Statistical Properties and Problems in Modeling the Bolivian Foreign Exchange Market

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STATISTICAL PROPERTIES AND PROBLEMS IN MODELLING THE BOLIVIAN FOREIGN EXCHANGE MARKET

by

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ABSTRACT

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The Bolivian foreign exchange market is explained in terms of the official and parallel exchange rates. The data covers the post hyper inflationary period from 1986 to 1992. The distribution of the rate of depreciation of the official and parallel exchange rates is long tailed and strongly departs from normality due to the existence of outliers. A market interactions model of the autoregressive kind is estimated using robust regression. This procedure produces M-parameter estimates using iteratively reweighted least squares. The robust method handles well the outlier problem and at the same time it reveals the true nature of the statistical properties of the data by not being able to produce white noise in the squared residuals. Both markets show a one-time break in the variance creating two periods of differential behavior, with one of them having GARCH properties. Robust unit root and cointegration tests also fail to produce white noise squared residuals due to the same phenomena. Further research requires the development of a robust procedure that could take care of the outlier and heteroskedasticity problems simultaneously.

STATISTICAL PROPERTIES AND PROBLEMS IN MODELLING
THE BOLIVIAN FOREIGN EXCHANGE MARKET

Chapter

CHAPTER I

IINTRODUCTION

In September 1985 the Bolivian economy was stabilized after a period of hyperinflation during the previous two years. The success of the stabilization program relied on several actions, all of which were based on the premise of the need to change the economic system. After stabilization Bolivia was a different country, it had moved from a state run economy towards a market economy. Among the required actions, probably the most important was its first step; the stabilization of the foreign exchange market. The success in the stabilization of this market went also along the lines of changing its system. Before the inflationary period, the exchange rate was set at a fixed official rate. During high inflation the government not only continued this policy but also increased its control over Dollar reserves in order to avoid depletion. This created an excess demand for Dollars which resulted in a highly volatile parallel exchange rate. In the end, as Sachs (1987) explains, the parallel exchange rate may have been the true force behind the hyperinflation.

Stabilization of the foreign exchange market was reached by introducing a managed float system, in which a specified minimum amount of Dollars is sold daily in an auction, therefore letting the daily rate be set according to market forces of supply and demand. The result was an almost complete realignment of the official and parallel rates. Almost complete because this last did not disappear. In fact, there is still demand not attended by the new system which discriminates against small firms and individuals who require amounts below the daily minimum. The sector of small firms is of considerable importance in the Bolivian case, it has been the most dynamic sector after stabilization with an important impact in the rest of the economy. The parallel exchange rate is set by market forces within this sector and therefore has a life of its own, probably to the point that it might be influencing the official rate itself. Furthermore, its observed sensitivity compared to the official rate is a permanent reminder that it is there and ready to take over any time the official system fails.

The objectives of this study are: 1) To determine the statistical properties of the official and parallel exchange rates data after stabilization, and 2) To identify appropriate techniques that better incorporate these properties into exchange rate modeling. The purposes of the objectives are: a) To improve our understanding of short-term dynamics and b) To determine how short-term dynamics relate to long-run equilibrium.

Several studies have been done in this direction in the literature, Rogalski and Vinso (1978) analyzed the behavior of several currencies and showed that the distribution of exchange rate changes is unimodal and has fatter tails than the normal distribution. Hsieh (1988) showed they are not independent and identically distributed, and distributions that vary over time better characterize the data. He found that Autoregressive Conditional Heteroskedastic (ARCH) models do well in capturing these data properties. Goodhart, Mcmahon and Ngama (1993) tested for unit roots with higher frequency exchange rate data (daily, hourly, and minute-to-minute), only to find non-stationarity as the dominant property. Meese and Rogoff (1983) already showed that a random walk model for exchange rates performed no worse than several univariate time series models but it always outperformed structural models based on their out-of-sample forecasting accuracy. Macdonald and Taylor (1993) found in dynamic error correction models a way out to outperform a random walk model. One characteristic of all these studies is that they use data from developed economies. Most developing countries have two exchange rate markets, the official and the parallel.[1] This characteristic requires the simultaneous analysis of both time series to understand the properties of the entire foreign exchange market. Agenor and Taylor (1993) studies 19 developing countries and found cointegration (steady – state relationships) between both exchange markets in 14 cases.

Chapter II presents some general aspects which characterize foreign exchange markets in developing economies. These will be used as a framework to guide the analysis and help with the interpretation of results. It is important to notice that the functioning of the foreign exchange market is explained strictly in terms of the relationship between the official and parallel exchange rates. Consideration of other important macroeconomic variables (expected inflation, real balances, and interest rates) was left out to keep the analysis simple. The purpose of Chapter III is to understand the data, and this is done in trying to establish the weekly expected rate of depreciation and its volatility. Chapter IV finds the short-run dynamics structure of the official and parallel markets and their interrelations. Also the existence of a steady-state or long term equilibrium relationship between both markets is studied. Then both of these findings are incorporated into an Error Correction Model whose structure gives an empirical approximation (in terms of the variables considered) as to how the Bolivian foreign exchange market works. Finally, conclusions are presented in Chapter V.

[1] It is important to differentiate between a parallel and a black market. This last one has an illegal nature, while the former does not. Many developing economies exhibit black markets, but this is not the Bolivian case after stabilization.

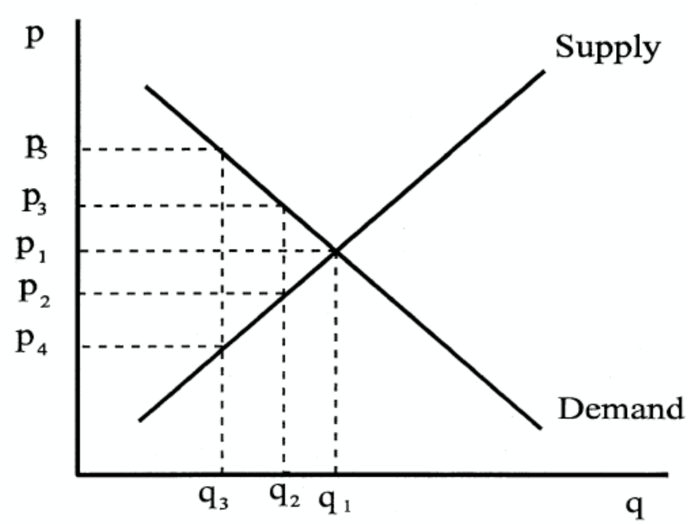
CHAPTER II

SOME GENERAL ASPECTS OF THE BOLIVIAN FOREIGN EXCHANGE MARKET

In any free market the price of a good is determined by supply and demand interactions. In this case the good being bought and sold is foreign currency, and its price is called the exchange rate because one unit of foreign currency is paid off with domestic currency. In the Bolivian case the dominant foreign currency is the U.S. Dollar and therefore the exchange rate is the price of one Dollar in terms of Bolivians.

As in any well-functioning market we will also have an equilibrium price and quantity; p_1 and q_1 in figure 1, with p_1 being the long-term equilibrium exchange rate. This equilibrium assumes there is no market intervention and therefore a parallel exchange rate does not exist.

Figure 1



The problem begins with government intervention in the market, first by gaining control over foreign exchange reserves, and second by influencing the price and amounts being sold.

Although in the Bolivian case the new system requires the selling of Dollars through and auction, which should reflect market forces, the government still, has control over the supply of Dollars and the rules of this auction. The rules are that bidding should be equal to or above a certain amount and a certain price.

The natural result from intervention is market distortion. Following figure 1, if the government sets the total quantity of Dollars to be sold at q_2 and sells at p_2, then this will create an excess of demand or demand non-attended. The first result is the appearance of a parallel market which supplies to the remaining demand but at the higher price p_3. The difference $p_3 - p_2$ is called the premium, and since activities in the parallel market are not illegal, this premium merely reflects the profit from providing this service. The second result is that as long as the premium is positive p_2, p_3, and q_2 represent the long-term equilibrium with market intervention. Now, it is possible that the government might be interested in changing this equilibrium, for example by setting the quantity to q_3. In this case the new long-term official and parallel equilibrium exchange rates would be p_4 and p_5 respectively. This points to a third result; when the equilibrium official exchange rate decreases (increases), the equilibrium parallel exchange rate increases (decreases). Hence, there is an inverse long-term relationship between these rates.

This result should not be interpreted as if the government has the exclusive power to influence long-term equilibrium. This last can also change due to slowly changing long-term supply and demand conditions.[2] In terms of short-term dynamics, give the continuous influence

[2] More specifically this is referring to the possibility of economic development and growth. As population grows together with the magnitude of economic activities, the demand and supply for foreign exchange will also increase (right shifts of supply and demand curves in figure 1).

of exogenous variables on the foreign exchange market,[3] it is suspected that both prices will be fluctuating around their long-term equilibrium, say p_2, p_3 and q_2. This leads to a fourth result, that whenever short-term disequilibrium occurs, the interaction of supply and demand will bring exchange rates back to their long-term equilibrium.

[3] This is referring to changes in non-economic variables (political instability), and other economic variables (inflation, budget deficit, money supply and demand, etc.) that affect the foreign exchange market. The magnitude of these changes can lead to sudden shocks, and the simultaneous change of many of them can cause erratic movements that might contradict the third result above.

CHAPTER III

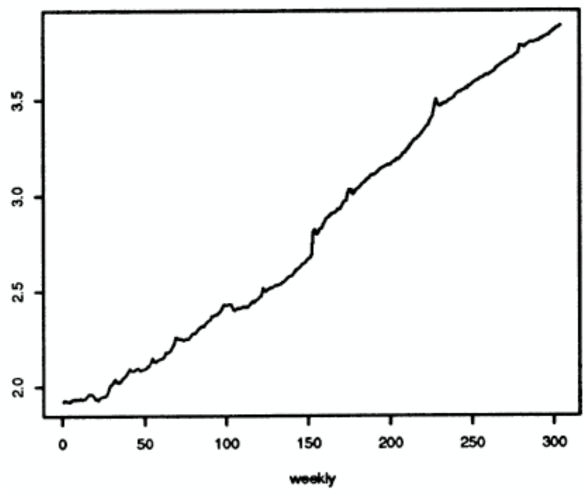
EXPECTED RATE OF DEPRECIATION AND ITS VOLATILITY

Data Description

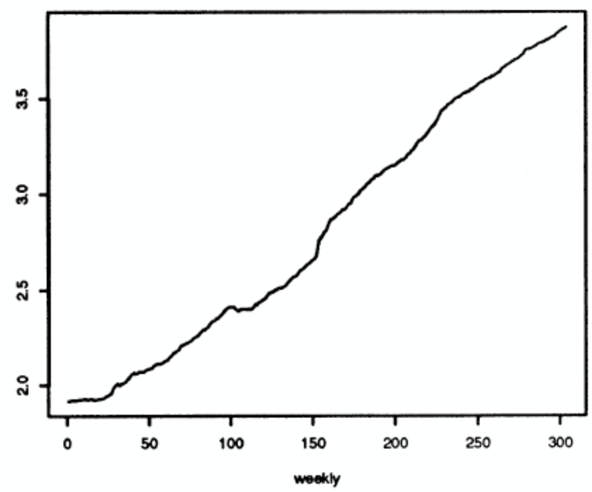
The data consist of weekly observations of the nominal official (O_t) and Parallel (P_t) price of one Dollar expressed in Bolivians. The observations belong to the after stabilization period from September 1986 to June 1992. Figures 2 and 3 are plots of these data. It can be seen that both series are increasing over time with similar behavior and probably moving together as our theory suggests. Each pair of observations (P_t, O_t) at any time t can be thought of representing figure 1 with the difference that it could be a disequilibrium or equilibrium situation. The fact that the exchange rate is increasing over time (regardless if P_t or O_t) means that more and more Bolivians are required to obtain one Dollar. This implies that the Bolivian currency is continuously loosing value in nominal terms, that is, it is depreciating against the Dollar. Figure 4 plots the premium $(P_t - O_t)$ and as expected it is always positive over time and probably fluctuating around a long term relationship as our theory also suggests.

The problem of dealing with nominal prices is that they are usually non-stationary due to a strong trend component. One way of solving this problem is by doing a regression on time and using the residuals of this regression as the detrended and therefore stationary series. Another approach for obtaining stationarity is to take a first difference of the data. In this study we will take the second approach. One reason for taking this approach is that it transforms the data into its rate of change form (or rate of growth) by taking the first difference of their logarithms. If the exchange rate is a function of time $e = f(t)$, then the rate of change is defined as $f'(t)/f(t)$. Another way of obtaining the same result is to transform the equation into logs, $\log e = \log f(t)$,

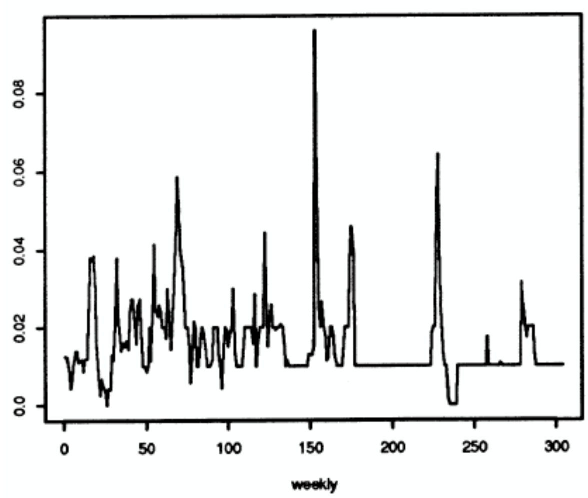
2. PARALLEL(P)



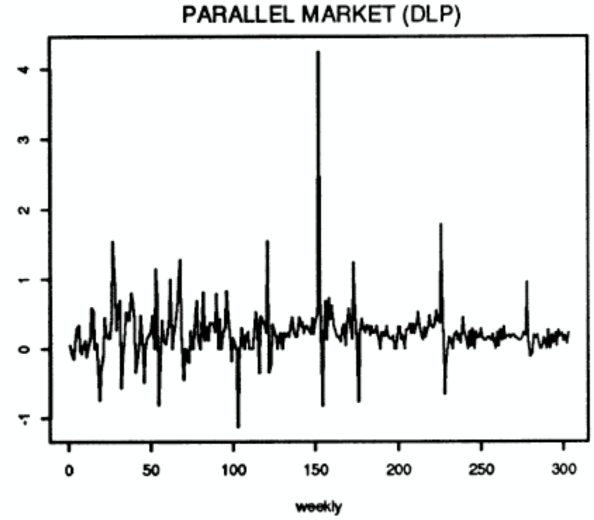
3. OFFICIAL(O)



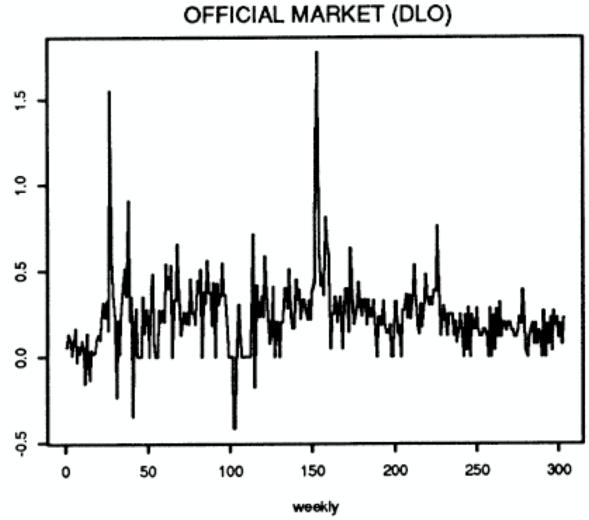
4. PREMIUM(P-O)



5. DEPRECIATION RATE
PARALLEL MARKET (DLP)



6. DEPRECIATION RATE
OFFICIAL MARKET (DLO)



whose derivative is also f'(t)/f(t). Given that we are dealing with discrete functions of time, the derivative of the log can be approximated as follows:

$$\frac{\Delta \log f(t)}{\Delta t} = \frac{\log f(t+1) - \log f(t)}{\Delta t}$$

Since the denominator is one, this derivative will equal the first difference of log f(t). More accurately it can be written as 100*log [f(t+1)/f(t)] to express percentage changes. Figures 5 and 6 are plots of the two series obtained using this procedure, where DLP_t and DLO_t represent the rate of change of the Parallel and Official exchange rates respectively.

Summary Statistics and Normality

Table 1 provides some summary statistics for DLP_t and DLO_t. The means indicate that on average the weekly growth rate for both DLP_t and DLO_t (or weekly expected rate of depreciation) is about 0.23%. They both seem to be moving at about the same rate. Their volatility is indicated by the standard deviation, which is greater for DLP_t then for DLO_t. This is also verified by the higher, range between the maximum and minimum values of DLP_t compared to DLO_t. Given some degree of market intervention in the official market, the government might try to avoid drastic changes in the official rate whenever possible. What is surprising is the magnitude of the standard deviation compared to the mean. In both cases the dispersion of the data implies that the foreign exchange market is an extremely risky activity, so much, that having knowledge of the expected value does not matter.

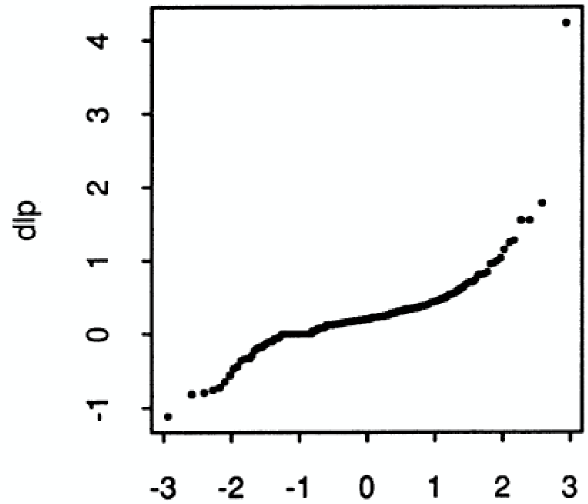
TABLE 1
DESCRIPTIVE STATISTICS

Variable	Mean	S.D.	Maximum	Minimum	Skewness	Kurtosis
DLP_t	0.2320	0.3983	4.2360	-1.1232	3.5836	37.2055
DLO_t	0.2333	0.2215	1.7755	-0.4175	2.1612	15.7335

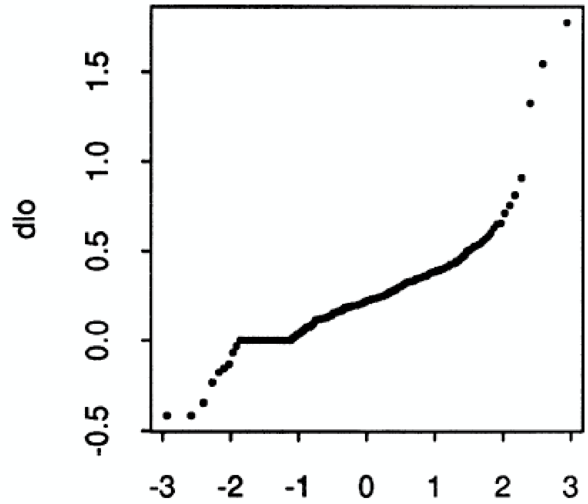
With respect to skewness and kurtosis, a healthy normal distribution should have its skewness value close to zero and kurtosis close to 3. Both DLP_t and DLO_t seem to have some degree of skewness and a strong degree of kurtosis, suggesting fat tailed distributions. This can be verified by the shape of the normal probability plots presented in figures 7 and 8. These plots together with the histograms of theses series (figures 9 and 10) suggest than the skewness, fat tails, and large standard deviations may be due to a few extreme observations. In order to account for this possible distortion, the statistics were re-estimated using three types of robust methods: An L-estimator (trimmed data) and two M-estimators (Huber and Tukey's Bisquare). For the first case the data was sorted and the 10% highest and lowest values were discarded, then a trimmed mean was computed with the remaining values. For the case of the M-estimates, the mean and standard deviation are obtained by solving the following two equations:

$$\sum_{i=1}^{n} \psi\left(\frac{y_i - \mu}{s}\right) = 0$$

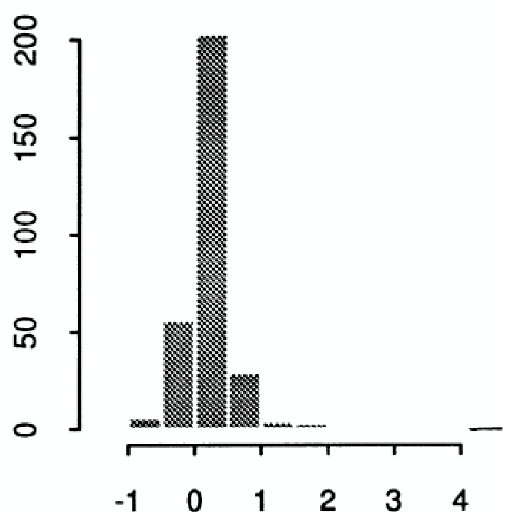
$$\sum_{i=1}^{n} \psi\left(\frac{y_i - \mu}{s}\right)^2 = \beta$$



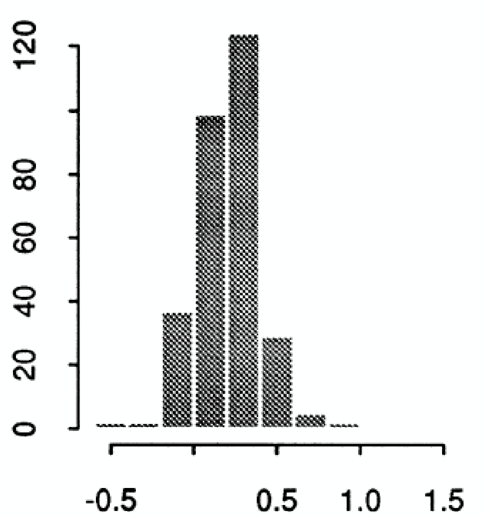
Quantiles of Standard Normal
7. Parallel Depreciation Rate



Quantiles of Standard Normal
8. Official Depreciation Rate



dlp
9. Parallel Depreciation Rate



dlo
10. Official Depreciation Rate

where μ = mean, s = standard deviation, and β depends on ψ. When $\psi(x) = x$ for $abs(x) < k$ or $k*sign(x)$ otherwise, then Huber's loss function is being used and $\psi(x)$ is its derivative. When $\psi(x) = (1 - (x/k)^2)^2 6x/k^2$ or otherwise, then Tukey's loss function is being used and $\psi(x)$ is its derivative.

Table 2 gives the summary statistics using the robust methods. The L- and M-estimators coincide around the same expected weekly rate of depreciation of 0.20% and 0.21% for DLP_t and DLO_t respectively, and both are lower than 0.23% previously found. Although the standard deviations dropped significantly, as expected, their size compared to the mean still suggest high dispersion or volatility of DLP_t and DLO_t. The result of DLP_t being more volatile than DLO_t is not as clear. Skewness is now close to zero implying symmetric distributions, but some degree of kurtosis still persists.

TABLE 2
ROBUST DESCRIPTIVE STATISTICS

Variable	Robust Method	Mean	S.D.	Skewness	Kurtosis
DLP_t	Trimmed	0.2088			
	Huber	0.2077	0.2054	-0.4420	1.2697
	Tukey	0.1999	0.1563	-0.6749	1.9659
DLO_t	Trimmed	0.2174			
	Huber	0.2178	0.1659	-0.3291	1.0381
	Tukey	0.2166	0.1503	-0.5387	1.1661

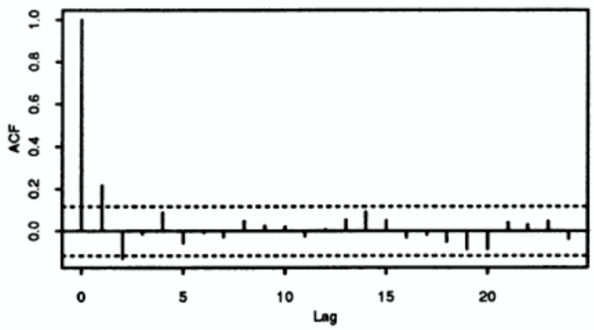
What robust techniques do is either downweight or completely ignore extreme observations or outliers which might be distorting the computed statistics. From an economist's point of view it does not make sense to eliminate historic information given that there is only one observation per time period, and at the same time the extreme observation could be the most important one for understanding the economic phenomena. From a statistician's point of view, the mean is just the average of some numbers and the standard deviation is the square root of the average of the squared deviations from the mean. Therefore for its computation it does not matter if it is time series or cross-sectional data, but if there are extreme observations those will distort the statistic. One approach to this problem is to think of extreme observations as shocks to the exchange rate market. These shocks could be a result of sudden changes in exogenous factors like new international economic conditions or new government regulations. They could also be a result of changes in non-economic variables like news of political instability or natural disasters. Therefore diminishing or eliminating their influence would have the effect of showing market operations under stable conditions, free of abrupt changes in exogenous factors.

Testing for Autocorrelation

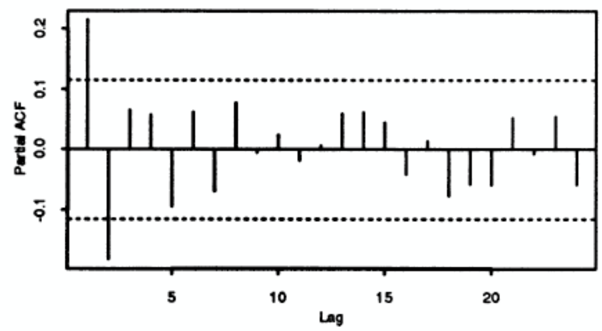
The computation of mean and standard deviation statistics above was based on one big assumption, that observations over time are independent of each other. That is, there is no autocorrelation and a sudden shock that occurred last week will not affect market's behavior in the present week. Figures 11 to 14 show autocorrelation and partial autocorrelation plots for the DLP_t and DLO_t series. The behavior of both series suggests autocorrelation of first and second orders. The Ljung-Box Q-statistic can be used to test the hypothesis that all autocorrelations are zero; that is, that the series is white noise. The Q-statistic is given by

$$Q = n(n+2)\sum_{j=1}^{p}\frac{r^2_j}{(n-j)}$$

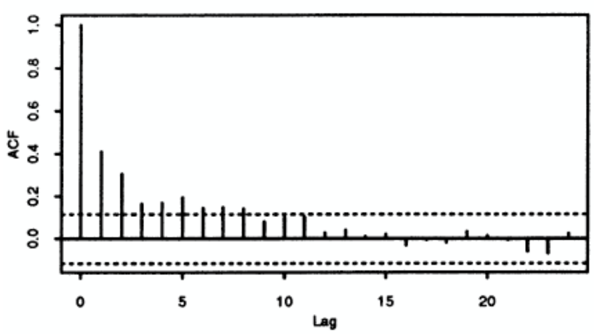
11. Series : Parallel.Depreciation.Rate.DLP



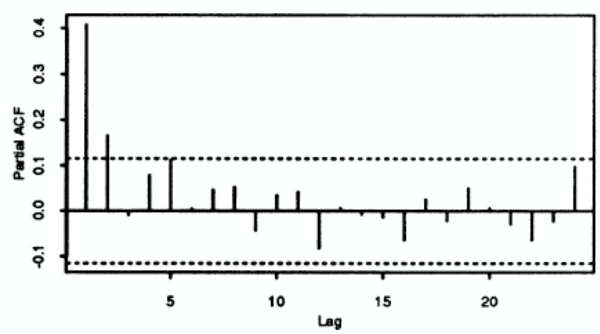
12. Series : Parallel.Depreciation.Rate.DLP



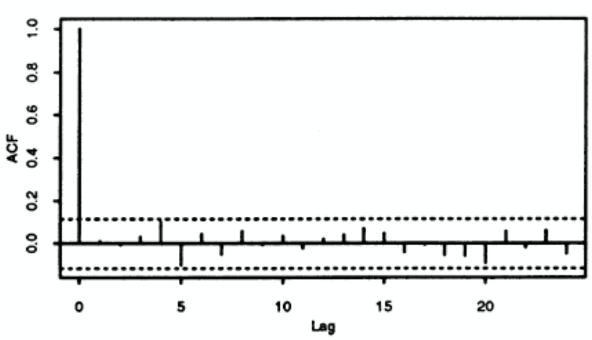
13. Series : Official.Depreciation.Rate.DLO



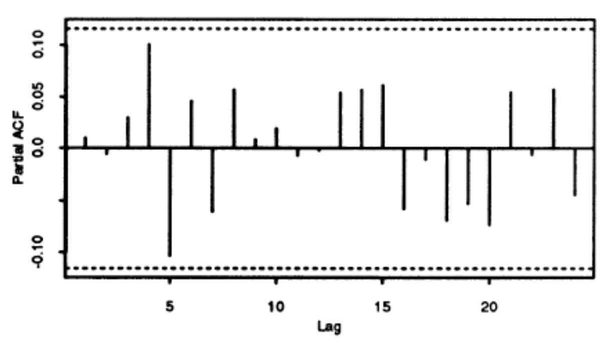
14. Series : Official.Depreciation.Rate.DLO



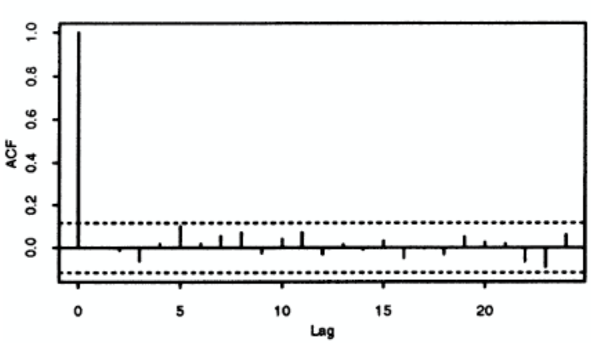
15. Series : Ols.Residuals.for.DLP



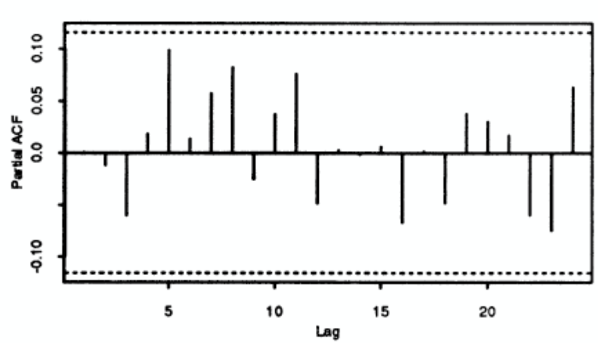
16. Series : Ols.Residuals.for.DLP



17. Series : Ols.Residuals.for.DLO



18. Series : Ols.Residuals.for.DLO



where r_j is the jth autocorrelation and n is the number of observations. Under the null hypothesis, Q is distributes as chi-squared, with degrees of freedom equal to the number of autocorrelations p. For $p = 10$ the 5% critical value for the chi-squared is 18.3, which is below our computed $Q = 23.31$ for DLP_t and $Q = 129.84$ for DLO_t, which suggests that neither series is white noise. This implies the past will have an effect in today's behavior of the foreign exchange market. Using the information from the autocorrelation and partial autocorrelation plots, Table 3 presents two AR(2) models that have been found to better represent the data. Figures 15 to 18 plot the autocorrelation and partial autocorrelation of the residuals from these regressions. Visual inspection shows a white noise series in both cases. The two Q-statistics computed on the residuals of both models show values less than 18.3, therefore we fail to reject the hypothesis that all autocorrelation are zero, that is, both sets of residuals are white noise series. Sometimes testing for autocorrelation on the squares of the residuals is a good indication of hidden autocorrelation, non-linearities and/or heteroskedasticity. $Q.res^2$ is the computed statistics, and in both cases it suggests again that the residual series are white noise; figures 19 and 20 show normal probability plots of these residuals. In both cases there is clear indication of extreme observations, one in figure 19 and at least three in figure 20. The structure of the two models (see Table 3), and the significance of their parameters as seen through their t-values, suggest that observation of market operations during the previous two weeks has an influence over current market operations. What is disappointing is the size of this influence. The first model explains only 7.7% of the variability of DLP_t, while the second explains 18.8% of the variability of DLO_t.

Figure 19

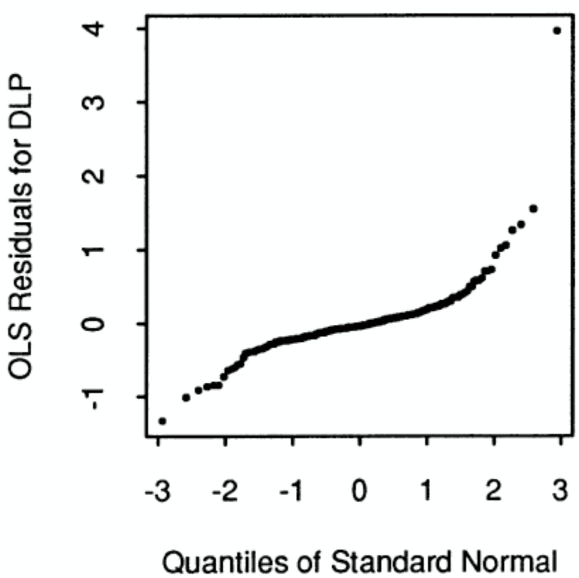


Figure 20

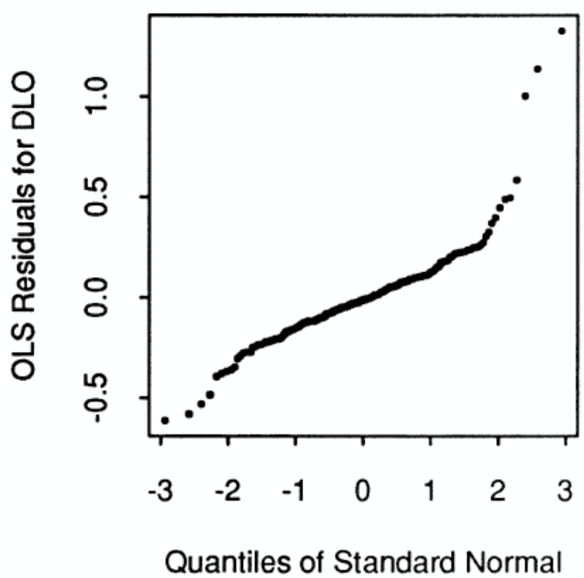


Figure 21

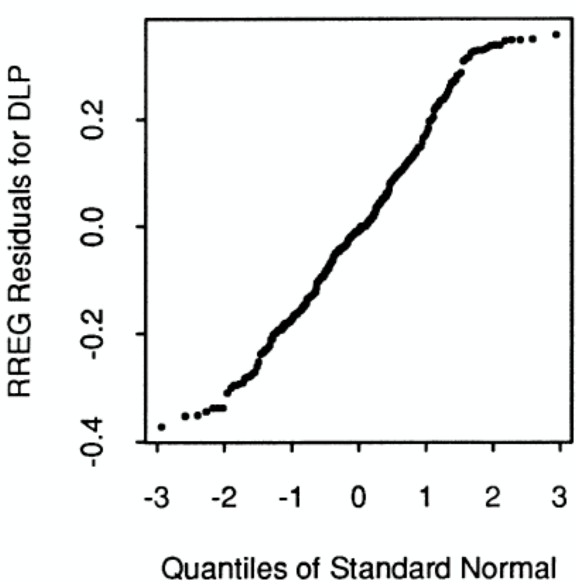


Figure 22

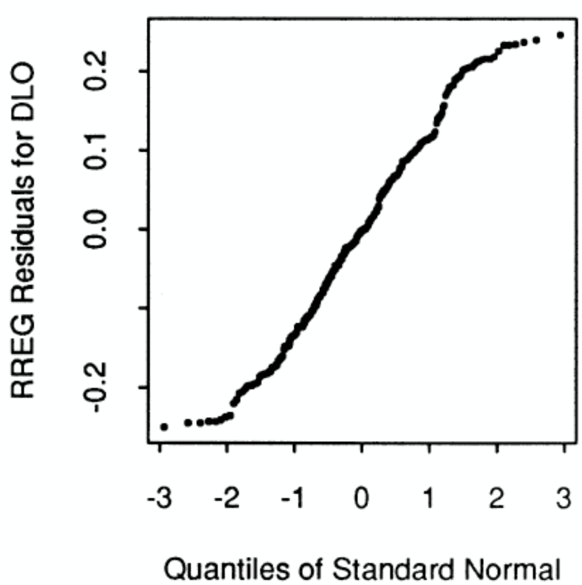


TABLE 3
OLS ESTIMATION OF AR(2) MODELS FOR DLP_t AND DLO_t

Depended Variable	Independent Variables	Coefficient	t-Statistic
DLP_t	C	0.2168	7.82
	DLP_{t-1}	0.2528	4.44
	DLP_{t-2}	0.1808	-3.17
$R^2 = 0.077$; Q = 9.32 (0.5020)*; $Q.res^2 = 0.37$ (0.9999)*			
DLO_t	C	0.1162	6.25
	DLO_{t-1}	0.3395	5.94
	DLO_{t-2}	0.1652	2.89
$R^2 = 0.188$; Q = 7.39 (0.6882)*; $Q.res^2 = 15.55$ (0.1132)*			

* P-value for the computed Q-Statistic.

The Problem of Influential Observations

The above findings must be taken with care because the few extreme observations could be having an important distorting effect on the estimated parameters, t-values and R^2. In order to address this issue influential observation diagnostics were performed. Appendix A shows plots of the values found in several detecting criteria. The plots A1 to A7 are related to the AR(2) model of DLP_t, while the plots A8 to A14 are related to the AR(2) model of DLO_t. The influential observations diagnostics used were the following: i) Studentized residuals, to observe departures from normality (Rstudent); ii) Elements of the diagonal of the Hat matrix, to find leverage points (Hdiag); iii) Deletion of individual observations to analyze the sensitivity of the covariance matrix (COVRATIO), also of the fit (DFFITS), and of the estimated coefficients (DFBETAS). Appendices B1 and B2 extract a list of the influential observations as detected by the cut-off criteria of each of these methods. The general conclusion from this analysis is again that the above autoregressive models contain a few influential observations that are affecting considerably the performance of OLS in estimating the AR(2) models.

Probably the best way to solve this problem is to investigate what happened in the economy those specific weeks in which the exchange rate experienced shocks. The quantity of foreign currency demanded not only depends on its price (the exchange rate) as was shown in figure 1, but also on other economic and non-economic variables. These other variables could be acting alone or together in their influence over the quantity bought and its price. Some of these variables could be having a one-time shock effect, like news, but this information is not readily available. Others could be affecting the exchange rate on weekly basis, like expected inflation, but a weekly Consumer Price Index from which actual inflation could be extracted does not exist. Other variables could be having a periodical effect, like payment of the December Bonus. This last one can be incorporated in the analysis with dummy variable. The December Bonus is a Christmas Bonus which any employed person receives by law. The Bonus is generally paid either one week before Christmas, during the Christmas week, or even the week after as many businesses depend on their Christmas sales to pay for it. Once it is received it is believed that a portion of it, if not all, will be saved in terms of foreign currency in order to protect it from inflation. To describe this situation, the dummy variable created contains numbers 1, 2 and 3 on those weeks of every year in the belief that it is in this last week when most of the people receive their Bonus. Figures 23 and 24 plot this series against DLP_t and DLO_t, and as expected it does capture several extreme observations. The problem of the remaining influential observations could be treated by use of statistical methods. The above models were re-estimated using a robust regression procedure. This procedure consist of iteratively reweighted least squares to approximate a robust fit, with residuals from the current fit passed through a weighing function to give weights for the next iteration. The weighing function is the converged Huber estimate followed by Tukey's Bisquare.[4]

The results where the following:

TABLE 4
RREG ESTIMATES OF AN AR(2) MODEL FOR DLP_t AND DLO_t

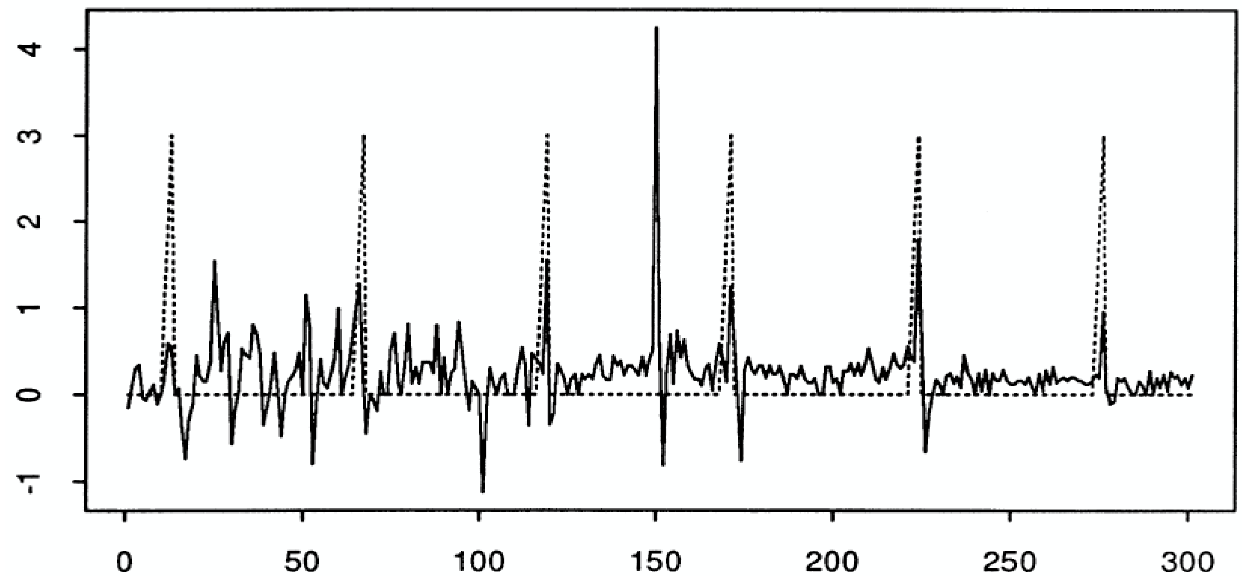
Depended Variable	Independent Variables	Coefficient	t-Statistic
DLP_t	C	0.1814	13.69
	DLP_{t-1}	0.2553	8.02
	DLP_{t-2}	-0.1689	-5.78
	DUM	0.1161	4.39
$R^2 = 0.2673$; Q = 24.33 (0.0006)*; $Q.res^2 = 83.68$ (9.5E-14)*			
DLO_t	C	0.1288	10.63
	DLO_{t-1}	0.2199	5.67
	DLO_{t-2}	0.1561	4.25
	DUM	0.0641	4.16
$R^2 = 0.2591$; Q = 23.58 (0.0087)*; $Q.res^2 = 19.55$ (0.0338)*			

* P-value for the computed Q-Statistic.

To better understand how the robust regression operated, figures 25 and 26 plot weights (values between 0 and 1) against DLP_t and DLO_t. The procedure was successful in downweighting or even eliminating observations that were previously identified as influential. The implication of this is complicated. An observation at time t (which includes DLP_t, DLP_{t-1},

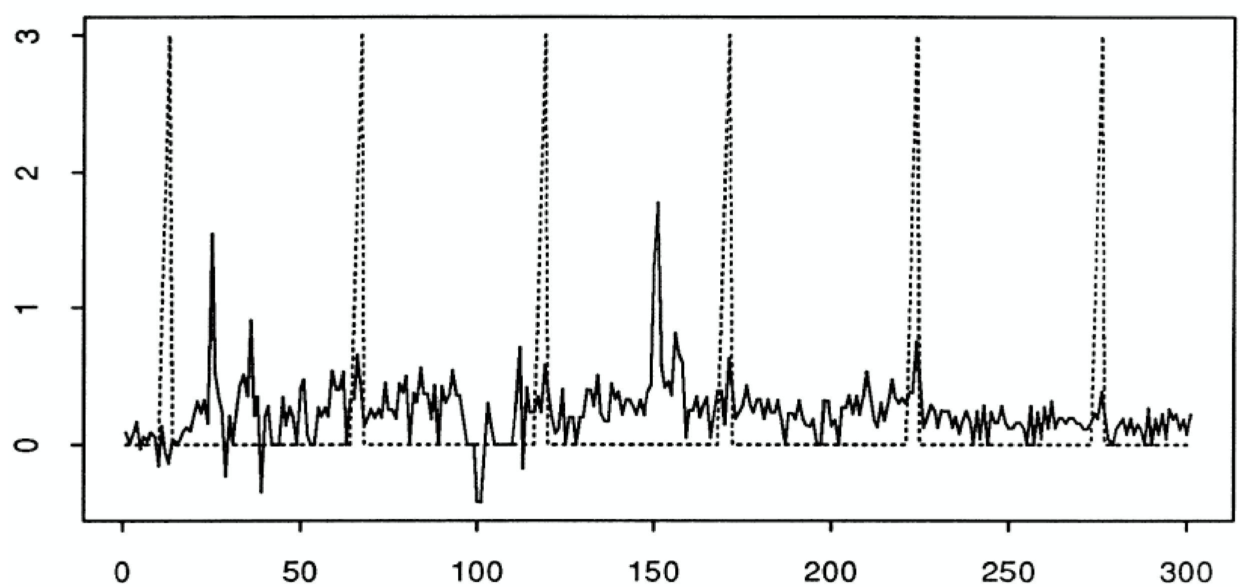
[4] This corresponds to the default method in Splus. Other alternative methods include Andrew's sine function and Hampel's three part redescending function.

Figure 23



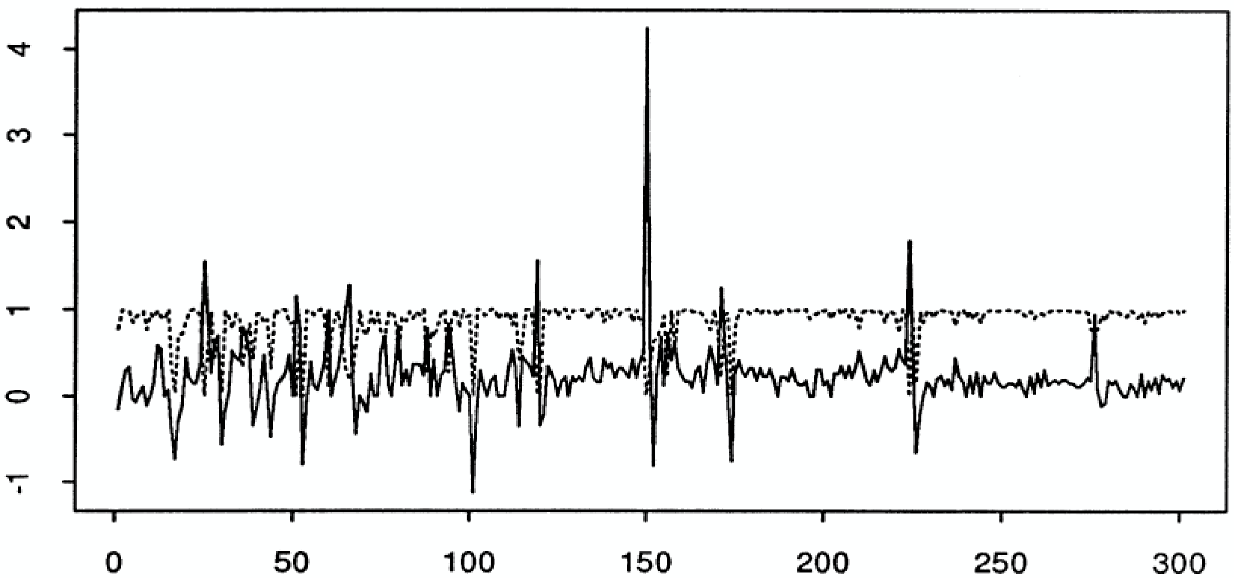
DLP vs THE BONUS DUMMY

Figure 24



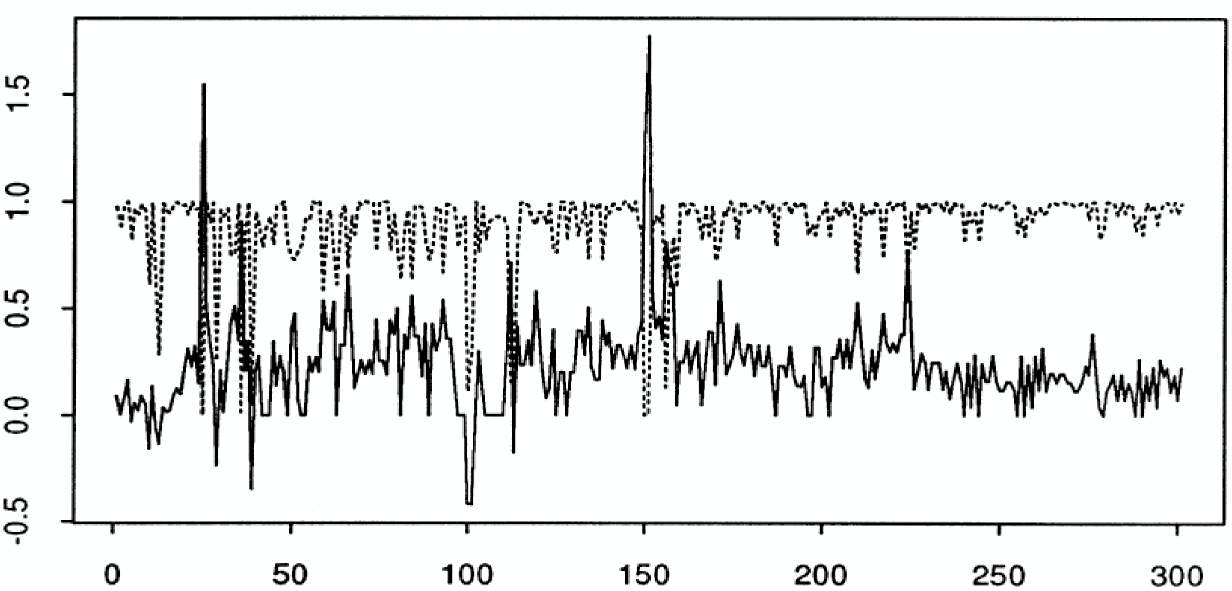
DLO vs THE BONUS DUMMY

Figure 25



ROBUST WEIGHTS vs DLP

Figure 26



ROBUST WEIGHTS vs DLO

DLP_{t-2}, DUM) was downweighted but at the same time it is still present in the observation at time t+1 and t+2 given the structure of the models. In a way no historic information has been lost with the benefit of greater accuracy. The downside of the method is that at the same time it downweighted, although slightly, almost all observations regardless of whether they were influential or not. An effect similar to smoothing the entire series. Again this can be interpreted as observing market operations under stable conditions, free of abrupt changes in exogenous variables.

Comparing to the previous estimates, the new values of the estimated parameters show slight changes and larger t-values. Also the December Bonus dummy is significant as expected. The surprise is in the increase in the explanatory power of the models, now they explain 26.7% and 25.9% of the variability of DLP_t and DLO_t respectively. But the Q-statistics on the residuals suggest these are not white noise series, therefore either further lags or other explanatory variables should be included. The normal probability plots of these weighted residuals (figure 21 and 22), show further departure from normality as they adopt an S shape suggesting short tails.

CHAPTER IV

MODELING THE BOLIVIAN FOREIGN EXCHANGE MARKET

Modeling Short-Term Dynamic Interrelationships among Markets

The second order autoregression models of the previous section suggest two main conclusions: i) Under conditions of market stability (represent by the robust regression results), observation of market operations in the previous two weeks can help explain about 26% of the variability of current market operations. ii) This phenomena occurs for both the official and the parallel market. However, as our theory in section II suggests, it is not possible that each market could operate independently of the other. The two markets form a system called the foreign exchange market, and therefore what happens in the official market must be affecting the parallel market and vice versa. This does not imply that the previous autoregressive models are wrong. It could be possible that people and small firms operating in the parallel market take their decisions base not only on observed operations in the previous two weeks in the parallel market, but also observed current and previous operations in the official market. Larger firms operating in the official market could be having similar behavior.

An autoregressive system was estimated to take these market interactions into account, together with the individual market dynamics. The system consists of two equations where the dependent variable in any equation is explained in terms of its own lagged values and the current and lagged values of the dependent variable in the second equation. Given the structure of the equations in the system, these will be referred to as the Market Interactions Model. Table 5 presents each equation estimated independently of the other using ordinary least squares (OLS) and robust regression (RREG). This last was done with the purpose of analyzing the impact of extreme observations.

The first equation includes DLO_t, DLO_{t-1}, DLO_{t-2} and DLO_{t-3} in explaining short-term dynamics of DLP_t. By their t-values under the RREG procedure we can conclude that all variables are significant in explaining the behavior of DLP_t. Notice the poor performance of OLS compared to the RREG procedure which not only shows that all variables included are significant, but also captures the significance of a third lag of both DLP_t and DLO_t. The explanatory power of the model increases considerably, now the variables included explain about 86% of the variability of DLP_t. The Q-statistic is smaller than the critical value of 18.3 for a 5% level, which suggest the residuals from the RREG procedure are white noise. But the Q-statistic for the squared residuals is not white noise which points to additional hidden autocorrelation, non-linearities and/or heteroskedasticity problems. The second equation includes DLP_t, DLP_{t-1} and DLP_{t-2} in explaining short-term dynamics of DLO_t. By their t-values under RREG only DLP_t, DLO_{t-1} and DLO_{t-2} are significant in explaining the behavior of DLO_t. The variables DLP_{t-1} and DLP_{t-2} appear as non-significant probably due to a multicollinearity problem. If this is the case then running the regression without DLP_{t-2} should help the performance of DLP_{t-1}. The explanatory power of the model increases too, now the variables included explain more than 60% of the variability of DLO_t. However, similarly to the above case, while the residuals are white noise by their Q-statistic, the Q-statistic for the squared residuals suggest these are not white noise.

TABLE 5
OLS AND RREG ESTIMATION OF THE MARKET

Depended Variable	Independent Variables	OLS Coefficient	OLS t-Statistic	RREG Coefficient	RREG t-Statistic
DLP_t	C	-0.0183	-0.62	0.0136	1.61
	DLP_{t-1}	-0.0632	-1.16	0.1368	6.27
	DLP_{t-2}	-0.2990	-5.43	-0.1701	-9.36
	DLP_{t-3}			-0.0284	-1.75
	DLO_t	0.9857	11.09	0.8551	31.53
	DLO_{t-1}	0.2391	2.26	-0.1263	-3.26
	DLO_{t-2}	0.1121	1.11	0.1371	4.69
	DLO_{t-3}			0.0830	3.05
	DUM	0.2002	5.91	0.1903	15.80
		$R^2 = 0.4616$ Q = 14.96 (0.1335) $Q.res^2 = 7.00$ (0.7254)		$R^2 = 0.8631$ Q = 5.79 (0.8325) $Q.res^2 = 35.92$ (8.6E-05)	
DLO_t	C	0.0817	5.23	0.0928	9.34
	DLO_{t-1}	0.0963	1.64	0.1672	4.35
	DLO_{t-2}	0.1423	2.58	0.1458	4.17
	DLP_t	0.3000	11.09	0.2738	15.75
	DLP_{t-1}	0.1036	3.51	-0.0197	-0.86
	DLP_{t-2}	0.0255	0.80	0.0155	0.77
	DUM	-0.0294	-1.49	0.0077	0.60
		$R^2 = 0.4706$ Q = 8.32 (0.5976) $Q.res^2 = 13.93$ (0.1762)		$R^2 = 0.6218$ Q = 5.62 (0.8461) $Q.res^2 = 27.68$ (0.0020)	

Note.- Numbers in parenthesis are p-values for the computed Q-Statistic.

Figure 27

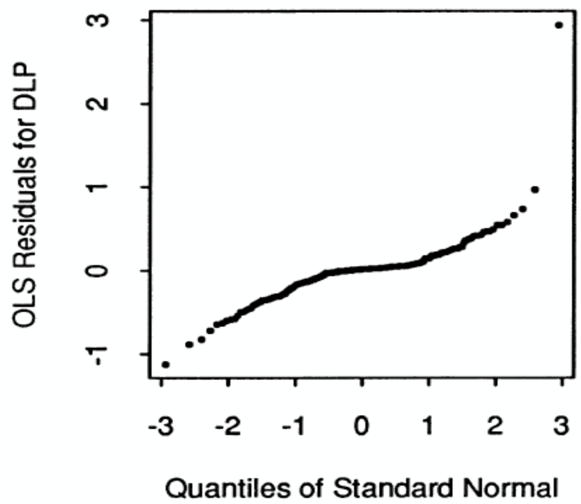


Figure 28

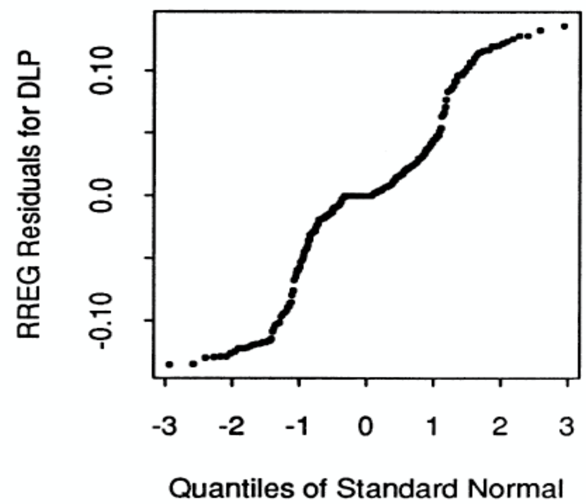


Figure 29

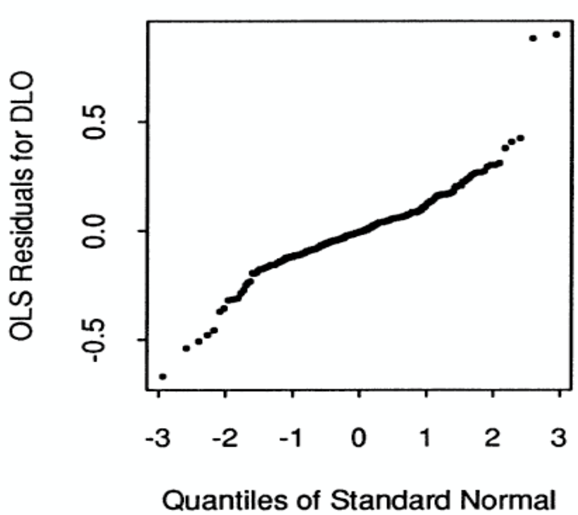
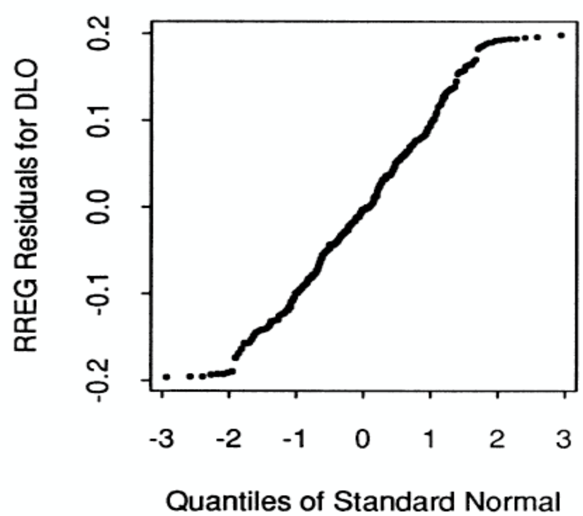


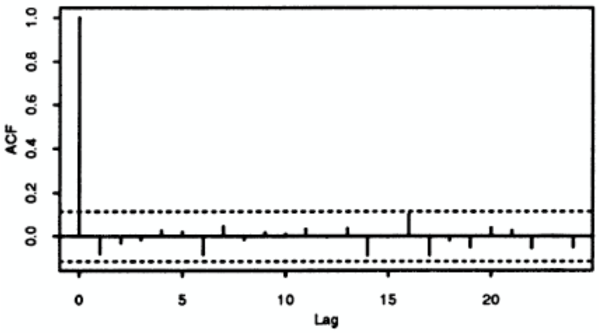
Figure 30



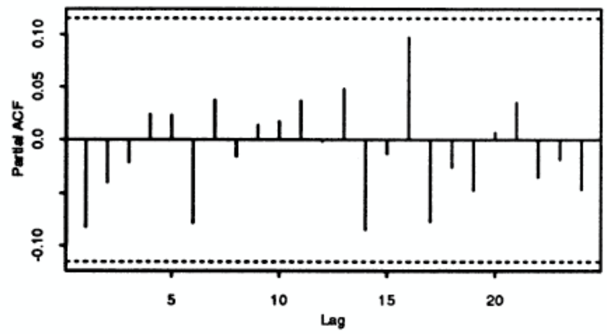
Figures 27 to 30 are normal probability plots of the residuals of the above regressions. The least squares residuals (figures 27 and 29) show departure from normality even when not considering the presence of extreme observations. These observations have an important distorting effect on the estimated statistics as was shown in table 3. The robust regression residuals (figures 28 and 30) are corrected from extreme observations, but they also show departure from normality. This departure is acute in the case of the residuals when DLP_t is the independent variable. Its normal probability plot (figure 28) suggests a short tailed bimodal distribution. The normal probability plot of the residuals when DLO_t is the independent variable (figure 30) suggests a short tail distribution. Therefore the computed t and Q statistics should be taken with care given that they are based on normality assumptions.

Further lags were included in the RREG procedure in order to diminish the problem of non-white noise residuals and non-normality. However, in both cases the Q-statistic for the squared residuals still showed non-white noise, and the newly estimated parameters tended to be not significantly different from zero. To understand why this could be happening figures 31 to 38 plot the autocorrelation and partial autocorrelation functions of the residuals and squared residuals from the RREG procedures. The squared residuals for DLP_t (figures 33 and 34) show an AR(1) variance generating process. The squared residuals for DLO_t (figures 37 and 38) show either an AR(4) or a more complicated ARMA generating process for the variance. The important aspect in both cases is that the variance is not constant over time, and therefore Generalized Autoregressive Conditional Heteroskedasticity or GARCH methods could be used to solve this problem. The idea is to apply ARMA techniques to model the variance of the residuals. Figures 39 and 40 plot these residuals for DLP_t and DLO_t, and they both show portions of the series with either the variance changing over time and large (small) changes of

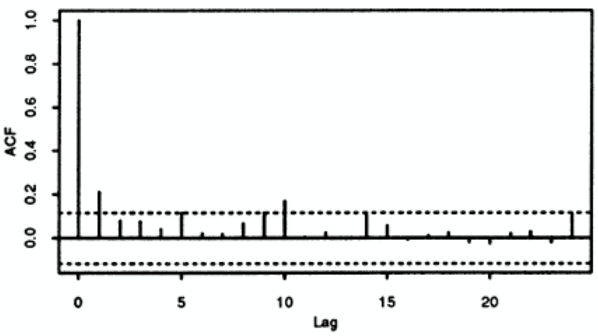
31. Series : RREG.Residuals.for.DLP



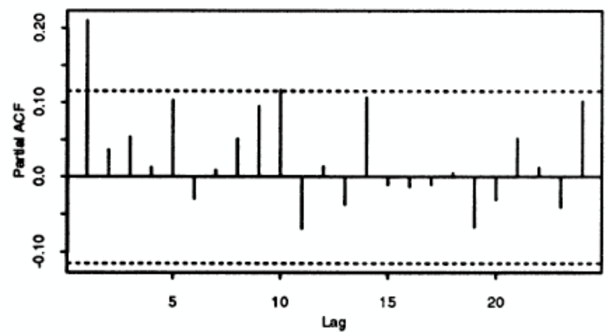
32. Series : RREG.Residuals.for.DLP



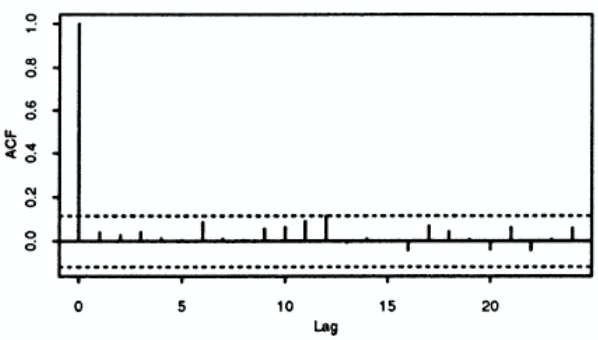
33. Series : RREG.SquareResid.for.DLP



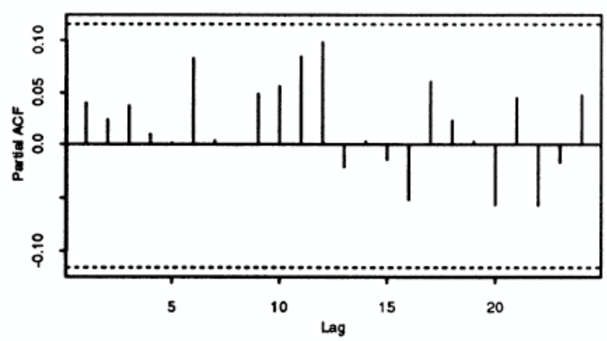
34. Series : RREG.SquareResid.for.DLP



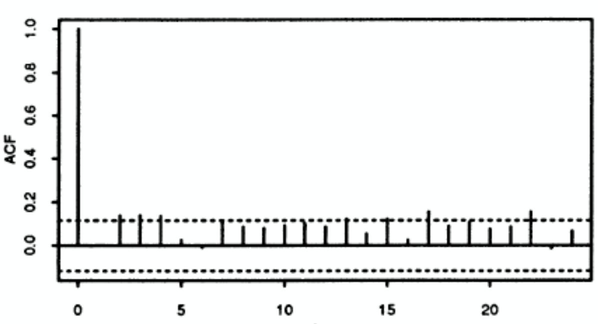
35. Series : RREG.Residuals.for.DLO



36. Series : RREG.Residuals.for.DLO



37. Series : RREG.SquareResid.for.DLO



38. Series : RREG.SquareResid.for.DLO

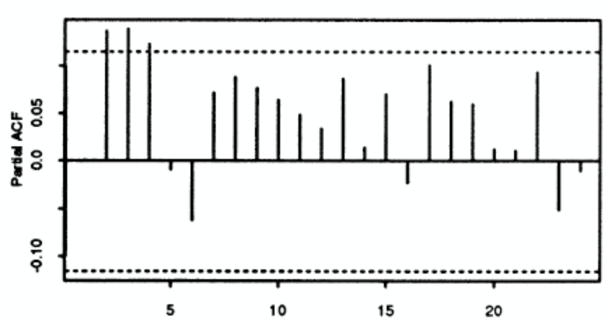


Figure 39: Market Interactions Model

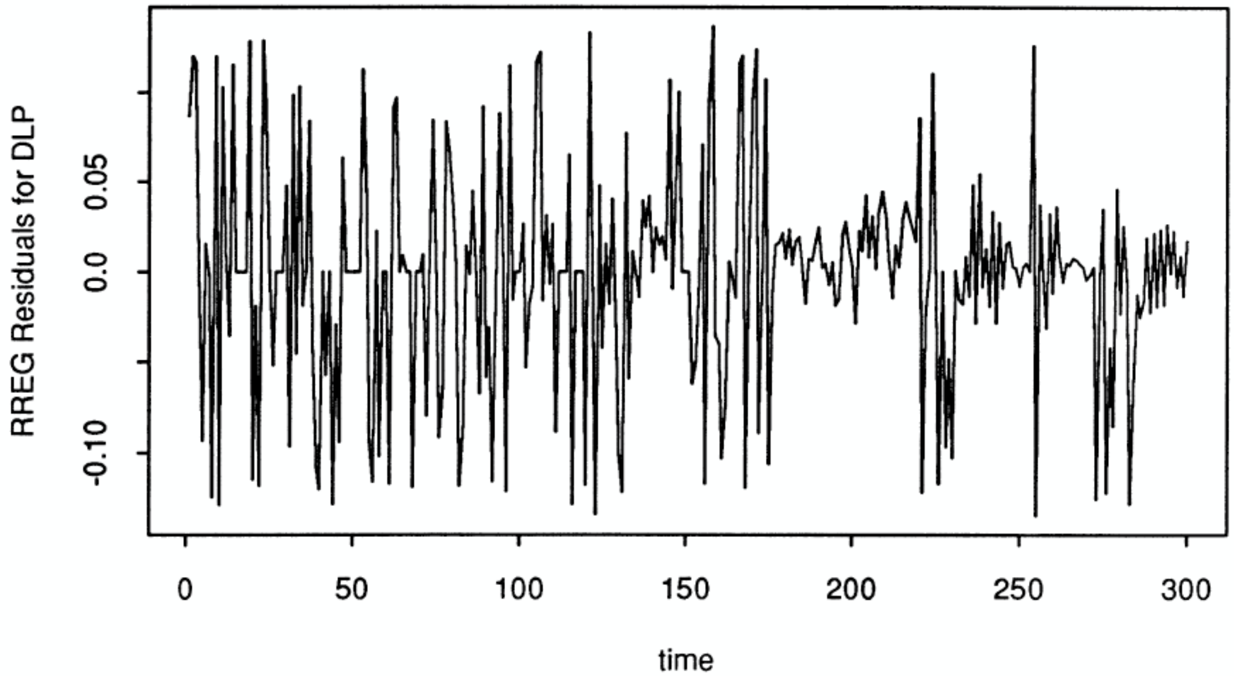
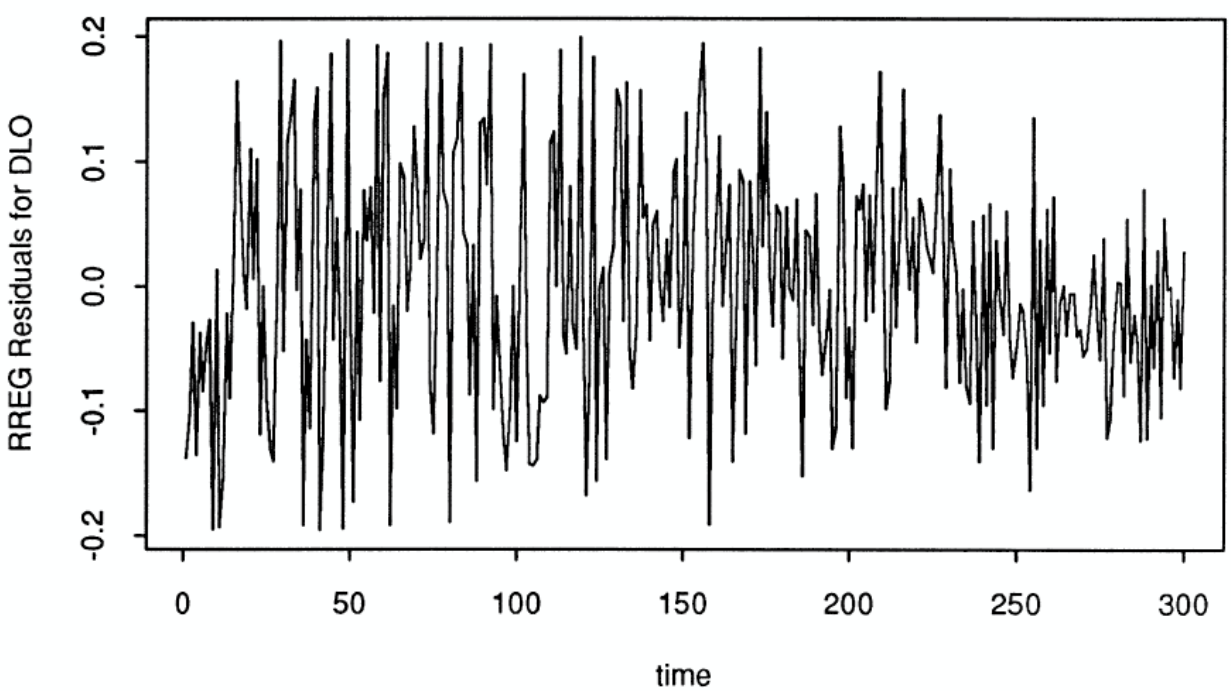


Figure 40: Market Interactions Model



either sign are "stylized facts" for many economic and financial variables. GARCH methods were originally introduced by Robert Engle only a decade ago, and they have been used successfully in taking account for these facts in the applied econometrics literature. Non-white noise square residuals could also be an indication of non-linearitires, and if this were the case then we have an inappropriate model, which is a more serious problem.

Because of the mentioned statistical problems the following interpretations can only be taken as preliminary. In general the estimated autoregressive system tends to confirm the existence of parallel and official market interactions coexisting with individual market behavior. The interaction seems to be stronger from the parallel market point of view. Current behavior in this market seems to be associated with the current and the previous three weeks of movements in the official market. While the official market is only associated with current behavior of the parallel market. The magnitude of adjustment also confirms this view. If the rate of depreciation in the official market increases by 1% in the current week, the parallel market automatically increases by 0.8551 of that 1% in the current week besides additional adjustments in the next three weeks. But if the 1% increase is experienced in the parallel market, the official market will only increase by 0.2738 of that 1% in the current week with no further adjustments. From this analysis we cannot say the official market drives the parallel market, but it is certainly close to that.

Testing for Long-Term Equilibrium Relationships among Markets

Probably the most important aspect predicted by the theory presented in Chapter II, is the existence of a steady-state or long-term relationship between the official and parallel markets. One plausible way of empirically testing it is through a co-integration test. The idea of cointegration can most easily be explained by considering the case of two non-stationary time series x_t and y_t. If this is the case, then it is generally true that a linear combination, $y_t - ax_t$, where a is a constant, will also be non-stationary. However, if the two series have the characteristic of being stationary only after a first difference (In which case both series are said to be integrated of order 1 or I(1), and at the same time there exists a value of "a" such that $y_t – ax_t$, is I(0) rather then I(1), then the series are said to be co-integrated of order 1,1 or CI(1,1). In other words there is some kind of steady-state relationship between the variables. In the case of $a = 1$, the steady-state relationship is such that y_t and x_t cannot drift too far apart.

In our case DLP_t and DLO_t correspond to a first difference of $LP_t = Log(P_t)$ and $LO_t = log(O_t)$. It is expected that LP_t and LO_t are nonstationary, while DLP_t and DLO_t are stationary. The Dickey-Fuller Unit Root Test will be used to verify these claims which were taken for granted in the previous section. Once this is done, a linear combination of LP_t and LO_t needs to be found, and then tested for co-integration. That is, use the Dickey-Fuller Unit Root test again to test if the linear combination is stationary. If this linear combination is I(0) then LP_t and LO_t would be CI(1,1) and a steady-state relationship would exist between them.

Testing for Unit Roots

Dickey and Fuller (1979) introduced a class of test statistic known as Dickey-Fuller DF-statistic to test if the following AR(1) process has a unit root:

$$y_t = \beta_1 + \rho y_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim \text{iid}\ (0, \sigma^2) \tag{1}$$

The test is of H_0: $\rho = 1$ or non-stationarity, against H_1: $\rho < 1$ or stationarity. If in fact $\rho = 1$, then the model becomes random walk with drift β_1:

$$y_t = \beta_1 + y_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim \text{iid}\ (0, \sigma^2) \tag{2}$$

In this case it would be appropriate to take a first difference of y_t to have a workable stationary series, and for this reason y_t is said to be a difference stationary or DF series. In the co-integration literature y_t would be referred to as an I(1) series. However, another method of obtaining a stationary series is to use the residuals of a regression of y_t on time t:

$$y_t = \beta_0 + \beta_1 t + u_t \tag{3}$$

where u_t now follows some ARMA process. In this case y_t is said to be a trend stationary or TS series. The difference between a DS and a TS series has important implications about the nature of the data generating process of y_t. If it is DS then it is always drifting away without bound, and any departure from the previous position is a permanent departure. If the series is TS then it is always fluctuating around a general trend, and any departure from trend is only temporary. Not surprisingly, the economic theory implication of whether a series is DS or TS has generated an incredible amount of work in the applied econometrics literature in the last decade.

The Dickey-Fuller procedure considers the two questions of stationarity versus non-stationarity and whether a series is DS or TS at the same time. We can nest the models (2) and (3) into one:

$$y_t = \beta_0 + \beta_1 t + u_t \qquad ut = u_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim \text{iid}\,(0, \sigma^2) \tag{4}$$

or

$$y_t = \beta_0 + \beta_1 t + \rho u_{t-1} + \varepsilon_t$$

$$y_t = \beta_0 + \beta_1 t + \rho(y_{t-1} - \beta_0 - \beta_1(t-1)) + \varepsilon_t$$

$$y_t = \beta_0\,(1-\rho) + \beta_1\rho + \beta_1(1-\rho)t + \rho y_{t-1} + \varepsilon_t$$

$$y_t = \alpha_0 + \alpha_1 t + \rho y_{t-1} + \varepsilon_t \tag{5}$$

where $\alpha_0 = \beta_0\,(1-\rho) + \beta_1\rho$ and $\alpha_1 = \beta_1(1-\rho)$. Now, when $\rho = 1$ then $\alpha_1 = 0$ and y_t becomes a random walk with drift. That is, y_t is not only non-stationary but also DS. When $\text{abs}(\rho) < 1$ then y_t is simply stationary, except when $\rho = 0$ in which case y_t becomes TS stationary. In terms of a stationary testing strategy, we can start with H_0: $\rho = 1$ for non-stationarity and DS, against H_1: abs(ρ) < 1 for stationarity. If H_0 is rejected then we can test H_0: $\rho = 0$ for non-stationarity and TS. However, equation (5) can be reparameterized by subtracting y_{t-1} from both sides.

$$y_t - y_{t-1} = \alpha_0 + \alpha_1 t + \rho y_{t-1} - y_{t-1} + \varepsilon_t$$
$$\Delta y_t = \alpha_0 + \alpha_1 t + \gamma y_{t-1} + \varepsilon_t \qquad (6)$$

where Δ is the first difference operator and $\gamma = \rho - 1$. Now H_0: $\rho = 1$ is equivalent to H_0: $\gamma = 0$ for non-stationarity and DS, and the test is against H_1: $\gamma < 0$ for stationarity. Of course the special case of $\gamma = -1$ imply TS stationary. In practice the suggested procedure is to use OLS to estimate the following equation:

$$\Delta y_t = \alpha_0 + \alpha_1 t + \gamma_1 y_{t-1} + \sum_{i=1}^{p-1} \gamma_{2i} \Delta y_{t-i} + \varepsilon_t$$

which differs from the above in that a summation of p additional differenced lags are included in order to ensure that the residuals series ε_t is white noise. This last equation is known as the Augmented Dickey-Fuller Test.

Table 6 presents the estimated equtions for testing if the series LP_t and LO_t contain a unit root. The test compares the t-ratio of the estimated γ_1 to its asymptotic distribution which has been tabulated by Dickey and Fuller (1981). In both cases the DF statistic is smaller than the critical value at the 5% and even 10% level (in absolute value), therefore we fail to reject the null of non-stationarity and conclude that LP_t and LO_t are non-stationary.

TABLE 6
OLS AND RREG UNIT ROOT TESTING OF LP_t AND LO_t

Depended Variable	Independent Variables	OLS Coefficient	OLS t-Statistic	RREG Coefficient	RREG t-Statistic
DLP_t	C	0.0119	1.6770	0.0042	1.3958
	T	3.8E-05	1.3073	7.7E-06	0.6224
	LP_{t-1}	-0.0154	-1.3516	-0.0038	-0.8028
	DLP_{t-1}	0.2596	4.5393	0.3335	10.2048
	DLP_{t-2}	-0.1701	-2.9585	-0.1482	-4.9563
	DLP_{t-3}			0.0636	2.4759
	DLP_{t-4}			0.0754	3.0663
		DF-Statistic = -1.3516 Q = 10.02 (0.4387) Q.res^2 = 0.38 (0.9999)		DF-Statistic = -0.8028 Q = 15.66 (0.1097) Q.res^2 = 47.69 (7.0E-07)	
DLO_t	C	0.0059	1.6517	0.0029	1.3314
	t	1.9E-05	1.2710	6.5E-06	0.6828
	LO_{t-1}	-0.0077	-1.3116	-0.0028	-0.7714
	DLO_{t-1}	0.3423	5.9821	0.2276	6.0668
	DLO_{t-2}	0.1725	3.0038	0.1131	3.0324
	DLO_{t-3}			-0.0071	-0.1896
	DLO_{t-4}			0.1443	3.8084
		DF-Statistic = -1.3116 Q = 8.28 (0.6015) Q.res^2 = 15.25 (0.1142)		DF-Statistic = -0.7714 Q = 8.95 (0.5368) Q.res^2 = 13.03 (0.2219)	

DF Critical Values: 1% -3.9934; 5% -3.4267; 10% -3.1362
Note.- Numbers in parenthesis are p-values for the computed Q-Statistic.

Notice that under OLS only two lags were enough to ensure white noise residuals. Under RREG four lags were necessary to achieve white noise residuals for the regression with DLO_t as the independent variable. The regression with DLP_t as the independent variable still points to hidden autocorrelation, non-linearities and/or heteroskedasticity problems when considering the Q-statistic of the squared residuals. Therefore the non-stationarity of LP_t can only be taken as preliminary.

Table 7 presents the estimated equations for testing if the series DLP_t and DLO_t contain a unit root. In both cases the ADF statistic is greater than the critical value al 1% level (in absolute value), therefore we reject the null of non-stationarity and conclude that DLP_t and DLO_t are stationary. Again, under OLS only two lags were enough to ensure white noise residuals. Under RREG three lags were necessary to achieve white noise residuals for the regression with DLO_t as the independent variable. The regression with DLP_t as independent variable still points to hidden autocorrelation, non-linearities and/or heteroskedasticity problems when considering the Q-statistic on the squared residuals. Therefore the stationarity of DLP_t can only be taken as preliminary.

TABLE 7
OLS AND RREG UNIT ROOT TESTING OF DLP_t AND DLO_t

Depended Variable	Independent Variables	OLS Coefficient	OLS t-Statistic	RREG Coefficient	RREG t-Statistic
ΔDLP_t	C	0.2254	4.5216	0.1810	7.9486
	T	-1.3E-04	-0.5253	-2.0E-04	-1.9829
	DLP_{t-1}	-0.8726	-9.7509	-0.6928	-13.5811
	ΔDLP_{t-1}	0.1345	1.8791	0.0205	0.4951
	ΔDLP_{t-2}	-0.0639	-1.1026	-0.1335	-4.3828
	ΔDLP_{t-3}			-0.0721	-2.9302
		DF-Statistic = -9.7509 Q = 9.30 (0.5038) $Q.res^2$ = 0.44 (0.9999)		DF-Statistic = -13.5811 Q = 15.25 (0.1232) $Q.res^2$ = 51.64 (1.3E-07)	
ΔDLO_t	C	0.1271	4.4109	0.1263	6.8639
	t	-6.1E-05	-0.4535	-0.0001	-0.9690
	DLO_{t-1}	-0.5017	-7.2881	-0.5301	-11.2140
	ΔDLO_{t-1}	-0.1582	-2.2919	-0.2446	-5.2764
	ΔDLO_{t-2}	0.0085	0.1473	-0.1328	-3.0489
	ΔDLO_{t-3}			-0.1405	-3.7347
		DF-Statistic = -7.2881 Q = 7.24 (0.7026) $Q.res^2$ = 16.04 (0.0985)		DF-Statistic = -11.2140 Q = 8.43 (0.5869) $Q.res^2$ = 13.22 (0.2116)	

DF Critical Values: 1% -3.9934; 5% -3.4267; 10% -3.1362
Note.- Numbers in parenthesis are p-values for the computed Q-Statistic.

Testing for Cointegration

The theory of cointegration suggests that if the variables LP_t and LO_t have been found to be integrated of order one I(1), that is, stationary only after a first difference, then they are said to be cointegrated if a linear combination of them, v_t, is stationary:

$$v_t = LP_t - \alpha - \beta^* LO_t$$

If v_t is stationary then the time paths of the two series tend to move roughly together instead of diverging without limit. If this is the case, it is said that the variables LP_t and LO_t are cointegrated in the sense that there exists a long-term equilibrium relationship between the two regardless of short-term volatility. The nature of this equilibrium relationship is described by the cointegrating vector. The testing procedure consists of running an OLS on the following equations:

$$LP_t = \alpha - \beta^* LO_t + v_t$$

$$LO_t = -\alpha/\beta + (1/\beta)^* LP_t + u_t$$

Then using the values of the estimated residuals v_t and u_t to perform a unit root test. Table 8 shows the results of this procedure.

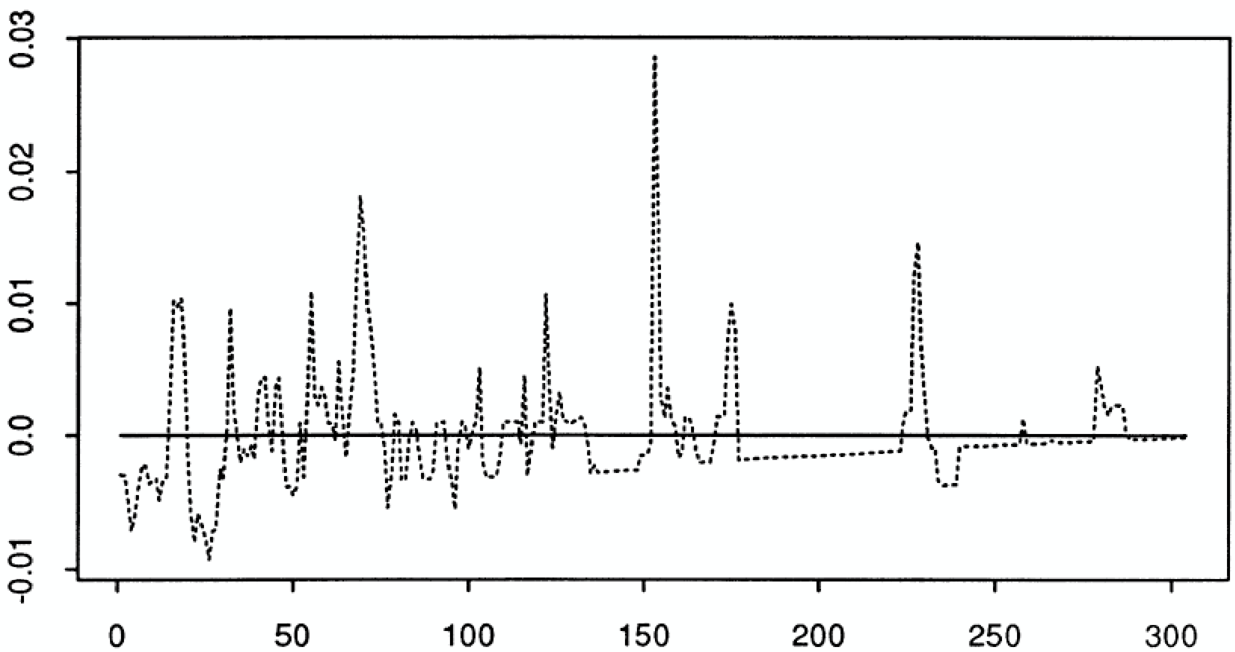
TABLE 8
OLS AND RREG UNIT ROOT TESTING OF v_t AND u_t

Depended Variable	Independent Variables	OLS Coefficient	OLS t-Statistic	RREG Coefficient	RREG t-Statistic
LP_t	C	0.01554	14.6462		
	LO_t	0.99056	964.2689		
Δv_t	C	7.3E-05	0.2167	-0.5E-03	4.9425
	t	-3.4E-07	-0.1768	1.6E-06	2.7226
	v_{t-1}	-0.3362	-6.7823	-0.2158	-12.3339
	Δv_{t-1}	0.1532	2.6400	0.1759	7.5175
	Δv_{t-2}	-0.0813	-1.4081	-0.0811	-3.8575
		DF-Statistic = -6.7823 Q = 11.78 (0.3000) Q.res^2 = 1.58 (0.9986)		DF-Statistic = -12.3339 Q = 3.08 (0.9794) Q.res^2 = 83.03 (1.2E-13)	
LO_t	C	-0.0153	-14.13		
	LP_t	1.0092	964.26		
Δu_t	C	-0.0001	-0.3394	0.0005	4.7199
	t	6.3E-07	0.3211	-1.4E-06	-2.4250
	u_{t-1}	-0.3361	-6.7810	-0.2156	-12.3354
	Δu_{t-1}	0.1531	2.6380	0.1762	7.5375
	Δu_{t-2}	-0.0814	-1.4089	-0.0812	-3.8652
		DF-Statistic = -6.7810 Q = 11.77 (0.3007) Q.res^2 = 1.58 (0.9986)		DF-Statistic = -12.3354 Q = 3.08 (0.9794) Q.res^2 = 83.54 (1.0E-13)	

DF Critical Values: 1% -3.9934; 5% -3.4267; 10% -3.1362
Numbers in parenthesis are p-values for the computed Q-Statistic.

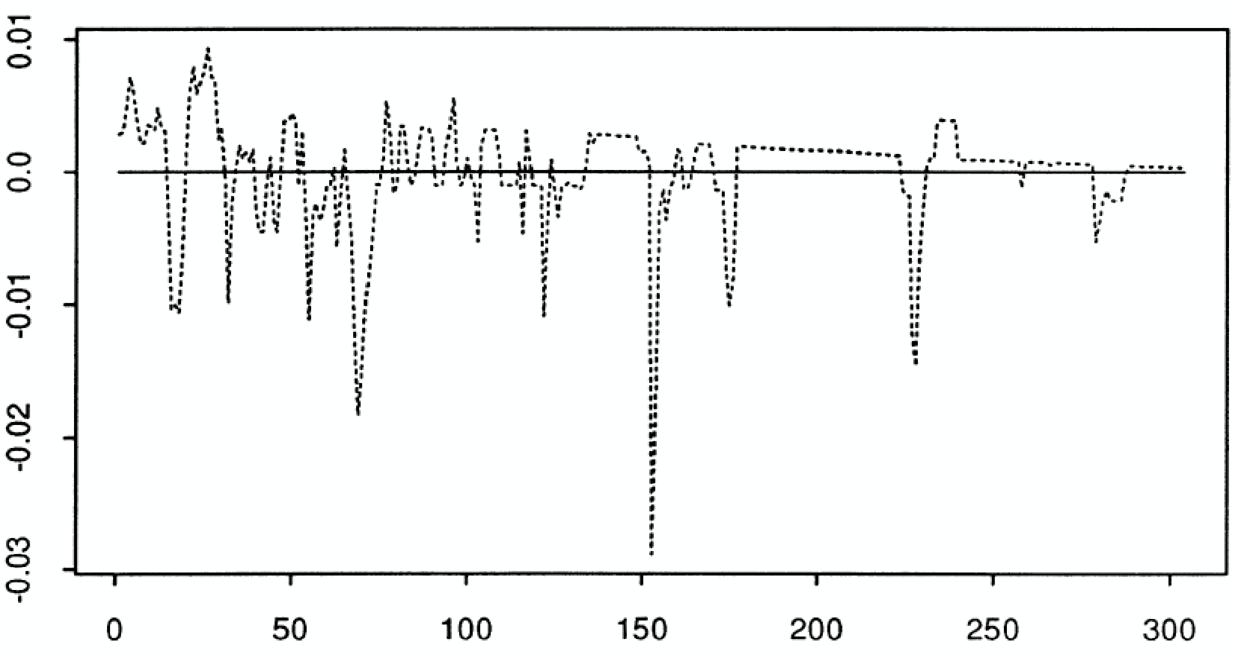
Under OLS the computed DF statistics are greater than the critical value at 1% level, therefore we reject the null hypothesis of non-stationarity for the residuals v_t and u_t. Under RREG this conclusion tends to be ratified but cannot be considered due to possible non-white noise, non-linearities and/or heteroskedasticity problems as suggested by the Q-Statistic of the regression squared residuals. Therefore again the stationarity of v_t and u_t can only be taken as preliminary. This result is not all that surprising if we examine more closely the plots of the estimated values of v_t and u_t (see figures 41 and 42). It is clear that both plots can be divided into two parts at around the 140^{th} week. The first part from week 1 to week 140 shows variability and it can be shown to be stationary. While the second part from week 140 on, except for a few extreme observations the series shows no variability and a mean that is slowly moving towards the zero line, more like a non-stationary series. Only the Q-statistic of the squared residuals from the RREG procedure was able to capture this. From a statistical point of view there is a problem in concluding whether the series v_t and u_t are stationary or not, the first part of the plots favor stationarity but the second part does not. From an economist's point of view the horizontal lines at $v_t = 0$ and $u_t = 0$ correspond to no departure from long-term equilibrium between LP_t and LO_t. The first part of the plots show how the foreign exchange market was continuously out of equilibrium and erratic, while on the second half there is less erratic behavior and rather a steady course toward full equilibrium. That is, it is precisely the second part of the plots that show the existence of a long-term equilibrium relationship!. In the short-term LP_t and LO_t could present differences in value and behavior but their time paths are moving together under a long-term equilibrium relationship, which is what the OLS result suggests!. In order to

Figure 41



OLS Residuals of v

Figure 42



OLS Residuals of u

continue the analysis, the preliminary although questionable result of v_t and u_t being stationary will be taken as an assumption.

Combining Short-Term Dynamics with Long-Term Equilibrium

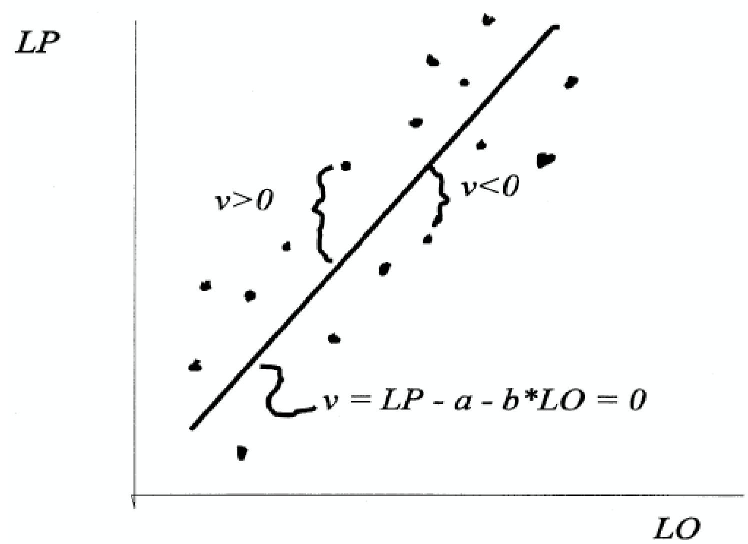
DLP_t and DLO_t might have specific economic names, those of rates of change of the exchange rate in each market, but this does not hide the fact that they are differenced data. The problem of differenced data is that it only retains the higher frequency components and eliminates the low frequency components. In economic terms this means the differenced data is a good representation of only the short-run dynamics of the exchange rates as the above analysis corroborates. The long-run component of it has been eliminated and therefore valuable information relative to equilibrium relationships is lost. An Error Correction Model is applicable to solve this problem by incorporating the cointegration results. The basic idea is to bring long-run relationships back into the analysis of short-run dynamics in the following way:

$$DLP_t = \theta_{11} - \theta_{12}{*}v_{t-1} + \text{lagged}\ (DLP_t, DLO_t) + \varepsilon_{t11}$$
$$DLO_t = \theta_{21} - \theta_{22}{*}u_{t-1} + \text{lagged}\ (DLP_t, DLO_t) + \varepsilon_{t21}$$

where ε_{t11} and ε_{t21} are white noise. These equations are known as the error correction form of the cointegrated variables. According to the Granger Representation Theorem if LP_t and LO_t are cointegrated it has been proved by Granger and Engle (1985) that there always exists the above data generating mechanism. The variables v_t and u_t correspond to the estimated "cointegrating relationship" found in the previous section, and $v_{t-1} = LP_{t-1} - \alpha - \beta{*}LO_{t-1}$ together with u_{t-1} enter the model by representing the out of equilibrium experienced in the previous week. It is expected that market forces will correct this disequilibrium by affecting the next period short-term behavior of both DLP_t and DLO_t. This is the way long-term equilibrium is incorporated back into modeling short-term dynamics.

Table 9 presents the estimated regressions. Under RREG the first equation shows v_{t-1} is a significant variable in explaining the variability of DLP_t. The negative sign of the parameter for v_{t-1} suggests a negative relationship between last week's disequilibrium and the current week's DLP_t. This was expected since disequilibrium situations are distances of points to the long-term equilibrium line, this line results from the case of $v_t = 0$ (see figure 43). If $v_t > 0$ (point above the line) then the rate of change in the parallel rate must drop next period to get back to equilibrium. If $v_t < 0$ (point below the line) then the rate of change must increase to get back to equilibrium.

Figure 43



The size of the estimated parameter also suggests that about 33% of the correction towards equilibrium is done within a week through DLP_t.[5] The greater the size of the parameter

[5] This argument assumes there is a direct causality link between DLP_t and the out of equilibrium position of LP_t respect to LO_t. This is what the theory of Chapter II also suggests.

TABLE 9
OLS AND RREG ESTIMATION OF AN ERROR CORRECTION MODEL

Depended Variable	Independent Variables	OLS Coefficient	OLS t-Statistic	RREG Coefficient	RREG t-Statistic
DLP_t	C	-0.0004	-1.80	-3.0E-05	-0.29
	DLP_{t-1}	-0.2554	-4.64	-0.2155	-9.19
	DLP_{t-2}	-0.1238	-2.27	-0.1235	-6.32
	DLP_{t-3}			-0.0018	-0.09
	DLO_t	0.9935	12.31	0.7672	25.10
	DLO_{t-1}	0.4358	4.40	0.1178	2.70
	DLO_{t-2}	0.0468	0.51	0.1074	2.99
	DLO_{t-3}			0.1579	5.05
	DUM	0.0022	7.21	0.0019	14.53
	v_{t-1}	-0.3724	-7.97	-0.3375	-18.37
		$R^2 = 0.5580$ Q = 10.94 (0.3622) $Q.res^2 = 0.97$ (0.9998)		$R^2 = 0.8923$ Q = 15.85 (0.1040) $Q.res^2 = 41.05$ (1.1E-05)	
DLO_t	C	0.0008	5.82	0.0009	9.92
	DLO_{t-1}	0.0044	0.07	0.1097	2.78
	DLO_{t-2}	0.1491	2.79	0.1526	4.42
	DLP_t	0.3440	12.31	0.3019	16.12
	DLP_{t-1}	0.1693	5.28	0.0426	1.66
	DLP_{t-2}	-0.0195	-0.60	-0.0198	-0.95
	DUM	-0.0005	-2.47	-0.0001	-0.79
	u_{t-1}	-0.1316	-4.53	-0.0900	-4.76
		$R^2 = 0.5054$ Q = 8.24 (0.6054) $Q.res^2 = 18.13$ (0.0528)		$R^2 = 0.7038$ Q = 6.70 (0.7534) $Q.res^2 = 7.59$ (0.6688)	

Note.- Numbers in parenthesis are p-values for the computed Q-Statistic.

of v_{t-1} the faster the correction. It is important to notice that the correction mechanism only works after something happened (big or small), therefore it can never help in predicting disequilibrium. The other variables remain significant as was found before. One problem remains with the robust regression in that the Q-statistic for the regression squared residuals still suggests non-white noise, non-linearities and/or heteroskedasticity problems.

Under RREG the second equation shows u_{t-1} is a significant variable in explaining the variability of DLO_t. By the size of its estimated parameter we can conclude that corrections toward equilibrium are done rather slowly; 9% of disequilibrium is restored within a week through DLO_t.[6] Also as found before, only DLP_t, DLO_{t-1} and DLO_{t-2} are the other variables which are significant in explaining the behavior of DLO_t.

In both equations the OLS procedure generates apparently clean results, but we know those estimated statistics have influential observation problems. Comparing these with the estimated statistics and tests from the RREG procedure is a good indication of how far off they are. But the RREG procedure has problems of its own specially in the first equation where non-white noise, non-linearities and/or heteroskedasticity problems still persist. The normal probability plots of the RREG residuals from both equations (figures 44 and 47) once again suggest departure from normality due to short tails. This question all computed statistics since they are based on normality assumptions. Figures 48 and 49 plot these residuals which are basically the same compared to figures 39 and 40 from the Market Interactions Model. The inclusion of the error correction term might be significant and important to include, but the problem of heteroskedasticity has remained intact.

[6] This argument also assumes there is a direct causality link between DLO_t and the out of equilibrium position of LO_t respect to LP_t.

Figure 44

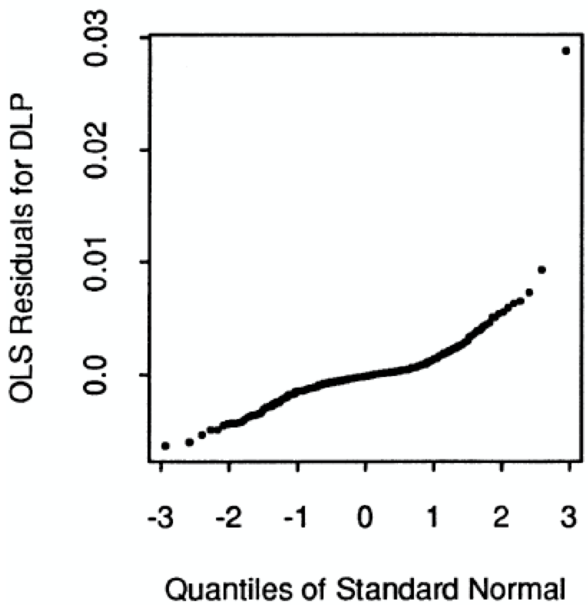


Figure 45

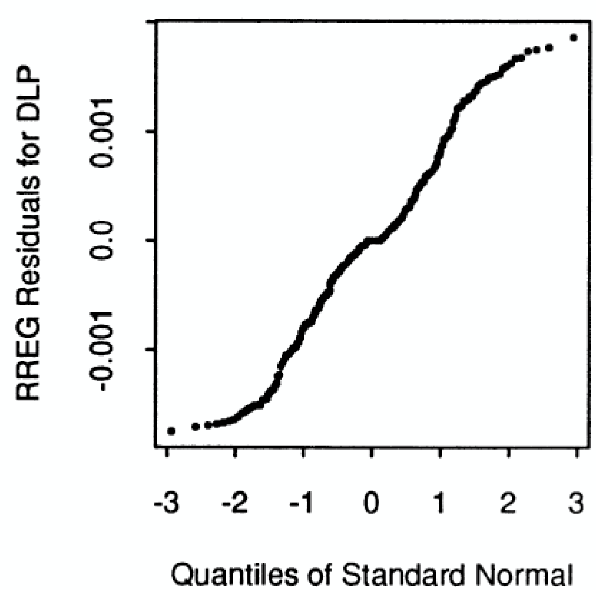


Figure 46

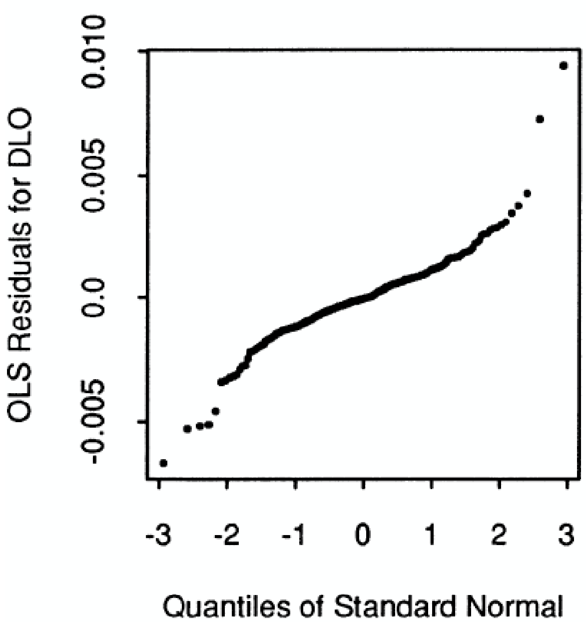


Figure 47

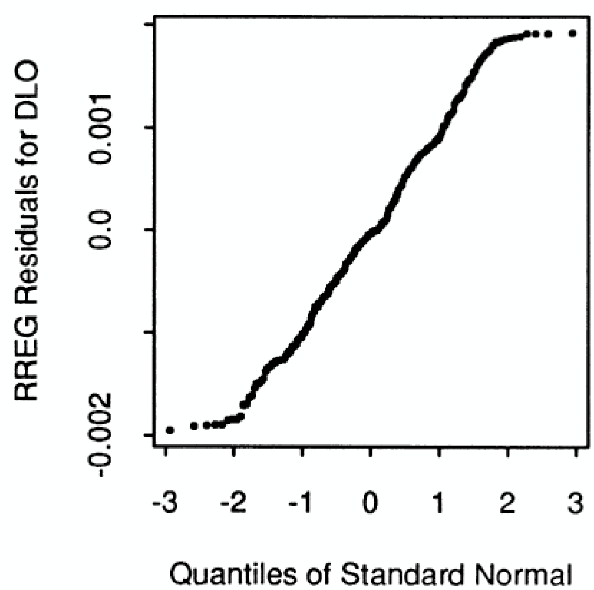


Figure 48: Error Correction Model

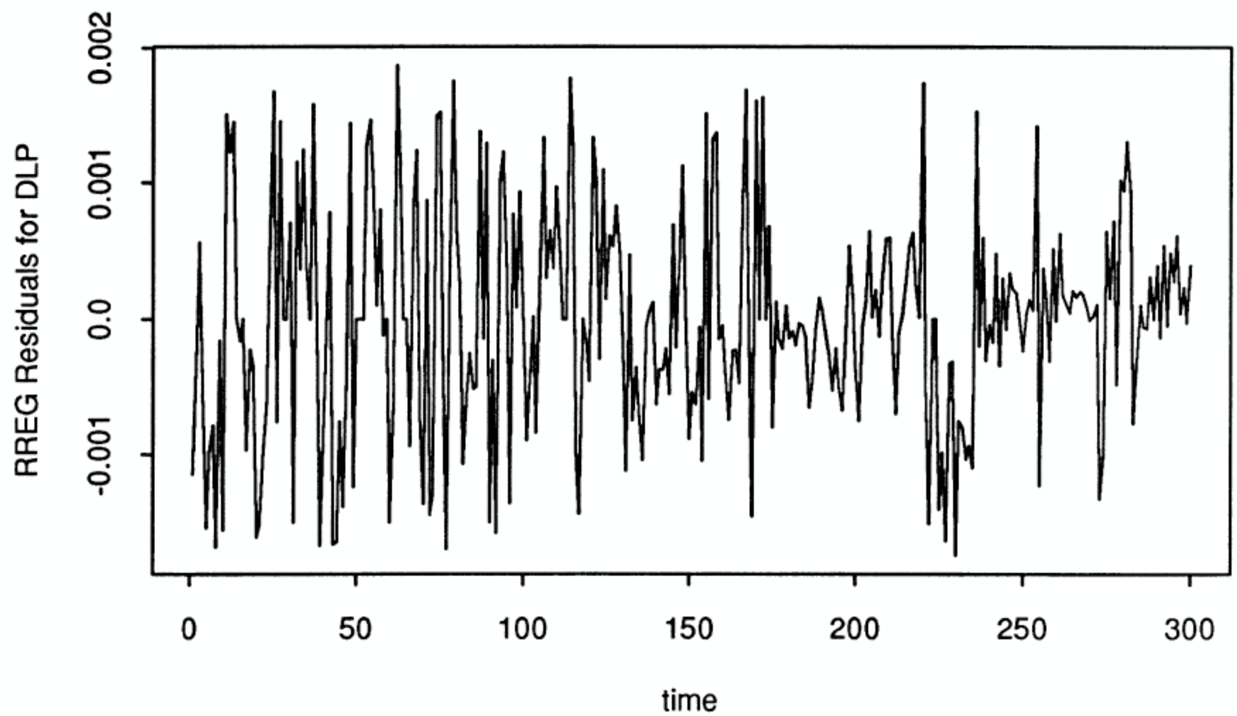
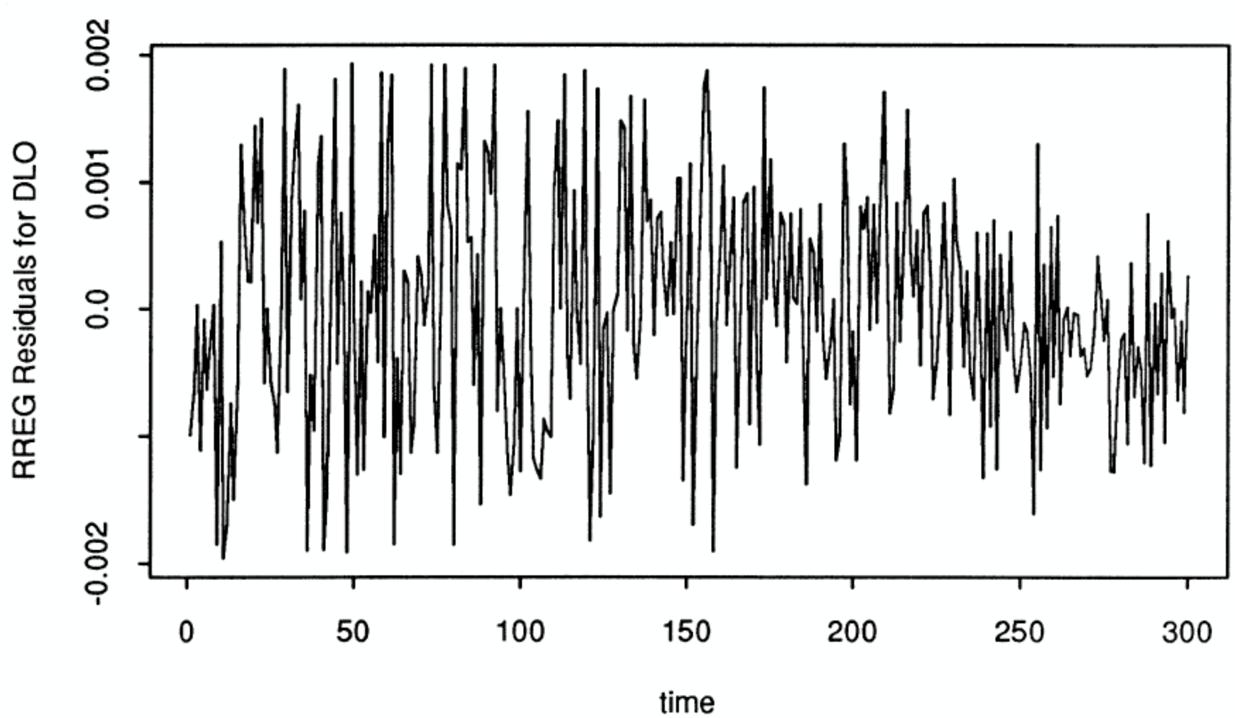


Figure 49: Error Correction Model



CHAPTER V

CONCLUSIONS

In terms of the statistical aspects of the study, the following conclusions are appropriate:

1. It does not make much sense to discover the type of distribution function that better represents the rate of change on the exchange rate. First, because most probably the observations are not independent over time and therefore some degree of autocorrelation would exist that should be taken into consideration. Second and most importantly, because other procedures, like regression analysis, can produce data generating processes that are richer in terms of understanding the underlying phenomena.

2. One important characteristic of the Bolivian exchange rate data is that the behavior of its rate of change (either DLP_t or DLO_t) is not always homogeneous over time. Rather, it experiences sudden large increases followed by large drops from time to time. This generates extreme observations which distorts the data and make the series appear highly volatile. Assuming that these extreme observations are a result of exogenous factors and not internally generated, the best way to treat them is by including other explanatory variables.

3. A statistical way of handling this problem is by using robust methods. The analysis does show important differences in the estimated statistics and tests compared to ordinary least squares estimates. The downside of the method is that it leaves up to a statistical procedure the decision of how to weight each observation, and surely some of these would be difficult to justify in economic terms. Other problems that appeared as the method was applied to this study are that (i) it revealed departures from normality due to short tails and, (ii) non-white noise squared residuals.

4. About the first problem, robust regression eliminated extreme observations and this had the effect of cutting the sides of the histogram of the data, thus creating short tails. Non-normality of the residuals question the estimated t-statistics and Q-statistics, but this problem may not be so serious considering the size of the time series. In large samples the central limit theorem holds implying that the estimated parameters are asymptotically normal and that tests involving these parameters are asymptotically valid even if the residuals are not normal. This assumes though that their variance is finite and that the basic assumptions of regression analysis hold.

5. This points to the second problem of possible non-white noise residuals, non-linearities and/or heteroskedasticity which is more serious. Large values for the Q-statistic of the squared residuals were found in regressions involving DLP_t and DLO_t as the independent variables, and also when LP_t and LO_t were being tested for cointegration. Non-linearity is an indication of an inappropriate model while heteroskedasticity could be treated with GARCH methods. These basically apply ARMA techniques to model the variance of the residuals. Plots of the residuals did show the variance changing over time or in clusters, which are characteristics typically modeles with GARCH methods. Heteroskedasticity appears to be the remaining underlying problem in both the Market Interactions Model and the Error Correction Model. However, the use of GARCH methods in this specific case requires first the implementation of a computational link between GARCH and Robust Regression. Non-white noise residuals also became evident during the cointegration analysis when the residuals u_t and v_t were being tested for stationarity. Besides the heteroskedastic residuals, these tests revealed another general characteristic of the exchange rate data in that it shows a first period of greater variability and a second period of less variability.

6. The combination of these three data characteristics: influential observations, heteroskedasticity and two different behavioral periods were at the heart of the statistical problems found in the present study.

The following are conclusions related to the economic aspects of the study, and obtained from the robust regression procedures as applied to an Error Correction Model. Given the statistical problems mentioned, these conclusions can only be taken as preliminary:

1. In general the model tends to verify the results predicted by economic theory. There is interaction between the parallel and the official markets, and there is a long-term relationship between the two.

2. About 89% of the variability of the rate of change of the parallel exchange rate can be explained in terms of its own past behavior, the current and past behavior of the official market, and it's out of equilibrium position respect to the official market.

3. About 70% of the variability of the rate of change of the official exchange rate can be explained in terms of its own past behavior, the current and past behavior of the parallel market, and it's out of equilibrium position respect to the parallel market.

4. There is a stronger association between the official market and the behavior of the parallel market than viceversa. The parallel market observes and adjusts to the current and previous two weeks of movements in the official market, and also to its long-term out of equilibrium position. While the official market only observes and adjusts to current behavior of the parallel market and its long-term out of equilibrium position. The magnitude of these adjustments is greater in the parallel market than in the official market. From this analysis we cannot say the official market drives the parallel market, but it is certainly close to that. But at

the same time this confirms the hypothesis that the dynamism of the small firms sector has a degree of influence over the official rate market.

Further research on this topic should include other explanatory variables under a better theoretical model. Probably the inclusion of the expected rate of inflation, expected real interest rate, and the rate of change of the economy's money supply would help in explaining exchange rate short-term dynamics. This would require testing for multivariate cointegration and the development of a multivariate error correction model. Also variables representing changing international economic conditions and variables representing changing non-economic conditions could help explaining many extreme observations experienced in the foreign exchange market. Since many variables would be involved, principal component analysis could be used to reduce them to a couple of variables.

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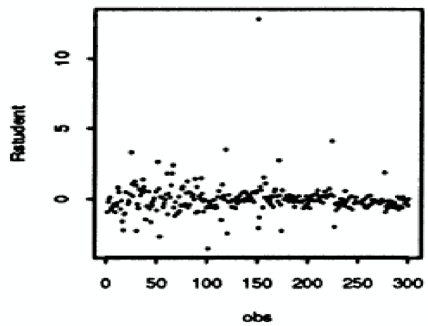
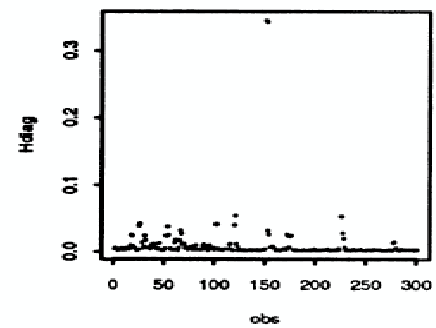
APPENDICES

APPENDIX A

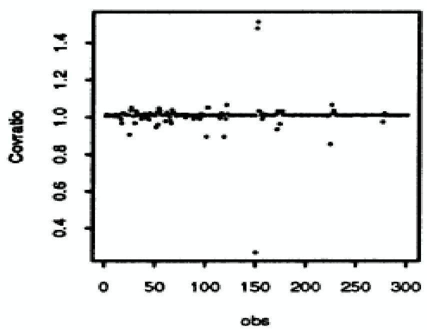
A1. RSTUDENT DLP



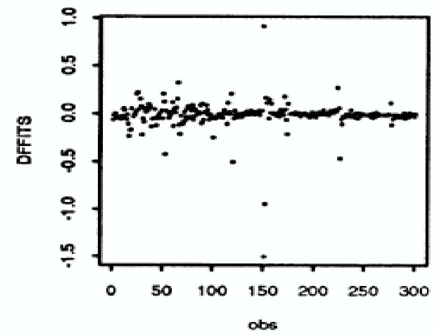
A2. HDIAG DLP



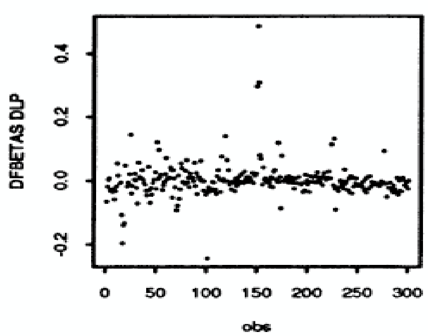
A3. COVRATIO DLP



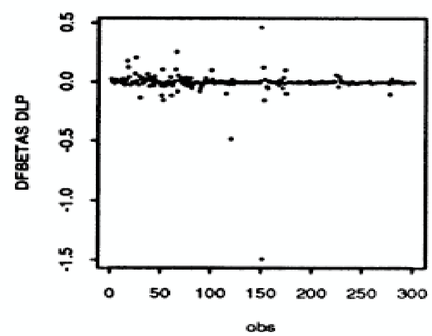
A4. DFFITS DLP



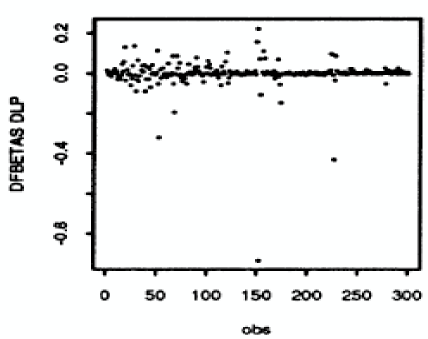
A5. CONSTANT TERM



A6. FIRST LAG

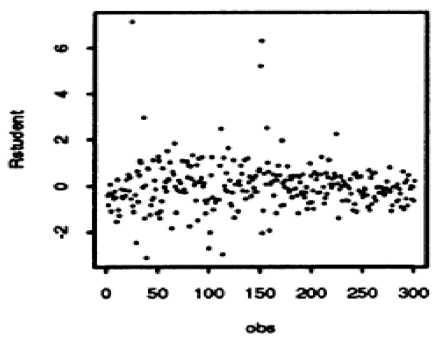


A7. SECOND LAG

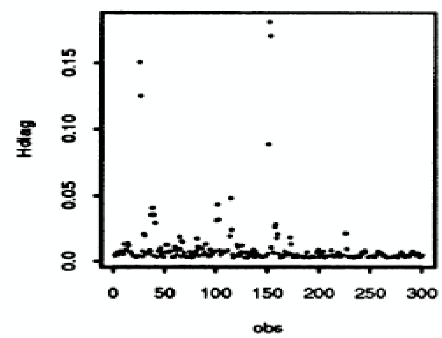


APPENDIX A (Continuation)

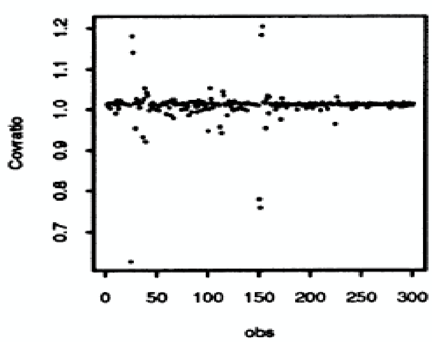
A8. RSTUDENT DLO



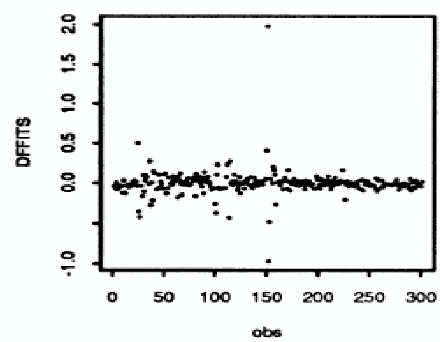
A9. HDIAG DLO



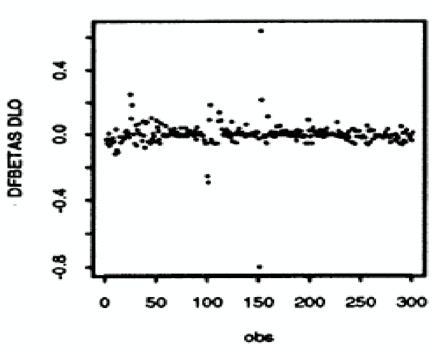
A10. COVRATIO DLO



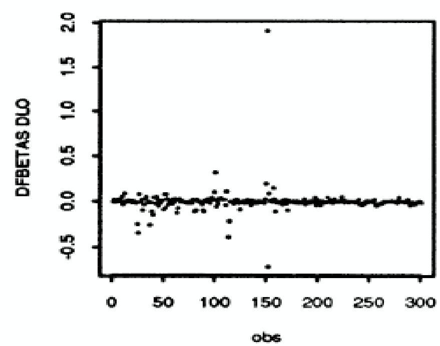
A11. DFFITS DLO



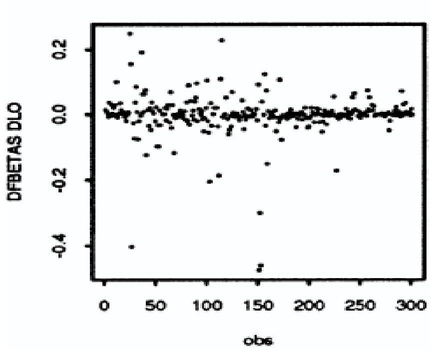
A12. CONSTANT TERM



A13. FIRST LAG



A14. SECOND LAG



APPENDIX B

B1: DLP_t Influential Observations

Obs.	Rstudent	Hdiag	COVRAT	DFFIT	DFBETAS1	DFBETAS2	DFBETAS3	Frequency
17	1			1	1	1		4
18					1	1		2
19					1		1	2
25	1		1		1			3
26		1				1		2
27		1	1					2
28							1	1
30	1					1		2
51	1		1		1		1	4
53	1			1		1	1	4
54		1	1					2
60	1							1
66				1		1		2
67		1	1					2
68		1					1	2
101	1		1	1	1			4
102		1	1					2
103		1	1					2
119	1		1		1			3
120	1	1		1		1		4
121		1						1
150			1		1		1	3
151	1	1	1	1	1	1	1	7
152		1	1	1	1	1	1	6
153		1				1		2
154		1						1
171	1		1		1			3
172		1						1
174	1						1	2
224	1		1	1				3
225		1	1					2
226		1		1	1		1	4
227		1						1
TOT	13	17	15	9	12	10	10	

APPENDIX B (Continuation)

B2: DLO_t Influential Observations

Obs.	Rstudent	Hdiag	COVRAT	DFFIT	DFBETAS1	DFBETAS2	DFBETAS3	Frequency
10					1			1
25	1		1	1	1	1	1	6
26		1	1	1		1	1	5
27		1	1	1	1		1	5
29	1		1					2
36	1		1	1			1	4
37		1		1		1		3
38		1	1					2
39	1		1					2
40		1	1			1		3
41		1					1	2
63						1		1
68							1	1
100	1		1	1	1			4
101	1	1		1	1	1		5
102		1	1					2
103		1			1		1	3
112	1		1	1	1		1	5
113	1		1	1		1		4
114		1	1	1		1	1	5
150	1		1	1		1		4
151	1	1	1	1	1	1	1	7
152	1	1	1	1	1	1	1	7
153		1	1	1	1		1	5
156	1		1				1	3
157		1				1		2
158		1						1
159				1			1	2
171	1							1
224	1							1
226							1	1
TOT	14	15	18	15	10	12	15	

APPENDIX C

```
Q.stat<-function(x) {
n_length(x)
y_acf(x, type=”correlation”, plot=F)
z_y$acf [2:11]
Q.BoxCox_n*sum(z^2)

w_0
for (i in 1:10) {
ac_y$acf [i+1]^2/(n-i-1)
w_w+ac
Q.LjungBox_n*(n+2)*w

yy_acf (x^2, type=”correlation”, plot=F)
zz_yy$acf [2:11]
QBC.sr_n*sum(zz^2)

ww_0
for (i in 1:10) {
aac_yy$acf [i+1]^2/(n-i-1)
ww_ww+aac
QLB.sr_n*(n+2)*ww

Return (Q.BoxCox, Q.LjungBox, QBC.sr, QLB.sr)
}
```

APPENDIX C

```
# LEAST SQUARES AND ROBUST REGRESSION ROUTINES

Regress<-function (x, y, z) {
a_lsfit (x, y)                              #Least squares regression of y on x
da_ls.diag(a)
ls.resid_a$residuals
tss_sum ((y-mean(y))^2)
rss_sum (a$residuals^2)
r2_1- (rss/tss)                             #Computation of R^2
tvalues_a$coef / da$std.err                 #t-values of estimated parameters
table_rbind (a$coef, t(tvalues))            #Matrix of coefficients and t-values
Qstat_Q.stat (a$residuals)                  #Q-statistics for ols residuals

ra_rreg (x, y)                              #Robust Regression of y on x
weights_ra$w
rreg.resid_ra$residuals*sqrt(ra$w)
rtss_sum (ra$w*(y-(sum(y*ra$w)/sum(ra$w)))^2)
rrss_sum(ra$w*ra$residuals^2)
rr2_1- (rrss/rtss)                          #Computation of R^2
rs2_rrss/(nrow(x) - ncol(x) -1)
v_diag(ra$w)
rcovar_rs2*solve(t (z) %*%v%*%z)
rvar_diag(rcovar)
rtvalues_ra$coef / sqrt(rvar)               #t-values of estimated parameters
rtable_rbind(ra$coef, rtvalues)             #Matrix of coefficients and t-values
Qrstat_Q.stat(ra$residuals*sqrt(ra$w))      #Q-statistics for rreg residuals

return(table, r2, Qstat, ls.resid, rtable, rr2, Qrstat, weights, rreg, rreg.resid)
}
```

APPENDIX C

```
# THIS PROGRAM DOES ADF UNIT ROOT TESTING
# x is the data to be tested for unit roots
# p is the number of lags to be considered
# The program automatically includes a constant, time and x1=x(-1)
# The program also includes dx=diff(x) in the LHS and up to dx(-p) on RHS
# Parameter are estimated by OLS
# The program automatically also produces robust regression estimates
# In both type of regressions Q-stats are computed

uroot<-fntion (x, p) {
n_length(x)
a_p+1
b_n-1
x1_x[a:b]
nn_n- (p+1)
time_c(1:nn)
xx_cbind(time, x1)
dx_diff(x)
m_length(dx)
dx0_dx [a:m]
for (i  in 1:p) {
a_p - i+1
b_m – i
xx_cbind(xx, dx[a:b])
}
xxx_cbind(rep(1, nn), xx)
y_regress(xx, dx0, xxx)
return(y)
}
```