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A Comparative Study of Teacher Opinion With Respect to the Mathematics Program in Grades Seven, Eight, and Nine in the Schools of Utah and California

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A COMPARATIVE STUDY OF TEACHER OPINION WITH RESPECT TO THE MATHEMATICS PROGRAM IN GRADES SEVEN, EIGHT AND NINE IN THE SCHOOLS OF UTAH AND CALIFORNIA

by

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Sidney W. Bingham

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INTRODUCTION

To live intelligently in our rapidly changing society, every member of our society needs to acquire knowledge and skills in the field of mathematics, not only in specialized fields, but in all areas of everyday living.

The mathematics program in the schools must therefore provide for instruction in functional mathematics for the future housewife, grocery clerk, farmer, waitress, unskilled laborer, as well as for those who will need a foundation for the higher mathematics needed in engineering, physics, chemistry, and scientific research.

Mathematical learning is a type of developmental learning in which each new process or understanding is related to and builds upon previously acquired skills and knowledge. Thus it is important, from the introductory course to the terminal course in mathematics, to teach for understanding of the concepts, the meaning of the terms used, and for skill and accuracy in the operations involved.

The great number of youth entering the schools makes it imperative that the mathematics program provides for a wide range of interests and abilities.

The National Academy of Sciences--National Research Council Division of Mathematics reports that

> . . . the general outline of the mathematical subjects to be studied has been rigidly fixed; as a result the present mathematical curriculum is clearly out of date.
> . . . growth of mass education has raised serious problems for instructional standards. The case of the old-line

> "standard" mathematics courses clearly indicates the problems which arise. Each year more students, more badly prepared, arrive to take the standard mathematics course. Under this impact the courses are slowly diluted in content and eventually replaced in whole or in part by new "general" courses. These new courses are intended to teach mathematics relevant to life situations; usually this means a lower mathematical content (16, p. 3).

Many students receive no formal training in mathematics beyond the ninth grade. The foundation needed for further study in mathematics is quite logically laid early in the secondary grades. Thus, in view of the above considerations, it is evident that much attention must be focused upon the mathematics program at the junior high school level if it is to contribute adequately to the broad purposes of education.

Definition of terms

The "junior high school level" as used in this paper will refer to grades seven, eight, and nine. Schools containing any or all of these grades and also containing grades below the seventh will be referred to as elementary schools; schools containing only these grades will be referred to as junior high schools; schools containing any or all of these grades and also containing grades beyond the ninth will be referred to as high schools.

Purpose of Study

The purpose of this study is to examine various aspects of the mathematics program in the schools of Utah at the junior high school level. Where there is sufficient information, the mathematics program in Utah will be compared with the corresponding program in California and with what is generally accepted as the essential elements of a satisfactory junior high school program in mathematics according to authorities in the field of mathematics education as

determined by an extensive coverage of the literature in the field.

It is hoped that this study may furnish data, reveal problems, and create interests that will lead to a more extensive evaluation of the entire mathematics program in the state of Utah, and perhaps elsewhere. It is also hoped that this study will reveal some of the strengths and weaknesses of the program as it exists; that educators may defend and build upon the strong points and may be encouraged to improve the weak points.

Scope of Study

This study will be limited to the opinion of the teachers with respect to the mathematics program in their respective schools, except as explained in the following paragraphs.

In the portion of the study dealing with the mathematics program in the schools of Utah, referred to as the Utah Study, the opinions are of the classroom teacher.

In the portion of the study dealing with California, referred to as the California Study, some opinions are those of the classroom teacher, some are those of the head of the department of mathematics, and a few are those of the school principal.

The author is mindful that the differences in the positions of those furnishing opinions may effect the validity of the comparison between states. However, information for a completely valid comparison is not immediately available.

The Utah study will further be limited to a descriptive analysis of the following aspects of the mathematics program in grades seven, eight, and nine of the schools of Utah.

I. With respect to teachers
   A. Teacher load

1. The number of sections taught per day
2. The number of sections of mathematics taught per day
3. The average number of students per class, in mathematics only

B. Preparation and experience
1. The number of years of teaching experience
2. The number of years teaching mathematics
3. The number of years of college completed
4. The degrees granted
5. Major subject (undergraduate, graduate)
6. Minor subject (undergraduate, graduate)
7. Approximate number of quarter hours of credit received in mathematics (undergraduate, graduate)

II. With respect to the mathematics courses offered
1. Title of the course
2. Major objectives of the course
3. Major divisions of course content
4. Major experiences designed to achieve the objectives of the course through its content
5. Why students enroll (required, elective)
6. Methods by which students are helped to develop their ability to apply skills learned in other courses to mathematics
7. Means used to determine the effectiveness of the course
8. What significant changes have taken place in the mathematics program during the past five years

The following items will be noted:

1. Differences in the mathematics program according to the size of schools, and according to the type of school--elementary school, junior high school, or high school,

2. Outstanding differences in objectives and methods of teachers according to the number of years of experience teaching mathematics, and

3. Outstanding differences in objectives and methods of teachers according to the number of hours of college credit received for mathematics.

The California study will be limited to a descriptive analysis of the following aspects of the arithmetic program in the seventh and eighth grades of the junior high schools of California:

I. With respect to the arithmetic courses offered
   1. Title of course
   2. Major objectives of the course
   3. Major divisions of course content
   4. Major experiences designed to achieve the objectives of the course through its content
   5. Why students enroll (required, elective)
   6. Methods by which students are helped to develop their ability to apply reading skills to mathematics
   7. Means used to determine the effectiveness of the course
   8. What significant changes have taken place in the mathematics program during the past five years.

Differences in the mathematics program according to the size of schools will be noted. Comparison of the mathematics programs in Utah and California will be limited to those aspects of the programs for which there is comparable data.

Method of Procedure

The "October Report of High School Principal, 1954" is a California State Board of Education questionnaire which was sent to all of the secondary schools of the state of California. "Part IV--Description of Particular Course in Mathematics" was concerned with various aspects of the mathematics program in the secondary schools of California (See table 1, Appendix). The questionnaire was originated and distributed by the California State Board of Education. The author of this paper was instrumental in classifying and analyzing the data of the questionnaire as it pertained to the arithmetic program in grades seven and eight of the junior high schools of California. The information obtained from the above questionnaire is the source of data for the portion of this study called the California study.

A similar questionnaire was prepared for the teachers of mathematics at the junior high school level in the schools of Utah (See Tables 2 and 3, Appendix).

In order to obtain accurate information concerning the names of teachers who were teaching mathematics in the seventh, eighth or ninth grades, a questionnaire was sent to each of the principals of the schools containing any or all of those grades, except for certain elementary schools where it was known that the teacher of a particular grade was the sole teacher of that grade. More than ninety-seven percent of the principals responded.

Each teacher in Utah who taught mathematics in grades seven, eight or nine, where names were available, was then sent a questionnaire. The returns on the questionnaire which was sent to the teachers are analyzed in Tables 4, 5, and 6 (See Appendix).

Data from questionnaires were then analyzed. Comparable results of the analysis were then compared with the corresponding results of the California Study and with the opinions of authorities in the field of mathematics education as determined by a study of available literature on the subject.

REVIEW OF LITERATURE

This section is a review of the literature concerning the opinions of authorities in the field of mathematics education with respect to what is generally considered essential to a satisfactory program in mathematics at the junior high school level.

Curricular offerings in mathematics

It is generally agreed that, because of varying local conditions and objectives and other practical considerations, there may be, and perhaps should be, some differences between schools in their curricular offerings in mathematics.

There are considerable differences in opinion as to what constitutes an adequate offering in mathematics at the junior high school level. Following are a few viewpoints related to this problem.

Douglass, in response to the query, "How rapidly should young people be pushed in the study of arithmetic?", observes:

> The result of over-acceleration in arithmetic should be obvious. Many children have had unhappy experiences with arithmetic. They feel inadequate in quantitative situations; they fear figures; they dread computation. Eventually they develop a bad attitude toward mathematics in general. This has been noticed by many teachers in the upper grades and in college. This attitude is not easily changed.
> . . . there is a trend toward slowing down a bit in the primary grades and giving more attention to the problem of seeing that pupils do not get lost in the very beginning (4, pp. 290-94).

Douglass is convinced that mathematical-ability grouping is needed. This might be done within a class or through separation into separate classes. He also feels that there is a necessity for remedial sections of mathematics in the junior high schools.

Grossnickle (8) recommends extension of the arithmetic program through the ninth grade and beyond because of the result of the practice of continuous promotion and the tendency to defer topics of arithmetic.

G. W. Myers of Columbia University

> . . . attacked the traditional organization of arithmetic, algebra, geometry, and trigonometry as being out of line with the demands of effective learning. He argued that the traditional organization creates artificial barriers, blocks, and learning difficulties. Moreover, he said that such organization may have appeal for the scholar and the mathematician, but that it is not a sound organization for the immature, beginning pupil (27, p. 81).

From the findings of his study, Beckman (1) is led to believe, though he realizes that other studies arrive at other conclusions,

that general mathematics is better than algebra for attaining mathematical literacy if the curriculum is well planned and if the teaching is effective.

Some feel that in the secondary schools there should be a "multi-track" program in mathematics. "Multi-track" has reference to separate programs for those who are preparing for careers in mathematics and science, for those who are above average mentally but intend to be in some activity requiring a limited amount of mathematics, for those who are below average mentally but could benefit by some training in mathematics, and for other such groups.

According to Irvin, "The Commission on Post-War Plans of the National Council of Teachers of Mathematics has recommended a double-track mathematics program,"--one, a program for those needing specialized mathematics and those who show special aptitude for mathematics and science and, two, a program for the average citizen (10, pp. 235-40).

While there is considerable disagreement as to how the mathematics offerings should be graded and organized most authorities agree to a large extent with the following outline of the recommendations by the National Council of Teachers of Mathematics as to the guiding principles to be applied in formulating and organizing the mathematics offering in the secondary schools.

1. The curriculum should include basic elements of arithmetic, algebra, geometry, graphic representation, and trigonometry.

2. For every type of pupil, a mathematics course must give constant attention to the foundations, while at the same time it stresses significant applications within the learner's potential range of

understanding and interest.

3. In grades seven through nine, the fundamental concepts, principles, and skills of mathematics must be introduced and developed in a carefully organized pattern, paying attention to logical considerations and psychological and pedagogical principles.

4. With retarded pupils, modifications are needed in the rate of progress and degree of comprehension.

5. The precise scope and degree of emphasis should vary with location of schools.

6. Psychological considerations should help determine the amount of work, speed of accomplishment, and the preparation of each unit or topic.

7. Mathematics is composed of a relatively large body of closely related abstract ideas. Its fundamentals must be learned as an organized sequence. Constant attention and real understanding are necessary for success in this field. Considerable practice over a period of months or years are needed for mastery and the ability to apply the results of mathematical training. Each forward step in the subject is, as a rule, a simple one. Thus, the following considerations are significant.

   a. Constant evaluation and guidance is needed.
   b. The importance of each day's work should be stressed.
   c. Understanding of concepts is the key to success.
   d. A properly constructed curriculum will give adequate attention to "overviews", motivating discussions, summaries, and organic reviews.
   e. There should be adequate practice but not mere drill.
   f. There should be spaced learning--periodic returns to the same topics.

8. Principles of arrangement of sequence of topics are:

a. Each topic should be arranged so as to contribute to the development of a unified mathematical picture.
b. The study should emphasize problem solving and modes of thinking, and should not become a mere sequence of formal and relatively abstract drills.
c. A complete or exhaustive treatment of a topic or unit is not always advisable, but it should not contain unrelated "odds and ends."
d. Sufficient background of prerequisite concepts and skills should, in general, be had before a new topic is introduced.
e. Generally, new topics should not be introduced prior to the time they may be needed or effectively applied (19, pp. 55-58).

Objectives

The course in mathematics should be built around properly planned objectives, general and specific. The objectives of a mathematics course should be consistent with the general objectives of education and with specific needs of the nation, state, community, and individuals involved.

The general objectives of education as determined in the recent White House Conference for Education are listed here as they appeared in the Herald Journal, December 4, 1955.

> . . . 14 things the schools should accomplish:
> 1. The fundamental skills of communication--reading, writing, spelling as well as other elements of effective oral writing (and written) expression; the arithmetical and mathematical skills, including problem solving. While schools are doing the best job in their history in teaching these skills, continuous improvement is desirable and necessary.
> 2. Appreciation of our democratic heritage.
> 3. Civil rights and responsibilities and knowledge of American institutions.
> 4. Respect and appreciation for human values and for the beliefs of others.
> 5. Ability to think and evaluate constructively and creatively.
> 6. Effective work habits and self-discipline.
> 7. Social competency as a contributing member of his family and community.
> 8. Ethical behavior based on a sense of moral and spiritual values.

9. Intellectual curiosity and eagerness for lifelong learning.
10. Aesthetic appreciation and self-expression in the arts.
11. Physical and mental health.
12. Wise use of time, including constructive leisure pursuits.
13. Understanding of the physical world and man's relation to it as represented through basic knowledge of the sciences.
14. An awareness of our relationships with the world community (9, p. 2).

The invariant frame of reference around which all objectives center are the physical universe, society, and the child (19, p. 21). However, the objectives within the frame of reference may vary with time and location.

The general objectives of mathematics are of two types, "to insure mathematical competence for the ordinary affairs of life, to the extent that this can be done as a part of general education" and "to provide sound mathematical training for future leaders in science, mathematics and other learned fields (12, pp. 33-36)."

While specific mathematical objectives may vary slightly between grades at the junior high school level, there is general agreement among authorities as to the desirability of establishing and working toward the following somewhat general goals during this period:

1. Proficiency in performing fundamental skills,
2. Comprehension of basic concepts,
3. Appreciation of significant meanings,
4. Development of desirable habits and attitudes,
5. Efficiency in making sound applications, and
6. Confidence in making intelligent and independent interpretations (2, p. 16).

It is felt that in the seventh and eighth grades and in general mathematics courses beyond the eighth grade greater emphasis should be placed upon the development of functional competence in mathematics, while in the ninth grade some emphasis should be shifted toward the development of mathematical power (17).

Major divisions of course content

As may be anticipated, there is no rigid, chartable program for the seventh, eighth, and ninth grades, especially the ninth, that would be universally accepted by authorities in the field of mathematics education.

The intention of the author in listing the following programs of course content is not to give the impression that any of these programs would be generally accepted, but to indicate what concepts, principles, and skills various authorities feel are desirable at the different levels of maturity.

The Joint Commission of the Mathematics Association of America and the National Council of Teachers of Mathematics favor, approximately, the following detailed outline of course content.

Seventh and eighth grade mathematics. The mathematical programs of the seventh and eighth grades are treated as a unit. They should include topics from arithmetic, informal geometry, graphic representation, algebra, and trigonometry; with some emphasis upon modes of thinking, habits, and attitudes; also historical backgrounds, correlation with life situations and other school activities.

I. Arithmetic
   A. Basic concepts and principles (7, 8) (The numbers enclosed in parenthesis indicate the grade level at which greater emphasis is probably desirable)
      1. Familiarity with the vocabulary of arithmetic

2. Understanding basic principles of arithmetic, such as dividend equal divisor times quotient plus remainder

B. Basic skills or techniques (7, 8)
   1. The four fundamental operations with whole numbers, fractions and decimals
   2. Skills and processes needed for social application of arithmetic
   3. Ability to use tables of measure

C. Using arithmetic in problem situations (7, 8)
   1. Development of problem-solving attitude
   2. Development of ability to analyze and solve arithmetical problems
   3. Study of practical problems such as
      a. numerical problems arising in the home, school, store, and community (7)
      b. everyday business problems--buying selling, profit (7)
      c. business or social problems demanding greater maturity--banking, investment, taxation, insurance (8)
      d. problems arising in science, shop, household arts (7, 8)

II. Informal geometry

A. Basic Concepts (7, 8)
   1. Familiarity with vocabulary of geometry
   2. Meaning of key concepts such as circle, angle, triangle, perpendicular
   3. Realization of approximation of physical measurements and drawings

B. Basic skills and techniques (7, 8)
   1. Use of ruler, compass, protractor, squared paper (7)
   2. How to draw basic figures of geometry (7)
   3. Constructions (7)
      a. an angle equal to a given angle
      b. parallel lines
      c. an equilateral triangle, given an angle
      d. an isosceles triangle, given base and a side
      e. related figures
   4. Constructions (8)
      a. perpendiculars
      b. bisectors
      c. common rectilinear figures such as rectangles, squares, right triangles, regular hexagons, related figures
      d. figures similar or congruent to a given figure
   5. Direct measurement of common figures (7, 8)
   6. Finding simple areas and volumes (7, 8)
   7. Solving applied problems involving above skills (7, 8)

8. Simple scale drawings (7, 8)
9. Simple indirect measurement (7, 8)
   a. use of scale drawings
   b. use of similar and congruent triangles
   c. the Pythagorean relation

C. Important geometric facts and relations (7, 8)
1. Introductory study of the geometry of size, shape, and position (7)
2. Informal study of symmetry (8)
3. Informal study of the Pythagorean relation (8)
4. Acquaintance with such geometric facts, properties, and relations as may be readily derived by informal methods, such as radii of a circle are equal (7), all right angles are equal (7), common rules of mensuration (7, 8), the area of a triangle depends upon the length of the base and the altitude

D. Discovering and testing geometric relations (7, 8)
1. Through the use of laboratory techniques, the pupil should begin to discover, to analyze, to verify geometric relationships

III. Graphic Representation
A. Intrepretation of pictorial or graphic charts (7)
B. Making bar graphs of simple statistical data (7, 8)
C. Making circle graphs of simple social problems (7, 8)
D. Making line graphs to show change (7, 8)
E. Making graphs based on economic data (8)

IV. Algebra (7, 8)
A. Confined to basic terms and concepts such as
1. The meaning of such terms as equation, formula, coefficient
2. Making a formula as a shorthand expression of a mathematical rule
3. Symbolic representation
4. Solving simple one or two step equations or formulas
5. Application of above skills to solution of simple problems arising in life situations

V. Trigonometry (7, 8)
A. Familiarity with basic vocabulary of trigonometry
B. Scale drawing
C. Making simple outdoor measurements.

Ninth grade mathematics. The mathematics program in the ninth grade should be a process of reviewing, applying, and extending the work of the seventh and eighth grades, in so far as arithmetic, geometry, and graphic representation are concerned.

I. Algebra (9)
   A. Basic concepts
      1. Basic vocabulary
      2. Understanding key concepts such as exponent, negative, positive, ratio
   B. Fundamental skills and techniques
      1. The four fundamental operations, involving
         a. positive and negative numbers
         b. algebraic monomials or simple polynomials
         c. algebraic fractions, mainly with monomial denominators
      2. Simple special products and factoring
      3. Powers and roots
   C. Fundamental principles
      1. A study of the principles governing the fundamental operations, such as the rules of order and grouping, the rules of signs and of exponents
      2. A study of principles used in the solution of equations
   D. Study of relationships and of dependence
      1. By tables
         a. interpreting tables of related number pairs
         b. making tables based on formulas
      2. By graphs
      3. By formulas
      4. By equations
   E. Using algebra in life situations and problems
      1. Translating quantitative statements into the language of algebra
      2. Using techniques of algebra to make precise generalizations according to given conditions
      3. Solving general verbal problems
      4. Applying techniques of algebra in solution of problems arising at home, school, or in the community
      5. Interpreting the solutions of equations, including negative values, where they have significance

II. Trigonometry (9)
   A. Reviewing necessary concepts and skills
   B. Simple indirect measurement
   C. Using natural functions, including tables

Seventh, eighth, and ninth grade mathematics. There are elements of course content that are about the same for the seventh, eighth, and ninth grades.

I. Mathematical modes of thinking, habits, attitudes, and types of appreciation (7, 8, 9)

1. The development of habits of accuracy and correctness
2. The development of habits of estimating and checking
3. Learning to interpret and analyze elementary problem situations
4. Neat preparation and arrangement of written solutions of suitable mathematical problems
5. The development of an interest in the study of simple quantitative relationships
6. Learning to appreciate the place of mathematics in everyday life
7. Learning to understand and apply relational thinking.

II. History of Mathematics
   1. The history of the development of many of the topics of mathematical interest should be used to stimulate interest, to broaden the understanding, and to aid retention and transfer by the student (7, 8, 9).

III. Correlated mathematical projects and activities (7, 8, 9)
   1. Many projects and activities that correlate mathematics and other school subjects and life situations should be a part of the course content at the junior high school level (19, pp. 78-91, pp. 248-251).

Remedial classes. The course content for remedial work or for slow pupils should vary according to the structure of the class. The planning for remedial work should be done according to the needs of the members of the class as determined by extensive testing and evaluation.

Cole (3, p. 225), an authority in adolescent psychology, claims that the reason, in part, for a large amount of failure or lack of success in secondary school subjects is that "these courses are totally inappropriate for about half the pupils." She claims that the answer to the problem is not a lowering of standards in present classes, rather a revision of the curriculum that will give unsuccessful students material with which to be successful. This implies

special course content for the slower pupils.

The Joint Commission lists one possible outline for slow pupils (19, pp. 252-53).

Classes for superior students. Some schools may find it desirable to have a special, flexible outline of course content for superior students. This outline should also be according to the needs, interests, and abilities of the class members as determined by extensive evaluation and testing.

It is recognized that for optimum results, even where grouping is in effect, it is desirable to do considerable individual instruction, extending the mathematical pursuits of individual pupils beyond the ordinary outline of course content.

Major experiences and materials of instruction that are designed to achieve objectives of a course in mathematics

The question of what experiences a child should have in connection with his acquisition of important mathematical skills and concepts and what aids should a teacher use in the process of helping him to acquire these skills and concepts probably has as many answers as there are teachers of mathematics. This section will attempt only to give a brief review of the literature that has been written on this subject, in order to indicate the nature of the activities that are felt to enhance learning in mathematics, and to list a few of the teaching aids used. Some evaluative comments concerning various experiences and teaching aids will be listed. Some publications that cover the subject quite extensively will also be listed.

Until some time during the nineteenth century it was generally felt that the teaching of mathematics was done satisfactorily by the

method of lecture, drill, and recitation. The main equipment needed for such teaching was a blackboard, a text, a piece of chalk, a straight edge, and perhaps a piece of string.

In fact, there is some evidence that the same method and the same equipment is standard in many of the classrooms where mathematics is taught today.

Much has been discovered during the past century concerning learning processes. There has been considerable research done in such areas related to learning as motivation, retention and transfer of learning, interests, needs, understanding, and the use of multisensory aids to learning.

Psychologists and authorities in mathematics education, as well as many classroom teachers of mathematics, recognize that activities do much to stimulate interest in mathematics, partly because they are a means of connecting the abstract operations of mathematics to the concrete things that pupils are familiar with, and because they show the usefulness of mathematics when properly applied.

> It may be taken as axiomatic that students will work most diligently and most effectively at tasks in which they are genuinely interested.
>
> As a rule, students readily become interested in things which are new or exciting, in things for which they can perceive practical values or applications to situations and fields of study in which they are already interested, and in things which involve puzzle elements or elements of mystery.
>
> . . . the possession of a background of related information tends to intensify interest in new work, . . .
>
> The elements of novelty, of usefulness, and of sheer intellectual curiosity are the primary stimuli for the awakening of interest (2, p. 126).

> All initial learning in the classroom should have its origin in some concrete problematic situation in which sensory activity can enter. All learning has a motor base, reinforced by sensory experience, from which concepts are formed by making discriminations. The capacity of a child to initiate this experience

> and to profit by it is a function of his intelligence. Not all children are endowed with the same basic capacity for initiating experience. Where the experience is lacking, the teacher must aid the learning by supplying proper sensory activity to motivate it (7, p. 26).

Several authorities feel that at the junior high school level pupils are unable to deal entirely with the abstract in mathematics, that pupils will learn and retain many of the principles and concepts of mathematics better if teachers plan interesting, meaningful experiences in a laboratory setting.

Reeve (25, p. 504) writes, "Although teachers of mathematics have been slow to transform their recitation rooms into laboratories and workshops, this is what really needs to be done to insure proper use of multi-sensory aids in teaching."

> Multi-sensory aids contribute much to the process of learning. While some of the values can be measured, there are immeasurable attitudes and responses which are recognized and appreciated by both the teacher and the pupils.
>
> A mathematics laboratory is well worth all the effort it takes to build and organize. Multi-sensory aids are part of the common-sense method of developing power to discover, to understand, and to use mathematics and mathematical laws in meeting the challenge of today (22, p. 25).

Much of the simpler equipment of the mathematics classroom or laboratory may be made by the pupils. It is felt that students who design and make such equipment, after studying its history and use, get the most fun and profit from it. Students take pride in things they make themselves, and it adds to the interest in using the equipment. Pupils tend to better understand the principles upon which the instruments are made if they make them themselves.

However, the fact that a method yields favorable results when moderately used, is no assurance that increased use yields increasingly

favorable results. Such is the case with laboratory activities and with the use of multi-sensory aids in teaching mathematics. There is danger in over-using these otherwise valuable tools. Teachers are warned that

> . . . While well-conducted laboratory work of the nature indicated can do much to supplement and enrich some parts of mathematical study, especially in the lower grades and the junior high school, it can never provide a complete foundation for mathematical work. . . . Experiments and applications are valuable in relating mathematical principles and processes to other fields, but they do little toward developing and clarifying the interrelations of the different parts of mathematics itself. They can be made to provide valuable supplementary work for purposes of enrichment of certain parts of mathematics, but beyond this, too much may not be expected . . . Experimentation which is mere busywork and which does nothing to develop understandings of principles or applications is practically worthless (2, p. 164).

Activities which may be used to enhance the learning of mathematics may be classified into two groups: one, those which take place in the classroom, and, two, those that take place outside of the classroom, whether they are activities which might be termed extra-curricular or those that are called non-school.

Following is an arbitrary grouping of some of the major experiences or activities which a pupil may encounter in the study of mathematics at the junior high school level.

I. Classroom experiences
   - A. Use of
     1. Textbooks
     2. Workbooks
     3. Laboratory equipment
   - B. Creation of, or construction of
     1. Bulletin board displays, posters
     2. Scrapbooks and notebooks
     3. Simple laboratory-type equipment
     4. Problems related to present mathematical interests
     5. General rules from specific mathematical principles

6. Mathematical games, puzzles, contests
7. Geometric figures and designs
8. Graphs, charts

C. Activities related to real life problems
1. Study income tax forms
2. Make out a bank deposit slip
3. Study newspapers for applications of mathematics

D. Seeing and hearing audio-visual aids
1. Films
2. Slides
3. Models
4. Flannel boards

E. Teacher presentations
1. Lectures
2. Demonstrations
3. Examinations
4. Reviews
5. Teacher-led discussions

F. Problem solving
1. Actual and simulated
2. Graphic and statistical
3. Oral and written
4. Individually and in groups

G. Blackboard work by students
1. Presentations
2. Problem solving

H. Group discussions

I. Drills for
1. Accuracy
2. Speed
3. Mastery of skills
4. Vocabulary
5. Retention of facts

J. Mathematical recreations
1. Games
2. Contests
3. Puzzles
4. Others

K. Presentations by
1. Students
   a. Reports
   b. Demonstrations
   c. Projects
2. Older students
3. Outside speakers
   a. Businessmen
   b. Professors
   c. Parents

II. Out-of-class experiences
A. Homework
1. Solution of problems--exercises

2. Getting information on uses of mathematics
B. Mathematics or science clubs
C. Field trips
D. Laboratory activities
   1. Measurement of heights, distances, areas
   2. Lay-out of baseball diamond and other game areas
E. Fairs, museums, or other display activities
F. Research, gathering mathematical data
G. Work experience
   1. At home
   2. At school
   3. In the community

Values and limitations of various experiences and materials of instruction

The experiences which are most widely encountered by pupils in mathematics classes are teacher presentations and the use of textbooks. Some teachers appear to get good results in mathematics through well-planned lectures. Others may use demonstrations, questioning, or other methods of teacher presentation with equally good results. It may be argued that particular methods are better than others, but it is axiomatic that the success of the mathematics instructor in the classroom will be no better than the ability of the teacher to impart learning to the students, independent of the methods used. Effective instruction cannot be guaranteed by any single, simple formula.

Textbooks. The mathematics textbook is an extremely important feature of the mathematics program because in many schools it largely determines the content and organization of the course of study. Indeed, in many cases it is the course of study. Thus, the selection of textbooks is an extremely important matter and one that is becoming increasingly complex with the great number of textbooks now on the market. The selection should be made after careful study of

available textbooks and after judging each according to a list of criteria for textbook evaluation which is suitable to the locality in which the textbooks will be used.

Many authorities recommend more than one textbook where possible, or where needed to accomplish objectives.

Workbooks. The following brief statement implies the values and limitations of workbooks.

> Generally speaking, . . . workbooks which are properly designed and which are used in appropriate situations and in appropriate ways embody certain features which may make them valuable aids in instruction (2, p. 102).

Audio-visual aids. While the term, audio-visual aids, may be perceived to include nearly all of the aids to teaching, in this paper it will be restricted to motion pictures, film strips, slides, models, flannel boards and such aids as are closely related to those just mentioned.

Many feel that few students are able to see clearly in their minds those things that they have not seen with their eyes.

Grossnickle (8) recommends wide use of audio-visual aids to lend to meaningful discovery of even the elementary concepts such as division.

Authorities generally agree that motion pictures are one of the best devices for illustrating effectively the practical applications of mathematics and for illustrating concepts which deal with continuous change, such as locus of points.

Motion pictures, film strips, and slides must be used with judgment, after they have been checked for appropriate and accurate content. Where the content is not accurate, it is frequently as easy to establish a false idea as it is a correct one. False ideas are

not easily erased from the mind.

It is desirable for the teacher to prepare the class for the film to be presented and to follow with appropriate discussions or other activities.

It should be kept in mind that interest in motion pictures on mathematical subjects may not be directly associated with interest in mathematics.

Drill. Drill is one of the experiences common to most of the students of mathematics. Butler and Wren observe that

> Many of the operations of mathematics need to be performed not only correctly but with reasonable facility and speed if they are to be very useful. Some of them need to be actually automatized. The acquisition of facility in such operations can be secured only through systematic and repeated practice, i. e., through drill.
> . . . however, understanding must go hand in hand with operational facility (2, p. 176).

Most authorities agree that understanding should preceed drill. Butler and Wren list principles of drill that should be useful as a guide to teachers (2, pp. 176-179).

Drill, while valuable if properly used, has limitations. It should be kept in mind that what is understood, those things that pupils learn meaningfully, stays with the learner better and is applied more successfully than what they "learn" by mere drill (15, p. 8)(28, chapter 2).

Review. Review should aim at the fixation and retention of facts, processes, concepts; and at the thoughtful organization of the details of the subject matter into a coherent "whole"; and the relationship of the various parts to each other.

Where students review by themselves the teacher should help plan

the review. Definite instructions on what and how to review are needed.

Drillbooks, workbooks, or textbooks that contain certain review exercises may be valuable aids to the teacher in a program for adequate maintenance of mathematics.

Clubs. The mathematics club offers:

1. Consideration of special interest matters,

2. A place for free exchange of mathematical ideas, help, and criticism,

3. An informal and social atmosphere not available in the classroom.

> Mathematics clubs provide an excellent means of stimulating and fostering mathematical study. Membership in these clubs is usually voluntary, and for this reason the clubs are composed mainly of students who have a real interest in mathematics and who desire to obtain a view of the subject which is somewhat different from that gained in the classroom (2, p. 134).

Puzzles, games, contests. There is much evidence that moderate and well-timed use of puzzles, games, contests, and other such recreational devices adds much of interest and zest to the courses especially in the junior high school. However, they should not be permitted to pre-empt an undue amount of the regular class time.

Homework. Instead of having a considerable amount of practice at home, with which pupils may and frequently do receive assistance which does more harm than good, there is a tendency to use the home as a source of information and data pertinent to the uses of arithmetic in every-day life (24, pp. 29-34).

Classroom equipment. Values and limitations of classroom equipment were discussed in the preceeding section. However, there is some

calculating equipment that has not been mentioned that might be used to a greater extent than it has been. Some authorities feel that the abacus should be introduced early in the elementary grades; and that the slide rule and the mechanical calculator might be introduced in the ninth grade. Calculators serve as a rapid means of checking computations and tend to relieve the drudgery sometimes associated with certain types of problems. Other advantages are also claimed by various writers.

Types of classroom equipment are numerous. Articles may range in cost from a fraction of a cent to several hundred dollars. For an extensive list of types and uses of such equipment the reader is referred to the following publications: (22), (25), and (29).

What mathematics should be required

There are few authorities on mathematics education who have made a definite statement as to what mathematics should be required of all pupils. Perhaps the most significant research in that area has been done by the Commission on Post-War Plans, a commission of the National Council of Teachers of Mathematics. In their final report they listed twenty-nine questions. The comment preceeding the list of questions was, "If you can say 'yes' to nearly all of them you can feel pretty secure when it comes to dealing with the problems of everyday affairs."

The list is as follows:

1. Computation. Can you add, subtract, multiply, and divide effectively with whole numbers, common fractions, and decimals?
2. Per Cent. Can you use per cents understandingly and accurately?
3. Ratio. Do you have a clear understanding of ratio?
4. Estimating. Before you perform a computation, do

you estimate the result for the purpose of checking your answer?

5. Rounding numbers. Do you know the meaning of significant figures? Can you round numbers properly?
6. Tables. Can you find correct values in tables; e.g. interest and income tax?
7. Graphs. Can you read ordinary graphs; bar, line, and circle graphs? The graph of a formula?
8. Statistics. Do you know the main guides that one should follow in collecting and interpreting data; can you use averages (mean, median, mode); can you draw and interpret a graph?
9. The Nature of a Measurement. Do you know the meaning of a measurement, of a standard unit, or the largest permissible error, of tolerance, and of the statement that "a measurement is an approximation"?
10. Use of Measuring Devices. Can you use certain measuring devices, such as an ordinary ruler, other rulers (graduated to thirty-seconds, to tenths of an inch, and to millimeters), protractor, graph paper, tape, caliper micrometer, and thermometer?
11. Square root. Can you find the square root of a number by table or by division?
12. Angles. Can you estimate, read and construct an angle?
13. Geometric concepts. Do you have an understanding of point, line, angle, parallel lines, triangle (right scalene, isosceles, and equilateral), parallelogram (including square and rectangle), trapezoid, circle, regular polygon, prism, cylinder, cone, and sphere?
14. The 3-4-5 relation. Can you use the Pythagorean relationship in a right triangle?
15. Construction. Can you, with ruler and compasses, construct a circle, a square, and a rectangle; transfer a line segment and an angle; bisect a line segment and an angle; copy a triangle; divide a line segment into more than two equal parts; draw a tangent to a circle; and draw a geometric figure to scale?
16. Drawing. Can you read and interpret, reasonably well, maps, floor plans, mechanical drawings, and blueprints? Can you find the distance between two points on a map?
17. Vectors. Do you understand the meaning of vector; and can you find the resultant of two forces?
18. Metric system. Do you know how to use the most important metric units (meter, centimeter, millimeter, kilometer, gram, kilogram)?
19. Conversion. In measuring length, area, volume, weight, time, temperature, angle, and speed; can you shift from one commonly used standard unit to another widely used standard unit; e.g., do you know the relation between yard and foot, inch and centimeter, etc.?

20. Algebraic symbolism. Can you use letters to represent numbers; i.e., do you understand the symbolism of algebra--do you know the meaning of exponent and coefficient?
21. Formulas. Do you know the meaning of a formula--can you, for example, write an arithmetic rule as a formula, and can you substitute given values in order to find the value of a required unknown?
22. Signed numbers. Do you understand signed numbers and can you use them?
23. Using the axioms. Do you understand what you are doing when you use the axioms to change the form of a formula or when you find the value of an unknown in a simple equation?
24. Practical formulas. Do you know from memory certain widely used formulas relating to areas, volumes, and interest; and to distance, rate, and time?
25. Similar triangles and proportion. Do you know the meaning of similar triangles, and do you know how to use the fact that in similar triangles the ratios of corresponding sides are equal? Can you manage a proportion?
26. Trigonometry. Do you know the meaning of tangent, sine, cosine? Can you develop their meaning by means of scale drawings?
27. First steps in business arithmetic. Are you mathematically conditioned for satisfactory adjustment to a first job in business; e.g., have you a start in understanding the keeping of a simple account, making change, and in arithmetic that illustrates the most common problems of communications and everyday affairs?
28. Stretching the dollar. Do you have the basis for dealing intelligently with the main problems of the consumer; e.g., the cost of borrowing money, insurance to secure adequate protection against the numerous hazards of life, the wise management of money, and buying with a given income so as to get good value as regards both quantity and quality?
29. Proceeding from hypothesis to conclusion. Can you analyze a statement in a newspaper and determine what is assumed, and whether the suggested conclusions really follow from the given facts or assumptions (18, pp. 315-339)?

In the above report, addressing high school students, the Commission says, "In order that too many doors do not close on you early in life, the minimum high school mathematics for you would seem to be one or two years of general mathematics."

The Joint Commission of the Mathematical Association of America

and the National Council of Teachers of Mathematics reports, "The Commission believes that mathematics should be required through the ninth school year, and beyond the ninth year in the case of competent students (19, p. 74)."

Douglass (4, pp. 290-94) feels that two years of mathematics beyond the eighth grade should be required if the proper mathematics offering is available. He recommends that election between algebra and general mathematics in the ninth grade should be on the basis of aptitude more than on ambition because it is less likely to change.

In a study reported by Irvin (10), two mathematics specialists recommended that mathematics subjects be ungraded at the secondary level. The study implied that where enrollment is elective it should be determined by (1) the student's past achievement, (2) the teacher's recommendation, (3) reading comprehension test results, (4) special interests and vocational preferences, (5) intelligence tests scores, and (6) parent's wishes.

There is an indication that authorities in mathematics education might generally agree upon the following: Where the mathematics program is adequate, every student who does not have a conspicuous lack of ability should be required to study mathematics through the ninth grade.

Methods by which students are helped to develop ability to apply skills learned in other courses to mathematics

If an objective of mathematics instruction is to insure mathematical competence for the ordinary affairs of life, mathematics teachers should obviously attempt to show the inter-relationship of mathematics and other courses of study, in so far as they apply to

the ordinary affairs of life. Perhaps the following somewhat typical incident, as related by Schult will illustrate the need for such correlation.

> One day in one of our home economics classes, the teacher asked the girls to make half a recipe which involved taking half of one-and-a-half cups of flour. Not one girl in the class was "sure" of the amount to use. After the home economics teacher complained to the mathematics teacher, the latter, as soon as the children returned to her, sent all the girls to the blackboard and instructed them to take half of one-and-a-half. Every one did it correctly.
>
> "Your home economics teacher said you could not do that in class today," said the mathematics teacher.
>
> "But," interrupted one little girl, "that was CUPS!"
>
> This indicated the need for relating the mathematics work, as far as possible, to actual experiences so that we will not have our mathematics knowledge in a vacuum (30, pp. 217-20).

The applied phases of mathematics correlation with life situations and other school activities should vary in accordance with the maturity of the pupils, the total allotment of time, community needs and interests, and other considerations. "Definite provision should be made, however, for the study of many life situations that are mathematically rich and also for school activities that have a mathematical bearing (19, p. 81)."

Reading. "The ability to comprehend the printed page is necessary in any subject. Therefor the mathematics teacher cannot escape being a teacher of reading also (30, p. 448)."

Every help the teacher can give the student in applying reading skills to reading in mathematics will yield favorable results in mathematics accomplishment.

> Experienced teachers know that much of the high mortality in arithmetic (and other mathematics) is due to defective reading on the part of the students. . . . the teacher must develop vocabulary meaning by extending the sense meaning of

> words, by fusing meanings into related ideas, by developing the ability of the students to react critically to the words and having them see implications. Among many techniques at her command she might ask questions which will help the students to interpret the printed page in the light of their experiences with the word and extending these experiences with the context at hand (31, p. 462).

Murray (31, p. 462) lists six kinds of readings encountered.

1. Reading to obtain facts
2. Reading to analyze facts
3. Reading to interpret facts
4. Reading to understand the mathematical processes
5. Reading to understand the subject-matter vocabulary
6. Reading to see the implications for applications to similar problem situations

Following are some suggested methods of improving the reading of problems:

1. Create readiness by discussion and demonstration of techniques used in reading literature and in reading an arithmetic problem.
2. Explain terminology.
3. Present problems that present a familiar social situation whenever possible.
4. Have drill in reading for comprehension.
5. Have pupils reproduce the problems, paying attention to pronunciation and vocabulary meaning.
6. Have pupils paraphrase the problem.
7. Have pupils state in their own words what the problem asks for and the information given.
8. Diagram the problem.
9. Practice problems involving different situations.

10. Have pupils create or formulate problems.

11. Encourage wide reading, i.e. variety of texts, business sections of newspapers, and world almanacs.

12. Others--For extensive and detailed methods, the reader is referred to a bibliography prepared by Charles H. Butler (31, pp. 452-59).

Courses other than reading. Many hundreds of pages have been written concerning the integration of mathematics with other school subjects. In a short summary as is intended in this paper, it is felt to be unwise to attempt a coverage of a topic of such magnitude. Interested readers are referred to the Seventeenth Yearbook of the National Council of Teachers of Mathematics (21). Nearly every volume of The Mathematics Teacher and School Science and Mathematics contains numerous articles of merit concerning integration.

Evaluation of the effectiveness of the mathematics course

Evaluation is one of the fine arts of teaching. It may be effective, subtle, and far reaching in the results obtained if properly used. The proper use of evaluation requires considerable skill and adeptness in evaluative procedures and in the interpretation of results.

> Mathematics teachers should realize that evaluation is a continuous process involving numerous personal conferences and observations of attitudes and behavior as well as including pencil and paper tests. Evaluation must be concerned with individual growth resulting from personalized instruction rather than with achievement comparisons between members of a group (10, pp. 235-40).

It is considered advisable to administer tests one of the first days that a teacher meets with the class to aid in determining where to start with instruction. Continuous evaluation may be effectively used to determine topics that need reteaching, to prepare plans for

review work, to find individual pupil's trouble areas, to determine individual and group progress, to help the pupil evaluate his own efforts, and in other areas.

Evaluation should attempt to measure more than subject-matter achievement. It should test the pupils' understanding of the number system, their appreciation of why we do what we do in processes with numbers, their ability to round off numbers, to make estimates and to judge whether or not their answers are reasonable. Tests should also attempt to evaluate the pupils' understanding of the vocabulary and concepts of mathematics, their ability to perform mathematical skills in various situations, and other important outcomes consistent with the objectives of the course (15, p. 26).

> Without measurement in terms of goals to be achieved, children can make little progress in creative work and will suffer the subsequent disappointment of frustrated efforts and failure (6, pp. 392-95).

While the above statement is somewhat controversial, it emphasizes evaluation in terms of objectives. It implies that pupils should be aware of the goals of their particular course and of their progress toward those goals. These principles are widely accepted.

In practice, there are two major types of pencil and paper tests most frequently used to measure progress in a course in mathematics. They are teacher-made tests and standardized tests. Some other types of written tests are those designed by textbook authors, departments of mathematics, local committees, or school district staffs. There are several types of commercially produced tests appearing on the market that vary in value and purpose.

It is becoming more common for teachers, through observations,

conferences, and interviews, to attempt to evaluate attitudes, appreciations, and other outcomes of instruction without the use of written tests.

Advantages of using teacher-made tests over standardized tests are: Teacher-made tests may be constructed to test only the material that has been covered in class, to test any area of the subject-matter regardless of scope, or to test individual progress in particular areas. They may be constructed to test material in connection with unique local situations. Teacher-made tests are relatively inexpensive and are always available. They are more flexible.

Unless care is taken, however, teacher-made tests may be poorly constructed and of little value.

Advantages of standardized tests are that they posses norms which provide for equitable comparisons. They are likely to be more valid and reliable with respect to the things they are designed to test. They usually measure skills and abilities that are worth measuring. Scoring and administering are relatively easy. A controversial advantage is that the teacher need not spend so much time on the testing program.

Morton points out that tests should not emphasize speed too much. Speed is a symptom of mastery, but no guarantee of it. Some pupils with good mastery will never display much speed. He also states,

> The development of better evaluation technics and the more skilful use of the technics we have present a real challenge to classroom teachers of the present time (15, pp. 27-28).

To emphasize the importance of the evaluation program, the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics proclaims:

> . . . professional maturity, the welfare of the pupils, and the future of mathematics in the secondary schools all demand an even deeper and broader study of instructional problems. In such study evaluation plays a fundamental role. . . . If it is superficial and narrow, the development of the place of mathematics in education is retarded. If it is thoroughly and comprehensively done, there is much hope for the future (19, p. 186).

Teacher load

It is universally recognized that the teacher who is not overloaded is potentially in a better position to do a good job of teaching than one who is. It is also recognized that an overload for one teacher may not be for another.

Some of the factors of teacher load are the number of periods a day that the teacher must teach, the number of preparations per day, the length of the class periods, the type of class (English, Music, etc.), the number of students in each class, and the teacher's out-of-class activities.

Because the conditions under which a good job of teaching can or cannot be done will vary from teacher to teacher, there are no absolute values for teacher load that can be established to apply to all teachers. There has been considerable research to try to establish desirable norms or standards; however, many of the findings of research have been controversial or contradictory (14, pp. 212-15).

Douglass and Noble developed what is frequently called the Douglass Formula to aid in determining standard teacher load (5, pp. 97-98).

Accrediting associations have established certain norms which they feel should not be exceeded if an optimum job of teaching is to be accomplished.

The popular consensus of opinion as to the desirable teaching load at the junior high school is approximately as follows:

1. Five class periods per day (at least one period free for preparation, planning, consultation, etc.),

2. One to three preparations per day,

3. Maximum of sixty-minute class periods, ninety-minute laboratory periods (this includes time spent changing classes),

4. Twenty-five to thirty students per class for academic classes (The number may be greater for such classes as band or physical education, but less for remedial classes),

5. Out-of-class responsibilities should be reasonable and in accordance with the ability of the teacher to assume such responsibilities.

Teacher preparation

> The task of the teacher now should be that of guiding and directing their (the pupils) work, stimulating them, encouraging them, helping them over hard spots, evaluating their progress, and in every way possible striving to get them to put forth their best efforts to achieve a permanent and functional mastery of the material upon which they are working (2, p. 167).

While this statement indicates the task of the teacher in but one area of his work, that of teaching such that students achieve the mastery of subject-matter and its application, it indicates something of the responsibility that a teacher should accept when entering the profession. It implies that the teacher should have certain qualities of a guidance specialist, a psychologist, a research expert, a social leader, and a subject-matter specialist. Perhaps careful analysis may reveal other implications, but these alone are sufficient to illustrate the importance of trying to attract the most capable people of our

society into the teaching profession.

Education authorities regret that in many districts salaries and other conditions are such that many capable people cannot be attracted into the teaching profession even though they would like to teach if conditions were suitable.

This controversial subject may not immediately appear to be relevant to the subject of teacher preparation, but in the opinion of the author, it appears to be one of the major influences in keeping the gap relatively wide between what is recognized as desirable in teacher preparation and that which is required.

It is recognized that preparation, though important, is only one factor which contributes to the capability of teachers. The Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics recognized five major qualities that are desirable in the mathematics teacher: (1) social and civic attributes, (2) general culture, (3) familiarity with educational problems and theories, (4) skill in instruction, and (5) knowledge of and interest in mathematics. The commission regards the latter quality as the most important of all, assuming the personality and character of the teacher to be satisfactory (19, p. 188).

The North Central Association Committee says:

> In view of what the Committee recognizes as inconclusive evidence regarding factors influencing teaching success, the Committee has taken the position that quality of teaching will be definitely related to the subject matter preparation which the teacher has received for the actual teaching assignment given him (11, pp. 3-10).

Following is a summary of what is generally considered to be desirable as a minimum of preparation for secondary school teachers of mathematics:

I. In Mathematics
   1. Courses including complete treatment of college algebra, analytic geometry, and six semester hours of calculus
   2. A course that examines somewhat critically Euclidean geometry, and gives brief introductions to projective geometry and non-Euclidean geometry, using synthetic methods (three semester hours)
   3. Advanced algebra, including work in theory of equations, mathematics of finance, and statistics (six semester hours). The course should examine somewhat critically the basic laws of algebra, irrational and complex numbers, and operations with them.
   4. Either directed reading or a formal course in the history of mathematics and its concepts.
   5. Additional work is recommended for those who teach the more advanced secondary courses.

II. In related fields (where the related field is not regarded as a teaching subject)
   1. An introductory course in physics, astronomy, or chemistry that makes use of some mathematics.

III. In professional preparation
   1. A course in specific methods of teaching mathematics. This course should be taught by someone well trained in mathematics, who has taught in the secondary schools
   2. A course in secondary education, consideration given to the philosophy and history of education
   3. A course in educational psychology
   4. A course in educational tests and measurement including some statistical methods
   5. Practice teaching in major field (about ninety class-hours recommended) (19, pp. 201-202).

In addition to the above, the teacher should have a broad general education.

THE UTAH STUDY

The questionnaire which was sent to the Utah teachers (See Table 2 and Table 3, Appendix) was patterned somewhat after a California questionnaire (See Table 1, Appendix), except that more information was asked for. The Utah questionnaire was distributed during the winter of 1956.

It is felt that the responses to the Utah questionnaire comprise a representative sample of the total population of Junior High Schools in the state of Utah. An analysis of the returns of the questionnaire with respect to the schools, student population, and teachers represented is indicated in Table 4, Table 5, and Table 6, respectively (See Appendix). In addition, the author's check as to the geographical location of the schools represented revealed that, geographically, the sample is representative of the total population.

In view of the above, the author believes that the findings of this study reveal the nature of the mathematics program at the junior high school level in the state of Utah, in so far as the opinions of the teachers, as expressed in the questionnaires, reveal the nature of the mathematics program in their respective classrooms. It is recognized that in many cases the information contained in the questionnaire does not reveal the nature of the program completely or accurately.

The statements in the following sections are all made with reservations consistent with the above conditions. While many observations

may be worded to imply that they accurately describe general conditions of the mathematics program throughout the state of Utah, it is realized that they describe the various aspects of the program with no greater accuracy than is represented in the responses to the questionnaire. It is the intention and hope of the author that the reader will interpret the results of this study with this in mind.

Analysis of Teacher Load

Number of Sections per day. As indicated in Table 7, 61.6 percent of the teachers at the junior high school level teach six or more sections per day. Six of the elementary teachers who were teaching several grades in the same room were reported as teaching twenty-one or more sections per day.

To clarify the meaning of the word "sections" as used here, the following examples are given: A teacher who teaches two classes of mathematics and three classes of science per day would be classified as teaching five sections per day. A teacher who teaches four subjects to each of three grades would be classified as teaching twelve sections per day.

There is an indication that larger schools tend to require the teachers to teach fewer sections per day than do smaller schools. (See Table 7)

Number of sections of mathematics taught per day. By comparing Table 7 and Table 8, it is evident that a large percentage of the teachers of mathematics, even in the larger schools, teach subjects other than mathematics. While 13.8 percent of the mathematics teachers teach one to four sections per day, 75.4 percent teach one to four sections of mathematics per day. Twenty-six percent of the

mathematics teachers teach only mathematics. Twenty-seven percent of those teaching only mathematics teach fewer than five sections per day.

Table 7. Number of sections per day that teachers are engaged in classroom instruction.

Type and Size of School	Number of Sections Per Day				
	1-4	5	6	7	more than 7
High Schools					
1-200	1	1	8	3	0
201-500	6	7	14	0	0
above 500	2	4	4	0	0
Junior High Schools					
1-300	8	5	9	0	0
301-850	5	15	50	1	0
above 850	3	21	21	1	0
Elementary Schools					
1-35	0	1	0	2	7
36-150	1	1	1	0	7
above 150	5	0	3	4	3
	31	55	110	11	17

Number of students per class of mathematics. Sixty-eight percent of the mathematics classes at the junior high school level have thirty or more students per class. Six-and-a-half percent contain forty or more students. The junior high schools tend to have larger mathematics classes than the other two types of schools at this level. Eighty-two percent of the junior high school mathematics classes contain thirty or more students, 7.9 percent contain forty or more.

Table 8. Number of sections of mathematics taught per day.

Type and Size of School	Number of sections per day							
	1	2	3	4	5	6	7	8
High Schools								
1-200	8	4	1	0	0	0	0	0
201-500	19	5	1	2	1	0	0	0
above 500	0	2	3	2	3	0	0	0
Junior High								
1-300	6	11	3	2	0	0	0	0
301-850	19	15	3	9	7	17	0	0
above 850	6	8	4	8	14	5	1	0
Elementary Schools								
1-35	0	3	0	0	3	1	1	2
36-150	0	3	5	2	0	0	0	0
above 150	0	8	5	2	0	0	0	0
	58	59	25	27	28	23	2	2

Table 9. Number of students per class of mathematics.

Type and Size of School	Students Per Class				
	Fewer than 25	25-29	30-34	35-39	40 or more
High School					
1-200	7	4	0	1	0
201-500	6	8	9	2	3
above 500	0	0	7	3	0
Junior High					
1-300	5	7	6	4	0
301-850	3	6	30	23	9
above 850	0	3	22	19	2
Elementary School					
1-35	10	0	0	0	0
36-150	8	0	1	1	0
above 150	3	1	7	4	0
	42	29	82	57	14

Analysis of teacher experience and training

Number of years of teaching experience. It was noticed that some teachers who listed one year of experience on this question stated that this was their first year of teaching, in response to Part II, question 8. Thus, a few of the teachers who are listed as having had from one to five years of experience should have been listed as having no previous experience. Because there was no way of determining how many first year teachers listed one year of experience, Table 10 is compiled exactly as reported in the questionnaire.

The distribution of teachers according to years of experience is as follows: During the first five years many teachers leave the profession, afterwards the decrease is gradual and relatively constant until age of retirement.

Table 10. Number of years of teaching experience of mathematics teachers at the junior high school level.

Type and Size of School	Years of Teaching Experience				
	0	1-5	6-15	16-30	above 30
High School					
1-200	1	7	4	1	0
201-500	3	7	6	10	3
above 500	0	4	0	4	2
Junior High					
1-300	2	6	5	6	3
301-850	4	27	15	21	4
above 850	5	17	11	12	2
Elementary School					
1-35	1	2	4	2	1
36-150	0	5	0	4	0
above 150	0	4	7	4	0
	16	79	52	64	15

Number of years of experience teaching mathematics. Fifty-and-seven-tenths percent of the mathematics teachers of Utah at the junior high school level have taught mathematics for fewer than six years. Fifty percent of the teachers report that they have taught mathematics for fewer years than their total number of years of teaching experience.

As may be seen by comparing the information in Table 10 and Table 11, there is a tendency to shift teachers of other subjects into mathematics classes. One teacher complained vigorously that he hates to teach mathematics, but that was his assignment. Others felt they were not sufficiently trained to teach mathematics but that they had been assigned to do so.

Table 11. Number of years of experience teaching mathematics.

Type and Size of School	Years of Experience Teaching Mathematics				
	0	1-5	6-15	16-30	above 30
High School					
1-200	1	8	3	1	0
201-500	4	11	9	4	0
above 500	0	4	0	4	2
Junior High					
1-300	3	8	5	3	3
301-850	6	31	17	16	1
above 850	5	18	16	6	1
Elementary School					
1-35	1	3	4	2	0
36-150	1	4	3	2	0
above 150	0	6	7	2	0
	21	93	64	40	7

The notices on the placement bureau bulletin board at the Utah

State Agricultural College (spring quarter, 1956) indicate that administrators request teachers to teach mathematics and to coach, or to teach shop and mathematics, or to teach mathematics and other subjects, sometimes not closely related. In view of the above considerations and because mathematics is a required subject at the junior high school level, it appears that evaluation of policies and practices with respect to the training and hiring of teachers that may teach mathematics should receive immediate attention.

Number of years of college completed. It appears that there is a slight tendency for larger schools to have teachers with more years of college training than do smaller schools. Small elementary schools have the greatest number of teachers with less than four years of college training. Forty-two percent of the teachers of mathematics report that they have completed five or more years of college study.

Table 12. Number of years of college completed by teachers of mathematics at the junior high school level.

Type and Size of School	Years completed					
	0-3	4	5	6	7	above 7
High School						
1-200	0	11	2	0	0	0
201-500	1	19	5	2	1	0
above 500	0	9	1	0	0	0
Junior High						
1-300	0	12	6	2	2	0
301-850	0	34	29	6	2	1
above 850	1	18	20	6	1	0
Elementary School						
1-35	5	3	1	0	0	0
36-150	1	8	1	0	0	0
above 150	1	7	5	2	0	0
	9	121	70	18	6	1

Degrees held by teachers of mathematics. While forty-two percent of the teachers of mathematics report having completed five or more years of college, sixteen percent report having been granted advanced degrees. Six percent of all teachers report less than a bachelor's degree. Twenty-two percent of the elementary teachers report less than a bachelor's degree.

Table 13. Degrees held by teachers of mathematics at the junior high school level.

Type and Size of School	Degrees held None	Two year normal certificate	Bachelor	Master	Doctor
High School					
1-200	0	0	13	0	0
201-500	2	0	22	4	0
above 500	1	0	9	0	0
Junior High					
1-300	2	0	14	5	1
301-850	0	0	61	11	0
above 850	1	0	35	11	0
Elementary School					
1-35	3	3	3	1	0
36-150	1	0	9	0	1
above 150	1	0	12	2	0
	11	3	178	34	2

Major field of study. The major fields of study were classified as follows:

1. Mathematics
2. Physical science: includes chemistry, physics, geology
3. Agriculture
4. Biological science: includes biology, physiology, psychology, zoology

5. Education
   A. Elementary
   B. Secondary
      With a teaching major in mathematics
      With a teaching major not in mathematics
      Physical science composite
6. Fine arts: includes art, English, languages, music, speech
7. Forestry and wildlife
8. Industrial arts
9. Physical education
10. Social science: includes business, economics, history, social science composite, sociology

Only eight percent of the teachers reporting a major field of study indicated that their undergraduate major was in mathematics. Twenty percent listed either a major or minor in mathematics. In these reports there were a few obvious contradictions. Some listed a minor in mathematics with fewer than five hours of college credit in mathematics. In all, twenty-five reported undergraduate minors in mathematics.

Twenty-two percent listed their undergraduate major as mathematics, physical science composite, physical science, or secondary education with a teaching major in mathematics. Fifty-eight percent reported a major in social science, physical education, fine arts, or agriculture, according to the above classification.

No teacher reported a graduate major in mathematics. Only three listed their graduate minor as mathematics--one of these indicated zero hours of graduate credit in mathematics. (See Table 14.)

Quarter hours of credit received in mathematics. Thirteen percent of the teachers reported no college credit in mathematics; sixty percent reported fewer than eighteen quarter hours of credit, the

Table 14. Tabulation of major fields of undergraduate study as reported by mathematics teachers.

Undergraduate Major	Number of majors		
	High School	Junior High	Elementary
Mathematics	2	14	1
Physical science	5	6	0
Agriculture	6	13	3
Biological science	1	11	0
Education			
Elementary	0	0	9
Secondary			
teaching major mathematics	1	1	1
teaching major not mathematics	2	6	1
physical science composite	2	14	0
Fine arts	4	22	4
Forestry and wildlife	0	3	0
Industrial arts	6	3	1
Physical education	9	10	1
Social science	10	32	9
	48	135	30

amount generally required for a minor in mathematics; ten percent indicated that they had received forty-two or more quarter hours of credit in mathematics, the approximate number of hours needed to satisfy the minimum amount of training in mathematics recommended by the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics (19, pp. 201-202). (See Table 15)

Even where teachers have little or no training in mathematics, there is little tendency for them to include courses in mathematics in connection with their graduate study. (See Table 16.)

Analysis of mathematics course

In order to analyze Part II of the questionnaire with efficiency and to represent the findings in tabular form with economy of space

Table 15. Quarter hours of credit in mathematics received by teachers of mathematics (Undergraduate).

Type and Size of School	Hours of credit				
	0	1-17	18-29	30-41	above 41
High School					
1-200	1	7	1	2	0
201-500	0	15	5	2	0
above 500	0	4	1	3	1
Junior High					
1-300	2	9	5	3	2
301-850	8	30	14	8	9
above 850	6	17	9	6	7
Elementary School					
1-35	3	6	1	0	0
36-150	5	3	1	0	1
above 150	3	9	2	0	1
	28	100	39	24	21

Table 16. Quarter hours of credit in mathematics received by teachers of mathematics (Graduate).

Type of school	Hours of Credit		
	1-5	6-17	above 17
High school	2	3	0
Junior high	7	8	3
Elementary	1	1	0
	10	12	3

it was felt necessary to codify and classify the information given in the questionnaires.

The meaning of the words as used in the tables of the following sections will be explained in the text preceding the tables.

It may be noted that some of the objectives, divisions of content,

experiences, and methods are quite general in nature, while others are more specific. For the most part, that is the way these items were reported. Of course, some grouping and editing was necessary to simplify the analysis and tabulation of the data.

Throughout this portion of the study comparison of objectives, methods, experiences, and other aspects of the mathematics program as to size of school, type of school, experience and training of teachers will be indicated only at the seventh and eighth grade level. It is felt that the programs are too dissimilar and that there are insufficient numbers to make reasonable comparisons at the ninth grade level.

Objectives, division of course content, experiences, methods, and means of evaluation were analyzed by the author, with respect to type of school, size of school, years of experience teaching mathematics, and college credit received by the teacher in the field of mathematics. The results of this analysis will be indicated but will not be shown in tabular form except to show where significant differences exist, the method of tabulation, or to show something that is felt to be of general interest at this time.

Title of Course. While exact titles of courses in mathematics varied, the courses may be quite accurately classified under the following titles:

1. Seventh Grade General Mathematics. The major emphasis in this course was on arithmetic.

2. Eighth Grade General Mathematics. The major emphasis in this course was on arithmetic. Some schools shifted emphasis, in varying degrees, toward remedial work or preparation for algebra.

3. Ninth Grade General Mathematics. The major emphasis in this course was on applied arithmetic. Some schools put considerable emphasis on review of arithmetic fundamentals and remedial work. Some schools put some emphasis on algebraic and geometric concepts.

4. Remedial Mathematics. There were four schools that reported classes of a strictly remedial nature. Responses were too few and too varied to make a valid analysis of the remedial mathematics program in Utah.

5. Business Mathematics. Five schools reported classes where the emphasis was mainly upon business arithmetic.

6. Ninth Grade Algebra, or First Course in Algebra. The major emphasis was upon algebra.

7. The Stewart school at the University of Utah offered courses with a variety of titles. The content was also varied. Many of the courses were ungraded.

Objectives. The objectives given in response to the questionnaire were coded and classified as follows:

1. Need: To assist each pupil to recognize the need for and the importance of mathematical skills

2. Basic Skills: To assist each pupil to develop, improve, and maintain basic mathematical skills and concepts including problem solving techniques. This grouping includes: fundamental processes, quantitative processes, the four basic operations, counting, standard units of measure, simple measurement, whole numbers, fractions, decimals, percentage, etc.

3. Application: To assist each pupil to apply basic mathematical skills and concepts in class and in actual life problems and to provide experiences which will assure ability to apply these skills

4. Desire. To stimulate within each pupil an interest in mathematics and the desire to learn and apply mathematical skills

5. Thought problems: To assist each pupil to develop the ability to analyze and work thought problems

6. Self-realization: To assist each pupil with respect to personal evaluation, self-realization, and improvement regarding mathematics

7. Advanced: To provide each pupil with growth in more complex mathematical concepts, spacial relationships, and geometry

8. Mind: To assist each pupil toward general improvement of mind. This grouping includes: techniques of thinking clearly, precisely, independently and in an organized manner; ability to analyze and make sound judgments based on factual data; capacity for abstract reasoning; ability to recognize and interpret mathematical situations; and such improvements of mind

9. Preparation: To provide a means of preparation for more advanced mathematics. This grouping includes: preparation for algebra and other more advanced high school mathematics and for college

10. Interest: To develop within each pupil a genuine interest in mathematics. This grouping includes: enjoyment of mathematics; overcome frustration, fear, and hatred for mathematics; enjoyment of a sense of accomplishment

11. Thought tool: To develop within each pupil an appreciation of numbers as a tool for organized thinking

12. Correlation: To assist each pupil to develop skill in other tool subjects: reading, writing, communication, and mathematical literacy

13. History: History and ancient applications of mathematics

14. Remedial: To provide individual adjustment for those who are handicapped through absences, and slow learning

15. Character: To assist each pupil toward general improvement of character. This grouping includes: personal improvement of working habits; accept responsibility; good citizenship; proper discipline; and such improvements of character

As may be seen in Table 17, there is little difference in the objectives of the course in mathematics between the seventh and eighth grades. The major emphasis is on objectives in connection with basic

Table 17. Objectives of seventh and eighth grade general mathematics.

	Seventh Grade					Eighth Grade				
	High School	Junior High	Elementary	Totals		High School	Junior High	Elementary	Totals	
Teachers Reporting	21	65	30	116	%	20	76	15	111	%
Objectives:										
1. Need	1	3	8	12	10.3	2	7	4	13	11.7
2. Basic skills	20	57	27	104	89.7	16	62	13	91	82.0
3. Applications	8	37	17	62	53.4	15	46	8	69	62.2
4. Desire	0	4	1	9	7.8	0	7	1	8	7.2
5. Thought problems	3	13	1	17	14.6	1	7	1	9	8.1
6. Self-realization	1	1	2	4	3.4	0	1	0	1	.9
7. Advanced	1	5	2	8	6.9	1	7	2	10	9.0
8. Mind	6	7	5	18	15.5	3	14	2	9	8.1
9. Preparation	2	5	1	8	6.9	3	15	1	19	17.1
10. Interest	0	0	0	0	0.0	1	0	0	1	0.9
11. Thought tool	0	1	0	1	0.9	0	0	0	0	0.0
12. Correlation	2	2	0	4	3.4	2	2	0	4	3.6
13. History	0	0	0	0	0.0	0	0	0	0	0.0
14. Remedial	0	2	0	2	1.7	0	0	0	0	0.0
15. Character	4	5	2	11	9.5	3	4	1	8	7.2

Table 18. Objectives of ninth grade general mathematics and algebra.

	Ninth Grade General Mathematics					Algebra			
	High School	Junior High	Elementary	Totals		High School	Junior High	Totals	
Teachers Reporting	14	41	3	58	%	12	23	35	%
Objectives:									
1. Need	0	1	0	1	1.7	2	1	3	5.7
2. Basic skills	11	36	3	50	86.2	7	15	22	62.9
3. Application	11	30	3	44	75.9	3	5	8	22.9
4. Desire	4	4	0	8	13.8	0	2	2	5.7
5. Thought Problems	0	0	0	0	0.0	1	1	2	5.7
6. Self-Realization	0	1	0	1	1.7	0	2	2	5.7
7. Advanced	2	7	2	11	19.0	0	2	2	5.7
8. Mind	3	6	0	9	15.5	4	11	15	42.9
9. Preparation	1	6	1	8	13.8	7	7	14	40.0
10. Interest	0	1	0	1	1.7	1	3	4	11.4
11. Thought tool	0	0	0	0	0.0	0	3	3	8.6
12. Correlation	0	0	0	0	0.0	0	3	3	8.6
13. History	0	0	0	0	0.0	0	0	0	0.0
14. Remedial	0	0	0	0	0.0	0	0	0	0.0
15. Character	2	5	0	7	12.1	1	2	3	8.6

Table 19. Objectives with relation to the number of quarter hours of credit in mathematics received by mathematics teachers.

	Quarter Hours of Credit in Mathematics									
	None		1-17		18-29		30-41		above 41	
Teachers reporting	22	%	66	%	26	%	16	%	15	%
Objectives:										
1. Need	1	4.5	8	12.1	2	7.7	0	0	0	0.0
2. Basic skills	20	90.9	57	86.4	24	92.3	13	81.2	15	100.0
3. Application	8	36.4	39	59.1	16	61.5	11	68.8	7	46.7
4. Desire	0	0.0	4	6.1	1	3.8	2	12.5	1	6.7
5. Thought problems	2	9.1	10	15.2	4	15.4	1	6.2	2	13.3
6. Self-realization	0	0.0	1	1.5	0	0.0	0	0.0	2	13.3
7. Advanced	1	4.5	7	10.6	2	7.7	0	0.0	1	6.7
8. Mind	1	4.5	5	7.6	4	15.4	3	18.8	1	6.7
9. Preparation	3	13.6	8	12.1	3	11.5	4	25.0	2	13.3
10. Interest	0	0.0	1	1.5	0	0.0	0	0.0	0	0.0
11. Thought tool	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0
12. Correlation	1	4.5	2	3.0	2	7.7	1	6.2	0	0.0
13. History	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0
14. Remedial	1	4.5	0	0.0	0	0.0	0	0.0	1	6.7
15. Character	0	0.0	3	4.5	4	15.4	1	6.2	4	26.7

skills and applications. There appears to be a slight increase in the eighth grade of attention to applications and preparation for higher mathematics, with a slight decrease in emphasis on basic skills and improvement of mind.

Ninth grade general mathematics shows a shift of emphasis toward application of mathematical skills. Algebra has decreased emphasis on application, but increased emphasis on improvement of mind and preparation for higher mathematics. (See Table 18.)

There are no outstanding differences revealed in objectives in relation to size of school, years of experience teaching mathematics, or training of teachers. Table 19 shows the analysis of objectives in relation to training of teachers.

Divisions of course content, seventh and eighth grades. The classification of divisions of course content was as follows:

1. Whole numbers: Basic operations with whole numbers
2. Fractions: Basic operations with common fractions
3. Decimals: Basic operations with decimal fractions
4. Percentage
5. Denominate number: (measures of time, weight, capacity, distance)
6. Geometry: Simple geometry (perimeters, areas, volumes, terms used)
7. Application: Applications to life problems (home, business, government, transportation, communication, housing problems, interest, banking)
8. Simple graphs
9. Metric system
10. Conversions
11. Using tools: Use of tools for constructions and measurements

12. Vocabulary: Vocabulary of arithmetic

13. Problems: Problem solving and analysis (word problems, equations)

14. Indirect measurement: Ratio, proportion, Pythagorean, relation

15. Square root

16. Algebra: Introduction to algebra (use of formulas, signed numbers, symbols)

17. Correlation: Correlation with other subjects

18. Drills: Drills for neatness, accuracy, and speed

19. Estimation: Estimation, approximate number, short cuts, aids

20. Drawings: Scale drawings

21. Remedial: Attention to individual needs, remedial activities

22. Appreciation: Appreciation activities

23. Review: Review previous years' work, general review

24. Numbers: Number systems, comparing numbers (See Table 20).

Divisions of course content ninth grade general mathematics.

Classification of divisions of course content was as follows:

1. Basic operations: Basic operations with whole numbers and fractions

2. Percentage

3. Denominate number: time, weight, capacity, distance

4. Geometry: simple geometry and measurement of geometric forms, perimeters, area, volume

5. Measurement: (unspecified)

6. Indirect measurement: ratio, proportion, Pythagorean relation

7. Metric system: The metric system and conversions

Table 20. Divisions of course content, seventh and eighth grades.

	Seventh Grade	Eighth Grade
Number of Teachers Reporting:	116	111
Content:		
1. Whole numbers	39	32
2. Fractions	93	47
3. Decimals	91	49
4. Percentage	99	72
5. Denominate numbers	59	23
6. Geometry	63	75
7. Application	68	79
8. Simple graphs	50	11
9. Metric system	0	3
10. Conversions	0	1
11. Using tools	4	3
12. Vocabulary	2	0
13. Problems	5	4
14. Indirect measurement	11	20
15. Square root	2	4
16. Algebra	10	54
17. Correlation	0	0
18. Drills	7	10
19. Estimation	1	0
20. Drawings	5	7
21. Remedial	4	4
22. Appreciation	0	0
23. Review	23	33
24. Numbers	15	3

8. Vocabulary: Vocabulary of mathematics

9. Roots and powers: mostly square roots and positive integral powers

10. Algebra: Introduction to algebraic concepts

11. Formal Geometry: Introduction to formal geometry

12. Trigonometry: Introduction to trigonometry

13. Higher Mathematics: Introduction to higher mathematics (unspecified)

14. Estimation: Estimation and approximate number

15. Scale drawings

16. Graphs: Graphs and charts, construction and use

17. Remedial: Remedial activities, individual and group

18. Number systems: Number systems, comparing numbers, understanding numbers

19. Review: Review of previous work

20. Application: Application to everyday problems: home, business, government, transportation, communication, housing problems, banking, financing, interest

21. Problems: Problem solving, thought problems

22. Equations: Using equations and formulas

23. Signed numbers

24. Correlation: Correlation activities, correlating mathematics and other subjects

25. Remedial: Attention to individual needs, remedial activities

26. Drill: Drill for speed accuracy, retention of skills

27. Statistics: Introduction to statistical mathematics

For tabulation of divisions of course content ninth grade general mathematics see Table 21.

Table 21. Divisions of course content, ninth grade general mathematics.

Total Number of Teachers Reporting: 58

Content:	No.	Content:	No.
1. Basic operations	24	15. Scale Drawings	1
2. Percentage	30	16. Graphs	10
3. Denominate number	7	17. Remedial	2
4. Geometry	25	18. Number systems	2
5. Measurement	10	19. Review	23
6. Indirect measurement	8	20. Application	47
7. Metric system	3	21. Problems	7
8. Vocabulary	1	22. Equations	8
9. Roots and powers	3	23. Signed numbers	5
10. Algebra	27	24. Correlation	1
11. Formal geometry	20	25. Remedial	0
12. Trigonometry	6	26. Drill	2
13. Higher mathematics	2	27. Statistics	3
14. Estimation	0		

Divisions of course content algebra. The classification of divisions of course content in algebra was as follows:

1. Review: Review of fundamentals of arithmetic
2. Problem solving: Analysis and solution of thought problems
3. Remedial: Remedial activities with individuals and groups
4. Vocabulary: Vocabulary of Algebra

5. Factoring: Special products and factoring
6. Powers and roots
7. Signed numbers
8. Literal numbers: Basic operations with simple literal number expressions
9. Equations: Solving equations with one unknown
10. Graphs
11. Formulas
12. Application: Application to life situations
13. Numbers: The number system, understanding numbers
14. Ratio: Ratio and proportion
15. Fractions: Operations with algebraic fractions
16. Quadratics: Solution of simple quadratic equations
17. Variation
18. Trigonometry: Introduction to concepts of numerical trigonometry
19. Dependence
20. Grouping: Operations with grouping symbols
21. Axioms: Understanding axioms, theorems, postulates, and such concepts
22. Simultaneous equations
23. History: Historical development of Algebra

There is a conspicuous absence of emphasis on the historical development of algebra, an area that might be used effectively to stimulate interest in algebra: There is little or no emphasis indicated on applications of algebra, understanding numbers, vocabulary of algebra, review of fundamentals of arithmetic, and remedial activities. All of these divisions are important in varying degree in an introductory

Table 22. Divisions of course content, Algebra.

	High School	Junior High	Totals
Number of Teachers Reporting	12	23	35
Objectives			
1. Review	1	1	2
2. Problem solving	3	3	6
3. Remedial	0	0	0
4. Vocabulary	0	0	0
5. Factoring	4	15	19
6. Powers and roots	4	13	17
7. Signed numbers	3	20	23
8. Literal numbers	7	15	22
9. Equations	9	18	27
10. Graphs	5	14	19
11. Formulas	6	11	17
12. Application	0	1	1
13. Numbers	0	0	0
14. Ratio	4	11	15
15. Fractions	4	10	14
16. Quadratics	5	12	17
17. Variation	0	6	6
18. Trigonometry	3	6	9
19. Dependence	0	2	2
20. Grouping	0	3	3
21. Axioms	0	1	1
22. Simultaneous equations	6	11	17
23. History	0	0	0

course in Algebra.

See Table 22 for tabulation of course content.

Experiences and materials designed to achieve objectives. Following is the classification of experiences designed to achieve the objectives of the course in mathematics:

1. Life situations: Activities related to real life problems
2. Audio-visual aids: special audio-visual aids such as: films, slides, motion pictures, flannel board, etc.
3. Blackboard: Blackboard work by students
4. Bulletin board: Bulletin board, student maintained
5. Discussion: Classroom discussion
6. Assignments: Regular or periodic assignment of problems
7. Drills: Drills for speed, accuracy, fundamental concepts
8. Field Trips
9. Games: Games, puzzles, contests and other recreational activities emphasizing the enjoyment of mathematics
10. Graphs: Constructing and using graphs
11. Homework
12. Correlation: Integration and correlation activities
13. Laboratory: Laboratory experiences, in class and out of class--using tools to draw and measure, constructing models, tools, etc.
14. Notebooks
15. Communication: Oral communication of mathematical concepts and problems, recitation
16. Outside speakers: Presentations by businessmen, skilled workmen, parents and other outside speakers
17. Problem solving: Problem solving, actual and simulated, oral and written

18. Scrapbooks: Student maintained scrapbooks and collections, individual or group

19. Student helpers: Better students helping slower students

20. Supervised study

21. Teacher presentations: Teacher presentations such as lectures, demonstrations, interest stories

22. Testing: Testing for self-evaluation, diagnosis, as a learning process

23. Textbooks: Use of textbooks for understanding and exercises

24. Workbooks: Use of workbooks, drillbooks, worksheets

25. Projects: Pupil reports, projects, demonstrations

26. Review: Review activities

The schools, in general, appear to be weak in the use of experiences and activities that stimulate interest in mathematics, such as creation of scrapbooks, presentations by outside speakers, correlation activities, and laboratory experiences.

The use of textbooks and homework are the most common experiences, with the possible exception of teacher presentations. While the tabulation (Table 23) does not indicate that teacher presentation is the most common experience, there is little doubt but what it is the case.

There were no significant differences indicated, in the experiences that students encounter, in terms of size or type of school, nor in training or experience of teachers.

Reasons for enrollment. All seventh and eighth grade students were required to enroll in mathematics. Following is a tabulation of reasons for enrollment in ninth grade mathematics courses. (See Table 24.)

Table 23. Experiences and materials of instruction designed to achieve the objectives of course in mathematics.

Grade:	7th	8th	9th General math.	9th Algebra
Number of Teachers Reporting:	116	111	58	35
Experiences				
1. Life situations	25	23	16	6
2. Audio visual aids	12	7	9	3
3. Blackboard	9	9	5	5
4. Bulletin board	3	2	1	1
5. Discussion	6	5	2	1
6. Assignments	21	23	9	6
7. Drills	20	8	5	2
8. Field trips	14	16	13	0
9. Games	39	26	7	3
10. Graphs	11	5	5	0
11. Homework	69	59	26	22
12. Correlation	3	1	0	1
13. Laboratory	12	13	5	1
14. Notebooks	7	8	5	2
15. Communication	3	2	2	0
16. Outside speakers	2	1	2	0
17. Problem solving	2	2	0	3
18. Scrapbooks	2	1	0	0
19. Student helpers	0	3	0	0
20. Supervised study	0	1	0	1
21. Teacher presentations	7	7	4	4
22. Testing	8	13	6	3
23. Textbooks	91	88	42	28
24. Workbooks	13	11	6	3
25. Projects	9	5	3	2
26. Review	1	4	1	0

As indicated in Table 24, there are schools where "some" are required to enroll in ninth grade mathematics. The means used to determine who should be required to enroll are tabulated in Table 25. The means most frequently given is listed in the table under "other." The means given was, one course of mathematics beyond the eighth grade is required for graduation from high school. In a sense, this might properly be listed as "elective" or "required of all", depending

Table 24. Reasons for enrollment in mathematics class, ninth grade.

Reasons for Enrollment	High School	Junior High	Elementary	Totals
9th grade general mathematics				
required of all	4	18	2	24
required of some	4	12	0	16
elective	6	11	1	18
	14	41	3	58
Algebra				
required of all	4	1	0	5
required of some	3	4	0	7
elective	5	18	0	23
	12	23	0	35

Table 25. Means used to determine who should enroll, when "some" are required to enroll in ninth grade mathematics.

	High School		Junior High	
Means used	General Mathematics	Algebra	General Mathematics	Algebra
Counseling	1	0	0	0
Aptitude tests	0	0	2	0
Prior record	1	1	4	3
Other	3	3	8	1

upon the situation. Over half of those listed under "other" are of this type.

Methods used to develop ability to apply reading skills to mathematics. Following is a list of the methods used to develop the students ability to apply reading skills to mathematics:

1. Vocabulary: Vocabulary study and spelling

2. Word Problems: Word problems--read, write, discuss, analyze, and solve

3. Create Problems: Have students create original word problems

4. Instruction: Direct teacher instruction, motivation, direction

5. Restate: Have students read written work or problems, then restate in own words

6. Reports: Student reports, oral and written

7. Questioning: Use of the question

8. Tests: Tests and evaluations

9. Translation: Exercises in translating symbols into words, words into symbols, words into ideas

10. Mental picture: Training to form mental picture of problems read

11. Matching: Matching exercises

12. Comprehension: Use of drills and exercises in reading comprehension, following instructions, understanding the problem

13. Student teaching: Student teaching within classroom

14. Research: Classroom study and activities in connection with reports, forms, mathematics concepts used in real life

15. None: No reading skills used

16. Oral reading: Oral reading, not necessarily mathematical material

There were no outstanding differences in the methods used as to type of school, size of school, training of teachers, or experience of teachers. An analysis of methods used in the different grades is shown in Table 26.

Sixty-seven respondents did not answer this question.

Table 26. Methods used to develop ability to apply reading skills to mathematics

Grade:	Seventh	Eighth	Ninth General Math.	Ninth Algebra
Number of Teachers Reporting	116	111	58	35
Methods:				
1. Vocabulary	27	24	9	9
2. Word Problems	60	46	23	14
3. Create Problems	7	6	1	0
4. Instruction	19	22	8	4
5. Restate	10	5	1	2
6. Reports	1	2	1	0
7. Questioning	3	3	0	0
8. Tests	5	6	1	1
9. Translation	2	2	3	2
10. Mental Picture	0	1	1	0
11. Matching	0	0	1	0
12. Comprehension	15	13	4	2
13. Student teaching	0	1	0	0
14. Research	7	8	3	0
15. None	1	2	1	1
16. Oral reading	7	12	6	2

Methods used to develop ability to apply skills learned in other courses to mathematics. Following is classification of methods used to help the student develop ability to apply skills learned in other courses to mathematics: (Tabulation is shown in Table 27.)

1. Geography: Using geographical knowledge in problems; time zones, latitude, longitude, distances between towns

2. Physical Science: Using knowledge of the physical sciences in problems; temperature, gear speeds, levers, metric measure

3. Writing: Drills in neatness and content of writing

4. Art: Use of art in construction of illustrative materials

5. Shop: Use of shop skills in construction of models

6. Home Economics: Creating problems in connection with materials, cooking

7. Social Studies: Using information on world trade and other situations in connection with graphs and problems

8. Business: Using business knowledge and situations in problems

9. Out-of-School: Using out-of-school situations for problems; using skills learned in scouting, farming, and other places in solution of problems

10. Research: Using research methods in mathematics; having students give reports on mathematically significant situations in other areas

11. Instruction: Direct instruction by teacher, explanation of application

12. Mechanical Drawing: Use of skills learned in mechanical drawing for scale drawing, graphing, illustrating

13. Health: Using skills and knowledge gained in health and physical education classes to create and aid in solution of problems

14. Workbooks: Using workbooks that correlate other subjects with mathematics

15. Create Problems: Let students create problems from experience in other (not specified) courses

16. Discussion: Class discussions of how skills learned in other classes might be used in mathematics

There were no outstanding differences in the methods used as to

Table 27. Methods used to develop ability to apply skills learned in other courses to mathematics.

Grade	Seventh	Eighth	Ninth General Math.	Ninth Algebra
Teachers Reporting	116	111	58	35
1. Geography	6	3	0	0
2. Physical Science	14	15	6	1
3. Writing	6	7	3	0
4. Art	5	6	1	0
5. Shop	9	19	10	3
6. Home Economics	5	10	6	1
7. Social Studies	6	9	4	0
8. Business	1	3	3	1
9. Out-of-School	3	1	0	0
10. Research	5	4	3	2
11. Instruction	15	19	13	10
12. Mechanical Drawing	1	0	1	0
13. Health	0	3	0	0
14. Workbooks	0	1	0	0
15. Create Problems	0	1	2	1
16. Discussion	2	1	0	0

type of school, size of school, or experience of teachers.

There was some difference in the number of methods reported per teacher with respect to the training of the teachers (See Table 28). The importance and implications of this difference is not immediately ascertainable. One indication, of questionable accuracy, may be that teachers with more training pay more attention to methods of correlation. In order to arrive at any concrete conclusions in this respect, more detailed study is needed.

One-hundred-sixty-two (50.6 percent) respondents did not answer this question. This does imply that mathematics teachers in general pay little attention to methods of integration and correlation of mathematics and other courses. If this is the case, it seems regrettable

Table 28. Methods used to develop ability to apply skills learned in other courses to mathematics with respect to mathematics training of teachers.

7th and 8th grade teachers, only one questionnaire per teacher	Quarter Hours of Credit Received in Mathematics				
	0	1-17	18-29	30-41	above 41
Number of Teachers Reporting	22	66	26	16	15
1. Geography	1	2	1	1	0
2. Physical science	3	5	7	2	2
3. Writing	1	2	2	1	1
4. Art	0	5	0	2	0
5. Shop	4	6	6	2	4
6. Home Economics	1	4	5	1	0
7. Social Studies	1	2	1	2	1
8. Business	0	2	1	1	2
9. Out-of-School	0	3	0	0	1
10. Research	0	2	0	1	1
11. Instruction	1	12	7	5	2
12. Mechanical Drawing	0	0	1	0	1
13. Health	0	1	0	0	0
14. Workbooks	0	1	0	0	0
15. Create Problems	0	1	0	1	1
16. Discussion	1	0	0	0	1
Total number of responses	13	48	31	19	18
Average number of methods reported per teacher	0.59	0.72	1.19	1.19	1.20

in view of the stimulation of interest and increased understanding of mathematics that may be accomplished through properly directed correlation activities.

Means of determining effectiveness of course. Following is the list of tests and methods of evaluation used to determine the effectiveness of the mathematics course:

1. SAT: Stanford Achievement Test
2. CAT: California Achievement Tests
3. Cooperative: Cooperative Arithmetic and Mathematics Tests
4. Functional: Functional evaluation--mathematics teacher observation of student behavior
5. IEPT: Iowa Every Pupil Tests
6. Iowa: Iowa Test--Educational development record
7. MAT: Metropolitan Achievement Tests--arithmetic
8. Teacher: Local teacher constructed
9. Textbook: Tests designed by textbook authors
10. Standard: Standardized tests (unspecified)
11. Other: Other tests (unspecified)
12. CMMT: California Mental Maturity Test
13. S-RT: Sangren-Reidy Test
14. H and N Tests
15. Breukner: Breukner Diagnostic Test
16. Student: Student designed tests
17. Self: Pupil self-evaluation

There were no outstanding differences in the evaluation program with respect to size of schools, type of schools, teacher training, or teacher experience.

Tabular analysis of methods of evaluation with respect to grades is found in Table 29.

Table 29. Tests and methods of evaluation used to determine effectiveness of course in mathematics.

Grade	Seventh	Eighth	Ninth General Math.	Ninth Algebra
Teachers Reporting	116	111	58	35
Means of Evaluation:				
1. SAT	3	2	1	0
2. CAT	12	16	4	1
3. Cooperative	0	0	1	0
4. Functional	41	39	22	10
5. IEPT	4	1	0	0
6. Iowa	1	1	0	0
7. MAT	1	1	0	0
8. Teacher	32	36	11	11
9. Textbook	29	34	6	6
10. Standard	17	24	11	8
11. Other	48	42	33	16
12. CMMT	0	1	1	0
13. S-RT	0	1	0	0
14. H and N Tests	2	2	0	0
15. Breukner	1	1	0	0
16. Student	1	2	2	0
17. Self	2	4	1	0

Changes in mathematics program during past five years.

Following is a list of changes reported to have taken place in the mathematics program during the past five years:

1. Over-crowding of classrooms resulting in less individual help available for students
2. More ability grouping in mathematics
3. Greater emphasis on application of mathematics to changing living conditions
4. Greater emphasis on individual needs and guidance
5. Greater emphasis on the need for and importance of mathematics
6. Greater emphasis on "meaning" in mathematics
7. Increased use of teaching aids, less "text-teaching"
8. Poorer preparation of students necessitating more need for review of fundamentals
9. Less emphasis on memorization
10. More emphasis on reasoning ability and thought problems
11. More short-cut methods
12. Better textbooks
13. More mental and oral arithmetic
14. Greater variation in methods
15. Increase in mathematics teachers teaching mathematics only
16. More pupil participation in discussion and instruction
17. Increased teacher load
18. Smaller mathematics budget
19. Increased interest in mathematics
20. Greater emphasis on fundamentals of mathematics
21. Decreased requirements for passing

22. Enlarged mathematics offering

23. More mathematics required

24. Recognition of need for remedial work, remedial classes established

25. More acceleration and challenge to better students

26. Algebra changed from tenth to ninth grade for better students

27. Better selection and guidance of students to enroll in algebra

28. More emphasis on effective reading to understand mathematics

The frequency of the above changes is tabulated in Table 30. The number entered under "changes" refers to the change bearing the corresponding number in the above list.

Only 148 of the 320 questionnaires tabulated listed any changes during the past five years.

The following changes were listed by about ten percent, or more, of those indicating changes: Greater emphasis on application of mathematics to changing living conditions, greater emphasis on individual needs and guidance, increased use of teaching aids, and better textbooks.

Table 30. Changes in mathematics program during past five years.

Changes	High School	Jr. High	Elementary	Total
1	1	5	0	6
2	0	1	0	1
3	11	19	11	41
4	5	12	6	23
5	2	3	2	7
6	0	9	2	11
7	4	14	5	23
8	1	5	0	6
9	0	3	1	4
10	3	6	0	9
11	0	3	0	3
12	3	5	7	15
13	0	3	0	3
14	1	1	5	7
15	0	5	0	5
16	1	6	2	9
17	0	3	0	3
18	0	2	0	2
19	2	3	3	8
20	1	1	2	4
21	1	1	0	2
22	1	5	0	6
23	2	2	0	4
24	1	0	3	4
25	0	1	2	3
26	2	1	0	3
27	1	3	0	4
28	2	0	0	2

THE CALIFORNIA STUDY

The "October Report of High School Principal, 1954" is a questionnaire which was distributed by the California State Department of Education to all secondary schools of California. Each school was required to file a copy of the report with the state department of education by the end of October, 1954.

"Part IV--Description of Particular Course in Mathematics", the portion of the above report with which this study is concerned, was analyzed during the summer of 1955.

Analysis of mathematics course

Title of course. Exact titles varied, but the following classification is relatively accurate.

1. Seventh Grade General Mathematics. The emphasis of this course was on arithmetic.

2. Eighth Grade General Mathematics. The emphasis of this course was on arithmetic. There was some attention paid to concepts of geometry and algebra.

3. Remedial Mathematics. This course was general, in nature of topics covered, but its emphasis was on remedial activity--working with individuals to correct the difficulties they were having with mathematics, especially the fundamental skills of arithmetic. The larger schools showed a tendency to have a greater number of remedial offerings than did the smaller schools.

Objectives. Following is a list showing the classification of

objectives as used in the California study. Table 31 shows the tabulation of these objectives as reported in the questionnaire.

1. Need: To assist each pupil to recognize the need for and the importance of mathematical skills

2. Basic Skills: To assist each pupil to develop, improve, and maintain basic mathematical skills and concepts, including problem solving techniques. This grouping includes: fundamental processes, quantitative processes, the four basic operations, counting, standard units of measure, simple measurement, whole numbers, fractions, decimals, percentage, etc.

3. Application: To assist each pupil to apply basic mathematical skills and concepts in class and in actual life problems and to provide experiences which will assure ability to apply these skills

4. Desire: To stimulate within each pupil an interest in mathematics and the desire to learn and apply mathematical skills

5. Thought Problems: To assist each pupil to develop the ability to analyze and work thought problems

6. Self-realization: To assist each pupil with respect to personal evaluation, self-realization, and improvement regarding mathematics

7. Advanced: To provide each pupil with growth in more complex mathematical concepts, spacial relationships, and geometry

8. Mind and Character: To assist each pupil toward general improvement of mind and character. This grouping includes: personal improvement of working habits; techniques of thinking clearly, precisely, independently, and in an organized manner; problem solving techniques; ability to analyze and make sound judgments based on factual data; capacity for abstract reasoning; ability to recognize and interpret mathematical situations; ability to follow directions; and such improvement of mind and character

9. Preparation: To provide a means of preparation for more advanced mathematics. This grouping includes: preparation for algebra and other more advanced high school mathematics and for college

10. Interest: To develop within each pupil a genuine interest in mathematics. This grouping includes:

Table 31. Objectives of seventh and eighth grade mathematics, California.

	Seventh Grade					Eighth Grade				
Size of School	1-300	301-850	above 850	Total	%	1-300	301-850	above 850	Total	%
Number of Schools	27	69	155	251		27	66	154	247	
Objectives										
1. Need	2	7	22	31	12.4	8	15	33	56	22.7
2. Basic Skills	27	67	154	248	98.8	27	66	154	247	100.0
3. Application	19	39	87	145	57.8	21	43	101	167	67.6
4. Desire	2	3	4	9	3.6	2	1	4	7	2.8
5. Thought Problems	5	13	21	39	15.5	4	6	13	23	9.3
6. Self-realization	1	5	2	8	3.2	0	2	4	6	2.4
7. Advanced	2	5	8	15	6.0	7	8	28	43	17.4
8. Mind and Character	5	27	32	64	25.5	7	27	49	83	33.6
9. Preparation	1	2	6	9	3.6	1	5	10	16	6.5
10. Interest	0	2	5	7	2.8	1	3	3	7	2.8
11. Thought tool	1	5	0	6	2.4	1	2	10	13	5.3
12. Correlation	2	14	35	51	20.3	4	15	35	54	21.9
13. History	0	1	0	1	0.4	0	0	0	0	0.0
14. Remedial	0	0	3	3	1.2	0	0	1	1	0.4

enjoyment of mathematics; overcome frustration, fear, and hatred for mathematics, enjoyment of a sense of accomplishment

11. Thought tool: To develop within each pupil an appreciation of numbers as a tool for organized thinking

12. Correlation: To assist each pupil to develop skill in other tool subjects: reading, writing, communication, and mathematical literacy

13. History: History and ancient applications of mathematics

14. Remedial: To provide individual adjustment for those who are handicapped through absences, and slow learning

Divisions of course content. Following is the classification of the divisions of course content used in the California study. Table 32 shows the tabulation of the data.

1. Whole numbers: Basic operations with whole numbers

2. Fractions: Basic operations with common fractions

3. Decimals: Basic operations with decimal fractions

4. Percentage

5. Denominate number: (measures of time, weight, capacity, distance)

6. Geometry: Simple geometry (perimeters, areas, volumes, terms used)

7. Application: Applications to life problems (home, business, government, transportation, communication, housing problems, interest, banking)

8. Simple graphs

9. Metric system

10. Conversions

11. Using tools: Use of tools for constructions and measurements

12. Vocabulary: Vocabulary of arithmetic

13. Problems: Problem solving and analysis (word

problems, equations)

14. Indirect measurement: ratio, proportion, Pythagorean relation

15. Square root

16. Algebra: Introduction to algebra (use of formulas, signed numbers, symbols)

17. Correlation: Correlation with other subjects

18. Drills: Drills for neatness, accuracy, and speed

19. Estimation: Estimation, approximate number, short cuts, aids

20. Drawings: Scale drawings

21. Remedial: Attention to individual needs, remedial activities

22. Appreciation: Appreciation activities

23. Review: Review previous years' work, general review

24. Numbers: Number systems, comparing numbers

Experiences and materials of instruction designed to achieve objectives. The classification of experiences and materials of instruction which were designed to achieve objectives of the mathematics course follow. Table 33 shows the tabulation of the data.

1. Life Situations: Activities related to real life problems

2. Audio-visual aids: Special audio-visual aids, such as motion pictures, slides, films, flannel board, etc.

3. Blackboard: Blackboard work by students

4. Bulletin board: student maintained

5. Checking: methods of, and correction of errors

6. Discussions

7. Assignments: Regular assignments

8. Drills: Drills for speed, accuracy, fundamentals, vocabulary

Table 32. Divisions of course content, seventh and eighth grades, California.

	Seventh Grade	Eighth Grade
Schools Reporting	251	247
Content		
1. Whole numbers	216	141
2. Fractions	232	151
3. Decimals	235	155
4. Percentage	132	232
5. Denominate number	152	161
6. Geometry	208	230
7. Application	160	176
8. Simple graphs	164	170
9. Metric system	5	55
10. Conversions	5	2
11. Using tools	40	45
12. Vocabulary	3	1
13. Problems	19	19
14. Indirect measurement	19	90
15. Square root	1	82
16. Algebra	30	178
17. Correlation	7	5
18. Drills	26	27
19. Estimation	13	27
20. Drawings	1	34
21. Remedial	1	1
22. Appreciation	0	1
23. Review	39	88
24. Numbers	25	2

9. Field trips

10. Forming rules: Forming general rules from specifics

11. Games: Games, puzzles, contests, and other activities emphasizing the enjoyment of mathematics

12. Graphs: Graphical representation of mathematics concepts, actual conditions, charts, constructions

13. Homework

14. Correlation: Integration and correlative activities

15. Interview: Interview older students and others to discover need for mathematics

16. Laboratory: Laboratory experiences--drawing and measuring with tools, etc.

17. Notebooks

18. Communication: Oral communication of mathematics problems and concepts, recitation

19. Outside speakers: Presentations by businessmen, skilled workmen, parents, and other outside speakers

20. Problem solving: Problem solving, actual and simulated--oral and written

21. Research: extended study in related areas

22. Scrapbooks

23. Student helpers: Better students helping slower students

24. Supervised study

25. Teacher Presentations: such as, lectures, demonstrations, interest stories

26. Testing: Testing--for diagnoses, achievement; as a learning process

27. Textbooks: Use of textbooks for understanding, problem solving, and exercises

28. Workbooks: Use of workbooks, drillbooks, worksheets

29. Projects: Pupil reports, demonstrations, projects

30. Review: Review activities

Reasons for enrollment. All seventh and eighth grade students are required to take mathematics. A few schools require only one semester of eighth grade mathematics.

In a few schools, where evaluation indicates the need for it, some students are required to enroll in a remedial class. Twenty-four schools reported remedial classes in their mathematics curriculum; twenty were in schools above 850 population, thirteen were located in Los Angeles City, twenty-one were located within Los Angeles County.

Table 33. Experiences and materials of instruction designed to achieve objectives, California.

	Seventh Grade	Eighth Grade
Number of Schools Reporting	251	247
Experiences		
1. Life situations	154	152
2. Blackboard	69	86
3. Audio-visual aids	19	16
4. Bulletin board	2	5
5. Checking	8	6
6. Discussions	29	16
7. Assignments	6	8
8. Drills	113	101
9. Field trips	5	26
10. Forming rules	1	1
11. Games	29	23
12. Graphs	55	54
13. Homework	20	6
14. Correlation	32	33
15. Interview	0	1
16. Laboratory	47	59
17. Notebooks	3	3
18. Communication	16	12
19. Outside	1	16
20. Problem solving	107	99
21. Research	2	7
22. Scrapbooks	3	13
23. Student helpers	1	2
24. Supervised study	10	2
25. Teacher Presentations	39	41
26. Testing	59	49
27. Textbooks	21	10
28. Workbooks	10	7
29. Projects	29	9
30. Review	17	9

Means used to determine who should enroll in remedial classes are tabulated in Table 34.

Analysis of Remedial Classes. Objectives, course content, experiences, methods, and evaluation were approximately the same for remedial classes and for regular classes. The greatest differences

Table 34. Means used to determine who should enroll in remedial classes, California (twenty-four schools reporting).

Means	Number of responses
Counseling	17
Placement tests	12
Prognosis tests	3
Diagnostic tests	11
Prior record	18
Election	6
Other	6

were in the following areas: More emphasis was placed upon enjoyment of mathematics and creating an interest in mathematics in the remedial classes. Also there was a difference in the reasons for enrollment as previously indicated.

Methods used to develop ability to apply reading skills to mathematics. The classification of methods used to develop the ability of the student to apply reading skills to mathematics is listed below. Table 35 is a tabulation of the data.

1. Vocabulary: Vocabulary study and spelling
2. Word problems: Word problems--read, write, discuss, analyze, and solve
3. Create problems: Have students create original word problems
4. Instruction: Direct teacher instruction, motivation, direction
5. Suitable books: Use books most suited to student's reading ability
6. Restate: Have students read written work or problems, then restate in own words
7. Reports: Student reports, oral and written
8. Questioning: Use of the question

9. Tests: Tests and evaluations

10. Translation: Exercises in translating symbols into words, words into symbols, words into ideas

11. Drills: Drills in copying or writing rules and problems

12. Mental pictures: Training to form mental picture of problems read

13. Matching: Matching exercises

14. Games

15. Comprehension: Use of drills and exercises in reading comprehension, following instructions, understanding the problem

16. Aids: Use of supplementary materials--dittos, supplementary texts, audio-visual aids--used for reading, drills, stimulation and information

17. Picture reading: Picture reading through graphs

18. Student teaching: Student teaching within classroom

19. Research: Classroom study and activities in connection with reports, forms, mathematics concepts used in real life

20. Oral reading: Oral reading, not necessarily mathematical material

Means of determining effectiveness of course. Following is a list of tests and methods of evaluation used in the seventh and eighth grades of California. A tabulation of the data is shown in Table 36.

1. SAT: Stanford Achievement Test

2. CAT: California Achievement Tests

3. CAMT: California Arithmetic and Mathematics Tests

4. Cooperative: Cooperative Arithmetic and Mathematics Tests

5. Functional: Functional evaluation--mathematics teacher observation of student behavior

6. Iowa: Iowa Test--Educational development record

Table 35. Methods used to develop ability to apply reading skills to mathematics, California.

	Seventh Grade	Eighth Grade
Schools Reporting	251	247
Methods		
1. Vocabulary	170	161
2. Word problems	184	172
3. Create problems	20	25
4. Instruction	73	86
5. Suitable books	6	2
6. Restate	25	24
7. Reports	14	23
8. Questioning	8	5
9. Tests	18	20
10. Translation	12	25
11. Drills	4	5
12. Mental pictures	3	8
13. Matching	2	0
14. Games	5	8
15. Comprehension	93	98
16. Aids	24	35
17. Picture reading	20	22
18. Student teaching	2	3
19. Research	60	79
20. Oral reading	29	37

7. MAT: Metropolitan Achievement Tests--Arithmetic

8. OAPT: Orleans Algebra Prognosis Test

9. Teacher: Local Teacher Constructed

10. District: School district staff constructed

11. Miscellaneous: Miscellaneous Arithmetic--Mathematics Tests (Not used by more than one school)

12. Textbook: Tests designed by Textbook authors

13. Standard: Standardized tests (unspecified)

14. Other: Other tests (unspecified)

15. Strathmore: Strathmore tests

16. LADT: Los Angeles Diagnostic Tests

17. W-MAT: Woody-McCall Arithmetic Tests

18. CMMT: California Mental Maturity Test

19. S-RT: Sangren-Reidy Test

20. L-CAFT: Lee-Clark Arithmatic Fundamental Test (See Table 36)

Table 36. Means of determining effectiveness of course, California.

	Seventh Grade	Eighth Grade
Number of Schools Reporting	251	247
Means of evaluation		
1. SAT	38	53
2. CAT	46	52
3. CAMT	28	28
4. Cooperative	2	2
5. Functional	135	142
6. Iowa	2	8
7. MAT	3	4
8. OAPT	2	9
9. Teacher	153	148
10. Districts	43	51
11. Miscellaneous	8	11
12. Textbook	52	43
13. Standard	55	41
14. Other	141	130
15. Strathmore	2	3
16. LADT	21	13
17. W-MAT	3	3
18. CMMT	5	5
19. S-RT	2	0
20. L-CAFT	5	5

Changes in mathematics program during past five years. Following is a list of changes reported to have taken place in the mathematics program during the past five years. Table 37 is a tabulation of data concerning the changes. The number listed in the table under "changes" refers to the change bearing the corresponding number in the following list.

1. More individualized instruction
2. More and better teaching aids: audio-visual aids, supplements for texts, better texts
3. More applications to real life situations
4. Mathematics made more meaningful through direct activity experience, graphical representation of life situations, use of community resources, and field studies resulting in less busy work
5. Better organization of subject matter resulting in such improvements as uniformity, topics arranged orderly and graded according to student ability, curriculum outlines, less repitition
6. More review, special training, or remedial classes and better guidance in that connection
7. More and better student aids, models, workbooks, supplementary texts, better texts
8. Teaching for a broader understanding of mathematics
9. More and better evaluation of students and student progress
10. More ability grouping
11. More emphasis placed on fundamentals of mathematics
12. Attempt to have teachers trained in mathematics teach mathematics
13. Reorganization or coordination of mathematics personnel or mathematics policy in district and in individual schools
14. Greater integration of mathematics and other subjects
15. Increased enrollment in mathematics

16. More awareness of the students for the need for mathematics

17. Poorer preparation of students for mathematics

18. Better records on student's ability, achievements

19. Greater use of puzzles, games, and similar activities to motivate interest in basic mathematics concepts

20. More emphasis placed on student needs and interests

21. Less drill

22. Greater stress of vocabulary and spelling

23. More mathematics required

24. More and better in-service training for teachers

25. No changes given

Table 37. Changes in mathematics program during past five years, California (525 schools reporting).

Changes	Frequency Reported	Changes	Frequency Reported
1	62	14	32
2	142	15	6
3	118	16	12
4	86	17	10
5	76	18	11
6	38	19	10
7	104	20	23
8	36	21	15
9	110	22	16
10	76	23	14
11	65	24	69
12	9	25	105
13	45		

Notable changes in the mathematics program in the junior high schools of California during the five years preceding October, 1954 are (in order of greatest frequency reported): More and better teaching aids, more applications to real life situations, more and better evaluation of students and student progress, more and better

"student" aids to learning, mathematics made more meaningful--less busy work, better organization and grading of subject-matter, more ability grouping, more and better in-service training for teachers, more emphasis placed on fundamentals of mathematics, and more individualized instruction.

SUMMARY AND CONCLUSION

The "October Report of High School Principal, 1954 is a California State Board of Education questionnaire which was sent to all of the secondary schools of the state of California. "Part IV--description of Particular course in Mathematics "was a part of the questionnaire which was concerned with various aspects of the mathematics program in the secondary schools of the state.

The author of this paper was instrumental in classifying and analyzing the data of the above questionnaire as it applied to the mathematics program of the seventh and eighth grades of the junior high schools.

A similar questionnaire, extended in scope to include information concerning the teachers, was sent to all teachers who were teaching mathematics in grades seven, eight, or nine in the schools of Utah.

In so far as the data submitted in the questionnaires represent a true picture of the mathematics porgrams in Utah and California at the junior high school level, the following conclusions may be drawn: With respect to teachers of mathematics at the junior high school level in Utah

1. Over half of the teachers exceed or are at the recommended maximum for the number of sections per day that they must teach. A study including also the number of teachers who have at least one free period per day would reveal more information with respect to teacher load.

2. Larger schools tend to require the teachers to teach fewer sections per day than do smaller schools, but teach more students per class.

3. Sixty-eight percent of the mathematics classes have thirty or more students per class. Eighty-three percent of the classes in the junior high schools have thirty or more students per class. Forty-one percent of the junior high schools have thirty-five or more students per class. Thus, there is considerable over-crowding in the mathematics classrooms, especially in the junior high schools. It appears that individual instruction and classroom activity would suffer, as a consequence.

4. Twenty-six percent of the mathematics teachers teach mathematics only.

5. Fifty percent of the teachers have taught mathematics for fewer years than their total years of teaching experiences. Fifty percent of all mathematics teachers have been teaching fewer than five years.

6. It appears that larger schools tend to have teachers with more years of college training than do smaller schools. Small elementary schools have the largest number of teachers of mathematics with fewer than four years of college training. Forty-two percent of the teachers report that they have completed five or more years of college.

7. Sixteen percent of the mathematics teachers report that they have advanced degrees, six percent report less than a bachelor's degree.

8. Only eight percent of the teachers of mathematics report

that their undergraduate major was in mathematics; twenty percent listed either a major or minor in mathematics. Twenty-two percent indicated that their undergraduate major was in mathematics, physical science composite, physical science, or secondary education with a teaching major in mathematics.

9. No mathematics teacher reported a graduate major in mathematics; only three listed a graduate minor in mathematics.

10. Thirteen percent of the mathematics teachers reported no college credit in mathematics; sixty percent listed fewer than eighteen quarter-hours of college credit, the usual number of hours required for a minor in mathematics. Ten percent indicated they had received forty-two or more quarter-hours of college credit in mathematics; the approximate number recommended as the minimum preparation for teachers of secondary mathematics.

11. There is little tendency for mathematics teachers to enroll in mathematics classes during their graduate program of study.

12. Administrators, in general, do not require training in mathematics as a prerequisite for the teaching of mathematics, probably because of the shortage of teachers trained in mathematics.

13. Policies and practices with respect to training and employment of teachers who may be required to teach mathematics at the junior high school level appear to be in need of evaluation and of revision. It is recommended that secondary school administrators and representatives of teacher training institutions unitedly evaluate the need for mathematics teachers, what training is deemed desirable, and what action should be taken to insure an adequate supply of properly trained teachers of mathematics at the junior high

school level.

14. The most outstanding deviation of the mathematics program from what is recognized by authorities of mathematics education as being satisfactory appears to be in the area of teacher training. More study, of a controlled nature, should be done in this area to ascertain the consequences of this deviation.

15. There is no comparable data on Utah and California schools in this section.

With respect to course in mathematics in Utah and California

1. In Utah, the major emphasis with respect to objectives is in connection with basic mathematical skills and applications. In the eighth grade, as compared to the seventh, there is a slight increase in emphasis upon applications and upon preparation for higher mathematics; there is a slight decrease in emphasis upon basic skills and improvement of mind. In California, there is a slight increase in emphasis upon improvement of mind in the eighth grade, as compared to the seventh. There is greater emphasis in California, as compared to Utah, upon correlation, within the mathematics class, with other tool subjects such as reading and writing.

2. In Utah, ninth grade general mathematics shows a shift of emphasis toward applications of mathematical skills, as compared to the eighth grade. In the teaching of Algebra there is decreased emphasis on applications, increased emphasis on improvement of mind and preparation for higher mathematics.

3. In Utah, the divisions of course content are similar for seventh and eighth grades. In the eighth grade, as compared to the seventh, there is decreased emphasis upon fractions, decimals,

denominate number and graphs; there is increased emphasis upon introduction to algebraic concepts. In California, comparing course content in the seventh and eighth grades, the eighth grade has less emphasis upon percentage, the metric system, indirect measurement, algebra, and scale drawings. Notable differences in the Utah and California schools exist with respect to emphasis upon certain divisions of course content. In the seventh grade California schools place greater emphasis upon the basic operations with whole numbers, simple geometry, graphs, and the use of tools for constructions and measurement. In the eighth grade California schools place greater emphasis upon the basic operations with whole numbers, fractions and decimals; percentage; denominate number; graphs; the metric system; and introduction to algebraic concepts.

4. In Utah, in ninth grade general mathematics, the divisions of course content are approximately the same as for the eighth grade except that there is slightly greater emphasis upon applications and basic concepts of algebra and trigonometry.

5. In ninth grade algebra, in Utah, there is little or no emphasis indicated on application of algebra, understanding numbers, vocabulary of algebra, review of fundamentals of arithmetic, and remedial activities. Many authorities consider these items important in an introductory course in algebra. Algebra courses appear to offer greater opportunity than ninth grade general mathematics for increasing the student's power in mathematical skills, but neglect to show areas of application for the increased knowledge and skills.

6. With respect to experiences and materials of instruction which are designed to achieve the objectives of the course in mathematics,

California schools indicate greater emphasis upon activities in connection with real life problems, audio-visual aids, construction and use of graphs, integration and correlation activities, laboratory experiences, and problem solving than do the Utah Schools. California schools place less emphasis upon daily or regular assignments, homework, and the use of textbooks.

7. Utah schools appear to be weak in the use of experiences and activities that stimulate interest in and increase understanding of mathematics, such as laboratory experiences, correlation activities, presentations by outside speakers, and creation of scrapbooks. It is indicated that homework and the use of textbooks are the most common experiences that students encounter in the mathematics classes at the junior high school level in Utah.

8. All students in Utah and California are required to enroll in mathematics courses in the seventh and eighth grades.

9. There is greater emphasis in California, especially in the Los Angeles area, than in Utah upon having remedial classes.

10. Objectives, course content, experiences, methods and evaluation were approximately the same for remedial classes as for regular classes, except in the remedial class greater emphasis was placed upon the enjoyment of mathematics and upon creating an interest in mathematics.

11. California schools pay more attention to applying reading skills and skills learned in other courses to mathematics than do Utah schools.

12. In Utah, there is an indication that teachers with greater mathematics training pay more attention to methods of correlating

mathematics with other subjects. This study reveals no other notable differences in objectives or methods of teachers according to teaching experience or mathematics training. A more detailed, better controlled study in this area would probably be of value.

13. California schools put greater emphasis upon evaluation than Utah schools, especially with respect to the use of standardized tests.

14. In Utah, prominent changes indicated in the mathematics program at the junior high school level during the past five years were: greater emphasis upon application of mathematics to changing living conditions and upon individual needs and guidance, increased use of teaching aids, and better textbooks.

15. Notable changes in the mathematics program in the junior high schools of California during the five years preceding October 1954 were listed as: more and better teaching aids, more applications to real life situations, more and better evaluation of students and student progress, more and better "student" aids to learning, mathematics made more meaningful--less busy work, better organization and grading of subject-matter, more ability grouping, more and better in-service training for teachers, more emphasis placed upon fundamentals of mathematics, and more individualized instruction.

16. It appears that the junior high schools of Utah are a little more "traditional" in their mathematics program than the schools of California. Schools of California place a little more emphasis upon applications of mathematics to life situations, correlation with other subjects, use of teaching aids, laboratory methods of instruction, and student activity to create interest in and understanding of mathematics. Utah schools place greater emphasis upon daily or regular assignments,

homework, and the use of the textbook.

17. Comparing the mathematics program at the junior high school level, in Utah, with what is recommended by authorities, it appears that the program is relatively strong with respect to course offerings, course content, and mathematics requirements. It appears to be weak in the following areas: The number of students per class exceeds the recommended maximum in some schools, especially in the junior high schools. Teaching aids, correlation activities, laboratory methods, and student activities that enhance interest in and understanding of mathematics do not appear to be used as generally as is recommended. Evaluation practices appear to be less adequate than is recommended. The academic training of teachers, with respect to mathematics, is far short of what is recommended.

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APPENDIX

Table 1. OCTOBER REPORT OF HIGH SCHOOL PRINCIPAL, 1954 (California)
Part IV--Description of Particular Course in Mathematics.

1. Title of course _____

2. List three major objectives of the course.

3. List the major divisions of the course content.

4. List a few of the major experiences that are designed to achieve the objectives of your mathematics course through its content.

5. Check the appropriate reason(s) why students enroll in this course.

_____Required of all _____Required of some _____Elective

A. If answer to item 5 is "required of some," explain what students are required to take this course.

B. Which of the following means are used to determine whether or not students need to, or are qualified to, enroll in this course?

_____Counseling
_____Placement tests
_____Prognostic tests
_____Diagnostic tests
_____Prior record
_____Other

6. List a few methods by which you help students to develop their ability to apply reading skills to mathematics.

7. By what means do you determine the effectiveness of this mathematics course? (Include standardized tests regularly used.)

8. List a few of the most significant changes that have taken place during the last five years of your mathematics program.

Table 2. Questionnaire to Utah Teachers, Part I.

INSTRUCTIONS FOR PREPARING THE QUESTIONNAIRE

1. Fill out "Part I".

2. Fill out one copy of "Part II" for each distinct course you teach. For example, one copy for each of the following: "Ninth Grade Arithmetic", Ninth Grade Remedial Mathematics", "Ninth Grade Algebra", "Eighth Grade General Mathematics", etc.

3. Please make your responses inclusive but brief, and as accurate as possible.

Part I--TEACHER INFORMATION

Teaching load

1. Number of sections (classes) you teach per day . . _____

2. Number of sections of mathematics you teach per day. _____

3. Average number of students per class (mathematics only)_____

Training and experience

4. Number of years of teaching experience . . . . _____

5. Number of years teaching mathematics . . . . _____

6. Number of years of college completed . . . . _____

Degrees granted _____

7. Undergraduate major_____ Graduate major_____

8. Undergraduate minor_____ Graduate minor_____

9. Approximate number of quarter-hours of credit received in mathematics (Multiply semester-hours of credit by $1\frac{1}{2}$)

Undergraduate . . . . _____

Graduate . . . . . _____

Table 3. Questionnaire to Utah Teachers, Part II.

Part II--DESCRIPTION OF PARTICULAR COURSE IN MATHEMATICS

(Please use additional pages or reverse side if necessary)

1. Title of course (use separate sheet for each distinct course)

   _____

   Number of sections that you teach per day with this title_____

   Total number of students in this (these) class(es) _____

2. List three major objectives of this course.

3. List the major divisions of the course content.

4. List a few of the major experiences and resource materials that are designed to achieve the objectives of this course. (i.e. daily homework, games, scrapbooks, field trips, use of texts--name texts used--etc.)

5. A. Check appropriate reason(s) why students enroll in this course.

   required of all ( ); required of some ( ); elective ( )

   B. If this course is "required of some", what means are used to determine which students are required to enroll in this course?

6. A. List any methods by which you help students to develop their ability to apply reading skills to mathematics.

   B. List any methods by which you help students develop their ability to apply skills learned in other courses to mathematics.

7. By what means do you determine the effectiveness of this mathematics course? (Include standardized tests regularly used.)

8. List the most significant changes that have taken place during the last five years in your mathematics program.

Table 4. Analysis of schools responding to Utah Questionnaire.

Size of Schools	Number of Schools	Number of Schools Reporting	Percent of Schools Reporting
Junior High School			
1-300	23	15	65
301-850	32	28	88
above 850	13	12	92
	68	55	81
High Schools			
1-200	17	13	76
201-500	21	18	86
above 500	6	6	100
	44	37	84
Elementary Schools			
1-35	22	12	55
36-150	22	8	36
above 150	19	13	68
	63	33	52

Table 5. Analysis of student population represented in response to Utah questionnaire.

Size of Schools	Population of Grades 7, 8, 9	Population Represented	Percentage of Population Represented
Junior High School			
1-300	2815	1467	52
301-850	6251	7489	46
above 850	13624	6141	45
	32690	15097	46
High Schools			
1-200	1181	584	49
201-500	2921	1128	39
above 500	1501	974	65
	5603	2686	48
Elementary Schools			
1-35	130	61	47
36-150	494	216	44
above 150	1553	706	45
	2177	983	45

Table 6. Analysis of teachers responding to Utah questionnaire.

Size of Schools	Number of Teachers	Number of Teachers Reporting	Percentage of Teachers Reporting
Junior High School			
1-300	46	23	50
301-850	140	72	51
above 850	87	48	55
	273	143	52
High Schools			
1-200	35	16	46
201-500	55	30	55
above 500	17	10	59
	107	56	52
Elementary Schools			
1-35	22	12	55
36-150	24	9	38
above 150	32	14	44
	78	35	45