1 would be more effective in raising the yield if some input of x_2 were also used, rather than none, then one may have to include an additional term which contained both x_1 and x_2 in a joint or cross product form such as x_1x_2 or $x_1^{.5}x_2^{.5}$ to the respective equations (1) and (2). Use of an additional cross product term is shown in equation (3).

Heady concludes by saying the problem is choosing which of the above alternative functions is "more appropriate" than others for the desired types of economic analysis. Direct statistical tests (analysis of variance and F tests) are available for determining whether a significant reduction in variance is obtained by including one more or less terms in an equation, such as the cross product or square root functions. However, direct tests are not available for choosing between widely used functions like Cobb-Douglas, cross product or square root functions, etc. Therefore, it is advisable that one use his logic and knowledge of the subject matter, as well as such statistical criteria as the greatest coefficient of determination or the smallest deviation from the regression in making this subjective decision. Furthermore, plotting the derived quantities against the sample observations may aid the choice and selection.

PRESENTATION OF DATA AND POSTULATED FUNCTIONAL RELATIONSHIP

The following methods were used to identify functional relationships between the inputs or independent variables and output yields or dependent variables for each crop. Independent variables used in this study were the amount of nitrogen residual (NR) and phosphorous residual (PR) in the soil, the amount of fertilizer nitrogen (NA) and fertilizer phosphorous (PA) added to the soil, and the amount of irrigation water applied (W). Methods of irrigation (M) was used as an independent variable for sugar beets, peas, and potatoes. In the data, set M = 1 for sprinkler and M = 0 for furrow irrigation. Furthermore, no fertilizers of nitrogen and phosphorous were applied to first and second year alfalfa. In equation form, the input-output relationship is depicted by equation (4) for sugar beet, pea and potatoe crops, and equation (5) for first and second year alfalfa respectively.

$$(4) \quad Y = f(NR,\ NA,\ PR,\ PA,\ M.\ W)$$

$$(5) \quad Y = f(NR, PR,\ W)$$

$$(6) \quad \frac{df(NR, PR \text{-----})}{dNR} = \frac{\text{Cost of the respective resource input}}{\text{Price of output of the respective crop}}$$

Y, the output of yield for sugar beets, peas and potatoes, was measured in tons per acre, pounds per acre and bushels per acre respectively, while the output of first and second year alfalfa was measured in tons per acre. The inputs in equation (4) and (5), the residual and applied fertilizers, were measured in pounds per acre and the irrigation water applied was measured in acre inches.

In equation (6), above, the optimum rate of resouces input was calculated by taking partial derivatives of Y with respect to each input. Each partial derivative or marginal product equation was set equal to the input cost ratio to the output crop price.

The cost of inputs such as applied fertilizers (nitrogen and phosphorous) and irrigation water were obtained from data compiled by the Economic Research Institute of Utah State University. Average market prices for the year 1970 were chosen. These input and output prices are given in Table 1.

It is assumed that the value of residual nitrogen and phosphorous in the soil is the same price as the applied fertilizers. This assumption and price adjustment for the current fertilizer application will receive more complete discussion in the results and summary section.

Table 1. The average input and output prices for the year 1970

Crops	Prices	Price of Nitrogen Residual (NR) Per Pound	Price of Fertilizer Nitrogen Applied (NA) Per Pound	Price of Phosphorous Residual (PR) Per Pound	Price of Fertilizer Phosphorous Applied (PA) Per Pound	Cost of Irrigation Water (W) Acre Inches
First year alfalfa	$24.00 per ton	0.10¢	--	0.09¢	--	S = 0.80¢[a] F = 0.40¢
Second year alfalfa	$24.00 per ton	0.10¢	--	0.09¢	--	S = 0.80¢ F = 0.40¢
Sugar Beets	$17.00 per ton	0.10¢	0.10¢	0.09¢	0.09¢	S = 0.80¢ F = 0.40¢
Potatoes	$1.56 per bushel	0.10¢	0.10¢	0.09¢	0.09¢	S = 0.80¢ F = 0.40
Peas	$0.06 per pound	0.10¢	0.10¢	0.09¢	0.09¢	S = 0.80¢ F = 0.40¢

[a]S = Sprinkler irrigation.
F = Furrow irrigation.

ESTIMATION OF PRODUCTION FUNCTIONS

Model Building Program

The estimated production surfaces are used as an aid in selecting the appropriate model for the production function. Hurst's model building program was used to gain a visual perspective of the effects on each crop yield at different levels and combinations of resource use. The procedure divides the observation for each variable and combination of variables into minimum, maximum, and then five given interval lengths from minimum to maximum. At the same time, the corresponding mean output for all combination groups is given. That is, these output means were computed along with the number of observations for each class interval of each input X in pairwise combinations. This allows simplifying three-dimensional figures which illustrate the main effect and two-way interaction effects of combinations of variables on crop yield.

Development of Production Surfaces

The model building program, as discussed above, was used to develop three-dimensional surfaces for each crop and pairwise input combinations. An illustration of this, in Figure 2, depicts sugar beet production surfaces; the production surfaces for the first and second year alfalfa, potatoes and peas are shown in Figures 4 to 7 in the Appendix (A, B, C, and E). Examination of Figure 2 indicates that independent positive input increments increased output. Also, there was a strong interaction effect between the rates of

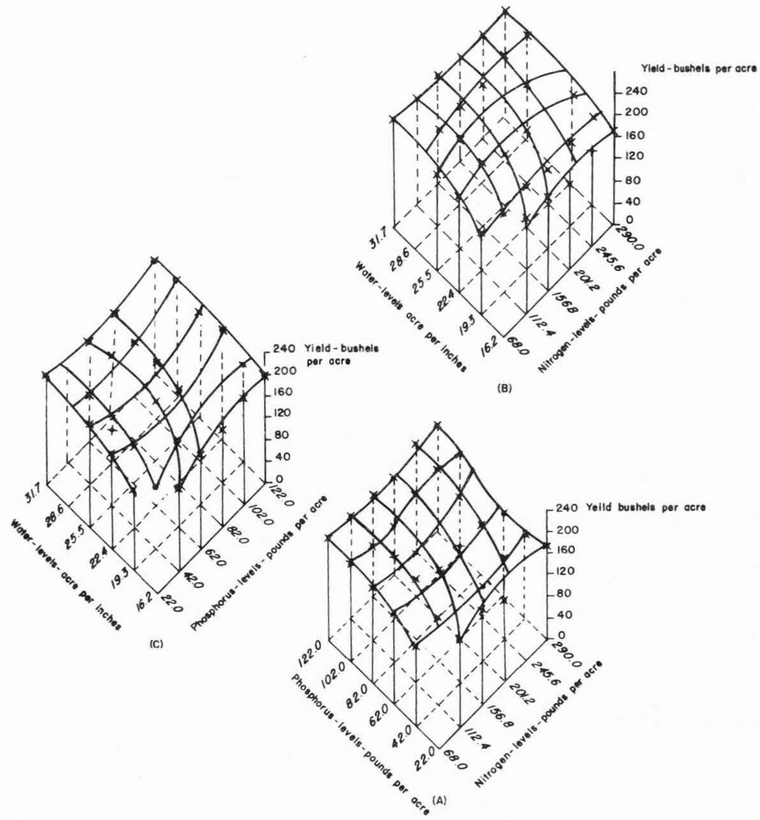

Figure 2. Main and two-way interaction effects of combinations of variables on sugar beet yields, 1954.

nitrogen-phosphorous and phosphorous-water use, illustrating that the joint action was more effective in increasing the yield than if only one input was used. In Figure 2 (a), holding nitrogen levels constant at intervals from 112.4 to 245.6 pounds per acre and at the same time varying phosphorous levels, it was observed that output increased at a decreasing rate. There was an apparent significant increase in yield at higher levels of phosphorus input. Hence, this cross section of the production surface appears to be in stage 2 of production. Similarly by interchanging these two variables, it was observed that from the phosphorous levels of 42.0 to 82.0 pounds per acre, the production surfaces appeared to exhibit increasing and constant rates and would appear to be in stage 1 or early stage 2. Then at the constant phosphorous level of 102.0 pounds per acre and varying the levels of nitrogen, the production surface appears to be increasing at a decreasing rate with a significant increase in yields. Interaction effects observed between the inputs of nitrogen-water and phosphorous-water use can be interpreted similarly from Figure 2 (b and c).

For each crop, the following observations were made: (a) Production surfaces tended to rise more rapidly as the fertilizers and water rates were increased when inputs were considered pairwise, acknowledging their joint effect. (b) These interaction effects exhibit the complementary nature of the resource inputs. (c) Some surfaces do not clearly indicate interaction effect due to the lack of observations.

Multiple Regression Analyses

Six variables were included in the multiple regression program for the crops potatoes, sugar beets and peas. Only three variables were included for the first and second year alfalfa crops, since no nitrogen or phosphorous was applied. Further, irrigation methods (M = 1 sprinkler and M = 0 for furrow) were not considered for these crops. Furthermore, in all regressions, as there was a range of intervals for the amounts of water application (W) and residuals of nitrogen (NR) and phosphorous (PR), linear and nonlinear terms were included. In contrast, the application of nitrogen (NA) and phosphorous (PA) only linear terms were used, since there were only two application rates (NA = 0 and 80 pounds per acre; PA = 0 and 44 pounds per acre). Hence, this program was designed to evaluate the contribution which each group of variables made towards explaining crop yield changes. Statistics generated by this program included calculated regression coefficients, coefficient of multiple determination (R^2), degrees of freedom (DF), and significance levels for each coefficient.

Derivation of Three Types of Production Functions

Three types of input-output response coefficients were estimated as discussed above. The production functions included were the estimated Cobb-Douglas, square root and polynomial forms. All included linear terms but differed variously by using exponents representing powers of 0.5 and 2.0, the first with powers of 0.5 termed a square root equation and the latter termed a polynomial. The results for each of the three production function equations, as estimated for first year alfalfa are as follows:

Cobb-Douglas

$$\text{(6)}\quad \ln y = \ln 1.406 + \underset{(0.030)}{0.052 \ln NR} + \underset{(0.016)}{0.032 \ln PR} + \underset{(0.066)}{0.016 \ln W}$$

Square root

$$\text{(7)}\quad Y = -33.08 - \underset{(0.127)}{0.150\ NR} + \underset{(0.026)}{0.023\ PR} + \underset{(0.488)}{0.285W}$$

$$-\underset{(0.0008)}{0.001\ NRPR} + \underset{(0.042)}{0.005\ NRW} + \underset{(0.002)}{0.003\ PRW}$$

$$+\underset{(5.910)}{6.085\ NR^{0.5}} - \underset{(2.701)}{1.377\ PR^{0.5}} + \underset{(11.286)}{9.307\ W^{0.5}}$$

$$+\underset{(0.363)}{0.154\ NR^{0.5}\ PR^{0.5}} - \underset{(0.983)}{1.151\ NR^{0.5}\ W^{0.5}} + \underset{(0.63)}{0.002\ PR^{0.5}\ W^{0.5}}$$

Polynomial

$$\text{(8)}\quad Y = 10.9054 - \underset{(0.015)}{0.0139\ NR} + \underset{(0.014)}{0.0046\ PR} - \underset{(0.133)}{0.2261\ W}$$

$$-\underset{(0.00002)}{0.000002\ NR^2} - \underset{(0.00004)}{0.00006\ PR^2} - \underset{(0.002)}{0.0021\ W^2}$$

$$-\underset{(0.00005)}{0.000008\ NRPR} + \underset{(0.0003)}{0.0005\ NRW} + \underset{(0.0003)}{0.0002\ PRW}$$

Y is the output of first year alfalfa,

NR is the nitrogen residual in the soil, and

W is the water applied.

Specification of the Model

This type of study calls for a production function or surface which is convex from above as in Figure 1, and which exhibits decreasing marginal productivity of the variable inputs. Two such functions are the equations of square root (7) and polynomial (8) with the properties of 2.0 and 0.5 degree parabola in all the variables (except the variables of applied fertilizers of nitrogen, and irrigation methods). Furthermore, these two equations have interaction terms included and are more effective in depicting the type of production relationship expected and tended to be consistent with the plotted production surfaces. The Cobb-Douglas function is less flexible in terms of elasticity and being homogenous degree one, it exhibits constant return to scale. Therefore, one should choose either the square root or the polynomial model depicting input-output relationships typical of stages 1 and 3 of the classical production function.

Analysis of regression and statistical results for all crops are shown in Tables 2-4. In the estimated polynomial equation, it was noted that if the linear terms have a negative coefficient, then the nonlinear term of the respective input should be positive and vice versa. This implies positive but not necessarily constant returns, and stage 1 and stage 3 of production are possible. For all crops, neither the polynomial nor the square root functions exhibits a superior fit, whether evaluated in terms of significance of the coefficients, coefficients of determination (R^2), or F values. Also, the polynomial form is characterized by a linear marginal product. Because no other functional

Table 2. Cobb-Douglas production surfaces estimated for potatoes, sugar beets, peas, first and second year alfalfa

$\ln y = \ln a + b_1 \ln x_1 + b_2 \ln x_2 ---- b_n \ln x_n$ [/a]

Independent Variable	Potatoes Regression Coefficient $\hat{B}i$	Potatoes Calculated F Value on $\hat{B}i$	Potatoes DF	Sugar Beets Regression Coefficient $\hat{B}i$	Sugar Beets Calculated F Value on $\hat{B}i$	Sugar Beets DF	Peas Regression Coefficient $\hat{B}i$	Peas Calculated F Value on $\hat{B}i$	Peas DF
lna	-3.922	--	255	2.271	--	255	8.468	--	255
lnNT	0.031	0.235	1	-0.019	0.164	1	-0.168	8.276[b]	1
lnPT	0.139	11.375[b]	1	0.249	54.675[b]	1	-0.008	0.043	1
lnW	0.391	73.165[b]	1	0.614	48.827[b]	1	0.212	0.228	1
	$R^2 = 0.2434$		3	$R^2 = 0.2810$		3	$R^2 = 0.1175$		3

Table 2. Continued

Independent Variable	First Year Alfalfa Regression Coefficient $\hat{B}i$	First Year Alfalfa Calculated F Values on $\hat{B}i$	DF	Second Year Alfalfa Regression Coefficient $\hat{B}i$	Second Year Alfalfa Calculated F Values on $\hat{B}i$	DF
a	1.406		255	0.375		255
lnNR	0.052	3.016[c]	1	0.069	4.094[b]	1
lnPR	0.032	3.782[b]	1	0.028	4.508[b]	1
lnW	0.016	0.056	1	0.342	17.063[b]	1
	$R^2 = 0.0271$		3	$R^2 = 0.0929$		3

[a] Functional form.
[b] On all production surface equation tables, indicates that these are significant at 5 percent levels of probability.
[c] Indicates significant at 10 percent levels of probability.
[d] Significant at 25 percent levels of probability.

Table 3. Square root production surface equation estimated for first and second year alfalfa, potatoes, sugar beet and peas

Independent Variable	First Year Alfalfa: Regression Coefficient $\hat{B}i$	First Year Alfalfa: Calculated F Value on $\hat{B}i$	DF	Second Year Alfalfa: Regression Coefficient $\hat{B}i$	Second Year Alfalfa: Calculated F Value on $\hat{B}i$	DF
a	-33.080	--	255	-44.40	--	255
NR	-0.150	1.39[d]	1	0.084	0.57	1
PR	0.023	0.75	1	-0.302	2.18[d]	1
W	0.285	0.34	1	-1.392	1.34[d]	1
NRPR	-0.001	1.77[d]	1	0.001	2.29[d]	1
NRW	0.005	1.71[d]	1	-4.005	1.07	1
PRW	0.0003	0.02	1	0.008	1.56[d]	1
$NR^{.5}$	6.085	1.06	1	-4.769	1.02	1
$PR^{.5}$	-1.377	0.26	1	11.081	3.07[c]	1
$W^{.5}$	9.307	0.68	1	13.341	0.82	1
$NR^{.5}PR^{.5}$	0.154	0.18	1	-0.296	3.46[c]	1
$NR^{.5}W^{.5}$	-1.151	1.37[d]	1	1.351	1.78[d]	1
$PR^{.5}W^{.5}$	0.002	0.001	1	-1.496	2.14[d]	1
	$R^2 = 0.1438$		12	$R^2 = 0.1438$		12

Table 3. Continued

Independent Variable	Potatoes Regression Coefficient $\hat{B}i$	Potatoes Calculated F Value on $\hat{B}i$	DF	Sugar Beets Regression Coefficient $\hat{B}i$	Sugar Beets Calculated F Value on $\hat{B}i$	DF	Peas Regression Coefficient $\hat{B}i$	Peas Calculated F Value on $\hat{B}i$	DF
a	-1684.340	--	255	-1802.390	--	255	3240.370	--	255
NR	-10.781	1.69[d]	1	-2.428	3.59[b]	1	-14.916	0.77	1
NA	-0.141	0.27	1	-0.021	0.01	1	-8.018	4.84[b]	1
PR	-11.208	6.80[b]	1	-0.066	6.03[b]	1	-25.690	0.02	1
PA	0.007	0.0002	1	0.587	0.97	1	4.968	0.54	1
M	4.943	0.62	1	6.126	1.16	1	-165.97	0.24	1
W	-0.005	57.31[b]	1	-30.748	11.31[b]	1	612.178	9.86[b]	1
$NR^{.5}$	163.399	1.39[d]	1	66.689	3.69[c]	1	484.077	0.44	1
$PR^{.5}$	78.289	0.89	1	166.020	12.44[b]	1	371.497	0.51	1
$W^{.5}$	296.769	8.41[b]	1	383.450	12.99[b]	1	-2787.080	1.84[d]	1
$NR^{.5}PR^{.5}$	5.455	0.36	1	-2.901	1.20	1	-7.836	0.04	1
$NR^{.5}W^{.5}$	7.711	0.62	1	0.339	0.004	1	-70.511	0.24	1
$PR^{.5}W^{.5}$	17.572	8.09[b]	1	-6.527	0.99	1	42.510	0.53	1
NAPA	0.002	0.09	1	0.0005	0.06	1	-0.079	1.98[d]	1
NAW	0.001	0.01	1	0.027	0.04	1	0.717	3.42[c]	1
PAW	-0.038	1.50[d]	1	-0.011	0.02	1	-0.165	0.05	1
	$R^2 = 0.5219$		12	$R^2 = 0.3928$		12	$R^2 = 0.2345$		12

[a] Functional form.
[b] On all production surface equation tables, indicates that these are significant at 5 percent levels of probability.
[c] Indicates significant at 10 percent levels of probability.
[b] Significant at 25 percent levels of probability.

Table 1. Polynomial production surface equation estimated for first and second year alfalfa, potatoes, sugar beets and peas

Independent Variable	First Year Alfalfa: Regression Coefficient $\hat{B}i$	First Year Alfalfa: Calculated F Value on $\hat{B}i$	DF	Second Year Alfalfa: Regression Coefficient $\hat{B}i$	Second Year Alfalfa: Calculated F Value on $\hat{B}i$	DF
a	10.9054	--	255	-3.4023	--	255
NR	-0.0139	0.913	1	0.0043	0.0169	1
PR	0.0046	0.104	1	0.0659	6.846[b]	1
W	-0.2261	2.889[c]	1	0.4946	1.048	1
NR^2	-0.000002	0.011	1	-0.000009	2.097[d]	1
PR^2	-0.00006	3.358[c]	1	-0.00003	2.553[d]	1
W^2	-0.0021	1.965[d]	1	-0.0069	0.729	1
NRPR	-0.000008	0.028	1	-0.0002	0.425	1
NRW	0.0005	2.357[d]	1	0.0005	0.233	1
PRW	0.0002	0.541	1	-0.0015	3.624	1
	$R^2 = .0469$		9	$R^2 = 0.1243$		9

Table 4. Continued

Independent Variable	Potatoes			Sugar Beets			Peas		
	Regression Coefficient $\hat{B}i$	Calculated F Value on $\hat{B}i$	DF	Regression Coefficient $\hat{B}i$	Calculated F Value on $\hat{B}i$	DF	Regression Coefficient $\hat{B}i$	Calculated F Value on $\hat{B}i$	DF
a	-397.202	--	255	-428.481	--	255	2,949.668	--	255
NR	-3.055	0.83	1	1.397	3.1[c]	1	8.149	0.169	1
NA	-0.131	0.23	1	-0.0411	0.002	1	-8.363	5.173[b]	1
PR	7.562	6.03[b]	1	7.177	15.2[b]	1	14.973	0.729	1
PA	-0.031	0.006	1	0.580	0.92	1	4.731	0.482	1
M	4.523	0.51	1	5.619	0.96	1	-206.664	3.629[b]	1
W	32.008	21.53[b]	1	24.374	18.2[b]	1	-136.10	0.556	1
NR^2	-0.018	0.92	1	-0.004	2.84[c]	1	-0.026	0.233	1
PR^2	-0.070	10.03[b]	1	-0.039	7.70[b]	1	-0.100	3.121[c]	1
W^2	-0.879	67.08[b]	1	-0.361	13.2[b]	1	12.800	3.378[c]	1
NRPR	0.0094	0.09	1	-0.011	1.70[d]	1	-0.033	0.059	1
NRW	0.042	0.55	1	-0.002	0.008	1	-0.472	0.142	1
PRW	0.121	5.01[b]	1	-0.047	0.90	1	0.306	0.247	1
NAPA	0.0021	0.150	1	0.0005	0.05	1	-0.082	2.035[d]	1
NAW	-0.0002	0.002	1	0.004	0.07	1	0.749	3.672[c]	1
PAW	-0.033	1.143	1	-0.011	0.16	1	-0.141	0.038	1
	$R^2 = 0.5103$		12	$R^2 = 0.3832$		12	$R^2 = 0.2272$		12

[a] Functional form.
[b] On all production surface equation tables, indicates that these are significant at 5 percent levels of probability.
[c] Indicates significant at 10 percent levels of probability.
[d] Significant at 25 percent levels of probability.

form exhibited superior fit and interpretation of the polynomial equation (8) was somewhat easier, it was chosen as the form to be used in deciding economic optimum for the first year alfalfa crop, as well as polynomial equation for all crops considered in this study.

DISCUSSION OF THE RESULTS AND ECONOMIC IMPLICATION

Economic Optima

The relationship derived from the above polynomial equation (8) (see p. 24) provides a basis for determining the optima input usage rates. Marginal productivity of each input in equation (8) is estimated to determine these input optimum rates. This is done by taking the first derivatives of the estimated production function with respect to NR, PR and W. Similarly derived estimates of the marginal productivity equations for other crops are shown in Table 6 of Appendix E. To obtain the estimates of input usage optimum rates, each marginal equation (9, 10 and 11) were set equal to the ratio of input price to output price and the system of equations as follows:

$$(9)\quad \frac{dy}{dNR} = -\,0.0139 + 0.000004\ NR - 0.000008\ PR + 0.0005\ W = \frac{10}{2400}$$

$$(10)\quad \frac{dy}{dPR} = -0.0046 - 0.000008\ NR - 0.00012\ PR + 0.0002\ W = \frac{10}{2400}$$

$$(11)\quad \frac{dy}{dW} = -0.2261 + 0.0005\ NR + 0.002\ PR - 0.0042 = \frac{10}{2400}$$

Data used in setting up the necessary price ratios for the system of equations were listed and their sources discussed in the earlier section of data presentation (pp. 17-18). Solving simultaneously the above system of equations, the optimum amounts were: nitrogen residual (NR) = 159.43 pounds per acre, phosphorous residual (PR) = 71.97 pounds per acre, and the amount of irrigation water to be applied (W) = 38.74 acre inches. Similarly, optimum inputs amounts were determined for all other crops considered in this study and these results are shown in second row of each crop strata in Table 5.

It was observed that some of the optimum input rates were much larger or smaller than expected, while some were negative which could reasonably be expected to be positive. For instance, the estimated optimum of large, small and negative results was observed for the crops as follows: (a) potatoes - NA = 354.96 pounds per acre, (b) water = 4.75 acre inches and W = 13.55 acre inches, (c) sugar beets - NA = -78.81 pounds per acre. Results of this nature suggest that further investigation into the estimated optimum rates is needed. Therefore, to check these results, further statistical analyses were carried out, including the estimation of probable minimum and maximum marginal physical productivities for each input. This system of equations was then used as a linear program problem. Using the confidence method, minimum and maximum marginal physical productivity for each of the inputs were determined as follows:

$$[\underbrace{b - t_{\alpha}\hat{\sigma}_b}_{\text{(Minimum)}} < \beta < \underbrace{b + t_{\alpha}\hat{\sigma}_b}_{\text{(Maximum)}}] = .90$$

where b = coefficient value of each marginal productivity,

t = test values taken at α= 95 percent, and

$\hat{\sigma}_b$ = standard error for each coefficient.

In this way, minimum and maximum bounds were set on each inputs marginal productivity. These respective bounds may be said to be greater than or equal

Table 5. Estimated optimum rates of fertilizer use and water application for first and second year alfalfa, potatoes, sugar beets and peas

Crop	Nitrogen Residual in Soil (NR) Pounds Per Acre	Nitrogen to be Applied (NA) Pounds Per Acre	Phosphorous Residual in Soil (PR) Pounds Per Acre	Phosphorous to be Applied (PA) Pounds Per Acre	Amount of Water to be Applied Inches Per Acre
First year Alfalfa					
Est. at Max. MPP's[a]	-104.18	--	142.35	--	-7.99
Estimated Optimum	159.43	--	71.97	--	38.74
Est. at Min. MPP's[b]	1398.35	--	-1281.31	--	-953.86
Second year Alfalfa					
Est. at Max. MPP's[a]	1138.55	--	203.11	--	-17.12
Estimated Optimum	63.49	--	25.23	--	33.00
Est. at Min. MPP's[b]	-58.13	--	-223.77	--	17.37
Potatoes					
Est. at Max. MPP's[a]	-1884.66	-149.05	120.63	-135.46	49.73
Estimated Optimum	134.65	354.96	84.49	86.10	25.29
Est. at Min. MPP's[b]	-178.18	-592.45	103.13	-252.36	34.89

Table 5. Continued

Crop	Nitrogen Residual in Soil (NR) Pounds Per Acre	Nitrogen to be Applied (NA) Pounds Per Acre	Phosphorous Residual in Soil (PR) Pounds Per Acre	Phosphorous to be Applied (PA) Pounds Per Acre	Amount of Water to be Applied Inches Per Acre
Sugar Beets					
Est. at Max. MPP's[a]	-3749.74	-1672637.0	-2353.67	22028464.0	12445.69
Estimated Optimum	64.19	-78.81	74.72	73.59	13.55
Est. at Min. MPP's[b]	-118.81	866.16	565.03	725.77	-212.43
Peas					
Est. at Max. MPP's[a]	-174.80	-601.73	-189.01	1304.28	-7.59
Estimated Optimum	48.28	31.22	52.36	-78.89	4.75
Est. at Min. MPP's[b]	85.29	-114.63	-225.11	-86.39	9.06

*Estimated at maximum marginal physical product.

**Estimated at minimum physical product.

to (for the upper bound) and less than or equal to (for the lower bound) the ratio of the respective variable input to crop price. Furthermore, establishing bounds on marginal productivity implies a set of bounds on the optimal resources allocation. Such a bounded solution will establish at least a 90 percent confidence interval on optimal use rates, however, because the joint probability distributions of the interaction terms are ignored (the distributions are considered completely dependent), the actual confidence level may be greater than 90 percent. For instance, it falls within the probability limits that one of the coefficients may be at the lower bound. However, to say that the marginal physical product is at the lower bound implies that all of the coefficients are at the lower bound simultaneously. Unless the distribution of the coefficients are completely dependent, the probability that all the coefficients would be at the lower bounds simultaneously would be much less than 90 percent. Therefore, the 90 percent confidence interval may be a much smaller interval than the calculated interval.

Hence, these systems of equations were treated as a linear programming problem, and an attempt was made to solve the system of bounded equations. This is illustrated for first year alfalfa as follows:

Marginal productivity of the input NR at maximum:

$$(12.0) \quad 0.0103 + 0.00006\ NR + 0.00026\ PR + 0.0005\ W \geq \frac{\text{Price of NR}}{\text{Price of alfalfa}}$$

Marginal productivity of the input NR at minimum:

$$(12.1) \quad -\ 0.038 + 0.00008\ NR - 0.003\ PR + 0.0003\ W \leq \frac{\text{Price of NR}}{\text{Price of alfalfa}}$$

Marginal productivity of the input PR at maximum:

$$(13.0)\quad 0.037 - 0.000004\ PR + 0.00026\ NR + 0.0008 \geq \frac{\text{Price of PR}}{\text{Price of alfalfa}}$$

Marginal productivity of the input PR at minimum:

$$(13.1)\quad - 0.028 - 0.0013\ PR - 0.0003\ NR + 0.0004 \leq \frac{\text{Price of PR}}{\text{Price of alfalfa}}$$

Marginal productivity of the input W at maximum:

$$(14.0)\quad -0.0061 + 0.0056\ W + 0.0005\ NR + 0.0008\ PR \geq \frac{\text{Price of W}}{\text{Price of alfalfa}}$$

Marginal productivity of the input W at maximum:

$$(14.1)\quad - 0.446 - 0.0006\ W + 0.0003\ NR + 0.0004\ PR \leq \frac{\text{Price of W}}{\text{Price of alfalfa}}$$

Solving the above sets of equations by the linear programming method resulted in an unbounded solution, perhaps the marginal value product (MVP) which is more than likely either horizontal or positively inclined to the X axis. Such cases are illustrated in Figure 3 (a) and (b). In either case, no solution exists unless arbitrary input constraints are imposed, since the marginal value product (MVP) does not intersect the resource supply or price line. In the case of Figure 3 (a), this would occur when the marginal productivity is constant; whereas in Figure 3 (b), the marginal productivity is increasing typical of stage 1 of the production. To get a bounded solution, the marginal value product should be negatively sloped as shown in Figure 3 (c), consistant with a case where the production function increases at a decreasing rate as would occur in stage 2 of production. Obviously, therefore, the unbounded solution could have been caused due to some of the inputs in stage 1 of production.

Although the confidence interval is at least 90 percent, the unbounded solution may be for a much higher confidence interval and to establish an exact

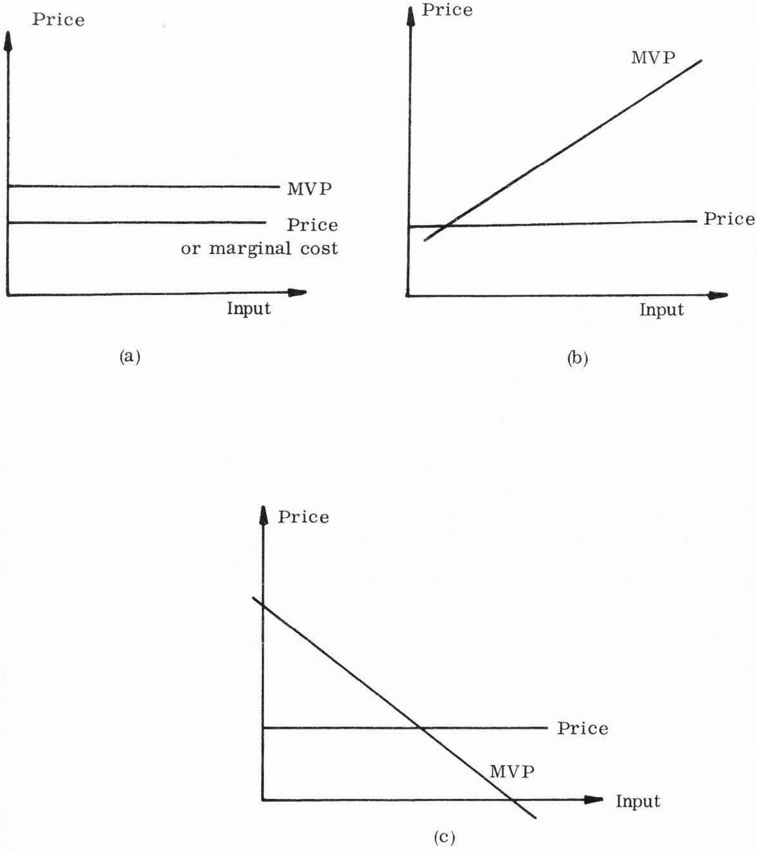

Figure 3. Illustration of unbounded, bounded solutions from marginal value product and price lines.

90 percent confidence interval, methods of stochastics programming would have to be used. However, no attempt was made to do this.[1] Within the framework of this present problem, one would say that the estimated marginal physical products implied no bounds on the optimal resources allocation.

Because no implied bounded solutions were determined, it may be useful to find out what some of these possible solutions are. Hence, three sets of solutions were determined as follows: (a) solution of the optimum was determined at the estimated marginal productivity, (b) solution of the optimum was determined at the maximum estimates of marginal productivity, and (c) solution of the optimum was determined at the minimum estimates of marginal productivity. These results are given in rows 1 and 3 of each crop strata in Table 5.

However, these solutions at the maximum estimates of the marginal productivity and the minimum estimates of the marginal productivity are not confidence intervals, but merely possible solutions. For instance, it was noted that some estimated optimum solutions were between two negative solutions of the maximum and minimum estimates marginal productivity, implying that in an equation, an increase in the value of coefficient may cause the solution value to decrease from the original equation solution, while in the same equation, a decrease in the value of coefficient can cause the solution to decrease compared to the original solution. These types of solutions could be expected and can be illustrated by the following system of equations:

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[1]Material derived from unpublished notes of John A. Tribble, Department of Economics, Utah State University, Logan, Utah.

(15.1) Estimated at Maximum MPP $-a'x + b' = d$

(15.2) Estimated optima $ax + b = d$

(15.3) $a''x + b'' = d$

Equation (15.2) is assumed to be the estimated optima at marginal productivity, where x is the variable input, and a and b are the coefficients of the marginal productivity. By equating these to d, the price ratio, the increase and decrease in the value of the coefficients from the original equation (15.2) can be shown for equations (15.1) and (15.3) as follows: (1) $a' > a < a''$ (2) $b' > b < b''$. Also, the assumed numerical values taken are: $[a' = 3] > [a = 1] < [a'' = -1]$ and $[b' = 2] > [b = 1] < [b = 0]$. Substituting in and solving the respective equations, the estimated solutions are as follows:

At estimates of maximum marginal physical productivity, from equation (15.1), $x = \dfrac{-b' + d}{a'} = \dfrac{-2 + 1}{3} = \dfrac{-1}{3}$.

At the estimates of marginal physical productivity, from equation (15.2), $x = \dfrac{-b + d}{a} = \dfrac{-1 + 1}{1} = 0$

At the estimates of minimum marginal physical productivity, from equation (15.3), $x = \dfrac{-b'' + d}{a''} = \dfrac{0 + 1}{-1} = -1$

Therefore, the above solutions indicate that it may be possible to obtain negative solutions at both the estimates of maximum and minimum marginal physical productivity.

SUMMARY AND CONCLUSIONS

Estimation of an agricultural crop production function provides a basic tool for an economic analysis, as well as for farm management decisions. Fertilizers and water applications play an important part in crop production. Since the economics of crop production is very important today, a need exists to determine to what extent product output can be increased by altering levels and combinations of water and fertilizers, and, also, to identify the optimal use of them. In addition, there is a need to know the rate of which inputs substitute for one another in the production of a given yield, in order to establish a basis for determining least-cost input combinations.

The Utah State Experiment Station and United States Department of Agriculture conducted agronomic field and laboratory studies in correlation with output performance studies. These studies provided estimates of output response for alfalfa, canning peas, potatoes and sugar beets using varying levels and combinations of water and fertilizers. The main objectives of this study were to:

1. From the experimental data made available, estimate the production functions for the four crops.

2. Apply output and input prices to translate physical outputs and inputs into monetary units.

3. Calculate the value of the marginal product for each input.

4. Determine each input's optimal levels and allocation.

Six inputs were employed for the potatoes, sugar beets and canning peas, whereas, for the first and second year alfalfa, only three variable inputs were employed. A model building program was used with these variable inputs and corresponding output data to obtain three dimensional production surfaces in pairwise combination of inputs to each crop's corresponding mean output. These production surfaces aided in selecting the production model.

A multiple regression model using linear, non-linear and interaction terms was employed in deriving three production functions for each crop. These terms were used on the basis of varying rates of input applications and what was observed from the three dimensional figures. Cogg-Douglas, square-root and polynomial functions were estimated for each model and the respective statistics analyzed. The problem of selecting a "best" model from the above three models was solved on the basis of economic theory, observed biologic physical production processes and observing the three dimensional production surfaces and statistical analyses. The polynomial form was selected as the "best" model for each crop.

Marginal productivity for each input for the different crops was calculated by taking first derivatives of each crop's polynomial function and with respect to their variable inputs. Using these, optimal rates of input were determined by equating them to the ratio of the input price to crop price and solving simultaneously. Input and output prices for the year 1970, as compiled by The Economic Research Institute, were used in this study. Qualifications of the results were required because of the non-significant statistical relationships including the F values of the regression coefficients and relatively low coefficients of determination (R^2) and also because some "optimal inputs values did not seem reasonable relative to observed rates." Further statistical analyses were carried out to determine the confidence interval (minimum and maximum) for each inputs marginal productivity.

These marginal productivity estimates were used to establish a system of inequalities of marginal physical product and price ratio (input-output price ratio). Then an attempt was made to use this system as a linear programming problem to solve for upper and lower bounds on the optimum levels of inputs. This resulted in unbounded solution. As an alternative, the above problem was rephrased as a system of equalities, and solved simultaneously to obtain optimal input levels at the marginal productivities maximum and minimum values. These estimates at minimum and maximum values are not confidence intervals.

Conclusions

Analysis of regression and statistical results including the F values of regression coefficients and coefficients of determination (R^2) for all crops gave mixed results and signal the necessity of giving careful qualification to any results obtained. For example, the highest coefficient of determination ($R^2 = 0.5103$) was for the crop potatoes and lowest ($R^2 = 0.0469$) was obtained for first year alfalfa. Together with these low coefficients of determination (R^2), the results of the linear programming problem tend to further mitigate the significance of the results. Of the three possible optimum solutions (at the minimum, estimated, and maximum marginal physical product), confidence in the estimated values were very low; however, the estimated optimum values of water input for the first and second year alfalfa were more effective in depicting the type of results expected and tended to be consistent with this inputs' average use. In other instances, the optimum values of inputs were found to be somewhat higher or lower than expected. The estimates would not be recommended for making policy decision, except in full recognition of their obvious limitations.

Despite the lack of general applicability of the results, the approach taken to estimate the different production functions and the economic analyses carried out were judged to be the correct one. Therefore, one could make further suggestions for establishing a more useful policy making tool; these are discussed in the following section of limitations and recommendations.

Limitations and Recommendations

One limitation is that experiments carried out in this study have been specifically designed to provide answers to agronomic questions and have not been a joint effort on the part of agronomists and economists to provide economic answers.

For instance, only two rates of each fertilizer application were included, which did not provide an adequate basis for economic analysis. Economic analysis would have been considerably improved if a variety of fertilization and soil moisture rates had been included in the experiment, because under such conditions, the input-output relationships could have been observed more clearly and possibly a better estimate of the production surfaces could have been obtained. Some further refinements which would have improved the analysis concern the treatment of fertilizer residuals. Instead of assuming a uniform residual (based on one year's residual), measurements of the residual at the end of each year should have been made. The output value attributable to this residual in future production could then be discounted to determine the present output value (present and future). As the study was conducted, the value of the residual (or potential output which could be produced with it) at the end of production year was considered to be equal to zero. Measuring the residual at the end of each production year and discounting would make it possible to estimate production function for each experimental period year.[1]

Concerning the economic aspects of the problem, there was the possibility that the model was improperly specified. This specification problem can

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[1]Present value of future income streams is equal to the sum of the discounted income increments:

$$P.V._T = \sum_{t=0}^{\infty} y_{T+t} \quad (1 + r)^{-t}$$

when $P.V._T$ is the present value in time period T, y_{T+t} is the income increment in time period $T + t$, and r is an interest rate. $P.V._T$ is the present value of an application of fertilizers. Y_{T+t} is the income generated from an application of fertilizer in time period T to production in time period $T + t$. t is the rate of interest charged to farmers for business loans. The difficult item to measure is y_{T+t}. For a fertilizer like nitrogen we might expect that this income increment would vary in cycles as with a nitrogen cycle, whereas, for a fertilizer such as phosphorus the income increment should decrease by a certain percentage each year.

take two forms. First, perhaps not all the relevant variables were accounted for. For instance, weather differences, some undetected physical factors could have accounted for output differences. Second, the form of production functions might have been a type not investigated, (constant elasticity, polynomial production function of higher powers, etc.) Further investigation might consider these different types of production functions.

The above considerations point up the need to conduct some part of fertility and water application research within a framework that would lead to some useful agronomic and economic analysis. This thesis study provides evidence of the necessity for joint agronomic-economic investigations.

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APPENDICES

Appendix A

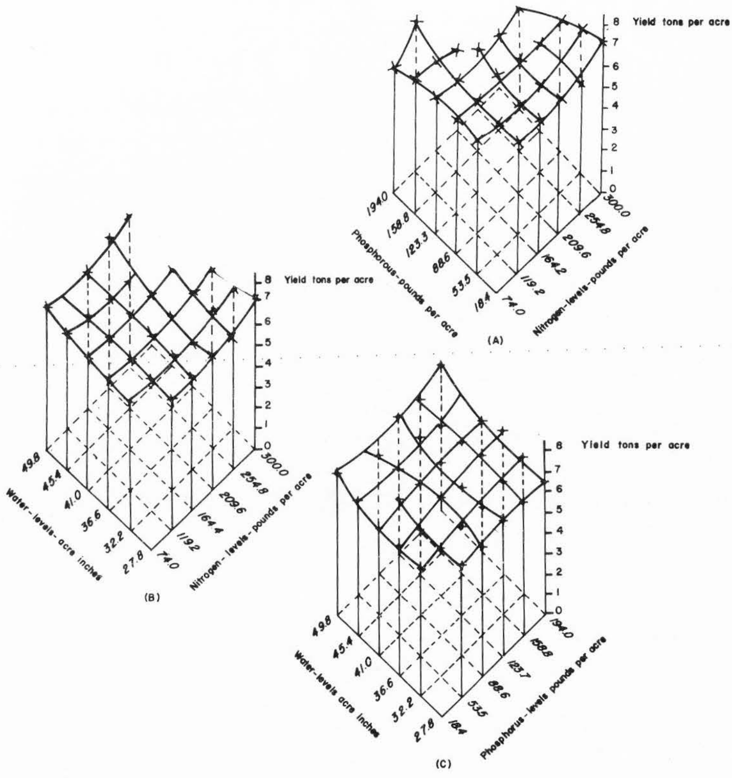

Figure 4. Main and two-way interaction effects of combinations of variables on first year alfalfa yields, 1954.

Appendix B

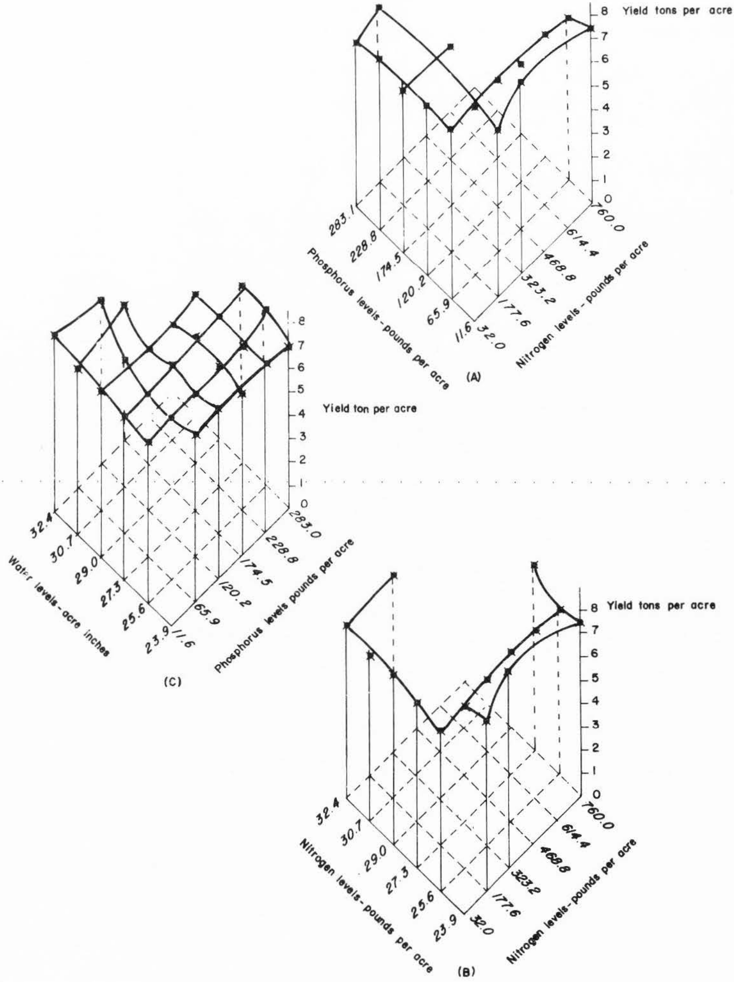

Figure 5. Main and two-way interaction effects of combinations of variables on second year alfalfa yields, 1954.

Appendix C

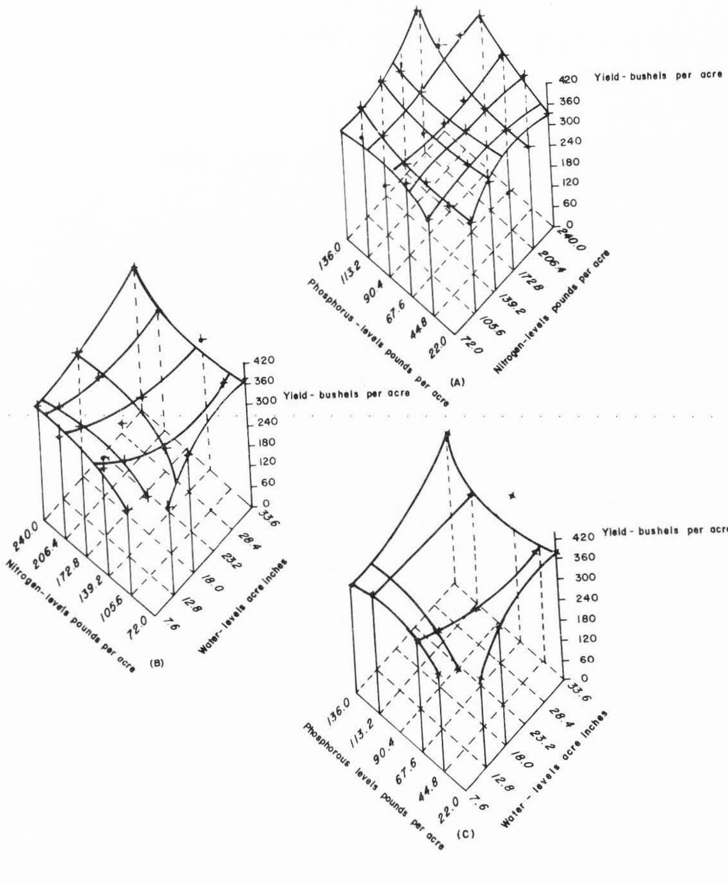

Figure 6. Main and two-way interaction effects of combinations of variables on potato yields, 1954.

Appendix D

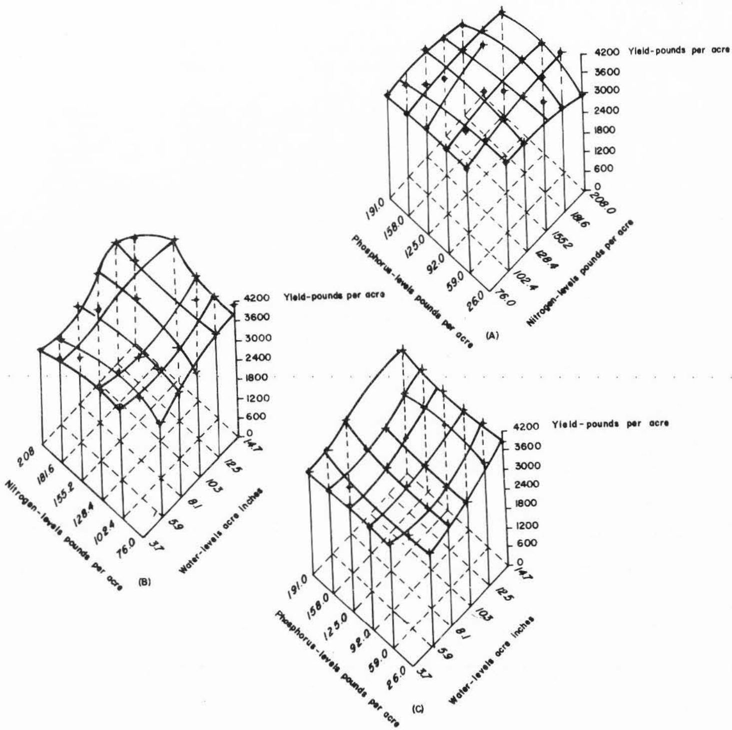

Figure 7. Main and two-way interaction effects of combinations of variables on pea yields, 1954.

Table 6. Estimated values of marginal productivity of each variable input for different crops

First Year Alfalfa					
MPP[a] - NR = -0.0139	-0.000004 NR		-0.000008 PR		+0.0005 W
MPP - PR = 0.0046	-0.000008 NR		-0.00012 PR		+0.0002 W
MPP - W = -0.2261	+0.0005 NR		+0.0002 PR		-0.0042 W
Second Year Alfalfa					
MPP - NR = 0.0043	-0.000018 NR		-0.0002 PR		+0.0005 W
MPP - PR = 0.0659	-0.0002 NR		-0.00006 PR		-0.0015 W
MPP - W = 0.4946	+0.0005 NR		-0.0015 PR		-0.0138 W
Potatoes					
MPP - NR = 3.055	-0.036 NR		+0.0094 PR		+0.042 W
MPP - NA = -0.131				+0.0021 PA	-0.0002 W
MPP - PR = 7.562	+0.0094 NR		-0.140 PR		+0.121 W
MPP - PA = -0.031		+0.0021 NA			-0.033 W
MPP - W = 32.008	+0.042 NR	-0.0002 NA	+0.121 PR	-0.033 PA	-1.758 W

Table 6. Continued

<u>Sugar Beets</u>					
MPP - NR = 1.397	-0.0084 NR		-0.011 PR		-0.0022 W
MPP - NA = -0.041				+0.0005 PA	+0.0036 W
MPP - PR= 7.177	-0.011 NR		-0.078 PR		-0.047 W
MPP - PA = 0.580		+0.0005 NA			-0.011 W
MPP - W = 24.374	-0.0022 NR	+0.0036 NA	-0.047 PR	-0.011 PA	-0.722 W
<u>Peas</u>					
MPP - NR = 8.149	-0.052 NR		-0.033 PR		-0.472 W
MPP - NA = -8.363				-0.082 PA	+0.749 W
MPP - PR = 14.973	-0.033 NR		-0.200 PR		+0.306 W
MPP - PA = 4.730		-0.082 NA			-0.141 W
MPP - W = -136.109	-0.472 NR	+0.749 NA	+0.306 PR	-0.141 PA	+25.604 W

[a]MPP--refers to the marginal physical productivity of the respective input.