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A Formula to Express Evapotranspiration as a Function of Soil Moisture and Evaporative Demands of the Atmosphere

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A FORMULA TO EXPRESS EVAPOTRANSPIRATION AS A FUNCTION OF SOIL MOISTURE AND EVAPORATIVE DEMANDS OF THE ATMOSPHERE

by

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ABSTRACT

A Formula to Express Evapotranspiration as a Function
Of Soil Moisture and Evaporative Demands
Of the Atmosphere

by

Aldo L. Norero, Doctor of Philosophy

Major Professor: Dr. Gaylen L. Ashcroft
Department: Soils and Meteorology

A mathematical expression was developed and tested which describes the relation between evapotranspiration and soil moisture. A general premise of this mathematical model is that the evapotranspiration-soil moisture relationship is determined by interaction of climatic, soil and plant factors.

The basic model is

$$\frac{dET_a}{d\Psi_s} = - k\varepsilon[1 - (ET_a/ET_{mx})]$$

in which ET_a is the actual evapotranspiration, Ψ_s is the total soil water potential, k is a proportionality coefficient, ε is the soil moisture extraction capacity of the atmosphere, and ET_{mx} is the evapotranspiration that would occur from a particular crop-soil unit when soil moisture was not limiting. From this model the following expression was derived:

$$ET_a = \left(\frac{1}{1 + \left(\frac{\Psi_s}{\Psi'_s}\right)^{\left[\frac{2.56}{\log (\Psi_{mi}/\Psi_{mx})}\right]}} \right) g\ E_o$$

where Ψ_{mx}, Ψ'_s, Ψ_{mi} are the soil potentials at which ET_a is equal to 95%, 50% and 5% of ET_{mx}, respectively; E_o is the evaporation from a free water surface and expressed the evaporative demands of the atmosphere. The term g is a proportionality coefficient equal to ET_{mx}/E_o.

A second formula was developed that expresses the same relationship in terms of soil water content, and was derived from the former by assuming a hyperbolic relationship between soil water potential and water content. These formulas, as well as various other models which are described in the literature, were tested using experimental data covering a wide range of climatic, soil and plant variables.

It was concluded that: (a) Most models advocated in the literature are only adequate to describe the relation between evapotranspiration and soil moisture under particular climatic, soil and plant conditions. (b) The formulas derived from the proposed model provide a good fit for the evapotranspiration-soil moisture relationship under widely different circumstances. If proper values are chosen for the coefficients, these formulas yield relations that are similar to several of the models taken from the literature. Consequently, the proposed mathematical expression appears to be a general model of the manner in which plants use soil water under different vegetative and environmental conditions. (c) It seems possible to predict in a comparative way the pattern of soil water utilization in a soil-plant-atmosphere system. This may be done from a knowledge of the relations between the coefficients of the formulas and climatic, soil and plant factors influencing evapotranspiration.

(129 pages)

INTRODUCTION

Water use by crops has been studied extensively under plentiful soil moisture conditions. In contrast, fewer investigations have dealt with water consumption by plants when soil moisture is limiting. Nevertheless, it is this latter condition that prevails much of the time.

Many formulas have been developed in which weather data are used to predict evaporation from well watered crops. These formulas have yielded good estimates in a seasonal and regional basis, but have been inadequate for short term estimates of water use under localized conditions. It is reasonable that most of these formulas are of limited application since they take little or no account of the soil moisture or the nature and condition of the plants. The soil, the plant and the atmosphere form a single system for the movement of water. Thus, evaporation of water from plants is the result of interactions of the three components of the system, and it cannot be characterized by any single component.

Better decisions will be possible in practical plant-water problems when the quantitative limits imposed by these three components of the single system are better understood and elucidated. Working mathematical models will play an important role in achieving this.

The purpose of this thesis study is to develop a general mathematical expression that would describe the relation between evapotranspiration and soil moisture under various climatic, soil and plant conditions.

REVIEW OF LITERATURE

Relations Between Soil Moisture and Plant Transpiration

Previous concepts

In the past more emphasis has been placed on individual elements of the soil-plant-atmosphere system than on the system as a whole.

Early investigators (Lawes, 1850) focused attention on the plant as the principal factor in water consumption. It was thought that production of a given amount of dry matter consumed a fixed amount of water. The "transpiration ratio" and the "water requirements" are terms that exemplify this concept (Briggs and Shantz, 1913).

Later, the soil was considered to be the major factor governing water use by plants. Knowledge about the energetics of soil water (Edlefsen, 1941) strengthened the idea of an "available soil moisture range" (Veihmeyer and Hendrickson, 1927). This concept is based on the assumption that plants can use only the water held in the soil between two arbitrarily defined energy limits--"field capacity" (FC) and the "permanent wilting point" (PWP). Field capacity is a property of the soil and corresponds to an energy status of 0.1-0.3 bars. The term PWP implies a plant phenomenon; however, it was held that practically all crops wilted permanently and could extract no more water when the soil moisture tension reached a prescribed limit, often 15 bars (Briggs and Shantz, 1912).

In later years, the climate has received considerable attention as the factor exerting major control over the evaporation of water from plant communities. This approach has yielded a large number of prediction formulas for water consumption using climatic data and based on both experimental and theoretical studies (A. S. C. E., 1966; A. S. A. E., 1966). Most of these formulas, however, are only applicable to particular soil moisture and plant conditions. These restrictions are specified in the term "potential transpiration." This was defined as "the rate of evaporation from an extended surface of short green crop actively growing completely shading the ground of uniform height, and not short of water" (Penman, 1956).

Present concept

During the last two decades several authors have pointed out that interactions among plant, soil and meteorological factors are important in controlling the actual evapotranspiration-soil moisture relationship. Relevant factors and conditions that are cited as important are: evaporative demand of the atmosphere, sensitivity of stomata, availability of soil water (Slatyer, 1957), hydraulic conductivity of the soil and density of root systems (Hagen et al., 1959), depth of rooting and the moisture retaining characteristics of the soil (Ashcroft and Taylor, 1953), root-leaf ratios (Parker, 1949), relative magnitude of the resistances to vapor flow through cuticle and stomata, and the effects of incipient drying on resistances to water movement through the plant (Rutler and Sands, 1958), diffusion resistance of leaves, nature of the canopy (Gates and Hanks, 1967), and previous evaporation history (Weatherley, 1951).

In recent literature evapotranspiration is considered as a dynamic process operating in a soil-plant-atmosphere continuum (van der Honert, 1948). This idea is depicted in Figure 1. The role of leaf water potential, Ψ_L, and the stomatal apparatus in regulating water flow through the soil-plant-atmosphere system is emphasized.

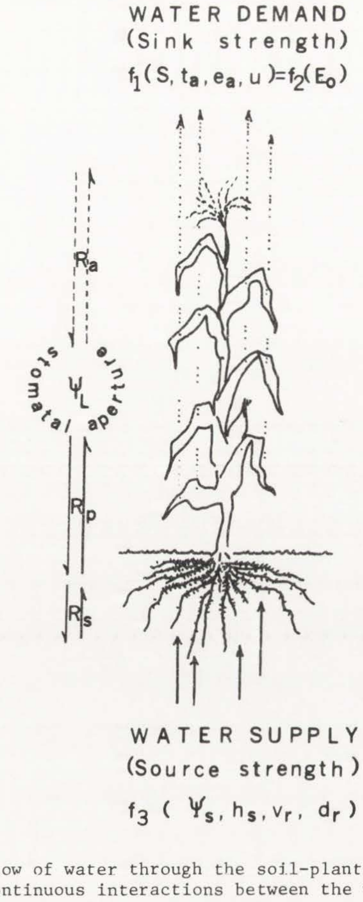

Figure 1. Flow of water through the soil-plant-atmosphere continuum. Continuous interactions between the water supplying power of the soil and the water demands of the atmosphere determine the water potential in the leaves. This, in turn, controls stomatal aperture which governs the water losses from plants. S, radiant energy; t_a, air temperature; e_a, vapor pressure of the air; u, wind; E_o, evaporation intensity from a free water surface; R_a, R_p, R_s, aerodynamic plant and soil resistances to the flow of water, respectively; Ψ_s, soil water potential; h_s, capillary conductivity; V_r, root volume; and d_r, root density. Solid and dotted arrows indicate liquid and vapor flows, respectively. Half arrows mean interactions.

According to Slatyer (1967), the value of Ψ_L is determined by the continuous interaction of a source strength (consisting of the soil water potential and the flow capacity to the roots) and a sink strength (in the form of a potential transpiration rate).

Thus, the main factors determining actual evaporation are: soil water potential (Ψ_s), hydraulic conductivity of the soil (h), root volume (V_r), root density (D_r), internal resistances in the plant (R_p), the critical level of Ψ_L for stomatal closure (crit Ψ_L), energy available to evaporate water (gE_o) and resistance to vapor flow from the foliage to the atmosphere (R_a).

Models Relating Soil Moisture to Evapotranspiration

Models based on experimental studies

The concept of upper and lower limits to moisture availability is a fundamental premise of nearly all models that are founded on experimental studies. These limits are considered universal characteristics of soils. There is, however, considerable controversy as to the degree of availability of the water held between these limits. Some authors have found transpiration to remain at its maximum until soil water is depleted to the wilting point value (PWP) (Veihmeyer and Hendrickson, 1955; Glover and Forsgate, 1964). In contrast, many other evidences indicate that transpiration is reduced as soil moisture tension increases (Kramer, 1949; Kozlowski, 1949; Blair et al., 1950; Army and Kozlowski, 1951; Bourdeau, 1958).

There are diverse opinions as to the functional relation between soil moisture and transpiration. Figure 2 shows a number of models that have been advocated. These are: (a) equal availability in the whole

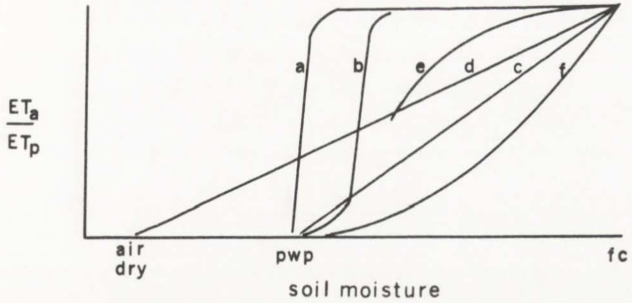

Figure 2. Models proposed to describe the relation between evapotranspiration and soil moisture. a, Veihmeyer and Hendrickson (1955); b, Penman (1949); c, Thornthwaite and Mathers (1955); d, Havens (1956); e, Pierce (1958); and f, Bahrani and Taylor (1961). ET_a, actual evapotranspiration; ET_p, potential evapotranspiration.

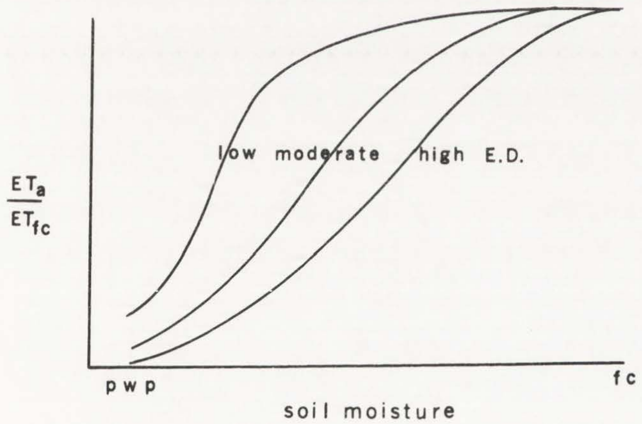

Figure 3. Model proposed by Shaw (1963) to describe the relation between evapotranspiration and soil moisture, as influenced by the evaporative demands of the atmosphere, E.D. ET_a, actual transpiration; ET_{fc}, evapotranspiration at field capacity.

range of soil moisture between field capacity and permanent wilting point, (Veihmeyer and Hendrickson, 1955; Glover and Forsgate, 1964); (b) equal availability in three fourths of the "available water range," and greatly reduced at the end of the range (Penman, 1949); (c) linear decrease in availability between field capacity and permanent wilting point (Thornthwaite and Mather, 1955; Wu, 1967; Halstead, 1954); (d) linear decrease in availability between field capacity and the air-dry value of soil moisture (Havens, 1956); and (e) a gradual and curvilinear decrease in relative evapotranspiration (Pierce, 1958; Eagleman and Decker, 1965; West and Perkman, 1953; Butler and Prescott, 1955; Knoerr, 1961).

The experiments of Denmead and Shaw (1962), Holmes and Robertson (1963) and of Zahner (1967) have partially resolved the contraversy illustrated in Figure 2. They have demonstrated how different evaporation intensities of the atmosphere and the physical properties of soils interact to control the relation between soil moisture and evapotranspiration. Shaw (1963) includes the influence of the evaporative demands of the environment in his model (Figure 3). Other studies that substantiate Shaw's model were conducted by Scholte-Ubing (1961); Holmes and Robertson (1963); Makkink and van Heemst (1956), and Closs (1958). In their models, which are basically similar, Holmes (1961) and Zahner (1967) include the influence of soil types (Figures 4 and 5).

In all models, except Havens' (Figure 2, curve d) and in certain cases Shaw's (Figure 3), the 15 bar tension value is considered the end point of transpiration. In all models the field capacity value is considered the moisture level at which uninhibited transpiration occurs.

Models based on theory

More basic and quantitative formulations than the aforementioned

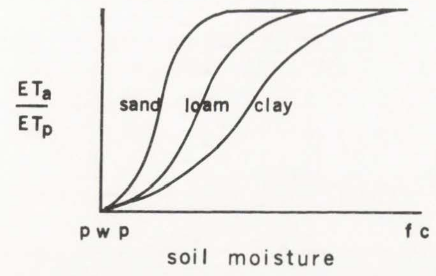

Figure 4. Model proposed by Holmes (1961) to describe the relation between evapotranspiration and soil moisture as influenced by soil types.

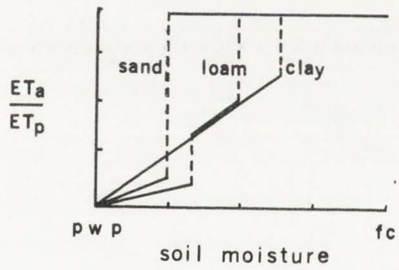

Figure 5. Model proposed by Zahner (1967) to describe the relation between evapotranspiration and soil moisture as influenced by soil types.

models have been developed from theoretical analysis of the factors involved in the flow of water through the soil-plant-atmosphere system. They are based either on applications of the classical flow equation (Philip, 1966; Gardner, 1960a, Visser, 1965; Cowan, 1965; Hallaire, 1964) or on the thermodynamics of irreversible processes (Sudnitzin, 1968). The essential assumptions of these models are the following: (a) the flow of water from soil to roots is determined by the water potential of the soil mass, the water potential at the root surface, the hydraulic conductivity of the soil and root density; (b) the leaf water potential (Ψ_L) and the resistances to liquid flow in the plant influence stomatal aperture, and hence, the capacity of the plant to transpire at the potential rate; (c) water flow and transpiration are proportional to the difference in water potential between the roots and the leaves, and the internal plant resistances are constant; (d) a critical value for the leaf water potential, crit Ψ_L, exists at which stomata close and reduce transpiration enough to prevent further decline in Ψ_L; (e) at values of Ψ_L larger than crit Ψ_L transpiration proceeds at the potential rate.

These models provide interesting evidence of the interaction between soil, plant and atmosphere factors and do much to harmonize apparently contradictory results. The influence of evaporative demands and of root density on transpiration is illustrated in Figures 6A and 6B after the model of Cowan (1965), and the effect of soil types is shown in Figure 7, after the model of Gardner and Ehlig (1963). Many field and laboratory studies confirm the main conclusions of these models (Makkink and van Heemst, 1956; Lemon et al., 1957; Scholte-Ubing, 1959; Bahrani and Taylor, 1961; Denmead and Shaw, 1962; Holmes and Robertson, 1959).

Some of their assumptions, however, are questionable. First, the

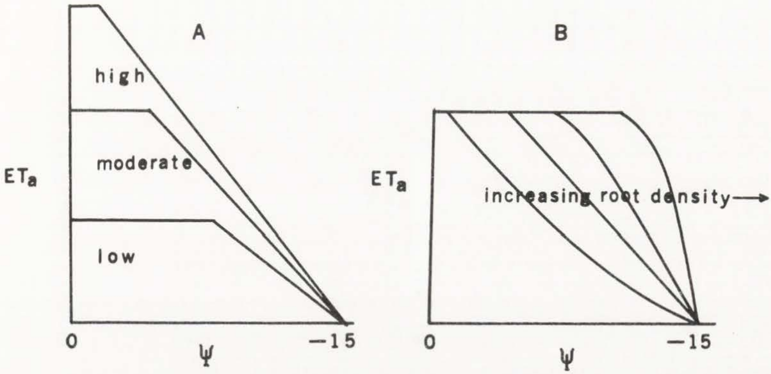

Figure 6. The influence of A, evaporative demands of the atmosphere, and B, of root density on the relation between evapotranspiration, ET_a, and soil water potential, Ψ, as predicted from a theoretical model of Cowan (1965).

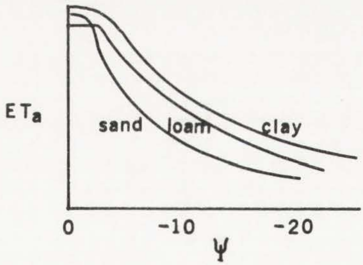

Figure 7. The influence of soil types on the relation between evapotranspiration, ET_a, and soil water potential, Ψ, as predicted from a theoretical model proposed by Gardner and Ehlig (1963).

water evaporated from a wet soil is usually excluded from these models, but unless the crop completely shades the ground the water evaporated from the soil can comprise an important part of the total evapotranspiration (Penman and Long, 1960). Second, transpiration does not always cease at a critical soil moisture tension, e.g. 15 bars in Cowan's model (Figures 6A and 6B); the soil can be dried to much higher tensions by evaporation and also by transpiration, much of which can be cuticular (Satoo and Namura, 1953). Third, the assumption that internal plant resistances remain constant in spite of changes in water stress is probably incorrect (Kramer, 1950; Ordin and Gairon, 1961; Rutler and Sands, 1958). Fourth, external surface resistances probably change as leaves fold or roll in response to water stress (Slatyer, 1967).

The use of theoretical models is largely limited to hypothetical cases because of the difficulties in actually measuring some of the quantities involved in the equations.

Summary

The current status of knowledge regarding the relation between evapotranspiration and soil moisture can be summarized as follows: (a) It is generally agreed that a reduction of soil water leads to reduced rates of evapotranspiration; the magnitude of this depression is variable and is conditioned by weather, crop and soil factors. (b) No universal agreement exists regarding the general relationship among these factors. (c) The most general expressions available to interpret or model the soil-plant-atmosphere system are based on theoretical considerations; the applicability of these expressions, however, is largely limited to hypothetical cases. (d) Though a great deal of understanding is gained by theoretical approaches, a need still exists for practical formulations.

STATEMENT OF THE PROBLEM

Theory

The purpose of this study is to develop and test a general mathematical expression for the relation between evapotranspiration and soil moisture.

Four basic principles guide the development of this expression.

First, when soil moisture is plentiful, evapotranspiration is essentially determined by the amount of energy available to evaporate the water. At this stage, actual evapotranspiration, ET_a, from a soil-crop unit is at maximum, ET_{mx}; this statement is represented mathematically as:

$$ET_a = pET_{mx} = f_1(ED) = gE_o \quad [1]$$

where:

p is a proportionality coefficient, in this case = 1,

ED is the evaporative demand of the environment as measured, for example, in terms of evaporation from a free water surface, E_o,

g is a proportionality constant expressing the ratio ET_{mx}/E_o.

Second, when the soil begins to dry and water is not conducted to the evaporating surfaces fast enough to meet the atmospheric demand, actual evapotranspiration falls behind the maximum rate. At this stage, soil moisture becomes a controlling factor:

$$ET_a = pET_{mx} \quad [2]$$

where:

$p < 1 = f_2(\text{soil moisture})_{\text{crop, soil}}$.

Thus, p expresses the degree of inhibition of ET_{mx} as a function of soil moisture for a given crop-soil unit.

Third, the shape of the drying curve, $f_2(SM)_{c,s}$, will be determined by interactions between the desiccating power of the atmosphere, the nature of the soil and the characteristics of the vegetation.

Fourth, for a given soil and crop unit, the relation between actual evapotranspiration and soil moisture will depend on the capacity of the soil-plant-atmosphere system to conduct water and on the magnitude of the evaporation deficits that the flow produces.

Mathematically, this expression becomes

$$\frac{dET_a}{d\Psi_s} = -k\varepsilon\left(1 - \frac{ET_a}{ET_{mx}}\right) \qquad [3]$$

in which:

$ET_a = E_s + T_{s,c}$ is the actual evapotranspiration;

E_s is the water evaporating directly from the soil;

$T_{s,c}$ is the sum of stomatal and cuticular transpiration from the plant surfaces;

Ψ_s is the total soil moisture potential;

k is a coefficient;

$\varepsilon = \frac{ET_a}{\Psi_s}$ is the soil moisture extraction capacity of the atmosphere,

and

ET_{mx} is the evapotranspiration that would occur from a particular crop-soil unit when $\Psi_s = 0$ and no other conditions are limiting.

The last term of equation [3], $1 - (ET_a/ET_{mx})$, is a relative evapotranspiration deficit. It defines an upper limit to the function, namely $ET_a = ET_{mx}$. When this happens, the relative evapotranspiration deficit is zero, and consequently $dET_a/d\Psi_s = 0$. This means that ET_a no longer is dependent on Ψ_s, but only the ED of the atmosphere.

The term ET_{mx} is considered the maximum possible ET in response to the evaporative demands of the atmosphere. It may differ from "potential evaporation," as defined previously, because it does not involve restrictions in the nature of the vegetative surface and it specifies a soil moisture level.

The variable $\varepsilon = ET_a/\Psi_s$ expresses an instantaneous ratio between a flux term and a potential term and, therefore, is called the "soil moisture extraction capacity of the atmosphere." It embodies soil, plant and atmospheric factors influencing the flow of water. It is equal to the product of an over-all transmissivity term for the soil-plant-atmosphere system and a relative driving force. The transmissivity term comprises a resistance to flow from the soil directly to the atmosphere, and a resistance to flow through the plant to the atmosphere that are linked in parallel. To show how soil, plant and atmospheric factors may interact to determine the value of ε, equation [4] is presented. A more comprehensive analysis of this equation is given in Appendix A.

$$\varepsilon = \left(\frac{V_s}{I_a^e + I_s^e} + \frac{V_p}{I_a^t + I_f^t + \eta\left(\frac{M_p}{M_r}\right)(I_p^t + I_s^t)}\right)\left(\frac{\Psi_{at}}{\Psi_s} - 1\right) \qquad [4]$$

in which:

I is impedance to the flow of water,

e is a superscript which denotes evaporation directly from the soil,

t is a superscript which donates transpiration

a is a subscript to denote boundary layer and turbulent conditions at the soil-atmosphere and plant-atmosphere interfaces, f is a subscript referring to cuticular and stomatal characteristics,

p is a subscript to denote internal plant characteristics influencing flow,

s is a subscript referring to soil,

at is a subscript referring to atmosphere,

V_s is the volume of soil contributing the water,

V_p is the volume of vegetation experiencing transpiration,

M_p is the dry mass of above-ground plant tissues,

M_r is the dry mass of plant roots,

η is a proportionality constant relating the densities of the soil and plant tissues and the mass fraction of the soil occupied by roots.

Clearly, it is the interaction of weather, plant and soil factors that determines the relation between ET_a and Ψ_s.

All impedances indicated in equation [4] change in an unknown manner and magnitude with water stress. Therefore, it is almost impossible to develop a unique relationship between ET_a and Ψ_s from equation [4] unless drastic simplifications are made.

Replacing ε by its identity, ET_a/Ψ_s, the integration of equation [3] yields the following expression:

$$ET_a = \left(\frac{c}{c + \Psi_s^k}\right) ET_{mx} \qquad [5]$$

in which c is a constant of integration. The procedure used to integrate equation [3] is given in Appendix B.

A similar expression is obtained if soil moisture tension, Ψ_s, is substituted by soil moisture content, Θ, defined here as a gravimetric percentage. An equation of fairly wide applicability relating Ψ_s and Θ is the following:

$$\Psi_s = a\Theta^{-b} \qquad [6]$$

in which a and b are empirical constants. Combining equations [5] and [6] gives

$$ET_a = \left(\frac{n}{n + \Theta^{-m}}\right) ET_{mx} \qquad [7]$$

in which

$$n = \frac{c}{a^k}$$

$$-m = -bk$$

The terms in parentheses in equations [5] and [7] correspond to functions of p(SM) in equation [2].

To test whether equations [5] and [7] are adequate to describe actual ET_a data, these equations are converted to logarithmic forms:

$$\log \left(\frac{ET_a}{ET_{mx} - ET_a}\right) = \log c - k \log \Psi_s \qquad [8]$$

and

$$\log \left(\frac{ET_a}{ET_{mx} - ET_a}\right) = \log n + m \log \Theta \qquad [9]$$

If equations [8] and [9] (and consequently, equations [5] and [7]) are adequate models, a plot of log $[ET_a/(ET_{mx} - ET_a)]$ vs log Ψ_s or log Θ should yield a straight line. The value of the constants are determined from the log plot.

If ET_{mx} is not known, it can be determined from the following expressions, which are derived from equations [5] and [7], respectively,

$$\frac{1}{ET_a} = \frac{1}{ET_{mx}} + \left(\frac{1}{cET_{mx}}\right)\Psi_s^k \qquad [10]$$

$$\frac{1}{ET_a} = \frac{1}{ET_{mx}} + \left(\frac{1}{nET_{mx}}\right)\Theta^{-m} \qquad [11]$$

A plot of $1/ET_a$ vs Ψ_s or vs Θ on arithmetic paper should yield an exponential or a hyperbolic curve, respectively. In the plot of equation [10], $1/ET_{mx}$ is the intersect of the curve to the ordinate axle, whereas in the plot of equation [11], $1/ET_{mx}$ is a horizontal asymptote.

Very often, actual evapotranspiration data, ET_a, are expressed in relation to evapotranspiration at an arbitrarily selected value of soil moisture, for example, at field capacity, ET_{fc}. If $ET_{fc} = ET_{mx}$ equations [5] and [7] remain unchanged. If $ET_{fc} < ET_{mx}$, however, they become modified by a factor $f = ET_{fc}/ET_{mx}$.

$$ET_a = \left(\frac{c/f}{c + \Psi_s^k}\right)ET_{fc} \qquad [12]$$

and

$$ET_a = \left(\frac{n/f}{n + \Theta^{-m}}\right)ET_{fc} \qquad [13]$$

To test the adequacy of equations [12] and [13], their logarithmic forms are used:

$$\log \left(\frac{ET_{fc}}{ET_a} - f\right) = \log (f/c) + k \log \Psi_s \qquad [14]$$

and

$$\log \left(\frac{ET_{fc}}{ET_a} - f\right) = \log (f/n) - m \log \Theta \qquad [15]$$

Therefore, a plot of $[(ET_{fc}/ET_a) - f]$ vs Ψ_s or Θ should yield a straight line on log paper.

Objectives

This thesis study has the following purposes:

First, to test the hypothesis that led to equation [3]; and second, to investigate how the coefficients in formulas [5], [7] and [13], derived from equation [3], vary with evaporative demands, soils and crops.

METHODOLOGY

Twenty-eight experiments were conducted to investigate the relation between evapotranspiration and soil moisture as influenced by plant species, stage of growth, soil and evaporative intensity of the environment. Table 1 lists the variables of each experiment.

Table 1. Various combinations of crops, soils, evaporative demands and growth stages used to test the formulas derived from equation [3]

Experiment number	Variables			
	Crop	Soil	Evaporative demand	Stage of growth
			mm day^{-1}	
1	Sunflower	Avon	6.7	a
2			4.0	a
3			2.1	a
4		Millville	6.7	a
5			4.0	a
6			2.1	a
7	Wheat	Avon	5.6	b
8			3.8	b
9			2.2	b
10		Millville	5.6	b
11			3.8	b
12			2.2	b
13	Lentil	Avon	4.8	c
14			2.9	c
15			2.2	c
16			2.2	d
17		Millville	4.8	c
18			2.9	c
19			2.2	c
20			2.2	d
21	Beans	Avon	8.0	e
22			4.5	e
23			8.0	f
24			4.5	f
25		Millville	7.3	e
26			3.8	e
27			7.3	f
28			3.8	f

[a] Eight to ten full size leaves.
[b] Onset of heading.
[c] Eight inches high.
[d] Fifteen inches high.
[e] Juvenile state.
[f] Onset of flowering.

Approximately 1000 cm^3 of soil were placed in nine 1200 cm^3 plastic containers. That volume corresponds to 785 grams of Avon clay loam and to 961 grams of Millville silt loam. The mechanical composition and moisture retaining characteristics of these soils are given in Table 2.

Table 2. Mechanical analysis and soil moisture constants of Avon clay loam and Millville silt loam

Soil type	Soil fraction			Moisture constant		
	Sand	Silt	Clay	1/10 bar	1/3 bar	15 bars
	%	%	%	%	%	%
Avon clay loam	23.6	47.3	29.1	35.2	30.5	16.8
Millville silt loam	16.2	57.2	26.6	27.1	24.2	8.3

The plants were grown under controlled conditions until they were large enough to initiate measurements. From the pots that were similar in morphological development, two to four pairs were selected that did not differ in daily evapotranspiration by more than 25 percent. One member of the pair was rewatered daily to a moisture content at which drainage from the pot ceased and was used as a control. The other member of the pair was allowed to deplete the soil moisture. The length of the drying cycle varied with pot and environmental condition. The experiment was concluded when evapotranspiration reached approximately one-tenth of the initial evapotranspiration or when the soil water potential was about -15 bars, whichever came first. The number of plants varied among experiments and the same plants were not necessarily used again in different environmental conditions. A mulch of paper or vermiculite was placed over each pot to reduce direct soil evaporation to less than 10 percent of the initial evapotranspiration.

All experiments were conducted in a large growth chamber that houses three benches (Figure 8). Different evaporation intensities in the chamber were established by controlling room temperature at selected values; however, there was no control for relative humidity. The evaporation intensities were measured with small evaporimeters having a surface exposure of 100 cm^2 (Figure 9). More details about the experimental arrangement and conditions are given in Figure 10.[1] A uniform and constant temperature throughout the chamber was achieved in every case. However, ventilation patterns varied in such a way that evaporation differed from bench to bench. These differences were, nevertheless, consistent and reproducible. This explains the various values of evaporative demands in Table 1 and Figure 10.

Evapotranspiration and soil moisture content were computed each day by weighing on a top loading balance of 5000 grams capacity that read directly to $\pm$ 0.05 grams (Figure 11). The measurements were taken immediately after the light period commenced.

The results of these experiments and also those of 20 other experiments obtained from different sources in the literature were used to test formulas [5], [7] and [13] which were derived from the basic model, expressed by equation [3]. The data of each experiment were subjected to a regression analysis (Snedecor, 1946). Examples of this and other computational procedures are given in Appendix D.

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[1] The experiments with beans are not shown in Figure 10, but the arrangement was similar to that indicated for the other crops.

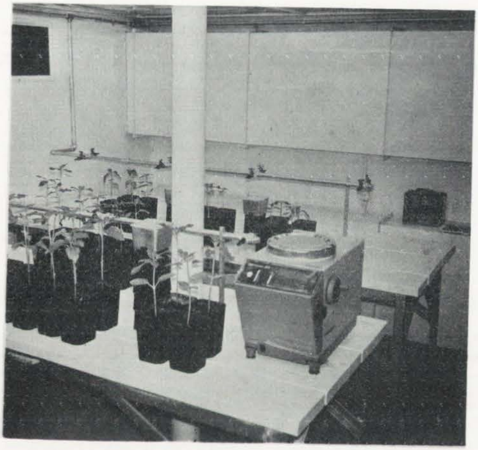

Figure 8. View of the growth chamber in which the experiments were conducted.

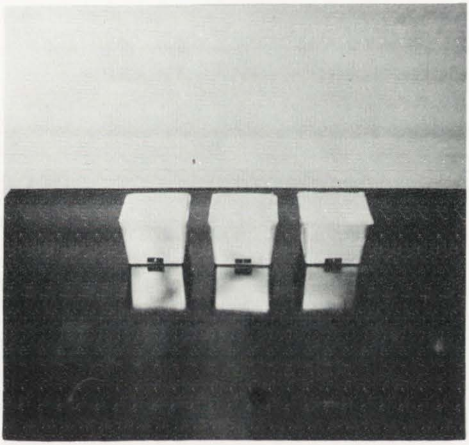

Figure 9. Evaporimeter used to measure the evaporative demands of the environment in the growth chamber.

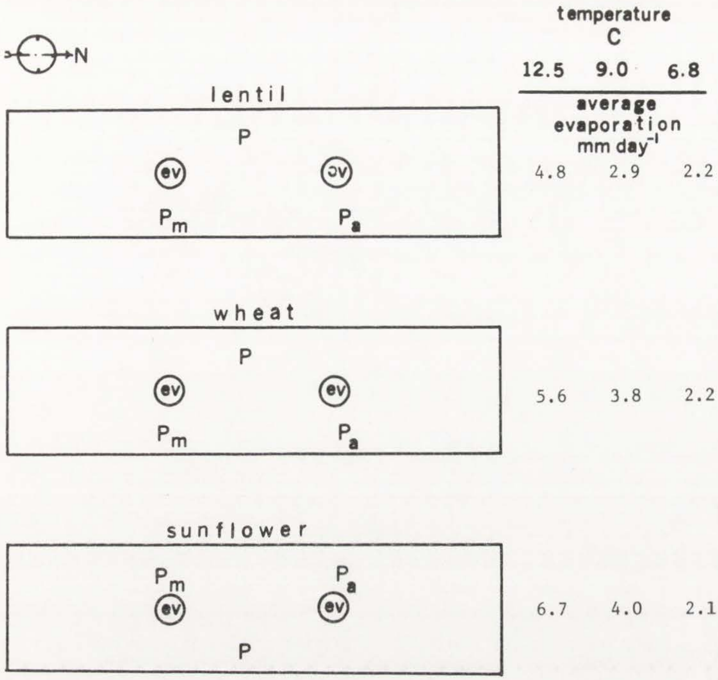

Figure 10. Experimental arrangement indicating the distribution of cultures and describing the conditions that were established in the growth chamber. ev, evaporimeters; P_m, pots with Millville silt loam; P_a, pots with Avon clay loam; P, pots with plants not included in the experimental run. The temperature was set at the values shown above. The respective evaporations are indicated as a function of location. Day and night temperatures were the same. Light period, 13 hrs.; dark period, 11 hrs. The plant corresponds to the south chamber of the Department of Botany, Utah State University.

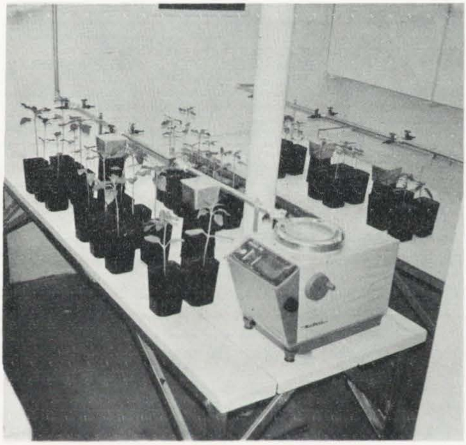

Figure 11. Top loading balance, Mettler P5, used for measuring daily evapotranspiration and soil moisture content.

RESULTS AND DISCUSSION

The General Character of Equation [3]

Equations [5], [7] and [13] which were derived from the basic model, equation [3], were used to analyze the data of 48 experiments. The results of these experiments are presented in Tables 5 through 21 which are included in Appendix C. In five of the experiments, soil moisture was reported in terms of potential. These were used to test equation [5]. Equations [7] and [13] were applied to the data of the other 43 experiments in which soil moisture was expressed as water content. Equation [7] was used in those cases where the evapotranspiration at field capacity, ET_{fc}, was equal to the maximum evapotranspiration, ET_{mx}. In a few cases, the moisture content at field capacity was not sufficient to satisfy the evaporative demands of the environment, and ET_{fc} was less than ET_{mx}. The data from these experiments were used to test equation [13].

The data of all 48 experiments were well described by equations [5], [7] and [13]. These results are shown in Figures 12 through 59.

In contrast, no model in Figure 2 could adequately express the results of all experiments. They did, however, fit particular sets of experimental data. For example, the results in Figure 44 followed model a; the results in Figure 45 agreed with model b; model c could be applied to the data of Figure 24; model d approached the results of Figure 58; and model e fitted the results of Figure 41.

There was a qualitative agreement between the model of Shaw (Figure 3) and the results given in Figures 60 through 63. The curves relating

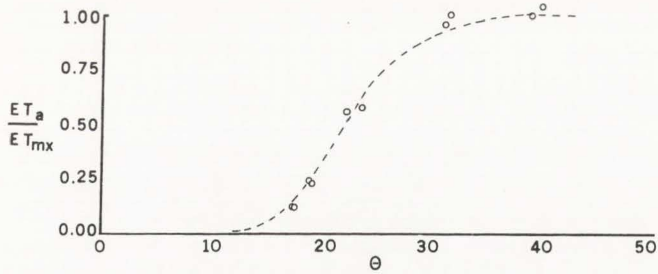

Figure 12. Relative evapotranspiration of sunflower as a function of moisture content of Avon soil under an evaporative demand of 6.7 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.985.

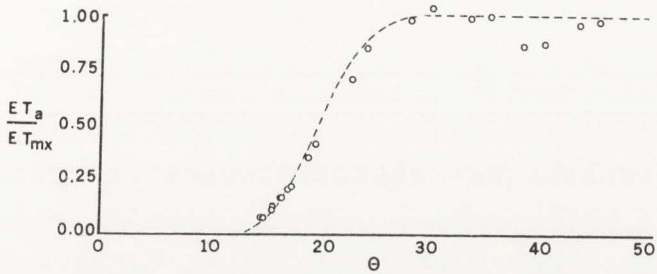

Figure 13. Relative evapotranspiration of sunflower as a function of moisture content of Avon soil under an evaporative demand of 4.0 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.939.

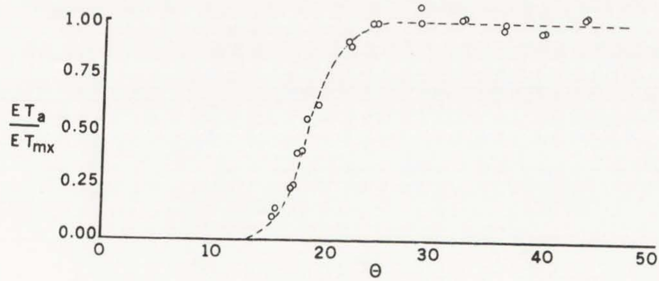

Figure 14. Relative evapotranspiration of sunflower as a function of moisture content of Avon soil under an evaporative demand of 2.1 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.994.

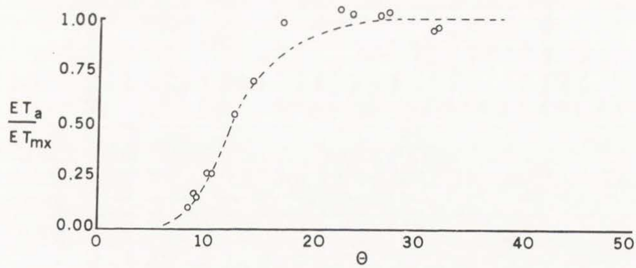

Figure 15. Relative evapotranspiration of sunflower as a function of moisture content of Millville soil under an evaporative demand of 6.7 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.989.

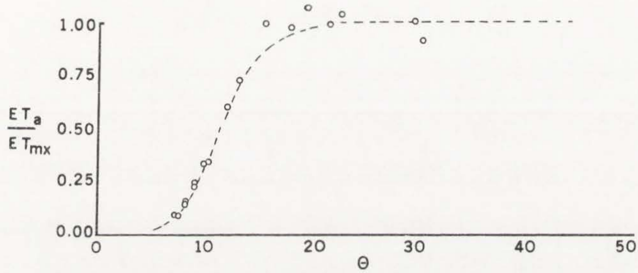

Figure 16. Relative evapotranspiration of sunflower as a function of moisture content of Millville soil under evaporative demand of 4.0 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.003.

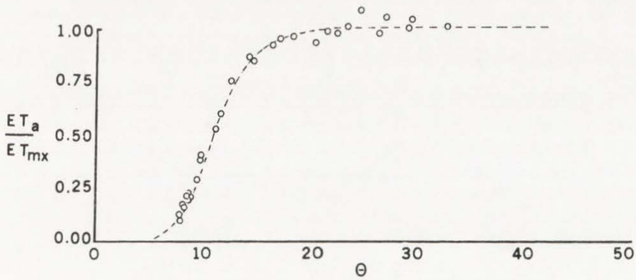

Figure 17. Relative evapotranspiration of sunflower as a function of moisture content of Millville soil under evaporative demand of 2.1 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.995.

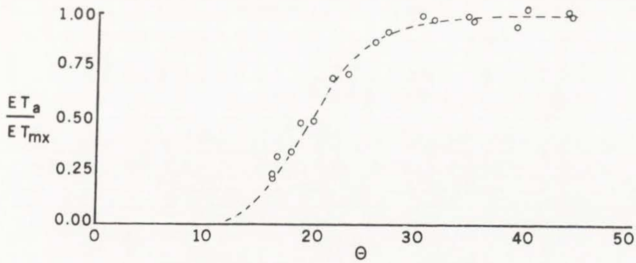

Figure 18. Relative evapotranspiration of wheat as a function of moisture content of Avon soil under an evaporative demand of 5.6 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.960.

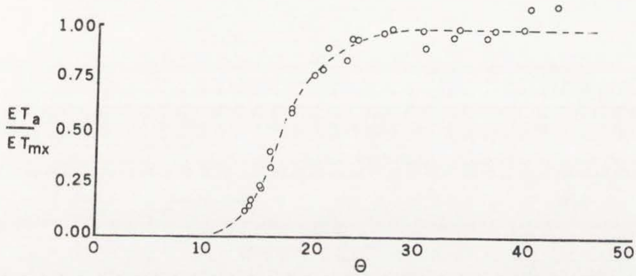

Figure 19. Relative evapotranspiration of wheat as a function of moisture content of Avon soil under an evaporative demand of 3.8 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.989.

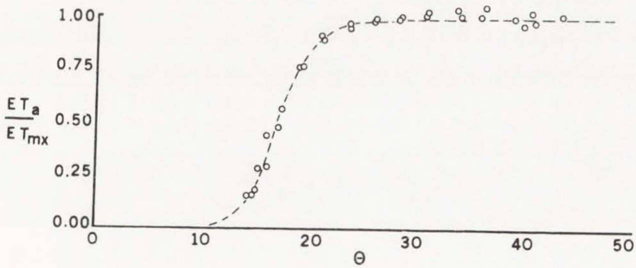

Figure 20. Relative evapotranspiration of wheat as a function of moisture content of Avon soil under an evaporative demand of 2.2 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.994.

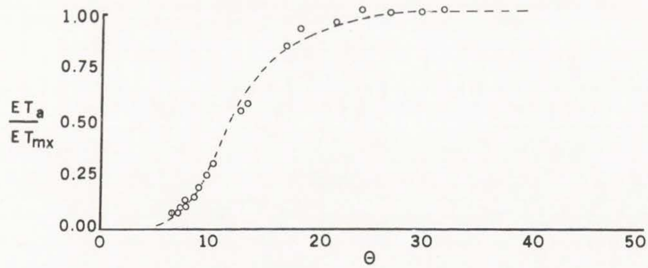

Figure 21. Relative evapotranspiration of wheat as a function of moisture content of Millville soil in an evaporative demand of 5.6 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.994.

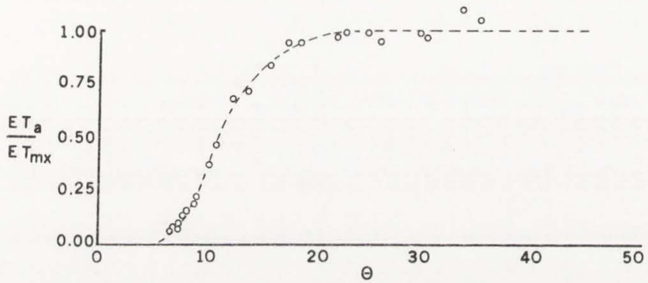

Figure 22. Relative evapotranspiration of wheat as a function of moisture content of Millville soil in an evaporative demand of 3.8 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.998.

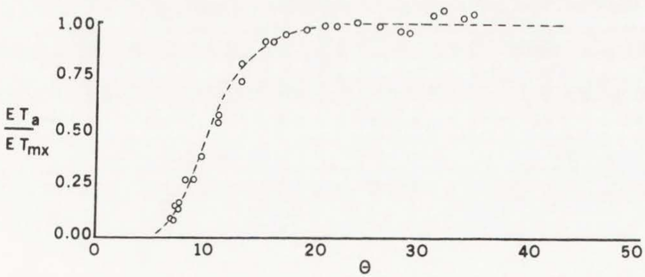

Figure 23. Relative evapotranspiration of wheat as a function of moisture content of Millville soil in an evaporative demand of 2.2 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.998.

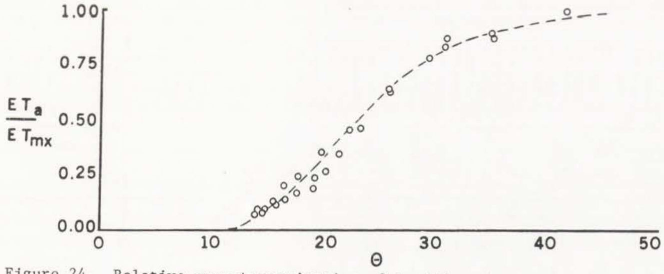

Figure 24. Relative evapotranspiration of lentil at an early stage of growth as a function of moisture content of Avon soil under an evaporative demand of 4.8 mm day^{-1}. Theoretical curve is shown in broken line. $r = 0.990$.

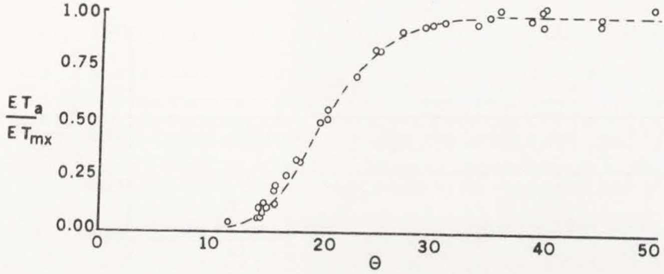

Figure 25. Relative evapotranspiration of lentil at an early stage of growth as a function of moisture content of Avon soil under an evaporative demand of 2.9 mm day^{-1}. Theoretical curve is shown in broken line. $r = 0.994$.

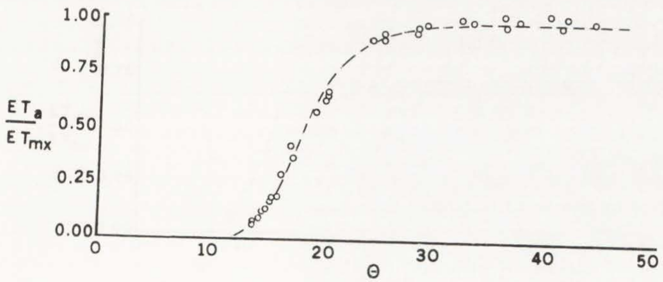

Figure 26. Relative evapotranspiration of lentil at an early stage of growth as a function of moisture content of Avon soil under an evaporative demand of 2.2 mm day $^{-1}$. Theoretical curve is shown in broken line. $r = 0.918$.

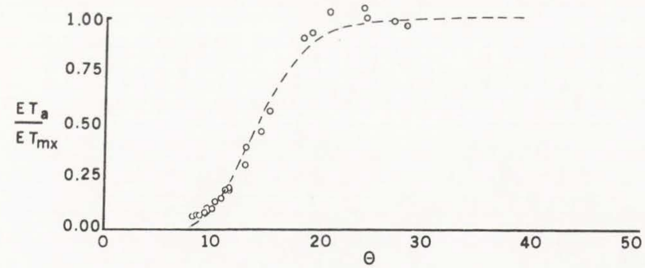

Figure 27. Relative evapotranspiration of lentil at an early stage of growth as a function of moisture content of Millville soil under an evaporative demand of 4.8 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.976

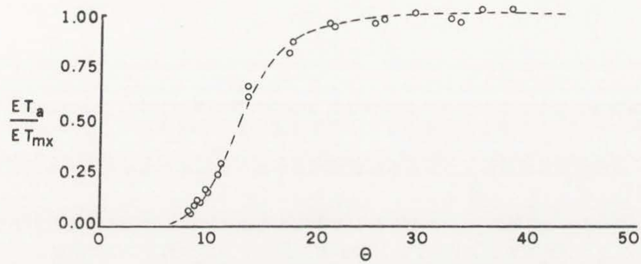

Figure 28. Relative evapotranspiration of lentil at an early stage of growth as a function of moisture content of Millville soil under an evaporative demand of 2.9 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.996.

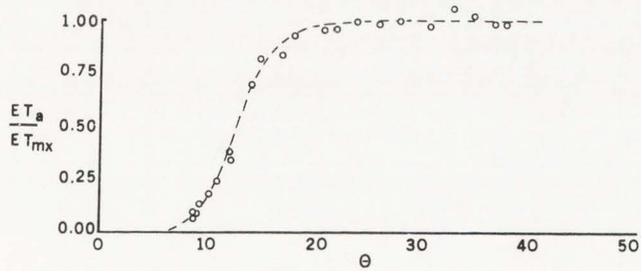

Figure 29. Relative evapotranspiration of lentil at an early stage of growth as a function of moisture content of Millville soil under an evaporative demand of 2.2 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.997.

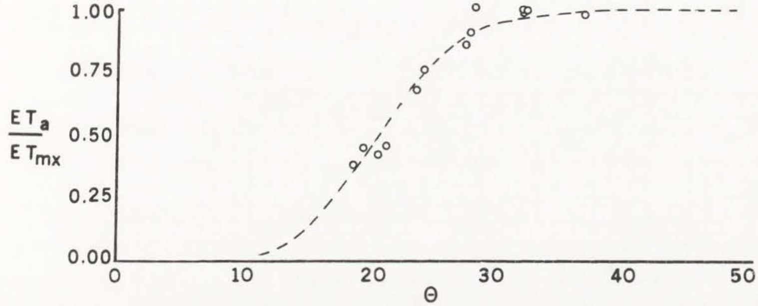

Figure 30. Relative evapotranspiration of lentil at a late stage of growth as a function of moisture content of Avon soil under an evaporative demand of 2.2 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.973

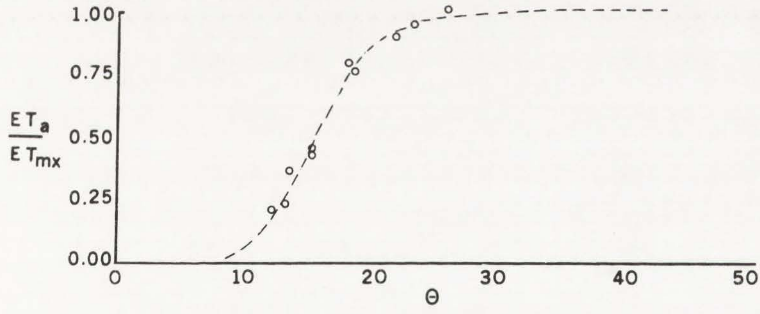

Figure 31. Relative evapotranspiration of lentil at a late stage of growth as a function of moisture content of Millville soil under an evaporative demand of 2.2 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.991.

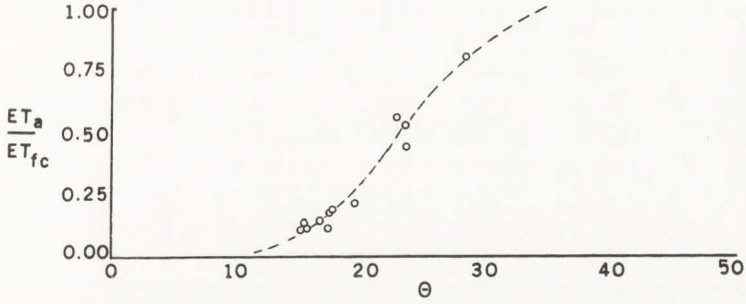

Figure 32. Relative evapotranspiration of bean at an early stage of growth as a function of moisture content of Avon soil under an evaporative demand of 8.0 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.979.

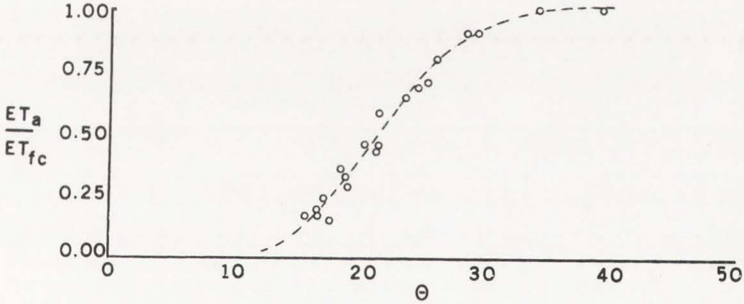

Figure 33. Relative evapotranspiration of bean at an early stage of growth as a function of moisture content of Avon soil under an evaporative demand of 4.5 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.985.

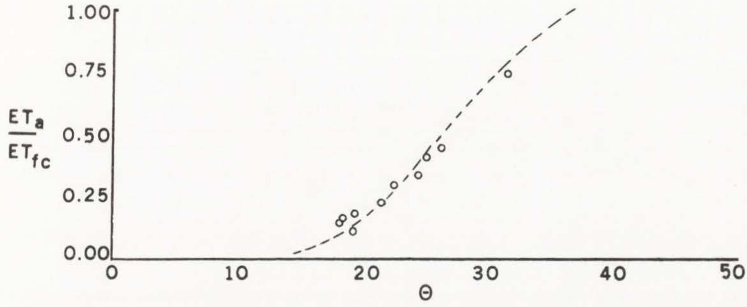

Figure 34. Relative evapotranspiration of bean at a late stage of growth as a function of moisture content of Avon soil under an evaporative demand of 8.0 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.976.

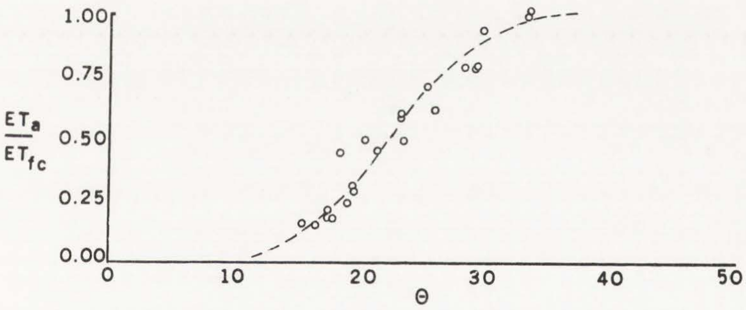

Figure 35. Relative evapotranspiration of bean at a late stage of growth as a function of moisture content of Avon soil in an evaporative demand of 4.5 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.974.

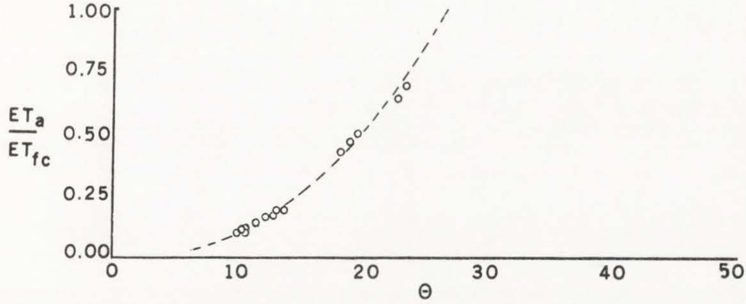

Figure 36. Relative evapotranspiration of bean at an early stage of growth as a function of moisture content of Millville soil under an evaporative demand of 7.3 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.995.

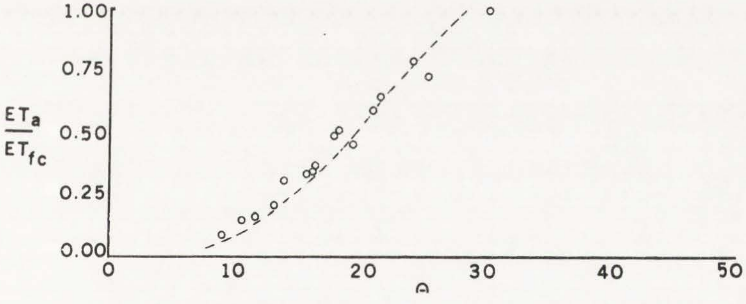

Figure 37. Relative evapotranspiration of bean at an early stage of growth as a function of moisture content of Millville soil under an evaporative demand of 3.8 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.977.

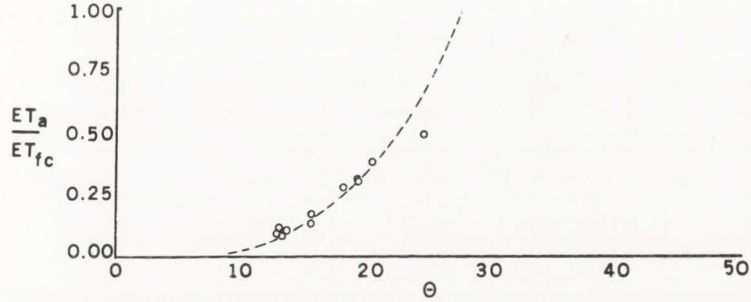

Figure 38. Relative evapotranspiration of bean at a late stage of growth as a function of moisture content of Millville soil under an evaporative demand of 7.3 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.976.

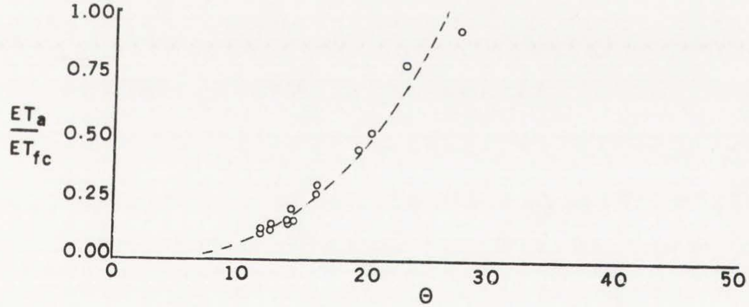

Figure 39. Relative evapotranspiration of bean at a late stage of growth as a function of moisture content of Millville soil under an evaporative demand of 3.8 mm day^{-1}. Theoretical curve is shown in broken line. r = 0.982.

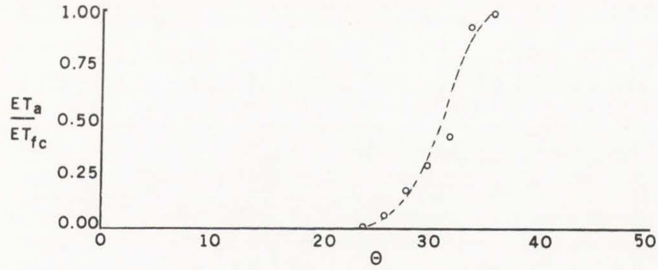

Figure 40. Relative evapotranspiration of corn as a function of moisture content. ET_a at field capacity, 6.4 mm day^{-1}. (Data of Denmead and Shaw, as given by Slatyer, 1967.) r = 0.984.

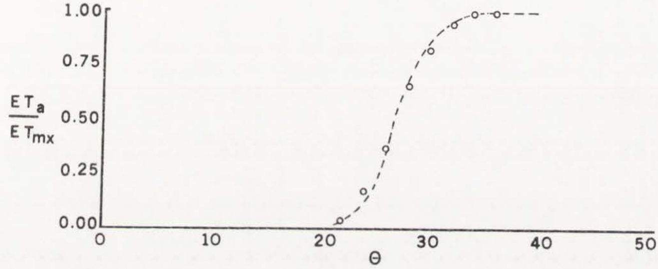

Figure 41. Relative evapotranspiration of corn as a function of moisture content. ET_a at field capacity, 5.6 mm day^{-1}. (Data of Denmead and Shaw, as given by Slatyer, 1967.) r = 0.994.

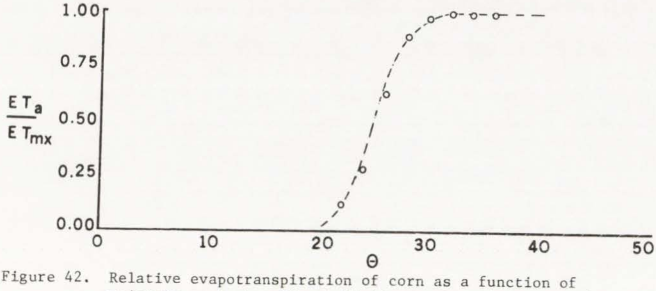

Figure 42. Relative evapotranspiration of corn as a function of moisture content. ET_a at field capacity, 4.1 mm day^{-1}. (Data of Denmead and Shaw, as given by Slatyer, 1967.) r = 0.988.

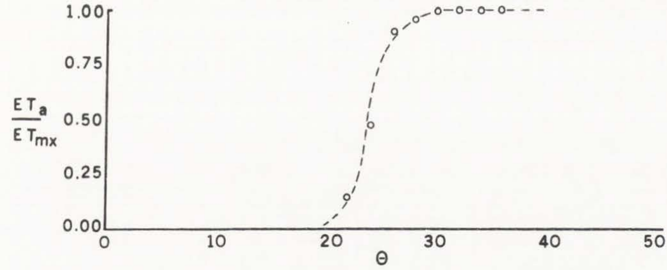

Figure 43. Relative evapotranspiration of corn as a function of moisture content. ET_a at field capacity, 3.3 mm day^{-1}. (Data of Denmead and Shaw, as given by Slatyer, 1967.) r = 0.994.

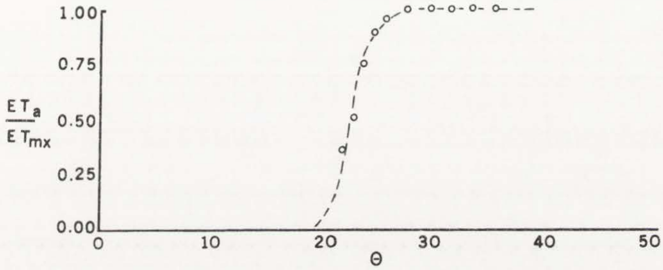

Figure 44. Relative evapotranspiration of corn as a function of moisture content. ET_a at field capacity, 2.0 mm day^{-1}. (Data of Denmead and Shaw, as given by Slatyer, 1967.) r = 0.991.

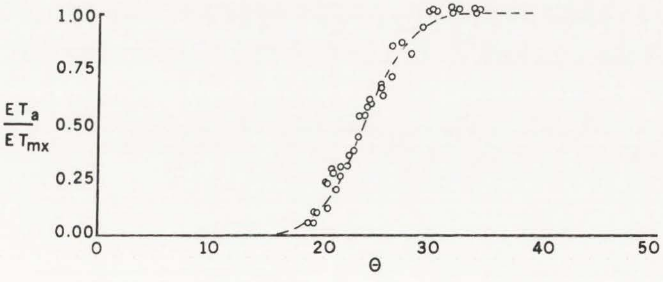

Figure 45. Relative evapotranspiration of oat as a function of moisture content, under variable greenhouse conditions. (After Sudnitzin, 1968.) r = 0.974.

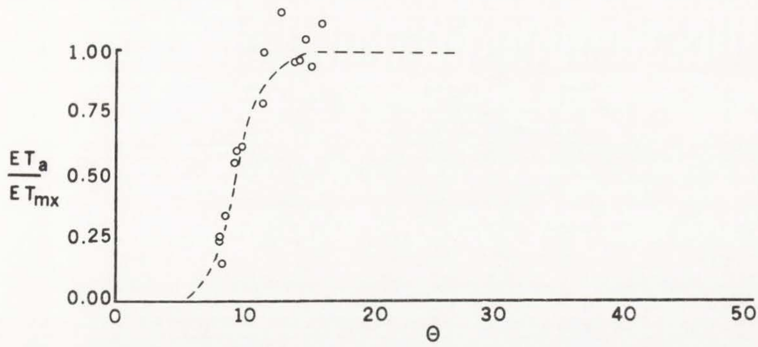

Figure 46. Relative evapotranspiration of Atriplex as a function of moisture content. Winter, greenhouse conditions. (After Palmer et al., 1964.) r = 0.934.

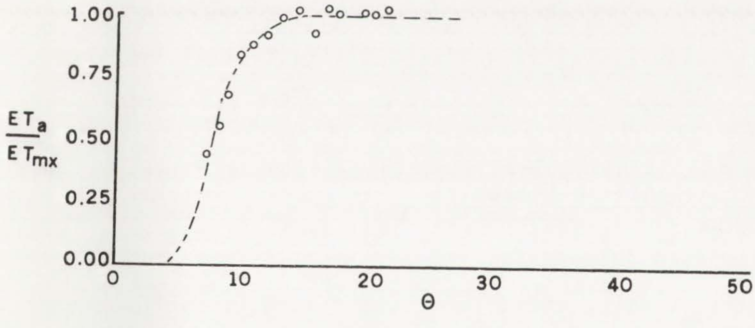

Figure 47. Relative evapotranspiration of Atriplex as a function of moisture content. Summer, greenhouse conditions. (After Palmer et al., 1964.) r = 0.987.

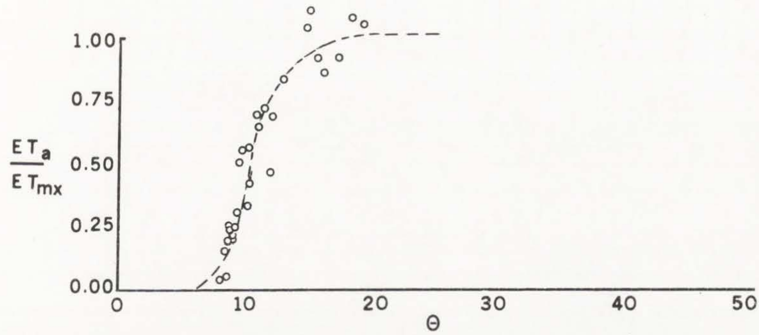

Figure 48. Relative evapotranspiration of cotton as a function of moisture content. Winter, greenhouse conditions. (After Palmer et al., 1964.) r = 0.869.

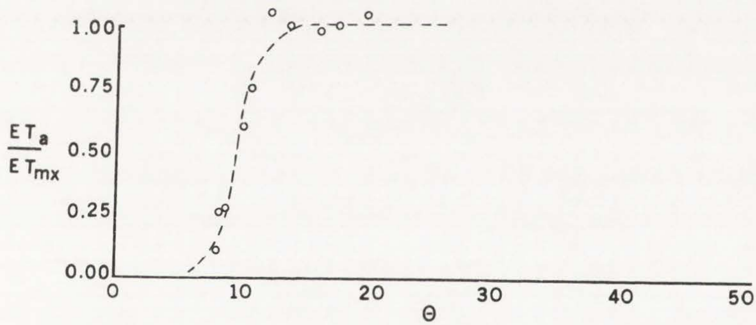

Figure 49. Relative evapotranspiration of cotton as a function of moisture content. Summer, greenhouse conditions. (After Palmer et al., 1964.) r = 0.967.

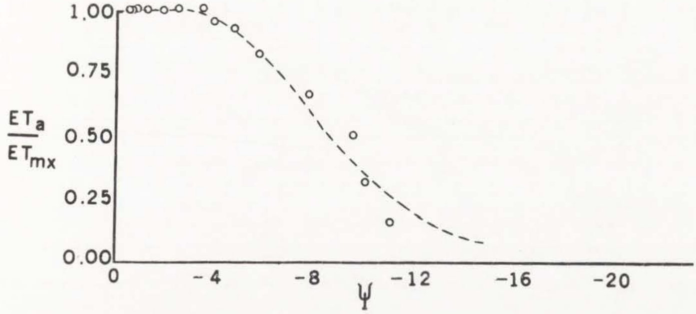

Figure 50. Relative evapotranspiration of alfalfa as a function of soil water potential, in bars, under field conditions in Arizona. (After Van Bavel, 1967.) r = 0.981.

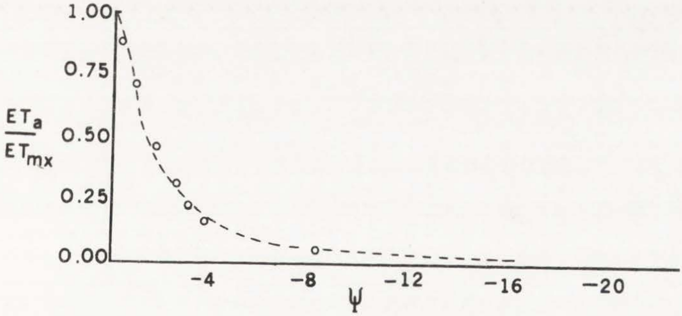

Figure 51. Relative evapotranspiration of alfalfa as a function of soil water potential, in bars, under field conditions in Utah. (After Bahrani and Taylor, 1961.) r = 0.989

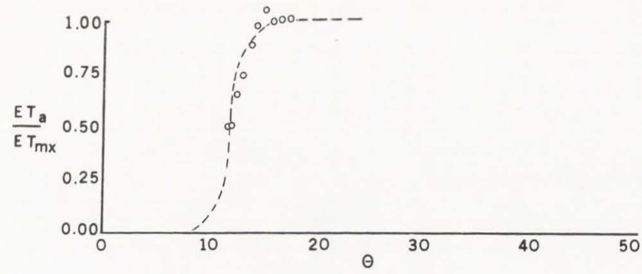

Figure 52. Relative evapotranspiration of ladino clover at an early stage of growth, as a function of moisture content under controlled conditions. (After Hagan et al., 1957.) r = 0.957.

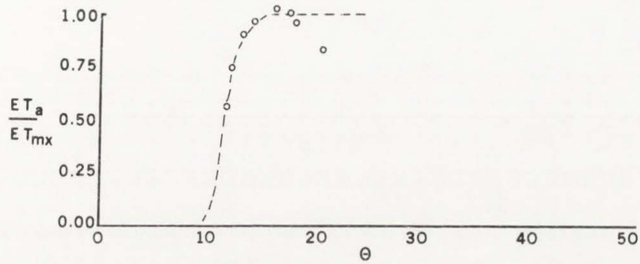

Figure 53. Relative evapotranspiration of ladino clover at an intermediate stage of growth, as a function of moisture content under controlled conditions. (After Hagan et al., 1957.) r = 0.990.

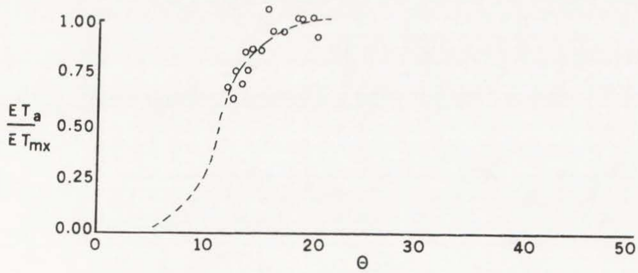

Figure 54. Relative evapotranspiration of ladino clover at a late stage of growth, as a function of moisture content under controlled conditions. (After Hagan et al., 1957.) r = 0.831.

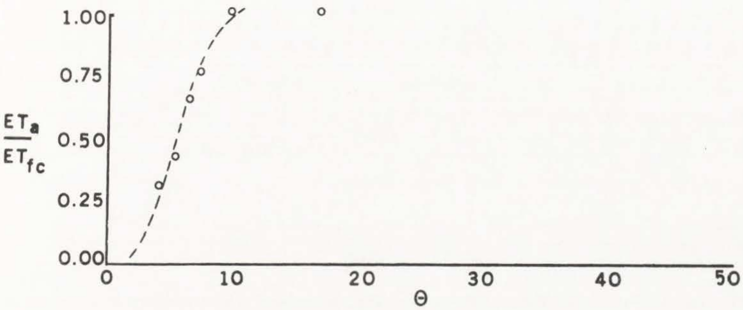

Figure 55. Relative evapotranspiration of pepper as a function of moisture content, under controlled conditions. (After Gardner and Ehlig, 1963.) r = 0.962.

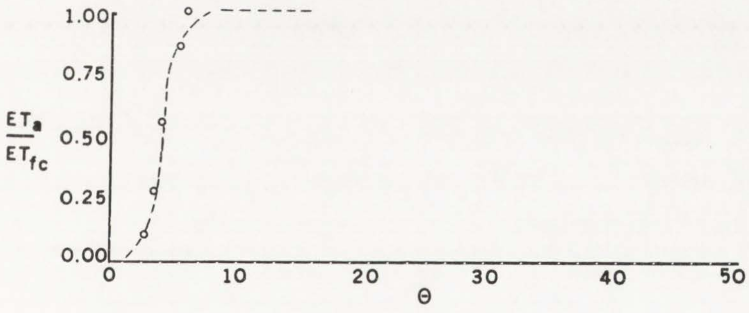

Figure 56. Relative evapotranspiration of birdsfoot trefoil as a function of moisture content, under controlled conditions. (After Gardner and Ehlig, 1963.) r = 0.999.

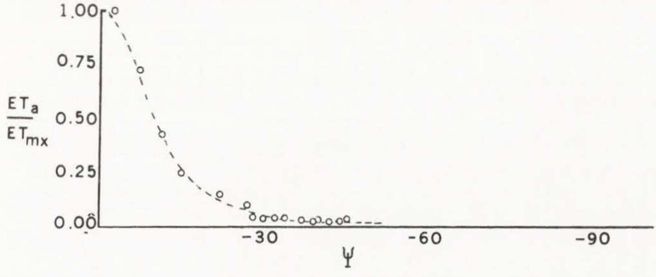

Figure 57. Relative evapotranspiration of tomato as a function of soil water potential, in bars, under greenhouse conditions. (After Slatyer, 1957.) r = 0.990.

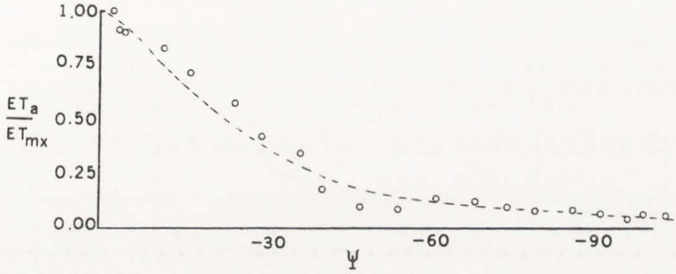

Figure 58. Relative evapotranspiration of privet as a function of soil water potential, in bars, under greenhouse conditions. (After Slatyer, 1957.) r = 0.971.

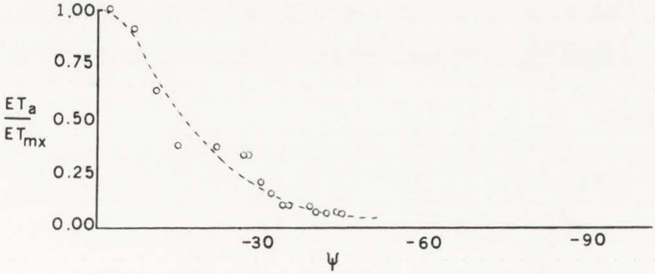

Figure 59. Relative evapotranspiration of cotton as a function of soil water potential, in bars, under greenhouse conditions. (After Slatyer, 1957.) r = 0.962.

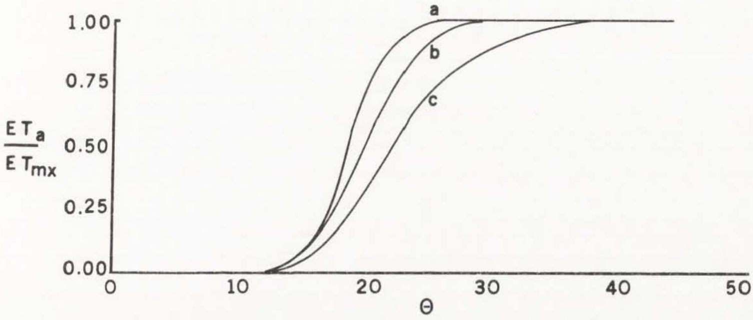

Figure 60. Relative evapotranspiration of sunflower as a function of moisture content of Avon clay loam at three different evaporative demands: a, 2.1 mm day^{-1} (Figure 14); b, 4.0 mm day^{-1} (Figure 13); c, 6.7 mm day^{-1} (Figure 12).

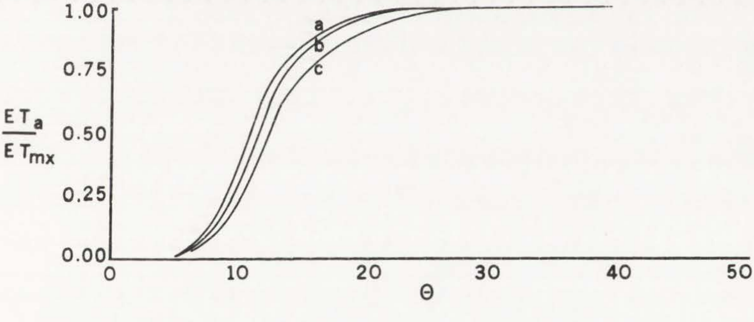

Figure 61. Relative evapotranspiration of sunflower as a function of moisture content of Millville silt loam at three evaporative demands: a, 2.1 mm day^{-1} (Figure 17); b, 4.0 mm day^{-1} (Figure 16); c, 6.7 mm day^{-1} (Figure 15).

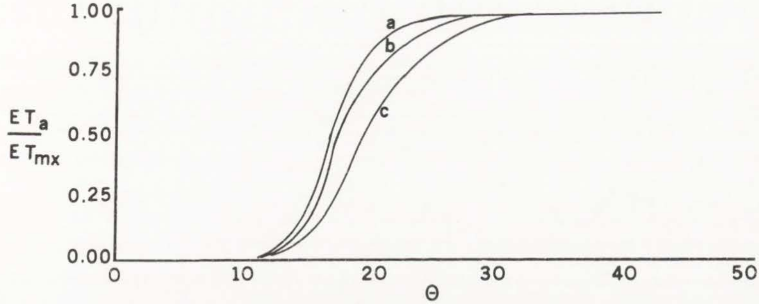

Figure 62. Relative evapotranspiration of wheat as a function of moisture content of Avon clay loam at three different evaporative demands: a, 2.2 mm day^{-1} (Figure 20); b, 3.8 mm day^{-1} (Figure 19); c, 5.6 mm day^{-1} (Figure 18).

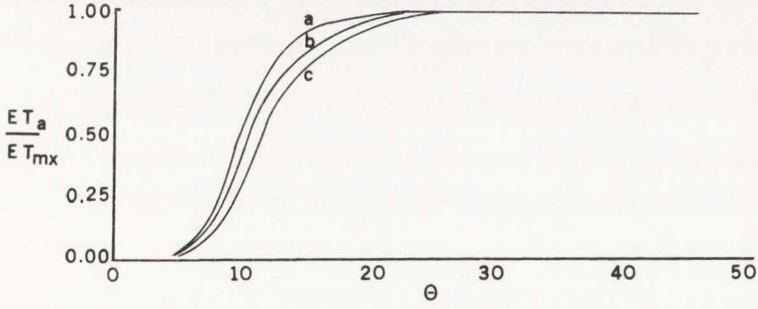

Figure 63. Relative evapotranspiration of wheat as a function of moisture content of Millville silt loam at three different evaporative demands: a, 2.2 mm day^{-1} (Figure 23); b, 3.8 mm day^{-1} (Figure 22); c, 5.6 mm day^{-1} (Figure 21).

evapotranspiration to soil moisture were displaced according to the evaporative demands of the environment in the manner predicted by Shaw's model. However, the relative displacement varied with the soil and the crop type.

There was also general agreement between the relations proposed in Holmes' model (Figure 4) and the curves obtained from equation [7]. However, a consistent difference due to soil types, as indicated in that model, was not found experimentally. The relative positions of the curves were influenced by the evaporative demand of the environment, as illustrated in Figures 64 and 65.

Equation [3] does not specify any rigid limits for the availability of soil water. On the other hand, field capacity, FC, and the permanent wilting point, PWP, are considered to be the maximum and minimum limits of moisture availability, respectively, for almost all models in Figures 2, 3, 4 and 5. The results (shown in Figures 12 through 59), however, indicate that the soil moisture at which evapotranspiration ceases may depart appreciably from the -15 bars value (PWP). Extreme examples are shown in Figures 51, 58 and 59. (See also Slatyer, 1967, p. 229.)

The evapotranspiration at field capacity, ET_{fc}, is often practically equal to the maximum evapotranspiration, ET_{mx}. In these cases, the moisture content at this suction value is adequate to sustain the flow of water demanded by the climatic environment. Under certain circumstances, however, this moisture level may be insufficient to meet the evaporation intensity of the atmosphere. This situation is exemplified in the experiment of Bahrani and Taylor (Figure 51), one of the experiments of Denmead and Shaw (Figure 40) and notably in the bean experiments (Figures 32 through 39). In the first two cases, the reason that

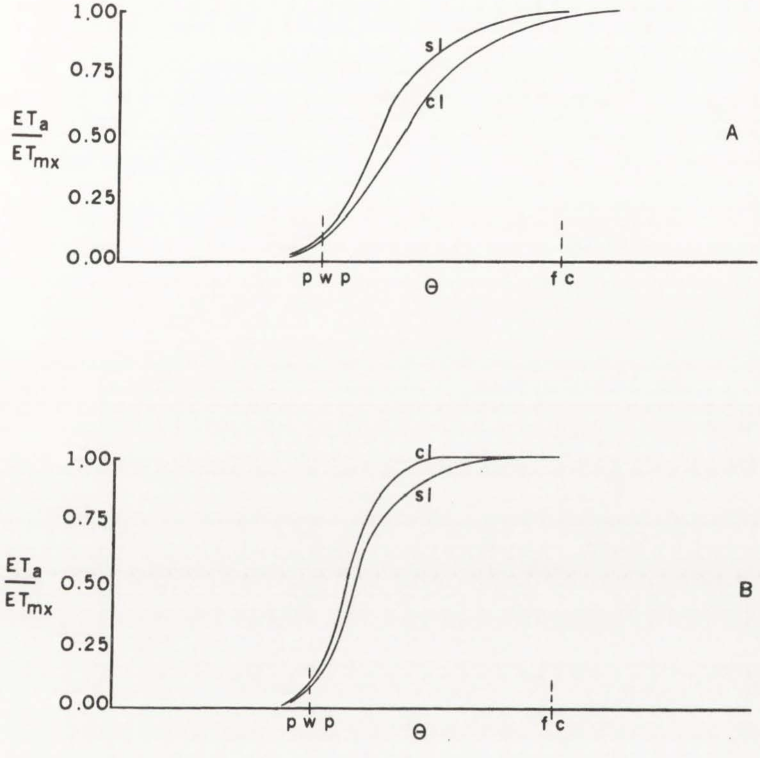

Figure 64. Relative evapotranspiration of sunflower as a function of moisture content in the "available range" as influenced by soil type. sl, Millville silt loam. cl, Avon clay loam. A, evaporative intensity of 6.7 mm day^{-1} (Figures 12 and 15). B, evaporative intensity of 2.1 mm day^{-1} (Figures 14 and 17).

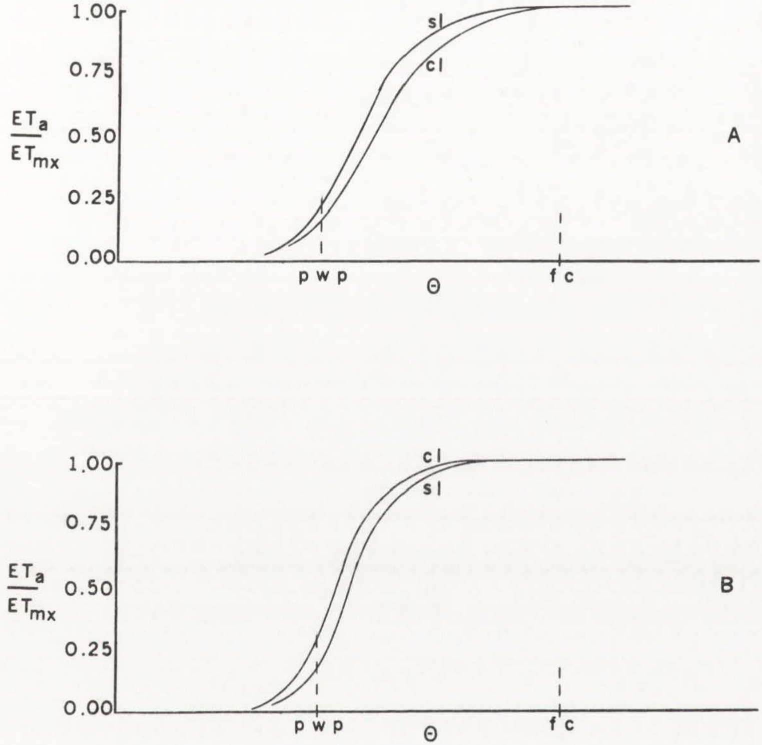

Figure 65. Relative evapotranspiration of wheat as a function of moisture content in the "available range" as influenced by soil type. sl, Millville silt loam. cl, Avon clay loam. A, evaporative intensity of 5.6 mm day^{-1} (Figures 18 and 21). B, evaporative intensity of 2.2 mm day^{-1} (Figures 20 and 23).

$ET_{fc} < ET_{mx}$ is because of the high evaporative demands to which the plants were subjected. Godard (1964, p. 361) has also presented evidence to support this contention. He showed that under severe atmospheric demands, transpiration of crops did not satisfy the climatic environment even when the roots were experiencing optimum water conditions. This happened particularly in days characterized by strong winds, carrying hot dry air. In the bean experiments, the cause for $ET_{fc} < Et_{mx}$ presumably lies in the high shoot/root ratio of the plants. This situation probably arose from root damage during transplanting. A large shoot/root ratio may decrease the plant's capacity for extracting soil water. This has been theoretically predicted by equation [4], and empirically demonstrated by Parker (1949) with other plant species.

The 48 experiments used to test the validity of equation [3] and various other models involved many variables: (a) 15 plant species, (b) different stages of growth, (c) various degrees of confinement of the root system (pots of various sizes, lysimeter and field conditions), (d) nine soil types, (e) field, greenhouse and growth chamber environments and (f) a wide range of evaporative demands of the air, from very low to excessively high. Consequently, the models shown in Figure 2 appear to be valid under particular circumstances. The models of Figures 3 and 4, although more flexible than those of Figure 2, are still limited in their application. Equations [5], [7] and [13] can be used to generate, with the proper choice of constants, a number of curves similar to the models of Figures 2, 3 and 4. Thus, the model represented by equation [3] is sufficiently general and versatile to describe the relation between evapotranspiration and soil moisture under many different circumstances.

The Meaning of the Coefficients in Equations [5][1] and [7]

The graphs (Figures 12 through 59) demonstrate the dynamic character of soil water availability, which is a direct contrast with the concept of a static range of available water defined by FC and PWP. As climatic, soil and plant conditions varied not only were these limits trespassed but the degree of water availability also varied. These changes were reflected in the value of the coefficients in equations [5] and [7].

The plots of equation [5] and [7] yield sigmoidal curves, which for the latter equation are symmetrical. These curves are characterized by maximum and minimum horizontal asymptotes, an inflection point and variable curvatures depending on the value of the coefficients. The maximum horizontal asymptote covers the range of soil moisture at which the evapotranspiration is practically equal to the maximum evapotranspiration. Therefore, it describes the soil moisture region in which evapotranspiration is determined primarily by the energy available in the aerial environment to evaporate water. The minimum horizontal asymptote, on the other hand, spans the range of soil moisture at which water extraction is practically nil, regardless of ambient conditions. The section of the curve between these asympotic ranges, corresponds to the region of soil moisture at which evapotranspiration progressively falls behind the maximum value. Its curvature, therefore, expresses the degree of restriction that soil water imposes on water flow through the soil-plant atmosphere system.

Consequently, the availability of soil water for evapotranspiration

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[1]Equation [13] is a particular case of equation [5]; consequently it is omitted in the following discussion.

may be described in a dynamic way in terms of three ranges: (a) a region of unrestricted availability, U, in which maximum evapotranspiration occurs; (b) a region of restricted availability, R, in which evapotranspiration is less than the maximum; and (c) a region of absolute unavailability, N, where practically no flow of water occurs from soil to atmosphere. The sum of regions U and R represents the total available range, T, (Figure 66). Furr and Reeves (1945) have proposed a somewhat similar scheme.

The meaning of the coefficients of equations [5] and [7] in terms of the availability of soil water as defined above may now be explained by a mathematical analysis of these equations.

The coefficients c and n

From equations [8] and [9] which are the logarithmic forms of equations [5] and [7], respectively,

$$\log\left(\frac{ET_a}{ET_{mx} - ET_a}\right) = \log c - k \log \Psi \qquad [8]$$

$$\log\left(\frac{ET_a}{ET_{mx} - ET_a}\right) = \log n + m \log \Theta \qquad [9]$$

it follows that,

$$\log 1 = 0 = \log c - k \log \Psi' \qquad [16]$$

and

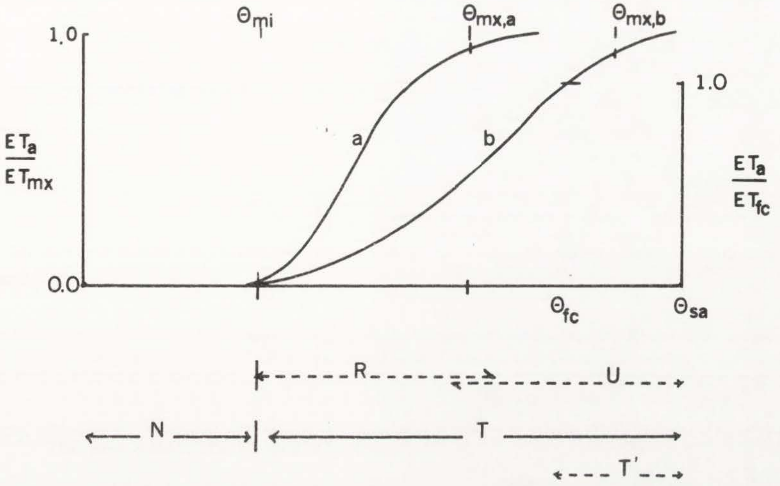
$$\log 1 = 0 = \log n + m \log \Theta' \qquad [17]$$

when Ψ' and Θ' are the values of Ψ and Θ at which $ET_a = 0.5\ ET_{mx}$. Therefore,

$$\log c = k \log \Psi' \qquad [18]$$

and

$$\log n = -m \log \Theta' \qquad [19]$$



ET_a, actual evapotranspiration = f_1(weather, soil moisture)$_{c,s}$
ET_{mx}, maximum evapotranspiration = f_2(weather)$_{crop}$
ET_{fc}, evapotranspiration at field capacity
Θ_{sa}, total water holding capacity of the soil = f_3(soil)
Θ_{fc}, moisture retained against the pull of gravity = f_3(soil)
Θ_{mi}, moisture content at which $ET_a = 0.05\ ET_{mx} = f_4(\text{crop})_{soil}$
Θ_{mx}, moisture content at which $ET_a = 0.95\ ET_{mx} = f_5(\text{crop, weather})_s$
a, a case where $\Theta_{fc} > \Theta_{mx}$ ($ET_{fc} = ET_{mx}$)
b, a case where $\Theta_{fc} < \Theta_{mx}$ ($ET_{fc} < ET_{mx}$)
U, range of unrestricted availability
R, range of restricted availability
T, total available moisture range, $\Theta_{sa} - \Theta_{mi}$ = U + R
T', range of transient availability, $\Theta_{sa} - \Theta_{fc}$
N, range of unavailable moisture

Figure 66. Dynamic classification of soil moisture availability.

Taking antilogs, we get

$$c = (\Psi')^{k} \qquad [20]$$

and

$$n = (\Theta')^{-m} \qquad [21]$$

Finally, combining equations [20] and [5], and equations [21] and [7], we have

$$ET_a = \left(\frac{1}{1 + (\frac{\Psi}{\Psi'})^{k}}\right) ET_{mx} \qquad [22]$$

and

$$ET_a = \left(\frac{1}{1 + (\frac{\Theta'}{\Theta})^{-m}}\right) ET_{mx} \qquad [23]$$

The new constants Ψ' and Θ', corresponding to the soil moisture values at which one-half of the maximum evapotranspiration occurs, can be identified as the infection points of the water availability curves. In the case of equation [7], Θ' is also the midpoint of R, the region of restricted water availability.[1] It may be noted that high values of Θ' or large potential values of Ψ' represent conditions in which a smaller proportion of the total available water, T, is readily utilized by a crop.

The coefficients k and m

Due to the asymptotic nature of the curves of equations [5] and [7], the limits between the ranges U, R and N are not abrupt. For practical purposes, however, arbitrary choices may be made. For example, a value of Ψ_{mx}, or Θ_{mx}, can be selected as the

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[1]The curves produced by equation [7] resemble the curves for the Gauss distribution used in statistics, some of whose properties they share.

limit between U and R, and defined as the value of Ψ, or Θ, at which $ET_a = 0.95\ ET_{mx}$. Similarly, a value of Ψ_{mi}, or Θ_{mi}, may be chosen as the limit between R and N, and defined as the value of Ψ, or Θ, at which $ET_a = 0.05\ ET_{mx}$. Then equation [3],

$$\frac{dET_a}{d\Psi_s} = -k\,\frac{ET_a}{\Psi_s}\left(1 - \frac{ET_a}{ET_{mx}}\right) \qquad [3]$$

can be integrated between the limits of $0.95\ ET_{mx}$ and $0.05\ ET_{mx}$,

$$\int_{0.05}^{0.95} \frac{d(ET_a/ET_{mx})}{(ET_a/ET_{mx})(1 - ET_a/ET_{mx})} = -k \int_{\Psi_{mi}}^{\Psi_{mx}} \frac{d\Psi_s}{\Psi_s} \qquad [24]$$

$$\log 361 = k \log \left(\frac{\Psi_{mi}}{\Psi_{mx}}\right) \qquad [25]$$

from which,

$$k = \frac{2.56}{\log \left(\frac{\Psi_{mi}}{\Psi_{mx}}\right)} \qquad [26]$$

The corresponding expression for m is

$$m = \frac{2.56}{\log \left(\frac{\Theta_{mx}}{\Theta_{mi}}\right)} \qquad [27]$$

The coefficient k and m, therefore, represent the relative amplitude of the soil moisture range in which evapotranspiration is restricted by water supply. A small value of m, for example, must arise from a large ratio Θ_{mx}/Θ_{mi} and would, therefore, indicate a comparatively wide range in soil moisture values in which utilization of soil water is restricted. If the value of m is increased as a result of narrowing the Θ_{mx}/Θ_{mi} ratio then there is an extension of soil moisture range in which water is readily utilized. It reaches its maximum extension at $\Theta_{mx}/\Theta_{mi} = 1$, when $m = \infty$, meaning that the entire range of available water is equally available and adequate to satisfy the evaporative demands of the atmosphere.

It follows from the foregoing considerations that a dynamic expression of soil water availability in terms of water used in evapotranspiration may be derived by combining equations [26] with [22], or equation [27] with [23]:

$$\frac{ET_a}{ET_{mx}} = \left\{\frac{1}{1 + \left(\frac{\Psi}{\Psi'}\right)^{\left[\frac{2.56}{\log\,(\Psi_{mi}/\Psi_{mx})}\right]}}\right\} \qquad [28]$$

$$\frac{ET_a}{ET_{mx}} = \left\{\frac{1}{1 + \left(\frac{\Theta'}{\Theta}\right)^{\left[\frac{2.56}{\log\,(\Theta_{mx}/\Theta_{mi})}\right]}}\right\} \qquad [29]$$

Equal availability throughout the entire available water range, T, is expressed by $ET_a/ET_{mx} = 1$; this occurs when $\Psi_{mi} = \Psi_{mx}$, or $\Theta_{mx} = \Theta_{mi}$, so that the second term of the denominator becomes zero. In cases in which $ET_a < ET_{mx}$, the relative proportions of the regions of unrestricted, U, and of restricted availability, R, as well as the pattern of water utilization in the latter are described by the values of the ratios of Ψ_{mi}/Ψ_{mx}, Θ_{mx}/Θ_{mi}, Ψ/Ψ' and Θ'/Θ.

Effect of climatic, soil and crop factors on the coefficients

Table 3 presents a summary of the variables involved in each experiment, the values of the coefficients of equations [22] and [23] obtained from regression analysis and values of Ψ', Ψ_{mx} and Ψ_{mi}.

The values of m fluctuated widely from a minimum of 2.9 to a maximum of 23.0; most values ranged between 5 and 10 depending on the climatic environment, soil, crop and plant condition. A much narrower range was exhibited by values of k.[1] The minimum and maximum values fluctuated between 1 and 2. The reason for this rather narrow range is that in most cases the changes in evapotranspiration occur under soil moisture potential values of -1 to -40 bars. From the relation $k = 2.56/\log(\Psi_{mi}/\Psi_{mx})$, it follows that k would generally vary around -1.6.

The manner in which the values of the coefficients will be affected by climatic, soil and plant factors can be predicted qualitatively from a study of equation [4]. This equation indicates the main factors involved in the flow of water through the soil-plant-atmosphere system, and how they interact to determine evapotranspiration. The analysis is best done by considering how these factors are related to the critical values of Ψ', Ψ_{mx} and Ψ_{mi}, which describe the pattern of water utilization.

The term Ψ'_s was previously defined as the Ψ_s value at which $ET = 0.5\ ET_{mx}$. Rewriting equation [4] in a simpler form and at the Ψ' value gives

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[1]It may be recalled that $m = (-k)(-b)$, where -b is a constant that varies greatly among different soils.

Table 3. Climatic, soil and plant variables involved in each experiment and values of coefficients in formulas [5], [7], [13], [28] and [29]

Ref	Plant	Growth stage	Soil	Roots	Env	ED	Coefficients					
							m	Θ'	k	Ψ'	Ψ_{mx}	Ψ_{mi}
						$\frac{mm}{day}$		%		bars	bars	bars
This thesis	Sunflower		c l	conf	g c	6.7	7.313	22.1	1.152	2.4	0.3	46
						4.0	10.446	19.4	1.646	5.6	1.3	46
						2.1	12.709	18.6	2.005	7.0	2.4	46
			s l	conf	g c	6.7	5.188	12.5	1.435	3.1	0.5	32
						4.0	5.519	11.5	1.525	4.2	0.7	32
						2.1	5.858	11.0	1.620	4.9	0.8	32
	Wheat		c l	conf	g c	5.6	7.062	20.0	1.112	4.5	0.4	90
						3.8	7.803	15.2	1.230	24.0	0.8	90
						2.2	9.353	17.3	1.474	11.5	1.6	90
			s l	conf	g c	5.6	4.606	11.7	1.274	3.9	0.4	42
						3.8	5.120	9.9	1.414	7.0	0.6	42
						2.2	5.292	10.6	1.463	5.6	0.7	42
	Lentil	yr	c l	conf	g c	4.8	4.988	23.3	0.787	1.7	0.0	46
						2.9	7.280	20.9	1.148	3.4	0.3	46
						2.2	7.641	18.4	1.205	8.0	0.3	46
		yr	s l	conf	g c	4.8	5.846	14.5	1.617	1.8	0.5	19
						2.9	5.963	13.4	1.648	2.4	0.5	19
						2.2	6.617	12.9	1.828	2.8	0.7	19

Table 3. Continued

Ref	Plant	Growth stage	Soil	Roots	Env	ED	Coefficients					
							m	Θ'	k	Ψ'	Ψ_{mx}	Ψ_{mi}
						mm/day		%		bars	bars	bars
		ol	c l	conf	g c	2.2	6.640	20.8	1.045	2.6	0.2	70
		ol	s l	conf	g c	2.2	6.138	15.2	1.699	1.5	0.7	24
	Bean	yr	c l	conf	g c	8.0	4.876	24.6[a]	0.768	1.2[b]	0.0-	70
						4.5	6.002	22.0[a]	0.944	2.5[b]	0.1-	70
		ol	c l	conf	g c	8.0	4.425	28.8[a]	0.697	0.4[b]	0.0-	70
						4.5	5.480	23.4[a]	0.865	1.6[b]	0.0-	70
		yr	s l	conf	g c	7.3	3.121	21.4[a]	0.862	0.5[b]	0.0-	48
						3.8	3.470	19.1[a]	0.958	0.7[b]	0.1-	48
		ol	s l	conf	g c	7.3	2.948	27.6[a]	0.815	0.1[b]	0.0-	48
						3.8	4.638	19.9[a]	1.282	0.6[b]	0.4-	48
(1)	Corn		s c l	conf	fld	6.4	13.770	31.6[a]	1.336	0.6[b]	0.7	58-
						5.6	16.346	26.6	1.587	2.7	1.4	58
						4.1	18.167	25.0	1.763	4.8	2.1	58
						3.3	21.035	23.9	2.040	7.0	3.2	58
						2.0	23.000	22.8	2.235	10.5	4.1	58
(2)	Oats		l c	conf	g	var	11.055	24.1	2.215	10.4	5.0	70
(3)	Atriplex			lys	g w	1.9	8.662	9.3	1.227	40	8.9	1100
					g s	6.2	6.183	8.0	0.877	95	1.3	1100

Table 3. Continued

Ref	Plant	Growth stage	Soil	Roots	Env	ED	Coefficients					
							m	Θ'	k	Ψ'	Ψ_{mx}	Ψ_{mi}
						$\frac{mm}{day}$		%		bars	bars	bars
	Cotton			lys	g w	2.9	8.018	10.7	1.139	17	1.0	180
					g s	9.2	9.363	9.7	1.328	29	2.1	180
(4)	Alfalfa			unc	fld	9.0			4.267	8.5	4.0	27-
(5)	Alfalfa		s l	unc	fld	11.8			1.501	1.3	0.2	26-
(6)	Ladino	yr	c l	conf	g c		16.384	12.3	2.290	10.0	6.1	80
	clover	y	c l	conf	g c		16.072	11.9	2.250	14.0	5.8	80
		ol	c l	conf	g c		5.794	11.3	0.810	18.0	0.6	80
(7)	Pepper		sa l	n-un	g		3.696	5.8[a]	1.406	6.0[b]	1.1	75
	Trefoil		sa l	n-un	g		5.803	6.3	2.210	4.7	3.8	55
(8)	Tomato		s c l	conf	g				2.342	9.9	2.329	46
	Privet		s c l	conf	g				1.694	20.0	2.209	380
	Cotton		s c l	conf	g				2.364	15.2	2.798	100

[a] Moisture content at which $(ET_{fc}/ET_a) - f = 1$. [b] Soil water potential at which $(ET_{fc}/ET_a) - f = 1$.

Key to symbols: Ref, references; env, environment; ED, evaporative demand; c l, clay loam; s l, silt loam; s c l, silty clay loam; l c, loamy clay; sa l, sandy loam; conf, confined; lys, in lysimeter; unc, unconfined; n-un, non uniform in container; g c, growth chamber; fld, field; g, greenhouse; g w, greenhouse, winter; g s, greenhouse, summer; yr, younger; y, young; ol, older.

(1) Denmead and Shaw, in Slatyer, 1967. (2) Sudnitzin, 1964. (3) Palmer et al., 1964. (4) van Bavel, 1967. (5) Bahrani and Taylor, 1961. (6) Hagan et al., 1957. (7) Gardner and Ehlig, 1963. (8) Slatyer, 1957.

$$\varepsilon' = \frac{0.5\ ET_{mx}}{\Psi'} = (k_s + k_p)\left(\frac{\Psi_{at}}{\Psi_s} - 1\right) \qquad [30]$$

in which

$$k_s = \frac{1}{I_a^e + I_s^e} \qquad [31]$$

is a coefficient for the transport of water from the soil directly to the atmosphere (evaporation, E_s) and

$$k_p = \frac{1}{I_a^t + I_f^t + \eta\left(\frac{M_p}{M_r}\right)\left(I_p^t + I_s^t\right)} \qquad [32]$$

is a coefficient for the transport of water from the soil to the atmosphere through the plant (transpiration, T). The other terms are as formerly defined.

We now note that

$$ET_{mx} \equiv (k_{ss} + k_{ps})\Psi_{at} \qquad [33]$$

in which k_{ss} and k_{ps} are the coefficients for the transfer of water through the soil-plant-atmosphere system when $\Psi_s = 0$, i.e. at moisture saturation and when other factors are not limiting absorption. They represent the maximum values of the transmission coefficients of the system.

Substituting equation [33] into equation [30] and solving for Ψ', gives

$$\Psi' = \left(\frac{1}{k_{ss} + k_{ps}} - \frac{0.5}{k_s + k_p}\right) ET_{mx} \qquad [34]$$

To show a more explicit relationship between Ψ' and the evaporative demands of the environment, ET_{mx} may be expressed in terms of E_o, the evaporation rate from a free water surface. Davis (1963) has proposed the following empirical and linear relation:

$$ET_{mx} = a + gE_o \qquad [35]$$

in which a and g are experimental coefficients and E_o is a measure of the evaporative intensity of the atmosphere. The coefficient a is often negligible.

Combining equations [34] and [35], we have

$$\Psi' = \left(\frac{1}{k_{ss} + k_{ps}} - \left(\frac{0.5}{k_s + k_p}\right)\right)(a + gE_o) \qquad [36]$$

Equation [36] states that under constant evaporative demands of the atmosphere, Ψ' will be proportional to $(k_s + k_p)$. It follows from equation [31] and [32], therefore, that some of the main factors contributing towards higher values of Ψ' will be: (a) a high capillary conductivity of the soil; (b) low resistances to water flow through the plant; (c) a large root/shoot ratio; (d) rather insensitive stomatal control; (e) a profuse root system and (f) morphological features of the foliage and of the soil surface that reduce the diffusion resistances through the boundary layer.

These conclusions can be verified in part with data of Table 3. Some experiments with lentils and beans were performed at two stages of growth under constant soil and ambient conditions. Because the plants were grown in relatively small pots, the greater vegetative growth at stage II was associated with larger shoot/root ratios than at stage I.

Most of the other soil, atmospheric and plant factors mentioned above must have remained fairly constant. Hence, the experiments carried out at stage II must show lower values of Ψ' than the experiments performed at stage I. Table 4 indicates that this was indeed verified.

Table 4. The effect of stage of growth of lentil and bean plants on the value of Ψ' and k under constant environment

Crop	Soil	Evaporative demand	Growth stage	Ψ'	k
		$mm\ day^{-1}$		bars	
Lentil	clay loam	2.2	early	8.0	1.205
	clay loam	2.2	late	2.6	1.045
Lentil	silt loam	2.2	early	2.8	1.828
	silt loam	2.2	late	1.5	1.699
Bean	clay loam	8.8	early	1.2	0.768
	clay loam	8.8	late	0.4	0.697
Bean	clay loam	4.5	early	2.5	0.944
	clay loam	4.5	late	1.6	0.865
Bean	silt loam	7.3	early	0.5	0.862
	silt loam	7.3	late	0.1	0.815
Bean	silt loam	3.8	early	0.7	1.282
	silt loam	3.8	late	0.6	0.958

An analysis similar to that which resulted in equation [36] leads to the equation

$$\Psi_{mx} = \left[\left(\frac{1}{k_{ss} + k_{ps}} \right) - \left(\frac{0.95}{k_s + k_p} \right) \right] (a + gE_o) \qquad [37]$$

and to conclusions similar to those drawn for equation [36]; i.e., under constant evaporative demands, the same factors which promote high values of Ψ' must also produce high values of Ψ_{mx}. This may be demonstrated indirectly by the values of k. Since $k = 2.56/\log(\Psi_{mi}/\Psi_{mx})$ and Ψ_{mi} is fairly constant for a given crop and soil, higher values of k must coincide with higher values of Ψ'. Table 4 shows that this was certainly the case.

Next, an analysis will be done to study how the coefficients are influenced by different evaporative demands. Evapotranspiration was previously expressed as follows:

$$ET_a = E_s + T_{c,s} = E_s + \left(\frac{\overline{\Psi}_L - \overline{\Psi}_s}{I_p^t + I_s^t} \right) \qquad [38]$$

By definition, $ET_a = 0.95\ ET_{mx}$ when $\overline{\Psi}_s = \Psi_{mx}$, so that

$$0.95\ ET_{mx} = E_s + \frac{(\text{crit}\ \Psi_L + \alpha) - \Psi_{mx}}{I_p^t + I_s^t} \qquad [39]$$

where crit Ψ_L is the critical leaf water potential at which incipient closure of stomata takes place,

$$\alpha = \Psi_L^{0.95} - \text{crit}\ \Psi_L \qquad [40]$$

and $\Psi_L^{0.95}$ is the leaf water potential associated with $ET_a = 0.95\ ET_{mx}$. If $\propto$ is negligible, as is shown, for instance, in graphical data presented by Gardner and Ehlig (1963), equation [39] simplifies to

$$0.95\ ET_{mx} = E_s + \left(\frac{\text{crit}\ \Psi_L - \Psi_{mx}}{I_p^t + I_s^t}\right) \qquad [41]$$

Introducing equation [35] and rearranging terms to solve for Ψ_{mx}, we have

$$\Psi_{mx} = \text{crit}\ \Psi_L - (I_p^t + I_s^t)(0.95_a + 0.95\ gE_o - E_s) \qquad [42]$$

Equation [42] states, therefore, that the value of Ψ_{mx} will be influenced by the critical leaf water potential at which stomata begin to close in response to loss in turgor. This value can vary from approximately -4 bars to -15 bars depending on the plant species (Gardner and Ehlig, 1963; Meriaux, 1964). The term Ψ_{mx} also depends on the evaporative demands of the atmosphere. In general, an increase in E_o will be associated with smaller values of Ψ_{mx}; i.e., evapotranspiration will be restricted at lower suction values. However, the influence of E_o on the value of Ψ_{mx} will be conditioned not only by the particular crit Ψ_L of the plant, but also by other plant and soil factors that influence the flow of water. These additional factors are expressed in the term $(I_p^t + I_s^t)$. For example, relatively minor changes in E_o may be associated with comparatively large changes in Ψ_{mx} if $(I_p^t + I_s^t)$ has a large value. Some conditions that may lead to these results are (a) slow permeabilities in the conducting tissues of the plant, (b) a sparse root system, and (c) a low capillary conductivity of the soil. In addition, the presence of E_s in equation [42] reveals that the effect of E_o on Ψ_{mx} will also depend on the relative magnitudes of the evaporation and transpiration fluxes.

A development similar to that which led to equation [42], results in the following expression for Ψ_{mi}

$$\Psi_{mi} = fi\ \Psi_L - (I_p^t + I_s^t)(0.05a + 0.05\ gE_o - E_s) \qquad [43]$$

where fi Ψ_L is the leaf water potential at which the loss of turgor is essentially complete. This value corresponds to the sum of the solute, Ψ_π, and the matric, Ψ_τ, potentials of the plant tissues. Since the former greatly exceeds the latter (Weibe, 1966), equation [43] can be written thus,

$$\Psi_{mi} = \Psi_\pi - (I_p^t + I_s^t)(0.05a + 0.05\ gE_o - E_s) \qquad [44]$$

Equation [44] predicts that soil water will be utilized to comparatively low potential values when: (a) plants contain large quantities of osmotically active substances, (b) the resistances to water flow in the plant and in the soil are comparatively low, and (c) when evaporative demands are low. Nevertheless, the product in the right hand side of the equation [44] is not nearly so variable as the equivalent product in equation [42]. Soils of widely different textures exhibit approximately the same capillary conductivities in the region between -10 to -100 bars potential (Gardner, 1960b). The value of these conductivities is also so extremely low that $I_s^t \gg I_p^t$, regardless of the rooting characteristics and water transmitting power of plant tissues. Consequently, equation [44] predicts that Ψ_{mi} will depend primarily upon the osmotic characteristics of the plants and will be rather insensitive to ambient conditions. This conclusion is essentially that of Slatyer's (1967).

This author holds that the final wilting of plants occurs, and water absorption ceases, when the soil moisture potential balances the water potential in plant leaves and the osmotic pressure of the leaf cell sap. Furr and Reeves (1949) also recognized that their "ultimate wilting percentage" was more closely related with the tolerance to desiccation of the wilting plant and that it was less indicative of the characteristics of the soil itself. Figures 60, 61, 62 and 63; 40 through 44; 46 and 47; 48 and 49, among others, show indeed that for a given crop-soil unit the curves obtained under different evaporation intensities converged toward the same final value. The values of Ψ_{mi}, associated with ultimate wilting and the end of the available water range, varied greatly among crops (Table 3). Xerophytic species like Atriplex, and plants typical of arid environments like cotton and privet exhibited the highest values. In contrast, typical mesophytic species like tomato, alfalfa, sunflower, etc. reached Ψ_{mi} at much lower suction values. However, a soil effect was also noted. This may be expected if the fertility of the soils is different, because mineral absorption will influence the osmotic characteristics of the plants (Slatyer, 1961).

Therefore, by recalling that $k = 2.56/\log(\Psi_{mi}/\Psi_{mx})$ and assuming Ψ_{mi} constant it can be expected that: (a) for a given crop-unit, k will increase as evaporation intensities decrease, and (b) the extent of that effect will vary with soil and crop conditions according with the water transmission properties of the particular system.

These conclusions agree with the experimental data summarized in Table 3.

SUMMARY AND CONCLUSIONS

A mathematical expression was developed and tested which describes the relation between evapotranspiration and soil moisture. The general premise of this mathematical model is that the evapotranspiration-soil moisture relationship is determined by interactions of climatic, soil and plant factors.

Four formulas were derived from this basic model. One of them expresses the relation between evapotranspiration and soil moisture in terms of total soil water potential. A second formula expresses the same relationship in terms of soil water content, and was developed from the former by assuming a hyperbolic relationship between soil water potential and water content. These two formulas describe actual evapotranspiration in relation to a maximum evapotranspiration, where maximum evapotranspiration is defined as the water removal that would occur when soil moisture exerted no control over the flow of water from the soil to the atmosphere. Two other formulas were derived as particular cases of the previous ones, e.g., the case where actual evapotranspiration is expressed in relation to the evapotranspiration that would occur at an arbitrarily selected value of soil moisture, such as the field capacity.

These formulas, as well as various other models which are described in the literature, were tested using the results of 48 experiments covering a wide range of climatic, soil and plant variables. Of the 48 experiments, 28 were specifically designed for this study to investigate the relation between soil moisture and evapotranspiration as influenced by evaporative demands of the environment, soil type, plant species and stage of growth. The experiments were conducted in a growth chamber under controlled conditions. The rest of the experiments were selected from the literature to obtain a wider variety of conditions for testing the models.

It was concluded that: (a) Most models advocated in the literature are only adequate to describe the relation between evapotranspiration and soil moisture under particular climatic, soil and plant conditions. The majority of the models do not specify these conditions and have, therefore, little predictive value. (b) The formulas derived from the proposed model provide a good fit for the evapotranspiration-soil moisture relationship under widely differing circumstances. If proper values are chosen for the coefficients, these formulas yield relations that are similar to several of the models taken from the literature. Consequently, the proposed mathematical expression appears to be a general model of the manner in which plants use soil water under different vegetative and environmental conditions. (c) As a result of the general character of the formulas, their coefficients may be used to characterize broad and dynamic ranges of water availability. (d) It seems possible to predict in a comparative way the pattern of water consumption in a soil-plant-atmosphere system. This may be done from a knowledge of the relations between the coefficients of the formulas and climatic, soil and plant factors influencing evapotranspiration. (e) The majority of the experiments analyzed in this study included results that were obtained in rather artificial environments. Hence, it will be necessary to investigate further the suitability of the formulas under field conditions.

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APPENDIXES

Appendix A

Neglecting the influence that simultaneous flows may have (Taylor, 1963), the flow of water from soil to roots can be considered to occur along gradients of water potentials (Gardner, 1966). Flow from roots to leaves can also be treated in this manner, at least in general or qualitative terms (Slatyer, 1967). Flow from leaf to air is in the vapor phase along gradients of water vapor concentration.

To treat these flows on a common basis it will be assumed that flow across each zone (soil-root, root-xylem, leaf-air) is proportional to the water potential difference and inversely proportional to the impedance that develops across it (Van den Honert, 1948; Visser, 1965). Then at any given instant, absorption of water by plant roots, Q, can be described by equation [45] (Gardner, 1966).

$$Q = \frac{\overline{\Psi}_L - \overline{\Psi}_s}{I_p^t + I_s^t} \qquad [45]$$

where,

Q is the rate of water absorption expressed in terms of volume of water absorbed per volume of soil per unit time,

I_p^t is the internal plant impedence (units depend on the units of Ψ),

I_s^t is the soil impedance (units will depend on the units of Ψ),

$\overline{\Psi}_L$ is the average water potential of the foliage,

$\overline{\Psi}_s$ is the average water potential of the bulk soil.

If the flow, Q, is expressed in terms of volume of water per unit time, Q', equation [45] becomes

$$Q' = \left(\frac{\overline{\Psi}_L - \overline{\Psi}_s}{I_p^t + I_s^t}\right) V_s \qquad [46]$$

in which Q' is the absorption flux and V_s is the volume of soil contributing the water.

Solving for $\overline{\Psi}_L$ yields

$$\overline{\Psi}_L = \left(\frac{I_p^t + I_s^t}{V_s}\right) Q' + \overline{\Psi}_s \qquad [47]$$

Similarly, the flow of water from the foliage to the atmosphere is expressed by equation [48]

$$T' = \left(\frac{\Psi_{at} - \overline{\Psi}_L}{I_a^t + I_f^t}\right) V_p \qquad [48]$$

in which:

T' is transpiration flux; i.e., volume of water transpired per unit time.

Ψ_{at} is water potential of the atmosphere.[1]

I_a^p is boundary layer and aerodynamic impedances of the foliage. Its value depends on the wind, morphological features of the foliage and surface characteristics of leaves.

I_f^t is the overall leaf impedance comprising both stomatal and cuticular impedances. Its value is determined by histological,

[1]The use of $\Delta\Psi$ to describe the driving force of the vapor phase is conceptually incorrect; the driving force is actually a difference between water vapor densities or vapor pressures between the interior of leaves and the air around them. The purpose of this usage, however, is to provide an analogy for the overall water transport and analyze the principal factors involved.

physiological and morphological characteristics of leaves and other plant organs.

V_p is the volume of vegetation.

Combining equations [47] and [48] gives

$$T' + \left(\frac{\Psi_{at} - \overline{\Psi}_s}{I_a^t + I_f^t}\right)V_p - \left(\frac{V_p}{V_s}\right)\left(\frac{I_p^t + I_s^t}{I_a^t + I_f^t}\right)Q' \qquad [49]$$

Any difference between T' and Q' must represent a flow from or to the plant tissues, S

$$T' - Q' = S = \left(\frac{dH}{dt}\right)M_p \qquad [50]$$

where:

S is the flux of water from or to the plant tissues,

dH/dt is the dehydration rate,

H is the gravimetric moisture content of plant tissues on a dry matter basis,

M_p is the total dry matter of plant tissues.

When T' > Q', S is positive; the tissues dehydrate yielding a volume of water per unit time. When T' < Q' (often at nighttime), S is negative; the tissues rehydrate as they absorb a volume of water per unit time.

Combining equations [49] and [50], so as to substitute for Q', yields

$$T' = \left(\frac{\Psi_{at} - \overline{\Psi}_s}{I_a^t + I_f^t}\right)V_p - T'\left(\frac{V_p}{V_s}\right)\left(\frac{I_p^t + I_s^t}{I_a^t + I_f^t}\right) + M_p\left(\frac{I_p^t + I_s^t}{I_a^t + I_f^t}\right)\left(\frac{V_p}{V_s}\right)\frac{dH}{dt} \qquad [51]$$

A critical value of $\overline{\Psi}_L$, crit Ψ_L, is adopted above which stomata remain fully open and transpiration proceeds at the maximum rate, T'_{mx}. Below crit Ψ_L, stomata progressively close. At this stage, transpiration is reduced to a degree which prevents further decline in $\overline{\Psi}_L$ and transpiration drops to the level of the prevailing absorption (Slatyer, 1967; Cowan, 1965). Under these conditions; i.e., $T' < T'_{mx}$, $T' = Q'$ and $\frac{dH}{dt} = 0$, equation [51] simplifies to

$$T' = \left(\frac{\Psi_{at} - \overline{\Psi}_s}{I_a^t + I_f^t}\right)V_p - T'\left(\frac{V_p}{V_s}\right)\left(\frac{I_p^t + I_s^t}{I_a^t + I_f^t}\right) \qquad [52]$$

Rearranging terms, and dividing through by $\overline{\Psi}_s$ produces

$$\frac{T'}{\Psi_s} = \left(\frac{V_p}{I_a^t + I_f^t + \left(\frac{V_p}{V_s}\right)\left(I_p^t + I_s^t\right)}\right)\left(\frac{\Psi_{at}}{\Psi_s} - 1\right) = \tau \qquad [53]$$

in which τ is defined as the "transpiration capacity" and expresses an instantaneous ratio between the transpiration flux and the soil water potential.

A similar equation can be written for evaporation directly from the soil, E_s. Assuming that the same volume of soil, V_s, acts as a source for E_s and $T_{s,c}$, the equation for evaporation is

$$E'_s = \left(\frac{\Psi_{at} - \overline{\Psi}_s}{I_a^e + I_s^e}\right)V_s \qquad [54]$$

in which:

E_s is the evaporation flux from the soil.

I_a^e is aerodynamic impedance to water vapor flow from the soil-air interface to the atmosphere. Its value depends on the wind and on the characteristics of the soil surface.

I_s^e is the soil impedance to liquid flow toward and through the capillaries that are transporting water to the surface.

By analogy with equation [53]

$$\frac{E'}{\bar{\Psi}_s} = \left(\frac{V_s}{I_a^e + I_s^e}\right)\left(\frac{\Psi_{at}}{\bar{\Psi}_s} - 1\right) = \nu \qquad [55]$$

wherein ν is defined as the "evaporation capacity" and expresses an instantaneous ratio between the evaporation flux and the soil water potential.

Combining equations [53] and [55] gives

$$\varepsilon = \tau + \nu = \left(\frac{ET_a}{\bar{\Psi}_s}\right) = \left(\frac{V_s}{I_a^e + I_s^e} + \frac{V_p}{I_a^t + I_f^t + \left(\frac{V_p}{V_s}\right)\left(I_p^t + I_s^t\right)}\right)\left(\frac{\Psi_{at}}{\bar{\Psi}_s} - 1\right) \qquad [56]$$

Some of the terms in equation [56] can be expanded further to illustrate the complexity of the interactions involved and make the influence of certain factors more obvious.

$$I_f^t = \frac{I_s I_e}{I_s + I_e} \text{ (Slatyer, 1967)} \qquad [57]$$

where:

I_s is stomatal impedance and

I_e is cuticular impedance. These impedances act in parallel.

$$I_p = I_r + I_x + I_m + I_i \text{ (Slatyer, 1967)} \qquad [58]$$

in which:

I_r is the impedance across the root cortex,

I_x is along xylem vessels,

I_m is through mesophyll,

I_i is intercellular spaces of leaves. These impedances are linked in series.

$$I_s^t = \frac{1}{BLh} \quad \text{(Gardner, 1966)} \qquad [59]$$

in which:

B is a constant that takes into account the actual geometry of water movement from soil to plant roots,

L is length of root per unit volume of soil.

$$h = \frac{a}{b + (\Psi_s)^n} \quad \text{(Gardner, 1958)} \qquad [60]$$

where:

h is capillary conductivity,

a, b and n are empirical constants.

The ratio a/b corresponds to the saturated hydraulic conductivity. Therefore, I_s^t can be expanded to

$$I_s^t = \frac{1}{a/_b BL} - \frac{\Psi_s^n}{aBL} = \text{sat } I_s^t + \frac{\Psi_s^n}{aBL} \qquad [61]$$

in which:

sat I_s^t corresponds to the soil impedance to water flow towards roots at moisture saturation.

By analogy,

$$I_s^e = \frac{1}{\beta ch} + I_c^e \qquad [62]$$

in which:

β is a constant that accounts for the geometry of flow toward capillary pores that conduct water to the surface,

C is length of effective capillaries per unit volume of soil,

I_c^e is impedance to water flow through the capillaries.

The terms β, C and I_c^e presumably depend on the structural condition of the soil.

$$I_s^e = \frac{1}{a/_b \beta C} + \frac{\psi_s^n}{a\beta C} + I_c^e = \text{sat } I_s^e + \frac{\psi_s^n}{a\beta C} + I_c^e \qquad [63]$$

$$\frac{V_p}{V_s} = \left(\frac{\rho_s}{\rho_p}\right)\left(\frac{M_p}{M_s}\right) = \propto\left(\frac{\rho_s}{\rho_p}\right)\left(\frac{M_p}{M_r}\right) = \eta\left(\frac{M_p}{M_r}\right) \qquad [64]$$

in which:

(M_p/M_r) is the shoot-root ratio,

ρ_s is the density of the soil,

ρ_p is the density of the plant tissues,

$\propto = M_r/M_s$ is the mass ratio of roots to soil.

Appendix B

Integration of equation [3] has been performed following the sequence outlined below:

$$\frac{dET_a}{d\Psi_s} = -k\left(\frac{ET_a}{\Psi_s}\right)\left(1 - \frac{ET_a}{ET_s}\right) \qquad [3] \text{ or } [65]$$

Separate variables

$$\frac{dET_a}{ET_a\left(1 - \frac{ET_a}{ET_{mx}}\right)} = -k\,\frac{d\Psi_s}{\Psi_s} \qquad [66]$$

Integrate

$$\int \frac{dET_a}{ET_a\left(1 - \frac{ET_a}{ET_{mx}}\right)} = \log c - k \log \Psi_s \qquad [67]$$

Complete the square of $ET_a[1 - (ET_a/ET_{mx})]$

$$ET_a\left(1 - \frac{ET_a}{ET_{mx}}\right) = -\frac{1}{ET_{mx}}\left(\left((ET_a) - \frac{1}{2}(ET_{mx})\right)^2 - \frac{1}{4}(ET_{mx})\right) \qquad [68]$$

Create the identity,

$$z = (ET_a - \frac{1}{2}ET_{mx}) \qquad [69]$$

Then

$$ET_a = z + \frac{1}{2}ET_{mx} \qquad [70]$$

Differentiate

$$\frac{dET_a}{dz} = 1 \qquad [71]$$

Partially rewrite equation [67]

$$\int \frac{dET_a}{ET_a\left(1 - \frac{ET_a}{ET_{mx}}\right)} = - ET_{mx} \int \frac{ET_a}{\left(ET_a - \frac{1}{2}ET_{mx}\right)^2 - \frac{1}{4}\left(ET_{mx}\right)^2} = ET_{mx} \int \frac{dz}{\left(\frac{1}{2}ET\right)^2 - z^2} \qquad [72]$$

From a table of integrals,

$$ET_{mx} \int \frac{dz}{\left(\frac{1}{2}ET\right)^2 - z^2} = \log \left(\frac{ET_a}{ET_{mx} - ET_a}\right) \qquad [73]$$

Rewrite equation [67] in full

$$\log \left(\frac{ET_a}{ET_{mx} - ET_a}\right) = \log c - k \log \Psi_s \qquad [74]$$

Take antilog

$$\frac{ET_a}{ET_{mx} - ET_a} = c\Psi^{-k} \qquad [75]$$

Rearrange terms

$$ET_a = \left(\frac{c}{c + \Psi^k}\right) ET_{mx} \qquad [5] \text{ or } [76]$$

Appendix C

Table 5. Moisture content, Θ, of Avon clay loam and relative evapotranspiration, ET_a/ET_{mx}, of sunflowers at three evaporation intensities, E_o

Evaporation intensity, E_o, mm day^{-1}					
6.7		4.0		2.1	
Θ	ET_a/ET_{mx} [a]	Θ	ET_a/ET_{mx} [b]	Θ	ET_a/ET_{mx} [c]
%		%		%	
53.7	0.923	51.9	1.142	44.2	1.016
52.7	0.935	50.5	1.217	44.0	1.015
46.3	0.981	45.6	0.963	40.3	0.963
45.1	1.018	43.8	0.947	40.1	0.952
40.5	1.033	40.3	0.857	36.6	0.979
39.2	1.009	38.6	0.857	36.5	0.964
31.9	0.996	35.6	0.991	32.9	1.044
31.2	0.954	33.6	0.986	32.8	1.035
23.8	0.551	30.2	1.031	28.9	0.995
23.8	0.567	28.3	0.964	28.9	1.071
19.4	0.221	24.5	0.840	25.0	0.983
19.3	0.229	23.1	0.700	24.8	0.969
17.7	0.124	19.9	0.400	22.5	0.881
17.6	0.140	19.1	0.329	22.3	0.909
		17.7	0.207	19.8	0.621
		17.4	0.197	18.8	0.550
		16.6	0.159	18.4	0.413
		16.3	0.159	17.9	0.389
		15.7	0.123	17.4	0.253
		15.5	0.109	17.1	0.243
		15.0	0.070	16.0	0.130
		14.8	0.068	15.6	0.099

[a] Data from two pairs of pots with an average ET_{mx} = 52.9 gr day^{-1} pot^{-1}.
[b] Data from two pairs of pots with an average ET_{mx} = 36.0 gr day^{-1} pot^{-1}.
[c] Data from two pairs of pots with an average ET_{mx} = 25.0 gr day^{-1} pot^{-1}.

Table 6. Moisture content, Θ, of Millville silt loam and relative evapotranspiration, ET_a/ET_{mx}, of sunflowers at three evaporation intensities, E_o

Evaporation intensity, E_o, mm day^{-1}					
6.7		4.0		2.1	
Θ	ET_a/ET_{mx} [a]	Θ	ET_a/ET_{mx} [b]	Θ	ET_a/ET_{mx} [c]
%		%		%	
31.9	0.946	33.3	1.120	33.0	1.000
31.5	0.940	32.9	1.210	29.9	1.030
27.0	1.027	29.9	0.995	29.3	1.000
26.4	1.015	28.8	0.900	27.2	1.040
23.9	1.032	25.7	0.895	26.7	0.968
22.3	1.048	25.7	0.840	24.9	1.080
17.4	0.982	22.8	1.020	23.7	1.000
14.6	0.706	21.8	0.968	22.6	0.967
12.8	0.543	19.3	1.060	20.6	0.977
10.6	0.270	18.1	0.955	20.6	0.920
10.2	0.266	15.6	0.980	18.6	0.952
9.1	0.149	13.2	0.717	17.4	0.932
9.0	0.160	12.3	0.580	16.6	0.914
8.2	0.115	10.5	0.331	14.6	0.842
		10.3	0.323	14.5	0.848
		9.2	0.213	12.8	0.742
		9.2	0.220	11.8	0.598
		8.4	0.152	11.1	0.523
		8.4	0.141	10.0	0.395
		7.9	0.125	9.9	0.366
		7.9	0.123	9.2	0.276
		7.4	0.075	8.8	0.218
				8.6	0.214
				8.1	0.167
				8.1	0.152
				7.8	0.074
				7.6	0.115

[a] Data from two pairs of pots with an average ET_{mx} = 52.2 gr day^{-1} pot^{-1}.
[b] Data from two pairs of pots with an average ET_{mx} = 35.8 gr day^{-1} pot^{-1}.
[c] Data from two pairs of pots with an average ET_{mx} = 26.0 gr day^{-1} pot^{-1}.

Table 7. Moisture content, Θ, of Avon clay loam and relative evapotranspiration, ET_a/ET_{mx}, of wheat at three evaporative intensities, E_o

Evaporation intensity, E_o, mm day^{-1}					
5.6		3.8		2.2	
Θ	ET_a/ET_{mx} [a]	Θ	ET_a/ET_{mx} [b]	Θ	ET_a/ET_{mx} [c]
%		%		%	
44.7	1.000	43.7	1.101	43.8	1.009
44.1	1.028	40.9	1.110	41.4	0.966
40.5	1.035	40.0	0.989	41.2	1.022
39.5	0.945	37.3	1.000	39.7	0.954
36.2	0.974	36.6	0.952	39.0	0.985
34.9	1.002	34.0	1.010	36.7	1.043
31.8	0.984	33.6	0.951	36.4	1.009
30.4	1.012	30.8	0.893	34.2	1.000
27.4	0.925	30.2	0.993	33.8	1.027
26.1	0.680	27.8	0.996	31.2	0.995
23.6	0.724	26.9	0.902	31.0	1.019
22.3	0.709	24.5	0.937	28.8	0.990
20.5	0.508	23.6	0.823	28.5	0.975
19.3	0.492	21.5	0.800	26.2	0.971
18.3	0.355	20.8	0.763	26.2	0.981
17.0	0.325	18.8	0.600	23.8	0.961
16.8	0.248	18.3	0.597	23.8	0.945
15.6	0.230	16.8	0.410	21.5	0.904
		16.5	0.331	21.4	0.879
		15.5	0.224	19.4	0.758
		15.2	0.254	19.2	0.751
		14.8	0.186	17.5	0.565
		14.4	0.114	17.2	0.470
		14.1	0.131	16.1	0.429
				16.1	0.294
				15.4	0.275
				15.0	0.171
				14.7	0.164
				14.7	0.149
				14.2	0.149

[a] Data from two pair of pots with an average ET_{mx} = 38.1 gr day^{-1} pot^{-1}.
[b] Data from two pair of pots with an average ET_{mx} = 28.8 gr day^{-1} pot^{-1}.
[c] Data from two pair of pots with an average ET_{mx} = 21.4 gr day^{-1} pot^{-1}.

Table 8. Moisture content, Θ, of Millville silt loam and relative evapotranspiration, ET_a/ET_{mx}, of wheat at three evaporative intensities, E_o

Evaporation intensity, E_o, mm day^{-1}					
5.6		3.8		2.2	
Θ	ET_a/ET_{mx} [a]	Θ	ET_a/ET_{mx} [b]	Θ	ET_a/ET_{mx} [c]
%		%		%	
31.7	1.010	34.8	1.052	35.4	1.026
29.7	0.995	33.2	1.101	33.6	1.007
26.7	0.990	30.2	0.962	32.0	1.049
24.0	1.005	29.3	0.985	30.8	1.030
21.8	0.950	25.8	0.949	28.6	0.947
18.3	0.921	25.0	0.985	28.0	0.961
17.1	0.850	22.4	0.988	25.5	0.980
13.2	0.580	21.8	0.972	24.1	1.000
12.9	0.544	18.7	0.943	22.2	0.977
10.1	0.301	17.4	0.930	21.3	0.976
9.8	0.263	15.4	0.844	19.0	0.960
8.7	0.192	13.5	0.723	18.5	0.942
8.4	0.151	12.3	0.667	15.9	0.898
7.6	0.130	10.4	0.457	15.8	0.903
7.5	0.115	9.8	0.374	13.4	0.796
7.1	0.097	8.6	0.215	12.9	0.724
6.9	0.089	8.4	0.194	11.1	0.565
6.4	0.076	7.8	0.164	10.6	0.534
		7.6	0.136	9.5	0.369
		7.2	0.107	8.8	0.275
		7.0	0.084	8.5	0.257
		6.8	0.098	7.8	0.147
		6.6	0.079	7.8	0.150
				7.4	0.138
				7.4	0.137
				6.9	0.092
				6.9	0.081

[a] Data from two pair of pots with an average ET_{mx} = 49.5 gr day^{-1} pot^{-1}.
[b] Data from two pair of pots with an average ET_{mx} = 36.8 gr day^{-1} pot^{-1}.
[c] Data from two pair of pots with an average ET_{mx} = 28.3 gr day^{-1} pot^{-1}.

Table 9. Moisture content, Θ, of Avon clay loam and relative evapotranspiration, ET_a/ET_{mx}, of lentils at three evaporation intensities, E_o, and at an early stage of growth

Evaporation intensity, E_o, mm day^{-1}					
4.8		2.9		2.2	
Θ	ET_a/ET_{mx} [a]	Θ	ET_a/ET_{mx} [b]	Θ	ET_a/ET_{mx} [c]
%		%		%	
47.7	0.980	49.6	1.038	44.8	0.992
42.2	1.000	45.0	0.987	42.5	1.016
36.4	0.914	44.8	0.972	41.8	0.979
35.5	0.885	44.0	1.015	41.0	1.026
31.4	0.870	40.4	0.955	38.2	0.997
31.1	0.827	40.0	1.027	37.2	0.977
29.6	0.795	39.0	0.984	36.6	1.020
26.3	0.613	36.0	1.019	33.6	0.985
26.2	0.638	34.8	0.985	33.4	1.011
23.8	0.470	34.0	0.962	33.2	1.000
22.8	0.459	31.4	0.967	29.6	0.977
21.9	0.344	29.5	0.955	29.3	0.963
20.4	0.279	29.3	0.944	29.0	0.970
20.1	0.349	27.5	0.919	26.2	0.931
19.6	0.228	24.8	0.841	25.7	0.904
18.4	0.188	24.6	0.825	24.9	0.900
18.1	0.243	22.8	0.714	21.2	0.679
18.0	0.172	20.5	0.545	20.9	0.648
17.0	0.159	20.4	0.524	20.8	0.630
16.9	0.146	19.6	0.512	19.8	0.583
16.7	0.209	17.8	0.334	18.2	0.428
15.9	0.129	17.7	0.325	18.0	0.373
15.9	0.139	17.3	0.262	17.2	0.286
15.0	0.109	16.2	0.131	16.4	0.192
14.9	0.113	16.2	0.185	16.3	0.180
14.3	0.094	16.0	0.200	16.3	0.167
14.1	0.101	15.5	0.118	15.7	0.139
		15.2	0.121	15.5	0.130
		15.0	0.091	14.9	0.091
		14.8	0.098	14.8	0.090
		14.5	0.057	14.6	0.091
		14.2	0.055	14.6	0.062
		12.2	0.047	14.5	0.070

[a] Data from three pair of pots with an average ET_{mx} = 51.7 gr day^{-1} pot^{-1}.
[b] Data from three pair of pots with an average ET_{mx} = 37.0 gr day^{-1} pot^{-1}.
[c] Data from three pair of pots with an average ET_{mx} = 31.7 gr day^{-1} pot^{-1}.

Table 10. Moisture content, Θ, of Millville silt loam and relative evapotranspiration, ET_a/ET_{mx}, of lentils at three evaporative intensities, E_o, and at an early stage of growth

Evaporation intensity, E_o, mm day^{-1}					
4.8		2.9		2.2	
Θ	ET_a/ET_{mx} [a]	Θ	ET_a/ET_{mx} [b]	Θ	ET_a/ET_{mx} [c]
%		%		%	
28.7	0.949	38.6	1.019	38.2	0.988
27.3	0.975	35.7	1.020	36.8	0.988
24.7	1.000	34.2	0.965	34.9	1.022
24.5	1.051	32.7	0.986	33.3	1.024
21.2	1.025	30.2	1.014	31.3	0.985
19.8	0.914	26.8	0.993	29.5	0.991
18.9	0.901	26.0	0.973	27.7	1.002
15.8	0.547	22.4	0.958	25.8	0.994
15.0	0.457	21.8	0.953	24.1	0.985
13.6	0.380	18.2	0.884	22.1	0.969
13.3	0.303	17.8	0.827	20.6	0.957
12.1	0.190	14.3	0.666	18.5	0.919
12.0	0.204	14.3	0.622	17.0	0.839
11.4	0.178	11.5	0.296	15.1	0.800
11.1	0.154	11.4	0.253	14.2	0.687
10.9	0.115	10.4	0.172	12.1	0.351
10.4	0.112	10.2	0.166	11.6	0.375
10.3	0.115	9.7	0.121	10.8	0.243
10.1	0.092	9.5	0.131	10.3	0.180
9.9	0.096	9.2	0.107	9.6	0.131
9.7	0.095	8.8	0.068	9.2	0.091
9.7	0.082	8.8	0.060	9.0	0.083
9.2	0.065			8.8	0.074
9.1	0.082				
8.8	0.061				
8.7	0.066				
8.3	0.063				

[a] Data from three pair of pots with an average ET_{mx} = 53.8 gr day^{-1} pot^{-1}.
[b] Data from two pair of pots with an average ET_{mx} = 42.1 gr day^{-1} pot^{-1}.
[c] Data from two pair of pots with an average ET_{mx} = 35.5 gr day^{-1} pot^{-1}.

Table 11. Moisture content, Θ, of Avon clay loam and Millville silt and relative evapotranspiration, ET_a/ET_{mx}, of lentils under an evaporation intensity of 2.2 mm day^{-1} and at a late stage of growth

Avon clay loam		Millville silt loam	
Θ	ET_a/ET_{mx}[a]	Θ	ET_a/ET_{mx}[b]
%		%	
36.7	0.977	26.0	1.000
32.4	1.000	23.4	0.942
32.2	1.000	22.0	0.900
31.5	1.000	18.7	0.758
27.7	1.011	18.3	0.796
27.7	0.900	15.1	0.462
27.7	0.855	15.0	0.430
24.3	0.760	13.3	0.359
23.7	0.676	12.9	0.225
23.0	0.636	11.8	0.205
21.4	0.458		
20.6	0.417		
20.1	0.467		
19.7	0.442		
18.6	0.384		

[a]Data from three pair of pots with an average ET_{mx} = 36.9 g day^{-1} pot^{-1}.

[b]Data from two pair of pots with an average ET_{mx} = 44.0 g day^{-1} pot^{-1}.

Table 12. Moisture content, Θ, of Avon clay loam and relative evapotranspiration, ET_a/ET_{fc}, of beans under two evaporation intensities, E_o, and at two stages of growth[a]

Evaporation intensity, E_o, mm day^{-1}							
8.0				4.5			
Stage of growth				Stage of growth			
I		II		I		II	
Θ	ET_a/ET_{fc}	Θ	ET_a/ET_{fc}	Θ	ET_a/ET_{fc}	Θ	ET_a/ET_{fc}
%		%		%		%	
28.8	0.795	32.1	0.746	39.9	1.000	34.4	1.010
25.2	0.615	26.6	0.451	34.7	0.995	34.1	0.988
23.7	0.518	25.5	0.407	29.6	0.885	30.5	0.935
23.7	0.438	24.8	0.337	29.2	0.902	29.8	0.794
23.3	0.458	22.7	0.300	26.4	0.785	29.7	0.775
19.6	0.207	22.0	0.227	25.6	0.697	28.9	0.775
17.7	0.179	19.7	0.185	25.1	0.678	26.5	0.614
17.7	0.113	19.7	0.119	23.9	0.636	25.5	0.695
17.6	0.167	18.8	0.165	21.9	0.421	24.2	0.565
17.6	0.142	18.7	0.151	21.7	0.573	23.9	0.477
17.2	0.148			21.7	0.437	23.8	0.585
16.2	0.123			20.6	0.447	21.9	0.439
15.8	0.122			19.4	0.269	21.3	0.468
15.6	0.139			19.2	0.327	20.2	0.293
15.6	0.122			18.9	0.338	20.1	0.283
				18.8	0.349	19.5	0.217
				17.9	0.134	19.3	0.428
				17.5	0.223	18.4	0.174
				16.7	0.178	18.1	0.204
				16.3	0.154	18.1	0.166
						16.8	0.143
						15.8	0.147

[a]Stage II corresponds to flower initiation; stage I corresponds to approximately 3/4 the vegetative development of stage II.

Table 13. Moisture content, Θ, of Millville silt loam and relative evapotranspiration, ET_a/ET_{fc}, of beans under two evaporation intensities, E_o, and at two stages of growth[a]

Evaporation intensity, E_o, mm day^{-1}							
7.3				3.8			
Stage of growth				Stage of growth			
I		II		I		II	
Θ	ET_a/ET_{fc}	Θ	ET_a/ET_{fc}	Θ	ET_a/ET_{fc}	Θ	ET_a/ET_{fc}
%		%		%		%	
23.6	0.678	24.7	0.493	30.9	0.990	27.9	0.918
23.1	0.633	20.9	0.378	25.9	0.730	23.5	0.781
19.8	0.477	19.7	0.305	24.8	0.788	20.6	0.505
19.3	0.457	19.6	0.305	24.7	0.788	19.7	0.430
18.5	0.417	18.6	0.271	22.2	0.646	16.6	0.256
14.3	0.194	16.1	0.131	21.7	0.589	16.6	0.290
13.6	0.189	15.9	0.176	19.8	0.475	14.5	0.174
13.2	0.180	13.9	0.122	18.7	0.505	14.4	0.192
13.1	0.167	13.3	0.111	18.2	0.492	14.1	0.136
12.5	0.155	13.2	0.125	16.8	0.372	13.1	0.136
11.6	0.139			16.7	0.338	12.8	0.105
10.8	0.118			16.1	0.330	12.0	0.134
10.7	0.116			14.0	0.307	12.0	0.099
10.7	0.108			13.7	0.204		
10.2	0.097			12.0	0.173		
				10.9	0.152		
				9.2	0.093		

[a]Stage II corresponds to flower initiation; stage I corresponds approximately to 3/4 the vegetative development of stage II.

Table 14. Moisture content, Θ, of Colo. silty clay loam and relative evapotranspiration, ET_a/ET_{fc}, of corn at five evaporative intensities, ET_{fc}[a]

Evaporation intensity, ET_{fc}[b] mm day^{-1}									
2.0		3.3		4.1		5.6		6.4[c]	
Θ	ET_a/ET_{fc}	Θ	ET_a/ET_{fc}	Θ	ET_a/ET_{fc}	Θ	ET_a/ET_{fc}	Θ	ET_a/ET_{fc}
%		%		%		%		%	
36	1.000	36	1.000	36	1.000	36	1.000	36	1.000
34	1.000	34	1.000	34	1.000	34	0.988	34	0.936
32	1.000	32	1.000	32	1.000	32	0.944	32	0.429
30	1.000	30	0.992	30	0.977	30	0.822	30	0.333
28	1.000	28	0.955	28	0.887	28	0.655	28	0.177
26	0.962	26	0.854	26	0.631	26	0.371	26	0.083
25	0.888	24	0.419	24	0.289	24	0.182	24	0.023
24	0.746	22	0.156	22	0.132	22	0.044		
23	0.507								
22	0.361								

[a]Data of Denmead and Shaw, extracted from graphs given in Slatyer (1967, p. 55).

[b]Evaporation intensities are given here in terms of evapotranspiration at field capacity; the first four values of ET_{fc} are presumably the same as $ET_{mx} = gE_o$.

[c]This value of ET_{fc} is presumably lower than ET_{mx} by a factor of 0.85, so that $ET_{fc} = 0.85\ gE_o$.

Table 15. Moisture content, Θ, of a loamy clay soil and relative evapotranspiration, ET_a/ET_{fc}, of oats under three different environments[a]

Θ	ET_a/ET_{fc}	Θ	ET_a/ET_{fc}
%		%	
34.6	1.000	(continued)	
34.3	0.985	24.1	0.530
34.0	1.020	23.7	0.520
32.7	1.015	23.5	0.420
32.1	0.990	23.0	0.367
31.8	1.020	22.7	0.340
30.5	1.000	22.5	0.289
30.2	1.010	21.9	0.300
29.8	1.000	21.9	0.265
29.4	0.915	21.5	0.190
28.5	0.795	21.3	0.265
27.5	0.835	21.0	0.280
26.8	0.822	20.8	0.220
26.8	0.694	20.6	0.115
25.8	0.640	20.5	0.220
25.8	0.610	19.8	0.080
25.5	0.661	19.4	0.100
24.5	0.585	19.4	0.040
24.3	0.556	18.9	0.050

[a]Data of Sudnitzin (1968), extracted from graph. Although the data correspond to three ambient conditions they were given together in the original reference because no major differences were noted between them.

Table 16. Soil moisture content, Θ, and relative evapotranspiration, ET_a/ET_{fc}, of cotton during two different seasons[a]

Season			
Winter (ET_{mx} = 2.9 mm day^{-1})		Summer (ET_{mx} = 9.2 mm day^{-1})	
Θ	ET_a/ET_{fc}	Θ	ET_a/ET_{fc}
%		%	
19.4	1.04	19.7	1.03
18.4	1.07	17.8	0.98
17.5	0.91	16.2	0.97
16.2	0.85	13.9	0.98
15.9	0.87	12.3	1.04
15.3	1.10	10.8	0.74
14.9	1.03	10.1	0.58
13.0	0.82	8.7	0.26
12.4	0.67	8.2	0.25
12.0	0.45	8.0	0.10
11.5	0.71		
11.0	0.63		
10.9	0.68		
10.4	0.41		
10.4	0.55		
10.2	0.32		
9.8	0.54		
9.7	0.48		
9.3	0.29		
9.2	0.24		
9.1	0.21		
9.0	0.20		
8.8	0.23		
8.5	0.18		
8.5	0.14		
8.5	0.04		
8.4	0.24		
8.2	0.03		

[a]Data extracted from graph given by Palmer et al. (1964).

Table 17. Soil moisture content, Θ, and relative evapotranspiration, ET_a/ET_{fc}, of Atriplex spp. during two different seasons[a]

Season			
Winter (ET_{mx} = 1.9 mm day^{-1})		Summer (ET_{mx} = 6.2 mm day^{-1})	
Θ	ET_a/ET_{fc}	Θ	ET_a/ET_{fc}
%		%	
21.2	1.07	21.7	1.03
20.0	1.06	20.6	1.01
18.5	0.80	19.8	1.02
17.9	0.98	17.9	1.02
16.0	1.11	17.0	1.03
15.2	0.94	15.9	0.93
14.7	1.05	14.6	1.02
14.3	0.97	13.3	0.99
13.8	0.96	12.1	0.92
12.8	1.16	10.9	0.88
11.5	1.00	9.9	0.84
11.3	0.79	9.0	0.68
9.5	0.63	8.4	0.56
9.4	0.60	7.7	0.44
9.0	0.56		
8.5	0.35		
8.4	0.26		
8.2	0.27		
8.2	0.16		
8.1	0.24		

[a]Data extracted from graphs given by Palmer et al. (1964).

Table 18. Soil water potential, Ψ_s, and relative evapotranspiration, ET_a/ET_{mx}, of alfalfa under two growing conditions

Arizona[a]		Utah[b]	
Ψ_s	ET_a/ET_{mx}	Ψ_s	ET_a/ET_{mx}
bars		bars	
0.55	1.000	0.29	0.856
0.74	1.000	0.43	0.860
0.92	1.000	0.91	0.707
1.38	1.000	1.76	0.458
1.94	1.000	2.59	0.303
2.49	1.000	3.14	0.221
3.51	1.000	3.70	0.163
3.97	0.960	8.14	0.049
4.78	0.922		
5.72	0.820		
7.75	0.657		
9.51	0.500		
10.06	0.333		
10.95	0.161		

[a]Data extracted from graph, Van Bavel (1967).
[b]Data extracted from graph, Bahrani and Taylor (1961).

Table 19. Moisture content, Θ, of clay loam soil and relative evapotranspiration, ET_a/ET_{mx}, of ladino clover at three stages of growth and under the same environmental conditions[a]

Stages of growth[b]					
I		II		III[c]	
$ET_{mx} = 1.16$ g hr^{-1}		$ET_{mx} = 1.62$ g hr^{-1}		$ET_{mx} = 2.60$ g hr^{-1}	
Θ	ET_a/ET_{mx}	Θ	ET_a/ET_{mx}	Θ	ET_a/ET_{mx}
%		%		%	
17.9	1.000	21.3	0.879	21.1	0.920
17.0	1.000	20.1	1.140	20.4	1.000
16.3	0.991	18.7	0.959	19.3	1.000
15.5	1.044	18.1	1.000	19.0	1.000
14.7	0.965	15.8	1.020	17.6	0.925
14.1	0.882	14.7	0.967	16.7	0.935
13.5	0.735	13.7	0.904	16.1	1.037
12.9	0.661	12.8	0.753	15.5	0.848
12.5	0.514	12.1	0.565	14.7	0.859
12.1	0.514			14.3	0.750
				14.1	0.839
				13.7	0.685
				13.1	0.747
				12.9	0.624
				12.5	0.672

[a]Data of Hagen et al. (1957), extracted from graphs.
[b]These are given in terms of evapotranspiration when fully watered because the differences noted are a result of the differences in size of the plants, the environment was the same.
[c]These data represent stages 3 and 4 of the original reference. They have been treated as one because of scarcity of data for a regression analysis.

Table 20. Soil moisture content, Θ^a, and relative evapotranspiration, ET_a/ET_{fc}, of pepper and birdsfoot trefoil plants, grown under the same ambient conditions

Pepper		Birdsfoot trefoil	
Θ	ET_a/ET_{fc} [b]	Θ	ET_a/ET_{fc} [c]
%		%	
6.3	1.000	8.8	1.000
5.7	0.835	7.5	0.763
4.5	0.552	6.9	0.654
3.6	0.274	5.5	0.418
3.0	0.105	4.2	0.290

[a] Data of Gardner and Ehlig (1963), extracted from graphs.
[b] $ET_{fc} = ET_{mx} = 23.7$ mm day^{-1}.
[c] $ET_{fc} = 0.856\ ET_{mx} = 37.3$ mm day^{-1}.

Table 21. Total soil moisture potential, Ψ_s, and relative evapotranspiration, ET_a/ET_{mx}, of tomato, cotton and privet grown under similar environmental conditions[a]

Crop					
Tomato		Cotton		Privet	
Ψ_s	ET_a/ET_{mx}[b]	Ψ_s	ET_a/ET_{mx}[c]	Ψ_s	ET_a/ET_{mx}[d]
bars		bars		bars	
2.5	1.000	2.5	1.000	3.1	1.000
7.0	0.717	7.0	0.879	3.9	0.905
10.9	0.428	10.9	0.619	5.0	0.877
14.8	0.259	14.8	0.371	11.7	0.820
21.8	0.162	21.8	0.366	17.1	0.707
27.3	0.104	27.3	0.333	24.9	0.565
28.2	0.065	28.2	0.333	29.6	0.424
29.6	0.052	29.6	0.198	36.6	0.339
32.4	0.059	32.4	0.148	41.3	0.184
34.3	0.052	34.3	0.111	48.3	0.099
36.6	0.039	36.6	0.111	54.5	0.090
38.6	0.033	38.6	0.086	61.5	0.141
40.5	0.039	40.5	0.068	68.6	0.127
42.1	0.033	42.1	0.074	74.8	0.099
43.6	0.033	43.6	0.068	79.5	0.085
44.7	0.039	44.7	0.063	86.5	0.090
				91.9	0.071
				96.6	0.042
				99.7	0.071
				103.6	0.065

[a]Data of Slatyer (1957), extracted from graphs.

[b]ET_{mx} = 223.8 gr day^{-1} $plant^{-1}$.

[c]ET_{mx} = 117.4 gr day^{-1} $plant^{-1}$.

[d]ET_{mx} = 101.7 gr day^{-1} $plant^{-1}$.

Appendix D

The following examples illustrate the kind of computations that were performed to obtain the theoretical curves shown in Figures 12 through 59.

Example 1

Test of equation [7]. Equation [7] was written thus,

$$ET_a = \left(\frac{n}{n + \Theta^{-m}}\right) ET_{mx} \qquad [7]$$

First, evaluate ET_{mx} using equation [11]. To do this, plot $1/ET_a$ vs Θ on arithmetic paper. For instance, from Table 5, Appendix C, under an evaporation intensity of 2.1 mm day^{-1}, we can recompute the original values of ET_a, hence $1/ET_a$. These values are given below,

Θ	$1/ET_a$
%	day pot gr^{-1}
44.2	0.0394
44.0	0.0394
40.3	0.0415
40.1	0.0420
36.6	0.0408
36.5	0.0415
32.9	0.0383
32.8	0.0386
28.9	0.0402
28.9	0.0373
25.0	0.0407
24.8	0.0413
22.5	0.0453
22.3	0.0440
19.8	0.0644
18.8	0.0727
18.4	0.0968
17.9	0.1030
17.4	0.1580

(continued)

Θ	$1/ET_a$
%	day pot gr^{-1}
17.1	0.1650
16.0	0.3080
15.6	0.4050

Figure 67 gives the plot of Θ vs $1/ET_a$. Extrapolation from the horizontal asymptote gives $1/ET_{mx} = 0.04$; i.e., $ET_{mx} = 25$ gr day^{-1} pot^{-1}. In this case, it is easy to find $1/ET_{mx}$ from inspection of the horizontal asymptote. In cases where this is less obvious $1/ET_{mx}$ can be computed from the following relationships:

$$\frac{1}{ET_{mx}} = \frac{\left(\frac{1}{ET_a}\right)_1 \left(\frac{1}{ET_a}\right)_2 - \left(\frac{1}{ET_a}\right)_3^2}{\left(\frac{1}{ET_a}\right)_1 + \left(\frac{1}{ET_a}\right)_2 - 2\left(\frac{1}{ET_a}\right)_3} \qquad [77]$$

The values $(1/ET_a)_1$ and $(1/ET_a)_2$ can be arbitrarily chosen, but $(1/ET_a)_3$ must be selected so that

$$\Theta_3 = \sqrt{\Theta_1 \cdot \Theta_2} \qquad [78]$$

in which Θ_1, Θ_2, and Θ_3 are $f(1/ET_a)_1$, $f(1/ET_a)_2$ and $f(1/ET_a)_3$, respectively. The necessary values are obtained from a smooth curve as shown in Figure 67.

Second, perform the computations shown in Table 22. It is not necessary to include the values of ET_a/ET_{mx} that approach 1.

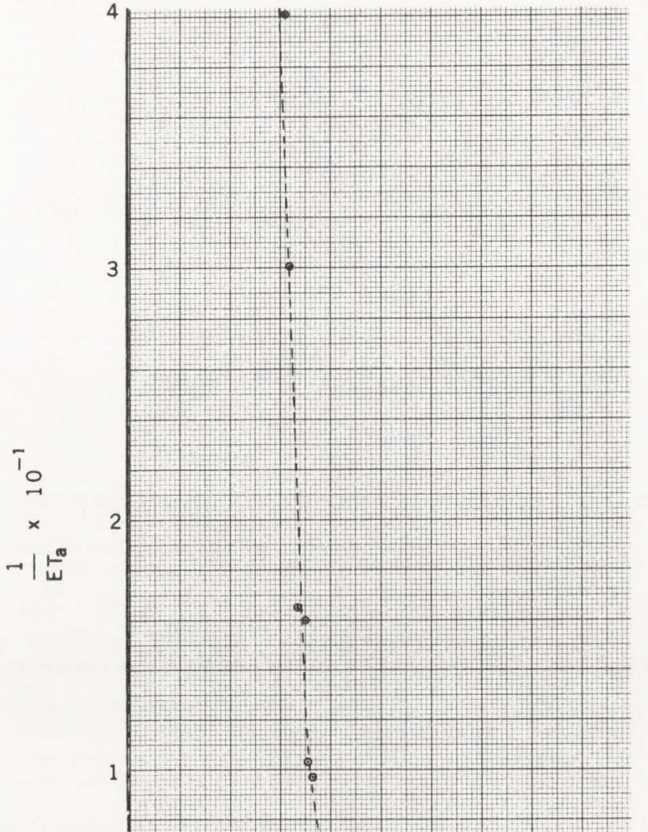

Figure 67. Plot of $1/ET_a$ vs soil water content. Data of Table 5, Appendix D, for an evaporation intensity of 2.1 mm day^{-1}.

Table 22. Regression analysis of the data in Table 5 for an evaporation intensity of 2.1 mm day^{-1}

$\frac{ET_a}{ET_{mx}}$	$\frac{ET_a}{ET_{mx} - ET_a}$	Θ	$\log \left(\frac{ET_a}{ET_{mx} - ET_a}\right)$ Y	$\log \Theta$ X
0.983	60.152	25.0	1.7792	1.3979
0.969	32.111	24.8	1.5066	1.3944
0.881	7.435	22.5	0.8713	1.3522
0.909	10.087	22.3	1.0036	1.3483
0.621	1.642	19.8	0.2154	1.2967
0.550	1.227	18.8	0.0888	1.2742
0.413	0.703	18.4	-0.1530	1.2648
0.389	0.638	17.9	-0.1952	1.2528
0.253	0.339	17.4	-0.4698	1.2406
0.243	0.321	17.1	-0.4935	1.2330
0.130	0.149	16.0	-0.8268	1.2041
0.099	0.110	15.6	-0.9586	1.1931

$SX = 15.4521$ $\qquad$ $SY = 2.3680$

$\bar{x} = 1.2876$ $\qquad$ $\bar{y} = 0.1973$

$SX^2 = 19.9518$ $\qquad$ $SY^2 = 9.3843$ $\qquad$ $S(XY) = 3.7431$

$\frac{(SX)^2}{12} = 19.8972$ $\qquad$ $\frac{(SY)^2}{12} = 0.4672$ $\qquad$ $\frac{(SX)(SY)}{12} = 3.0492$

$Sx^2 = 0.0546$ $\qquad$ $Sy^2 = 8.9172$ $\qquad$ $Sxy = 0.6939$

$m = 12.709$

$r = 0.994$**

$\hat{Y} = 12.709 X - 16.167$

A plot of log $[ET_a/(ET_{mx} - ET_a)]$ vs log Θ on arithmetic paper is shown in Figure 68, The regression line computed in Table 22 is also given. This line is used to generate the sigmoidal curves showing the theoretical relationship between soil moisture content, Θ, and the relative evapotranspiration, ET_a/ET_{mx}.

Example 2

Test of equation [5]. Equation [5] was written thus:

$$ET_a = \left(\frac{c}{c + \Psi_s^k}\right) ET_{mx} \qquad [5]$$

First, evaluate ET_{mx}, if it is unknown, using equation [10] which was derived from equation [5]. To do this, plot $1/ET_a$ vs Ψ_s on arithmetic paper. An exponential curve should result that does not pass through the origin of the coordinates. Extrapolation to the ordinate axle gives $1/ET_{mx}$. This may be done by simple inspection of the smooth curve drawn through the data provided the curve does not have to be extended too far beyond the observations.

Second, perform the type of computations shown in Table 22. In this case, however, the term X will represent the logarithm of the Ψ_s values. A plot of log $[ET_a/(ET_{mx} - ET_a)]$ vs log Ψ_s on arithmetic paper should yield a straight line of negative slope equal to k. The regression analysis will give the line of best fit from which the theoretical relationship between soil water potential, Ψ_s, and relative evapotranspiration, ET_a/ET_{mx}, is obtained.

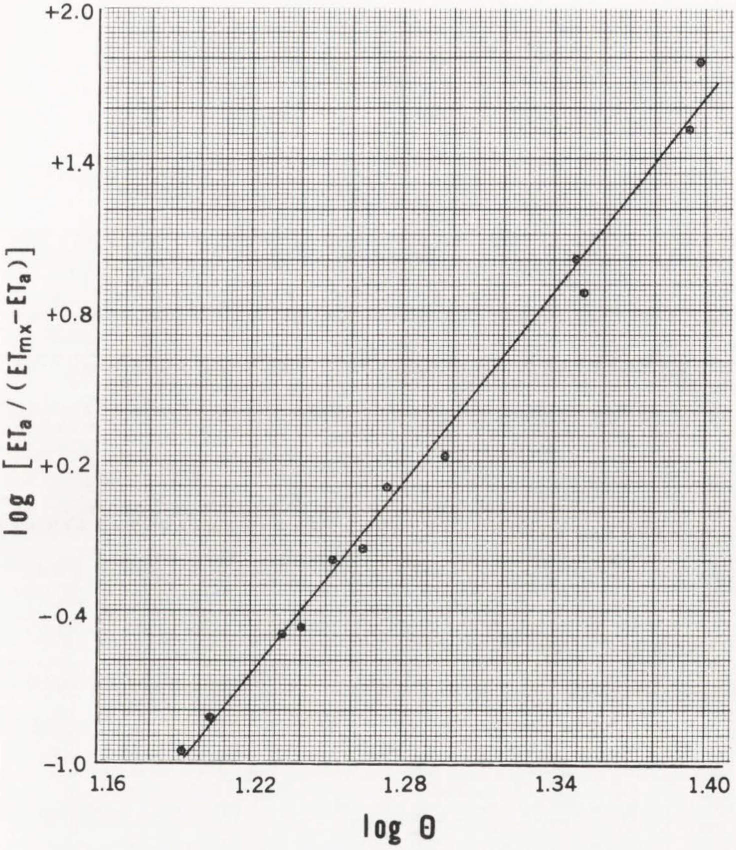

Figure 68. Plot of log $[ET_a/(ET_{mx} - ET_a)]$ vs log soil water content for the data given in Table 22.

VITA

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