An Application of Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Modelling on Taiwan's Time-Series Data: Three Essays

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AN APPLICATION OF AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY (ARCH) AND GENERALIZED AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY (GARCH) MODELLING ON TAIWAN'S TIME-SERIES DATA: THREE ESSAYS

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Modelling on Taiwan's Time-Series Data: Three Essays

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In this dissertation, three essays are presented that apply recent advances in time-series methods to the analysis of inflation and stock market index data for Taiwan. Specifically, ARCH and GARCH methodologies are used to investigate claims of increased volatility in economic time-series data since 1980.

In the first essay, analysis that accounts for structural change reveals that the fundamental relationship between inflation and its variability was severed by policies implemented during economic liberalization in Taiwan in the early 1980s. Furthermore, if residuals are corrected for serial correlation, evidence in favor of ARCH effects is weakened. In the second essay, dynamic linkages between daily stock returns and daily trading volume are explored. Both linear and nonlinear dependence are evaluated using Granger causality tests and GARCH modelling. Results suggest significant unidirectional

Granger causality from stock returns to trading volume. In the third essay, comparative analysis of the frequency structure of the Taiwan stock index data is conducted using daily, weekly, and monthly data. Results demonstrate that the relationship between mean return and its conditional standard deviation is positive and significant only for high-frequency daily data.

(140 pages)

INTRODUCTION

In his article "Autoregressive Conditional Heteroskedasticity (ARCH) with Estimates of the Variance of UK Inflation," Engle (1982) developed the ARCH model allowing the conditional variance of ordinary least-squares (OLS) residuals to change over time as a function of past error. Bollerslev (1986) extended the ARCH model by generalizing the autoregressive representation to account for an infinite lag structure. Bollerslev's model is typically referred to as the generalized autoregressive conditional heteroskedastic model (GARCH). The ARCH and GARCH models formulate time-varying conditional variances in time-series data and have proven to be effective tools in modelling temporal behavior of economic variables (see Engle, 1983; Engle and Bollerslev, 1986; Cosimano and Jansen, 1988; Welch, 1989).

This dissertation presents three essays that employ ARCH and GARCH methodologies to investigate time-series analysis of inflation and stock price index data for Taiwan. While previous studies have not incorporated Taiwan's time-series data into analysis, Taiwan provides an interesting arena to research for three reasons. First, Taiwan has made remarkable economic progress over the last several decades with an annual average economic growth rate of 8.36% in the past decade and per capita GNP of U.S. $10,215 in 1992. Second, Taiwan has become the world's thirteenth largest trading country with a foreign exchange reserve estimated at $90 billion in 1993. Third, Taiwan liberalized economic institutions in the early 1980s; thus, sufficient data are available for researchers to evaluate the effect of economic liberalization on economic phenomena.

This dissertation contains three different essays. The first essay addresses the issue of inflation. In this essay we explore the fundamental relationship between average monthly inflation and its variability between January 1971 and June 1992, and then we determine if the inflation and its variability fit the ARCH/GARCH processes. The second essay explores the dynamic linkage between stock returns and trading volume in the Taiwan Stock Market. It investigates both linear (Granger causality test) and nonlinear (GARCH modelling) dependence. This essay also applies several other econometric techniques such as the unit root test, cointegration test, and Lagrange multiplier test. The third essay addresses the empirical relationship between stock returns and volatility in Taiwan using daily, weekly, and monthly returns on the Taiwan Stock Exchange Index from January 1967 to September 1994. The final section provides an overview of the three essays and of their contribution to the current body of empirical literature that employs ARCH and GARCH methodologies.

ESSAY 1: ECONOMIC LIBERALIZATION, STRUCTURAL CHANGE, AND THE MEAN-VARIANCE LINKAGE OF INFLATION—TAIWAN'S EXPERIENCE

Abstract

This essay explores the fundamental relationship between average monthly inflation and its variability in Taiwan between January 1971 and June 1992. Chow test results suggest significant evidence of a structural change in inflation behavior beginning in 1982, a period of economic liberalization in Taiwan. Analysis that accounts for structural change reveals that the fundamental relationship between inflation and its variability was severed by policies implemented during economic liberalization in the early 1980s. In addition, ARCH and GARCH effects fail to be significant when structural change is accounted for.

I. Introduction

Jaffee and Kleinman (1977) demonstrated that the welfare cost of uneven inflation is an increasing function of both the expected inflation rate and of the expected dispersion of inflation rates over time and over commodities. Friedman (1977, pp. 465-66), in his Nobel Lecture, argued that "higher rates of inflation are generally associated with higher variability of inflation and presumably greater uncertainty about future rates." As a result of Friedman's assertion and the implications of Jaffee and Kleinman's work for the welfare cost of inflation, many attempts have been made to empirically validate the relationship between expected rates of inflation and its variance.

Most empirical studies have confirmed a positive relationship between the level of inflation and its intertemporal variability for a broad cross section of countries (see Okun, 1971; Logue and Willett, 1976; Jaffee and Kleinman, 1977; Foster, 1978; Fischer, 1981; Hafer and Heyne-Hafer, 1981; Katsimbris and Miller, 1982; Pagan, Hall, and Trivedi, 1983; Welch, 1989; Chowdhury, 1991). However, prior to Engle's (1982) application of autoregressive conditional heteroskedasticity (ARCH) techniques to U.K. inflation data, empirical analysis of the conditional mean-variance relationship lacked a methodology to incorporate the joint estimation of expectations in the level and variance of inflation.

Engle (1982) demonstrated that in the U.K., ARCH techniques improved inflation variance forecasts relative to traditional ordinary least squares (OLS) estimation. In a subsequent publication, Engle (1983, p. 292) again used an ARCH model to investigate the conditional mean-variance relationship using U.S. price data and found that a "high rate of inflation does not necessarily imply a high variance of inflation."[1] Engle's results using U.S. data appear to contradict Friedman's armchair empiricism and prior work by Okun (1971), Logue and Willett (1976), Foster (1978), and others. However, Engle's results have not escaped criticism either. In particular, Cosimano and Jansen (1988) argued that a more complete specification of autoregressive behavior in the reduced-form inflation equation largely eliminates evidence of ARCH-type residuals. While Cosimano and Jansen reached the same general conclusion as Engle, that inflation levels and variance are unrelated, they cited work by Holland (1984) to argue that ARCH effects are largely a result of model misspecification.

In an attempt to reconcile apparently contradictory results, Ball and Cecchetti (1990, p. 216) presented a comprehensive analysis of permanent and transitory movement in inflation for a cross section of 40 countries. Their central finding was that "the level of inflation has a much stronger effect on the variance of permanent shocks than on the variance of temporary shocks, and thus a stronger effect on uncertainty at long horizons." Ball and Cecchetti's results were particularly useful in accentuating the social cost of inflation. Since the added risk in long-term contracts must be compensated for, high inflation variance distorts the allocation of resources between risk compensation and productive enterprise.

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[1]By plotting the conditional mean and standard deviation of inflation, Engle (1982) showed that the variance of inflation was uncorrelated with the current level of inflation--that high inflation in one period did not lead to greater uncertainty about inflation in the next period.

The purpose of this essay is to employ recent data (1971-1992, henceforth, period W) to investigate empirically whether inflation "uncertainty" has increased with the inflation rate in Taiwan.

The rapid economic development in Taiwan is one of the few success stories of third-world development. While Taiwan has experienced average growth rates of 8.53% over the last 20 years, it has maintained a relatively low level of inflation compared with other developing countries. Studies by Ball and Cecchetti (1990), Buck (1990), and Chowdhury (1991) found evidence of significant positive correlation between a country's rate of inflation and its variability. However, previous research has not incorporated an analysis of Taiwan's inflation experience. Taiwan provides an interesting case study for two reasons: First, it has experienced high growth rates and relatively modest rates of inflation; and, second, Taiwan liberalized economic institutions in the early 1980s, and sufficient time-series data are available to assess the effect of liberalization on the basic economic linkage between inflation rates and inflation variability.

The essay is organized as follows. Section II describes how inflation uncertainty is modeled and measured. Section III presents the framework for evaluating the relationship between the conditional mean and conditional variance of inflation. Section IV presents the empirical results, and section V provides a summary of our analysis.

II. Measuring Inflation Uncertainty

According to rational expectations theory, individuals efficiently process all relevant and available information in making a forecast of a future period's rate of inflation.

Estimates of next period's inflation can be thought of as the mean of some underlying probability distribution, conditioned on the information generation process. Inflation uncertainty, then, arises from a lack of full information about how the future price level is determined. In theory, each individual's forecast of inflation uncertainty can be compared ex post by observing the range of the confidence interval bounds for a constant level of confidence. For example, an individual may have predicted at the end of 1992 that 1993 inflation had a 95% probability of being between 2% and 4%. If the same individual's 95% confidence interval for 1994 inflation (forecast made at the end of 1993) is wider, say 3% to 6%, then his or her uncertainty about 1994 inflation is greater than it was for 1993 inflation. The analysis presented above deals with inflation uncertainty for a representative individual. In practical application, the level of an individual's uncertainty about inflation is not directly observable (see Okun, 1971; Logue and Willett, 1976; Fischer, 1981; Engle, 1982), so we use the variance of inflation around its conditional mean as a proxy for inflation "uncertainty." An implicit assumption in this type of analysis is that variance need not be constant but may vary over time.

We assumed that the inflation rate is a random variable (Engle, 1982, 1983), and it has a nonstochastic unconditional mean and variance at each point in time. Individual economic agents form expectations of inflation based on their own information sets. Let p_t be the inflation rate at time t, Θ_{t-1} be the information available in time t-1, π_t be the conditional mean of p_t, and h_t be the conditional variance of p_t around π_t. Here, we attempted to measure π_t and h_t where:

(1) $E\left(p_t \mid \theta_{t-1}\right) = \pi_t$

(2) $E\left(\left(p_t - \pi_t\right)^2 \mid \theta_{t-1}\right) \equiv h_t$

The strength of these measures is that the conditional means and variances can be estimated jointly using conventionally specified models for economic variables.

III. Modelling the Mean-Variance Relationship

Following Holland (1984), we assumed that the variance of inflation was a function of lagged values of the conditional mean (π_t). Under this maintained hypothesis, we first evaluated the ARCH process introduced by Engle (1982) and, second, discussed the GARCH process presented in Bollerslev (1986). Each of the above procedures was based on the test of heteroskedasticity developed by Breusch and Pagan (1979).

In general, the reduced-form inflation model is specified as follows:

(3) $p_t = \alpha_0 + \alpha B(L) p_{t-1} + \beta B(L) m_{t-1} + \gamma B(L) w_{t-1} + \delta D_t + \lambda T + \varepsilon_t$

where p_t is inflation rate at time t, m_{t-1} is monthly money supply (M1) at time t-1, w_{t-1} is monthly manufacturing wage rate at time t-1, D_t is a dummy variable reflecting shocks to the system resulting from energy supply restrictions in 1973 and 1979, T is a time trend, and ε_t is an error term, $\varepsilon_t \sim N\left(0,\ h_t\right)$. B(L) is the back-shift operator where,

$\alpha B(L) = \alpha_1 + \alpha_2 L + \alpha_3 L^2 + \ldots + \alpha_k L^{k-1}$, $\beta B(L) = \beta_1 + \beta_2 L + \beta_3 L^2 + \ldots + \beta_l L^{l-1}$,

and $\gamma B(L) = \gamma_1 + \gamma_2 L + \gamma_3 L^2 + \ldots + \gamma_m L^{m-1}$. From equation (1) it is possible to represent the conditional mean of inflation (π_t) by the deterministic portion of equation (3) as follows:

(4) $$\pi_t = \alpha_0 + \alpha B(L)p_{t-1} + \beta B(L)m_{t-1} + \gamma B(L)w_{t-1} + \delta D_t + \lambda T .$$

From equation (2) we specified a linear relationship between the variance of inflation and lagged values of its conditional mean as follows:

(5) $$h_t = \Gamma_0 + \Gamma B(L)\pi_{t-1} ,$$

where $\Gamma B(L) = \Gamma_1 + \Gamma_2 L + \dots + \Gamma_n L^{n-1}$.

Following Cosimano and Jansen (1988), we used Hsiao's (1981) final prediction error (FPE) criterion to determine the appropriate lag length for each explanatory variable in (4). The FPE criterion chooses values for k, *l*, m to minimize the asymptotic mean square error (MSE) of the residuals. Choosing k, *l*, and m so as to minimize the FPE statistic is analogous to applying an F-test with varying significance levels.

Under this specification one can test whether the variance of inflation is dependent on the level of inflation by regressing the squared residuals from the OLS estimate of equation (3) (which assumes homoskedasticity) on the lagged estimated values of inflation. The test statistic NR^2 is distributed as $\chi^2(q)$ under the null hypothesis of homoskedasticity of ε_t. If the calculated test statistic exceeds its critical value, one rejects the null hypothesis and concludes that the variance of inflation depends on the level of inflation. One can also use the F-statistic to test the null hypothesis that $\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = \dots = \Gamma_n = 0$. If the null hypothesis is not rejected, one can conclude that inflation is unrelated to its variability (or that no significant relationship exists between the level of inflation and its variance). If heteroskedasticity is present, and if $\Sigma\Gamma > 0$ (where $\Sigma\Gamma$ is the sum of the coefficients of the lagged values of the expected inflation rate), then a positive relationship exists between the variability of inflation and its conditional mean.

The ARCH model presented by Engle (1982, 1983) assumed that the conditional variance of inflation at time t (h_t) was a function of past sample variances.

$$(6) \qquad E\left((p_t - \pi_t)^2 | \theta_{t-1}\right) \equiv h_t = \Lambda_0 + \Lambda B(L)\,\varepsilon_{t-1}^2$$

where $\Lambda_0 > 0$, $\Lambda_i > 0$, $i = 1, 2, \ldots q$, and $\Lambda B(L) = \Lambda_1 + \Lambda_2 L + \Lambda_3 L^2 + \ldots + \Lambda_q L^{q-1}$. Engle (1982, 1983) also presented a Lagrange multiplier test for ARCH process against the null hypothesis that H_0: $\Lambda_1 = \Lambda_2 = \Lambda_3 = \ldots = \Lambda_q = 0$, or ARCH(0). The test statistic NR^2, where R^2 is from the auxiliary regression (equation 6), is distributed as $\chi^2(q)$. The test procedure derived in Engle (1982, 1983) turned out to be just the same as for the general class of heteroskedasticity tests obtained by Breusch and Pagan (1979). If we reject the null hypothesis, then an ARCH effect exists.

Bollerslev (1986) expanded the ARCH model by generalizing the autoregressive representation to account for an infinite lag structure. Bollerslev's model was typically referred to as the generalized autoregressive conditional heteroskedastic model (GARCH). Bolerslev's representation assumed that the conditional variance of inflation at time t (h_t) is a function of past sample variance and lagged conditional variances. The GARCH(p, q) process is then given by:

$$(7) \qquad E\left((p_t - \pi_t)^2 | \theta_{t-1}\right) \equiv h_t = \Lambda_0 + \Lambda B(L)\,\varepsilon_{t-1}^2 + \kappa B(L)\,h_{t-1}^2$$

where $\Lambda_0 > 0$, $\Lambda_i > 0$, $i = 1, 2, \ldots, q$, $\kappa_j > 0$, $j = 1, 2, \ldots, p$,

$\Lambda B(L) = \Lambda_1 + \Lambda_2 L + \Lambda_3 L^2 + \ldots \Lambda_q L^{q-1}$, and $\kappa B(L) = \kappa_1 + \kappa_2 L + \kappa_3 L^2 + \ldots \kappa_p L^{p-1}$.[2] For $p = 0$, the GARCH(p, q) process reduces to an ARCH(q) process; and for $p = q = 0$, ε_t is simply a white noise. Bollerslev suggests a Lagrange multiplier test for GARCH(p, 0) against GARCH(p, q) (details see Bollerslev, 1986).[3] For simplicity, in this study, we tested only a GARCH(1, 1) process. According to Engle and Bollerslev (1986), if $\Lambda_1 + \kappa_1 = 1$ in the GARCH(1, 1) process, then the model is known as IGARCH (integrated GARCH), which implies persistence of the conditional variance over all future horizons and also an infinite variance of the unconditional distribution of ε_t.

IV. Data and Empirical Results

Figures 1 and 2 depict Taiwan's history of monthly inflation. Casual observation suggests a possible structural break in the series between 1981 and 1982, with a relatively high rate of inflation between January 1971 and December 1981 (an average yearly rate of 8.7% without taking into account the outlier) and a relatively low period of inflation between January 1982 and June 1992 (an average yearly rate of 2.1%). According to

[2]The nonnegativity constraints associated with the parameters in the h_t equation are necessary to satisfy certain regularity conditions associated with the ARCH and GARCH models.

[3]Autocorrelation and partial autocorrelation functions of the innovation series are typically used when identifying and checking the time-series behavior of ARMA models. Bollerslev (1986) pointed out that these same functions, as applied to the squared residual series, can be useful for identifying and checking the time-series behavior of the conditional variance equation of the GARCH model.

Figure 1

Monthly Consumer Price Index Plot

(January 1971 - June 1992)

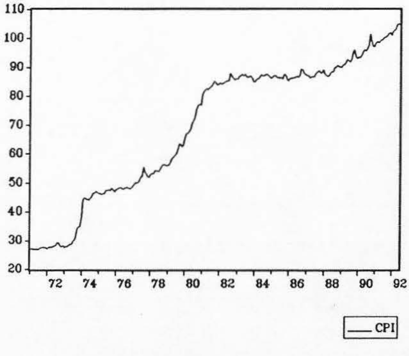
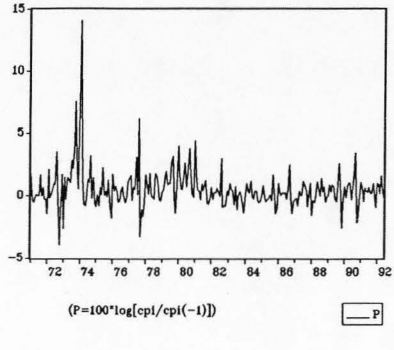


Figure 2

Monthly Inflation Rate



(P=100*log[cpi/cpi(-1)])

Chang (1991), the low inflation experience in the second subperiod is likely a result of two factors. First, the Taiwan government has exercised strict control of money supply during a period of economic liberalization. Second, increases in labor productivity have consistently outpaced wages. Additional factors may include the rarity of fiscal deficits, drops in the prices of imports, and lower import tariffs. The striking difference between these two subperiods raised the question whether or not we could pool them together in the regression analysis. In order to answer this question, we constructed a Chow test (and a Goldfeld-Quandt test) for a structural break in 1982. The resulting F-statistic and likelihood ratio statistic are reported in Table 1, and both reject the hypothesis that the second subperiod belongs to the same regression as the first subperiod at the 1% level. These results left us with two subperiods for analysis, January 1971 to December 1981 (henceforth, period I), and January 1982 to June 1992 (henceforth, period II). These subperiods capture what we might a priori suppose is the high mean-high variance of the 1970s and the low mean-low variance of the 1980s. Based on the Chow test results, we estimated a separate set of equations for the first and second subperiod. In addition, we estimated the model using the entire sample period to assess the effect of not accounting for structural breaks in the model on time-series properties of the estimators.

Our empirical investigation was separated into three sections. First, we analyzed the data series for the presence of unit roots. Second, we used the FPE criterion to establish optimal lag lengths for estimation of the reduced form equations. Finally, we explored ARCH and GARCH properties of the estimated variance.

Table 1

TESTS FOR STRUCTURAL CHANGE IN TAIWAN REDUCED-FORM INFLATION MODELS (3), (8): (1971.01-1981.12 VERSUS 1982.01-1992.06)

A. Tests for Change in Parameters (Chow Test):

Equation (3)	F-statistic	2.7214	Probability	0.0069
	Likelihood ratio	22.2374	Probability	0.0045
Equation (8)	F-statistic	2.7526	Probability	0.0023
	Likelihood ratio	31.1728	Probability	0.0010

B. Tests on Variance (Goldfeld-Quandt Test):

$H_0: \sigma_1^2 = \sigma_2^2$
$H_a: \sigma_1^2 \neq \sigma_2^2$

Period 1 (1971.01-1981.12): $SSR_1 = 379.40$
Period 2 (1982.01-1992.06): $SSR_2 = 88.94$

G-Q Test = $(SSR_1/d.f)/(SSR_2/d.f) = 3.88^*$

* : Denotes significance at the 1% level.

SSR : Denotes sum of squared residuals.

It is widely recognized that many macroeconomic time-series contain unit roots (dominated by stochastic trends) (see, for example, Nelson and Plosser, 1982; Stock and Watson, 1986). Unit root tests are important in examining the stationarity of a time-series because a nonstationary regressor invalidates many standard empirical results and thus requires special treatment. The test of unit root nonstationarity is performed by using a testing procedure proposed by Dickey and Fuller (1979, 1981) and by Said and Dickey

(1980).[4] Test results summarized in Table 2 confirm the presence of unit roots in the data series used in our analysis. Therefore, our model employs the first difference of the log of the data series.

We estimated structural equations (equation 3) for each subperiod under investigation by using Hsiao's (1981) FPE criterion to determine the optimum lag length for each explanatory variable. Optimum lag lengths were 2, 1, and 1 for p_{t-1}, m_{t-1}, and w_{t-1}, respectively, in the first subperiod and 10, 1, and 1 for p_{t-1}, m_{t-1}, and w_{t-1}, respectively, in the second subperiod. Table 3 presents estimation results, where p_t is the first difference of the log of the monthly consumer price index, m_{t-1} is the lagged value of the first difference of the log of the monthly money supply (M1), w_{t-1} is the lagged value of the first difference of the log of the monthly manufacturing wage rate, and D_t is a dummy variable taking a value of one for 1973.01-1974.12 and 1979.01-1982.12 to capture the oil supply shocks occurring in 1973 and 1979.[5] For both subperiods, the coefficient on m_{t-1} is insignificant while the coefficient on p_{t-2} is statistically significant. The oil shock dummy variable is only statistically significant for the first period.

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[4]Schwert (1989) compared the performance of alternative unit root tests and concluded that the augmented version of the Dickey-Fuller tests were superior to various alternatives, including the Phillips-Perron test, in the presence of an autoregressive moving average process of unknown order.

[5]Here we did not incorporate the T (trend) term in our empirical analysis due to two reasons: first, we found that coefficients on trend data were not statistically significant, and, second, as Nelson and Kang (1984) pointed out, it is better to use regressions in first differences rather than regressions in levels with T (time trend) as an extra explanatory variable.

Table 2

UNIT ROOT TESTS FOR DATA SERIES USING THE
AUGMENTED DICKEY-FULLER TEST

1. Whole sample period: 1971.01-1992.06

	τ_τ	Mackinnon Critical Values:		Integrated Order of (q)
p_t	-1.0633			
$D(p_t)$	-10.6068*	-3.9037	(1%)	1
m_t	-0.7154			
$D(m_t)$	-12.8989*	-3.3935	(5%)	1
w_t	-0.7108			
$D(w_t)$	-17.5277*	-3.1225	(10%)	1

2. First subperiod: 1971.01-1981.12

	τ_τ	Mackinnon Critical Values:		Integrated Order of (q)
p_t	-1.2659			
$D(p_t)$	-7.2847*	-4.0314	(1%)	1
m_t	-1.3323			
$D(m_t)$	-9.3132*	-3.445	(5%)	1
w_t	-0.0385			
$D(w_t)$	-13.0542*	-3.1471	(10%)	1

3. Second subperiod: 1982.01-1992.06

	τ_τ	Mackinnon Critical Values:		Integrated Order of (q)
p_t	-0.6218			
$D(p_t)$	-10.3656*	-4.0331	(1%)	1
m_t	-2.0347			
$D(m_t)$	-8.8609*	-3.4458	(5%)	1
w_t	-1.6349			
$D(w_t)$	-11.7672*	-3.1476	(10%)	1

* : Denotes significance at each percentage level.

D : Denotes the first difference operator.

Table 3

PARAMETER ESTIMATES FOR REDUCED-FORM INFLATION MODEL (3) (DEPENDENT VARIABLE IS p_t)

Variables	Period I OLS[3.I]	Period II OLS [3.II]	Variables	Period I OLS[3.I]	Period II OLS [3.II]
p_{t-1}	0.511	0.008	m_{t-1}	-3.526	-2.411
	(5.526)*	(0.091)		(0.827)	(0.969)
p_{t-2}	-0.245	-0.292	W_{t-1}	12.185	-0.261
	(2.732)*	(3.136)*		(3.698)*	(0.132)
p_{t-3}		-0.199	D_t	0.782	0.081
		(2.058)*		(2.323)*	(0.297)
p_{t-4}		0.013	α_0	0.162	0.375
		(0.125)		(0.695)	(3.361)*
p_{t-5}		-0.079			
		(0.749)	R^2	0.288	0.173
p_{t-6}		-0.106			
		(1.066)	adj-R^2	0.259	0.078
p_{t-7}		-0.036			
		(0.366)	F-stat	9.974*	1.811
p_{t-8}		-0.098	SE	1.756	0.891
		(1.016)			
p_{t-9}		-0.239	MSE	0.0021	0.0015
		(2.543)*			
p_{t-10}		-0.039	SSR	379.40	88.94
		(0.415)			

* : Significant at the 5% level.
Note : The number in parentheses denotes t-statistics.

Table 4 presents estimation results for equation (5). These results suggest the presence of heteroskedasticity in the first subperiod but not in the second subperiod. Based on heteroskedasticity test results, weighted least squares (WLS) analysis is conducted with w_{t-1} as the weight. Table 4 also presents the new estimates for subperiod I obtained from WLS. The coefficient on m_{t-1} is still insignificant and has a negative sign. A comparison

Table 4

PARAMETER ESTIMATES FOR REDUCED-FORM INFLATION MODEL (3) (WEIGHTED LEAST SQUARE, DEPENDENT VARIABLE = p_t) AND TESTS FOR INFLATION UNCERTAINTY USING REGRESSION MODEL (5) OF INFLATION EXPECTATIONS (DEPENDENT VARIABLE = h_t)

Variables	Period I WLS(3.I)	Variables	Period I OLS (5.I) lag (4)	Period I OLS (5.I) lag (12)	Period II OLS (5.II) lag (4)	Period II OLS (5.II) lag (12)
p_{t-1}	0.664	π_{-1}	2.669	2.450	0.399	0.334
	(8.728)*		(3.458)*	(2.740)*	(1.009)	(0.759)
p_{t-2}	-0.473	π_{-2}	-1.055	-1.575	-0.087	-0.035
	(5.809)*		(1.342)	(1.741)	(0.199)	(0.070)
		π_{-3}	1.142	1.721	-0.619	-0.209
			(1.455)	(1.919)	(1.395)	(0.379)
		π_{-4}	0.524	0.642	-0.106	-0.476
			(0.678)	(0.716)	(0.262)	(0.837)
m_{t-1}	-8.699	π_{-5}		0.897		0.698
	(1.863)			(0.965)		(1.243)
w_{t-1}	24.428	π_{-6}		1.304		-0.224
	(13.362)*			(1.405)		(0.404)
D_t	1.497	π_{-7}		-0.316		-0.076
	(4.429)*			(0.341)		(0.137)
		π_{-8}		-0.453		0.254
				(0.492)		(0.450)
		π_{-9}		0.458		-0.486
				(0.524)		(0.856)
		π_{-10}		-1.085		-0.938
				(1.229)		(1.701)
		π_{-11}		-0.204		0.300
				(0.224)		(0.594)
		π_{-12}		-1.275		-0.887
				(1.414)		(1.934)
α_0	0.091	α_0	0.113	0.597	0.803	1.034
	(0.269)		(0.087)	(0.384)	(4.745)*	(5.085)*
R^2	0.867	R^2	0.129	0.198	0.059	0.165
adj-R^2	0.861	adj-R^2	0.100	0.106	0.027	0.066
SE	7.481	SE	9.059	9.328	1.381	1.292
F-stat	158.387*	F-stat	4.46*	2.14*	1.837	1.666
		D-W	1.756	1.704	1.996	2.093
		$\sum\Gamma$	3.279	2.556	-0.401	-1.741
			(+)	(+)	(-)	(-)
		N*R^2	16.125*	23.16*	7.198	18.81

* : Significant at the 5% level.

Note : The number in parentheses denotes t-statistics.

of the coefficients on dummy variables (1.497 vs 0.081) suggests that the first oil shock had a more powerful impact on p_t than the second oil shock. This may indicate that individuals are rational in adapting their expectations based on previous experience.

Heteroskedasticity tests based on the structure of variance in equation (5) support the hypothesis of dependence of inflation variability on the level of inflation for the first subperiod. However, for the second subperiod we failed to reject the null hypothesis of homoskedasticity and concluded that no significant relationship exists between inflation and its variability (weaker negative relationship). These results confirm the findings of Logue and Willett (1976), Hafer and Heyne-Hafer (1981), and Chowdhury (1991) that a weak relationship exists between inflation and its variability for countries with low average inflation rates (below 5%). Logue and Willett (1976) implied that the lack of a statistically significant relationship between the inflation and its variability might indicate that "the nations are better able to conduct internal monetary and fiscal policy, thus limiting the variability and level of inflation" (p. 155). Chang (1991) reported that during the second subperiod, the 8th and 9th Economic Development Plans were implemented and a series of policies were announced to liberalize economic structure in Taiwan.[6]

The ARCH process presented by Engle (1982, 1983) maintained a hypothesis that the residuals from the reduced-form inflation model were uncorrelated. Since serially correlated residuals may, when squared, give results that look like the ARCH model, it was

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[6]According to Yu (1991), in order to achieve a moderate of economic growth with only mild inflation, the government has adopted the following measures: a tight money policy, economic liberalization, the Six-Year National Development Plan, and the Statute for Upgrading of Industries.

important to carry out diagnostic tests on residuals from equation (3) to help ensure that the residuals were not correlated. We performed three tests to evaluate the residuals from the reduced form equations: the Godfrey test for serial correlation, the ARCH(q) test, and the GARCH(1, 1) test. Table 5 presents the results.

The Godfrey test for serial correlation shows strong evidence of serial correlation on residuals from subperiod I.[7] Autocorrelograms constructed for the set of residuals from subperiod I suggested evidence of an AR(3) process. Rewriting the reduced-form inflation equation to include the AR(3) specification results in the following:

$$(8) \qquad p_t = \alpha_0 + \alpha B(L) p_{t-1} + \beta B(L) m_{t-1} + \gamma B(L) w_{t-1} + \delta D_t + \varepsilon_t$$

$$\varepsilon_t = e_t + \rho_1 \varepsilon_{t-1} + \rho_2 \varepsilon_{t-2} + \rho_3 \varepsilon_{t-3}, \quad e_t \sim N(0, h_t)$$

$$h_t = \Lambda_0 + \Lambda B(L) e_{t-1}^2, \quad (AR(3)\text{-}ARCH(q)) \text{ or}$$

$$h_t = \Lambda_0 + \Lambda B(L) e_{t-1}^2 + \kappa B(L) h_{t-1}^2, (AR(3)\text{-}GARCH(p, q))$$

ARCH tests in Table 5 indicate that correction for the AR(3) process in subperiod I residuals eliminates significant evidence of an underlying ARCH process. Table 6 reports the results of equation (8) for subperiod I. We can see that m_{t-1} has a positive sign, which is consistent with economic theory; however, it remains an insignificant contributor to explaining variation in inflation. The dummy variable that accounts for the oil supply shock is statistically significant. The coefficients of ρ are all significant.

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[7]According to Green (1990), the Durbin-Watson test is not likely to be valid when there is a lagged dependent variable in the equation. The statistic will usually be biased toward finding no autocorrelation, the issue has been studied by Nerlove and Wallis (1966). So, in this study, we used the Godfrey (1978) test for serial correlation on residuals.

Table 5

SPECIFICATION TESTS FOR EQUATIONS (3) AND (8)

A. Godfrey Lagrange Multiplier Test for Serial Correlation

	Equation (3)			Equation (8), AR(3)		
Order of Serial Correlation	Whole Period N^*R^2	Period I N^*R^2	Period II N^*R^2	Whole Period N^*R^2	Period I N^*R^2	Period II N^*R^2
1	16.52*,**	14.88*,**	0.25	2.19	0.07	NA
2	16.58*,**	15.76*,**	5.03	2.37	0.09	NA
3	26.95*,**	16.08*,**	12.23*	3.00	2.75	NA
5	30.32*,**	23.37*,**	15.61*	3.55	6.08	NA
6	30.92*,**	24.06*,**	15.79*	10.00	8.30	NA
7	31.35*,**	24.24*,**	17.61*	10.05	8.31	NA
10	31.71*,**	24.42*,**	20.16*	11.97	9.18	NA
11	32.11*,**	26.51*,**	20.25*	12.55	9.99	NA
12	32.12*,**	27.65*,**	20.53	13.18	10.12	NA

B. ARCH Test

	Equation (8), AR (3)		Equation (3)		Tabulated	
Order of ARCH	Whole Period N^*R^2	Period I N^*R^2	Period I N^*R^2	Period II N^*R^2	$\chi_{5\%}^2(q)$	$\chi_{1\%}^2(q)$
1	17.06*,**	2.03	5.34*	0.00	3.84	6.63
2	19.50*,**	2.73	6.25*	0.30	5.99	9.21
3	27.94*,**	4.49	6.98	0.74	7.82	11.34
4	31.91*,**	6.26	12.38*	1.40	9.49	13.28
5	33.53*,**	6.41	14.41*	1.60	11.07	15.09
6	33.50*,**	7.68	14.31*	1.66	12.59	16.81
7	33.37*,**	7.84	14.23*	2.92	14.07	18.48
8	33.82*,**	8.19	14.41	3.99	15.51	20.09
12	41.37*,**	11.77	15.75	10.95	21.03	26.22
16	45.68*,**	15.68	16.22	14.05	26.30	32.00
20	55.92*,**	20.87	17.28	27.91	31.41	37.57
24	64.20*,**	26.31	19.62	34.08	36.42	42.98

C. GARCH (1, 1) Test: 1971.01-1992.06 (Whole Period)

$$\begin{array}{lllllllll} p_t = 0.232 & + 0.182\, p_{t-1} & - 0.147\, p_{t-2} & - 0.044\, p_{t-3} & - 0.075\, p_{t-4} & - 2.331\, m_{t-1} & + 1.809\, w_{t-1} & + 0.620\, D_t \\ \quad (2.231) & (2.162) & (-1.750) & (-0.516) & (1.038) & (-1.252) & (1.245) & (3.863) \end{array}$$

$$R^2 = 0.209$$

$$\text{adj-}R^2 = 0.187$$

$$\begin{array}{lll} h_t = 0.225 + & 0.317\, e_{t-1}^2 & + 0.590\, h_{t-1}^2 \\ \quad (2.750) & (3.615) & (5.953) \end{array}$$

*,** : Significant at the 5% and 1% levels.

Table 6

PARAMETER ESTIMATES FOR REDUCED-FORM INFLATION MODEL (8)

Variables	Dependent Variable = p_t Whole Period (8.a), AR(3)	Period I (8.b), AR(3)
p_{t-1}	1.273	1.143
	(7.385)†	(10.345)*
p_{t-2}	-0.721	-0.389
	(2.262)*	(3.669)*
p_{t-3}	0.224	
	(0.806)	
p_{t-4}	-0.005	
	(0.042)	
m_{t-1}	0.957	1.598
	(0.535)	(0.432)
w_{t-1}	10.771	16.706
	(5.368)*	(5.024)*
D_t	0.215	0.259
	(2.550)*	(2.034)*
α_0	-0.088	-0.193
	(1.829)	(1.817)
ρ_1	-0.967	-0.792
	(5.378)*	(6.267)*
ρ_2	-0.628	-0.600
	(4.159)*	(4.635)*
ρ_3	-0.317	-0.392
	(3.455)*	(4.038)*
R^2	0.335	0.416
adj-R^2	0.307	0.376
F-stat	12.014*	10.439*
SE	1.366	1.626

* : Significant at the 5% level.

† : The number in parentheses denotes t-statistics.

As a comparative analysis, we used identical test procedures to evaluate evidence of a mean-variance relationship over the entire sample period. Results for the entire sample period indicated significant evidence of a positive relationship between conditional mean and variance, as well as strong ARCH and GARCH(1, 1) effects[8,9] (see Tables 5, 6, and 7). Following Engle (1983), we plotted the conditional mean and variance of inflation in Figure 3. Figure 3 further supports our results that high inflation of the 1970s was associated with high variability of inflation, and low inflation of the 1980s was associated with low variability of inflation. It appears that our results confirm the results of Cosimano and Jansen (1988) for analysis of ARCH effects in the subsamples, while contradicting their results in the full sample. Our results are probably most consistent with the findings of Ball and Cecchetti (1990), which demonstrate that uncertainty about long-term rather than short-term inflation is more significantly affected by changes in the mean level of inflation.

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[8]From Table 5 we know $h_t = \Lambda_0 + \Lambda_1 e_{t-1}^2 + \kappa_1 h_{t-1}^2 = 0.225 + 0.317 e_{t-1}^2 + 0.59 h_{t-1}^2$. The GARCH coefficients, Λ_1 and κ_1, were also statistically significant. These results provided strong evidence that inflation volatility can be categorized by a GARCH(1, 1) specification. Since the estimates of the autoregressive parameter κ_1 are greater than Λ_1, and the sum of these parameters is smaller than unity, both processes are likely to be stationary.

[9]We know that the volatility persistence is measured by the sum of $\Lambda_1 + \kappa_1$. A more intuitive way of measuring volatility persistence is the half life of a shock (HL) calculated as:

$$HL = \frac{\log(0.5)}{\log(\Lambda_1 + \kappa_1)} .$$

The HL was approximately 7.1 months for the research period. This means that a shock to volatility diminishes to half its original size in 7.1 months.

Table 7

PARAMETER ESTIMATES FOR REDUCED-FORM INFLATION (3) (DEPENDENT VARIABLE = p_t) AND TESTS FOR INFLATION UNCERTAINTY USING REGRESSION MODELS (5) OF INFLATION EXPECTATIONS (DEPENDENT VARIABLES = h_t) (OLS = ORDINARY LEAST SQUARE)--FOR THE WHOLE SAMPLE PERIOD

Variables	Whole Period OLS(3.W)	Whole Period WLS(3.W)
p_{t-1}	0.388	0.325
	(6.068)*	(5.195)*
p_{t-2}	-0.186	-0.419
	(-2.751)*	(-6.508)*
p_{t-3}	0.015	0.534
	(0.223)	(7.782)*
p_{t-4}	0.134	0.258
	(2.158)*	(2.904)*
m_{t-1}	-0.775	2.333
	(-0.312)	(0.818)
w_{t-1}	5.877	5.581
	(3.010)*	(5.261)*
D_t	0.666	1.891
	(2.926)*	(5.362)*
α_0	0.099	0.021
	(0.836)	(0.116)
R^2	0.243	0.829
adj-R^2	0.221	0.824
F-stat	11.209*	169.093*
SE	1.441	7.783

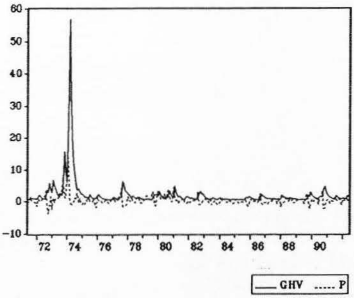
Table 7--CONTINUED

Variables	Whole Period OLS (5.W) lag(4)	Whole Period OLS (5.W) lag(12)	Whole Period WLS (5.W) lag(4)	Whole Period WLS (5.W) lag(12)
π_{-1}	1.483	1.253	-3.183	-3.305
	(2.569)*	(1.926)	(-4.227)*	(-3.931)*
π_{-2}	-0.011	-0.108	2.507	2.756
	(-0.021)	(-0.164)	(3.502)*	(3.247)*
π_{-3}	1.522	2.075	4.761	5.769
	(2.769)*	(3.159)*	(6.642)*	(6.799)*
π_{-4}	0.532	1.382	3.747	4.612
	(0.921)	(2.021)*	(4.971)*	(5.221)*
π_{-5}		1.206		-0.048
		(1.640)		(-0.051)
π_{-6}		0.472		0.464
		(0.636)		(0.483)
π_{-7}		-0.510		-2.502
		(-0.692)		(-2.626)*
π_{-8}		-1.068		-0.295
		(-1.418)		(-0.303)
π_{-9}		-0.459		1.594
		(-0.651)		(1.748)
π_{-10}		-1.237		-2.386
		(-1.207)		(-1.801)
π_{-11}		0.196		1.237
		(0.403)		(1.966)*
π_{-12}		-0.392		-0.719
		(-1.048)		(-1.489)
α	0.129	0.498	-0.574	-0.256
	(0.232)	(0.838)	(-0.792)	(-0.333)
R^2	0.106	0.265	0.341	0.384
adj-R^2	0.091	0.105	0.330	0.353
F-stat	7.21*	3.365*	31.545*	11.982*
SE	6.354	6.38	8.289	8.245
$\sum\Gamma$	3.525	2.807	7.833	7.174
	(+)	(+)	(+)	(+)
$N*R^2$	26.145*	36.207*	84.909*	93.312*

* : Significant at the 5% level.

Figure 3

Conditional Mean and Variance of Inflation Plots



V. Conclusion

This paper investigates the relation between inflation and its variability in Taiwan for the period of January 1971 to June 1992. The empirical evidence presented here finds a significant positive relationship between inflation and its variability in Taiwan for analysis over the full sample period. This result is consistent with those found in most of the literature. However, this strong relationship broke down when the whole sample period was divided into two subperiods. The mean-variance relationship seemed to be significant in periods of high inflation (January 1971 to December 1981) but not in periods of relatively low inflation (January 1982 to June 1992). These results tend to confirm the findings of Logue and Willett (1976), Hafer and Heyne-Hafer (1981), and Chowdhury (1991). Furthermore, we found that correction for serial correlation on residuals (by using the Godfrey test) from our reduced-form inflation model eliminated evidence of ARCH effects in subsample analysis.

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ESSAY 2: THE DYNAMIC LINKAGE BETWEEN STOCK RETURNS AND TRADING VOLUME IN THE TAIWAN STOCK MARKET

Abstract

This essay explores the dynamic linkages between daily stock returns and daily trading volume in a small stock market, the Taiwan Stock Exchange in Taiwan, during the period of September 7, 1988 through December 13, 1993. We investigated both linear (Granger causality test) and nonlinear (GARCH modelling) dependence. Chow test results suggest significant evidence of a structural change in both stock returns and trading volume on October 1, 1990, an ending period of the big bear market for recent Taiwan stock market history. Our empirical evidence indicates significant unidirectional Granger causality from stock returns to trading volume, which is not consistent with earlier United States results. This variation in the results is explained by the relative low trading volume, small size of the Taiwan market, and cross-country differences.

I. Introduction

For the past two decades, a substantial amount of empirical research has been undertaken to investigate the linkage between stock returns and trading volume in the United States and other major industrial countries of the world. Beginning with Osborne (1959), this linkage has been studied from a variety of perspectives. Granger and Morgenstern (1963) investigated the relationship between price indices and aggregate exchange volume by using spectral analysis of weekly data from 1939-1961. They found no relation between movement in a Securities and Exchange Commission composite price index and the aggregate level of volume on the New York Stock Exchange (NYSE). Data from two individual stocks also showed no price-volume relation. Ying (1966) examined the relation between the Standard and Poor's index of daily closing prices of 500 common stocks and total daily volume on the NYSE. He found that large increases in volume were usually accompanied by large price changes, that large volumes were typically associated with an increase in price, and that small volumes usually accompanied price declines. Ying (1966) was the first to document the positive price-volume relationship. Similar positive correlation has been reported, for example, by Rogalski (1978), Harris (1986) and Comiskey, Walking and Meek (1987). Wood, McInish and Ord (1985) and Harris (1986) found a positive relationship between absolute price changes and trading volume. Copeland (1976), Epps and Epps (1976), Jennings, Starks and Fellingham (1981) and Tauchen and Pitts (1983) discussed the price-volume relationship from the theoretical point of view. All of them supported the positive price-volume relation.[1] Most of this research has attempted to theoretically model and/or empirically determine a contemporaneous relationship. However, Smirlock and Starks (1988) and Martikainen et al. (1994) took a different approach in that they examined the lag relationship between stock returns and trading volume by using the notion of Granger-causality. Both of their results showed a significant bidirectional feedback between stock return and trading volume.

The purpose of this essay is to follow Smirlock and Starks (1988) and Martikainen et al. (1994) by exploring the linear relationship between stock returns and trading volume in a small stock market, the Taiwan Stock Exchange, using the notion of Granger (1969) causality. In addition, the essay examines the nonlinear dependence in terms of the (G)ARCH methodology.

While previous studies have not incorporated the Taiwan Stock Market into their analysis, Taiwan provides an interesting arena to explore the dynamic linkage between stock returns and trading volume for two reasons. First, Taiwan has made remarkable economic progress and enjoyed an annual average economic growth rate of 8.36% in the past decade. The per capita GNP grew to US$10,215 in 1992. Due to the continual growth of its economy, the liquidity provided by a huge accumulation of foreign exchange reserves, the relatively low bank interest rate, and huge hot money inflow in the 1986-1988 period, the Taiwan securities market became more active than in the past. In addition,

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[1] A survey by Karpoff (1987) provided an excellent review of the literature as well as an extensive bibliography. He summarized the following stylized facts regarding the price-volume relationship in the U.S. stock market: (1) Volume is positively related to the magnitude of the price change, and (2) volume is positively related to the price change per se.

revision of Securities and Exchange Laws on January 1988 was a significant event. The revision had broad implications, including the removal of restrictions on the establishment of new securities firms, licensing of foreign securities houses, deregulation on the participation of foreign institutional investors, and deregulation of restrictions on margin financing. As a result, both domestic and foreign participation in the securities market have increased tremendously. Table 8 briefly summarizes the underlying institutional structure and activity in the Taiwan stock market for selected years.

This essay is organized as follows. Section II presents a formal characterization of Granger's definition of causality; the ARCH (Engle, 1982) and GARCH (Bollerslev, 1986) models are presented. Section III reviews the data and discusses the summary statistics of the data series. In section IV, the empirical results are presented, and conclusions are contained in section V.

II. Methodology

Granger-Causality, Unit Root, and Cointegration Tests

Granger's (1969) definition of causality is based upon the predictability of a time series. If all available past information allows us to predict Y_{1t} better than we can when all past information except Y_{2t} is used, then by Granger's definition, Y_{2t} "Granger" causes Y_{1t}. We can briefly summarize Granger's definition of causality and feedback as follows. Let u_t be all the available information in the universe, $(u_t - Y_{it})$ denote all available

Table 8

THE SITUATION OF THE TAIWAN STOCK MARKET (1988-1992)

Item	1988	1989	Year 1990	1991	1992
Number of firms listed	163	181	199	221	256
Total market capitalization (NT$ billion)	3,383	6,174	2,682	3,184	2,545
The dollar value of total common share traded (NT$ billion)	7,868	25,408	19,031	9,683	5,917
The number of total common shares traded (billion)	101.4	220.6	232.3	175.9	107.6
Price limits	3%	3%	3%	7%	7%
Turnover rate (%)[a]	332.6	590.1	506.1	321.9	180.0
Number of open brokerage accounts (1,000)	1,606.2 (8.06)[b]	4,208.5 (20.9)	5,033.1 (24.7)	5,162.9 (25.1)	5,070 (24.4)
Brokerage fees and other costs	0.15% of total trading share value for each transaction				
Trading hours	M-F: 9:00 a.m. - 12:00 noon; Sat: 9:00 a.m. - 11:00 a.m.			Total 17 hours	
Tax on capital gains (Transation tax rate is between 1% to 2%)	Suspended in 1974 and was reinstated on Jan 1989. The ceiling for individuals on tax-exempt stock sales is NT$3 million (around US$111 thousand)				
Supplement indices	A: 2 section indices.[c] B: 8 industry indices				

[a] $\text{Turnover Rate} = \dfrac{\text{Total trade volume}_t}{\text{Total shares of all listed companies}_t}$

[b]The number in parentheses denotes the ratio of number of open brokerage accounts to population.

[c]Two section indices are categories A and B. Eight industry indices are: cement, food, plastics & chemicals, textiles, electric & machinery, pulp & paper, construction, and banking & finance.

Source: Security of Exchange Commission, Ministry of Finance, no date.

information apart from the specified series Y_{it}, Y_{*t} (Y_{1t} or Y_{2t}) be characterized as a stationary stochastic process, and $\sigma^2(Y_{*t}|u_t)$ represent the minimum prediction error variance of Y_t given u_t. Causality is then defined as follows:

(1) If $\sigma^2(Y_{1t}|u_t) < \sigma^2(Y_{1t}|u_t\text{-}Y_{2t})$, we say Y_{2t} is causing Y_{1t}, denoted by $Y_{2t} \Rightarrow Y_{1t}$.

(2) If $\sigma^2(Y_{2t}|u_t) < \sigma^2(Y_{2t}|u_t\text{-}Y_{1t})$, we say Y_{1t} is causing Y_{2t}, denoted by $Y_{1t} \Rightarrow Y_{2t}$.

Feedback is defined as follows:

(3) If $\sigma^2(Y_{1t}|u_t) < \sigma^2(Y_{1t}|u_t\text{-}Y_{2t})$ and $\sigma^2(Y_{2t}|u_t) < \sigma^2(Y_{2t}|u_t\text{-}Y_{1t})$,

(both (1) and (2) hold), then $Y_{1t} \Leftrightarrow Y_{2t}$, and we say there will be feedback between Y_{1t} and Y_{2t}.

Granger causality implicitly assumes that information relevant to prediction is contained only in the data series Y_t (Y_{1t} or Y_{2t}). If an unspecified third variable, say X_t, enters the model, which causes both Y_{1t} and Y_{2t}, it may give rise to spurious causality when true causality between Y_{1t} and Y_{2t} does not exist. It is also important to note that the above-mentioned condition is a necessary but not sufficient condition to conclude unidirectional causality (for detail, see Granger, 1969). In the context of a stock returns/trading volume relationship, the Granger causality test involves estimation of the following two regression models:

(4) $R_t = \theta_{11}^m(L)R_t + \theta_{12}^n(L)V_t + a + u_t$

(5) $V_t = \theta_{21}^m(L)R_t + \theta_{22}^n(L)V_t + b + v_t$

where

(6) $\theta_{ij}^m(L) = \sum_{l=1}^{M_{ij}} \theta_{ijl}L^l, \qquad \theta_{ij}^n(L) = \sum_{l=1}^{N_{ij}} \theta_{ijl}L^l,$

and L represents the lag operator such that $LR_t = R_{t-1}$. In our application, R_t and V_t are stock returns and trading volume, respectively, and u_t and v_t are error terms where $E[ut, us] = 0$, $E[vt, v_s] = 0$, $E[ut,vs] = 0$ for all $t \neq s$. From regression equations (4) and (5), unidirectional causality from V_t to R_t is implied if the estimated coefficients on the lagged V_t variables in equation (4) are statistically different from zero as a group (based on a standard F-statistic) and if the set of estimated coefficients on the lagged R_t variables in equation (5) is not statistically different from zero. On the other hand, R_t causes V_t if the estimated coefficients on the lagged R_t variables in equation (5) are statistically different from zero as a group and if the set of estimated coefficients on the lagged V_t variables in equation (4) are not statistically different from zero. Bidirectional causality or feedback between V_t and R_t would exist if the set of estimated coefficients on the lagged V_t variables in equation (4) were statistically significant as a group and the set of estimated coefficients on the lagged R_t variables in equation (5) were also statistically significant as a group.

Granger's definition of causality is based upon the incremental predictability criterion; however, Zellner (1979) considered Granger's definition illogical because he claimed that it lacked the requirement of "full" information. Prior to Zellner's criticism, Sargent (1976) had developed a new version of Granger's test that avoided the requirement of "full information." According to Sargent (1976), if all available past information on both R_t and V_t can help us to predict R_t better than using only all past information of R_t, then we say V_t causes R_t; otherwise, it is better to predict R_t by using only past information of R_t. In Sargent's opinion, any omission of relevant past information could result in a false conclusion. However, Sargent never specified any criterion of choosing the optimum system lag. Hsiao (1979a, 1979b, 1981) suggested transforming each variable into an autoregressive model and using Akaike's (1969a, 1969b) final prediction error (FPE) to determine the lag period. This procedure is known as the stepwise Granger-causality technique, which provides a statistical criteria for choosing the optimum lag length using past information. We followed Hsiao (1979a, 1979b, 1981) and Fawson and Chang (1994), and used full information maximum likelihood (FIML) estimation to obtain efficient parameter estimates for the system of equations (4) and (5).

Following Hsiao (1979a, 1979b, 1981) and Fawson and Chang (1994), we chose the optimal lag length of the autoregressive model by minimizing the final prediction error (FPE). The FPE criterion is specified as follows:

$$FPE = \left[(T + k)/(T - k)\right](SSR/T),$$

where T is the number of observations, k is the number of parameters estimated, and SSR is the sum of squared residuals. By using this criterion, a lag p was chosen such that $FPE(p) = \min \{ FPE(k) | \; k = 1, 2, ..., m \}$. As Singh and Talwar (1982) pointed out, the FPE criterion attempted to balance the "cost" of increased variance when a higher order was selected and the "cost" of coefficient bias when a lower order was selected.[2]

Combining the definition of causality and using the FPE criterion, we followed Hsiao's (1979a, 1979b, 1981) sequential procedure for identifying the above bivariate

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[2]In a paper examining the problems encountered in choosing lag lengths, Thornton and Batten (1985) found Hsiao's method to be superior to both arbitrary lag length selection and several other systematic procedures for determining lag length.

autoregressive model.[3] First, we selected an optimal lag for the single-dimensional autoregressive process (this entailed selection of the optimal value for m in equation 4 and n in equation 5). The optimal lag in the single-dimension is then imposed as one searches for the optimal lag in other dimensions (this entailed selection of the optimal value for n in equation 4 and m in equation 5, conditioned on optimal lags established in step 1).

To test for causality, we compared the FPE with V_t omitted from equation (4), $FPE(m^*)$, to the FPE with V_t included in equation (4), $FPE(m^*,n^*)$. If $FPE(m^*) < FPE(m^*,n^*)$, then trading volumes do not Granger-cause stock returns and a one-dimensional autoregressive representation for R_t is used. If $FPE(m^*) > FPE(m^*,n^*)$, trading volumes Granger-cause stock returns and the optimal model for predicting R_t is the one including m lagged R_t and n lagged V_t. We then repeated the procedure for the V_t (trading volumes) process, treating R_t (stock returns) as the manipulated variable. We carried out a similar test with trading volumes as the dependent variable. Finally, we combined all single equation specifications (those represented by equations 4 and 5) in order to identify the system.

The above equations assume that variables investigated are stationary over time. In this study we applied the augmented Dickey-Fuller test (ADF) as recommended by

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[3]Hsiao (1979a, 1979b, 1981) pointed out that using the final prediction error to determine the lag length was equivalent to using a series of F tests with variable levels of significance.

Engle and Granger (1987) and Schwert (1989) to test the stationarity of data series.[4] The test is the t-statistic on ϕ in the following regression:

$$(7) \qquad dY_t = \alpha_0 + \phi Y_{t-1} + \sum_{i=1}^{n} \psi_i dY_{t-i} + \varepsilon_t ,$$

where d is the first-difference operator, ε_t is a stationary random error, Y_t is the series under consideration, and n is large enough to ensure that ε_t is a stationary random error (white noise). The null hypothesis is that Y_t (R_t or V_t) is a nonstationary series, and it is rejected when ϕ is significantly negative. In practice we do not know the appropriate order of the autoregression, n. In our study, we followed the suggestion of Engle and Yoo (1987) and used the Akaike (1974) information criterion (AIC) to determine the optimal specification of equation (7).[5] The criterion is defined as:

$$(8) \qquad AIC(q) = T \ln\left(\frac{SSR}{T}\right) + 2q ,$$

where T is the sample size to which the model is fitted, SSR is the sum of squared residuals, and q is the number of parameters--equal to n + 2. By using this method, we determined the appropriate order of the model by computing equation (7) over a selected grid of values of n and finding that value of n at which the AIC attains its minimum. The

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[4]Schwert (1989) compared the performance of alternative unit root tests, and concluded that the augmented version of the Dickey-Fuller tests was superior to various alternatives, including the Phillips-Perron test, in the presence of an autoregressive moving average process of unknown order. In this study, we included a constant but not time trend in the test as recommended by Dickey, Bell, and Miller (1986).

[5]AIC attempts to minimize the optimal lag length selection, while FPE attempts to maximize the optimal lag length selection.

distribution of the ADF statistic is nonstandard, and, accordingly, we used the critical values tabulated by MacKinnon (1990).

Once a unit root has been confirmed for a data series, the question is whether or not some long-run equilibrium relationship exists between stock returns and trading volume. Thus, we estimated the following cointegrating regressions:

(9) $$R_t = \gamma_1 V_t + \alpha_0 + \epsilon_{1t}, \qquad V_t = \gamma_2 R_t + \beta_0 + \epsilon_{2t}.$$

R_t and V_t are said to be cointegrated, if ϵ_{it} (i = 1, 2) are stationary, $\epsilon_{it} \sim I(0)$. Engle and Granger (1987) pointed out that the cointegrating regression measures the long-run relationship between time-series variables, and the residuals measure short-run disequilibria. The null hypothesis of the cointegration test is that the series formed by the residuals of each of the cointegrating regressions are not stationary. This means that the original data series, R_t (stock returns) and V_t (trading volume), do not have a common root and, therefore, are not cointegrated. To test the null hypothesis of nonstationarity of the series of residuals, Engle and Granger (1987) have proposed several test statistics for testing the null of no-cointegration, in this essay we used the ADF tests.[6] The test is the t-statistic on σ in the following regression:

(10) $$d\epsilon_t = \sigma\epsilon_{t-1} + \sum_{i=1}^{n}\kappa_i d\epsilon_{t-1} + \eta_t,$$

where d is the first-difference operator, ϵ_t is the error from the cointegration equation, η_t is a stationary random error, and the null hypothesis of nonstationarity is rejected when σ

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[6]Here we only focused on bivariate variables, so the ADF test was appropriate enough to perform the cointegration test.

is significantly negative. Here we still used AIC to determine the appropriate order of the autoregression, n.

Engle and Granger (1987) showed that if two nonstationary variables are cointegrated, then a vector autoregression in the first differences is misspecified. This means that the presence of cointegration between R_t (stock returns) and V_t (trading volume) can cause the Granger causality tests of equations (4) and (5) to be misspecified. Therefore, it is necessary to test for cointegration before running the causality tests.[7] Engle and Granger (1987) pointed out tht if the cointegrating regressions on the nonstationary variable R_t and V_t produces a stationary error term, then this error term must be included as an additional variable to the causality test regression.

GARCH Modelling

We know that the Granger-causality tests are based on the linear dependence between variables in equation. In order to explore the nonlinear relationship between stock returns and trading volume, following the method established by Martikainen et al. (1994), we employed (G)ARCH models. The motivation for our analyzing the nonlinear relationship between stock returns and trading volume follows from Ross (1989) and Martikainen et al. (1994), who showed that the variance of price changes was related directly to the rate of flow information. Furthermore, Lamoreux and Lastrapes (1990)

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[7]According to Tano (1993), the use of cointegration, error-correction modelling in the Granger causality models is important because of the possibility of the spurious comovement between the returns and trading volumes. The cointegration analysis attempted to identify conditions under which relationships are not spurious. Unlike the standard Granger causality, which may not detect any causal relationship between variables under consideration, with the ECM, cointegration ensures that Granger causality exists, at least in one direction.

showed that trading volume contains significant information in explaining stock return and volatility in the U.S. stock market by incorporating volume series into the GARCH variance equation.

Engle (1982) was the first to develop the ARCH model allowing the conditional variance to change over time as a function of past error. The strength of the ARCH technique is that the conditional mean and variance can be estimated jointly using traditional models for economic variables. We can express the model for stock return, R_t, as follows:

$$
\begin{gathered}
R_t = a_0 + \varepsilon_t \\
\varepsilon_t | \psi_{t-1} \sim N(0, h_t) \\
h_t = b_0 + c_j(L)\varepsilon_{t-j}^2 \,,\ j = 1, 2, \ldots, q \\
b_0 > 0 \,,\ \Sigma c_j > 0
\end{gathered}
\tag{11}
$$

The above model is called ARCH(q). The ARCH model presented by Engle (1982, 1983) also maintains a hypothesis that residuals from the reduced-form model are uncorrelated. Serially correlated residuals may, when squared, give results that look like the ARCH model. Bollerslev (1986) expanded the ARCH model by generalizing the autoregressive representation to account for an infinite lag structure. Bollerslev's model was typically referred to as the generalized autoregressive conditional heteroskedastic model (GARCH). Bollerslev's representation assumed that the conditional variance of stock return at time $t(h_t)$ was a function of past sample variance and lagged conditional variances. The conditional variance in GARCH(p,q) can be defined as follows:

$$
h_t = b_0 + b_i(L)\,h_{t-i} + c_j(L)\varepsilon_{t-J}^2 , \tag{12}
$$

where $b_0 > 0$, $\Sigma b_i > 0$, $i = 1, 2, \ldots, p$, $\Sigma c_j > 0$, $j = 1, 2, \ldots, q$.[8] For $p = 0$, the GARCH(p, q) process reduces to an ARCH(q) process, and for $p = q = 0$, ε_t is simply a white noise. Bollerslev suggested a Lagrange multiplier test for GARCH(p, 0) against GARCH(p, q).[9] Engle (1982) and Bollerslev (1986) also allowed for the inclusion of exogenous variables in the conditional mean and variance. Here we incorporated the volume series in the conditional variance of stock return to investigate their nonlinear relationship.

Bollerslev (1987) and Akgiray (1989) suggested that one lagged conditional variance term appeared to model conditional variance adequately. Our study applied the GARCH(1, 1) model. According to Engle and Bollerslev (1986), if $b_1 + c_1 = 1$ in the GARCH(1, 1) process, then the model was known as IGARCH (integrated GARCH), which implied persistence of the conditional variance over all future horizons and also an infinite variance of the unconditional distribution of ε_t (for detail, see Engle and Bollerslev, 1986). The presence of near-integrated GARCH (or $b_1 + c_1$ being close to but slightly less than unity) has been found by Bollerslev (1987), Baillie and Bollerslev (1989), Baillie and DeGennaro (1990), and Fawson, Glover and Chang (1994) for a number of financial market series.

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[8]The nonnegativity constraints associated with the parameters in the h_t equation are necessary to satisfy certain regularity conditions associated with the ARCH and GARCH models.

[9]Autocorrelation and partial autocorrelation functions of the innovation series are typically used when identifying and checking the time-series behavior of ARMA models. Bollerslev (1986) pointed out that these same functions, as applied to the squared residual series, can be useful for identifying and checking the time-series behavior of the conditional variance equation of the GARCH models.

III. Data Description and Summary Statistics of Data Series

Daily data from the Taiwan Stock Exchange (TSE) are used in this paper.[10] The sample extends from September 7, 1988 to December 13, 1993 for a total of 1,500 observations.[11] The stock return, R_t, was calculated by the logarithmic difference of the stock market index. That is, $R_t = [\log(P_t) - \log(P_{t-1})]$, where P_t denotes the level of the stock market index at time t.[12] Ajinkya and Jain (1989) pointed out there are three basic trading volume measures studied in the literature: the number of common stock shares traded (see Grundy, 1985; Harris, 1986); the dollar value of common shares traded (see Lakonishok and Vermaelen, 1986); and the dollar value of shares traded as a fraction of the total dollar value of the company's shares outstanding (see Ajinkya, Atiase and Gift, 1988). We employed the second trading volume measure (in logarithmic form) in our analysis.

Figures 4, 5, 6, and 7 show the data series plot during the research period.[13] Casual observation suggests a possible structural change in both stock returns and dollar value

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[10]We would like to thank Mr. Reming Yu, a financial analyst from Core Pacific Securities Investment Trust Co., Ltd, who kindly offered the data for our study.

[11]The time period of September 7, 1988 through December 13, 1993 was chosen for several reasons. First, trading activity was very heavy through this period. Second, the turnover rate was very high during the period. Third, the number of open brokerage accounts was at a record high (above 5 million) in this period.

[12]The P_t is a value-weighted index of virtually all shares traded. Two hundred fifty-six companies were listed on the Taiwan Stock Exchange at the end of 1992. The market-value-weighted formula is defined by:

Current Index = (Current AMV/Base AMV)*Base Index

where AMV stands for the aggregate market value, and the base date and the base index are 1966 = 100.

[13]In 1990, the Taiwan Stock Exchange reached an annual and all-time high of 12,495.34 on February 10, only to sink to 2,560.47 on Oct 1, 1990--a fall of over 80% in less than eight months. The frenetic trading volume on the Taiwan Stock Exchange is primarily based on rumor, stock manipulation, and speculation of individual investors. An average share trades ownership five times a year, the highest turnover rate in the world.

Figure 4

Taiwan Daily Stock Index

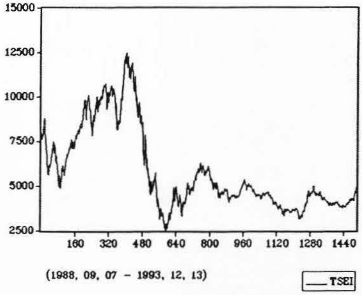


Figure 5

Taiwan Daily Stock Returns

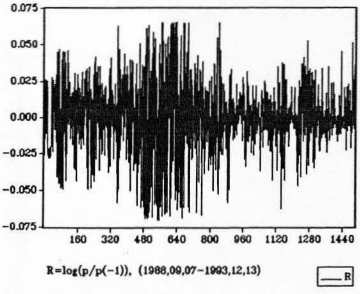


Figure 6

Dollar Values of Total Shares Traded

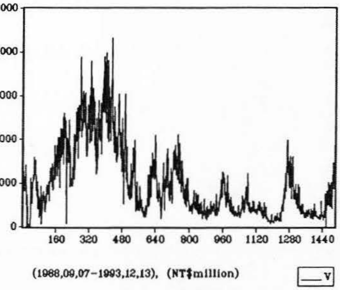
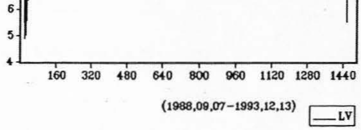


Figure 7

Dollar Values of Total Shares Traded in

Logarithmic Form



of trading volume during October 1990. Prior to October 1990, the market exhibits high volatility of stock return (the average annual rate of stock price fluctuation is 160%) and extremely high dollar value of common shares traded (a record high level of US$941 billion by January 1990). After October 1990, the market exhibited a relatively low volatility of stock return (the average annual rate of stock price fluctuation was under 100%) and low dollar value of common shares traded (average US$219 billion).

The relatively high volatility of stock return and high dollar value of common shares traded in the first subperiod are likely a result of several factors. First, Taiwan has made remarkable economic progress and enjoyed an annual average economic growth rate of 8.36% in the past decade. Second, the revision of the Securities and Exchange Law on January 1988 was a significant event. Third, according to Hsu and Liu (1991), the high volatility of this subperiod can be attributed to its small size as well as to the lack of alternative channels for investment. Certainly, the "herd mentality" of Taiwan investors has also attributed to this volatility.[14]

For the second subperiod, the relatively low volatility of stock return and low dollar value of common shares traded are likely a result of several factors. First, through the education of financial consultants and the experience of a bear market (from January 1990 to October 1990), investors became more rational in adjusting their portfolios to changing market conditions. Second, following the global trend of privatization, the government has

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[14]During this period, the investors are not rational enough to adjust their investment portfolios, and the so-called "news on the street" seems to dominate their decisions on investment. The investors often play the so-called "chasing after price" game. When the prices are going up/down, we can see a herd of people flow into the market. They seem to follow the "buy high and sell low" philosophy, which is opposite to that of the rational investors.

adopted a policy of gradually privatizing government-owned enterprises.[15] Third, in order to support the huge capital requirement of the Six-Year National Development Plan, the issue of a huge amount of new government bonds created a new wave of investment on bond market.[16]

The striking difference between these two subperiods raised the question of whether or not we could pool them together in the regression analysis. In order to answer this question, we constructed a Chow test for a structural break on October 1, 1990. As mentioned earlier, the ARCH model presented by Engle (1982, 1983) also maintained a hypothesis that the residuals from the reduced-form model are uncorrelated. Serially correlated residuals may, when squared, give results that look like the ARCH model. Before constructing the Chow test, we identified a separate set of reduced-form models for returns (R_t) and volumes (V_t) as follows:

$$(13) \qquad \begin{array}{l} R_t = a_0 + \varepsilon_t - a_1\varepsilon_{t-1} - a_3\varepsilon_{t-3} \\ \quad \varepsilon_t \sim N(0, h_t) \end{array} ,$$

$$(14) \qquad \begin{array}{l} V_t = b_0 + b_1 V_{t-1} + b_2 V_{t-2} + b_5 V_{t-5} + b_9 V_{t-9} + b_{11} V_{t-11} + \varepsilon_t \\ \quad \varepsilon_t \sim N(0, h_t) \end{array}$$

The resulting F-statistic and likelihood ratio statistic (see Table 9) both reject the hypothesis that the second subperiod belongs to the same regression as the first subperiod at the 5% level. These results left us with two subperiods for analysis, September 7, 1988 to October 1, 1990 (henceforth, period I), and October 2, 1990 to December 13, 1993 (henceforth, period II).

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[15] This privatization has provided more investment opportunities for both institutional and individual investors and, in the long run, helps stabilize the security market.

[16] The issue of new bonds by government not only creates another investment channel for the investors but also cools down the overheated stock market.

Table 9

TESTS FOR STRUCTURAL CHANGE IN TAIWAN STOCK RETURNS AND TRADE VOLUME (SEPTEMBER 7, 1988-OCTOBER 1, 1990 VERSUS OCTOBER 2, 1990-DECEMBER 13, 1993)

Tests for Change in Parameters (Chow Test):

Equation (13)			
F-statistics	4.3814	Probability	0.0044
Likelihood ratio	13.1393	Probability	0.0043
Equation (14)			
F-statistics	8.55564	Probability	0.0000
Likelihood ratio	50.87190	Probability	0.0000

Table 10 reports the summary statistics for the stock return and volume series used in our study. We found that average daily return was -0.032% and -0.177% for the whole sample period and for the first subperiod, respectively. One justification for these negative returns was that the recent bear market seemed to dominate the recent stock returns. Table 10 also shows that the stock returns were leptokurtic with the exception of subperiod I. The Jarque-Bera test[17] led to the rejection of normality of daily stock returns

[17]The Jarque-Bera test was used for testing normality and was given by

$$JB = T\left[\frac{M_3^2}{6M_2^3} + \frac{1}{24}\left(\frac{M_4}{M_2^2} - 3\right)^2\right] \sim \chi^2(2)$$

where $M_i = \sum_{i=1}^{T} \frac{e_t^i}{T}, \quad i = 0, 2, 3, 4$

Table 10

SUMMARY STATISTICS OF TAIWAN STOCK RETURNS
AND VOLUME SERIES

	R_t	$\|R_t\|$	R_t^2	V_t	V_t^2
Mean (W)	-0.000322	0.017652	0.000579	10.46872	110.45479
(I)	-0.001770	0.022257	0.000793	10.99509	121.91735
(II)	0.000642	0.014586	0.000437	10.11781	102.81309
Standard	0.024075	0.016367	0.000975	0.92799	18.60852
Deviation	0.028139	0.017283	0.001091	1.01337	19.49461
	0.020898	0.014971	0.000861	0.66597	13.34223
Maximum	0.065769	0.070447	0.004963	12.28397	150.89600
	0.065402	0.070447	0.001091	12.28397	150.89600
	0.065769	0.068861	0.047419	11.57033	133.87240
Minimum	-0.070447	0.000012	0.000000	4.89035	23.91551
	-0.070447	0.000013	0.000000	4.89035	23.91551
	-0.068861	0.000013	0.000000	5.50939	30.35336
Skewness	-0.230807	1.285039	2.479943	-1.06599	-0.47723
	-0.303500	0.898770	1.962426	-2.85701	-2.06782
	0.017345	1.663305	3.038515	-0.36279	-0.04418
Kurtosis	3.815494	4.046744	8.871250	6.97233	4.07314
	2.805053	3.077642	6.333544	14.47737	9.34998
	4.876462	5.463879	12.250230	4.77093	3.23356
Jarque- Bera N-	54.8457* *	480.9901**	3,689.542**	1,270.3**	128.9147**
Test stat	10.1444**	80.7946**	661.819**	4,109.5**	1,435.6461**
	132.0867**	642.6385**	4,593.636**	137.4**	2.3384
L-B Q(12)	42.29**	1,996.87**	1,947.14**	10,807.95**	11,778.67**
	31.80**	544.53**	652.86**	3,480.78**	3,856.70**
	14.23	1,024.20**	1,024.51**	5,884.37**	6,147.16**
L-B Q(24)	71.15**	3,421.98**	3,358.73**	17,055.91**	19,404.12**
	48.05**	788.51**	1,008.90**	4,482.90**	5,293.72**
	35.10	1,734.39**	1,667.46**	9,786.86**	10,185.38**
L-B q(60)	140.41**	7,027.02**	6,910.22**	28,666.71**	34,102.49**
	89.08**	1168.73**	1,535.07**	5,896.53**	7,435.67**
	96.39**	3,178.96**	3,062.01**	12,809.26**	13,300.33**

Table 10--CONTINUED

Autocorrelation at Lags	1	2	3	4	5	6
Whole period						
R_t	0.107*	-0.010	0.110*	0.004	-0.029	-0.010
$\|R_t\|$	0.320*	0.359*	0.375*	0.343*	0.345*	0.295*
R_t^2	0.310*	0.359*	0.376*	0.341*	0.345*	0.274*
V_t	0.901*	0.849*	0.808*	0.793*	0.795*	0.781*
V_t^2	0.918*	0.877*	0.844*	0.825*	0.820*	0.810*
First period						
R_t	0.146*	-0.013	0.138*	0.011	-0.022	-0.004
$\|R_t\|$	0.293*	0.313*	0.321*	0.263*	0.320*	0.214*
R_t^2	0.287*	0.350*	0.337*	0.294*	0.344*	0.221*
V_t	0.880*	0.797*	0.736*	0.728*	0.741*	0.717*
V_t^2	0.896*	0.829*	0.782*	0.761*	0.762*	0.744*
Second period						
R_t	0.056**	-0.016	0.070**	-0.015	-0.050	-0.019
$\|R_t\|$	0.275*	0.329*	0.359*	0.343*	0.295*	0.288*
R_t^2	0.287*	0.322*	0.376*	0.345*	0.301*	0.277*
V_t	0.862*	0.821*	0.780*	0.742*	0.725*	0.715*
V_t^2	0.880*	0.840*	0.799*	0.763*	0.743*	0.733*

* and ** : denote significance at the 5% and 1% levels, respectively.
R_t and V_t represent daily returns and volumes, respectively.
L-B Q(k) represents the Ljung-Box test for autocorrelations up to k lags.
The null hypothesis tested is that all autocorrelations up to k lags are jointly zero.

for the Taiwan stock market, which was consistent with the earlier results on Taiwan data as well.[18] The time-series dependence of squared returns indicated that, in addition to linear dependence, nonlinear dependence was also found in daily Taiwan stock returns (see Lee and Ohk, 1990). Using Ljung-Box Q-statistics, we investigated the autocorrelation of the stock return, R_t, and logarithmic volume series, V_t.[19] The test statistics indicated significant autocorrelation in daily Taiwan stock market returns and trading volume. Regarding stock returns, the results were consistent with those found by Lee, Pettit and Swankoski (1990), Lee and Ohk (1990), Ng, Chang and Chou (1990), and Fawson et al. (1994) for the Taiwan stock market. First-order autocorrelation of 0.107 (0.146 and 0.056 for subperiods I and II, respectively) indicated that about 1.15% (2.13% and 0.31% for subperiods I and II, respectively) of the daily return variation was predictable by using only the preceding day's returns. The Ljung-Box Q-statistics for the plain stock returns and for the squared stock returns were all highly significant, indicating the possible presence of

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[18]According to Judge et al. (1988, p. 891), the skewness of a distribution refers to its degree of symmetry (or lack of it), whereas the kurtosis of a distribution is influenced by the peakness of the distribution and the thickness of its tails. The measure of skewness and kurtosis are given by $\sqrt{b_1} = (\mu_3/\sigma^3)$ and $b_2 = (\mu_4/\sigma^4)$, respectively. The Jarque-Bera test is a joint test of whether or not estimates of $\sqrt{b_1}$ and/or $(b_2 - 3)$ are significantly different from 0. Under the null hypothesis, the Jarque-Bera statistic has an asymptotic $\chi^2(2)$ distribution with two degrees of freedom. It is a well-known fact the data are distributed normally when the coefficients of skewness and kurtosis are 0 and 3, respectively. A coefficient of kurtosis larger than 3 indicates the data series are leptokurtic and have a fat tail.

[19]The Ljung-Box Q-statistic is given by:

$$Q_{LB} = n(n+2) \sum_{j=1}^{k} \frac{r_j^2}{n-j},$$

where r_j is the j-th autocorrelation and n is the number of observations. Under the null hypothesis that all of the autocorrelations are zero, Q-statistics are distributed as chi-squared, with degrees of freedom equal to the number of autocorrelations, k.

time-varying risk premium and time-varying volatility. Further, we found that autocorrelation in absolute returns was higher than in plain returns. This indicated that small price changes were followed by small price changes, and large price changes were followed by large price changes. This result was consistent with those found in most stock market return literature (see Fama, 1965; Chou, 1988; Booth et al., 1992; Martikainen et al., 1994; Fawson et al., 1994).

In the trading volume series, we found that significant autocorrelation existed. The first-order autocorrelation statistics of 0.901 (0.88 and 0.862 for the first and second subperiods, respectively), revealed that as much as about 81.1% (77.4% and 74.3% for the first and second subperiods, respectively) of the volume figures was predictable by yesterday's volumes. This significant autocorrelation in the volume series remained large for the six lags investigated.[20] This finding was consistent with those of earlier U.S. studies. As Ajinkya and Jain (1989) pointed out, significant autocorrelation in trading volume could arise when all traders do not trade within one day on information they used to rebalance their investment portfolios. Some investors could adjust their holdings later than others either because they came to realize the information later or they chose to trade only periodically in order to minimize their indirect and direct transaction costs. The skewness and kurtosis figures and Jarque-Bera test also indicated the nonnormality of trading volume data, which was consistent with Ajinkya and Jain (1989). The significant autocorrelation on the squared volumes revealed nonlinear dependence on the trading

[20]Previous studies on the Taiwan stock market have not incorporated the trading volume into analysis; we will be the pioneers on this issue.

volume series. The autocorrelation was also higher for squared series than for plain volumes. Further, the Ljung-Box Q-statistics for the volume level and for the squared volumes were all highly significant, indicating the possible presence of time-varying volatility.

As mentioned earlier, the standard Granger-causality tests were based on stationary data series. Thus, we applied the augmented Dickey-Fuller test (ADF) as recommended by Engle and Granger (1987) and Schwert (1989) to test the stationarity of data series. Regarding stock returns (see Table 11), the results indicate that the null hypothesis of nonstationarity was clearly rejected for the return series (-19.3907 with 2 lags). Regarding trading volume, the null hypothesis of nonstationarity was also rejected in the context of level series (-5.9132 with 24 lags). This means that both stock returns and trading volume were integrated of order zero [or I(0)]. Since no cointegration between stock returns and trading volumes was found, we used the standard Granger-causality tests to test the linear dependence between stock returns and trading volumes.

IV. Empirical Results

Table 12 presents the FPEs resulting from treatment of each variable (R_t or V_t) as a one-dimensional autoregressive process, with the maximum m assumed to 12. Table 12 also reports the smallest FPEs of R_t and V_t. We assumed that each of the R_t and V_t variables was a controlled variable or output and treated the other variable as the manipulated variable or input. Holding the order of the autoregressive operator on the

Table 11

TIME-SERIES PROPERTIES OF TAIWAN DAILY STOCK RETURNS AND TRADE VOLUME DATA

	ADF Test Levels	AIC(n)
R_t	-19.3907*	-11,177.72(2)
V_t	-5.9132*	-3,324.71(24)

* : Denotes significance at the 1%, 5% and 10% levels.

Note : The augmented Dickey-Fuller (ADF) test is based on the following regression:

$$dY_t = \alpha_0 + \phi Y_{t-1} + \sum_{i=1}^{n} \psi_i dY_{t-i} + \varepsilon_t$$

where d is the first-difference operator, and ε_t is a stationary random error. The summation runs to n, where n is based on Akaike (1974) information criterion (AIC) to determine the optimal specification of the above equation. All series are transformed to logarithms form.

Table 12

THE FPE FROM FITTING A ONE-DIMENSIONAL AUTOREGRESSIVE MODEL FOR TAIWAN DAILY STOCK RETURNS AND VOLUME DATA

The Order of Lags	FPE of Returns * 10^{-6} Whole	First	Second	FPE of Volumes * 10^{-6} Whole	First	Second
1	573.99	779.06	436.83	162,102.0	231,390.0	113,160.9
2	574.86	782.04	437.69	15,5946.9	230,131.3	102,828.7
3	568.23*	768.35*	436.26*	155,070.6	229,784.1	101,955.7
4	569.11	771.33	437.09	150,993.4	217,838.1	101,963.1
5	569.47	773.88	438.36	146,391.9	210,385.7	101,180.9
6	570.45	777.47	438.12	146,537.6	211,135.4	100,296.1
7	572.35	783.61	439.61	147,001.1	212,470.1	99,894.3
8	572.46	784.89	438.34	146,749.8	211,827.8	99,626.9
9	573.19	786.34	439.08	146,458.1	212,628.1	99,538.3
10	573.53	786.64	439.96	146,746.5	211,713.3	99,226.4*
11	573.96	787.72	440.89	145,328.5*	206,776.2*	99,445.5
12	574.11	788.10	441.012	146,032.5	207,834.5	99,509.2

* : Indicates the minimum FPE.

controlled variable to the one we have already specified in Table 12, we computed the FPEs of the controlled variable by varying the order of lags of the manipulated variable from 1 to 12. Table 13 presents the order that gave us the smallest FPE. Based on the FPE criterion reported in Tables 12 and 13, the system of equations (4) and (5) for the whole sample period are specified as follows:

$$\begin{pmatrix} \text{Log } R_t \\ \text{Log } V_t \end{pmatrix} = \begin{pmatrix} \theta_{11}^{3}(L) & 0 \\ \theta_{12}^{10}(L) & \theta_{22}^{11}(L) \end{pmatrix} \begin{pmatrix} \text{Log } R_t \\ \text{Log } V_t \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} u_t \\ v_t \end{pmatrix}.$$

Table 13

THE OPTIMUM LAGS OF THE MANIPULATED VARIABLE AND THE FPE OF THE CONTROLLED VARIABLE AND GRANGER CAUSALITY (G-C) RESULT

Controlled Variable[a]	Manipulated Variable	The Optimum Lag of Manip-ulated Variable	FPE	Granger-Causality Result
Whole period:				
R_t (6)	V_t	2	568.347	$R_t \rightarrow V_t$
V_t (14)	R_t	10	134,130.6	
Period I:				
R_t (6)	V_t	1	768.81	$R_t \rightarrow V_t$
V_t (14)	R_t	9	197668.2	
Period II:				
R_t (6)	V_t	4	436.391	$R_t \rightarrow V_t$
V_t (13)	R_t	6	85557.13	

[a] : The number in parentheses indicates the order of autogressive operator in the controlled variable.

These results suggested unidirectional Granger-causality from stock returns, R_t, to trading volumes, V_t. The results implied that knowledge of the behavior of volume cannot marginally improve the conditional stock return forecasts based on past stock return forecasts alone. Further, these results also indicated that positive (negative) stock returns have increased (decreased) the investor's interest in Taiwan stock leading to increased (decreased) trading volume. However, these results seemed not to be consistent with those found by Smirlock and Starks (1988) using U.S. data. They found significant bidirectional feedback between volumes and price changes. One justification for this difference was the relatively low trading volume. Others are the small size of the Taiwan market and cross-country differences. Our results seemed to favor the suggestion of Karpoff (1987) that the size of the market may affect the price-volume relation. As Hsiao (1979a, 1979b, 1981) pointed out, the bidirectional causal relation between variables can also be further investigated by using separate equations (equations 4 and 5) estimated with their optimum lag structures. Table 14 reports the estimated parameter of regression equation (4) in which the current stock return is explained in terms of its past history. Due to the existence of unidirectional Granger causality from stock returns to trading volumes, equation (4) does not include the past trading volumes as explanatory variables. The equation seems to have a good fit for the stock returns due to the short optimal lag structure and significant t-statistics for almost all of the estimated parameters with the exception of second lagged

Table 14

ESTIMATED PARAMETERS FROM EQUATIONS (4) AND (5)

Independent Variable	Dependent Variable R_t			Dependent Variable V_t		
	Whole	First	Second	Whole	First	Second
Constant	-0.0002	-0.0013	0.0005	0.4576	0.729	0.734
	(-0.3675)	(-1.1082)	(0.823)	(3.8034)*	(3.054)*	(4.292)*
R_{t-1}	0.1121	0.1565	0.058	4.1627	3.251	5.585
	(4.3563)*	(3.8519)*	(1.728)	(10.2189)*	(4.835)*	(11.614)*
R_{t-2}	-0.0351	-0.0581	-0.020	0.5335	0.193	1.826
	(-1.3569)	(1.4136)	(0.60)	(1.2658)	(0.279)	(3.539)*
R_{t-3}	0.1153	0.1486	0.074	0.9577	1.022	1.366
	(4.4815)*	(3.6557)	(2.222)*	(2.2698)*	(1.482)	(3.631)*
R_{t-4}				0.8386	0.719	1.440
				(1.9775)*	(1.032)	(2.765)*
R_{t-5}				-0.7098	-1.317	0.839
				(1.6714)	(1.887)	(1.618)
R_{t-6}				-0.8136	-1.216	0.928
				(1.9071)	(1.736)	(1.809)
R_{t-7}				-1.1547	-1.195	
				(2.7217)*	(1.713)	
R_{t-8}				-0.7997	-0.712	
				(1.9071)	(1.026)	
R_{t-9}				-0.7891	-1.219	
				(1.8907)	(1.768)	
R_{t-10}				-0.7869		
				(1.8977)		
V_{t-1}				0.5689	0.646	0.391
				(21.4597)*	(15.308)*	(11.337)*
V_{t-2}				0.1328	0.106	0.191
				(4.3661)*	(2.098)*	(5.185)*
V_{t-3}				-0.0605	-0.136	0.042
				(1.9798)*	(2.710)*	(1.121)*
V_{t-4}				0.0374	0.059	-0.032
				(1.2231)	(1.182)	(0.854)
V_{t-5}				0.2028	0.302	0.036
				(6.6312)*	(5.988)*	(0.966)
V_{t-6}				0.0864	0.069	0.076
				(2.7866)*	(1.327)	(2.090)*
V_{t-7}				-0.0289	-0.094	0.075
				(0.9476)	(1.854)	(2.062)*
V_{t-8}				0.0152	0.045	0.024
				(0.4944)	(0.881)	(0.671)
V_{t-9}				0.0946	0.126	0.038
				(3.0959)*	(2.517)*	(1.087)

Table 14--CONTINUED

Independent Variable	Dependent Variable R_t Whole	First	Second	Dependent Variable V_t Whole	First	Second
V_{t-10}				0.0411	-0.003	0.087
				(1.3663)	(0.053)	(2.755)*
V_{t-11}				-0.1334	-0.187	
				(5.1585)*	(4.572)*	
R^2	0.025	0.044	0.008	0.849	0.824	0.814
F-stat	12.8045*	9.121*	2.653*	394.576*	108.41	241.58
$\Sigma\alpha$	0.192	0.246	0.112	1.438	-0.477	11.985
$\Sigma\beta$	NA	NA	NA	0.956	0.933	0.9272
T	1,500	600	900	1,500	600	900

* : Denotes significance at the 5% level.

Note : Number in parentheses denotes t-statistic.

period.[21] The sum of the coefficients of the lagged returns for equation (4) ($\Sigma\alpha_i = 0.192 > 0$) indicates that a positive relationship exists between current stock returns and lagged stock returns. Table 14 also reports the estimated parameters of regression equation (5), which explains the current trading volume in terms of its past history and the past history of stock returns. The equation seems to fit very well for the volume series, though there is a long optimal lag structure. Seven out of 11 of the coefficients of lagged volume,R_{t-1} , for equation (5) are significant. The sum of the coefficients of the lagged returns for equation (5) ($\Sigma\alpha_i = 1.438 > 0$) indicates that a positive relationship exists between lagged stock returns and current trading volumes. Further, the sum of the coefficients of the lagged volumes for equation (5) ($\Sigma\beta_j = 0.956 > 0$) indicates that a

[21]According to Efficient-Markets Hypothesis (EMH), under a weak form of efficiency, information on historical price trends is of no value for the prediction of either the magnitude or direction of price changes. Apparently, our results seem to violate this hypothesis. This result indicates the Taiwan stock market is not efficient.

positive relationship exists between lagged trading volume and current trading volume. In order to gauge whether our results were specific to a particular time period, we conducted the same tests for both subperiods I and II. Based on the FPE criterion reported in Tables 12 and 13, the system of equations (4) and (5) for subperiods I and II is specified as follows:

Subperiod I (September 7, 1988-October 1, 1990)

$$\begin{pmatrix} \text{Log } R_t \\ \text{Log } V_t \end{pmatrix} = \begin{pmatrix} \theta_{11}^{3}(L) & 0 \\ \theta_{12}^{9}(L) & \theta_{22}^{11}(L) \end{pmatrix} \begin{pmatrix} \text{Log } R_t \\ \text{Log } V_t \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} u_t \\ v_t \end{pmatrix}.$$

Subperiod II (October 2, 1990-December 13, 1993)

$$\begin{pmatrix} \text{Log } R_t \\ \text{Log } V_t \end{pmatrix} = \begin{pmatrix} \theta_{11}^{3}(L) & 0 \\ \theta_{12}^{6}(L) & \theta_{22}^{10}(L) \end{pmatrix} \begin{pmatrix} \text{Log } R_t \\ \text{Log } V_t \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} u_t \\ v_t \end{pmatrix}.$$

The results above for subperiods I and II also suggest unidirectional Granger-causality from stock returns, R_t, to trading volumes, V_t. Again, these results imply that knowledge of the behavior of volume cannot marginally improve the conditional stock return forecasts based on past stock return forecasts alone for both subperiods. The trading volume plays no important role on the conditional stock return forecast. Table 14 also reports the estimated parameters for regression equations (4) and (5) for the two subperiods. For equation (4), the equation seems to have a good fit for the stock returns due to the short optimal lag structure. However, t-statistics were only significant for the first lagged period of stock returns. This indicated that yesterday's stock returns can affect today's stock returns for subperiod I. For subperiod II, the coefficients for equation (4) were only significant at the third lagged period for stock returns. This indicated that current stock returns can be affected by the stock returns of three days ago. The sum of the coefficients of the lagged returns for equation (4) ($\Sigma\alpha_i = 0.246 > 0$ and $\Sigma\alpha_i = 0.112 > 0$) for subperiods I and II, respectively, also indicate that a positive relationship exists between current stock returns and lagged stock returns. Equation (5) also seems to fit very well for the volume series during both subperiods, though there is a long optimal lag structure. Interestingly, we found that the sum of the coefficients of the lagged returns for equation (5) ($\Sigma\alpha_i = -0.4765 < 0$) for the first subperiod indicated a negative relationship between lagged stock returns and current trading volume. For the second subperiod, we found $\Sigma\alpha_i = 11.9849 > 0$ (for equation 5) indicated a positive relationship between lagged stock returns and current trading volumes. Further, the sum of the coefficients of the lagged volumes for equation (5) ($\Sigma\beta_j = 0.9334 > 0$ and $\Sigma\beta_j = 0.9272 > 0$) for subperiods I and II, respectively, indicated that a positive relationship exists between lagged trading volumes and current trading volumes.

Following Martikainen et al. (1994), our study also presents the Pearson correlation coefficient between the two variables. Table 15 offers the cross-correlations between stock returns and trading volume including their lagged values. The contemporaneous correlation between the variables is significantly positive, 0.125 (0.188 and 0.138, respectively, for subperiods I and II). This is consistent with most of the U.S. studies reporting a positive stock return-volume relationship. The results from Table 15 also show that the cross-correlation between the current stock return and lagged trading volume

Table 15

CROSS-CORRELATION OF RETURNS AND VOLUME SERIES

	Cross (R_t, V_{t-i}) (i = 0, ..., 10)				Cross (V_t, R_{t-i}) (i = 0, ..., 10)		
	Whole	First	Second		Whole	First	Second
V_t	0.125*	0.188*	0.138*	R_t	0.125*	0.188*	0.138*
V_{t-1}	0.043	0.093*	0.049	R_{t-1}	0.184*	0.227*	0.242*
V_{t-2}	0.020	0.068	0.022	R_{t-2}	0.143*	0.188*	0.188*
V_{t-3}	0.005	0.051	0.005	R_{t-3}	0.131*	0.178*	0.170*
V_{t-4}	-0.010	0.037	-0.014	R_{t-4}	0.130*	0.176*	0.170*
V_{t-5}	-0.006	0.043	-0.008	R_{t-5}	0.098*	0.139*	0.137*
V_{t-6}	-0.001	0.041	0.011	R_{t-6}	0.090*	0.121*	0.132*
V_{t-7}	-0.008	0.020	0.019	R_{t-7}	0.071*	0.100*	0.108*
V_{t-8}	-0.008	0.018	0.023	R_{t-8}	0.063*	0.090*	0.103*
V_{t-9}	0.011	0.046	0.035	R_{t-9}	0.060*	0.085*	0.103*
V_{t-10}	-0.001	0.030	0.022	R_{t-10}	0.063*	0.091*	0.109*

* : Denotes significance at the 5% level.

R_t and V_t denote daily returns and volumes, respectively.

(10 lags) are not significant and the cross-correlation between current trading volume and lagged stock returns (10 lags) are all significant. These results further support our Granger-causality test result (R_t Granger cause V_t).

Stock Return Prediction

As we know the Granger-causality tests concentrate on linear lead-lag relationships between stock returns and trading volume. Following Lamoreux and Lastrapes (1990) and Martikainen et al. (1994), we incorporated trading volume into the GARCH variance equation to investigate whether trading volume contributes significant information in predicting stock returns and volatility in the Taiwan stock market. Furthermore, we investigated whether stock returns can offer useful information about future trading volume.

Earlier results suggest significant autocorrelation in Taiwan stock returns. To capture the autocorrelation characteristic in stock returns, MA(1) and MA(3), we incorporated error structures into the stock return model (Bollerslev, 1987).[22] We can express the conditional stock return and variance models as follows:

$$
\begin{aligned}
& R_t = a_0 + \varepsilon_t - a_1 \varepsilon_{t-1} - a_3 \varepsilon_{t-3} \\
(15) \quad & \varepsilon_t \sim N(0, h_t) \\
& (A)\ h_t = b_0 + b_1 h_{t-1} + c_1 \varepsilon_{t-1}^2 \\
& (B)\ h_t = b_0 + b_1 h_{t-1} + c_1 \varepsilon_{t-1}^2 + c_2 V \varepsilon_{t-1}^2
\end{aligned}
$$

[22]While earlier U.S. studies reported that only one moving-average term was typically used, our tests indicated that MA(1) and MA(3) were needed in the Taiwan stock market.

where Model (A) represents the results from the basic modeling of conditional stock returns and variance. Model (B) introduces an exogenous variable into the conditional variance, which captures the potential nonlinear dependence from lagged volume to the stock returns. The residuals from Model (A) can be interpretated as an unpredictable component for the returns. As such, the most recent (squared) residuals derived from the GARCH model based on the volume equation enters into the conditional variance of stock returns to evaluate the nonlinear relation between stock returns and trading volume.[23] The volume models are presented later in this study.

Table 16 reports GARCH results for stock return prediction over the whole sample period and both the first and second subperiods.[24] The coefficients of the MA terms were all statistically significant for both subperiods and the overall period. The GARCH coefficients, b_1 and c_1, were also statistically significant for both subperiods and the overall period for both Models (A) and (B). The estimated GARCH(1, 1) parametrization

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[23]According to Ng and Pirrong (1994), the inclusion of $e_{t-1}{}^2$ instead of e_{t-1} or $|e_{t-1}|$ in the empirical work produces uniformly superior results. To avoid the possible simultaneity bias, we introducedlagged squared residuals derived from GARCH model based on volume equation into the conditional variance of the stock returns to explore their nonlinear relation. As Karpoff (1987) pointed out, if the volume was not exogenous, any study that regressed return volatility on volume was subject to this simultaneity bias.

[24]We estimated the models by using TSP-International (version 4.2) software package. The nonlinear maximum-likelihood estimates were based on the Berndt-Hall-Hall-Hausman algorithm.

Table 16

GARCH RESULTS: RETURN PREDICTION FOR EQUATION (15)

	Return (Model (A))			Return (Model (B))		
	Whole	I	II	Whole	I	II
a_0	-0.0003	-0.0017	0.0006	-0.0003	-0.0017	0.00058
	(0.4902)	(1.5613)	(0.9311)	(0.6041)	(1.506)	(0.8391)
a_1	0.09131	0.1541	0.0663	0.0909	0.1493	0.0466
	(3.4689)*	(3.8095)*	(1.998)*	(3.358)*	(3.6263)*	(2.2077)*
a_3	0.0841	0.1408	0.0824	0.0836	0.1388	0.0441
	(3.3427)*	(3.4296)*	(2.481)*	(3.299)*	(3.3619)*	(2.377)*
b_0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(4.3117)*	(2.0192)	(4.3011)*	(4.3213)*	(1.909)	(4.3133)*
b_1	0.8857	0.8265	0.9174	0.8852	0.8273	0.9190
	(4.05103)*	(19.082)	(2.65970)*	(58.755)*	(18.622)*	(80.865)*
c_1	0.0977	0.1465	0.0573	0.0932	0.1477	0.0585
	(7.268)*	(3.4787)*	(24.134)*	(7.119)*	(3.4245)*	(5.742)*
c_2				0.000001	0.000001	0.00001
				(0.09321)	(0.3206)	(0.8021)
$b_1 + c_1$	0.9834	0.973	0.974	0.9784	0.975	0.9775
L - L	3,683.17	1 349.53	2 349.98	3 684.19	1 350.09	2 350.95
LR(2) for H_0: $b_1 = c_1 = 0$	452.34*	124.92*	297.84*			
LR(1) for H_0: $c_2 = 0$				2.04	1.12	1.94
L-B Q(12)	5.36	6.75	6.25	5.21	6.51	5.92
L-B Q(24)	32.98	22.0	25.67	30.17	21.57	22.32
L-B $Q^2(12)$	1,623.5*	937.58*	937.58*	1,543.2*	917.8*	934.2*
L-B $Q^2(24)$	2,849.6*	1,541.63*	1,541.6*	2,810.7*	1,379.4*	1,377.6*

* : Denotes significance at the 5% level.
L-L denotes log-likelihood.
L-B Q(k) represents the Ljung-Box test for autocorrelations up to k lags. The null hypothesis tested is that all autocorrelations up to k lags are jointly zero.

indicated a near-integrated GARCH process with persistent conditional variance.[25] These results also provided strong evidence that daily stock return volatility can be characterized by GARCH(1, 1) specification. Since the estimates of the autoregressive parameters b_1 were always greater than c_1, and the sum of these two parameters was smaller than unity, both processes were likely to be stationary (Bollerslev, 1987). The GARCH results of Table B.9 are consistent with previous findings for stock returns (see Akgiray, 1989; Ng et al., 1990). That is, the time-series of stock returns exhibit significant levels of second-order dependence, and they cannot be modeled as white noise processes.

To further support these findings, we employed a formal test of the GARCH hypothesis that conditional forecast variances are nonconstant. We did this by performing a standard likelihood ratio test in which, under the null hypothesis, the parameters of b_1 and c_1 were constrained to zero. The alternative hypothesis was that the model follows a GARCH form. The appropriate statistic was twice the difference of the maximized values of the log-likelihood functions for the unconstrained and constrained models, respectively, which has a chi-square distribution with two degrees of freedom under the null hypothesis. The results of the log-likelihood ratio tests presented in Table 16 lend support to our finding that stock returns follow a GARCH form for both Models (A) and (B). Regarding

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[25]We know that volatility persistence is measured by the sum of $b_1 + c_1$, a more intuitive way of measuring volatility persistence is the half life of a shock (HL) calculated as:

$$HL = \frac{\log(0.5)}{\log(b_1 + c_1)}$$

The HL was approximately 41 days for the overall period, and 25 days and 26 days for subperiods I and II, respectively.

Model (A), the Ljung-Box Q-statistics (k = 12, 24) on residuals (ε_t) for both subperiods and the overall period were all insignificant at the 5% level, indicating that no serial correlation had been detected. However, we found significant autocorrelation in squared residuals (ε_1^2). For Model (B), apparently the lagged squared volume residual contained no prediction ability of current stock return volatility for both subperiods or the overall period. This result was implied by c_2 having a low value and insignificant t-statistic. Also the log-likelihood function value was not significantly improved.

Table 16 reports the log-likelihood ratio test under the null hypothesis that c_2 was constrained to zero. These results further indicated that the lagged squared volume residual played no important role in predicting current stock return volatility for both subperiods or overall period. This result was not consistent with those found by Lamoreux and Lastrapes (1990) using U.S. data and Martikainen et al. (1994) using Finnish data. Furthermore, the Ljung-Box Q-statistics (k = 12, 24) on residuals (ε_t) for both two subperiods and the overall period were all insignificant at the 5% level, indicating that no serial correlation had been detected. We found significant autocorrelation only in the squared residuals (ε_t^2).[26]

Trading Volume Prediction

As our earlier findings indicated, strong autocorrelation existed on trading volumes. To capture the autocorrelation structures of the trading volume series, we fitted a

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[26]As Bollerslev (1987) suggested, this absence of serial correlation in the conditional first moments, coupled with the presence of serial correlation in the conditional second moments, was one of the implications of the GARCH(p, q) model.

time-series distributed lag model to the volume data. Specifically, we considered the following models:

$$
\begin{aligned}
& V_t = a_0 + a_i B(L) V_t + \varepsilon_t \\
& \text{where } \; i = 1, 2, 3, \ldots, 12 \\
(16) \qquad & \varepsilon_t \sim N(0, h_t) \\
& (A)' \;\; h_t = b_0 + b_1 h_{t-1} + c_1 \varepsilon_{t-1}^2 \\
& (B)' \;\; h_t = b_0 + b_1 h_{t-1} + c_1 \varepsilon_{t-1}^2 + c_2 Re_{t-1}^2
\end{aligned}
$$

We selected the model based on the significance of the regression parameters and results of the residual autocorrelation tests. We found that lags of 1, 2, 5, 9, and 11 were significant and also passed the residual test criterion for the overall sample period. The appropriate lags are 1, 5, 9, and 11, and 1, 2, 6, and 10, respectively, for subperiods I and II. Here, Model (A)′ represented the basic modeling of conditional trading volume and variance. Model (B)′ investigated whether or not the stock return can offer useful information about future trading volume and volatility prediction in the Taiwan stock market. Again, in Model (B)′, we introduced an exogenous variable into the conditional variance, which captured the potential nonlinear dependence form lagged stock return to the trading volume.

Following the previous procedure on stock return prediction, the most recent (lagged squared) residuals derived from the GARCH model based on the returns equation entered into the conditional variance of trading volumes. This allowed us to investigate the nonlinear relation between trading volume and stock returns. Table 17 reports the GARCH results regarding trading volume prediction for the whole sample period and the

Table 17

GARCH RESULTS: VOLUME PREDICTION FOR EQUATION (16)

	Volume (Model (A))			Volume (Model (B))		
	Whole	I	II	Whole	I	II
a_0	0.0200	0.4566	0.294	0.0259	1.123	0.207
	(0.2117)	(2.1471)*	(1.491)	(0.641)	(7.934)*	(1.258)
a_1	0.6401	0.7697	0.628	0.6405	0.753	0.6889
	(19.82)*	(22.71)*	(13.41)*	(19.46)*	(21.18)*	(18.17)*
a_2	0.1832		0.1716	0.1838		0.1532
	(5.2903)*		(3.806)*	(5.362)*		(4.072)
a_5	0.031	0.1133		0.0302	0.0881	
	(3.9637)*	(2.4972)*		(3.0379)*	(2.350)*	
a_6			0.0684			0.0439
			(2.4834)*			(2.094)*
a_9	0.1219	0.077		0.1182	0.052	
	(4.662)*	(1.961)*		(4.416)*	(2.108)*	
a_{10}			0.1019			0.0892
			(6.2902)*			(5.1975)*
a_{11}-	0.0216	-0.005		-0.0214	-0.008	
	(4.8592)*	(3.712)		(4.912)*	(2.985)*	
b_0	0.0822	0.0107	0.0878	0.0774	0.057	0.0442
	(10.915)*	(9.652)*	(50.046)*	8.468)*	(3.087)*	(11.139)*
b_1	0.7867	0.7763		0.7738	0.508	
	(38.081)*	(26.691)*		(37.627)*	(13.565)*	
c_1	0.1469	0.1186	0.0655	0.1443	0.254	0.0823
	(6.1686)*	(3.8338)*	(2.035)*	(6.069)*	(4.500)*	(2.595)*
c_2				2.8934	31.001	135.252
				(3.866)*	(15.425)*	(17.082)*
$b_1 + c_1$	0.9336	0.894	0.065	0.9181	0.762	0.082
L - L	-326.07	-152.71	-192.93	-323.24	-121.02	-151.201
LR (2) for H_0: $b_1 = c_1 = 0$	688.98*	424.82*	59.92*			
LR (1) for H_0: $c_2 = 0$				5.66*	63.38*	83.458*
L - B Q(12)	13.70	12.57	6.29	14.91	15.12	8.21
L - B Q(24)	35.33	30.12	21.37	37.23	32.21	22.79
L - B $Q^2(12)$	173.18*	76.75*	54.98*	187.12*	78.82*	57.91*
L - B $Q^2(24)$	227.81*	110.18*	57.31*	230.19*	112.34*	60.34*

* : Denotes significance at the 5% level.

L - L denotes the log-likelihood.

L - B Q(k) represents the Ljung-Box test for autocorrelations up to k lags. The null hypothesis tested is that all autocorrelations up to k lags are jointly zero.

first subperiods. Regarding the second subperiod, we only fitted ARCH(1) on the conditional variance. Due to singularity of the data, we could not estimate the coefficient of the lagged conditional variance. The coefficients of the lagged volume terms were all statistically significant for both subperiods and the overall period. The GARCH coefficients, b_1 and c_1, were also statistically significant for both the first subperiod and the overall period for both Model (A)′ and (B)′. The ARCH coefficient, c_1, was also statistically significant for both Models (A)′ and (B)′ during the second subperiod.

The log-likelihood ratio test statistics in Model (A)′ were highly significant in both subperiods and in the overall sample, indicating the existence of a conditional variance in the trading volume series in the Taiwan stock market. Regarding Model (A)′, the Ljung-Box Q-statistics (k = 12, 24) on residuals (ε_t) for both subperiods and the overall sample were all insignificant at the 5% level, indicating that no serial correlation had been detected. However, we found significant autocorrelation in squared residuals (ε_t^2).

For Model (B)′, the lagged squared return residual contributed to prediction of current trading volume volatility for both subperiods and the overall sample. This followed from the parameter estimate, c_2, having a large value and significant t-statistic. Also the log-likelihood function value was clearly improved. This result indicated that the stock return contained information on future trading volume.

To further support our results, we evaluated likelihood ratio test statistics under the null hypothesis that c_2 was constrained to zero. Log-likelihood ratio test results reported in Table 17 further indicated that the lagged squared return residual played a significant role in predicting current trading volume volatility for both subperiods and over the whole sample. Furthermore, Ljung-Box Q-statistics (k = 12, 24) on residuals (ε_t) for both subperiods and the overall period were all insignificant at the 5% level, indicating that no serial correlation had been detected. We found significant autocorrelation only in the squared residuals (ε_t^2) [see footnote 23].

V. Summary and Conclusions

This study explored the dynamic linkage between daily stock return and daily trading volume in a small stock market, the Taiwan Stock Exchange in Taiwan, during the period of September 7, 1988 through December 13, 1993. We investigated both linear (Granger causality test) and nonlinear (GARCH modeling) dependence. Chow test results suggested significant evidence of a structural change in both stock return and trading volume on October 1, 1990, an ending period of a bear market for recent Taiwan stock market history. Before testing the causality relationships, we investigated the unit root and cointegration tests. Our empirical evidence indicated significant unidirectional Granger causality from stock returns to trading volume. Similar findings were also supported by the nonlinear (G)ARCH models. However, these results were not consistent with earlier U.S. results (feedback exists). This variation in the results is explained by the relative low trading volume, small size of the Taiwan market, and cross-country differences.

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ESSAY 3: STOCK RETURNS AND VOLATILITY IN THE TAIWAN STOCK EXCHANGE

Abstract

This essay models the empirical relationship between stock returns and volatility in Taiwan using daily, weekly, and monthly returns on the Taiwan Stock Exchange Index from January 1967 to September 1994. Chow test results suggest significant evidence of a structural shift between 1987 and 1989 for all three data series studied. We modelled the stock returns and volatility using both ARCH and GARCH processes. Based on AIC criterion and diagnostic tests on normalized residuals ($\varepsilon_t/\sqrt{h_t}$), we found that GARCH(1, 1) is the most appropriate to evaluate stock return volatility for the Taiwan Stock Exchange. Furthermore, we used a GARCH in the mean model to examine the relationship between mean return and its conditional standard deviation. Our results showed the relationship between mean return and its conditional standard deviation was positive and significant only for the high frequency daily data set. Weekly and monthly data demonstrated a positive but insignificant relationship.

I. Introduction

The ability to forecast stock return volatility is a very important issue for investors. While most researchers agree that volatility is predictable in the stock market (see Bollerslev, Chou and Kroner 1992), they differ on how this volatility predictability should be modeled. It has long been argued that using the standard deviation of percentage changes in stock index as a measure of volatility is inappropriate. A significant drawback of this method is that it measures the total variability of excess returns and not the ex-ante uncertainty regarding them and, as a consequence, leads to inconsistent estimates.[1]

Stock returns are usually leptokurtic (fat-tail) in their distribution. This means that there is a greater proportion of large (and/or small) price changes compared to the proportion expected of a data set that is normally distributed. Under these conditions, a simple standard deviation of percentage changes in stock index is not an appropriate measure of stock return volatility. The autoregressive conditional heteroskedastic (ARCH) model recognizes this temporal dependence in the second moment of stock returns and exhibits a leptokurtic distribution for the unconditional errors from the stock returns generating process. Engle (1982) was the first to develop the ARCH model, allowing the conditional variance to change over time as a function of past error. The ARCH model provides a way of formalizing the observation that large changes tend to be followed by large changes (of either sign), and small by small, leading to contiguous periods of

[1]The basic assumption for this method was that the variance for stock return was constant. However, empirical evidence showed this assumption was unrealistic (see Pindyck, 1984; Poterba and Summer, 1986).

volatility and stability. The strength of the ARCH technique is that the conditional mean and variance can be estimated jointly using traditionally specified models.

By straightforward generalization, Bollerslev (1986) expanded the ARCH model to a GARCH process, which provided a more flexible framework to capture various dynamic structures of conditional variance. Another important extension of ARCH was the "(G)ARCH-in-mean" or "(G)ARCH-M" model, allowing the conditional variance to be a determinant of the mean to capture time-varying properties of the risk premium. Engle, Lilien and Robins (1987) used this model and found strong evidence of this link between risk and return in the term structure of interest rates. Regarding stock returns, French, Schwert and Stambauch (1987) used a GARCH-in-mean model and found evidence that the expected market risk premium was positively related to the predictable volatility of stock returns. They examined both daily and monthly returns on the NYSE stock index for the period from January 1928 to December 1984. Using the same source of data, but for a shorter period (weekly data for the period July 1962 to December 1985), Chou (1988) also found support for French et al.'s (1987) claim of a positive relationship between the predictable components of stock return and volatility. Stenius (1991), using monthly return data from the Helsinki stock exchange for the period from February 1949 to June 1988, also found support for French et al.'s results. Ng, Chang and Chou (1990) used daily return data for the U.S. S&P 500 Index, the Tokyo Price Index, the Korea Composite Stock Price Index, the Taiwan Stock Exchange Weighted Stock Price Index, and the SET Index of Thailand from January 1985 to December 1987 and also found a significant positive relationship between stock return and volatility for each market studied with the exception of the Taiwan market (negative and significant). On the other hand, Baillie and DeGennaro (1990) studied both daily data for the period January 1, 1970 to December 22, 1987 and monthly data for the period February 1928 to December 1984 on the NYSE stock index and concluded that any relationship between expected return and variance (or standard deviation) was weak. Poon and Taylor (1992) presented further support for Baillie and DeGennaro's (1990) findings. They used daily data for the period January 1969 to December 1989, and weekly and monthly data for the period January 1965 to December 1989 in the U.K. market.

Our study used ARCH and GARCH (and/or ARCH-M and GARCH-M) models to analyze stock return and volatility using daily (for the period January 5, 1967 to September 26, 1994), weekly, and monthly (for the period January 1967 to September 1994) Taiwan data. In this context, our study was similar to that performed by Poon and Taylor (1992) using U.K. data. This study sets out to model and address the empirical relationship between stock return and volatility in Taiwan; this represents the first attempt to validate a return-volatility relationship in the Taiwan Stock Exchange using such a long research period.

Taiwan provides an interesting arena to research for two reasons. First, Taiwan has made remarkable economic progress over the last several decades with an annual average economic growth rate of 8.36% in the past decade and per capita GNP of US$10,215 in 1992. Due to the continual growth of Taiwan's economy, the liquidity provided by a huge accumulation of foreign exchange reserves, the relatively low bank interest rate, and huge hot money inflow during the 1986-1988 period, the Taiwan securities market saw significant gains in the volume of activity. Second, a revision of the Securities and Exchange Law governing trade on the Taiwan Stock Exchange in January of 1988 had broad implications, which included the removal of restrictions on the establishment of new securities firms, allowance for the setup of foreign securities houses, deregulation of the participation of foreign institutional investors, and deregulation of restrictions on margin financing. Furthermore, the last decade has seen a significant increase in the integration of world capital markets. In light of pressure for incorporating developing economy stock markets into global investment strategies, studies on thin security markets have also proliferated. Empirical results from smaller markets, such as the Taiwan Stock Exchange, are of great importance to global fund investors who may be planning to invest in the Taiwan stock market.

The remainder of this paper is organized as follows. In section II we briefly describe the Taiwan Stock Exchange. Section III presents the main elements of the (G)ARCH approach. Section IV reviews the data and presents the summary statistics of the data series. Section V presents the empirical results. The conclusion is contained in section VI.

II. Taiwan Stock Exchange

The Taiwan Stock Exchange (TSE) is a small stock market compared to giants such as the New York Stock Exchange (NYSE). It is the second largest market in the Western Pacific Rim countries after Japan. Its capitalization value was about 3,000 (NT$) billion (2-3% of the total capitalization of NYSE) at the end of 1993, and its average annual turnover rate was about 169.85% (a record-high among the world stock markets). Taiwan Stock Exchange was founded in 1962, but trading was thin until the early 1980s. The number of companies listed was only 18 at the beginning of 1962. However, at the end of 1993, the number of companies listed had grown to 280. Table 18 presents annual data on volume, trade amount, number of companies, and turnover rate for the years 1967 to 1993. Trading volume, trading amount, and turnover rate have been somewhat erratic since 1988 (a period of the revision of the Securities and Exchange Law).

Trading at the TSE occurs from Monday to Saturday. The trading time period is from 9:00 a.m. to 12:00 noon Monday through Friday and 9:00 a.m. to 11:00 a.m. on Saturday. The most widely used market indicator for the TSE is the Taiwan Stock Exchange Index (TSEI). Apart from TSEI, there are several more narrowly defined indices labeled as categories A and B and an eight-industry index that includes cement, food, plastics and chemicals, textiles, electric and machinery, pulp and paper, construction, and banking and insurance. The TSEI is a value-weighted index of virtually all shares traded. The market-value-weighted formula is defined by:

current index = (current AMV/base AMV)*base index ,

where AMV stands for the aggregate market value. The base date and the base index are 1966 = 100. The transaction cost is the lowest in the world. For example, the transaction cost for an investment of 10,000 (U.S.$) is about 15 (U.S.$).[2] According to Rhee, Chang

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[2]The brokerage commission rates for share transactions in Taiwan are 0.15% in Taipei and 0.2% in other cities. Here, the Taipei rate was used.

Table 18

THE SITUATION OF THE TAIWAN STOCK EXCHANGE (1962-1993)

Year	Trading Volume/Shrs	Trading Amount(NT$)	Number of Companies	Turnover Rate(%)
	---------------(millions)---------------			
1967	797	5,429	38	134.18
1968	667	7,669	40	86.07
1969	442	4,213	42	48.25
1970	1,350	10,865	42	107.01
1971	1,275	23,598	45	94.48
1972	1,896	54,050	49	133.68
1973	3,997	87,090	63	187.96
1974	2,798	43,586	64	100.88
1975	6,645	130,336	68	192.18
1976	7,251	145,941	77	152.73
1977	10,498	172,177	82	165.11
1978	24,119	361,644	87	293.60
1979	13,037	205,488	96	126.78
1980	11,495	162,112	102	107.84
1981	13,197	209,216	107	103.07
1982	10,243	133,875	113	67.64
1983	23,868	363,844	119	142.79
1984	18,163	324,475	123	95.40
1985	14,533	195,227	127	68.08
1986	39,040	675,656	130	162.11
1987	76,857	2,668,632	141	267.47
1988	101,350	7,868,020	163	332.63
1989	220,560	25,407,960	181	590.14
1990	232,280	19,031,300	199	506.04
1991	175,930	9,682,730	221	321.90
1992	107,590	5,917,080	256	180.00
1993[1]	141,952	7,105,258	280	180.33

$$\text{Turnover rate} = \frac{\text{Total trade volume}_t}{\text{Total shares of all listed companies}_t}.$$

[1]Data available until October 1993.

Source: Security of Exchange Commission, Ministry of Finance.

Figure 10

Taiwan Weekly Stock Index

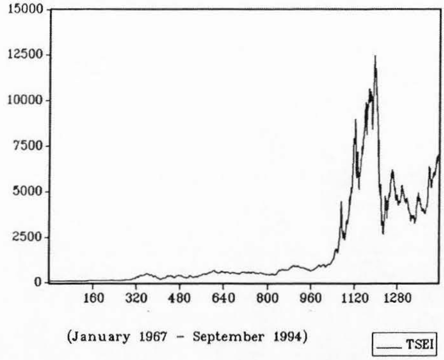


Figure 11

Taiwan Weekly Stock Returns, R = log [p/p (-1)]

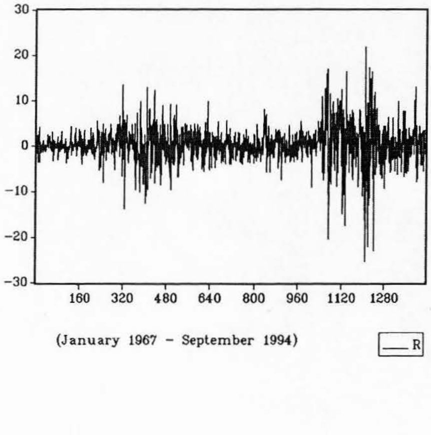


Figure 12

Taiwan Monthly Stock Index

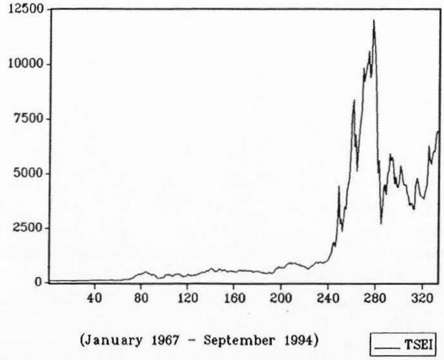
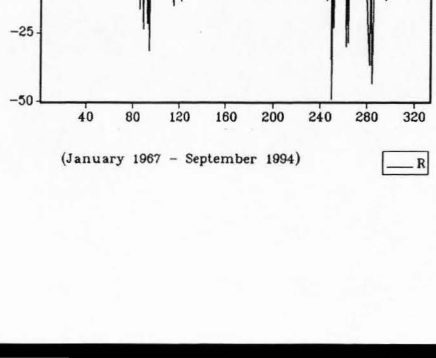


Figure 13

Taiwan Monthly Stock Returns, R = log [p/p (-1)]



2,60.47—a decrease of over 80% in less than 8 months. In the stock return series (see Figures 9, 11, and 13), there appears to be a clustering of stock return fluctuations. This behavior has been observed in many financial data series and was reported in Chou (1988), Booth et al. (1992), Martikainen et al. (1994), and Fawson et al. (1994). Some have suggested that such behavior is typically evident in series that exhibit persistent effects resulting from shocks to the data generation process.

Casual observation of Figures 8, 10, and 12 suggests a possible structural shift in the series between 1987 and 1989, with a relatively low price index and volatility between 1967 and 1988 and a relatively high price index and volatility between 1989 and 1994. The striking difference between these two subperiods raised the question of whether or not we can pool them together in the regression analysis. In order to answer this question, we constructed a Chow test for a structural break on August 5, 1987 for daily data, the second week of September 1988 for weekly data, and May 1989 for monthly data.[7]

As mentioned earlier, the ARCH model presented by Engle (1982, 1983) also maintained a hypothesis that the residuals from the reduced-form model were uncorrelated, since serially correlated residuals may, when squared, give results that look like the ARCH model. Before constructing the Chow test, a separate set of reduced-form models for average daily, weekly, and monthly returns (R_t) were specified as follows:

[7]The time periods were chosen for two reasons. First, we found that stock returns became more volatile after these three periods. Second, different combinations of chosen periods have been experimented with and the statistics (F-statistic and likelihood ratio statistic) showed a significant structural break in these three periods.

Daily data set:

(3) $$R_t = \alpha_0 + \alpha_1 R_{t-1} + \alpha_3 R_{t-3} + \alpha_4 R_{t-4} + \alpha_9 R_{t-9} + \alpha_{10} R_{t-10} + \alpha_{11} R_{t-11} + \varepsilon_t .$$

Weekly data set:

(4) $$R_t = \alpha_0 + \alpha_1 R_{t-1} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} .$$

Monthly data set:

(5) $$R_t = \alpha_0 + \varepsilon_t .$$

The resulting F-statistic and likelihood ratio statistic (see Table 19) both rejected the hypothesis that the second subperiod belonged to the same regression as the first

Table 19

TEST FOR STRUCTURAL CHANGE IN TAIWAN STOCK RETURN MODELS

Test for change in parameters (Chow test)

Daily data set: (January 5, 1967 to August 4, 1987 versus August 5, 1987 to September 26, 1994)

F-statistic	3.3279*	Probability	0.0015
Likelihood ratio	23.3023*	Probability	0.0015

Weekly data set: (1st week of January 1967 to 1st week of September 1988 versus 2nd week of September 1988 to 3rd week of September 1994)

F-statistic	3.6429*	Probability	0.0058
Likelihood ratio	14.5793*	Probability	0.0057

Monthly data set: (January 1967 to May 1989 versus June 1989 to September 1994)

F-statistic	3.9467*	Probability	0.0095
Likelihood ratio	12.7643*	Probability	0.0096

* : Denotes significance at the 5% level.

subperiod at the 5% level. These results left us with two subperiods for analysis, January 5, 1967 to August 4, 1987 (henceforth period I), and August 5, 1987 to September 26, 1994 (henceforth period II) for the daily data set; the first week of January 1967 to the first week of September 1988, and the second week of September 1988 to the third week of September 1994 for the weekly data series; and January 1967 to May 1988, and June 1988 to September 1994 for the monthly data series.

Table 20 presents the unconditional mean and variance of stock returns for each year. The market crash in the United States in 1987 seemed not to have had much effect on the Taiwan stock market. Stock returns and volatility in the Taiwan stock market both reached their peaks in 1987 with high (annual) positive returns and volatility of 6.76% and 25.45%, respectively. However, the Taiwan stock market crash in 1990 stands out with the second largest negative return of -6.28%, next only to the -7.85%, which happened at the ending period of the first oil crisis in 1974. The 1990 crash was associated with a sharp increase in volatility that has gradually dampened since then. In fact, price volatility in 1992 was no greater than it was in the two years immediately after the crash of the Taiwan stock market in 1990.

Tables 21, 22, and 23 report the summary statistics for the stock return series used in our study. We found the average for the overall sample period were: daily—5.39% (5.26% and 5.76% for the two subperiods, respectively), weekly—30.25% (39.74% and -3.54% for the two subperiods, respectively), and monthly—129.45% (173.46% and -54.83% for the two subperiods, respectively).

Table 20

UNCONDITIONAL MEAN AND VARIANCE OF TAIWAN STOCK MARKET RETURNS IN TERMS OF ANNUAL SERIES (1967-1993)

Year	Mean	Variance
1967	0.0106	0.0239**
1968	0.0006	0.0248
1969	0.0028	0.0388
1970	0.0093	0.0312
1971	0.0076	0.0680
1972	0.0436	0.0591
1973	0.0647	0.0538
1974	-0.0785**	0.1337
1975	0.0447	0.1275
1976	0.0100	0.1168
1977	0.0159	0.0752
1978	0.0139	0.0761
1979	0.0264	0.0746
1980	0.0013	0.0544
1981	-0.0011	0.0363
1982	-0.0181	0.0336
1983	0.0451	0.0743
1984	0.0079	0.0481
1985	-0.0003	0.0589
1986	0.0182	0.0373
1987	0.0676*	0.2546*
1988	0.0652	0.1863
1989	0.0526	0.0934
1990	-0.0628	0.2401
1991	0.0013	0.1107
1992	-0.0258	0.0741
1993	0.0265	0.0926
1967-1993 average	0.0121	0.1051

*, ** : Denotes the highest and lowest value, respectively.

Table 21

SUMMARY STATISTICS OF DAILY TAIWAN STOCK MARKET RETURNS
(January 5, 1967 to September 26, 1994)

		R_t	$\lvert R_t \rvert$	R_t^2
Mean	(W)	0.0539	1.0546	2.3608
	(I)	0.0526	0.8321	1.3758
	(II)	0.0576	1.7086	5.2559
SD	(W)	1.5357	1.1175	5.3146
	(I)	1.1718	0.8267	2.8069
	(II)	2.2924	1.5289	8.8095
Maximum	(W)	6.5771	7.0447	49.6277
	(I)	6.2643	6.2643	39.2415
	(II)	6.5771	7.0447	49.6277
Minimum	(W)	-7.0447	0.0000	0.0000
	(I)	-5.1994	0.0000	0.0000
	(II)	-7.0447	0.0000	0.0000
Skewness	(W)	-0.2348	2.0256	4.5916
		(0.0273)[s1]	(0.0273)	(0.0273)
	(I)	-0.0439	1.7914	3.9288
		(0.0316)	(0.0316)	(0.0316)
	(II)	-0.2645	1.3065	2.7198
		(0.0542)	(0.0542)	(0.0542)
Kurtosis	(W)	6.1391	8.0193	29.2456
		(0.0546)[s2]	(0.0546)	(0.0546)
	(I)	5.1774	6.6002	23.4306
		(0.0632)	(0.0632)	(0.0632)
	(II)	3.8337	4.3658	10.6593
		(0.1084)	(0.1084)	(0.1084)
J-B-N	(W)	3,375.01*	13,937.59*	259,009.00*
	(I)	1,187.01*	6,448.41*	119,766.90*
	(II)	82.91*	739.27*	7,505.36*
L-B Q(6)	(W)	155.89*	6,923.42*	7,212.72*
	(I)	109.04*	2,339.76*	2,145.68*
	(II)	79.19*	1,361.16*	1,424.99*
L-B Q(12)	(W)	255.59*	13,088.36*	13,683.25*
	(I)	179.31*	4,027.04*	3,528.99*
	(II)	92.85*	2,621.46*	2,723.71*

Table 21--CONTINUED

		R_t	$\|R_t\|$	R_t^2
L-B Q(24)	(W)	485.28*	23,147.59*	24,118.03*
	(I)	159.01*	6,459.26*	5,370.65*
	(II)	125.18*	4,472.82*	4,674.23*
Autocorrelation:				
Lag (1)	(W)	0.114*	0.364*	0.375*
	(I)	0.065*	0.247*	0.267*
	(II)	0.151*	0.309*	0.317*
Lag (2)	(W)	0.007	0.395*	0.412*
	(I)	-0.003	0.272*	0.271*
	(II)	0.015	0.352*	0.368*
Lag (3)	(W)	0.109*	0.402*	0.413*
	(I)	0.093*	0.275*	0.252*
	(II)	0.121*	0.376*	0.376*
Lag (4)	(W)	0.046*	0.383*	0.388*
	(I)	0.061*	0.255*	0.238*
	(II)	0.033	0.341*	0.344*
Lag (5)	(W)	0.016	0.371*	0.385*
	(I)	0.037*	0.230*	0.209*
	(II)	-0.001	0.336*	0.348*
Lag (6)	(W)	0.000	0.355*	0.341*
	(I)	-0.000	0.247*	0.221*
	(II)	0.004	0.291*	0.281*

* : Denotes significance at the 5% level.
R_t represents daily stock market returns.
W, I, and II denote: whole sample period, first subperiod, and second subperiod, respectively.
The s1 and s2 denote standard errors. The standard errors of the skewness and kurtosis are $(6/T)^{0.5}$ and $(24/T)^{0.5}$, respectively.
J-B-N denotes Jarque-Bera normality test.
L-B Q(k) represents the Ljung-Box test for autocorrelations up to k lags. The null hypothesis tested is that all autocorrelations up to k lags are jointly zero.

and Ageloff (1990), this low transaction cost in Taiwan explains why the TSE is one of the busiest markets in the world.

III. Methodology

ARCH Modelling

Engle (1982) was the first to develop the ARCH model, allowing the conditional variance to change over time as a function of past error. The ARCH model provides a way of formalizing the observation that large changes tend to be followed by large changes (of either sign), and small by small, leading to contiguous periods of volatility and stability. The strength of the ARCH techniques is that the conditional means and variance can be estimated jointly using traditionally specified models. We can express the model for stock return, R_t, as follows:

$$
\begin{aligned}
& R_t = \alpha_0 + \beta_0\sqrt{h_t} + \alpha_i X_t + \varepsilon_t \\
& \varepsilon_t \mid \psi_{t-1} \sim N(0, h_t) \\
(1) \qquad & h_t = b_0 + c_j(L)\varepsilon_{t-j}^2, \; j = 1, 2, \ldots, q \\
& b_0 > 0, \; \Sigma c_j > 0
\end{aligned}
$$

where X_t is a vector of variables that may include lagged dependent variables and contemporaneous variables, ε_t is a conditionally normal disturbance term, ψ_{t-1} is the information set available at time t-1, and h_t is a variance function with arguments ε_{t-1}^2, $\varepsilon_{t-2}^2, \ldots, \varepsilon_{t-q}^2$. The above model is called ARCH(q)-M (ARCH-in-mean, where the conditional standard deviation appears in the conditional mean) to capture time-varying properties of the risk premium. It is a further extension of the ARCH process (see Engle et al., 1987). If the constraint $\beta_0 = 0$ is imposed, then the model is reduced to the standard ARCH(q) model. The ARCH model presented by Engle (1982) maintains a hypothesis that the residuals from the reduced-form model are uncorrelated, since serially correlated residuals may, when squared, give results that look like the ARCH model. Engle (1982) also presented a Lagrange multiplier test for the ARCH process against the null hypothesis that H_0: $c_1 = c_2 = c_3 = \ldots = c_q = 0$, or ARCH(0). The test statistic, TR^2, where R^2 is from the auxiliary regression (equation 1), is distributed as $\chi^2(q)$. If we reject the null hypothesis, then an ARCH effect exists.

GARCH Modelling

Bollerslev (1986) expanded the ARCH model by generalizing the autoregressive representation to account for an infinite lag structure. Bollerslev's model is typically referred to as the generalized autoregressive conditional heteroskedastic model (GARCH). The GARCH model assumes that the conditional variance of stock return at time t (h_t) is a function of past sample variance and lagged conditional variances. The conditional variance in GARCH(p, q) can be defined as follows:

$$
\begin{aligned}
R_t &= \alpha_0 + \beta_0\sqrt{h_t} + \alpha_i X_t + \varepsilon_t \\
\varepsilon_t | \psi_{t-1} &\sim N(0, h_t) \qquad (2) \\
h_t &= b_0 + b_i(L) h_{t-i} + c_j(L)\varepsilon^2_{t-J}
\end{aligned}
$$

where $b_0 > 0$, $\Sigma b_i > 0$, $i = 1, 2, \ldots, p$, $\Sigma c_j > 0$, $j = 1, 2, \ldots, q$.[3] According to Bollerslev (1986), this equation was used to accommodate the nonlinear dependence phenomenon

[3]The nonnegativity constraints associated with the parameters in the h_t equation were necessary to satisfy certain regularity conditions associated with the ARCH and GARCH models.

plus possible persistence in the conditional variance. The above model is called GARCH(p, q)-M (GARCH-in-mean, where the conditional standard deviation appears in the conditional mean) to capture time-varying properties of the risk premium. It represents a further extension of the ARCH process (see Engle et al., 1987). Again, if the constraint $\beta_0 = 0$ is imposed, then the model reduces to the standard GARCH(p, q) model. For $p = 0$, the GARCH(p, q) process reduces to an ARCH(q) process, and for $p = q = 0$, ε_t is simply a white noise. Bollerslev suggested a Lagrange multiplier test for GARCH(p, 0) against GARCH(p, q).[4] In this study, the ARCH(1) (and/or ARCH(1)-M), ARCH(2) (and/or ARCH(2)-M), ARCH(3) (and/or ARCH(3)-M), GARCH(1,1) (and/or GARCH(1,1)-M), GARCH(2,1) (and/or GARCH(2,1)-M), and GARCH(1,2) (and/or GARCH(1,2)-M) models[5] are applied here. According to Engle and Bollerslev (1986), if $b_1 + c_1 = 1$ (or $b_1 + b_2 + c_1 = 1$ and $b_1 + c_1 + c_2 = 1$ in the GARCH(2,1) and GARCH(1,2) processes, respectively) in the GARCH(1,1) process, then the model is known as IGARCH (integrated GARCH), which implies persistence of the conditional variance over all future horizons and also an infinite variance of the unconditional distribution of ε_t. The presence of near integrated GARCH (or $b_1 + c_1$ being close to but slightly less than unity) has been found by Bollerslev (1987), Baillie and Bollerslev (1989), Baillie and DeGennaro (1990), and Fawson, Glover and Chang (1994) for a number of financial market series.

---

[4]Autocorrelation and partial autocorrelation functions of the innovation series are typically used when identifying and checking the time-series behavior of ARMA models. Bollerslev (1986) pointed out that these same functions, as applied to the squared residual series, can be useful for identifying and checking the time-series behavior of the conditional variance equation of the GARCH models.

[5]Following most of the literature, we excluded (G)ARCH models with $p + q > 4$.

IV. Data Description and Summary Statistics of Data Series

We used daily data for the period January 5, 1967 to September 26, 1994 (a total of 8,041 observations), weekly data for the period of the first week of January 1967 to the third week of September 1994 (a total of 1,433 observations), and the monthly data for the period January 1967 to September 1994 (a total of 333 observations) on the Taiwan Stock Exchange (TSE) index.[6] Since the objective of this study was to model nonlinear dependence in stock returns, we expected that dividend adjustment would not affect our results. French et al. (1987) and Poon and Taylor (1992) have already shown that dividend adjustment has little or no effect on the estimates of their models. In our study, we calculated the stock return, R_t, by the logarithmic difference of the stock market index. That is, $R_t = [\log (P_t) - \log (P_{t-1})]$, where P_t denoted the level of the stock market index at time t. Figures 8 through 13 show the data series plot for daily, weekly, and monthly market stock indices and returns during the research period. During 1988, the TSE market price index (see Figures 8, 10, and 12) jumped from 2,843.87 to 8,402.93. This constituted a 195.47% increase in average stock prices over a 12-month period. This spike in stock price behavior in 1988 was due, in large part, to Securities and Exchange Law revisions. The market price index continued to climb for the next two consecutive years, reaching an annual and all-time high of 12,95.34 on February 10. Eight months later it had slipped to

[6]We would like to thank Mr. Reming Yu, a financial analyst from Core Pacific Securities Investment Trust Co., Ltd., who kindly offered the data for our study.

Figure 8

Taiwan Daily Stock Index

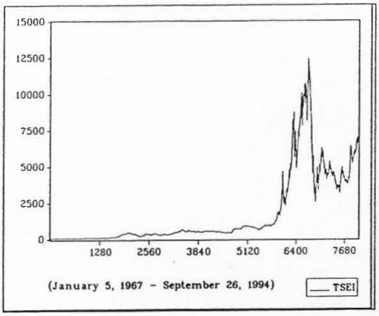


Figure 9

Taiwan Daily Stock Returns, R = log [p/p(-1)]

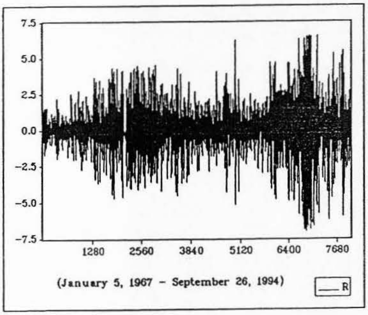


Table 22

SUMMARY STATISTICS OF WEEKLY TAIWAN STOCK MARKET RETURNS
(1st week of January 1967 to 3rd week of September 1994)

		R_t	$\lvert R_t \rvert$	R_t^2
Mean	(W)	0.3025	2.8401	16.948
	(I)	0.3974	2.4124	11.701
	(II)	-0.0354	4.3623	35.633
SD	(W)	4.1071	2.9813	45.1112
	(I)	3.3989	2.4259	28.3225
	(II)	5.9787	4.0812	77.4118
Maximum	(W)	22.0261	25.3418	642.2115
	(I)	17.0884	20.4489	418.1604
	(II)	22.0261	25.3419	642.2115
Minimum	(W)	-25.3418	0.0014	0.0000
	(I)	-20.4489	0.0058	0.0000
	(II)	-25.3418	0.0014	0.0000
Skewness	(W)	-0.3008	2.6028	7.0303
		(0.0647)[s1]	(0.0647)	(0.0647)
	(I)	0.1207	2.2993	6.6619
		(0.0732)	(0.0732)	(0.0732)
	(II)	-0.4103	2.1447	4.5563
		(0.1382)	(0.1382)	(0.1382)
Kurtosis	(W)	8.2182	13.2101	69.0087
		(0.1294)[s2]	(0.1294)	(0.1294)
	(I)	6.8984	11.0566	67.1346
		(0.1465)	(0.1465)	(0.1465)
	(II)	5.6768	8.8571	27.6413
		(0.2765)	(0.2765)	(0.2765)
J-B-N	(W)	1,646.335*	7,836.995*	271,772.9*
	(I)	710.688*	4,008.795*	199,878.3*
	(II)	102.561*	689.553*	9,030.6*
L-B Q(6)	(W)	63.64*	934.76*	603.37*
	(I)	88.66*	685.37*	614.18*
	(II)	10.62	101.48*	84.52*
L-B Q(12)	(W)	72.33*	1,823.26*	1,228.11*
	(I)	95.97*	1,215.01*	966.10*
	(II)	18.86	199.27*	189.52*

Table 22--CONTINUED

		R_t	$\|R_t\|$	R_t^2
L-B Q(24)	(W)	99.88*	3,010.97*	1,783.76*
	(I)	109.99*	1,629.17*	1,152.20*
	(II)	30.45	234.14*	217.93*
Autocorrelation				
Lag (1)	(W)	0.129*	0.387*	0.379*
	(I)	0.211*	0.355*	0.326*
	(II)	0.033	0.317*	0.358*
Lag (2)	(W)	0.152*	0.351*	0.288*
	(I)	0.143*	0.310*	0.228*
	(II)	0.155*	0.285*	0.264*
Lag (3)	(W)	0.059*	0.341*	0.254*
	(I)	0.108*	0.324*	0.295*
	(II)	-0.001	0.242*	0.179*
Lag (4)	(W)	0.006	0.346*	0.291*
	(I)	0.043	0.325*	0.408*
	(II)	-0.034	0.252*	0.182*
Lag (5)	(W)	0.031	0.275*	0.160*
	(I)	-0.022	0.333*	0.323*
	(II)	0.079	0.057	0.015
Lag (6)	(W)	-0.011	0.256*	0.135*
	(I)	0.000	0.257*	0.175*
	(II)	-0.024	0.108	0.052

* : Denotes significance at the 5% level.
R_t represents weekly stock market returns.
W, I, and II denote: whole sample period, first subperiod, and second subperiod, respectively.
The s1 and s2 denote standard errors. The standard errors of the skewness and kurtosis are $(6/T)^{0.5}$ and $(24/T)^{0.5}$, respectively.
J-B-N denotes Jarque-Bera normality test.
L-B Q(k) represents the Ljung-Box test for autocorrelations up to k lags. The null hypothesis tested is that all autocorrelations up to k lags are jointly zero.

Table 23

SUMMARY STATISTICS OF MONTHLY TAIWAN STOCK MARKET RETURNS
(January 1967 to September 1994)

		R_t	$\lvert R_t \rvert$	R_t^2
Mean	(W)	1.2945	7.2426	112.3235
	(I)	1.7346	6.7077	96.7232
	(II)	-0.5483	9.4826	177.6499
SD	(W)	10.5348	7.7491	267.3887
	(I)	9.6987	7.2057	243.8647
	(II)	13.4225	9.4405	343.8481
Maximum	(W)	40.6413	49.3442	2,434.855
	(I)	40.6413	49.3442	2,434.855
	(II)	30.4035	43.5327	1,895.092
Minimum	(W)	-49.3442	0.0044	0.0000
	(I)	-49.3442	0.1221	0.0149
	(II)	-43.5327	0.0044	0.0000
Skewness	(W)	-0.4098	2.2273	4.7427
		(0.1342)[s1]	(0.1342)	(0.1342)
	(I)	-0.2794	2.4069	5.5587
		(0.1493)	(0.1493)	
	(II)	-0.3975	1.6309	3.0274
		(0.3062)	(0.3062)	(0.3062)
Kurtosis	(W)	6.9427	9.0038	37.1723
		(0.2685)[s2]	(0.2685)	(0.2685)
	(I)	7.7893	10.6263	42.8341
		(0.2987)	(0.2987)	(0.2987)
	(II)	4.5552	5.2809	12.9422
		(0.6123)	(0.6123)	(0.6123)
J-B-N	(W)	224.336	773.146	12,223.88
	(I)	259.626	908.216	19,098.85
	(II)	8.135	42.246	361.35
L-B Q(6)	(W)	3.94	262.21	191.02
	(I)	3.45	246.11	175.08
	(II)	6.71	29.05	26.86
L-B Q(12)	(W)	11.66	336.69	210.12
	(I)	17.64	351.71	213.44
	(II)	15.81	30.82	29.21

Table 23--CONTINUED

		R_t	$\|R_t\|$	R_t^2
L-B Q(24)	(W)	28.72	423.43	232.67
	(I)	26.01	406.31	235.56
	(II)	23.99	33.84	32.93
Autocorrelation:				
Lag (1)	(W)	0.085	0.381*	0.376*
	(I)	0.047	0.400*	0.415*
	(II)	0.162	0.286*	0.265*
Lag (2)	(W)	0.007	0.437*	0.450*
	(I)	0.030	0.451*	0.462*
	(II)	-0.063	0.363*	0.398*
Lag (3)	(W)	0.002	0.408*	0.317*
	(I)	-0.051	0.404*	0.281*
	(II)	0.092	0.375*	0.360*
Lag (4)	(W)	0.038	0.323*	0.208*
	(I)	0.002	0.384*	0.268*
	(II)	0.100	0.056	0.042
Lag (5)	(W)	0.016	0.279*	0.213*
	(I)	0.081	0.333*	0.211*
	(II)	-0.149	0.246*	0.176
Lag (6)	(W)	-0.052	0.201*	0.186*
	(I)	-0.019	0.339*	0.248*
	(II)	-0.158	0.063	0.012

* : Denotes significance at the 5% level.
R_t represents monthly stock market returns.
W, I, and II denote: whole sample period, first subperiod, and second subperiod, respectively.
The s1 and s2 denote standard errors. The standard errors of the skewness and kurtosis are $(6/T)^{0.5}$ and $(24/T)^{0.5}$, respectively.
J-B-N denotes Jarque-Bera normality test.
L-B Q(k) represents the Ljung-Box test for autocorrelations up to k lags. The null hypothesis tested is that all autocorrelations up to k lags are jointly zero.

Apparently, the standard deviation was higher in the second subperiod than in the first subperiod for daily, weekly, and monthly data series. This further supports the higher variability of stock returns in the second subperiod than in the first subperiod, which is consistent with those observed in Figures 9, 11, and 13. Also, Tables 21, 22, and 23 show that stock returns are leptokurtic.

Regarding the daily data set, the unconditional sample skewness for actual daily returns, absolute value of daily returns, and squared daily returns exceeded the normal value of zero by 8 (1 and 5 for two subperiods, respectively), 74 (57 and 24 for two subperiods, respectively), and 168 (124 and 50 for two subperiods, respectively) standard errors, respectively. Similarly, the sample kurtosis for actual daily returns, absolute value of daily returns, and squared daily returns exceeded the normal value of three by approximately 112 (82 and 35 for two subperiods, respectively), 147 (104 and 40 for two subperiods, respectively), and 536 (371 and 98 for two subperiods, respectively) standard errors, respectively. For the weekly data set, the unconditional sample skewness for actual weekly returns, absolute value of weekly returns, and squared weekly returns exceeded the normal value of zero by 5 (2 and 3 for two subperiods, respectively), 40 (31 and 15 for two subperiods, respectively), and 109 (91 and 33 for two subperiods, respectively) standard errors, respectively. Similarly, the sample kurtosis for actual weekly returns, absolute value of weekly returns, and squared weekly returns exceeded the normal value of three by approximately 64 (47 and 21 for two subperiods, respectively), 102 (75 and 32 for two subperiods, respectively), and 533 (458 and 100 for two subperiods, respectively) standard errors, respectively. For the monthly data set, the unconditional sample skewness for actual monthly returns, absolute value of monthly returns, and squared monthly returns exceeded the normal value of zero by 3 (2 and 1 for two subperiods, respectively), 17 (16 and 5 for two subperiods, respectively), and 35 (37 and 10 for two subperiods, respectively) standard errors, respectively. Similarly, the sample kurtosis for actual monthly returns, absolute value of monthly returns, and squared monthly returns exceeded the normal value of three by approximately 26 (26 and 7 for two subperiods, respectively), 34 (36 and 9 for two subperiods, respectively), and 138 (143 and 21 for two subperiods, respectively) standard errors, respectively.[8] The statistics showed that daily, weekly, and monthly returns were negatively skewed and that daily data were more leptokurtic than those of weekly and monthly data. The Jarque-Bera test[9] also led to rejection of normality on the Taiwan stock market for daily, weekly, and monthly data sets. This result was consistent with previous studies that used Taiwan stock return data (see Lee and Ohk,

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[8]The standard errors for skewness and kurtosis are $(6/T)^{0.5}$ and $(24/T)^{0.5}$, respectively.

[9]The Jarque-Bera test was used for testing normality and is given by:

$$JB = T\left[\frac{M_3^2}{6M_2^3} + \frac{1}{24}\left(\frac{M_4}{M_2^2} - 3\right)^2\right] \sim \chi^2(2)$$

$$\text{where} \quad M_i = \sum_{i=1}^{T} \frac{e_t^{\,i}}{T}, i = 0, 2, 3, 4$$

1990; Fawson et al., 1994).[10] Using the Ljung-Box Q-statistics, we also investigated the autocorrelation of the daily, weekly and monthly stock returns, R_t. The figures indicated significant autocorrelation for the Taiwan stock market return using both daily and weekly data but not for the monthly data series. In fact, we expected the high frequency series to provide a high possibility of significant serial correlation. Significant positive first-order autocorrelation was consistent with results reported by Lee and Ohk (1990), Ng et al. (1990), and Fawson et al. (1994) on the Taiwan stock market. The time-series dependence of squared returns (for daily, weekly, and monthly series) also indicated that, in addition to linear dependence, nonlinear dependence was also found in Taiwan stock returns. The Ljung-Box Q-statistics for the actual daily stock returns and for the squared daily stock returns were all highly significant, indicating the possible presence of time-varying risk premium and time-varying volatility in daily data series. This autoregressive nature of the squared returns (for daily, weekly, and monthly series) further supported the use of the GARCH (and/or ARCH) model for the variance process of the stock returns data. Further, we found that the autocorrelation coefficient in absolute value of returns and squared returns was higher than in actual returns (for daily, weekly, and monthly series). This indicated that small price changes tend to be followed by small price changes, and large

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[10]According to Judge et al. (1988, p. 891), the skewness of a distribution referred to its degree of symmetry (or lack of it), whereas the kurtosis of a distribution was influenced by the peakness of the distribution and the thickness of its tails. The measure of skewness and kurtosis were given by $\sqrt{b_1} = (\mu_3/\sigma^3)$ and $b_2 = (\mu_4/\sigma^4)$, respectively. The Jarque-Bera test was a joint test of whether or not estimates of $\sqrt{b_1}$ and/or $(b_2 - 3)$ were significantly different from 0. Under the null hypothesis, the Jarque-Bera statistic had an asymptotic $\chi^2(2)$ distribution with two degrees of freedom. It is a well-known fact that the time-series data distributed normally with the coefficient of skewness and kurtosis 0 and 3, respectively. The coefficient of kurtosis larger than 3 indicated the data series was leptokurtic and had a fat tail.

price changes tend to be followed by large price changes. This result was consistent with those found in most of the literature (see Chou, 1988; Booth et al., 1992; Martikainen et al., 1994; Fawson et al., 1994).

V. Empirical Results

As is shown in Engle (1982, 1983), there is a formal test for the presence of ARCH. Tables 24, 25, and 26 report the results of the ARCH test for a qth order ARCH process (maximum $q = 12$). Values of the LM test statistics TR^2 and F-statistics are given. The R^2 is the squared multiple correlation coefficient resulting from the auxiliary regression (see equation 1), and T is the number of the observations in the data set. TR^2 is distributed as chi-squared with a q degree of freedom. F-statistics are from the same regression with degrees of freedom of (q-1, T-q) under the null hypothesis of $c_1 = c_2 = \ldots, c_q = 0$. Results show clear evidence of an ARCH effect in the daily, weekly, and monthly stock returns. As we found in the previous section, there existed a structural shift in data series. Tables 24, 25, and 26 also report the values of the LM test statistics TR^2 and F-statistics for both subperiods. Statistics suggest a significant ARCH effect in both two subperiods for data series studied.

To account for a structural shift in data series, we incorporated a dummy variable into the model. The reduced-form stock return and conditional variance models for three different series were specified as follows:

Table 24

LAGRANGE MULTIPLIER TEST FOR THE ARCH EFFECT ON DAILY DATA SERIES

(1) Whole sample period (January 5, 1967 to September 26, 1994)

q	1	2	3
F-stat	1,243.16*	1,071.36*	863.04*
TR^2	1,076.71*	1,691.76*	1,958.35*
q	4	5	6
F-stat	704.93*	597.05*	507.28*
TR^2	2,087.52*	2,176.95*	2,207.98*
q	7	8	9
F-stat	462.91*	411.16*	383.22*
TR^2	2,309.56*	2,334.56*	2,413.88*
q	10	11	12
F-stat	350.64*	319.65*	298.87*
TR^2	2,441.97*	2,446.73*	2,480.69*

(2) First subperiod (January 5, 1967 to August 4, 1987)

q	1	2	3
F-stat	439.72*	379.77*	296.07*
TR^2	409.76*	674.33*	773.89*
q	4	5	6
F-stat	245.94*	203.36*	177.93*
TR^2	845.49*	869.87*	906.84*
q	7	8	9
F-stat	153.71*	142.16*	127.86*
TR^2	912.93*	956.76*	966.41*
q	10	11	12
F-stat	117.09*	107.72*	100.25*
TR^2	980.64*	990.54*	994.71*

(3) Second subperiod (August 5, 1987 to September 26, 1994)

q	1	2	3
F-stat	211.79*	195.04*	159.72*
TR^2	192.04*	327.84*	388.57*
q	4	5	6
F-stat	130.06*	111.16*	94.41*
TR^2	415.21*	437.62*	444.29*
q	7	8	9
F-stat	89.96*	79.47*	75.89*
TR^2	482.31*	485.94*	513.13*
q	10	11	12
F-stat	69.78*	63.53*	59.79*
TR^2	521.44*	522.15*	532.62*

* : Denotes significance at the 5% level.

Table 25

LAGRANGE MULTIPLIER TEST FOR THE ARCH EFFECT ON WEEKLY DATA SERIES

(1) Whole sample period (1st week of January 1967 to 3rd week of September 1994)

q	1	2	3
F-stat	234.41*	124.82*	96.35*
TR^2	201.64*	212.89*	240.96*
q	4	5	6
F-stat	76.34*	61.49*	52.45*
TR^2	252.27*	253.79*	258.83*
q	7	8	9
F-stat	46.19*	41.93*	41.03*
TR^2	264.72*	272.87*	294.78*
q	10	11	12
F-stat	39.79*	36.55*	36.78*
TR^2	312.79*	315.44*	339.05*

(2) First subperiod (1st week of January 1967 to 1st week of September 1988)

q	1	2	3
F-stat	74.08*	55.73*	48.81*
TR^2	69.56*	101.51*	132.06*
q	4	5	6
F-stat	48.79*	42.72*	35.58*
TR^2	166.52*	179.78*	179.81*
q	7	8	9
F-stat	30.45*	30.77*	27.51*
TR^2	179.71*	202.52*	203.58*
q	10	11	12
F-stat	24.73*	23.64*	21.68*
TR^2	203.56*	212.09*	212.29*

(3) Second subperiod (2nd week of September 1988 to 3rd week of September 1994)

q	1	2	3
F-stat	42.59*	22.65*	16.54*
TR^2	37.71*	39.88*	43.27*
q	4	5	6
F-stat	12.44*	10.54*	8.61*
TR^2	43.49*	45.78*	45.14*
q	7	8	9
F-stat	8.26*	7.32*	7.17*
TR^2	49.72*	50.42*	54.71*
q	10	11	12
F-stat	7.42*	6.73*	7.73*
TR^2	61.48*	61.42*	73.38*

* : Denotes significance at the 5% level.

Table 26

LAGRANGE MULTIPLIER TEST FOR THE ARCH EFFECT ON MONTHLY DATA SERIES

(1) Whole sample period (January 1967 to September 1994)

q	1	2	3
F-stat	45.65*	56.46*	38.81*
TR^2	40.33*	84.71*	86.76*
q	4	5	6
F-stat	29.83*	23.75*	20.02*
TR^2	88.48*	88.29*	89.17*
q	7	8	9
F-stat	17.25*	14.99*	13.24*
TR^2	89.65*	89.36*	89.04*
q	10	11	12
F-stat	11.89*	10.74*	10.17*
TR^2	89.09*	88.79*	91.02*

(2) First subperiod (January 1967 to May 1989)

q	1	2	3
F-stat	37.04*	48.37*	32.02*
TR^2	32.76*	71.61*	71.29*
q	4	5	6
F-stat	23.87*	19.07*	16.47*
TR^2	71.11*	71.18*	73.19*
q	7	8	9
F-stat	14.23*	12.35*	11.11*
TR^2	73.72*	73.45*	74.19*
q	10	11	12
F-stat	10.09*	9.15*	9.34*
TR^2	74.87*	74.82*	80.81*

(3) Second subperiod (June 1989 to September 1994)

q	1	2	3
F-stat	2.83*	5.87*	4.16*
TR^2	2.79**	10.29*	10.96*
q	4	5	6
F-stat	3.44*	2.68*	2.35*
TR^2	12.02*	11.91*	12.56**
q	7	8	9
F-stat	1.92**	1.74	1.56
TR^2	12.27**	12.81*	13.09
q	10	11	12
F-stat	1.41	1.29	1.23
TR^2	13.36	13.62	14.29

* : Denotes significance at the 5% level. ** : Denotes significance at the 10% level.

Daily data series:

$$
(6)\quad
\begin{aligned}
R_t &= \alpha_0 + \beta_0\sqrt{h_t} + \alpha_1 R_{t-1} + \alpha_3 R_{t-3} + \alpha_4 R_{t-4} + \\
&\quad \alpha_9 R_{t-9} + \alpha_{10} R_{t-10} + \alpha_{11} R_{t-11} + \gamma_0 D_t + \varepsilon_t \,, \\
&\varepsilon_t | \psi_{t-1} \sim N(0, h_t) \,, \\
h_t &= b_0 + c_j(L)\varepsilon_{t-j}^2 + \gamma_1 D_t \sim \text{ARCH}(q)\text{-M} \\
h_t &= b_0 + b_i(L)h_{t-i} + c_j(L)\varepsilon_{t-j}^2 + \gamma_1 D_t \sim \text{GARCH}(p, q)\text{-M}
\end{aligned}
$$

Weekly data series:

$$
(7)\quad
\begin{aligned}
R_t &= \alpha_0 + \beta_0\sqrt{h_t} + \alpha_1 R_{t-1} + \\
&\quad \gamma_0 D_t + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} \,, \\
&\varepsilon_t | \psi_{t-1} \sim N(0, h_t) \,, \\
h_t &= b_0 + c_j(L)\varepsilon_{t-j}^2 + \gamma_1 D_t \sim \text{ARCH}(q)\text{-M} \\
h_t &= b_0 + b_i(L)h_{t-i} + c_j(L)\varepsilon_{t-j}^2 + \gamma_1 D_t \sim \text{GARCH}(p, q)\text{M}
\end{aligned}
$$

Monthly data series:

$$
(8)\quad
\begin{aligned}
R_t &= \alpha_0 + \beta_0\sqrt{h_t} + \gamma_0 D_t + \varepsilon_t \,, \\
&\varepsilon_t | \sim N(0, h_t) \,, \\
h_t &= b_0 + c_j(L)\varepsilon_{t-j}^2 + \gamma_1 D_t \sim \text{ARCH}(q)\text{-M} \\
h_t &= b_0 + b_i(L)h_{t-i} + c_j(L)\varepsilon_{t-j}^2 + \gamma_1 D_t \sim \text{GARCH}(p, q)\text{M}
\end{aligned}
$$

We specified the above reduced-form models based on the significance of the regression parameters and results of the residual autocorrelation test. To estimate the parameters of the above models, $\phi = (\alpha_0, \alpha_1, \ldots, b_0, b_1, \ldots, c_1, \ldots, \beta_0, \gamma_0, \gamma_1)$, for a sample of T daily (and/or weekly and monthly) returns, the conditional log-likelihood function was evaluated by:

$$L_T(\phi) = \sum_{t=1}^{T} \log f_t(\phi)$$

(9) $$f_t(\phi) = -\frac{1}{2}\log h_t - \frac{1}{2}\frac{\varepsilon_t^2}{h_t}$$

where $$\varepsilon_t = R_t - \alpha_0 - \alpha_i(L)R_{t-i} - \gamma_0 D_t \sim \text{(G)ARCH}$$

$$\varepsilon_t = R_t - \alpha_0 - \beta_0\sqrt{h_t} - \alpha_i R_{t-i} - \gamma_0 D_t \sim \text{(G)ARCH - M}$$

Numerical maximization of the above log-likelihood function followed the Berndt, Hall, Hall, and Hausman algorithm (see Berndt et al., 1974). Models were estimated by using the TSP-International (version 4.2) software package. Several (G)ARCH model specifications have been fitted to the Taiwan index returns using the above maximum likelihood procedures.

Table 27 reports the parameters estimates of ARCH(3), ARCH(3)-M, DARCH(3), DARCH(3)-M, GARCH(1,1), DGARCH(1,1), and GARCH(2,1)-M models for the daily stock returns. In regarding the ARCH(3) model, the coefficients of c_1, c_2, and c_3 are all statistically significant. The estimate of $c_1 + c_2 + c_3$ (0.787, a measure of persistence) is less than unity, indicating second-order stationarity for the stock return process. The β_0, representing the relationship between the mean return and its conditional standard deviation in ARCH(3)-M model, was positive and significant. The γ, taking into account the structural shift in data series, was found significant only in the conditional variance function (the estimate of γ_1 reported in Table 27) but not in the mean function (the estimate of γ_0 was not reported in this study due to insignificance of t-statistics). This indicates the structural change only occurred in the variance generating process but not in the mean generating process.

Table 27

ESTIMATES OF MODELS FOR TAIWAN DAILY STOCK MARKET RETURNS
(Whole sample period: January 5, 1967 to September 26, 1994)

	ARCH(3)	ARCH(3)-M	DARCH(3)	DARCH(3)-M
α_0	0.0459	-0.7721	0.0377	0.0156
	(4.1469)*	(-15.2055)*	(3.3224)*	(0.5571)
β_0		0.6664		0.0419
		(17.1129)*		(1.6191)
α_{-1}	0.0986	0.1459	0.0892	0.1132
	(10.1225)*	(17.8455)*	(8.1396)*	(11.8449)*
α_{-3}	0.0866	0.1507	0.0915	0.1145
	(10.0921)*	(19.0184)*	(9.7675)*	(13.8997)*
α_{-4}	0.0267	0.0257	0.0296	0.0375
	(3.8372)*	(3.6991)*	(3.7125)*	(5.3249)*
α_{-9}	0.0147	0.0259	0.0158	0.0069
	(2.0899)*	(3.8711)*	(2.0713)*	(1.0223)
α_{-10}	0.0266	-0.7014	0.0198	0.0139
	(3.8198)*	(-.0386)	(2.5606)*	(2.0378)*
α_{-11}	0.0277	0.0169	0.0327	0.0294
	(3.8976)*	(2.6619)*	(4.1983)*	(4.2771)*
b_0	0.5989	0.9783	0.5076	0.4628
	(39.5151)*	(17.1129)*	(35.6459)*	(39.4156)*
c_1	0.2175	0.0857	0.1882	0.1672
	(15.5749)*	(15.4797)*	(14.1836)*	(15.9804)*
c_2	0.2817	0.1064	0.2486	0.2093
	(17.7937)*	(17.0147)*	(15.7545)*	(17.6216)*
c_3	0.2876	0.1072	0.2294	0.2108
	(18.3425)*	(15.8711)*	(14.9647)*	(16.9231)*
γ_1			1.2429	0.9367
			(15.6313)*	(17.2229)*
$c_1+c_2+c_3$	0.7868	0.3012	0.6662	0.5873
L-L	-13,279.3	-13,645.6	-13,109.8	-13,147.9
LR(3)				
H_0:	2,897.2*			
$c_1=c_2=c_3=0$				
AIC	6,697.28	6,932.02	6700.33	6710.29
m3	-0.1237	-0.2435	-0.1016	-0.0971
m4	4.8658	4.5253	4.3811	4.3392
J-B-N	1,184.63	857.299	651.636	612.324

Table 27--CONTINUED

	GARCH(1, 1)	DGARCH(1, 1)	GARCH(2, 1)-M
α_0	0.0242	0.0229	-0.5116
	(2.3344)*	(2.7813)*	(-8.5359)*
β_0			0.4346
			(9.2866)*
α_{-1}	0.0741	0.0747	0.1394
	(6.4543)*	(6.4469)*	(17.2167)*
α_{-3}	0.0807	0.0813	0.1169
	(7.1194)*	(7.1341)*	(15.9164)*
α_{-4}	0.0301	0.0302	0.0311
	(2.6403)*	(2.6368)*	(4.1887)*
α_{-9}	0.0148	0.0143	0.0276
	(1.3194)	(1.2664)	(4.0439)*
α_{-10}	0.0099	0.0101	-0.0003
	(0.8873)	(0.9061)	(-0.0403)
α_{-11}	0.0316	0.0313	0.02333
	(2.8043)*	(2.8121)*	(3.5198*
b_0	0.0105	0.0138	0.5977
	(10.0892)*	(10.2821)*	(18.5421)*
c_1	0.0883	0.0949	0.0725
	(20.0643)*	(18.5812)*	(14.0215)*
c_2			0.0711
			(8.7339)*
b_1	0.9094	0.8969	0.4171
	(228.1821)*	(175.3962)*	(15.7032)*
γ_1		0.0385	
		(5.6358)*	
c_1+b_1	0.9977	0.9918	
$c_1+c_2+b_1$			0.5607
L-L	-12,761.9	-12,749.0	-13,675.7
LR(2)			
H_0:	3,932*		
$c_1=b_1=0$			
LR(3)			
H_0:			2,104.4*
$c_1=c_2=b_1$			
AIC	6,709.68	6,711.29	6,765.37
m3	-0.0449	-0.0492	-0.2288
m4	4.3582	4.2584	4.4579
J-B-N	619.615	532.849	780.8844

* : Denotes significance at the 5% level. L-L denotes log-likelihood.
LR(q) denotes the log-likelihood ratio test. AIC denotes Akaike (1974) information criterion.
m3 and m4 refer to the skewness and kurtosis of the normalized residuals ($\varepsilon_t/\sqrt{h_t}$), respectively.
J-B-N denotes Jarque-Bera normality test on the normalized residuals ($\varepsilon_t/\sqrt{h_t}$).

By looking at the estimate of $c_1 + c_2 + c_3$ (0.662, a measure of persistence) from DARCH(3),[11] apparently the magnitude was decreased relative to that of ARCH(3). As Lamoureux and Lastrapes (1990) pointed out, a (G)ARCH model that does not account for the structural change will pick up high persistence. Our result seemed to be consistent with these findings. The GARCH(1, 1) coefficients, b_1 and c_1 (reported in Table 27), were also statistically significant. The estimated GARCH(1, 1) parametrization indicated a near-integrated GARCH process with persistent conditional variance. These results also provided strong evidence that daily stock return volatility can be characterized by a GARCH(1, 1) specification. Since the estimate of the autoregressive parameter b_1 was greater than c_1, and the sum of these two parameters (0.9977) was smaller than unity, both processes were likely to be stationary (see Bollerslev, 1987). The γ_1, taking into account the structural shift in the conditional variance function (for DGARCH(1, 1)) also showed significance (reported in Table 27) but not in the mean function (the estimate of γ_0 was not reported in this study due to insignificance of t-statistics). The estimate of $c_1 + b_1$ (0.9918) from DGARCH(1, 1) also supports Lamoureux and Lastrapes's (1990) finding that a (G)ARCH model that does not account for the structural change will pick up high persistence. Following French et al. (1987), we also estimated the GARCH(2, 1)-M model on daily returns series. Our results regarding the GARCH(2, 1)-M model seemed to outperform French et al's. (1987) findings. We did not find any negative coefficients on our conditional variance model. The β_0, representing the relationship between the mean

[11] Since we incorporated the dummy variable into the conditional variance function to take into account the structural change, we called this DARCH(q) or DGARCH(p, q) process.

return and its conditional standard deviation in the GARCH(2, 1)-M model, was positive and significant, which was similar to those found in French et al. (1987). The GARCH (and/or ARCH) results of Table 27 were consistent with previous findings for stock returns (see Akgiray, 1989; Ng et al., 1990), i.e., the time-series of daily stock returns exhibited significant levels of second-order dependence, and they would not be modelled as white noise processes. To further support these findings, we used a formal test of the GARCH (and/or ARCH) hypothesis that conditional forecast variances were nonconstant. We performed a standard likelihood ratio test in which, under the null hypothesis, the parameters of b_1 and c_1 (and/or c_1 and c_2 and c_3 for ARCH(3)) were constrained to zero. The alternative hypothesis was that the model followed a GARCH (and/or ARCH) form. The appropriate statistic was twice the difference of the maximized values of the log-likelihood functions for the unconstrained and constrained models, respectively, which would have a chi-square distribution with two (3 for $p + q = 3$) degrees of freedom under the null hypothesis. The results of the log-likelihood ratio tests presented in Table 27 lended support to our findings that the daily stock return follows a GARCH (and/or ARCH) form. A comparison of the coefficients of skewness and kurtosis (for the normalized residuals $\varepsilon_t/\sqrt{h_t}$) reported in Table 27 and those reported in Table 21 for the original daily return series reveals that ARCH(3), ARCH(3)-M, DARCH(3), DARCH(3)-M, GARCH(1, 1), DGARCH(1, 1), and GARCH(2,1)-M models have taken care of most of the "excess" fat-tail and skewness in the daily returns series. Regarding the ARCH(3)-M, DARCH(3)-M, and GARCH(2, 1)-M models, the parameter β_0, representing the relationship between the market mean return and its conditional standard deviation, was positive and significant in its estimate. This result was not consistent with those found in Chou (1988), Baillie and DeGennaro (1990), Bottazzi and Corradi (1991), Poon and Taylor (1992), and Cochran and Mansur (1993). However, it was consistent with those found in Ng et al. (1990) and Lee and Ohk (1990).

It was not possible to deduce which of these seven models was most preferable because the likelihood functions were not nested. We applied the Akaike (1974) information criterion (AIC) and diagnostic test on residuals to deduce which model was most preferable. The AIC was defined as follows:

(10) $$AIC(q) = T\ln\left(\frac{SSR}{T}\right) + 2q ,$$

where T is the sample size to which the model is fitted, SSR is the sum of squared residuals, and q is the number of parameters, equal to n + 2. According to the figures of AIC reported in Table 27, the ARCH(3) gave us the minimum AIC figure. However, if we look at the diagnostic test on normalized residuals, $\varepsilon_t/\sqrt{h_t}$, it seemed that GARCH(1, 1) and DGARCH(1, 1) outperformed the rest of the seven models, since these two models captured more of the excess skewness and kurtosis (fat-tail) of the data. According to Hsieh (1989), this was indicative of proper GARCH model fitting. We knew that the volatility persistence was measured by the sum of $c_1 + b_1$ (for GARCH(1, 1)), another more intuitive way of measuring volatility persistence is the half-life of a shock (HL)[12] calculated as

[12]According to Lamoureux and Lastrapes (1990), half-life (HL) measures the period of time (number of days, weeks, or months) over which a shock to volatility diminishes to half its original size.

$$(11) \quad HL = \frac{\log(0.5)}{\log(b_1 + c_1)} .$$

The HL was approximately 301 days in our GARCH(1, 1) model. However, the HL was reduced to 84 days after we incorporated the dummy variable into the conditional variance equation (DGARCH(1, 1)). This finding was somewhat larger than those found in Baillie and DeGennaro's (1990) U.S. studies (69 days) and Poon and Taylor's (1992) U.K. studies (26 days).

Table 28 reports parameter estimates of ARCH(3), DARCH(3), GARCH(1, 1), GARCH(1, 1)-M, DGARCH(1, 1), DGARCH(1, 1)-M, and GARCH(2, 1)-M models for the weekly stock returns. Regarding the ARCH(3) model, the coefficients of c_1, c_2 and c_3 are all statistically significant. The estimate of $c_1 + c_2 + c_3$ (0.8978, a measure of persistence) was also less than unity, indicating second-order stationarity for the return process. The coefficients of the MA terms (θ_{-1} and θ_{-2}) were all statistically significant. The estimated γs, taking into account the structural shift in data series, was significant only in the conditional variance function (the estimate of γ_1 reported in Table 28).

By looking at the estimate of $c_1 + c_2 + c_3$ (0.7695, a measure of persistence) from DARCH(3), the magnitude decreased relative to that of ARCH(3). The GARCH(1, 1) coefficients, b_1 and c_1 (reported in Table 28), were also statistically significant. The estimated GARCH(1, 1) parameterization indicated a near-integrated GARCH process with persistent conditional variance. These results provided strong evidence that weekly stock return volatility can be characterized by a GARCH(1, 1) specification. Since the estimate

Table 28

ESTIMATES OF MODELS FOR TAIWAN WEEKLY STOCK MARKET RETURNS
(1st week of January 1967 to 3rd week of September 1994)

	ARCH(3)	DARCH(3)	GARCH(1, 1)	GARCH(1, 1)-M
α_0	-0.5606	-0.6094	-0.9049	-1.0465
	(-1,7633)	(-1.7251)	(-1.8378)	(-2.0259)*
β_0				0.0661
				(1.0147)
α_{-1}	2.5221	2.7349	3.7083	3.6275
	(2.4412)*	(2.3607)*	(2.3017)*	(2.2553)*
θ_{-1}	-2.4359	-2.6504	-3.6266	-3.5465
	(-2.3359)*	(-2.1718)*	(-2.2459)*	(-2.1999)*
θ_{-2}	-0.2322	-0.2577	-0.3685	-0.3593
	(-2.7198)*	(-2.6764)*	(-2.7417)*	(-2.7007)*
b_0	3.3929	3.1502	0.1357	0.1361
	(12.6492)*	(12.4927)*	(3.2871)*	(3.3559)*
c_1	0.3021	0.2645	0.1209	0.1201
	(7.3591)*	(6.7965)*	(8.5547)*	(8.5273)*
c_2	0.3544	0.3044		
	(9.3592)*	(7.8616)*		
c_3	0.2413	0.2007		
	(6.8053)*	(5.9587)*		
b_1			0.8741	0.8747
			(63.7821)*	(64.3231)*
γ_1		7.5611		
		(4.7923)*		
c_1+b_1			0.995	0.9948
$c_1+c_2+c_3$	0.8978	0.7695		
L-L	-3,731.26	-3,711.87	-3,665.69	-3,665.15
LR(2)				
H_0:			714.00*	
$c_1=b_1=0$				
LR(3)				
H_0:	582.86*			
$c_1=c_2=c_3=0$				
AIC	4,009.14	4,010.65	4,005.21	4,009.57
m3	-0.118132	-0.105741	-0.005416	-0.005921
m4	4.154434	3.896437	3.791781	3.796456
J-B-N	82.50275	50.40459	37.25623	32.41321

Table 28--CONTINUED

	DGARCH(1, 1)	DGARCH(1, 1)-M	GARCH(2, 1)-M
α_0	-0.8926	-1.0297	-0.3788
	(-1.8175)	(-2.0127)*	(-0.7893)
β_0		0.0651	0.0983
		(1.0047)	(1.7831)
α_{-1}	3.6555	3.5843	1.2772
	(2.2701)*	(2.2303)*	(2.8241)*
θ_{-1}	-3.5721	-3.5012	-1.1335
	(-2.2125)*	(-2.1728)*	(-2.7281)*
θ_{-2}	-0.3608	-0.3528	-0.0763
	(-2.7066)*	(-2.6721)*	(-2.1887)*
b_0	0.1806	0.1806	0.0218
	(3.3849)*	(3.4303)*	(1.1164)
c_1	0.1303	0.1295	0.0605
	(7.5881)*	(7.5109)*	(3.2264)*
c_2			0.0688
			(2.5724)*
b_1	0.8556	0.8562	0.8521
	(44.6338)*	(44.5042)*	(80.3761)*
γ_1	0.4311	0.8562	
	(2.2726)*	(2.2714)*	
c_1+b_1	0.9859	0.9857	
$c_1+c_2+b_1$			0.9814
L-L	-3,663.15	-3,662.63	-3,678.54
AIC	4,007.02	4,011.58	4,022.49
m3	-0.030862	-0.031079	0.047566
m4	3.691939	3.695891	4.096579
J-B-N	28.67387	29.00295	71.98541

* : Denotes significance at the 5% level.
L-L denotes log-likelihood.
LR(q) denotes the log-likelihood ratio test.
AIC denotes Akaike (1974) information criterion.
m3 and m4 refer to the skewness and kurtosis of the normalized residuals ($\varepsilon_t/\sqrt{h_t}$), respectively.
J-B-N denotes Jarque-Bera normality test on the normalized residuals ($\varepsilon_t/\sqrt{h_t}$).

of the autoregressive parameter b_1 was greater than c_1, and the sum of these two parameters (0.995) was smaller than unity, both processes were likely to be stationary (see Bollerslev, 1987). The γ_1, taking into account the structural shift in the conditional variance function (for DGARCH(1, 1)), was significant. The estimate of $c_1 + b_1$ (0.9859) from DGARCH(1, 1) provided further support for Lamoureux and Lastrapes's (1990) finding that a (G)ARCH model that does not account for the structural change would pick up high persistence. The β_0, representing the relationship between the mean return and its conditional standard deviation in the GARCH(1, 1)-M model, was positive but insignificant in its estimate.

We also estimated a GARCH(2, 1)-M model on weekly data series. We found no negative coefficients in the conditional variance model. However, the β_0, representing the relationship between the mean return and its conditional standard deviation in GARCH(2, 1)-M model, was positive but insignificant in its estimate, which was not consistent with those found in French et al. (1987). The GARCH (and/or ARCH) results of Table 28 were also consistent with previous findings for stock returns (see Akgiray, 1989; Ng et al., 1990), i.e., the time-series of weekly stock returns exhibited significant levels of second-order dependence, and they could not be modelled as white noise processes. We applied the same likelihood ratio test as performed above with daily data. Results of the log-likelihood ratio tests presented in Table 28 lent support to our findings that the weekly stock returns followed a GARCH (and/or ARCH) form. Furthermore, a comparison of the coefficients of the skewness and kurtosis (for the normalized residuals $\varepsilon_t/\sqrt{h_t}$) reported in Table 28 and those reported in Table 22 for the original weekly return series also revealed that ARCH(3), DARCH(3), GARCH(1, 1), GARCH(1, 1)-M, DGARCH(1, 1), DGARCH(1, 1)-M, and GARCH(2, 1)-M models have taken care of most of the "excess" fat-tail and skewness in the weekly returns series. Regarding the GARCH(1, 1)-M, DGARCH(1, 1)-M, and GARCH(2, 1)-M models, the parameter β_0, representing the relationship between the market mean return and its conditional standard deviation, was positive but insignificant in its estimate. This result was not consistent with those found in our daily data series. We applied the same Akaike (1974) information criterion (AIC) and diagnostic test on residuals to deduce which model was the most preferable to modeling the weekly stock return and volatility. We found that GARCH(1, 1) not only gave us the minimum AIC but also captured more of the excess skewness and kurtosis (fat-tail) of the data. The half life found in GARCH(1, 1) and DGARCH(1, 1) were about 138 weeks and 48 weeks, respectively, which were higher than those found in U.S. (18 weeks) and U.K. (49 weeks) studies (see Poon and Taylor, 1992).

Table 29 reports the parameter estimates of ARCH(3), ARCH(3)-M, DARCH(3), DARCH(3)-M, GARCH(1, 1), and DGARCH(1, 1) models for the monthly stock returns. The results were similar to those found in daily and weekly data series. The γ_1, taking into account the structural shift in the conditional variance function (for DARCH(3) and DGARCH(1, 1)), was significant (reported in Table 29), but γ_1 for the mean function was insignificant.

Regarding the ARCH(3)-M and DARCH(3)-M models, the parameter β_0, representing the relationship between the mean return and its conditional standard

Table 29

ESTIMATES OF MODELS FOR TAIWAN MONTHLY STOCK MARKET RETURNS
(January 1967 to September 1994)

	ARCH(3)	ARCH(3)-M	DARCH(3)	DARCH(3)-M
α_0	0.7889	-0.0069	0.7966	-0.3842
	(2.7127)*	(-0.0092)	(2.5741)*	(-0.4714)
β_0		0.1264	0.1969	
		(1.1849)	(1.6264)	
b_0	16.4315	17.6095	13.4544	13.4471
	(3.6849)*	(4.4771)*	(3.5594)*	(3.4806)*
c_1	0.2275	0.1768	0.2709	0.2321
	(3.4284)*	(3.5471)*	(3.6806)*	(3.1876)*
c_2	0.3271	0.2515	0.2507	0.2642
	(4.3931)*	(4.5896)*	(3.5196)*	(3.5265)*
c_3	0.4260	0.4649	0.3828	0.4041
	(4.0545)*	(4.8125)*	(3.9317)*	(4.2232)*
γ_1			53.5400	53.2119
			(3.2216)*	(3.2141)*
$c_1+c_2+c_3$	0.9806	0.8932	0.9044	0.9003
L-L	-1,165.56	-1,165.76	-1,165.76	-1,156.05
LR(3)				
H_0:	173.56*			
$c_1=c_2=c_3=0$				
AIC	1,573.28	1,577.06	1,575.25	1,583.04
m3	0.182374	0.196931	0.15034	0.173702
m4	3.391119	3.289357	3.388131	3.223641
J-B-N	2.729068	2.279068	2.300056	1.628804

Table 29--CONTINUED

	GARCH(1, 1)	DGARCH(1, 1)
α_0	0.8889	0.8648
	(2.7462)*	(2.7358)*
b_0	1.4566	2.4786
	(2.1904)*	(2.5014)*
c_1	0.1571	0.2046
	(5.3384)*	(4.6309)*
b_1	0.8188	0.7437
	(31.0908)*	(175.3962)*
γ_1		12.2631
		(2.1871)*
c_1+b_1	0.9759	0.9483
L-L	-1,164.44	-1,161.25
LR(2)		
H_0:	175.8*	
$c_1=b_1=0$		
AIC	1,571.01	1,573.06
m3	0.008242	0.020555
m4	3.269409	3.322197
J-B-N	0.695137	1.006651

* : Denotes significance at the 5% level.
L-L denotes log-likelihood.
LR(q) denotes the log-likelihood ratio test.
AIC denotes Akaike (1974) information criterion.
m3 and m4 refer to the skewness and kurtosis of the normalized residuals ($\varepsilon_t/\sqrt{h_t}$), respectively.
J-B-N denotes Jarque-Bera normality test on the normalized residuals ($\varepsilon_t/\sqrt{h_t}$).

deviation, was positive but insignificant in its estimate. This result was similar to those found in our weekly data series. The results of the log-likelihood ratio tests presented in Table 29 also lent support to our findings that the monthly stock return followed a GARCH (and/or ARCH) form. A comparison of the coefficients of the skewness and kurtosis (for the normalized residuals $\varepsilon_t/\sqrt{h_t}$) reported in Table 29 and those reported in Table 22 for the original monthly return series also revealed that ARCH(3), ARCH(3)-M, DARCH(3), DARCH(3)-M, GARCH(1, 1), and DGARCH(1, 1) models have taken care of most of the "excess" fat-tail and skewness in the monthly returns series. In terms of the AIC value and the diagnostic test on residuals, we found that GARCH(1, 1) not only gave the minimum AIC but also captured more of the excess skewness and kurtosis (fat-tail) of the data. The HL found in GARCH(1, 1) and DGARCH(1, 1) were about 28 months and 13 months, respectively, which were also higher than those found in Poon and Taylor's (1992) U.K. studies (3 months) and Baillie and DeGennaro's (1990) U.S. studies (9 months). An interesting finding in Table 29 is that the (G)ARCH models seemed to model the monthly returns quite well. If we look at the Jarque-Bera normality test reported in Table 29, the figures indicate that the normalized residuals ($\varepsilon_t/\sqrt{h_t}$) we normally distributed.

From Tables 27, 28, and 29, we found out the sum of $c_1 + b_1$, a measure of persistence, from the GARCH(1, 1) models are 0.9977, 0.995, and 0.9759, respectively. This result suggested that as the frequency of returns became higher, the series approached an integrated process. Similar observations were found in French et al. (1987), Chou (1988), Baillie and DeGennaro (1990), and Poon and Taylor (1992).

VI. Conclusion

This study represents our attempt to expand previous work by Fawson and Chang (1994) by using ARCH and GARCH (and/or ARCH-M and GARCH-M) models to analyze stock returns and volatility with daily (for the period January 5, 1967 to September 26, 1994), weekly and monthly (for the period January 1967 to September 1994) Taiwan data. This approach was similar to that used by Poon and Taylor (1992) using U.K. data. This study modelled and addressed the empirical relationship between stock returns and volatility in the Taiwan context for the first time covering such a long research period. Chow test results suggested significant evidence of a structural shift between 1987 and 1989 for all three data series studied. Based on AIC criterion and diagnostic tests on normalized residuals ($\varepsilon_t/\sqrt{h_t}$), we found that GARCH(1, 1) was the most appropriate model of stock return volatility of the Taiwan Stock Exchange. Furthermore, we used GARCH in mean models to examine the relationship between mean return and its conditional standard deviation. Our results showed the β_0, representing the relationship between mean market return and its conditional standard deviation, was positive and significant only for the high frequency daily data set. Regarding the weekly and monthly data set, this relationship was also positive but insignificant.

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CONCLUSION

This dissertation explored the application of ARCH and GARCH methods to economic time-series data from Taiwan. It consisted of three essays. The first essay addressed the issue of inflation, and the second and third essays focused on the return behavior of the Taiwan stock market.

Essay 1 explored the fundamental relationship between average monthly inflation and its variability between January 1971 and June 1992. Chow test results suggested significant evidence of a structural change in inflation behavior beginning in 1982; a period of economic liberalization in Taiwan. Analysis, which accounts for structural change, revealed that the fundamental relationship between inflation and its variability was severed by policies implemented during economic liberalization in the early 1980s. In addition, ARCH and GARCH effects failed to be significant when structural change was accounted for.

Essay 2 investigated the dynamic linkage between daily stock returns and daily trading volume in the Taiwan stock market during the period of September 7, 1988 to December 13, 1994. We investigated both linear (Granger causality test) and nonlinear (GARCH modelling) dependence. Chow test results suggested significant evidence of a structural change in both stock returns and trading volume on October 1, 1990, an ending period of a long bear market for the Taiwan stock market. We also applied econometric techniques such as a unit root test, a cointegration test, and a Lagrange multiplier test. Empirical evidence indicated significant unidirectional Granger causality from stock returns to trading volume, which was not consistent with earlier U.S. results. This result was explained by relative low trading volume, small size of the Taiwan market, and cross-country differences.

Essay 3 represented an attempt to model and address the empirical relationship between stock returns and volatility in the Taiwan Stock Exchange Index from January 1967 to September 1994. Chow test results suggested significant evidence of a structural shift between 1987 and 1989 for all three data series studied. The statistics showed that daily, weekly, and monthly returns were negatively skewed, and the Jarque-Bera test also led to the rejection of normality of returns for the daily, weekly, and monthly price index in the Taiwan stock market. These results were consistent with results from previous studies using Taiwan stock return data (see Lee, Pettit and Swankoski, 1990; Lee and Ohk, 1990; Fawson, Glover and Chang, 1994). Both the Lagrange multiplier test and the likelihood ratio test indicated that stock returns followed a GARCH (and/or ARCH) form. Based on the AIC criterion and diagnostic tests on normalized residuals ($\varepsilon_t/\sqrt{h_t}$), we found that GARCH(1, 1) is the most appropriate to model stock return volatility of the Taiwan Stock Exchange. Furthermore, we used a GARCH-in-mean model to examine the relationship between mean return and conditional standard deviation. Our results showed the coefficient representing the relationship between mean return and its conditional standard deviation to be positive and significant only for the high frequency daily data set. Regarding the weekly and monthly data set, this relationship was also positive but insignificant.

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