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DEVELOPMENT AND VALIDATION OF A WAIVER TEST FOR THE MATH  
CONTENT COURSES REQUIRED OF ELEMENTARY EDUCATION  
MAJORS AT UTAH STATE UNIVERSITY

by

Marvin Nelson Tolman

A dissertation submitted in partial fulfillment  
of the requirements for the degree

of

DOCTOR OF EDUCATION

in

Curriculum Development and Supervision

Approved:

UTAH STATE UNIVERSITY  
Logan, Utah

1975

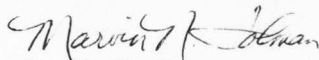
## ACKNOWLEDGMENTS

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Marvin N. Tolman

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## ABSTRACT

Development and Validation of a Waiver Test For the  
Math Content Courses Required of Elementary  
Education Majors at Utah State  
University

by

Marvin Nelson Tolman, Doctor of Education

Utah State University, 1975

Major Professor: Dr. Bryce E. Adkins

Department: Elementary Education

Purpose and Procedures

The purpose of this paper was to construct and validate an instrument which could effectively identify students enrolled in the elementary teacher training program at Utah State University whose mathematical competency equals or exceeds the standard for completion of the required math content courses prior to taking those courses. It was the hope of the writer that such students would be allowed to waive those courses and substitute courses representing areas of greater need for the individual student.

Care was taken in the construction and validation of the instrument to assure content validity with college-level texts designed for such courses and elementary texts which the teacher is expected to use in the teaching situation.

Preliminary forms of the instrument constructed in the study were administered to pilot groups. An item analysis was performed to determine the level of difficulty and discriminating power of individual test items as well as the ability of the distractors to distract. The final form consisted of 58 multiple-choice test items.

### Results

Using Kuder-Richardson reliability formula number 20, the reliability coefficient of the instrument was .87, which indicates a high level of reliability for a screening test. The variance was 82.72, resulting in a standard deviation of 9.10.

Correlating the test results, when used as a pretest in the above-mentioned courses, with final grades in the course resulted in a correlation coefficient which was significant at the .01 level of significance, indicating high predictive validity of the instrument with actual success in the course.

(143 pages)



CHAPTER I  
INTRODUCTION

Background of the study

Utah State University requires of its elementary education majors a two-course sequence in math content dealing with the real number system. This equates to approximately one-half the amount recommended by the Committee on Undergraduate programs in Mathematics (1960). No requirement exists at this time with regard to algebra or geometry. The math department at the university offers a course in geometry which is designed for pre-service elementary teachers. Students are encouraged to take this course but are not required to do so.

Some students enter the teacher education program at Utah State University having already taken courses in which the content covered in the required math courses, Mathematics for the Elementary School Teacher, has been considered. The University Council on Teacher Education (1973) at USU has begun dealing with this problem by issuing the following directive:

The Department of Elementary Education is requested to identify instruments for determining competencies in Mathematics, Educational Psychology, and Human Growth and Development during the coming school year. (University Council on Teacher Education, 1973, unpagged)

The utilization of such a test would enable identification of students for whom it would be repetitious to take

the required math content courses. This requirement could then be waived for those students who have the requisite knowledge and skills.

It is believed by some faculty members in both the math department and the elementary education department that the geometry course should be a required part of the curriculum for pre-service elementary teachers. The question of priority among math and other areas, however, is a source of disagreement. Perhaps if a waiver test was available for the basic Math 201-202 series, the course in geometry could be justified as a required course. This would bring USU one step closer to the recommendations of the Committee on the Undergraduate Program in Mathematics.

#### Purpose of the study

The purpose of this study was to construct and validate an instrument which could effectively identify students enrolled in the elementary teacher training program at USU whose mathematical competency equals or exceeds that required for Math 201-202 prior to taking those courses.

#### Objectives of the Study

The specific objectives of this study were as follows:

1. To review the literature concerned with tests designed to waive math course requirements of undergraduate elementary education programs.

2. To determine whether a test was already available which would serve adequately as a waiver test for the math content courses required of elementary education majors at Utah State University. If a test had been available it would have been used and norms would have been established for it for this purpose. Since such a test was not located the following steps became objectives for the study:

a. To construct and validate an instrument for the purpose stated above in "Purpose of the Study."

b. To establish norms for the test and identify a reasonable pass-fail criterion to recommend for use with the instrument when it is used as a waiver test for the math content courses required of elementary education majors at USU.

#### Definition of terms

1. CUPM. The Committee on Undergraduate Programs in Mathematics of the Mathematics Association of America.

2. Math Content Course. A course designed to teach the conceptual structure of mathematics, generally taught by the mathematics department of a college or university.

3. Math Methods Course. A course oriented toward methods of teaching mathematics in the elementary classroom.

4. Waiver Test. A test designed to identify students who have sufficient competencies in a particular area that one or more specific courses, otherwise required, would not be required of them.

5. Pre-service teachers. People who are training to become teachers but who have not yet completed the requirements for the teaching certificate.

6. Math 201-202. Mathematics content course designed for elementary education majors, involving sets, logic, and development of the real number system.

7. Math 301. A course designed to be the equivalent of Math 201 and Math 202 combined.

CHAPTER II  
REVIEW OF THE LITERATURE

Much interest has been expressed to the writer concerning the idea of a waiver test for math content courses required of elementary education majors by educators in many parts of the United States. Ten percent of the respondents to the questionnaire referred to in Chapter III have requested copies of the results of this research.

The literature published since 1960 has been searched and many teacher training institutions throughout the nation have been contacted (see Chapter III pp. 22-23). From this effort, no literature was discovered which dealt directly with tests designed to waive math content course requirements for prospective elementary teachers who may have acquired adequate mathematical competence prior to entering a teacher education program.

The year 1960 was selected as a logical beginning for the time span involved in this review because by that time the effect of the "modern mathematics" movement was being felt across the nation and the transition from "traditional" to "modern" mathematics had begun.

The focus of the review of literature for this dissertation is upon the following areas: (1) some of the problems which lie in the programs designed for the mathematical training of elementary teachers; (2) mathematical

requirements of elementary education majors; and (3) the effectiveness of programs designed for the mathematics training of elementary teachers.

#### Some curricular problems

One of the problems which colleges and universities must face is that of deciding which courses will best prepare a student in his chosen field. This problem has been compounded in the outlining of courses for the mathematics training of elementary teachers since the advent of the space age, when greater emphasis was placed on the training of scientists and mathematicians in the United States. L. J. Maconi (1972) stated that updating curricular requirements for pre-service teacher education is one of the most frustrating experiences in curriculum development at the college level.

Gerald R. Rising (1969) claims that training of teachers to teach mathematics is being very poorly done. He attributes this to a lack of leadership, direction, and finances, and to professors being more interested in expending their efforts toward their own professional growth and status than toward meeting the students' needs. The teacher education phase of professional education, according to Rising, is the lowest of the low on the status scale today.

The effectiveness of mathematics content courses designed for elementary teachers is weakened by the fact that very few instructors of these courses have had experience

in teaching in the elementary school themselves.

A study made by Robold (1965) on the background of college instructors of mathematics for prospective elementary-school teachers indicated that less than one-fourth of the instructors had taught in elementary school and still fewer had been certified to teach in elementary school. Less than one-tenth had all three of the following qualifications: (1) expressed an interest in elementary-school teacher education; (2) had the equivalent of a master's degree in mathematics; and (3) had certified for and experience in elementary-school teaching. The mathematical knowledge acquired in these courses could no doubt be made more meaningful to participants if the courses were taught by someone who not only had adequate background in mathematics but who also had taught in the elementary school.

If instructors with such dual qualifications were employed, perhaps it would be more logical to combine the math content courses with the math methods courses. Phillips (1968) studied the feasibility of a combined content-methods course as compared to the content course traditionally taught by the math department and methods course taught by the education department. He concluded that the combined content-methods course is a more meaningful and efficient approach to the teaching of mathematics to elementary teachers. This idea was supported by Hunkler and Quast (1973), who found that the attitudes of teachers

toward mathematics are also improved through the use of a combined content-methods course.

Another weakness in the training program for elementary-school teachers has been pointed out by Cadenhead and Newell (1973) in a study of the TTT (Training Teacher Trainers) Project conducted at Auburn University and funded by the U. S. Office of Education. One of the claims of this study is that:

For many years prospective teachers have been admonished to consider individual differences of the learner in the learning process. Yet while these prospective teachers themselves were learning how to become teachers they remained a part of a group, with little attention given to their own individuality. Such situations, where college professors have not modeled the kind of behavior they advocate, have no doubt contributed to group-oriented rather than individual-oriented education at all levels of the education enterprise. (Cadenhead and Newell, 1973, p. 51)

It seems that one logical place for individual differences to be considered would be to have some means of screening out those students for whom a given course would be repetitious of previous experience. Clarence Phillips (1968) stated, after studying the training of prospective elementary math teachers:

Specific objectives must be formulated in terms of major categories, such as operational skills in arithmetic and algebra, meaning and understanding in arithmetic and algebra, etc. The knowledge of students entering the elementary education curriculum should be evaluated in terms of these objectives, and courses beginning where the students are (in knowledge, not high school math credits) set up so as to achieve the prescribed goals with the greatest economy of time. (Phillips, 1968, p. 297)



It was recommended by Clark (1974) that training institutions in the state of Utah develop evaluation materials to measure the basic mathematical competencies required of prospective elementary teachers and offer remedial courses for those needing help in the fundamental mathematical concepts.

Clark's research also identifies many characteristics of effective elementary mathematics teachers, only one of which is a knowledge of that which is to be taught. In the opinion of the writer, this points out another reason to identify those who already have the background given in the math content courses--so they can carry on with material which is of greater need to the individual.

#### What should be required?

Robert Edward Rays (1967) concluded that there appears to be little agreement between instructors as to what topics should be covered in mathematics preparatory courses for elementary-school teachers. Marilyn Ray (1967) claimed that except for limited coverage of geometry and graphs, the textbooks used in college mathematics and methods courses adequately introduce the mathematical content most frequently emphasized in elementary schools. She recommends, however, that more background be given in algebra and geometry. Jane O. Swafford (1972) reports that Northern Michigan University accomplishes this by offering a 30-hour math major specifically designed for the prospective elementary-school teacher.

Educational Services Incorporated (1963), reporting on the Cambridge Conference on School Mathematics, stated that fragments of algebra, number theory, geometry, and probability will appear in the classroom and the teacher should therefore have more than fragmental knowledge of these. "We propose, as a rough criterion, that the teacher should know enough about the mathematical background to be pleased, instead of being embarrassed, by nearly all the questions that an eager and able student is likely to ask." (p. 199)

In planning the curriculum for the prospective elementary teacher, cautions Henry VanEngen (1972), there is great danger in overemphasizing the structural aspects of mathematics. Mathematics, he says, for mathematics sake, is not "their thing." He also expressed that there must be some relation between what prospective teachers learn in mathematics courses for elementary teachers and what they are going to teach. Weaver and Gibb (1964) also had this concern at the time of their study and stated that, "Too frequently, teachers have not studied the kind of mathematical content that they are being called upon to teach." (p. 281)

Some rather interesting opinions of in-service elementary teachers as to the mathematics training needs of pre-service teachers were reported by Sherrill (1973). Of 137 teachers surveyed, most felt that two content courses and two methods courses should be required in the

undergraduate curriculum. The study also reviewed the requirements of 25 accredited colleges and universities in the same state. Seven of these required two math content courses, one required three content courses, and none required two methods courses. The study continues with a review of state requirements for elementary teacher certification. It reports that, of the fifty states plus the District of Columbia, one requires 6-9 total hours of math, ten require 4-6 hours, eleven require 1-3 hours, and twenty-nine states require no math courses for certification.

In an effort to encourage institutions of higher education to gear up to meet the challenge of preparing teachers to teach mathematics, the Committee on the Undergraduate Program in Mathematics (CUPM) recommended minimum standards for the training of teachers on all levels. The CUPM (1960) recommendations for Level I (teachers of elementary-school mathematics) included the following: (1) a two-course sequence devoted to the structure of the real number system and its subsystems; (2) a course devoted to the basic concepts of algebra; and (3) a course in informal geometry.

CUPM has defined a "course" to mean a three-semester-hour course or equivalent. These requirements were to be in addition to courses in methods of teaching mathematics. It was recommended by CUPM that elementary teachers acquire training in addition to the twelve semester hours when possible. They recognize that the materials in these

courses might, in a sense, duplicate material studied in high school by the prospective teacher, but they urge that this material be covered again, this time from a more sophisticated, college-level point of view. The exact length of the training program is intended to vary according to the strength of the individual's preparation.

Since the CUPM recommendations were issued in 1960, efforts have been made by CUPM and others to determine the impact of these recommendations on the college curriculum. A CUPM (1967) report revealed that by 1966, the number of colleges requiring no math of their elementary education majors had dropped from 22.7 percent to 8.1 percent. Colleges requiring five or more semester hours had increased from 31.8 percent to 50.1 percent. Progress was reported to be slow, with some backsliding taking place. Of 715 colleges whose requirements were recorded in both surveys, 58 had decreased the number of required hours of mathematics.

After studying the extent of implementation of CUPM Level I recommendations, Fisher stated:

...this writer recommends that every institution in the United States preparing elementary school teachers should make immediate plans for the adoption and implementation of the Level I CUPM recommendations.

While the results of this survey indicate that significant progress has been made, we are in no position to be self-satisfied. Rather, circumstances suggest the advisability of an extension of efforts in this regard. (Fisher, 1967, p. 197)

Hunkler (1971) took a new look at the implementation of the CUPM recommendations by surveying teachers to find

out how much mathematics training they had actually acquired instead of how much had been required of them. Out of 211 teachers sampled, only one had satisfied the minimum recommendation of 12 semester hours. Ninety per cent had completed less than six hours and 60 per cent had not completed any hours in the college math courses recommended by CUPM.

Many institutions, although accepting the CUPM recommendations as desirable, have not made these recommendations a reality in their program of courses for elementary education majors. The following conclusions were drawn by Pitts (1974) after studying the mathematics content courses required of undergraduate elementary majors in the United States: (1) the CUPM recommendations of 12 semester hours has been accepted as desirable by more than half of the institutions; (2) the CUPM content unit recommendations for elementary majors in mathematics have not been implemented by more than half of the institutions; and (3) the CUPM Level I recommendations of a course in geometry has not been implemented by more than half of the institutions.

A survey was made of four institutions in Indiana by Smith (1971) to determine whether the specific topics recommended by CUPM were being taught in mathematics courses designed for preparation of elementary-school teachers. He found that all the topics listed under the CUPM course title "Structure of the Number System" were presented by

all the instructors. Ten out of the fourteen topics listed under the CUPM course title "Geometry" were presented by all instructors. None of the topics listed under the CUPM course title "Algebra" were presented by the majority of instructors in each of the four institutions.

The mathematical understandings of prospective elementary teachers in several colleges having different mathematics requirements were compared by Withnell (1968). He concluded that prospective elementary teachers are not being satisfactorily prepared by a three, six, or nine semester hour mathematics requirement. He suggested that the number of courses actually needed for satisfactory preparation of prospective elementary teachers may be far beyond the current recommendations.

Not all educators agree on this matter, however. Miller (1969) expressed the opinion that inasmuch as elementary teachers must be competent in mathematics, science, reading, social studies, language arts, health, music, physical education, etc., it is unrealistic to require 12 semester hours in mathematics alone. Since many teachers will be working in other specialized areas, Miller considers it a waste of both student and instructor manpower to require that depth of mathematical study. He proposed that training institutions offer a special minor, at both the undergraduate and graduate level, which bypasses the traditional sequence.

Are elementary teachers  
adequately competent to  
teach mathematics?

Although no test has been located in the literature which has been standardized and widely used for measuring the mathematical competency of pre-service elementary teachers, there are some instruments which have received limited use for this purpose, as well as for in-service teachers. Those which have been located in various studies are referred to in the following paragraphs.

The Utah Elementary Teachers Mathematical Competency Evaluation Instrument, developed by Clark (1974), was used to measure the mathematical competency of elementary-school teachers in selected Utah school districts. The test consisted of a revision of the 1969 test developed by Lita Schwartz and the 1965 test from the School Mathematics Study Group. A sample of 354 teachers from 12 Utah school districts were tested with a resulting mean score of 29 correct out of 65 problems.

Prospective elementary teachers in Georgia were tested by Creswell (1964). The sample included 313 students near graduation at eight teacher-trainer institutions. This was 72 percent of the estimated elementary graduates from the eight institutions. The measuring device used was the Metropolitan Achievement Test, Advanced Arithmetic Form AM 1959 edition. The test consists of 45 computation items and 48 items dealing with concepts and problem solving. The results were as follows: 81.6 percent scored at or

above the ninth grade level on the computation section and 90 percent scored at or above the ninth grade level on concepts and problem solving.

The mathematical status of elementary education majors at the University of Missouri was measured by Reys (1968) using the Algebra Level of the Contemporary Mathematics Test, a test designed for high school students who were completing one year of contemporary algebra. Data was collected from 234 elementary education majors who were enrolled in a math content and/or a math methods course. They were pre-tested during the first week of the semester and post-tested in the final week. There were significant gains in scores from pre- to post-test for both the content course and the methods course students. Post-test mean scores, however, were significantly below the means of eighth- and ninth-grade students on the same test.

A study was done by Koeckeritz (1970) involving 147 high school sophomores, 154 college freshmen, 171 college seniors (elementary education majors), and 153 in-service teachers, all in South Dakota, to analyze mathematical and professional knowledge of present and future elementary teachers. The instrument used was Callahan's Test of Professional and Mathematical Knowledge for Elementary Teachers. On the mathematical concept knowledge section, Koeckeritz found no significant difference among the four groups. Group mean scores ranged from 49.4 percent for college seniors to 44.1 percent for high school sophomores.



The differences were not significant at the .01 level. Koeckeritz concluded that according to his findings, neither years of experience, courses completed, nor level of training had any significant effect on attainment on a mathematical concept test.

Forty-one teachers from six suburban Philadelphia school districts were tested by Melson (1965). These teachers represented 17 colleges and universities in seven states, both public and private institutions, large and small, some devoted solely to teacher training and some offering training in many fields. The school districts involved were superior districts with high academic standards, desirable salary schedules, excellent locations, and frequently received national recognition for leadership in educational endeavors. The instrument used contained 33 test items. The median score was 36 percent. Two subjects scored above 75 percent, 27 scored between 25 and 50 percent, and 12 scored below 25 percent.

Frank Smith (1967) administered Melson's test to two groups of elementary education majors. In Smith's study the test was given as a pre-test at the beginning of the math methods course and as a post-test at the end. The pre-test scores were slightly higher than those obtained by Melson and the post-test scores were significantly higher, with 90 percent of the students scoring above 50 percent correct and 45 percent scoring above 75 percent correct.

Teachers in 100 elementary schools in Colorado were tested by Harper (1964). Those who had acquired a course in "modern mathematics" performed significantly better on the test than those who had not had such a course. Those who had no "modern math" but had six or more hours of college mathematics performed at a significantly higher level than those who had no "modern math" and less than six hours of college math.

Using a test constructed by himself, Kenney (1965) tested 356 teachers in California. On the 50-item test the scores ranged from 5-44 correct. Thirty-six subjects had 40-44 correct and four had less than 10 items correct. The median score was 29.7 items correct. Kenney designed his own test because he was unable to find an instrument in the literature to measure the understandings which seemed appropriate.

With a sample of 310 elementary teachers representing many schools and districts, Kipps (1968) tested for the ability to understand concepts used in new math curricula. The school districts involved were widely varied in size, geographic location, teacher experience, pupil population, etc. The mean score was 68 percent. Dividing the subjects according to amount of formal mathematics coursework, one group having received one-half year or less and the other group more than one-half, the mean score of each group was 68 percent. The amount of formal coursework had no significant effect on performance on the test.

McKillip and Mahaffey (1972) evaluated the knowledge, of 71 teachers in Atlanta, Georgia, of the content and methods of teaching elementary school mathematics. They found that those with more college math courses scored significantly better on the test than those with fewer math courses. Going beyond that, they even suggested that the influence of the teacher's mathematical preparation from his or her own elementary school background seems highly significant.

An instrument designed for evaluating North Carolina elementary-school teachers' understanding of contemporary arithmetic was designed and used by Griffin (1967). Griffin found that the subjects understood less than one-half of the total arithmetic topics covered by his test and only one-third of the 27 topics pertaining to "modern mathematics." He found a positive relationship between semester hours earned in college mathematics and teachers' understanding of arithmetic.

After studying the competence in geometry of undergraduate elementary education majors in Oregon, Bailey (1970) concluded that university programs for training elementary teachers in geometry concepts are unsuccessful. This would likely be attributable, in some degree, to the fact that most elementary education majors do not take the geometry courses designed for them since these courses are usually in the elective category rather than required.

Bailey also found no positive relationship between number of term hours of college mathematics and performance on the criterion test. His results did show a positive relationship, however, between performance on the test and term hours of math content courses designed for elementary education majors.

A survey by R. E. Reys (1967), of in-service teachers, indicates that those desiring additional training in mathematics prefer methods courses over content courses by nearly a two to one majority.

Further comparison of the effect of math methods courses and math content courses were made by Cox (1970). She found that a positive relationship exists between competence level and the number of hours in math methods courses for elementary teachers of grades three and six. According to her study, a positive relationship does not exist between competence level and the number of hours of college mathematics courses. Cox also tested the relationship of teachers' knowledge of mathematics and pupils' achievement in mathematics. The study showed no significant effect at the .01 level.

### Summary

The literature reviewed indicates, in general, that elementary teachers do not perform well on diagnostic tests in mathematics, particularly on "modern" math concepts. The results, however, are widely varied and

inconclusive and the instruments used are as yet unproven. This fact is demonstrated by the inconsistency with which the various studies indicate whether or not a positive relationship exists between the number of mathematics content courses taken and performance on the instruments. There is not sufficient evidence at this point to either accept or reject many of the conclusions that have been drawn.

## CHAPTER III

### PROCEDURES

The purpose of this chapter is to describe the procedures which were used in the research: (1) in an attempt to locate an existing test which had been designed for use as a waiver test for the mathematics content courses required of undergraduate elementary education majors; (2) in constructing and validating a suitable instrument; and (3) in analyzing the data.

#### Search for an existing test

In an effort to locate an existing instrument which would be suitable to use as a waiver test for the math content courses required of elementary education majors at Utah State University, the following steps were taken:

1. Burros' Sixth and Seventh Mental Measurements Yearbooks were searched.
2. The most recent 10 years of Education Index and Research in Education were reviewed.
3. Personal interviews were held with professors in the departments of mathematics, elementary education, and psychology at Utah State University.
4. The Counseling and Testing Services at USU were consulted.
5. Departments of mathematics at the University of Utah, Weber State College, Brigham Young University, and Southern Utah State College were contacted by mail.
6. The mathematics specialist of the Utah State

Department of Public Instruction was contacted by telephone.

7. Braswell's (1972) "Math Tests Available in the United States" was reviewed.

8. A letter of inquiry was written to Educational Testing Service in Princeton, New Jersey.

From the above search no test was discovered which was designed and content-validated for use as a waiver test for mathematics content courses required of elementary education majors.

In a continuing effort to locate an adequate measuring device, questionnaires were sent to 100 colleges and universities. These were randomly selected from the 434 institutions, outside the state of Utah, accredited by the National Council for Accreditation of Teacher Education for training elementary-school teachers as listed by the American Council on Education (1968).

Letters were written to those institutions reporting use of locally-developed tests, requesting copies of the tests and information regarding content validity and reliability. Permission was requested in each case to use all or part of the test at USU.

#### Construction and validation of an instrument

In the process of constructing and validating an instrument to meet the objectives of this study, the following steps were taken:

1. A content analysis was made of three college-level texts designed for use in mathematics content courses for elementary majors and three elementary mathematics series. The college-level texts used in the content analysis were Wheeler (1973), Peterson (1971), and Forbes (1971). The elementary texts used were the fourth, fifth, and sixth grade math textbooks published by Houghton-Mifflin (1972 edition), Addison-Wesley (1971 edition), and Holt, Reinhart and Winston (1970 edition). Wheeler's text was selected because it was the current text for Math 201-202 at USU. Peterson and Forbes were added in order to increase the objectivity of the content findings and were selected on the recommendation of one instructor of Math 201-202 as being representative of texts used for math content courses designed for prospective elementary teachers. The three elementary textbook series used were selected on the basis of being currently the three most widely-used mathematics series in the elementary schools of Utah. The purpose of this analysis was to identify topics and measure the emphasis placed on each topic. Emphasis was judged on the number of pages devoted to each topic.

2. Six persons, selected to represent the teacher training institutions of the state of Utah and the Utah State Department of Public Instruction, and who have had a great deal of experience in the mathematics training of elementary-school teachers, were asked to serve as a "panel of experts" to assist in the study. All six of these accepted



the request. The panel consisted of three individuals with a mathematics-oriented background and three with an elementary education-oriented background. Five of these were professors at teacher training institutions in the state of Utah, and the other was the mathematics specialist for the Utah State Department of Public Instruction. The panel members were selected on the basis of their position and their role in the mathematics training of elementary-school teachers.

3. A list of the 100 topics with greatest emphasis in the textbooks analyzed was sent to the panel of experts. They were asked to rate each topic, on a scale of one to five, according to the emphasis which they considered appropriate for that topic in the test being constructed.

4. On the basis of ratings given by the panel of experts and ratings derived from analyzing the content of textbooks, 60 mathematical topics were identified for representation in the test (see chapter 4, p.32 for reasoning in selecting 60 topics). Based on the realization that there are many more than 60 elementary-school mathematics topics which are important and the necessary limitations on the length of the test, stratification was used in selection to assure that each general area received representation which is appropriate according to the ratings given. Relative textbook emphasis and panel ratings were given equal consideration in the selection of these topics.

5. A pool of test items, comprised of three items for

each of the 60 selected topics, was written and submitted to the panel of experts. The panel was asked to rank the problems within each set of three according to first, second, and third choice. In ranking the items, the experts were asked to consider such characteristics as mathematical correctness, clarity, and appropriateness for the waiver test. Comments and suggestions were also solicited.

6. Based on the rankings of the panel, one test item out of each set of three was screened out. Revisions were made, as suggested by the experts, and a preliminary test was constructed using the remaining 120 items. Due to time limitations in administering the test, the preliminary form was divided into two parts, called Form A and Form B. Each of these two parts consisted of 60 items; one from each of the 60 selected topics. The purpose for constructing two preliminary forms was to determine the difficulty and discriminating power of more items than were needed for the final form of the test. Thereby the more appropriate item from each pair could be selected for inclusion in the final form.

7. Preliminary test forms A and B were each administered to a pilot group consisting of 71 students currently enrolled in math content courses designed for elementary education majors at Utah State University and Brigham Young University.

8. An item analysis was performed on the test items of the preliminary form to determine the level of diffi-

culty and discriminating power of each item, as suggested by Borg and Gall (1971), and the ability of the "distractors" to distract. The index of difficulty was determined by computing the percentage of the pilot group missing the item. Discriminating power of each item was determined by computing the difference between the percentage of high scorers (top 27 percent) missing the item and the percentage of low scorers (bottom 27 percent) missing the item, resulting in the index of discrimination. The worth of the distractors was determined by comparing the number of high scorers (top 50 percent) to select each of the alternatives with the number of low scorers (bottom 50 percent) to select each of the alternatives.

9. On the basis of the item analysis, one test item was screened from each of the 60 selected topics. Priority was given items with high discriminating power and with effective distractors.

10. Due to the amount of time required to administer the test, it was felt that the length should be decreased. After examining the remaining 60 problems, items A-12 and B-33 were considered to be of least value to the test. Item A-12 had the lowest index of discrimination and three of the panel members expressed the opinion that this item was meaningless, that the terms "multiplier" and "multiplicand" can be used interchangeably with the first and second factors. Therefore item A-12 was discarded. Item B-33 had a low index of discrimination and its removal did

not seriously weaken the representation of fractions compared with textbook emphasis, so this item was also removed from the test. Elimination of additional items seemed to be unjustified.

11. The final form of the test was constructed from the remaining 58 problems. Revisions were made where it seemed advisable and the final form of the test was administered to a second pilot group, again for the purpose of item analysis, to verify the effectiveness of the test items. The second pilot group was similar to the first, consisting of 65 students enrolled in mathematics content courses designed for elementary education majors at Utah State University and Brigham Young University.

12. The final form of the instrument was administered to the sample. All students enrolled in Math 202 at USU during spring and fall quarters of 1974 were tested during the last week of the course for the purpose of gathering data to be used in establishing norms for the test. The instrument was administered to all students enrolled in Math 301 (combined 201-202) at the beginning of summer quarter 1974, and the performance was compared with student success in the course, for the purpose of establishing predictive validity of the instrument.

#### Analysis of data

Kuder-Richardson formula 20 was employed to determine internal consistency of the test by analysis of individual

test items.

Results of the test administered to the students of Math 301 were correlated with the final test scores and the final grades achieved by those students. Pearson Product-Moment correlation was employed for this analysis to provide information about the predictive validity of the test.

## CHAPTER IV

## RESULTS

A valid and reliable test to be used to determine eligibility to waive the mathematics content courses required of elementary education majors was not found as a result of the extensive search described in Chapter III. As a consequence, development and validation of such a waiver test was undertaken. The purpose of this chapter is to present the findings from the questionnaire and from the step-by-step process of constructing and validating the Test of Mathematical Fundamentals, an instrument designed for use as a waiver test of the math content courses required of elementary education majors at Utah State University.

Questionnaire findings

Responses were received from 91 of the 100 colleges and universities to which questionnaires were sent. Of those responding, 65 reported that they use no waiver test at all for the mathematics courses required of elementary education majors; 10 use the mathematics section of the College Level Examination Program (CLEP), published by National Testing Service; and 16 use waiver tests which were developed locally.

In response to letters written to the 16 institutions

reporting use of locally-developed tests, 11 institutions replied: Two reported using the regular course final examination as a waiver test and did not send copies; one does not really use a test but waives the course requirement on the judgement of the student's advisor; and eight institutions submitted copies of their waiver tests. Of the eight institutions submitting tests, none had collected information as to content validity or reliability of the test. Seven of these granted permission to use their tests, all or in part, at Utah State University. One institution withheld such permission and requested that the test items not be used. This request was honored in the search for items for the pool of test items.

#### Rating and Selection of topics

A page-by-page analysis was made of the textbooks which were involved in the study (see chapter III, p. 24 ). All topics treated in the texts were listed. Topics which were generally treated in clusters, rather than individually, such as points, segments, rays, and lines, were listed in clusters (hereafter, "topic clusters" and "topics" are referred to simply as "topics"). Each topic was assigned a rating of zero to five, according to the emphasis on that topic in the textbooks. The following method was used in assigning the ratings:

1. Any topic which was treated by only one of the textbooks analyzed was considered relatively inappropriate and disregarded for the purposes of this test.

2. For the remaining topics, of which there were 100, ratings of 0.5 to 5.0 were assigned, according to emphasis in the selected elementary textbook series.

3. Ratings of 0.5 to 5.0 were assigned according to emphasis in the selected college textbook series.

4. The mean textbook rating for each topic was computed by adding the elementary textbook rating to the college-level textbook rating and dividing the total by two.

5. The list of topics was submitted to and rated by the panel of selected experts.

6. An overall rating for each topic was determined by computing the average of the mean textbook rating and the mean expert rating. The overall ratings ranged from 0.5 to 4.0. Table 1 (see Appendix A) lists the textbook rating, panel rating, and overall rating for each of the 100 topics.

To conform to the normal length of class periods, an effort was made to construct a test which would require approximately 45 minutes to administer. On the basis of expert opinion it was decided to allow an average of 45 seconds per item in determining the time required to administer the instrument. Thus, the test was to consist of approximately 60 items. It was arbitrarily decided that the items would be constructed to represent the 60 topics receiving the highest overall textbook-expert ratings.

In order to assure appropriate representation from each general area the following procedure for stratification was used:



1. The total rating points for the set of 100 topics (261.2) was divided by 60 (the number of topics to be represented in the test). The resulting mean was 4.4.

2. For each multiple of 4.4 rating points in any given general area, that general area was allowed one topic for representation in the test. Priority within each general area was given those topics with the highest ratings.

Total rating points for each general area and the number of topics for representation on the test from that general area are listed in Table 2 (Appendix A).

The 60 topics chosen for representation on the test are given in Table 3. In the general area of "Graphing," further investigation showed that the concepts covered by the texts under the topic "Tables and graphs" were very similar to concepts under "Graphing coordinates." Therefore, even though the overall rating was slightly lower, "Graphing negative numbers" was selected for representation in the test in preference to "Tables and graphs."

#### Pool of test items

The pool of test items, consisting of 180 items (three for each of the 60 topics), were rated by the panel of experts as indicated in Table 4. In each group, the item which ranked third (listed in the right-hand column

of Table 4) was screened out. The numerals in Table 4 identify the items as numbered in the pool. For a complete set of the pool of test items, see Appendix D.

#### Assembling the preliminary form of the test

Of the two remaining problems for each topic, the first (by rank) was arbitrarily assigned to Form A of the preliminary test and the second was assigned to Form B (see Table 5). Revisions were made in the items to be used according to the recommendations of the panel of experts. Twenty-two items were revised and five items were replaced before being included in the preliminary form. Revised and replaced items are identified in Table 5. Preliminary test forms A and B are given in Appendix E.

#### Item analysis of preliminary form

Item analysis of the preliminary form of the test, following administration of the test to the first pilot group, produced information regarding discriminating power and level of difficulty for each item, as well as ability of the distractors to distract. Table 6 gives the index of discrimination and index of difficulty for each item. For each item number, the Form A item and the Form B item relate to the same topic.

#### Assembling final form of test

In selecting problems for the final form of the test, one item out of each pair of items (each pair representing

one of the 60 selected topics) was screened out on the basis of the item analysis. In determining which item of each pair should be retained for the final form, the characteristics of high discriminating power, level of difficulty nearest .50, and effectiveness of distractors were given priority in that order. Items selected for the final form are listed in Table 7. The 11 test items which were revised at this point are marked with asterisks in Table 7. For a complete copy of the final test form, see Appendix F.

Asterisks in Table 6 mark the index of discrimination and the index of difficulty of those items chosen for the final form of the test. For reasons explained in chapter 3 (p. 27), items A-12 and B-33 were not included in the final form, thus the length of the instrument was reduced to 58 items. Item 53-A was selected in preference to item 53-B because the difference in discriminating power was only .01 and item 53-A was superior in terms of both item difficulty (see Table 6) and effectiveness of distractors.

Dividing the final form items, from the first item analysis, into categories of discrimination as described in Ebel (1965), we find that 19 items or 63.8 per cent are highly discriminating (index of .40 or higher), 20 items or 34.5 per cent are fair discriminators (.20 to .39), one item or 1.7 per cent is a low discriminator (.01 to .19), and no items are zero or negative discriminators (see Table 9).

### Item analysis of final form

Information regarding item analysis of the final form of the instrument, following administration to the second pilot group, is given in part in Table 8. The same information is given in Table 10 for the second item analysis as is given in Table 9 for the first item analysis, showing that 19 items or 32.8 per cent are high discriminators, 26 items or 44.8 per cent are fair discriminators, 12 items or 20.7 per cent are low discriminators, one item or 1.7 per cent is a zero discriminator, and no items are negative discriminators.

### Data analysis results

Reliability. Analysis of the norm group tests, using the Kuder-Richardson reliability formula number 20, yielded a reliability coefficient of .87. See Table 11 for data on test items, as used in computing the reliability coefficient.

Variance and standard deviation. The variance of the norm group was 82.72, resulting in a standard deviation of 9.10.

Predictive validity. Of 19 students completing the requirements for Math 301, which was the group used in gathering information about the predictive validity of the test, 14 had been present at the beginning of the quarter on the day the Test of Mathematical Fundamentals was administered.

No final examination was given in the course, but four tests were given throughout the course. Total points received on the four tests were therefore substituted for the final exam score for purposes of correlation.

The correlation between scores on the Test of Mathematical Fundamentals at the beginning of the course and scores on the four tests given in the course was .742, which is significant at the .01 level. For data used in this correlation see Table 12.

To correlate the scores on the Test of Mathematical Fundamentals with final grades, grade points were used, allowing four points for A, three points for B, and two points for C. No student received a final grade lower than C. The correlation between scores on the Test of Mathematical Fundamentals at the beginning of the course and final grades was .719, also significant at the .01 level. See Table 13 for data used in this computation.

Mean. The mean raw score of the norm group was 33.5. This equates to 57.8 percent. See Table 14 for frequency distribution of norm group raw scores.

## CHAPTER V

## SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The purpose of this chapter is to summarize the procedures and findings of the research and to present conclusions and recommendations based on the results of the study.

Summary of procedures

A search was conducted for an existing test which was constructed as a waiver test for math content courses designed for elementary education majors. No such test was located.

An instrument was constructed and validated, using the following steps:

1. A content analysis was made of the 4th, 5th, and 6th grade levels of the three elementary mathematics series most widely used in the state of Utah and of three college-level texts designed for math content courses for elementary teachers.
2. The 100 topics of greatest emphasis in the textbooks analyzed were rated by a panel of experts. The 60 most pertinent topics were selected on the basis of these ratings and ratings assigned according to emphasis in elementary and college-level textbooks.
3. A pool of test items, comprised of three items for each of the 60 selected topics, was written and submitted to the panel of experts. Based on rankings given by the panel, one test item on each topic was screened out.

4. Preliminary tests were constructed from the remaining items and administered to a pilot group.

5. An item analysis was performed on the preliminary test items to determine level of difficulty and discriminating power of each item as well as the ability of the distractors to distract.

6. On the basis of the item analysis, one test item was screened from each of the 60 selected topics. Two more items were eliminated to reduce the time required to administer the test.

7. The final form of the test was constructed from the remaining 58 items. This was administered to a second pilot group and an item analysis was again performed on the test items.

8. The final form of the instrument was administered to the sample.

The test appeared to be strong in discriminating power when compared with the standards recommended by Ebel (1965). See page 36 for percentages of high, fair, and low discriminators.

#### Summary of findings

The reliability coefficient of the test was .87, using Kuder-Richardson reliability formula number 20. The variance was 82.72 and the standard deviation was 9.10.

In testing the Math 301 group, during summer quarter 1974, for establishing predictive validity of the test,

the correlation between scores on the Test of Mathematical Fundamentals and actual success on the course was significant at the .01 level.

The mean raw score of the norm group, tested during spring and fall quarters of 1974, was 33.5, or 57.8 percent.

### Conclusions

Based on the findings of this study the following conclusions can be reached:

1. The Test of Mathematical Fundamentals will give acceptably reliable and consistent results. According to Garrett and Woodworth (1958), if a test is to be used to make individual diagnoses, it should have a reliability coefficient of .90 or higher. Otherwise, .60 is an acceptable reliability coefficient. The Test of Mathematical Fundamentals is not intended for individual diagnoses, but is designed for use as a screening test. Therefore, according to the standards given above, a reliability coefficient of .87 indicates high reliability of the instrument.

2. The test has high predictive validity and can be relied upon as a predictor of success in Math 201-202 or Math 301. Correlation between scores on the Test of Mathematical Fundamental at the beginning of Math 301 and actual success in the course was significant at the .01 level. The small size of the sample, however, must be considered.



3. Inasmuch as great care was exercised to assure content validity with college and elementary textbooks related to Math 201-202 and the mean percentage score of the norm group was 57.8, one of two possible conclusions could be reached.

a. The test does not have high content validity with the course despite its content validity with related textbooks.

b. In general, students of the sample did not learn well many of the textbook-related concepts taught during the course.

#### Recommendations

Based on the conclusions derived from this study the following recommendations are made:

1. The Test of Mathematical Fundamentals should be considered an acceptably reliable instrument.

2. The test should be accepted as having high content validity with textbooks related to Math 201-202.

3. The content taught in Math 201-202 should be examined for content validity with related textbooks. If the course content is found to correlate highly with textbook content, it is recommended that the Test of Mathematical Fundamentals be used as a waiver test with the criterion score set at or near the norm group mean raw score of 33.5. If the course content is not found to correlate highly with textbook content, one of two alternatives

should be elected.

a. The course content should remain unchanged and the Test of Mathematical Fundamentals should not be considered valid as a waiver test for the course.

b. The course content should be altered to correlate with related textbooks and the Test of Mathematical Fundamentals should be used as a waiver test for the course, with new norms being established.

4. If the Test of Mathematical Fundamentals is to be used as a waiver test for Math 201-202 it is recommended that the criterion score be adjusted as new norms are established, particularly after any substantial alteration of course content or change in concept emphasis in the course.

5. Testing, for the data reported herein, was done during regular 50-minute class periods. Therefore, if the Test of Mathematical Fundamentals is used as a waiver test for Math 201-202, using the current normative data as a criterion, it is recommended that a time limit of 50 minutes be imposed. If the test is to be used without limit on testing time, new data should be collected and the criterion adjusted as necessary.

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## APPENDIXES

## APPENDIX A

## Tables



Table 1. Mathematical topics and their ratings from textbook content analysis, ratings of panel of experts, and overall ratings

Topic	Textbook Rating	Panel Rating	Overall Rating
<u>SETS</u>			
Union	1.5	3.5	2.5
Intersection	2.0	3.5	2.7
Null, infinite, subset	1.5	3.5	2.5
One-to-one correspondence	1.0	3.5	2.2
Cartesian products	1.0	3.0	2.0
Relations (reflexive, symmetric, transitive)	1.0	2.2	1.6
<u>NUMBER THEORY</u>			
Even & odd numbers	0.5	3.5	2.0
Factors, greatest common factor	3.0	4.2	3.6
Primes, composites	2.5	4.0	3.2
Multiples, least common multiple	2.0	4.2	3.1
Divisibility	1.5	3.7	2.6
<u>ADDITION</u>			
Without regrouping	2.4	3.2	2.8
Regrouping	2.4	3.5	2.9
Money	0.8	2.0	1.4
Estimating	0.8	3.5	2.1
Properties	2.5	3.5	3.0
<u>SUBTRACTION</u>			
Without regrouping	0.8	3.3	2.0
Regrouping	3.2	4.0	3.6
Money	0.8	2.0	1.0
Properties	0.0	3.5	1.7
<u>MULTIPLICATION</u>			
Repeated addition	3.2	3.0	3.1
Powers of 10 as factors	0.8	3.5	2.1
Single digit factor and a factor of two or more digits	2.4	3.8	3.1

Table 1 (continued)

<u>Topic</u>	<u>Textbook Rating</u>	<u>Panel Rating</u>	<u>Overall Rating</u>
<u>MULTIPLICATION, cont.</u>			
Two factors of two or more digits	2.4	3.8	3.1
Money	0.8	2.0	1.4
Estimating	0.8	4.2	2.5
Multiplication-division relationship	1.6	4.5	3.0
Multiples of 10, 100, & 1000 as factors	2.4	3.2	2.7
Properties (comm., assoc., iden.)	2.0	3.6	2.8
Distributive property of mult. over addition	3.2	2.0	2.7
<u>DIVISION</u>			
Repeated subtraction	0.8	4.0	2.4
1-digit divisor & 1-digit quo.	2.4	3.0	2.7
1-digit divisor & 2-digit quo.	2.4	3.0	2.7
1-digit divisor & quotient of three or more digits	1.6	2.5	2.0
Involving multiples of 10, 100, and 1000	1.6	2.1	1.8
Divisors of 2 or more digits	2.4	3.6	3.0
Money	0.8	2.0	1.4
Estimating	1.6	3.8	2.7
Checking	0.8	3.5	2.1
Averages	1.6	3.5	2.5
<u>PERCENT</u>	2.5	3.8	3.1
<u>MEASUREMENT</u>			
Length	1.0	3.5	2.2
Area	3.5	4.0	3.7
Volume	2.0	4.0	3.0
Perimeter, circumference	2.5	3.8	3.1
Liquid measure	0.5	3.1	1.8
Converting units	0.5	3.6	2.0
Metric system	1.0	4.0	2.5
Weight, time, temperature	0.5	3.6	2.0

Table 1 (continued)

Topic	Textbook Rating	Panel Rating	Overall Rating
<u>FRACTIONS</u>			
Meaning	1.5	4.8	3.1
Equivalent fractions	3.5	4.2	3.8
Renaming to higher and lower terms	0.5	4.0	2.2
Reducing to lowest terms	2.0	3.8	2.9
Adding fractions and mixed numerals	4.0	3.8	3.9
Least common denominator	0.5	4.0	2.2
Mixed numerals and improper fractions	2.0	3.8	2.9
Subtracting fractions & mixed numerals	3.0	3.8	3.4
Multiplying fractions & mixed numerals	4.0	4.0	4.0
Dividing fractions	2.5	3.8	3.1
Terms	1.0	3.5	2.2
<u>DECIMALS</u>			
Fractions and decimals	2.0	3.6	2.8
Adding and subtracting	2.0	3.5	2.7
Multiplying	2.5	3.8	3.1
Dividing	2.5	3.8	3.1
Repeating and terminating	1.5	3.7	2.6
Estimating	1.0	3.8	2.4
Irrational numbers	1.0	3.1	2.0
<u>GRAPHING</u>			
Graphs on the number line	0.5	4.0	2.2
Graphing coordinates	3.0	3.6	3.3
Functions	2.0	3.5	2.7
Graphing negative numbers	1.5	3.3	2.4
Tables and graphs	1.5	3.8	2.6

Table 1 (continued)

Topic	Textbook Rating	Panel Rating	Overall Rating
<u>SPECIAL TOPICS</u>			
Bases other than ten	3.5	3.2	3.3
Other number systems (Egyptian & Roman)	2.0	1.8	1.9
Place value	2.5	4.7	3.6
Exponents	2.0	4.1	3.0
Ratio & scale drawings	1.5	3.0	2.2
Clock & modular arithmetic	2.5	2.1	2.3
Negative numbers	3.5	3.5	3.5
Probability	3.5	3.3	3.4
Sequences	0.5	2.6	1.5
Rounding numbers	1.0	3.5	2.2
Inequalities	3.5	3.2	3.3
Functions	2.0	3.7	2.8
Time, rate, distance	1.0	3.5	2.2
Mean, median, mode	2.0	3.0	2.5
Cardinal, ordinal, whole, natural, & real numbers	2.0	3.5	2.7
Congruency	1.5	3.1	2.3
Propositions & implications (inverse, converse)	2.0	3.1	2.5
Casting out nines	1.0	2.5	1.7
Standard deviation	1.0	2.0	1.5
Normal frequency curve	1.0	2.2	1.6
<u>GEOMETRY</u>			
Simple closed curves	0.5	3.8	2.2
Circle, radius, diameter, chord	1.0	3.9	2.5
Angles	3.0	3.9	3.5
Space figures	2.5	3.9	3.2
Points, segments, rays, & lines	2.0	3.8	2.9
Plane figures	4.0	4.0	4.0
Parallel and perpendicular	1.0	3.9	2.5
Symmetry, reflections, similar patterns	2.5	3.6	3.1

Table 2. Total overall rating points acquired by each general area and number of topics to be represented on the test

General Area	Total Rating Points	Number of Topics on Test
Sets	13.5	3
Number Theory	14.5	3
Addition	12.2	3
Subtraction	8.3	2
Multiplication	26.5	6
Division	23.3	5
Percent	3.1	1
Measurement	20.3	5
Fractions	33.7	8
Decimals	18.7	4
Graphing	13.2	3
Special Topics	50.0	12
Geometry	<u>23.9</u>	<u>5</u>
Totals	261.2	60

Table 3. Topics chosen for representation on the test

General Area	Topic or Topic Cluster
SETS	1. Union 2. Intersection 3. Null, infinite, subset
NUMBER THEORY	4. Factors, greatest common factor 5. Primes, composites 6. Multiples, least common multiple
ADDITION	7. Without regrouping 8. Regrouping 9. Properties
SUBTRACTION	10. Without regrouping 11. Regrouping
MULTIPLICATION	12. Repeated addition 13. Single-digit factor and a factor of two or more digits

Table 3 (continued)

General Area	Topic or Topic Cluster
MULTIPLICATION, cont.	14. Two factors of two or more digits 15. Multiplication-division relationship 16. Properties (comm., assoc., iden.) 17. Distributive property of mult. over addition
DIVISION	18. 1-digit divisor & 1-digit quo. 19. 1-digit divisor & 2-digit quo. 20. Divisors of 2 or more digits 21. Estimating 22. Averages
PERCENT	23. Percent
MEASUREMENT	24. Length 25. Area 26. Volume 27. Perimeter, circumference 28. Metric system
FRACTIONS	29. Meaning 30. Equivalent fractions 31. Reducing to lowest terms 32. Adding fractions and mixed numerals 33. Mixed numerals and improper fractions 34. Subtracting fractions & mixed numerals 35. Multiplying fractions & mixed numerals 36. Dividing fractions
DECIMALS	37. Adding and subtracting 38. Multiplying 39. Dividing 40. Repeating and terminating
GRAPHING	41. Graphing coordinates 42. Functions 43. Graphing negative numbers
SPECIAL TOPICS	44. Bases other than ten 45. Place value 46. Exponents 47. Clock & modular arithmetic 48. Negative numbers

Table 3 (continued)

General Area	Topic or Topic Cluster
SPECIAL TOPICS, cont.	49. Probability
	50. Inequalities
	51. Functions
	52. Mean, median, mode
	53. Cardinal, ordinal, whole, natural, & real numbers
	54. Congruency
GEOMETRY	55. Propositions & implications (inverse, converse)
	56. Angles
	57. Space figures
	58. Points, segments, rays, & lines
	59. Plane figures
	60. Symmetry, reflections, similar patterns

Table 4. Ranking, by panel of experts, of pool of test items

Topic	Items by Rank		
	First	Second	Third
<u>SETS</u>			
Union	2	1	3
Intersection	6	5	4
Null, infinite, subset	8	9	7
<u>NUMBER THEORY</u>			
Factors, greatest common factor	11	10	12
Primes, composites	15	14	13
Multiples, least common multiple	16	17	18
<u>ADDITION</u>			
Without regrouping	20	19	21
Regrouping	22	23	24
Properties	26	27	25
<u>SUBTRACTION</u>			
Without regrouping	29	28	30
Regrouping	31	32	33
<u>MULTIPLICATION</u>			
Repeated addition	35	34	36
Single-digit factor and a factor of two or more digits	37	38	39

Table 4 (continued)

Topic	Items by Rank		
	First	Second	Third
<u>MULTIPLICATION, cont.</u>			
Two factors of two or more digits	40	41	42
Multiplication-division relationship	44	43	45
Properties (comm., assoc., iden.)	47	48	46
Distributive property of mult. over addition	51	50	49
<u>DIVISION</u>			
1-digit divisor & 1-digit quo.	52	54	53
1-digit divisor & 2-digit quo.	56	55	57
Divisors of 2 or more digits	58	60	59
Estimating	62	63	61
Averages	65	64	66
<u>PERCENT</u>			
	69	67	68
<u>MEASUREMENT</u>			
Length	71	72	70
Area	74	73	75
Volume	76	77	78
Perimeter, circumference	79	81	80
Metric system	82	84	83
<u>FRACTIONS</u>			
Meaning	87	85	86
Equivalent fractions	89	88	90
Reducing to lowest terms	92	93	91
Adding fractions and mixed numerals	96	94	95
Mixed numerals and improper fractions	99	97	98
Subtracting fractions and mixed numerals	100	102	101
Multiplying fractions and mixed numerals	103	104	105
Dividing fractions	107	106	108
<u>DECIMALS</u>			
Adding and subtracting	110	109	111
Multiplying	112	113	114
Dividing	117	115	116
Repeating and terminating	120	118	119
<u>GRAPHING</u>			
Graphing coordinates	123	121	122
Functions	125	126	124
Graphing negative numbers	128	127	129



Table 4 (continued)

Topic	Items by Rank		
	First	Second	Third
<u>SPECIAL TOPICS</u>			
Bases other than ten	130	131	132
Place value	134	133	135
Exponents	137	138	136
Clock & modular arithmetic	141	439	140
Negative numbers	143	144	142
Probability	145	147	146
Inequalities	150	149	148
Functions	153	152	151
Mean, median, mode	155	156	154
Cardinal, ordinal, whole, natural, & real numbers	158	159	157
Congruency	162	160	161
Propositions & implications (inverse, converse)	164	165	163
<u>GEOMETRY</u>			
Angles	166	168	167
Space figures	171	170	169
Points, segments, rays, & lines	172	174	173
Plane figures	176	175	177
Symmetry, reflections, similar patterns	179	180	178

Table 5. Cross-reference of test items from pool of items to preliminary forms A and B

Form A No.	Pool No.	Form B No.	Pool No.
1	2	1	1
2	6	2	5
3	8	3	9*
4	11	4	10*
5	15	5	14*
6	16	6	17
7	20	7	19**
8	22	8	23**
9	26*	9	27
10	29*	10	28
11	31	11	32
12	35*	12	34
13	37	13	38*
14	40	14	41
15	44	15	43*

Table 5 (continued)

Form A No.	Pool No.	Form B No.	Pool No.
16	47*	16	48
17	51	17	50
18	52	18	54**
19	56	19	55
20	58	20	60
21	62	21	63
22	65	22	64*
23	69	23	67
24	71	24	72
25	74	25	73
26	76	26	77
27	79	27	81**
28	82	28	84
29	87	29	85
30	89*	30	88
31	92	31	93
32	96*	32	94
33	99*	33	97*
34	100	34	102
35	103	35	104
36	107	36	106
37	110	37	109
38	112	38	113
39	117	39	115
40	120	40	118*
41	123	41	121
42	125*	42	126*
43	128	43	127
44	130	44	131
45	134	45	133
46	137	46	138*
47	141	47	139
48	143	48	144
49	145	49	147
50	150	50	149
51	153	51	152
52	155	52	156*
53	158	53	159
54	162	54	160
55	164	55	165
56	166	56	168*
57	171	57	170
58	172	58	174
59	176*	59	175**
60	179	60	180*

\*Items which were revised before being placed in the preliminary form

\*\*Items which were replaced

Table 6. Discriminating power and level of difficulty obtained from item analysis of test items on preliminary forms A and B

Item No.	Index of Discrimination		Index of Difficulty	
	Form A	Form B	Form A	Form B
1	.16	.21*	.08	.16*
2	.16	.21*	.07	.51*
3	.79*	.11	.41*	.08
4	.11	.32*	.18	.38*
5	.32*	.26	.17*	.79
6	.74*	.37	.48*	.44
7	.32*	.05	.34*	.08
8	.52*	.27	.61*	.37
9	.48*	.27	.30*	.14
10	.11*	.11	.30*	.14
11	.31*	.05	.79*	.08
12	.06*	.05	.69*	.89
13	-.16	.37*	.38	.58*
14	.63*	.00	.42*	.87
15	.32*	.26	.17*	.31
16	.43	.53*	.31	.20*
17	.42*	.11	.38*	.11
18	.27*	.16	.13*	.14
19	.26*	.00	.51*	.17
20	.26*	.16	.66*	.27
21	.21*	.21	.27*	.23
22	.37*	.37	.24*	.17
23	.53*	.37	.37*	.34
24	.37*	.11	.55*	.07
25	.58*	.10	.72*	.03
26	.32*	.10	.59*	.41
27	.58*	.26	.32*	.41
28	.16	.21*	.75	.59*
29	.63*	.06	.32*	.90
30	.43*	.21	.31*	.14

Table 6 (continued)

Item No.	Index of Discrimination		Index of Difficulty	
	Form A	Form B	Form A	Form B
31	.26*	.26	.10*	.08
32	.43	.47*	.27	.21*
33	.11	.26*	.28	.14*
34	.43	.68*	.31	.44*
35	.68*	.37	.30*	.23
36	.16	.42*	.10	.54*
37	.16	.47*	.10	.58*
38	.32	.47*	.23	.63*
39	.05	.58*	.92	.35*
40	.16	.43*	.21	.32*
41	.68*	.37	.28*	.45
42	.42	.48*	.75	.72*
43	.37	.63*	.16	.62*
44	.53	.58*	.46	.46*
45	.37	.63*	.37	.48*
46	.42	.52*	.13	.57*
47	.53*	.37	.58*	.79
48	.22	.68*	.17	.52*
49	.32*	.26	.25*	.30
50	.16	.58*	.52	.54*
51	.32	.68*	.23	.71*
52	.32	.32*	.77	.71*
53	.47*	.48	.49*	.55
54	.69*	.32	.68*	.63
55	.37	.79*	.59	.54*
56	.16	.42*	.89	.49*
57	.05	.37*	.79	.83*
58	.53	.69*	.54	.51*
59	.58*	.26	.54*	.85
60	.31	.42*	.76	.63*

\*Items selected for final form of test

Table 7. Items selected for the final form from preliminary forms A and B

Final Form Item No.	Prelim. Form Item No.	Final Form Item No.	Prelim. Form Item No.
1	B-1	31	B-32
2	B-2	32	B-34
3	A-3	33	A-35*
4	B-4*	34	B-36
5	A-5	35	B-37
6	A-6	36	B-38
7	A-7	37	B-39
8	A-8*	38	B-40*
9	A-9	39	A-41*
10	A-10	40	B-42
11	A-11*	41	B-43
12	B-13	42	B-44
13	A-14	43	B-45
14	A-15	44	B-46*
15	B-16*	45	A-47
16	A-17	46	B-48
17	A-18	47	A-49
18	A-19	48	B-50*
19	A-20	49	B-51
20	A-21*	50	B-52
21	A-22	51	A-53
22	A-23	52	A-54
23	A-24	53	B-55
24	A-25*	54	B-56
25	A-26	55	B-57
26	A-27	56	B-58
27	B-28*	57	A-59
28	A-29*	58	B-60
29	A-30		
30	A-31		

\*Items which were revised before being placed in the final form

Table 8. Discriminating power and level of difficulty obtained from item analysis of test items on final form and on preliminary form

Item No.	Index of Discrimination		Index of difficulty	
	Final Form	Prelim. Form	Final Form	Prelim. Form
1	.17	.21	.14	.16
2	.28	.21	.46	.51
3	.61	.79	.31	.41
4	.17	.32	.25	.38
5	.06	.32	.14	.17
6	.28	.74	.42	.48
7	.23	.32	.35	.34
8	.67	.52	.51	.61
9	.11	.48	.18	.30
10	.22	.11	.34	.30
11	.22	.31	.83	.79
12	.11	.37	.35	.58
13	.39	.63	.34	.42
14	.00	.32	.20	.17
15	.06	.53	.14	.20
16	.50	.42	.32	.38
17	.51	.27	.22	.13
18	.22	.26	.37	.51
19	.39	.26	.52	.66
20	.16	.22	.17	.27
21	.27	.37	.23	.24
22	.34	.53	.48	.37
23	.39	.37	.49	.55
24	.22	.58	.79	.72
25	.28	.32	.46	.59
26	.50	.58	.37	.32
27	.22	.21	.79	.59
28	.22	.63	.43	.32
29	.39	.43	.40	.31
30	.12	.26	.09	.10

Table 8 (continued)

Item No.	Index of Discrimination		Index of difficulty	
	Final Form	Prelim. Form	Final Form	Prelim. Form
31	.34	.47	.17	.21
32	.34	.68	.43	.44
33	.50	.68	.32	.30
34	.67	.42	.46	.54
35	.33	.47	.39	.58
36	.22	.47	.69	.63
37	.67	.58	.35	.35
38	.50	.43	.57	.32
39	.39	.68	.32	.28
40	.11	.48	.65	.72
41	.39	.63	.63	.62
42	.44	.58	.18	.46
43	.45	.63	.52	.48
44	.33	.52	.49	.57
45	.17	.53	.46	.58
46	.72	.68	.55	.52
47	.45	.32	.45	.25
48	.45	.58	.48	.54
49	.61	.68	.72	.71
50	.17	.32	.80	.71
51	.55	.47	.68	.49
52	.22	.69	.82	.68
53	.56	.79	.63	.54
54	.45	.42	.39	.49
55	.44	.37	.83	.83
56	.39	.69	.52	.51
57	.16	.58	.66	.54
58	.33	.42	.65	.63

Table 9. Discrimination report on final test form items as determined by first item analysis (first pilot group)

Index	Item No.	Index	Item No.
.00		.41	
.01		.42	16,34,54,58
.02		.43	29,38
.03		.44	
.04		.45	
.05		.46	
.06		.47	31,35,36,51
.07		.48	9,40
.08		.49	
.09		.50	
.10		.51	
.11	10	.52	8,44
.12		.53	15,22,45
.13		.54	
.14		.55	
.15		.56	
.16		.57	
.17		.58	24,26,37,42,48,57
.18		.59	
.19		.60	
.20		.61	
.21	1,2,27	.62	
.22	20	.63	13,28,41,43
.23		.64	
.24		.65	
.25		.66	
.26	18,19,30	.67	
.27	17	.68	32,33,39,46,49
.28		.69	52,56
.29		.70	
.30		.71	
.31	11	.72	
.32	4,5,7,14,25,47,50	.73	
.33		.74	6
.34		.75	
.35		.76	
.36		.77	
.37	12,21,23,55	.78	
.38		.79	3,53
.39		.80	
.40		.81	



Table 10. Discrimination report on final test form items as determined by second item analysis (second pilot group)

Index	Item No.	Index	Item No.
.00	14	.40	
.01		.41	
.02		.42	
.03		.43	
.04		.44	42, 55
.05		.45	43, 47, 48, 54
.06	5, 15	.46	
.07		.47	
.08		.48	
.09		.49	
.10		.50	16, 26, 33, 38
.11	9, 12, 40	.51	17
.12	30	.52	
.13		.53	
.14		.54	
.15		.55	51
.16	20, 57	.56	53
.17	1, 4, 45, 50	.57	
.18		.58	
.19		.59	
.20		.60	
.21		.61	3, 49
.22	10, 11, 18, 24, 27, 28, 36, 52	.62	
.23	7	.63	
.24		.64	
.25		.65	
.26		.66	
.27	21	.67	8, 34, 37
.28	2, 6, 25	.68	
.29		.69	
.30		.70	
.31		.71	
.32		.72	46
.33	35, 44, 58	.73	
.34	22, 31, 32	.74	
.35		.75	
.36		.76	
.37		.77	
.38		.78	
.39	13, 19, 23, 29, 39, 41, 56	.79	
		.80	

Table 11. Data on test items as used in computing Kuder-Richardson reliability formula number 20

x = Test item  
 y = Number of correct responses  
 p = Proportion of correct responses  
 q = Proportion of incorrect responses

x	y	p	q	pq	x	y	p	q	pq
1	39	.886	.114	.1007	30	38	.864	.136	.1178
2	27	.614	.386	.2371	31	42	.955	.045	.0434
3	28	.636	.364	.2314	32	23	.523	.478	.2495
4	33	.750	.250	.1875	33	30	.682	.318	.2169
5	38	.864	.136	.1178	34	25	.568	.432	.2454
6	30	.682	.318	.2169	35	27	.614	.386	.2371
7	28	.636	.364	.2314	36	25	.568	.432	.2454
8	29	.659	.341	.2247	37	33	.750	.250	.1875
9	32	.727	.273	.1983	38	29	.659	.341	.2247
10	35	.795	.205	.1627	39	26	.591	.409	.2417
11	6	.136	.864	.1178	40	17	.386	.614	.2371
12	22	.500	.500	.2500	41	18	.409	.591	.2417
13	23	.523	.478	.2495	42	26	.591	.409	.2417
14	36	.818	.182	.1488	43	18	.409	.591	.2417
15	33	.750	.250	.1875	44	16	.364	.636	.2314
16	32	.727	.273	.1983	45	12	.273	.727	.1983
17	43	.977	.023	.0222	46	23	.523	.478	.2495
18	29	.659	.341	.2247	47	24	.545	.455	.2479
19	23	.523	.478	.2495	48	29	.659	.341	.2247
20	35	.795	.205	.1627	49	10	.227	.773	.1756
21	38	.864	.136	.1178	50	11	.250	.750	.1875
22	24	.545	.455	.2479	51	18	.409	.591	.2417
23	26	.591	.409	.2417	52	13	.295	.705	.2082
24	15	.341	.659	.2247	53	15	.341	.659	.2247
25	25	.568	.432	.2454	54	24	.545	.455	.2479
26	28	.636	.364	.2314	55	12	.273	.727	.1983
27	12	.273	.727	.1983	56	28	.636	.364	.2314
28	26	.591	.409	.2417	57	22	.500	.500	.2500
29	34	.773	.227	.1756	58	11	.250	.750	.1875

$$\sum pq = 11.9202$$

Table 12. Data used in computing reliability coefficient  
 $x$  = Number correct on Test of Mathematical Fundamentals out of 58 possible  
 $y$  = Total points on tests given throughout course

n	x	$x^2$	y	$y^2$	xy
1	54	2916	356	126,736	19,224
2	44	1936	361	130,321	15,884
3	42	1764	349	121,801	14,658
4	42	1764	324	104,976	13,608
5	41	1681	350	122,500	14,350
6	41	1681	364	132,496	14,924
7	34	1156	350	122,500	11,900
8	32	1024	361	130,321	11,552
9	27	729	329	108,241	8,883
10	26	676	353	124,609	9,178
11	22	484	278	77,284	6,116
12	20	400	313	97,969	6,260
13	19	361	291	84,681	5,529
14	12	144	183	33,489	2,196
Totals	456	16,716	4562	1,517,924	154,262

Table 13. Data used in computing reliability coefficient  
 $x$  = Number correct on Test of Mathematical Fundamentals out of 58 possible  
 $y$  = Final course grade A = 4 B = 3 C = 2

n	x	$x^2$	y	$y^2$	xy
1	See Table 12		4	16	216
2			4	16	176
3			4	16	168
4			3	9	126
5			4	16	164
6			4	16	164
7			4	16	136
8			4	16	128
9			3	9	81
10			4	16	104
11			2	4	44
12			3	9	60
13			3	9	57
14			2	4	24
Totals			48	172	1684

Table 14. Frequency distribution of norm group raw scores

x	f	x	f
49	1	33	2
48	1	32	2
47	1	31	1
46	2	30	5
44	1	29	1
42	2	27	1
41	3	23	1
40	1	22	1
39	1	21	1
38	2	19	2
37	2	17	1
36	2	15	1
35	3	12	1
34	2		

## APPENDIX B

Questionnaire and Accompanying Letter

Questionnaire

In our program for elementary education majors:

- We require no math content courses.  
 We require math content courses but do not use a waiver test.  
 We require math content courses and we use a waiver test of the following types:  
 Published  
 Unpublished  
 Developed locally

\_\_\_\_\_  
Name                      Institution                      Department

Letter Accompanying Questionnaire

Dear Sirs:

We are considering the use of a waiver test for the math content courses required of our elementary education majors. Before developing such a test, we are attempting to determine the practices of other institutions preparing elementary school teachers and whether a test already exists which would adequately serve this purpose.

We would be grateful if you would make the appropriate checks on the back of the enclosed self-addressed postcard and return it to us by November 20.

Sincerely,

*Bryce E. Adkins*  
*Marvin N. Tolman*

Bryce Adkins, professor  
of education

Marvin N. Tolman, graduate  
assistant

## APPENDIX C

List of 100 Mathematical Topics  
Rated by Panel of Experts,  
Accompanying Letter  
and  
Note Explaining Topic List



Letter Accompanying List of Topics

Dear \_\_\_\_\_,

In response to a request from the University Council on Teacher Education of USU, we are attempting to develop and validate an instrument for measuring the mathematical competence of our preservice elementary school teachers. As one check of the content validity of the test, we wish to utilize a panel of experts. We know that you have had a great deal of experience in the mathematical training of elementary school teachers and we would be grateful if you would be willing to serve on this panel. If you accept, we ask that you: (1) Respond to the enclosed materials, and (2) Evaluate a pool of test items.

A pool of test items will be constructed, based on the responses of the panel of experts to the enclosed materials and on the content analysis referred to therein. Following evaluation by the panel of experts, the pool of items will be tested for discriminatory power and items for the test will be selected from the pool. Data will then be gathered on the test itself.

If you feel inclined to assist us as requested, we would greatly appreciate receiving your response to the enclosed materials at your earliest convenience, hopefully by Feb. 15.

Thanks.

Sincerely,

Bryce Adkins, professor  
of education

Marvin N. Tolman, graduate  
assistant

In a preliminary effort to determine the math topics which should be tested by an instrument designed to measure the mathematical competence of preservice elementary school teachers, we have analyzed the content of nine elementary math textbooks and three college texts. The elementary texts used were the 4th, 5th, and 6th grade levels of Addison-Wesley 1971, Houghton Mifflin 1972, and Holt, Rinehart and Winston 1970. The college texts were:

Wheeler, Ruric, Modern Math: An Elementary Approach, Third edition, Brooks-Cole Publishing Co., 1973.

Peterson, John M., Basic Concepts of Elementary Mathematics, Prindle, Weber and Schmidt, Inc., 1971.

Forbes, Jack E., and Robert E. Eicholz, Mathematics for Elementary Teachers, Addison-Wesley Publishing Co., 1971

We have identified the topics included in these sources. Considering the purpose of our test, however, we feel that we would be in error to judge the topic-by-topic emphasis for the test strictly upon the emphasis placed on the topics by the textbooks. Therefore, we feel a need for a rating on these topics from our panel of experts. On the following pages, please circle a number to rate each topic according to the emphasis which you feel would be appropriate for the test mentioned above. A rating of one indicates least significance to the test and a rating of five indicates greatest significance.

Space is provided on the last page in case there are topics missing which you feel should be included or for other comments or suggestions you may wish to make.

SETS

	Least Emphasis			Greatest Emphasis	
Union	1	2	3	4	5
Intersection	1	2	3	4	5
Null, infinite, subset	1	2	3	4	5
One-to-one correspondence	1	2	3	4	5
Cartesian products	1	2	3	4	5
Relations (reflexive, symmetric, transitive)	1	2	3	4	5

NUMBER THEORY

Even & odd numbers	1	2	3	4	5
Factors, greatest common factor	1	2	3	4	5
Primes, composites	1	2	3	4	5
Multiples, least common multiple	1	2	3	4	5
Divisibility	1	2	3	4	5

ADDITION

Without regrouping	1	2	3	4	5
Regrouping	1	2	3	4	5
Money	1	2	3	4	5
Estimating	1	2	3	4	5
Properties	1	2	3	4	5

SUBTRACTION

Without regrouping	1	2	3	4	5
Regrouping	1	2	3	4	5
Money	1	2	3	4	5
Properties	1	2	3	4	5

MULTIPLICATION

Repeated addition	1	2	3	4	5
Powers of 10 as factors	1	2	3	4	5
Single digit factor and a factor of two or more digits	1	2	3	4	5
Two factors of two or more digits	1	2	3	4	5
Money	1	2	3	4	5
Estimating	1	2	3	4	5
Multiplication-division relationship	1	2	3	4	5
Multiples of 10, 100, & 1000 as factors	1	2	3	4	5
Properties (comm., assoc., iden.)	1	2	3	4	5
Distributive property of mult. over addition	1	2	3	4	5

<u>DIVISION</u>	Least Emphasis			Greatest Emphasis	
Repeated subtraction	1	2	3	4	5
1-digit divisor & 1-digit quo.	1	2	3	4	5
1-digit divisor & 2-digit quo.	1	2	3	4	5
1-digit divisor & quotient of three or more digits	1	2	3	4	5
Involving multiples of 10, 100, and 1000	1	2	3	4	5
Divisors of 2 or more digits	1	2	3	4	5
Money	1	2	3	4	5
Estimating	1	2	3	4	5
Checking	1	2	3	4	5
Averages	1	2	3	4	5
<u>PERCENT</u>	1	2	3	4	5
<u>MEASUREMENT</u>					
Length	1	2	3	4	5
Area	1	2	3	4	5
Volume	1	2	3	4	5
Perimeter, circumference	1	2	3	4	5
Liquid measure	1	2	3	4	5
Converting units	1	2	3	4	5
Metric system	1	2	3	4	5
Weight, time, temperature	1	2	3	4	5
<u>FRACTIONS</u>					
Meaning	1	2	3	4	5
Equivalent fractions	1	2	3	4	5
Renaming to higher and lower terms	1	2	3	4	5
Reducing to lowest terms	1	2	3	4	5
Adding fractions and mixed numerals	1	2	3	4	5
Least common denominator	1	2	3	4	5
Mixed numerals and improper fractions	1	2	3	4	5
Subtracting fractions & mixed numerals	1	2	3	4	5
Multiplying fractions & mixed numerals	1	2	3	4	5
Dividing fractions	1	2	3	4	5
Terms	1	2	3	4	5

<u>DECIMALS</u>	Least Emphasis			Greatest Emphasis	
Fractions and decimals	1	2	3	4	5
Adding and subtracting	1	2	3	4	5
Multiplying	1	2	3	4	5
Dividing	1	2	3	4	5
Repeating and terminating	1	2	3	4	5
Estimating	1	2	3	4	5
Irrational numbers	1	2	3	4	5
<u>GRAPHING</u>					
Graphs on the number line	1	2	3	4	5
Graphing coordinates	1	2	3	4	5
Functions	1	2	3	4	5
Graphing negative numbers	1	2	3	4	5
Tables and graphs	1	2	3	4	5
<u>SPECIAL TOPICS</u>					
Bases other than ten	1	2	3	4	5
Other number systems (Egyptian & Roman)	1	2	3	4	5
Place value	1	2	3	4	5
Exponents	1	2	3	4	5
Ratio & scale drawings	1	2	3	4	5
Clock & modular arithmetic	1	2	3	4	5
Negative numbers	1	2	3	4	5
Probability	1	2	3	4	5
Sequences	1	2	3	4	5
Rounding numbers	1	2	3	4	5
Inequalities	1	2	3	4	5
Functions	1	2	3	4	5
Time, rate, distance	1	2	3	4	5
Mean, median, mode	1	2	3	4	5
Cardinal, ordinal, whole, natural, & real numbers	1	2	3	4	5
Congruency	1	2	3	4	5
Propositions & implications (inverse, converse)	1	2	3	4	5
Casting out nines	1	2	3	4	5
Standard deviation	1	2	3	4	5
Normal frequency curve	1	2	3	4	5

GEOMETRY

	Least Emphasis			Greatest Emphasis	
Simple closed curves	1	2	3	4	5
Circle, radius, diameter, chord	1	2	3	4	5
Angles	1	2	3	4	5
Space figures	1	2	3	4	5
Points, segments, rays, & lines	1	2	3	4	5
Plane figures	1	2	3	4	5
Parallel and perpendicular	1	2	3	4	5
Symmetry, reflections, similar patterns	1	2	3	4	5

COMMENTS:

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## APPENDIX D

Pool of Test Items and Accompanying  
Note of Explanation

On the basis of ratings given by our panel of experts and ratings derived from analyzing the content of textbooks, 60 mathematical concepts have been identified for representation in our test. Realizing that there are many more than 60 concepts which are important and realizing the necessary limitations on the length of the test, stratification was used in selection to assure that each general area received representation which is appropriate according to the ratings given.

The final form of the test will include 60 items. These items will represent the 60 different concepts mentioned above. Three test items have been written for each of the 60 concepts. Two of the three items in each case will be screened out, hopefully leaving the most effective and appropriate items for the test.

As the first step in the screening process, we desire that you rank each set of three problems according to your first, second, and third choice. The numbers 1, 2, and 3 are typed at the right of each problem for your convenience. Please circle or check one of these for each test item. If in some cases the three problems do not differ in appropriateness in your opinion, so indicate.

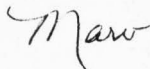
In ranking the test items, please consider such characteristics as appropriateness for this test, mathematical soundness, and clarity. If you notice errors or have comments or suggestions, notations of these will be appreciated. Please keep in mind that only one out of each set of three items will be used in the test.

The second screening step will be to test the items with students to determine the level of difficulty, discriminating power, and ability of the distractors to distract. Revisions will be made as deemed necessary both before and after the second step.

We hope to begin the second step in screening the test items next week and would certainly appreciate your prompt response to these materials.

Thanks for your expert assistance with this project. If you desire a copy of the final results of this study, please indicate.

Sincerely,



Marvin N. Tolman



Sets: Union

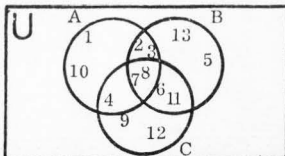
1. Given that  $A = \{w, x, y, z\}$  and  $B = \{u, v, w, x\}$ , which of the following is true: 1 2 3

- (a)  $A \cup B = \{w, x\}$  (c)  $A \cup B = \{u, v, w, w, x, x, y, z\}$   
 (b)  $A \cup B = \{u, v, w, x, y, z\}$  (d) None of these

2. Given that  $A = \{5, 2, 1, 4\}$ ,  $B = \{4, 7, 1, 3\}$ ,  $C = \{1, 2, 3, 6\}$ , which of the following is true: 1 2 3

- (a)  $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7\}$  (d)  $A \cup B \cup C = \emptyset$   
 (b)  $A \cup B \cup C = \{1, 1, 1, 2, 2, 3, 3, 4, 4, 5, 6, 7\}$  (e) None of these  
 (c)  $A \cup B \cup C = \{1\}$

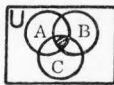
3. Considering this Venn diagram, which of the following is true: 1 2 3



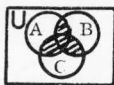
- (a)  $n(A \cup B) = 2$   
 (b)  $n(A \cup B) = 7$   
 (c)  $n(A \cup B) = 6$   
 (d)  $n(A \cup B) = 13$   
 (e) None of these

Sets: Intersection

4. Which of the following Venn diagrams illustrates  $(A \cap B) \cap C$ : 1 2 3



(a)



(b)



(c)

- (d) None of these

5. Given that  $A = \{v, w, x, s\}$ ,  $B = \{v, w, x, y\}$ , and  $C = \{r, w, x\}$ , which of the following is false: 1 2 3

- (a)  $(A \cup C) \cup B = A \cup (C \cup B)$  (d)  $A \cap (C \cup B) = (A \cap C) \cup (A \cap B)$   
 (b)  $(A \cap C) \cap B = A \cap (C \cap B)$  (e) None of these  
 (c)  $A \cup (C \cap B) = (A \cup C) \cap (A \cap B)$

6. If Set  $A = \{a, b, c, d, c\}$  and Set  $B = \{d, e, f, g\}$ , what is  $A \cap B$ : 1 2 3

- (a)  $\{a, b, c, d, e, f, g\}$  (d)  $\{a, b, c, d, d, e, e, f, g\}$   
 (b)  $\{d, c\}$  (e) None of these  
 (c)  $\{a, b, c, f, g\}$

Sets; Null, infinite, subset, etc.

7. Let  $A = \{3, 4\}$ ,  $B = \{3, 4, 5\}$ , and  $C = \{1, 2, 3, 4\}$ . Which of the following statements is true: 1 2 3
- (a)  $B \subset A$  (d)  $A \not\subset B$   
 (b)  $C \in \emptyset$  (e) None of these  
 (c)  $\emptyset \not\subset A$
8. Given that Set  $A = \{1, 2, 3, 4\}$  and Set  $B = \{5, 6, 7\}$ , which of the following is true: 1 2 3
- (a)  $A \cap B = \emptyset$  (d)  $A \cap B = \{\emptyset\}$   
 (b)  $A \cup B = \{ \}$  (e) None of these  
 (c)  $A \cap B \neq \{ \}$
9. If Set  $A = \{1, 2, 3, 4, \dots\}$ , which of the following is true: 1 2 3
- (a) Set A is the set of whole numbers (d) Set A is infinite  
 (b)  $\{9, 10, 11\} \notin A$  (e) None of these  
 (c)  $n(A) = 4$

Number Theory: Factors, GCF

10. Which of the following is a false statement: 1 2 3
- (a) The greatest common factor of 84 and 120 is 21.  
 (b) The greatest common factor of 198 and 210 is 6.  
 (c) The greatest common factor of 270 and 504 is 18.  
 (d) None of these
11. The phrase  $3 \times 3 \times 4$  could be described as: 1 2 3
- (a) A set of factors of 36 (d) The greatest common factors of 36  
 (b) A set of prime factors of 36 (e) None of these  
 (c) A prime factorization of 36
12. Which of the following is the greatest common factor of 486, 126, and 396: 1 2 3
- (a) 18 (d) 6  
 (b) 12 (e) None of these  
 (c) 36

Number Theory: Primes and composites

13. There are how many composite numbers less than 20 : 1 2 3  
(a) 12 (d) 10  
(b) 15 (e) None of these  
(c) 9
14. Which of the following is a prime number: 1 2 3  
(a) 1 (d) 91  
(b) 87 (e) None of these  
(c) 117
15. Resolve 330 into prime factors: 1 2 3  
(a)  $3 \times 11 \times 10$  (d)  $2 \times 3 \times 5 \times 11$   
(b)  $6 \times 11 \times 5$  (e) None of these  
(c)  $2 \times 15 \times 11$
- 

Number Theory: Multiples, LCM

16. What is the least common multiple of 56 and 21: 1 2 3  
(a) 336 (c) 1  
(b) 168 (d) None of these
17. What is the least common multiple of 18, 12, and 15 : 1 2 3  
(a) 3240 (d) 360  
(b) 180 (e) None of these  
(c) 3
18. Considering the relationship between the numbers 12 and 36, the 12 could 1 2 3  
be considered:  
(a) A multiple (d) The greatest prime number less  
(b) A factor than 36  
(c) The least common multiple (e) None of these

Addition: Without regrouping

19. In which of the following addition problems is regrouping necessary: 1 2 3
- (a)  $21 + 36$  (d) All of the above  
 (b)  $32 + 11 + 25$  (e) None of the above  
 (c)  $44 + 12 + 33$
20. Which of the following is true: 1 2 3
- (a) The sum of 43 and 26 contains seven tens  
 (b) The sum of 27 and 62 contains nine tens  
 (c) The sum of 3842 and 2037 contains nine hundreds  
 (d) The sum of 27,932 and 312,063 contains eight ten-thousands  
 (e) None of these
21. Which of the following is true: 1 2 3
- (a) If one addend is five and the other addend is six, the sum is two more than twice the first addend  
 (b) If three addends are twenty-one, thirty-six, and fifteen, the sum is one less than twice the second addend  
 (c) If one addend is fifty-six and the other addend is fifty-three, there are no hundreds in the sum  
 (d) If the sum is sixty-six, each of three addends could be twenty-two  
 (e) None of these

Addition: Regrouping

22. In the addition exercise below, which of the following could be possible digits for the  $\blacktriangle$ : 1 2 3
- (a) 5 (d) All of the above  
 (b) 2 (e) None of these  
 (c) 6
- $$\begin{array}{r}
 \square 9 \square \\
 + \square 5 \square \\
 \hline
 \square \blacktriangle \square
 \end{array}$$
23. In which of the following addition problems is regrouping necessary: 1 2 3
- (a)  $8367 + 2312$  (d)  $4273 + 2514 + 3102$   
 (b)  $1741 + 3217 + 4001$  (e) None of these  
 (c)  $6003 + 2454 + 3631$
24. Which of the following addition problems has a sum of 3999: 1 2 3
- (a)  $1372 + 2426 + 211$  (d)  $660 + 738 + 2142 + 459$   
 (b)  $1052 + 501 + 2346$  (e) None of these  
 (c)  $1250 + 2436 + 313$

Addition: Properties

25. Which of the following is the identity element for addition: 1 2 3  
 (a) 1 (d)  $\emptyset$   
 (b) 0 (e) None of these  
 (c) 10
26. Which property of whole numbers justifies the following statement: 1 2 3  
 $(4 + 3) + 5 = 5 + (4 + 3)$   
 (a) Distributive (d) Associative property of addition  
 (b) Closure (e) None of these  
 (c) Commutative property of addition
27. Which number sentence illustrates the associative property of addition: 1 2 3  
 (a)  $5 + (6 + 7) = (5 + 6) + 7$  (d)  $3 \times 7 = (3 \times 5) + (3 \times 2)$   
 (b)  $(2 + 3) + 4 = 4 + (2 + 3)$  (e) None of these  
 (c)  $3 + (7 + 1) = 3 + (1 + 7)$
- 

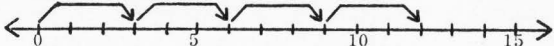
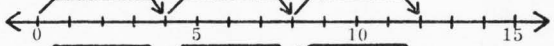
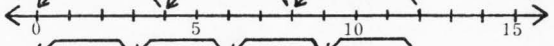
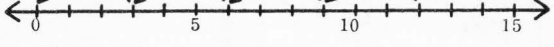
Subtraction: Without regrouping

28. What is the value of  $9 - (5 - 2)$ : 1 2 3  
 (a) 2 (c) Both a and b are correct  
 (b) 6 (d) None of these
29. Which is correct: 1 2 3  
 (a)  $37 - 6 = (30 - 7) - 6$  (c)  $48 - 7 = (40 + 8) - 7$   
 $= 30 - (7 - 6)$   $= 40 + (8 - 7)$   
 $= 31$   $= 41$   
 (b)  $34 - 6 = (20 + 14) - 6$  (d)  $42 - 7 = (30 + 12) - 7$   
 $= 20 - (14 + 6)$   $= 30 + (12 - 7)$   
 $= 28$   $= 25$   
 (e) None of these
30. Which is correct: 1 2 3  
 (a)  $14 - 8 = 8 - 14$  (d)  $(18 - 5) - 2 = 25 - (12 + 2)$   
 (b)  $15 - 3 \neq 17 - 5$  (e) None of these  
 (c)  $(9 - 6) - 3 = 9 - (6 - 3)$

Subtraction: Regrouping

31. Which of the following represents the proper regrouping for the minuend in order to solve  $624 - 478$ : 1 2 3
- (a)  $600 + 10 + 14$  (d)  $500 + 110 + 14$   
 (b)  $500 + 120 + 4$  (e) None of these  
 (c)  $500 - 120 - 4$
32. For the problem  $8002 - 3738$ , which is the correct answer: 1 2 3
- (a) 5264 (d) 4266  
 (b) 4364 (e) None of these  
 (c) 5736
33. In the problem  $384 - 296$ , what is the value of the minuend minus the difference: 1 2 3
- (a) 296 (d) 384  
 (b) 98 (e) None of these  
 (c) 208

Multiplication: Repeated addition

34. In which case does the repeated addition represent the multiplication: 1 2 3
- (a)  $9 + 9 + 9 + 9 + 9 = 9 \times 5$  (d) All of the above  
 (b)  $5 + 5 + 5 + 5 + 5 + 5 = 5 \times 6$  (e) None of these  
 (c)  $7 + 7 + 7 + 7 = 4 \times 7$
35. Which of the following best illustrates  $3 \times 5$ : 1 2 3
- (a)  $3 + 3 + 3 + 3 + 3$   
 (b)  $5 + 5 + 5$   
 (c) Both a and b illustrate  $3 \times 5$  equally well
36. Which of the following best illustrates  $4 \times 3$ : 1 2 3
- (a) 
- (b) 
- (c) 
- (d) 
- (e) All of these represent  $4 \times 3$  equally well

Multiplication: Single digit factor and a factor of two or more digits

37. In the multiplication problem  $4 \times 17$ , the 4 tells: 1 2 3
- (a) How many sets are being considered  
 (b) How many are in each set  
 (c) How many in all  
 (d) None of these
38. In the product of  $6 \times 75,374$  the five means: 1 2 3
- (a) five hundreds (d) five hundred thousands  
 (b) five thousands (e) None of these  
 (c) five millions
39. The product of 8 and 6047 is: 1 2 3
- (a) 48,376 (d) 6055  
 (b) 50,476 (e) None of these  
 (c) 755 R7
- 

Multiplication: Two factors of two or more digits

40. In the multiplication exercise shown below, which horizontal expression best illustrates through expanded notation the multiplication of the partial products:  $248$  1 2 3
- $$\begin{array}{r} 248 \\ \times 26 \\ \hline \end{array}$$
- (a)  $(6 \times 8) + (6 \times 4) + (6 \times 2) + (20 \times 3) + (20 \times 4) + (20 \times 2)$   
 (b)  $(6 \times 8) + (6 \times 40) + (6 \times 200) + (20 \times 8) + (20 \times 40) + (20 \times 200)$   
 (c)  $(6 \times 8) + (6 \times 40) + (6 \times 200) + (2 \times 8) + (2 \times 40) + (2 \times 200)$   
 (d)  $(6 \times 8) + (6 \times 4) + (6 \times 2) + (2 \times 8) + (2 \times 4) + (2 \times 2)$   
 (e) None of these
41. When you multiply 426 by the 3 in the exercise shown here, you will get a number that is how large compared with the final answer: 1 2 3
- |                          |                        |                               |
|--------------------------|------------------------|-------------------------------|
| (a) One-twelfth as large | (d) Ten times as large | 426                           |
| (b) Five-sixths as large | (e) None of these      | <u><math>\times 36</math></u> |
| (c) Twice as large       |                        |                               |
42. The five in the product of  $408 \times 531$  represents: 1 2 3
- (a) Five hundreds (d) Five hundred-thousands  
 (b) Five thousands (e) There is no five in the product  
 (c) Five ten-thousands

Multiplication: Mult.-Division relationship

43. The inverse of  $3 \times 8 = 24$  is: 1 2 3  
 (a)  $8 \times 3 = 24$  (d) Both b and c are correct  
 (b)  $24 \div 8 = 3$  (e) None of these  
 (c)  $24 \div 3 = 8$
44. The inverse of multiplication is: 1 2 3  
 (a) Addition (c) Division  
 (b) Subtraction (d) None of these
45. Multiplication and division are \_\_\_\_\_ operations: 1 2 3  
 (a) Equal (d) Equivalent  
 (b) Additive (e) None of these  
 (c) Inverse
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Multiplication: Properties (comm., assoc., iden.)

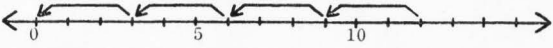
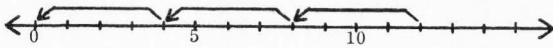
46. Which of the following sets contain an identity element for multiplication: 1 2 3  
 (a)  $\{1, 2, 3, 4, 5\}$  (d) All of the above  
 (b) The even whole numbers (e) None of these  
 (c)  $\{x \mid x \text{ is a whole number greater than } 10\}$
47. If  $x$  and  $y$  are any two whole numbers, then  $x \cdot y$  is a whole number. 1 2 3  
 This statement defines which of the following properties:  
 (a) Associative property of multiplication  
 (b) Closure property for multiplication  
 (c) Identity property for multiplication  
 (d) None of these
48. The statement  $4 \times 5 \times 6 = 6 \times 5 \times 4$  illustrates which of the following 1 2 3  
 mathematical properties:  
 (a) Associative property of multiplication  
 (b) Commutative property of multiplication  
 (c) Closure property of multiplication  
 (d) None of these



Multiplication: Distributive property

49. Which of the following illustrates the distributive property of multiplication over addition: 1 2 3
- (a)  $(a \cdot b) + (a \cdot c) = (b \cdot a) + (c \cdot a)$   
 (b)  $(a \cdot b) + c = c + (a \cdot b)$   
 (c)  $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$   
 (d) None of these
50. The mathematical sentence  $2 \times 36 = (2 \times 30) + (2 \times 6)$  illustrates which of the following: 1 2 3
- (a) The associative property of multiplication  
 (b) The distributive property of multiplication over addition  
 (c) The commutative property of multiplication  
 (d) Closure  
 (e) None of these
51. Which of the following illustrates the distributive property of multiplication over addition: 1 2 3
- (a)  $4 \times 5 \frac{1}{3} = (4 \times 5) + (4 \times \frac{1}{3})$       (d)  $(7 \times 2) \times 4 = 4 \times (7 \times 2)$   
 (b)  $(8 \times 9) \times 4 = 8 \times (9 \times 4)$       (e) None of these  
 (c)  $8 \times 6 = 6 \times 8$

Division: 1-digit divisor and 1-digit quotient

52. Which of the following are ways of expressing twelve divided by four: 1 2 3
- (a)  $12 \div 4$       (c)  $12/4$   
 (b)  $4 \overline{)12}$       (d) Both (a) and (b)  
 (e) Choices (a), (b), and (c) are all correct
53. In the equation  $54 \div 9 = 6$ , the divisor is: 1 2 3
- (a) 6      (c) 54  
 (b) 9      (d) None of these
54. Which of the following illustrates  $12 \div 3$ : 1 2 3
- (a) 
- (b) 
- (c) Both (a) and (b)  
 (d) None of these

Division: 1-digit divisor and 2-digit quotient

55. When 222 is divided by eight, the remainder is: 1 2 3
- (a) 5 (d) 0  
 (b) 7 (e) None of these  
 (c) 6
56. If the quotient is 32, the divisor is 6, and the remainder is 2, what is the dividend: 1 2 3
- (a) 5 (d) 194  
 (b) 192 (e) None of these  
 (c) 44
57.  $\square \div 4 = 36$  1 2 3
- (a) 40 (d) 9  
 (b) 12 (e) None of these  
 (c) 144
- 

Division: Divisors of 2 or more digits

58. Which statement best explains the 4 in the quotient of the following division problem? 1 2 3
- $$24 \overline{)1080} \begin{array}{r} 45 \\ \end{array}$$
- (a) There are at least forty 24's in 1080  
 (b) Two goes into ten four times  
 (c) There are four 24's in 108  
 (d) None of these
59.  $78000 \div 300 = \square$  1 2 3
- (a) 26000 (d) 26  
 (b) 2600 (e) None of these  
 (c) 260
60.  $12,831 \div \square = 47$  1 2 3
- (a) 273 (c) 2730  
 (b) 603,057 (d) None of these

Division: Estimating

61. Which of the following are true: 1 2 3
- (a)  $90 \times 7 < 699$  (c) Both (a) and (b)  
 $100 \times 7 > 699$  (d) None of these
- (b)  $90 \times 7 > 699$   
 $100 \times 7 < 699$
62. What is the largest multiple of 10 that will make this sentence true: 1 2 3  
 $n \times 60 < 5723$
- (a) 70 (d) 80  
(b) 90 (e) None of these  
(c) 60
63. Which is the best estimate for  $986 \div 29$ : 1 2 3
- (a) 50 (d) 30  
(b) 40 (e) None of these  
(c) 20
- 

Division: Averages

64. What is the average of 1784, 1596, 1851, 1679, 2048, and 1860: 1 2 3
- (a) 1850 (d) 1903  
(b) 1803 (e) None of these  
(c) 1800
65. Which of the following is true: 1 2 3
- (a) The average of 8 and 36 is 44 (d) The average of 12 and 38 is 25  
(b) The average of 14 and 24 is 10 (e) None of these  
(c) The average of 6 and 30 is 180
66. Another name for arithmetic mean is: 1 2 3
- (a) Difference (d) Product  
(b) Average (e) None of these  
(c) Sum

Percent

67. What percent of 150 is 6? 1 2 3  
(a) 4% (c) 25%  
(b) 900% (d) None of these
68. In the exercise  $15\%$  of  $200 = 30$ , the "percentage" is: 1 2 3  
(a) 200 (c) 30  
(b) 15 (d) None of these
69. If a jacket which normally sells for \$50 is on sale for \$35, how much is the discount: 1 2 3  
(a) 15% (d) 35%  
(b) 30% (e) None of these  
(c) 70%
- 

Measurement: Length

70. One mile is equal to: 1 2 3  
(a) 5282 feet (d) 1750 yards  
(b) 1670 yards (e) None of these  
(c) 5820 feet
71. One inch is approximately: 1 2 3  
(a) 25 cm. (d) 250 cm.  
(b)  $2\frac{1}{2}$  cm. (e) None of these  
(c) 30 cm.
72. Finding the distance from one point to another involves what kind of measure: 1 2 3  
(a) Length  
(b) Area  
(c) Volume  
(d) None of these

Measurement: Area

73. To determine the amount of carpet needed to cover a floor involves what kind of measure: 1 2 3
- (a) Volume (c) Length  
(b) Area (d) None of these
74. If the edges of a cube are four inches long, what is the surface area of the cube: 1 2 3
- (a) 4 square inches (d) 96 square inches  
(b) 16 square inches (e) None of these  
(c) 64 square inches
75. What is the area of an obtuse triangle if its base is 6 inches and its height is 7 inches: 1 2 3
- (a) 26 square inches (d) 21 square inches  
(b) 42 square inches (e) None of these  
(c) 13 square inches
- 

Measurement: Volume

76. What is the volume of a rectangular prism with the following measurements?  $l = 8$   $h = 6$   $w = 4$  1 2 3
- (a) 18 cubic units (d) 192 cubic units  
(b) 18 square units (e) None of these  
(c) 192 square units
77. A rectangular prism has edges that measure  $9 \times 11 \times 35$  inches. If a gallon contains 231 cubic inches of water, how many gallons will the container hold: 1 2 3
- (a) 7 (d) 15  
(b) 11 (e) None of these  
(c) 3465
78. To determine the volume of a rectangular prism, which of the following formulas would you use: 1 2 3
- (a)  $5 + 5 + 5 + 5$  (d)  $l \times w \times h$   
(b)  $l \times w$  (e) None of these  
(c)  $\pi r^2$

Measurement: Perimeter, circumference

79. The perimeter of a polygon is computed by the following method: 1 2 3  
 (a) Length times width  
 (b) Totalling the lengths of all of its sides  
 (c) Length times width times height  
 (d) None of these
80. Which of the following formulas is used in computing the circumference of a circle? 1 2 3  
 (a)  $2\pi r$  (c)  $d \times \pi r$   
 (b)  $\pi r^2$  (d) None of these
81. Which formula would be used to compute the perimeter of a football field: 1 2 3  
 (a)  $l \times w$  (d)  $\pi r^2$   
 (b)  $s + s + s + s$  (e) None of these  
 (c)  $l \times w \times h$
- 

Measurement: Metric System

82. One meter is equal to: 1 2 3  
 (a) 10 centimeters (d) 10 dekameters  
 (b) 1000 decimeters (e) None of these  
 (c) 100 millimeters
83. One kilometer is: 1 2 3  
 (a) Greater than one mile  
 (b) Less than one mile  
 (c) Equal to one mile
84. If a horse walks 8 kilometers per hour, how far will the horse walk in 3 hours: 1 2 3  
 (a) 240,000 meters (c) 24000 dekameters  
 (b) 2400 hectometers (d) 240 decimeters  
 (e) None of these

Rational Numbers: Meaning

85. In the division exercise  $1/3 \div 1/2 = 2/3$ , the  $2/3$  represents: 1 2 3  
 (a)  $2/3$  of a whole (d)  $2/3$  of two-thirds  
 (b)  $2/3$  of one-half (e) None of these  
 (c)  $2/3$  of one-third
86. Which of the following is a rational number? 1 2 3  
 (a)  $1/4$  (d) Choices (a), (b), and (c) are all rational numbers  
 (b) 25% (e) None of these  
 (c) .4
87. The rational number  $4/5$  means: 1 2 3  
 (a) 4 out of 5 equivalent disjoint sets  
 (b) 4 parts of an object which has been divided into 5 equal parts  
 (c) 4 divided by 5  
 (d) Any of the above could be correct  
 (e) None of these
- 

Rational Numbers: Equivalent fractions

88. Given  $5/6 \approx x/18$ , what is the value of x? 1 2 3  
 (a) 17 (d) 16  
 (b) 5 (e) None of these  
 (c) 6
89. Given the fractions  $a/b$  and  $c/d$ , a test for equivalency is whether or not: 1 2 3  
 (a)  $a \times d = b \times c$  (d)  $a \times d = c \times d$   
 (b)  $a \times c = b \times d$  (e) None of these  
 (c)  $a \times b = c \times d$
90. The fractions  $2/3$ ,  $4/6$ ,  $8/12$ , and  $10/15$  are called: 1 2 3  
 (a) Like fractions (d) Equivalent fractions  
 (b) Equal fractions (e) None of these  
 (c) Mixed fractions

Rational Numbers: Reducing to lowest terms

91. Which of the following is a reduced fraction: 1 2 3  
(a)  $6/3$  (d)  $2/4$   
(b)  $24/27$  (e) None of these  
(c)  $12/15$
92. When the rational number  $15/45$  is reduced to lowest terms, the result is: 1 2 3  
(a)  $1/4$  (d)  $5/15$   
(b)  $3/9$  (e) None of these  
(c)  $1/3$
93.  $2/3$  is the reduced form of which of the following: 1 2 3  
(a)  $15/25$  (d)  $75/120$   
(b)  $3/4$  (e) None of these  
(c)  $45/60$
- 

Rational Numbers: Adding fractions and mixed numerals

94. Compute:  $2/3 + 2/5 = ?$  1 2 3  
(a)  $4/15$  (d)  $10/5$   
(b)  $1\ 1/15$  (e) None of these  
(c)  $4/8$
95.  $2\ 2/3 + 3\ 4/9 = ?$  1 2 3  
(a)  $5\ 5/12$  (d)  $11/12$   
(b)  $5\ 10/18$  (e) None of these  
(c)  $15/9$
96. In order to solve the problem  $5/9 + 7/15$  it is necessary to find: 1 2 3  
(a) The least common multiple of 9 and 15  
(b) The greatest common factor of 9 and 15  
(c) The greatest common factor of 5 and 7  
(d) The least common multiple of 5 and 7  
(e) None of these



Rational Numbers: Mixed numerals and improper fractions

97.  $25 \frac{7}{8}$  is a: 1 2 3
- (a) Whole number (d) Irrational number  
 (b) Mixed numeral (e) None of these  
 (c) Natural number
98. Which of the following is an improper fraction; 1 2 3
- (a)  $\frac{6}{5}$  (d) Both (a) and (b)  
 (b)  $\frac{9}{9}$  (e) None of these  
 (c)  $\frac{8}{10}$
99. Which of the following terms describes the rational number  $\frac{23}{21}$ ; 1 2 3
- (a) Cardinal number (d) Proper fraction  
 (b) Improper fraction (e) None of these  
 (c) Mixed numeral

Rational Numbers: Subtracting fractions and mixed numerals

100.  $\frac{1}{7} - \frac{7}{50} = ?$  1 2 3
- (a)  $\frac{1}{700}$  (d)  $\frac{43}{350}$   
 (b)  $\frac{1}{350}$  (e) None of these  
 (c)  $\frac{3}{70}$
101. With which problem is regrouping necessary? 1 2 3
- (a)  $\frac{2}{5} - \frac{1}{5}$  (d)  $7 \frac{3}{10} - 4 \frac{7}{10}$   
 (b)  $7 \frac{3}{8} - 4 \frac{1}{8}$  (e) None of these  
 (c)  $9 \frac{7}{16} - 3 \frac{5}{16}$
102. Which of the following contains an error; 1 2 3
- (a)  $7 \frac{1}{2} = 7 \frac{2}{4} = 6 \frac{6}{4}$  (c)  $8 \frac{4}{9} = 8 \frac{8}{18} = 7 \frac{26}{18}$   
 $\underline{-2 \frac{3}{4}} = \underline{2 \frac{3}{4}} = \underline{2 \frac{3}{4}}$   $\underline{-5 \frac{5}{6}} = \underline{5 \frac{15}{18}} = \underline{5 \frac{15}{18}}$   
 $4 \frac{3}{4}$   $2 \frac{11}{18}$
- (b)  $6 \frac{2}{3} = 6 \frac{8}{12}$  (d)  $4 \frac{7}{10} = 4 \frac{21}{30}$   
 $\underline{-1 \frac{7}{12}} = \underline{1 \frac{7}{12}}$   $\underline{-2 \frac{11}{15}} = \underline{2 \frac{22}{30}}$   
 $5 \frac{1}{12}$   $2 \frac{1}{30}$
- (e) None of these

Rational Numbers: Multiplying fractions and mixed numerals

103. Multiply  $3 \frac{1}{3}$  by  $1 \frac{1}{5}$  1 2 3
- (a)  $\frac{6}{8}$  (d) 4  
 (b)  $\frac{30}{18}$  (e) None of these  
 (c)  $\frac{16}{8}$
104.  $\frac{3}{5} \cdot \frac{3}{5} = ?$  1 2 3
- (a)  $\frac{9}{25}$  (d)  $\frac{6}{10}$   
 (b)  $\frac{9}{5}$  (e) None of these  
 (c)  $\frac{3}{25}$
105. What is the value of "a" in the following sentence: 1 2 3  
 $\frac{2}{7} \times \frac{7}{2} = (2 \times \frac{1}{7}) \times (7 \times \frac{1}{2}) = a \times b$
- (a)  $\frac{14}{14}$  (d)  $\frac{2}{2}$   
 (b)  $\frac{7}{2}$  (e) None of these  
 (c) 14
- 

Rational Numbers: Dividing Fractions

106.  $\frac{1}{3} \div 3 \frac{1}{6} = ?$  1 2 3
- (a)  $\frac{2}{19}$  (d)  $\frac{19}{2}$   
 (b)  $\frac{19}{18}$  (e) None of these  
 (c)  $\frac{2}{3}$
107.  $\frac{4 \frac{3}{5}}{3 \frac{1}{6}}$  means 1 2 3
- (a)  $4 \frac{3}{5} \div 3 \frac{1}{6}$  (d)  $4 \frac{3}{5} - 3 \frac{1}{6}$   
 (b)  $3 \frac{1}{6} \div 4 \frac{3}{5}$  (e) None of these  
 (c)  $3 \frac{1}{6} \times 4 \frac{3}{5}$
108. Which of the following has a quotient of  $1 \frac{1}{2}$ : 1 2 3
- (a)  $2 \frac{1}{4} \div 3 \frac{3}{8}$  (d) Both (b) and (c)  
 (b)  $3 \frac{3}{8} \div 2 \frac{1}{4}$  (e) None of these  
 (c)  $2 \frac{3}{4} \div \frac{6}{11}$

Decimals: Adding and Subtracting

109. What is the hundreds digit in the answer to  $3375.49 - 861.1$ ? 1 2 3  
(a) 9 (d) 2  
(b) 1 (e) None of these  
(c) 4
110.  $495.738 + 367.2 =$  1 2 3  
(a) 49941.0 (d) 862.938  
(b) 499.410 (e) None of these  
(c) 86.2938
111. When subtracting decimal numerals: 1 2 3  
(a) The number of decimal places in the difference is equal to the sum of the decimal places in the minuend and subtrahend.  
(b) The number of decimal places in the difference is equal to the product of the decimal places in the minuend and subtrahend.  
(c) The decimals in the minuend, subtrahend, and difference are lined up with each other.  
(d) The place value of the digits in the difference is not dependent upon the position of the decimal.  
(e) None of these.
- 

Decimals: Multiplying

112.  $0.7 \times 0.02 = ?$  1 2 3  
(a) 0.14 (d) 14  
(b) 0.014 (e) None of these  
(c) 1.4
113.  $5/1000$  of 0.16 is: 1 2 3  
(a) .8 (d) .0008  
(b) .08 (e) None of these  
(c) .008
114. When multiplying decimal numerals, the position of the decimal in the product is determined by: 1 2 3  
(a) Multiplying the number of positions to the right of the decimal in the two factors.  
(b) Adding the number of positions to the right of the decimal in the two factors.  
(c) Multiplying the number of positions to the left of the decimal in the two factors.  
(d) Adding the number of positions to the left of the decimal in the two factors.  
(e) None of these

Decimals: Dividing

115. Which of the following division examples will result in the quotient 25.2? 1 2 3
- (a)  $.5 \overline{)126}$  (d)  $.0005 \overline{)126}$   
 (b)  $.05 \overline{)126}$  (e) None of these  
 (c)  $.005 \overline{)126}$
116. The quotient of  $326.6 \div .71$  is: 1 2 3
- (a) .046 (d) 4.6  
 (b) .46 (e) None of these  
 (c) 460
117. In the division  $.04 \overline{)5638}$ , moving the decimals to the positions indicated 1 2 3  
 by the " $\wedge$ " has the following effect:
- (a) Both the divisor and dividend are multiplied by 100 and the quotient is 10 times greater.  
 (b) The divisor, dividend, and quotient are all multiplied by 100.  
 (c) The value of the quotient is reduced.  
 (d) It has the effect of multiplying the entire problem by one.  
 (e) None of these

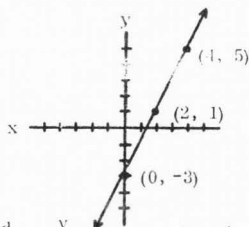
Decimals: Repeating and terminating

118. Expressed as a decimal, the fraction  $1/6$  is exactly equal to: 1 2 3
- (a) .16 (d)  $.16\overline{6}$   
 (b)  $.16\overline{7}$  (e) None of these  
 (c)  $.16\overline{6}$
119. Numbers whose decimal equivalents terminate are: 1 2 3
- (a) All real numbers  
 (b) All rational numbers  
 (c) All rational numbers which have a 2 or a 5 in the denominator  
 (d) All rational numbers which have no other prime factors of their denominators than 2 and 5.  
 (e) None of these
120. In performing the division indicated by  $4/9$  the resulting quotient will be a: 1 2 3
- (a) Terminating decimal  
 (b) Completed decimal  
 (c) Proper fraction  
 (d) Repeating decimal  
 (e) None of these

Graphing: Graphing Coordinates

121. Which of the following is plotted at the right:

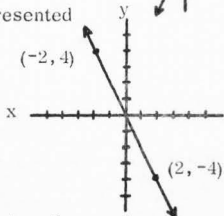
- (a)  $y = 2x - 3$   
 (b)  $y = 2x + 3$   
 (c)  $y = 3x - 2$   
 (d)  $y = 3x + 2$   
 (e) None of these



1 2 3

122. Which of the following equations is represented graphically:

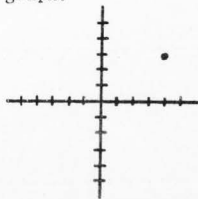
- (a)  $x = 2y$   
 (b)  $y = -2x$   
 (c)  $x = -2y$   
 (d)  $y = 2x - 1$   
 (e) None of these



1 2 3

123. Which of the following identifies the point on the graph:

- (a) (3, 4)  
 (b) (4, 3)  
 (c) Both (a) and (b)  
 (d) None of these



1 2 3

Graphing: Functions

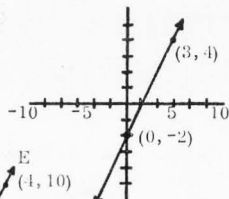
124. If a line passes through the points (0, -2), (1, 0), and (2, 2) on a coordinate system, it represents which of the following functions:

- (a)  $f(n) = 2n$   
 (b)  $f(n) = 2n - 2$   
 (c)  $f(n) = 2n + 2$   
 (d)  $f(n) = n - 1$   
 (e) None of these

1 2 3

125. Which of the following functions is represented on the graph:

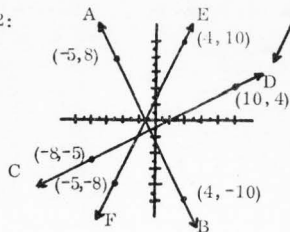
- (a)  $f(n) = 2n + 2$   
 (b)  $f(n) = n + 3$   
 (c)  $f(n) = 2n - 2$   
 (d)  $f(n) = n - 2$   
 (e) None of these



1 2 3

126. Which line represents
- $f(n) = 2n + 2$
- :

- (a)  $\overleftrightarrow{AB}$   
 (b)  $\overleftrightarrow{CD}$   
 (c)  $\overleftrightarrow{EF}$   
 (d) None of these



1 2 3

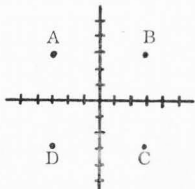
Graphing: Graphing negative numbers

127. Identify the quadrant or axis in which the point (4, -3) is found: 1 2 3

- (a) First quadrant (d) Negative horizontal axis  
 (b) Positive vertical axis (e) None of these  
 (c) Third quadrant

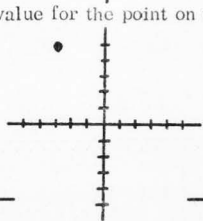
128. Which point on the graph represents (-3, -3): 1 2 3

- (a) A  
 (b) B  
 (c) C  
 (d) D  
 (e) None of these



129. Which of the following represents the x-axis value for the point on the graph: 1 2 3

- (a) 3  
 (b) -3  
 (c) 5  
 (d) -5  
 (e) None of these

Special Topics: Bases other than ten

130. Solve the following:  $43_{\text{five}} = \text{---}_{\text{ten}}$  1 2 3

- (a) 110 (d) 36  
 (b) 18 (e) None of these  
 (c) 23

131. Which set of numerals are not listed in a consecutive counting order: 1 2 3

- (a) 110, 111, 112 (Base 3) (d) 101, 110, 111 (Base 4)  
 (b) 14, 20, 21 (Base 5) (e) None of these  
 (c) 100, 101, 110 (Base 2)

132.  $\square \times 20_{\text{three}} = 300_{\text{four}}$  1 2 3

- (a)  $20_{\text{four}}$  (d) All of the above  
 (b)  $10_{\text{eight}}$  (e) None of these  
 (c)  $1000_{\text{two}}$

Special Topics: Place value

133. The numeral one million, one hundred one thousand, ten, and three thousandths, is written as follows: 1 2 3  
 (a) 1,100,110.003 (d) 1,101,010.03  
 (b) 1,101,010.0003 (e) None of these  
 (c) 1,100,001,010.003
134. Which of the following is arranged in order of decreasing value: 1 2 3  
 (a) .005, .05, .051, .501, .53 (d) .501, .53, .051, .05, .005  
 (b) .53, .501, .051, .05, .005 (e) None of these  
 (c) .005, .05, .051, .53, .501
135. In a place value system the value represented by a given symbol: 1 2 3  
 (a) Is always the same as the face value of the symbol  
 (b) Is independent of the base  
 (c) Is independent of the position of the symbol  
 (d) Depends on both the position of the symbol and the base  
 (e) None of these

Special Topics: Exponents

136.  $8^0 = \square$  1 2 3  
 (a) 0 (d) 80  
 (b) 1 (e) None of these  
 (c) 8
137. What is the value of  $2^5$ : 1 2 3  
 (a) 10 (d) 64  
 (b) 32 (e) None of these  
 (c) 16
138. An exponent: 1 2 3  
 (a) Tells how many times to add a number  
 (b) Tells the size of the base  
 (c) Tells how many times a number is to be used as a factor  
 (d) Is an expanded form of multiplication  
 (e) None of these

Special Topics: Clock and modular arithmetic

139. In 5-minute clock arithmetic,  $1^{-1} = \square$  1 2 3  
 (a) 0 (d) 3  
 (b) 1 (e) None of these  
 (c) 2
140. Which of the following is true: 1 2 3  
 (a)  $5 \equiv 9 \pmod{3}$  (d)  $34 \equiv -7 \pmod{13}$   
 (b)  $7 + 2 \equiv 3 \pmod{6}$  (e) None of these  
 (c)  $4 \equiv -4 \pmod{6}$
141. Using 5-minute clock arithmetic, which of the following is a true sentence: 1 2 3  
 (a)  $2 + 3 = 0$  (d)  $1 + 5 = 2$   
 (b)  $4 - 4 = 4$  (e) None of these  
 (c)  $2 - 3 = 1$

Special Topics: Negative numbers

142. The number represented by  $-(-6)$  is: 1 2 3  
 (a) The additive inverse of 6  
 (b) The additive inverse of the additive inverse of 6  
 (c) A negative number  
 (d) All of the above are correct  
 (e) None of these
143.  $(-x) + (-y) = \square$  : 1 2 3  
 (a)  $x - y$   
 (b)  $y - x$   
 (c)  $-(x + y)$   
 (d)  $-x + y$   
 (e) None of these
144.  $\frac{3}{-8}$  is equal to: 1 2 3  
 $\frac{2}{-3}$   
 (a)  $9/16$  (d)  $-9/16$   
 (b)  $-4$  (e) None of these  
 (c)  $4$



Special Topics: Probability

145. On the single toss of a pair of dice, what is the probability that their sum is 5 if you are told that one of the dice came up 3; 1 2 3
- (a)  $1/18$  (d)  $1/9$   
 (b)  $1/6$  (e) None of these  
 (c)  $2/12$
146. One card is drawn from each of two sets of four cards (each set numbered 1 through 4). What is the probability that the sum of the numbers is equal to seven; 1 2 3
- (a)  $1/4$   
 (b)  $1/8$   
 (c)  $1/16$   
 (d) None of these
147. A box contains four black, seven white, and three red balls. If one ball is drawn, what is the probability that it is red; 1 2 3
- (a)  $3/7$  (d)  $1/7$   
 (b)  $2/7$  (e) None of these  
 (c)  $3/14$

Special Topics: Inequalities

148. If  $a < b$ , then; 1 2 3
- (a)  $ac < bc$  (d)  $-a < -b$   
 (b)  $c - a < c - b$  (e) None of these  
 (c)  $a + c > b + c$
149. Which of the following could be made true by using the symbol "<"; 1 2 3
- (a)  $5/9 \bigcirc 2/3$  (d)  $6/21 \bigcirc 10/35$   
 (b)  $9/7 \bigcirc 5/7$  (e) None of these  
 (c)  $11/20 \bigcirc 8/15$
150. Which of the following symbols could be used to make this sentence true:  $8 + 4 \bigcirc 2 \cdot 7$  1 2 3
- (a)  $\neq$  (d) All of the above  
 (b)  $<$  (e) None of the above  
 (c)  $\leq$

Special Topics: Functions

151. Given  $f(n) = 3n - 6$ , complete the following:  $f(3)$   1 2 3  
 (a) 15 (d) 3  
 (b) 9 (e) None of these  
 (c) 6
152. Which open sentence defines the function whose members  $\{x, f(x)\}$  are shown at right: 1 2 3
- | $x$ | $f(x)$ |
|-----|--------|
| 0   | 3      |
| 1   | 5      |
| 2   | 7      |
| 3   | 9      |
- (a)  $f(x) = x + 3$   
 (b)  $f(x) = 2x + 2$   
 (c)  $f(x) = 3x + 2$   
 (d)  $f(x) = 2x + 3$   
 (e) None of these
153. If  $f(n) = -4n - 3$  and  $n = 5$ , what is  $f(n)$ : 1 2 3  
 (a) 17 (d) 23  
 (b) 5 (e) None of these  
 (c) 2
- 

Special Topics: Mean, median, mode

154. Given the data 7, 5, 6, 10, 6, the mode is: 1 2 3  
 (a) 5 (d) 6  
 (b) 17 (e) None of these  
 (c) 10
155. Given the data 3, 7, 5, 9, 15, 10, 5, which of the following is a true statement: 1 2 3  
 (a) Mean = 7 (d) Median = 7  
 (b) Median = 27 (e) None of these  
 (c) Mode = 15
156. Given the data 10, 5, 15, 9, 5, 7, 3, the number 27 is the: 1 2 3  
 (a) Mean  
 (b) Median  
 (c) Mode  
 (d) None of these

Special Topics: Cardinal, ordinal, whole, natural, and real numbers

157. Which of the following statements correctly defines the set of integers: 1 2 3
- (a) The set of all positive and negative rational numbers  
 (b) The set of all positive whole numbers  
 (c) The set of all natural numbers and their additive inverses  
 (d) None of these
158. The **rational** number  $1/2$  is found in which of the following: 1 2 3
- (a) The set of natural numbers (d) The set of real numbers  
 (b) The set of integers (e) None of these  
 (c) The set of whole numbers
159. In the sentence, "John was the 15th person selected for a scholarship," 1 2 3  
 what kind of number is 15:
- (a) Ordinal (c) Irrational  
 (b) Cardinal (d) None of these
- 

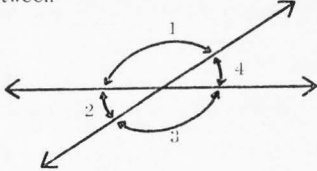
Special Topics: Congruency

160. Which of the following phrases best describes the term "congruent": 1 2 3
- (a) Is related to (d) Same size and shape as  
 (b) Smaller than (e) None of these  
 (c) Larger than
161.  $\overline{AB} \cong \overline{EF}$  means: 1 2 3
- (a)  $\overline{AB}$  is approximately equal to  $\overline{EF}$   
 (b)  $\overline{AB}$  is congruent to  $\overline{EF}$   
 (c)  $\overline{AB}$  and  $\overline{EF}$  are parallel  
 (d)  $\overline{AB}$  is perpendicular to  $\overline{EF}$   
 (e) None of these
162. Two triangles are congruent if: 1 2 3
- (a) The small triangle fits perfectly inside the large triangle  
 (b) They are both equilateral triangles  
 (c) When one is placed over the other they fit exactly  
 (d) The sides of one triangle are parallel to the sides of the other triangle  
 (e) None of these

Special Topics: Propositions and Implications

163. Given the implication, "If  $p$  then  $q$ " and given that  $q$  is true, which of the following would be a valid conclusion: 2 3
- (a)  $p$  is true  
 (b)  $p$  may be either true or false  
 (c)  $p$  is false  
 (d)  $p$  is neither true nor false  
 (e) None of these
164. Given the implication, "If you like mathematics, then you like this course," which of the following is the converse of that implication: 2 3
- (a) If you do not like mathematics, you do not like this course.  
 (b) If you like this course, you like mathematics.  
 (c) If you do not like this course, you do not like mathematics.  
 (d) None of these
165. Given the proposition, "All students are intelligent," which of the following best describes the statement, "If you are not intelligent, then you are not a student." 1 2 3
- (a) Negation (d) Contrapositive  
 (b) Converse (e) None of these  
 (c) Inverse

Geometry: Angles

166. Given angles 1, 2, 3, and 4 as shown below, which term best describes the relation between  $\angle 2$  and  $\angle 4$ : 1 2 3
- (a) Supplementary angles  
 (b) Complementary angles  
 (c) Vertical angles  
 (d) Supplementary adjacent angles  
 (e) None of these
- 
167. A  $45^\circ$  angle would be classified as which of the following: 1 2 3
- (a) Acute angle (d) Straight angle  
 (b) Obtuse angle (e) None of these  
 (c) Right angle
168. The corner of a window forms an angle of: 1 2 3
- (a)  $45^\circ$  (d)  $180^\circ$   
 (b)  $60^\circ$  (e) None of these  
 (c)  $90^\circ$




Geometry: Space figures

169. A polyhedron with 12 faces is a: 1 2 3
- (a) Tetrahedron (d) Dodecahedron  
 (b) Octahedron (e) None of these  
 (c) Icosahedron
170. A rectangular pyramid has how many faces: 1 2 3
- (a) 5 (d) 3  
 (b) 4 (e) None of these  
 (c) 6
171. Which of the following space figures has six edges: 1 2 3
- (a) Triangular prism (d) Cube  
 (b) Tetrahedron (e) None of these  
 (c) Rectangular pyramid
- 







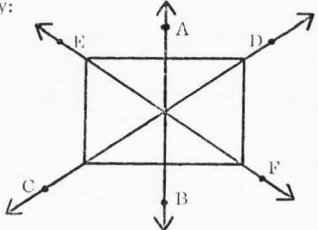


Geometry: Points, segments, rays, and lines

172. Given three points not on a straight line, how many planes can contain all three points: 1 2 3
- (a) zero (d) Infinite number  
 (b) three (e) None of these  
 (c) one
173. A line segment containing points A and B could be referred to symbolically as: 1 2 3
- (a)  $\overleftrightarrow{AB}$  (d)  $\overline{\overline{AB}}$   
 (b)  $\overrightarrow{AB}$  (e) None of these  
 (c)  $\overleftarrow{AB}$
174. Which of the following is true of a ray: 1 2 3
- (a) Extends on and on in two directions  
 (b) Has one end point and extends on and on in one direction  
 (c) Is a three-dimensional object  
 (d) Is a point in space  
 (e) None of these

Geometry: Plane figures

175. Which of the following types of triangles has an altitude endpoint which is not a vertex: 1 2 3
- (a) Right triangle (d) Isosceles triangle  
 (b) Obtuse triangle (e) None of these  
 (c) Acute triangle
176. Which of the following is a polygon: 1 2 3
- (a)  (d) All of the above  
 (b)  (e) None of these  
 (c) 
177. A 7-sided plane figure is called: 1 2 3
- (a) Hexagon (d) Decagon  
 (b) Octagon (e) None of these  
 (c) Nonagon

Geometry: Symmetry, reflections, similar patterns

178. Given  $\triangle ABC \sim \triangle DEF$ . If  $AB = 8$ ,  $BC = 4$ , and  $DE = 12$ , what is  $EF$ : 1 2 3
- (a) 6 (d) 12  
 (b) 8 (e) None of these  
 (c) 4
179. Which of the lines running through the rectangle is a line of symmetry: 1 2 3
- (a)   
 (b) Both  and   
 (c) , , and   
 (d) None of these
- 
180. Which of the following terms describes the relationship between these two triangles: 1 2 3
- (a) Symmetry   (d) All of the above  
 (b) Reflections (e) None of these  
 (c) Similar patterns

## APPENDIX E

Preliminary Test Forms A and B

1. Given that  $A = \{1, 2, 4, 5\}$ ,  $B = \{1, 3, 4, 7\}$ ,  $C = \{1, 2, 3, 6\}$ , which of the following is most appropriate for expressing the union of A, B, and C:
- (a)  $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7\}$  (d)  $A \cup B \cup C = \{ \}$   
 (b)  $A \cup B \cup C = \{1, 1, 1, 2, 2, 3, 3, 4, 4, 5, 6, 7\}$  (e) None of these  
 (c)  $A \cup B \cup C = \{1\}$
2. If Set  $A = \{a, b, c, d, e\}$  and Set  $B = \{d, e, f, g\}$ , what is  $A \cap B$ :
- (a)  $\{a, b, c, d, e, f, g\}$  (d)  $\{a, b, c, d, d, e, e, f, g\}$   
 (b)  $\{d, e\}$  (e) None of these  
 (c)  $\{a, b, c, f, g\}$
3. Given that Set  $A = \{1, 2, 3, 4\}$  and Set  $B = \{5, 6, 7\}$ , which of the following is true:
- (a)  $A \cap B = \emptyset$  (d)  $A \cap B = \{\emptyset\}$   
 (b)  $A \cup B = \{ \}$  (e) None of these  
 (c)  $A \cap B \neq \{ \}$
4. "3 x 3 x 4" could be described as:
- (a) A factorization of 36 (d) Both (a) and (c)  
 (b) A set of prime factors of 36 (e) None of these  
 (c) A prime factorization of 36
5. Resolve 330 into prime factors:
- (a)  $3 \times 11 \times 10$  (d)  $2 \times 3 \times 5 \times 11$   
 (b)  $6 \times 11 \times 5$  (e) None of these  
 (c)  $2 \times 15 \times 11$
6. What is the least common multiple of 56 and 21:
- (a) 336 (d) 1  
 (b) 168 (e) None of these
7. Which of the following is true:
- (a) The sum of 43 and 26 contains seven tens  
 (b) The sum of 27 and 62 contains nine tens  
 (c) The sum of 3842 and 2037 contains nine hundreds  
 (d) The sum of 27,932 and 312,063 contains eight ten-thousands  
 (e) None of these



8. In the addition exercise below, which of the following could be possible digits for the  $\blacktriangle$  :

- (a) 5                      (d) All of the above  
 (b) 2                      (e) None of these  
 (c) 6

$$\begin{array}{r}
 \square 9 \square \\
 + \square 5 \square \\
 \hline
 \square \blacktriangle \square
 \end{array}$$

9. Which property of whole numbers justifies the following statement:

$$(4 + 3) + 5 = 5 + (4 + 3)$$

- (a) Distributive property of multiplication over addition  
 (b) Closure for addition of whole numbers  
 (c) Commutative property of addition  
 (d) Associative property of addition  
 (e) None of these

10. Which of the following properly shows subtraction by expanded notation:

$$\begin{aligned}
 \text{(a)} \quad 37 - 6 &= (30 - 7) - 6 \\
 &= 30 - (7 - 6) \\
 &= 31
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad 48 - 7 &= (40 + 8) - 7 \\
 &= 40 + (8 - 7) \\
 &= 41
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad 34 - 6 &= (20 + 14) - 6 \\
 &= 20 - (14 + 6) \\
 &= 28
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad 42 - 7 &= (30 + 12) - 7 \\
 &= 30 + (12 - 7) \\
 &= 25
 \end{aligned}$$

- (e) None of these

11. Which of the following represents the proper regrouping for the minuend in order to solve  $624 - 478$ :

- (a)  $600 + 10 + 14$   
 (b)  $500 + 120 + 4$   
 (c)  $500 - 120 - 4$

- (d)  $500 + 110 + 14$   
 (e) None of these

12. Which of the following best illustrates  $3 \times 5$ :

- (a)  $3 + 3 + 3 + 3 + 3$   
 (b)  $5 + 5 + 5$   
 (c) Both a and b illustrate  $3 \times 5$  equally well  
 (d) Neither (a) nor (b) illustrates  $3 \times 5$

13. In the multiplication problem  $4 \times 17$ , the 4 tells:

- (a) How many sets are being considered  
 (b) How many are in each set  
 (c) How many in all  
 (d) None of these

14. In the multiplication exercise shown below, which horizontal expression best illustrates through expanded notation the multiplication of the partial products: 248

$$\begin{array}{r} 248 \\ \times 26 \\ \hline \end{array}$$

- (a)  $(6 \times 8) + (6 \times 4) + (6 \times 2) + (20 \times 8) + (20 \times 4) + (20 \times 2)$   
 (b)  $(6 \times 8) + (6 \times 40) + (6 \times 200) + (20 \times 8) + (20 \times 40) + (20 \times 200)$   
 (c)  $(6 \times 8) + (6 \times 40) + (6 \times 200) + (2 \times 8) + (2 \times 40) + (2 \times 200)$   
 (d)  $(6 \times 8) + (6 \times 4) + (6 \times 2) + (2 \times 8) + (2 \times 4) + (2 \times 2)$   
 (e) None of these
15. The inverse of multiplication is:
- (a) Addition (c) Division  
 (b) Subtraction (d) None of these
16. If  $x$  and  $y$  are any two whole numbers, then  $x \cdot y$  is a whole number. This statement defines which of the following properties:
- (a) Associative property of multiplication  
 (b) Closure property for multiplication  
 (c) Identity property for multiplication  
 (d) Commutative property for multiplication  
 (e) None of these
17. Which of the following illustrates the distributive property of multiplication over addition:
- (a)  $4 \times 5 \frac{1}{3} = (4 \times 5) + (4 \times \frac{1}{3})$  (d)  $(7 \times 2) \times 4 = 4 \times (7 \times 2)$   
 (b)  $(8 \times 9) \times 4 = 8 \times (9 \times 4)$  (e) None of these  
 (c)  $8 \times 6 = 6 \times 8$
18. Which of the following are ways of expressing twelve divided by four:
- (a)  $12 \div 4$  (c)  $12/4$   
 (b)  $4 \overline{)12}$  (d) Both (a) and (b)  
 (e) Choices (a), (b), and (c) are all correct
19. If the quotient is 32, the divisor is 6, and the remainder is 2, what is the dividend:
- (a) 5 (d) 194  
 (b) 192 (e) None of these  
 (c) 44

20. Which statement best explains the 4 in the quotient of the following division problem?  $24 \overline{)1080}^{45}$
- (a) There are at least forty 24's in 1080  
 (b) Two goes into ten four times  
 (c) There are four 24's in 108  
 (d) None of these
21. What is the largest multiple of 10 that will make this sentence true:  
 $n \times 60 < 5723$
- (a) 70  
 (b) 90  
 (c) 60  
 (d) 80  
 (e) None of these
22. Which of the following is true:
- (a) The average of 8 and 36 is 44  
 (b) The average of 14 and 24 is 10  
 (c) The average of 6 and 30 is 180  
 (d) The average of 12 and 38 is 25  
 (e) None of these
23. If a jacket which normally sells for \$50 is on sale for \$35, how much is the discount:
- (a) 15%  
 (b) 30%  
 (c) 70%  
 (d) 35%  
 (e) None of these
24. One inch is approximately:
- (a) 25 cm.  
 (b) 2 1/2 cm.  
 (c) 30 cm.  
 (d) 250 cm.  
 (e) None of these
25. If the edges of a cube are four inches long, what is the surface area of the cube:
- (a) 4 square inches  
 (b) 16 square inches  
 (c) 64 square inches  
 (d) 96 square inches  
 (e) None of these
26. What is the volume of a rectangular prism with the following measurements?  $l = 8$   $h = 6$   $w = 4$
- (a) 18 cubic units  
 (b) 18 square units  
 (c) 192 square units  
 (d) 192 cubic units  
 (e) None of these

27. The perimeter of a polygon is computed by the following method:
- (a) Length times width
  - (b) Totalling the lengths of all of its sides
  - (c) Length times width times height
  - (d) None of these
28. One meter is equal to:
- (a) 10 centimeters
  - (b) 1000 decimeters
  - (c) 100 millimeters
  - (d) 10 dekameters
  - (e) None of these
29. The rational number  $\frac{4}{5}$  means:
- (a) 4 out of 5 equivalent disjoint sets
  - (b) 4 parts of an object which has been divided into 5 equal parts
  - (c) 4 divided by 5
  - (d) Any of the above could be correct
  - (e) None of these
30. Given the fractions  $\frac{a}{b}$  and  $\frac{c}{d}$ , a test for equivalency is:
- (a)  $a \times d = b \times c$
  - (b)  $a \times c = b \times d$
  - (c)  $a \times b = c \times d$
  - (d)  $a \times d = c \times d$
  - (e) None of these
31. When the rational number  $\frac{15}{45}$  is reduced to lowest terms, the result is:
- (a)  $\frac{1}{4}$
  - (b)  $\frac{3}{9}$
  - (c)  $\frac{1}{3}$
  - (d)  $\frac{5}{15}$
  - (e) None of these
32. In order to solve the problem  $\frac{5}{9} + \frac{7}{15}$  it is helpful to find:
- (a) The least common multiple of 9 and 15
  - (b) The greatest common factor of 9 and 15
  - (c) The greatest common factor of 5 and 7
  - (d) The least common multiple of 5 and 7
  - (e) None of these
33. Which of the following terms describes the rational number  $\frac{23}{21}$ :
- (a) Cardinal number
  - (b) Improper fraction
  - (c) Mixed number
  - (d) Proper fraction
  - (e) None of these

34.  $1/7 - 7/50 = ?$

- (a)  $1/700$   
 (b)  $1/350$   
 (c)  $3/70$

- (d)  $43/350$   
 (e) None of these

35. Multiply  $3 \frac{1}{3}$  by  $1 \frac{1}{5}$

- (a)  $6/8$   
 (b)  $30/18$   
 (c)  $16/8$

- (d) 4  
 (e) None of these

36.  $\frac{4 \frac{3}{5}}{3 \frac{1}{6}}$  means

- (a)  $4 \frac{3}{5} \div 3 \frac{1}{6}$   
 (b)  $3 \frac{1}{6} \div 4 \frac{3}{5}$   
 (c)  $3 \frac{1}{6} \times 4 \frac{3}{5}$

- (d)  $4 \frac{3}{5} - 3 \frac{1}{6}$   
 (e) None of these

37.  $495.738 + 367.2 =$

- (a) 49941.0  
 (b) 499.410  
 (c) 86.2938

- (d) 862.938  
 (e) None of these

38.  $0.7 \times 0.02 = ?$

- (a) 0.14  
 (b) 0.014  
 (c) 1.4

- (d) 14  
 (e) None of these

39. In the division  $.04 \overline{) 5638}$ , moving the decimals to the positions indicated by the "▲" has the following effect:

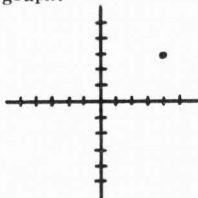
- (a) Both the divisor and dividend are multiplied by 100 and the quotient is 10 times greater.  
 (b) The divisor, dividend, and quotient are all multiplied by 100.  
 (c) The value of the quotient is reduced.  
 (d) It has the effect of multiplying the entire problem by one.  
 (e) None of these

40. In performing the division indicated by  $4/9$  the resulting quotient will be a:

- (a) Terminating decimal  
 (b) Completed decimal  
 (c) Proper fraction  
 (d) Repeating decimal  
 (e) None of these

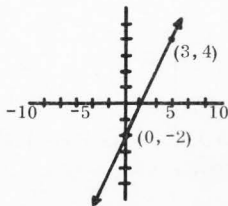
41. Which of the following identifies the point on the graph:

- (a) (3, 4)  
 (b) (4, 3)  
 (c) Both (a) and (b)  
 (d) None of these



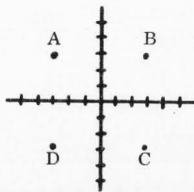
42. The line on the coordinate system at the right is the graph of which of the following functions:

- (a)  $f(n) = 2n + 2$       (d)  $f(n) = n - 2$   
 (b)  $f(n) = n + 3$       (e) None of these  
 (c)  $f(n) = 2n - 2$



43. Which point on the graph represents (-3, -3):

- (a) A  
 (b) B  
 (c) C  
 (d) D  
 (e) None of these



44. Solve the following:  $4^3_{\text{five}} = \text{---ten}$

- (a) 110      (d) 36  
 (b) 18      (e) None of these  
 (c) 23

45. Which of the following is arranged in order of decreasing value:

- (a) .005, .05, .051, .501, .53      (d) .501, .53, .051, .05, .005  
 (b) .53, .501, .051, .05, .005      (e) None of these  
 (c) .005, .05, .051, .53, .501

46. What is the value of  $2^5$ :

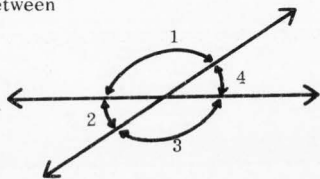
- (a) 10      (d) 64  
 (b) 32      (e) None of these  
 (c) 16

47. Using 5-minute clock arithmetic, which of the following is a true sentence:
- (a)  $2 + 3 = 0$  (d)  $1 + 5 = 2$   
 (b)  $4 - 4 = 4$  (e) None of these  
 (c)  $2 - 3 = 1$
48.  $(-x) + (-y) = \square$  :
- (a)  $x - y$   
 (b)  $y - x$   
 (c)  $-(x + y)$   
 (d)  $-x + y$   
 (e) None of these
49. On the single toss of a pair of dice, what is the probability that their sum is 5 if you are told that one of the dice came up 3:
- (a)  $1/18$  (d)  $1/9$   
 (b)  $1/6$  (e) None of these  
 (c)  $2/12$
50. Which of the following symbols could be used to make this sentence true:  $8 + 4 \bigcirc 2 \cdot 7$
- (a)  $\neq$  (d) All of the above  
 (b)  $<$  (e) None of the above  
 (c)  $\leq$
51. If  $f(n) = 4n - 3$  and  $n = 5$ , what is  $f(n)$ :
- (a) 17 (d) 23  
 (b) 5 (e) None of these  
 (c) 2
52. Given the data 3, 7, 5, 9, 15, 10, 5, which of the following is a true statement:
- (a) Mean = 7 (d) Median = 7  
 (b) Median = 27 (e) None of these  
 (c) Mode = 15
53. The rational number  $1/2$  is found in which of the following:
- (a) The set of natural numbers (d) The set of real numbers  
 (b) The set of integers (e) None of these  
 (c) The set of whole numbers

54. Two triangles are congruent if:
- The small triangle fits perfectly inside the large triangle
  - They are both equilateral triangles
  - When one is placed over the other they fit exactly
  - The sides of one triangle are parallel to the sides of the other triangle
  - None of these
55. Given the implication, "If you like mathematics, then you like this course," which of the following is the converse of that implication:
- If you do not like mathematics, you do not like this course.
  - If you like this course, you like mathematics.
  - If you do not like this course, you do not like mathematics.
  - None of these

56. Given angles 1, 2, 3, and 4 as shown below, which term best describes the relation between  $\angle 2$  and  $\angle 4$ :

- Supplementary angles
- Complementary angles
- Vertical angles
- Supplementary adjacent angles
- None of these



57. Which of the following space figures has six edges:
- |                         |                   |
|-------------------------|-------------------|
| (a) Triangular prism    | (d) Cube          |
| (b) Tetrahedron         | (e) None of these |
| (c) Rectangular pyramid |                   |
58. Given three points not on a straight line, how many planes can contain all three points:
- |           |                     |
|-----------|---------------------|
| (a) zero  | (d) Infinite number |
| (b) three | (e) None of these   |
| (c) one   |                     |



59. Which of the following is a polygon:



(1)



(2)



(3)



(4)

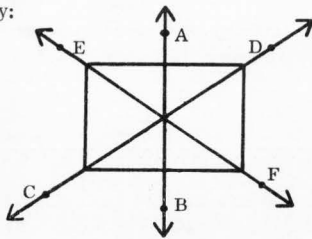


(5)

- (a) All of these  
 (b) Number (4) only  
 (c) Numbers (1) and (2) only  
 (d) All except (4)  
 (e) None of these

60. Which of the lines running through the rectangle is a line of symmetry:

- (a)  $\overleftrightarrow{AB}$   
 (b) Both  $\overleftrightarrow{CD}$  and  $\overleftrightarrow{EF}$   
 (c)  $\overleftrightarrow{AB}$ ,  $\overleftrightarrow{CD}$ , and  $\overleftrightarrow{EF}$   
 (d) None of these



1. Given that  $A = \{w, x, y, z\}$  and  $B = \{u, v, w, x\}$ , which of the following is most appropriate for expressing the union of  $A$  and  $B$ :
- (a)  $A \cup B = \{w, x\}$  (c)  $A \cup B = \{u, v, w, w, x, x, y, z\}$   
 (b)  $A \cup B = \{u, v, w, x, y, z\}$  (d) None of these
2. Given that  $A = \{v, w, x, s\}$ ,  $B = \{v, w, x, y\}$ , and  $C = \{r, w, x\}$ , which of the following is false:
- (a)  $(A \cup C) \cup B = A \cup (C \cup B)$  (d)  $A \cap (C \cup B) = (A \cap C) \cup (A \cap B)$   
 (b)  $(A \cap C) \cap B = A \cap (C \cap B)$  (e) None of these  
 (c)  $A \cup (C \cap B) = (A \cup C) \cap (A \cap B)$
3. If Set  $A = \{1, 2, 3, 4, \dots\}$ , which of the following is true:
- (a) Set  $A$  is finite (d) Set  $A$  is infinite  
 (b)  $\{9, 10, 11\} \notin A$  (e) None of these  
 (c)  $n(A) = 4$
4. Which of the following is the greatest common factor of 84 and 120:
- (a) 21 (d) 840  
 (b) 12 (e) None of these  
 (c) 1
5. Which of the following is a prime number:
- (a) 1 (d) 91  
 (b) 87 (e) None of these  
 (c) 117
6. What is the least common multiple of 18, 12, and 15:
- (a) 3240 (d) 360  
 (b) 180 (e) None of these  
 (c) 3
7. In the process of adding  $273 + 416$ , which of the following steps contains an error:
- (a)  $(200 + 70 + 3) + (400 + 10 + 6)$   
 (b)  $(200 + 400) + (70 + 10) + (3 + 6)$   
 (c)  $600 + 80 + 9$   
 (d) 689  
 (e) None of these

8. In adding  $8367 + 2712$  which of the following steps contains an error:
- (a)  $(8000 + 300 + 60 + 7) + (2000 + 700 + 50 + 4)$   
 (b)  $(8000 + 2000) + (300 + 700) + (60 + 50) + (7 + 4)$   
 (c)  $(8000 + 2000 + 1000) + (60 + 50 + 10) + 1$   
 (d)  $(11,000 + 120 + 1 = 11,121)$   
 (e) None of these
9. Which number sentence illustrates the associative property of addition:
- (a)  $5 + (6 + 7) = (5 + 6) + 7$  (d)  $3 \times 7 = (3 \times 5) + (3 \times 2)$   
 (b)  $(2 + 3) + 4 = 4 + (2 + 3)$  (e) None of these  
 (c)  $3 + (7 + 1) = 3 + (1 + 7)$
10. What is the value of  $9 - (5 - 2)$ :
- (a) 2 (c) Both a and b are correct  
 (b) 6 (d) None of these
11. For the problem  $8002 - 3738$ , which is the correct answer:
- (a) 5264 (d) 4266  
 (b) 4364 (e) None of these  
 (c) 5736
12. In which case does the repeated addition represent the multiplication:
- (a)  $9 + 9 + 9 + 9 + 9 = 9 \times 5$  (d) All of the above  
 (b)  $5 + 5 + 5 + 5 + 5 + 5 = 5 \times 6$  (e) None of these  
 (c)  $7 + 7 + 7 + 7 = 4 \times 7$
13. When we multiply  $6 \times 75,374$  the five in the answer means:
- (a) five hundreds (d) five hundred thousands  
 (b) five thousands (e) None of these  
 (c) five millions
14. When you multiply 426 by the 3 in the exercise shown here, you will get a number that is how large compared with the final answer:
- (a) One-twelfth as large (d) Ten times as large      426  
 (b) Five-sixths as large (e) None of these       $\times \underline{36}$   
 (c) Twice as large

15. The inverse operation of  $3 \times 8 = 24$  is:
- (a)  $8 \times 3 = 24$  (d) Both b and c are correct  
(b)  $24 \div 8 = 3$  (e) None of these  
(c)  $24 \div 3 = 8$
16. The statement  $4 \times 5 \times 6 = 6 \times 5 \times 4$  illustrates which of the following mathematical properties:
- (a) Associative property of multiplication  
(b) Commutative property of multiplication  
(c) Closure property of multiplication  
(d) None of these
17. The mathematical sentence  $2 \times 36 = (2 \times 30) + (2 \times 6)$  illustrates which of the following:
- (a) The associative property of multiplication  
(b) The distributive property of multiplication over addition  
(c) The commutative property of multiplication  
(d) Closure  
(e) None of these
18. Which of the following are correct:
- (a)  $27 \div (9 \div 3) = 9$  (d) All of the above  
(b)  $(27 \div 9) \div 3 = 9$  (e) None of these  
(c)  $(27 \div 9) \div 3 = 27 \div (9 \div 3)$
19. When 222 is divided by 8, the remainder is:
- (a) 5 (d) 0  
(b) 7 (e) None of these  
(c) 6
20.  $12,831 \div \square = 47$
- (a) 273 (c) 2730  
(b) 603,057 (d) None of these
21. Which is the best estimate for  $986 \div 29$ :
- (a) 50 (d) 30  
(b) 40 (e) None of these  
(c) 20

22. What is the average of 12, 38, 57, 12, 71, and 32:
- (a) 6 (d) 12  
(b) 37 (e) None of these  
(c) 222
23. What percent of 150 is 6?
- (a)  $4\%$  (c)  $25\%$   
(b)  $900\%$  (d) None of these
24. Finding the distance from one point to another involves what kind of measure:
- (a) Length  
(b) Area  
(c) Volume  
(d) None of these
25. To determine the amount of carpet needed to cover a floor involves what kind of measure:
- (a) Volume (c) Length  
(b) Area (d) None of these
26. A rectangular prism has edges that measure  $9 \times 11 \times 35$  inches. If a gallon contains 231 cubic inches of water, how many gallons will the container hold:
- (a) 7 (d) 15  
(b) 11 (e) None of these  
(c) 3465
27. If a rectangular-shaped piece of rangeland is 6 miles long and 4 miles wide, which of the following correctly determines the perimeter of the piece of land:
- (a)  $6 \times 4 = 24$  (d)  $6 \div 4 = 1.5$   
(b)  $6 + 4 = 10$  (e) None of these  
(c)  $(2 \times 6) + (2 \times 4) = 20$
28. If a horse walks 8 kilometers per hour, how far will the horse walk in 3 hours:
- (a) 240,000 meters (c) 24000 dekameters  
(b) 2400 hectometers (d) 240 decimeters  
(e) None of these

29. In the division exercise  $1/3 \div 1/2 = 2/3$ , the  $2/3$  represents:
- (a)  $2/3$  of a whole  
(b)  $2/3$  of one-half  
(c)  $2/3$  of one-third
- (d)  $2/3$  of two-thirds  
(e) None of these
30. Given  $5/6 \approx x/18$ , what is the value of  $x$ ?
- (a) 17  
(b) 5  
(c) 6
- (d) 16  
(e) None of these
31.  $2/3$  is the reduced form of which of the following:
- (a)  $15/25$   
(b)  $3/4$   
(c)  $45/60$
- (d)  $75/120$   
(e) None of these
32. Compute:  $2/3 + 2/5 = ?$
- (a)  $4/15$   
(b)  $1 \frac{1}{15}$   
(c)  $4/8$
- (d)  $10/6$   
(e) None of these
33.  $25 \frac{7}{8}$  is a:
- (a) Whole number  
(b) Mixed number  
(c) Natural number
- (d) Irrational number  
(e) None of these
34. Which of the following contains an error:
- (a)  $7 \frac{1}{2} = 7 \frac{2}{4} = 6 \frac{6}{4}$   
 $\frac{-2 \frac{3}{4} = 2 \frac{3}{4} = 2 \frac{3}{4}}{4 \frac{3}{4}}$
- (b)  $6 \frac{2}{3} = 6 \frac{8}{12}$   
 $\frac{-1 \frac{7}{12} = 1 \frac{7}{12}}{5 \frac{1}{12}}$
- (c)  $8 \frac{4}{9} = 8 \frac{8}{18} = 7 \frac{26}{18}$   
 $\frac{-5 \frac{5}{6} = 5 \frac{15}{18} = 5 \frac{15}{18}}{2 \frac{11}{18}}$
- (d)  $4 \frac{7}{10} = 4 \frac{21}{30}$   
 $\frac{-2 \frac{11}{15} = 2 \frac{22}{30}}{2 \frac{1}{30}}$
- (e) None of these
35.  $3/5 \cdot 3/5 = ?$
- (a)  $9/25$   
(b)  $9/5$   
(c)  $3/25$
- (d)  $6/10$   
(e) None of these

36.  $1/3 \div 3\ 1/6 = ?$

- (a)  $2/19$  (d)  $19/2$   
 (b)  $19/18$  (e) None of these  
 (c)  $2/3$

37. What is the hundreds digit in the answer to  $3375.49 - 861.1$ ?

- (a) 9 (d) 2  
 (b) 1 (e) None of these  
 (c) 4

38.  $5/1000$  of  $0.16$  is:

- (a) .8 (d) .0008  
 (b) .08 (e) None of these  
 (c) .008

39. Which of the following division examples will result in the quotient  $25.2$ ?

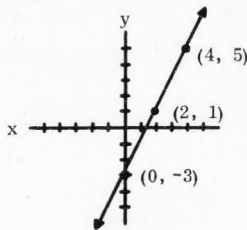
- (a)  $.5 \overline{)126}$  (d)  $.0005 \overline{)126}$   
 (b)  $.05 \overline{)126}$  (e) None of these  
 (c)  $.005 \overline{)126}$

40. Expressed as a decimal,  $1/6$  is equal to:

- (a) .16 (d)  $.16\overline{6}$   
 (b) .167 (e) None of these  
 (c)  $.16\overline{6}$

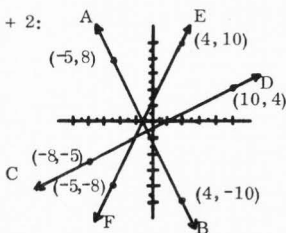
41. Which of the following is plotted at the right:

- (a)  $y = 2x - 3$   
 (b)  $y = 2x + 3$   
 (c)  $y = 3x - 2$   
 (d)  $y = 3x + 2$   
 (e) None of these



42. Which line is the graph of  $f(n) = 2n + 2$ :

- (a)  $\overleftrightarrow{AB}$   
 (b)  $\overleftrightarrow{CD}$   
 (c)  $\overleftrightarrow{EF}$   
 (d) None of these



43. Identify the quadrant or axis in which the point  $(4, -3)$  is found:

- (a) First quadrant  
 (b) Positive vertical axis  
 (c) Third quadrant  
 (d) Negative horizontal axis  
 (e) None of these

44. Which set of numerals are not listed in a consecutive counting order:

- (a) 110, 111, 112 (Base 3)  
 (b) 14, 20, 21 (Base 5)  
 (c) 100, 101, 110 (Base 2)  
 (d) 101, 110, 111 (Base 4)  
 (e) None of these

45. The numeral one million, one hundred one thousand, ten, and three thousandths, is written as follows:

- (a) 1, 100, 110.003  
 (b) 1, 101, 010.0003  
 (c) 1, 100, 001, 010.003  
 (d) 1, 101, 010.03  
 (e) None of these

46. A natural number exponent:

- (a) Tells how many times to add a number  
 (b) Tells the size of the base  
 (c) Tells how many times a number is to be used as a factor  
 (d) Is an expanded form of multiplication  
 (e) None of these

47. In 5-minute clock arithmetic,  $1-4 = \square$

- (a) 0  
 (b) 1  
 (c) 2  
 (d) 3  
 (e) None of these



48.  $\frac{\frac{3}{-8}}{-3}$  is equal to:

- (a)  $9/16$  (d)  $-9/16$   
 (b)  $-4$  (e) None of these  
 (c)  $4$

49. A box contains four black, seven white, and three red balls. If one ball is drawn, what is the probability that it is red:

- (a)  $3/7$  (d)  $1/7$   
 (b)  $2/7$  (e) None of these  
 (c)  $3/14$

50. Which of the following could be made true by using the symbol "<":

- (a)  $5/9 < 2/3$  (d)  $6/21 < 10/35$   
 (b)  $9/7 < 5/7$  (e) None of these  
 (c)  $11/20 < 8/15$

51. Which open sentence defines the function whose members  $[x, f(x)]$  are shown at right:

- (a)  $f(x) = x + 3$   
 (b)  $f(x) = 2x + 2$   
 (c)  $f(x) = 3x + 2$   
 (d)  $f(x) = 2x + 3$   
 (e) None of these

x	f(x)
0	3
1	5
2	7
3	9

52. Given the data 10, 5, 15, 11, 5, 7, 3, the number 8 is the:

- (a) Mean  
 (b) Median  
 (c) Mode  
 (d) None of these

53. In the sentence, "John was the 15th person selected for a scholarship," what kind of number is 15:

- (a) Ordinal (c) Irrational  
 (b) Cardinal (d) None of these

54. Which of the following phrases best describes the term "congruent":

- (a) Is related to (d) Same size and shape as  
 (b) Smaller than (e) None of these  
 (c) Larger than

55. Given the proposition, "All students are intelligent," which of the following best describes the statement, "If you are not intelligent, then you are not a student."

- (a) Negation (d) Contrapositive  
 (b) Converse (e) None of these  
 (c) Inverse

56. The corner of a rectangular window forms an angle of:

- (a)  $45^{\circ}$  (d)  $180^{\circ}$   
 (b)  $60^{\circ}$  (e) None of these  
 (c)  $90^{\circ}$

57. A rectangular pyramid has how many faces:

- (a) 5 (d) 3  
 (b) 4 (e) None of these  
 (c) 6

58. Which of the following is true of a ray:

- (a) Extends on and on in two directions  
 (b) Has one end point and extends on and on in one direction  
 (c) Is a three-dimensional object  
 (d) Is a point in space  
 (e) None of these

59. Which of the following can you be sure lie in one plane:



(1)



(2)



(3)



(4)



(5)

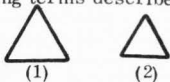


(6)

Rectangle Triangle Circle Rectangular prism Angle Three rays from one point

- (a) Numbers (1), (2) and (3) only (d) Number (5) only  
 (b) All except (4) and (6) (e) None of these  
 (c) Numbers (5) and (6) only

60. Which of the following terms describes the relationship between these two triangles:



- (a) Number (1) is symmetric to number (2)
- (b) Number (2) is the reflection of number (1)
- (c) Number (2) is similar to number (1)
- (d) All of the above
- (e) None of these

## APPENDIX F

Final Form of the  
Test of Mathematical Fundamentals

## TEST OF MATHEMATICAL FUNDAMENTALS

1. Given that  $A = \{w, x, y, z\}$  and  $B = \{u, v, w, x\}$ , which of the following is most appropriate for expressing the union of A and B:
- (a)  $A \cup B = \{w, x\}$  (c)  $A \cup B = \{u, v, w, w, x, x, y, z\}$   
 (b)  $A \cup B = \{u, v, w, x, y, z\}$  (d) None of these
2. Given that  $A = \{v, w, x, s\}$ ,  $B = \{v, w, x, y\}$ , and  $C = \{r, w, x\}$ , which of the following is false:
- (a)  $(A \cup C) \cup B = A \cup (C \cup B)$  (d)  $A \cap (C \cup B) = (A \cap C) \cup (A \cap B)$   
 (b)  $(A \cap C) \cap B = A \cap (C \cap B)$  (e) None of these  
 (c)  $A \cup (C \cap B) = (A \cup C) \cap (A \cap B)$
3. Given that Set A =  $\{1, 2, 3, 4\}$  and Set B =  $\{5, 6, 7\}$ , which of the following is true:
- (a)  $A \cap B = \emptyset$  (d)  $A \cap B = \{\emptyset\}$   
 (b)  $A \cup B = \{\}$  (e) None of these  
 (c)  $A \cap B \neq \{\}$
4. Which of the following is the greatest common divisor of 84 and 120:
- (a) 21 (d) 840  
 (b) 12 (e) None of these  
 (c) 1
5. Resolve 330 into prime factors:
- (a)  $3 \times 11 \times 10$  (d)  $2 \times 3 \times 5 \times 11$   
 (b)  $6 \times 11 \times 5$  (e) None of these  
 (c)  $2 \times 15 \times 11$
6. What is the least common multiple of 56 and 21:
- (a) 336 (c) 1  
 (b) 168 (d) None of these
7. Which of the following is true:
- (a) The sum of 43 and 26 contains seven tens  
 (b) The sum of 27 and 62 contains nine tens  
 (c) The sum of 3842 and 2037 contains nine hundreds  
 (d) The sum of 27,932 and 312,063 contains eight ten-thousands  
 (e) None of these

8. In the base 10 addition exercise below, which of the following could be possible digits for the  $\blacktriangle$  :

- (a) 5                      (d) All of the above  
 (b) 2                      (e) None of these  
 (c) 6

$$\begin{array}{r}
 \square 9 \square \\
 + \square 5 \square \\
 \hline
 \square \blacktriangle \square
 \end{array}$$

9. Which property of whole numbers justifies the following statement:

$$(4 + 3) + 5 = 5 + (4 + 3)$$

- (a) Distributive property of multiplication over addition  
 (b) Closure for addition of whole numbers  
 (c) Commutative property of addition  
 (d) Associative property of addition  
 (e) None of these

10. Which of the following properly shows subtraction by expanded notation:

$$\begin{aligned}
 \text{(a) } 37 - 6 &= (30 - 7) - 6 \\
 &= 30 - (7 - 6) \\
 &= 31
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) } 48 - 7 &= (40 + 8) - 7 \\
 &= 40 + (8 - 7) \\
 &= 41
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } 34 - 6 &= (20 + 14) - 6 \\
 &= 20 - (14 + 6) \\
 &= 28
 \end{aligned}$$

$$\begin{aligned}
 \text{(d) } 42 - 7 &= (30 + 12) - 7 \\
 &= 30 + (12 - 7) \\
 &= 25
 \end{aligned}$$

- (e) None of these

11. Which of the following represents the proper renaming of 624 in solving  $624 - 478$ :

- (a)  $600 + 10 + 14$   
 (b)  $500 + 120 + 4$   
 (c)  $400 + 210 + 14$

- (d)  $500 + 110 + 14$   
 (e) None of these

12. When we multiply  $6 \times 75,374$  the five in the answer means:

- (a) five hundreds  
 (b) five thousands  
 (c) five millions

- (d) five hundred thousands  
 (e) None of these

13. In the multiplication exercise shown below, which horizontal expression best illustrates through expanded notation the multiplication of the partial products: 248

$$\begin{array}{r}
 248 \\
 \times 26 \\
 \hline
 \end{array}$$

- (a)  $(6 \times 8) + (6 \times 4) + (6 \times 2) + (20 \times 8) + (20 \times 4) + (20 \times 2)$   
 (b)  $(6 \times 8) + (6 \times 40) + (6 \times 200) + (20 \times 8) + (20 \times 40) + (20 \times 200)$   
 (c)  $(6 \times 8) + (6 \times 40) + (6 \times 200) + (2 \times 8) + (2 \times 40) + (2 \times 200)$   
 (d)  $(6 \times 8) + (6 \times 4) + (6 \times 2) + (2 \times 8) + (2 \times 4) + (2 \times 2)$   
 (e) None of these

14. The inverse of multiplication is:
- (a) Addition (c) Division  
(b) Subtraction (d) None of these
15. The statement  $(4 \times 5) \times 6 = 6 \times (5 \times 4)$  illustrates which of the following mathematical properties:
- (a) Associative property of multiplication  
(b) Commutative property of multiplication  
(c) Closure property of multiplication  
(d) None of these
16. Which of the following illustrates the distributive property of multiplication over addition:
- (a)  $4 \times 5 \frac{1}{3} = (4 \times 5) + (4 \times \frac{1}{3})$  (d)  $(7 \times 2) \times 4 = 4 \times (7 \times 2)$   
(b)  $(8 \times 9) \times 4 = 8 \times (9 \times 4)$  (e) None of these  
(c)  $8 \times 6 = 6 \times 8$
17. Which of the following are ways of expressing twelve divided by four:
- (a)  $12 \div 4$  (c)  $12/4$   
(b)  $4 \overline{)12}$  (d) Both (a) and (b)  
(e) Choices (a), (b), and (c) are all correct
18. If the quotient is 32, the divisor is 6, and the remainder is 2, what is the dividend:
- (a) 5 (d) 194  
(b) 192 (e) None of these  
(c) 44
19. Which statement best explains the 4 in the quotient of the following division problem?
- $$24 \overline{)1080} \begin{array}{r} 45 \\ \underline{96} \\ 120 \\ \underline{120} \\ 0 \end{array}$$
- (a) There are at least forty 24's in 1080  
(b) Two goes into ten four times  
(c) There are four 24's in 108  
(d) None of these
20. What is the largest multiple of 10 that will make this sentence true:  
 $n \times 60 < 5723$
- (a) 70 (d) 80  
(b) 90 (e) None of these  
(c) 100

21. Which of the following is true:
- (a) The average of 8 and 36 is 44      (d) The average of 12 and 38 is 25  
(b) The average of 14 and 24 is 10      (e) None of these  
(c) The average of 6 and 30 is 180
22. If a jacket which normally sells for \$50 is on sale for \$35, how much is the discount:
- (a) 15%      (d) 35%  
(b) 30%      (e) None of these  
(c) 70%
23. One inch is approximately:
- (a) 25 cm.      (d) 250 cm.  
(b) 2.5 cm.      (e) None of these  
(c) 30 cm.
24. If the edges of a cube are four inches long, what is the surface area of the cube:
- (a) 48 square inches      (d) 96 square inches  
(b) 16 square inches      (e) None of these  
(c) 64 square inches
25. What is the volume of a rectangular prism with the following measurements?  $l = 8$     $h = 6$     $w = 4$
- (a) 18 cubic units      (d) 192 cubic units  
(b) 18 square units      (e) None of these  
(c) 192 square units
26. The perimeter of a polygon is computed by the following method:
- (a) Length times width  
(b) Totalling the lengths of all of its sides  
(c) Length times width times height  
(d) None of these
27. If a horse walks 8 kilometers per hour, how far will the horse walk in 3 hours:
- (a) 240,000 meters      (c) 24 dekameters  
(b) 2400 hectometers      (d) 240 decimeters  
(e) None of these

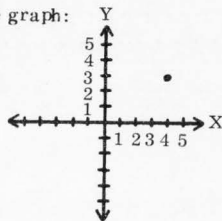


28. The rational number  $\frac{4}{5}$  may be interpreted as:
- 4 out of 5 equivalent disjoint sets
  - 4 parts of an object which has been divided into 5 equal parts
  - 4 divided by 5
  - Any of the above could be correct
  - None of these
29. Given the fractions  $\frac{a}{b}$  and  $\frac{c}{d}$ , a test for equivalency is:
- $a \times d = b \times c$
  - $a \times c = b \times d$
  - $a \times b = c \times d$
  - $a \times d = c \times d$
  - None of these
30. When the rational number  $\frac{15}{45}$  is reduced to lowest terms, the result is:
- $\frac{1}{5}$
  - $\frac{3}{9}$
  - $\frac{1}{3}$
  - $\frac{5}{15}$
  - None of these
31. Compute:  $\frac{2}{3} + \frac{2}{5} = ?$
- $\frac{4}{15}$
  - $1 \frac{1}{15}$
  - $\frac{4}{8}$
  - $\frac{10}{6}$
  - None of these
32. Which of the following contains an error:
- $$\begin{array}{r} 7 \frac{1}{2} = 7 \frac{2}{4} = 6 \frac{6}{4} \\ -2 \frac{3}{4} = 2 \frac{3}{4} = 2 \frac{3}{4} \\ \hline 4 \frac{3}{4} \end{array}$$
  - $$\begin{array}{r} 6 \frac{2}{3} = 6 \frac{8}{12} \\ -1 \frac{7}{12} = 1 \frac{7}{12} \\ \hline 5 \frac{1}{12} \end{array}$$
  - $$\begin{array}{r} 8 \frac{4}{9} = 8 \frac{8}{18} = 7 \frac{26}{18} \\ -5 \frac{5}{6} = 5 \frac{15}{18} = 5 \frac{15}{18} \\ \hline 2 \frac{11}{18} \end{array}$$
  - $$\begin{array}{r} 4 \frac{7}{10} = 4 \frac{21}{30} \\ -2 \frac{11}{15} = 2 \frac{22}{30} \\ \hline 2 \frac{1}{30} \end{array}$$
  - None of these
33. Multiply  $3 \frac{1}{3}$  by  $1 \frac{1}{5}$
- $3 \frac{1}{15}$
  - $\frac{30}{18}$
  - $\frac{16}{8}$
  - 4
  - None of these
34.  $\frac{1}{3} \div 3 \frac{1}{6} = ?$
- $\frac{2}{19}$
  - $\frac{19}{18}$
  - $\frac{2}{3}$
  - $\frac{19}{2}$
  - None of these

35. What is the hundreds digit in the answer to  $3375.49 - 861.1$ ?
- (a) 9 (d) 2  
 (b) 1 (e) None of these  
 (c) 4
36.  $5/1000$  of  $0.16$  is:
- (a)  $.8$  (d)  $.0008$   
 (b)  $.08$  (e) None of these  
 (c)  $.008$
37. Which of the following division examples will result in the quotient  $25.2$ ?
- (a)  $.5 \overline{)126}$  (d)  $.0005 \overline{)126}$   
 (b)  $.05 \overline{)126}$  (e) None of these  
 (c)  $.005 \overline{)126}$
38. Expressed as a decimal,  $1/6$  is exactly equal to:
- (a)  $.16$  (d)  $.1\overline{66}$   
 (b)  $.167$  (e) None of these  
 (c)  $.16\overline{6}$

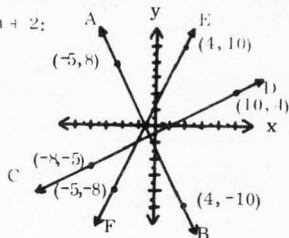
39. Which of the following identifies the point on the graph:

- (a) (3, 4)  
 (b) (4, 3)  
 (c) Both (a) and (b)  
 (d) None of these



40. Which line is the graph of  $f(n) = 2n + 2$ :

- (a)  $\overleftrightarrow{AB}$   
 (b)  $\overleftrightarrow{CD}$   
 (c)  $\overleftrightarrow{EF}$   
 (d) None of these



41. Identify the quadrant or axis in which the point (4, -3) is found:
- (a) First quadrant (d) Negative horizontal axis  
 (b) Positive vertical axis (e) None of these  
 (c) Third quadrant

42. Which set of numerals is not listed in consecutive counting order:
- (a) 110, 111, 112 (Base 3) (d) 101, 110, 111 (Base 4)  
 (b) 14, 20, 21 (Base 5) (e) None of these  
 (c) 100, 101, 110 (Base 2)
43. The numeral one million, one hundred one thousand, ten, and three thousandths, is written as follows:
- (a) 1,100,110.003 (d) 1,101,010.03  
 (b) 1,101,010.0003 (e) None of these  
 (c) 1,100,001,010.003
44. A natural number exponent:
- (a) Tells how many times to add a number to itself  
 (b) Tells the size of the base  
 (c) Tells how many times a number is to be used as factor  
 (d) Is an expanded form of multiplication  
 (e) None of these
45. Using 5-minute clock arithmetic, which of the following is a true sentence:
- (a)  $2 + 3 = 0$  (d)  $1 + 5 = 2$   
 (b)  $4 - 4 = 4$  (e) None of these  
 (c)  $2 - 3 = 1$
46.  $\frac{\frac{3}{-8}}{-3}$  is equal to:
- (a)  $9/16$  (d)  $-9/16$   
 (b)  $-4$  (e) None of these  
 (c)  $4$
47. On the single toss of a pair of dice, what is the probability that their sum is 5 if you are told that one of the dice came up 3:
- (a)  $1/18$  (d)  $1/9$   
 (b)  $1/6$  (e) None of these  
 (c)  $2/12$
48. Which of the following could be made true by replacing the  $\bigcirc$  with the symbol " $<$ ":
- (a)  $5/9 \bigcirc 2/3$  (d)  $6/21 \bigcirc 10/35$   
 (b)  $9/7 \bigcirc 5/7$  (e) None of these  
 (c)  $11/20 \bigcirc 8/15$

49. Which open sentence defines the function whose members  $[x, f(x)]$  are shown at right:

- (a)  $f(x) = x + 3$   
 (b)  $f(x) = 2x + 2$   
 (c)  $f(x) = 3x + 2$   
 (d)  $f(x) = 2x + 3$   
 (e) None of these

$x$	$f(x)$
0	3
1	5
2	7
3	9

50. Given the data 10, 5, 15, 11, 5, 7, 3, the number 8 is the:

- (a) Mean  
 (b) Median  
 (c) Mode  
 (d) None of these

51. The rational number  $1/2$  is found in which of the following:

- (a) The set of natural numbers  
 (b) The set of integers  
 (c) The set of whole numbers  
 (d) The set of real numbers  
 (e) None of these

52. Two triangles are congruent if:

- (a) The small triangle fits perfectly inside the large triangle  
 (b) They are both equilateral triangles  
 (c) When one is placed over the other they fit exactly  
 (d) The sides of one triangle are parallel to the sides of the other triangle  
 (e) None of these

53. Given the proposition, "All students are intelligent," which of the following best describes the statement, "If you are not intelligent, then you are not a student."

- (a) Negation  
 (b) Converse  
 (c) Inverse  
 (d) Contrapositive  
 (e) None of these

54. The corner of a rectangular window forms an angle of:

- (a)  $45^\circ$   
 (b)  $60^\circ$   
 (c)  $90^\circ$   
 (d)  $180^\circ$   
 (e) None of these

55. A rectangular pyramid has how many faces:

- (a) 5  
 (b) 4  
 (c) 6  
 (d) 3  
 (e) None of these

56. Which of the following is true of a ray:

- (a) Extends on and on in two directions
- (b) Has one end point and extends on and on in one direction
- (c) Is a three-dimensional object
- (d) Is a point in space
- (e) None of these

57. Which of the following is a polygon:



(1)



(2)



(3)



(4)



(5)

- (a) All of these
- (b) Number (4) only
- (c) Numbers (1) and (2) only
- (d) All except (4)
- (e) None of these

58. Which of the following terms describes the relationship between these two triangles:



(1)



(2)

- (a) Number (1) is symmetric to number (2)
- (b) Number (2) is the reflection of number (1)
- (c) Number (2) is similar to number (1)
- (d) All of the above
- (e) None of these

## VITA

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Doctor of Education

Dissertation: Development and Validation of a Waiver Test  
For the Math Content Courses Required of Elementary  
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Major Field: Elementary Education

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