5-1971

Linear Programming as a Decision Tool in Cheese Manufacturing Plants

Mark S. Huber
Utah State University

Recommended Citation

Huber, Mark S., "Linear Programming as a Decision Tool in Cheese Manufacturing Plants" (1971). *All Graduate Theses and Dissertations*. 4202.
https://digitalcommons.usu.edu/etd/4202

LINEAR PROGRAMMING AS A DECISION TOOL IN CHEESE MANUFACTURING PLANTS

by

Mark S. Huber

A thesis submitted in partial fulfillment
of the requirements for the degree

of

MASTER OF SCIENCE

in

Agricultural Economics

Approved:

UTAH STATE UNIVERSITY
Logan, Utah

1971

ACKNOWLEDGMENTS

I would like to express sincere appreciation to Dr. Allen LeBaron, my committee chairman, for his encouragement in the development of this thesis and for the confidence he has expressed in me during the time I have been working on this degree. A special thanks to Dr. Darwin Nielsen for being on the committee and for his time spent in helping to set up the linear program models and in interpreting the results. I would like to thank Dr. C. A. Ernstrom, not only for being on my committee and helping with this thesis, but also for his counseling and teaching during my undergraduate years.

I would like to thank the management of Cache Valley Dairy association and Hi-Land Dairyman Association for the privilege of using their plants for sampling purposes and for volunteering the cost data for each of their products; also those persons involved in the management of Utah State University's Dairy Products Laboratory for the use of their facilities.

I would like to express my appreciation to my parents who have always encouraged and supported me in school, as well as other worthy projects undertaken during my life.

Finally, to my wife, Colleen, I express my deepest gratitude and love for her patience, encouragement, and the many long hours she has spent in typing all drafts of this thesis, and for the months of work to support us financially.

Mark S. Huber

Mark S. Huber

TABLE OF CONTENTS

LIST OF TABLES

LIST OF TABLES (Continued)

LIST OF TABLES (Continued)

LIST OF FIGURES

Figure Page

ABSTRACT

Linear Programming as a Decision Tool
In Cheese Manufacturing Plants

by

Mark S. Huber, Master of Science

Utah State University, 1971

Major Professor: Dr. Allen LeBaron
Department: Agricultural Economics

This thesis considered the potential benefits of employing linear programming in cheese manufacturing plants as a decision tool for management. Its potential has been enhanced by the recent approval of acid orange 12 as a chemical for testing the percent protein in milk; therefore, a practical test is now available for monitoring protein as well as milk fat in milk manufacturing and fluid milk plants.

Seven models, each one differing only in the milk fat and protein percentages or means of standardizing the cheese milk, were manipulated individually and simultaneously to test the managerial benefits of linear programming under various plant and market conditions. Each model consisted of five cheese activities or variables, two butter activities, three powder activities, and a selling activity for each product produced.

The maximum price that could be paid the farm producer per hundredweight of milk and the minimum wholesale price per pound of manufactured product, to cover variable costs were determined for each variety of cheese and composition of milk.

There was a definite interaction between each of the activities. This caused the cost to produce a pound of cheese to vary according to the alternative uses for milk, cream, skimmilk, and whey.

When the simulated plant was being utilized at or near full capacity and the cheese milk was standardized with non fat dry milk powder, total cheese yield increased as did total profits. When the plant was not being utilized to full capacity, profits were higher by not standardizing.

(108 pages)

INTRODUCTION

This thesis is a preliminary study of the potential benefits of employing linear programming in cheese manufacturing plants. Linear programming has been used extensively in recent years as a management tool for allocation of resources in production and manufacturing processes. French and others (6, 17) have demonstrated the procedures and shown the benefits that could be realized from linear programming in many food industries. The cheese industry appears to be a good linear program candidate because virtually all plants have more than one process or activity, thereby requiring management decisions about proper allocation of milk fat and protein to various cheese varieties or other products. At the same time linear programming is conceptually suitable because cheese yield is a function of the amount of milk fat and protein in the milk. As yet, however, there is little evidence that managers of cheese plants have actually succeeded in tying business decisions to mathematical models.

One reason why linear programming has not been employed in the cheese industry is because the Walker casein test, the Association of Official Analytical Chemists (A.O.A.C.) casein test, and the Kjeldahl test for total protein are either slow, relatively expensive, or the variance is large between duplicate samples. Recently, however, acid orange 12 was approved as a dye-binding chemical for measuring total protein in fluid milk. This has an advantage over the other tests because it is a rapid, low cost procedure with a high degree of repeatability (15).

It is recognized that the protein fraction, casein, would be a more appropriate variable to use than total protein in estimating cheese yield since it is the protein fraction that is recovered in cheese production. However, McDowall (10) considered the total protein approach to estimating cheese yield as accurate as the A.O.A.C. casein test.

Given an approved dye-binding method now available for use in the dairy industry, a practical barrier (a functional test for protein) to introducing linear programming has been eliminated. The movement of protein, as well as milk fat, can be monitored and accounted for in cheese production (7, 11, 15). This implies that management can now allocate milk with different ratios of milk fat and protein to the cheese variety that has the greatest profit margin for that ratio.

The data used to estimate the cheese yield coefficients for this study were obtained by sampling milk and cheese from Cache Valley Dairy, Hi-Land Dairy's Richmond plant, and Utah State University's Dairy Products Laboratory. Coefficients were estimated by correlating total pounds of milk fat and protein in milk with the total pounds of cheese solids obtained from that milk by means of linear regression techniques and by adjusting the moisture content to yield total pounds of cheese. Equations were derived for this study instead of using those of other researchers for reasons discussed in the literature review summary section of this thesis.

Five varieties of cheese (Cheddar, aged Cheddar, Monterey, Swiss, and high fat Cheddar) were used as activities in setting up the linear program matrix. The cheese yielding coefficients estimated from the regression equations became the output coefficients in the matrix. Other activities used in the linear program are: (a) sweet cream butter,

(b) whey cream butter, (c) non fat dry milk powder (NDM), (d) whey skim powder, (e) blend powder (contains 50% skimmilk solids and 50% whey skim solids[1]), (f) a milk separating activity, and (g) a whey separating activity. It was impossible to get an accurate measurement, from the plants, of the amount of cream and skim used in these latter activities. Therefore, the coefficients used in this linear program model were derived by assuming a 100% recovery of all solids would occur. One selling activity was also entered in the matrix for each product produced. The variable cost for each activity was obtained from the three cheese plants where the data were obtained. The selling prices are the wholesale prices for each product at the time this study was undertaken.

There are different amounts of milk fat and protein recovered in the whey from each variety of cheese. However, this study monitors only the whey from Cheddar cheese as it flows from whey to whey butter, whey powder, and blend powder. Cheddar cheese whey is used because the emphasis on collecting data is on Cheddar, and because the amounts of milk fat and protein in the different wheys are relatively constant.

Seven separate models are developed and compared during the study. Each of the models are identical except in milk composition (milk fat and protein percentages vary or the input milk is standardized by different means) and therefore, the output or yield coefficients are different in each case. The milk is standardized to yield a fat in the dry matter (F.D.M.) of 50% for Cheddar, aged Cheddar, and Monterey cheese. The high fat Cheddar is standardized to yield an F.D.M. of 60%, and Swiss cheese 45.5%.

---

[1] Whey skim solids are the solids in the whey after the fat has been removed.

This study examines five ways in which linear programming can aid management:

(a) The maximum price management should pay for milk to produce a given cheese.

(b) The minimum wholesale price for which management could sell a pound of cheese and still cover variable costs.

(c) The optimum product mix under various market and plant conditions when milk of each composition, as defined by the seven models, is considered individually.

(d) The optimal allocation of milk with different compositions of milk fat and protein simultaneously entering the plant.

(e) The relative value of milk for different quantities used by the plant.

Other management aims could be postulated and tested, but the ones chosen cover a broad spectrum designed to give an indicative review of possibilities and potentials inherent in the linear programming method when applied to cheese manufacturing.

REVIEW OF LITERATURE

Yield formulas

There have been many attempts made to predict cheese yield from milk composition. Each formula has a slight variation in procedure and each is based on a different age of the cheese.[2]

McDowall (10) quotes a publication by Van Slyke and Price where they estimated the yield of cheese per 100 pounds of milk to be 1.63 [casein + fat]. Sometime later McDowall (10) analyzed Van Slyke's data and, by use of regression analysis, estimated the yield per 100 pounds of milk to be 1.04 + 1.4 [casein + fat]. Van Slyke and Price altered their data to yield a 37% moisture cheese. Using this same "casein + milk fat" approach, McDowall analyzed his own data. The first formula was based on the Walker procedure for determining casein and the second was by the A.O.A.C. procedure.

(a) Walker casein test--14 day old cheese--

yield of cheese per 100 pounds milk = 1.42 + 1.33 [casein + fat] (10).

(b) A.O.A.C. casein test--14 day old cheese--

yield of cheese per 100 pounds milk = 2.29 + 1.20 [casein + fat] (10).

(c) A.O.A.C. casein test--green cheese--

yield of cheese per 100 pounds milk = 2.32 + 1.22 [casein + fat] (10).

---

[2] It is interesting to note that many of these formulas were derived in an effort to price cheese milk. For example, in 1936, McDowall (10) went to great efforts to predict cheese yield using formulas of his own as well as other researchers. He then discussed each formula and its relative value as a basis for milk payment.

The "casein + milk fat" approach is based on the assumption that the casein and milk fat are in a somewhat constant proportion, and that milk fat gives the same amount of yield as does casein. Equal emphasis is placed on casein and milk fat so that it is possible, at least in theory, to remove part of the milk fat for an alternative and more profitable use, causing the fat in the dry matter (F.D.M.) to be less than the legal standard of 50%. Babcock, Farrington, *et. al.* (10) criticized this procedure because "casein + milk fat" is not a marketable commodity.

Veale criticized the "casein + milk fat" formulas as well as a fat based formula as a basis for milk payment because ". . . Friesian milk (3.47% fat) was underpaid by all formulae, Ayrshire milk (3.77% fat) about justly paid, and Jersey milk (4.44% fat) overpaid in all cases." (4) Its ability to predict cheese yield can, therefore, be criticized for the same reasons.

Perhaps the formulas that have gained the greatest attention are those based on a ratio of casein to milk fat. In 1897, Babcock (2) estimated the yield of cheese per 100 pounds of milk to be 1.1 [fat] + 2.5 [casein] where the casein was determined by the A.O.A.C. test. His theory was that 1.0 pounds of milk fat yielded 1.1 pounds of cheese, and that 1.0 pounds of casein yielded 2.5 pounds of cheese. The added yield was moisture retention.

McDowall (10) gives formulas based on the Walker casein test and the A.O.A.C. casein test for green cheese and cheese 14 days old. The formulas based on green cheese are as follows:

(a) Walker casein test--

yield per 100 pounds milk = 1.171 [fat] + 2.177 [casein] (10).

(b) A.O.A.C. casein test--

yield per 100 pounds milk = 1.070 [fat] + 2.346 [casein] (10).

(c) Total protein--

yield per 100 pounds milk = 1.222 [fat] + 1.633 [total protein] (10).

The formulas based on the 14 day old cheese are as follows:

(a) Walker casein test--

yield per 100 pounds milk = 1.189 [fat] + 2.084 [casein] (10).

(b) Walker casein test and pasteurized milk--

yield per 100 pounds milk = 1.077 [fat] + 2.257 [casein] (10).

(c) A.O.A.C. casein test--

yield per 100 pounds milk = 0.984 [fat] + 2.419 [casein] (10).

(d) Total protein--

yield per 100 pounds milk = 1.138 [fat] + 1.672 [total protein] (10).

McDowall (10) derived these equations by algebraically manipulating the original linear regression equations which were based on a protein to fat ratio. For example, the green cheese equation for total protein was:

yield of cheese per pound of fat = 1.222 + 1.633 [total protein/fat].

A fourteen day formula was given as well as a green cheese formula because, in earlier times, shrinkage due to moisture loss was very great the first 14 days after production. Today the cheese is wrapped in materials that stop moisture loss, and therefore, the relevant formula becomes the green cheese formula.

McDowall derived these formulas with the idea that they could be used as a basis for payment of milk. The casein to fat or total protein to fat ratios have the advantage over the "casein + fat" and fat yield formulas for the pricing of milk because there is a direct relationship between yield and the ratios.

Price and Van Slyke (23) developed the equation from which the industry has probably gained the greatest use. Their equation is:

$$\text{yield} = \frac{\left[0.93\ F + C - 0.1\right] 1.09}{1.0 - W}$$

where yield = pounds of cheese per hundred pounds of milk,

F = pounds of fat per hundred pounds of milk,

C = pounds of casein per hundred pounds of milk,

W = pounds of water per pound of cheese.

Shelton and Meaney (4) estimated the yield of cheese to be

$$\text{yield} = \left[\text{fat} - \frac{4 \times \text{fat}}{100}\right] + \left[\text{casein} - \frac{4 \times \text{casein}}{100} + \frac{22 \times \text{casein}}{100}\right] \times 2.26$$

based on the following premises: (a) a loss of 4% of the fat in the whey, (b) a loss of 4% of the casein in the whey, (c) a retention in the cheese of non-casein solids-not-fat (S.N.F.) equivalent to 22% of the casein, and (d) a cheese moisture content equivalent to 126% of the S.N.F. retained.

There have been many other formulas derived that are based on constituents other than casein, total protein, and milk fat. Van Dam and Janse (4) estimate the yield by fat and solids-not-fat.

Yield = fat + 1/3 solids-not-fat.

This has the disadvantage that casein is the only constituent of the solids-not-fat that remains in any amount in the cheese. There is also little response in yield when the ratios of fat to solids-not-fat constituents vary.

Spildo (4) and Cranfield and Blood (4) have estimated formulas based on calories. The formulas are essentially:

yield = $2\frac{1}{4}$ fat + lactose + protein.

The problem involved in this approach is in calculating the calories, and that alone outweighs any accuracy which might be obtained (4).

Bergman and Joost (4) derived a formula based on the fat and protein in the milk and on the water and salt in the cheese.

$$\text{Yield} = \frac{91\,F + 77\,P + 40}{100 - S + W}$$

where F = % of fat in milk,

P = % of protein in milk,

W = % of water in cheese,

S = % of salt in water.

J. G. Davis in his book Cheese (4) discusses six formulas proposed by other authors for the standardization of milk to achieve the proper F.D.M.

(a) Schulz and Kay estimated the yield of cheese to be:

$$\text{yield} = \left[\text{milk fat} - \text{whey fat}\right]\% + \left[0.75 + \frac{0.825\,Wff}{100 - 1.1\,Wff}\right]P$$

where Wff = the moisture content of the fat-free cheese,

P = protein content of the milk, assuming that 75% of the protein goes into the cheese.

(b) Korolew suggests the following formula.

$$f = \frac{b/a \times r \times y}{100 - y}$$

where a = portion of fat retained in cheese,

b = portion of not-fat constituents in cheese,

r = solids-not-fat content of milk,

y = required F.D.M. percentage,

f = fat content of milk after adjustment.

(c) Pettersen and Eikeland derived a formula by using Gouda cheese.

$$f = \frac{[0.75\ P + 0.08\ M]\ [y + 1]}{0.9\ [100 - (y + 1)]}$$

where f = fat content of milk after adjustment,

y = required F.D.M. percentage,

P = protein content of milk,

M = non-fat and non-protein solids in milk.

This formula assumes that 90% of the fat, 75% of the protein, and 8% of the non-fat and non-protein solids are retained in the cheese.

(d) Valen derived the following formula:

$$f = \frac{P + 0.2688}{0.01181\ [100 - F] + 0.01P + 0.002}$$

where f = fat content of milk after adjustment,

P = protein content of milk,

F = fat content of milk for cheese of 48% F.D.M. in Norway.

(e) Jakubowski and Bijok said that

$$f = \frac{C\ [1.03] \times s/p \times y + 0.9\ w}{100 - y}$$

where f = fat content of milk after adjustment,

y = required F.D.M.,

C = casein content of milk,

S = solids-not-fat % in cheese,

p = protein % in cheese,

w = fat % in whey.

(f) Jakubowski and Bijok then recommended the following simplified formula for Polish conditions:

$$f = 0.01\ y\ [2\ C + 1.1]$$

where f = fat content of milk after adjustment,

y = required F.D.M.,

C = casein in milk.

Alone, these various results provide no basis for choosing which of the many formulas presented in this review is the most accurate. If it is assumed that they are all equally accurate; perhaps McDowall's formula would be the most appropriate because of its simplicity, or Price and Van Slyke's formula because of its acceptance by the industry. Casein and milk fat are the two constituents of greatest importance in cheese yield and, as stated earlier, casein is a more important variable than total protein in predicting cheese yield. However, McDowall (10) felt that total protein is as accurate as the A.O.A.C. casein test in making this estimate. The general criticism of each of the formulas is that they either assume cheese moisture to be constant or that moisture content of cheese is completely dependent upon the ability of protein to bind water.

The moisture content of cheese is in three forms, (a) bound chemically, (b) loosely bound or absorbed, and (c) free moisture, or mechanically attached (4). Davis (4) says the final moisture content of cheese can be expressed mathematically.

$$H_2O = f_1 \left[t_1 \; T_1 \; pH_1\right] + f_2 \left[t_2 \; T_2 \; pH_2\right] + \ldots$$

where H_2O = final moisture content of cheese,

t_i = time,

T_i = temperature,

pH_i = the acidity,

f_i = a function of these controlling factors,

i = a particular phase of the cheese making process.

If what Davis says is correct, then cheese yield, including moisture, could be determined by the regression technique where coefficients are derived for milk fat, protein, and one coefficient for time, temperature, and pH for each phase of the cheese making process. If consideration is not given to time, temperature, and pH, then it is possible to question the accuracy of each of these formulas in estimating total cheese yield.

Factors affecting cheese yield

There have been many studies which indicate an increase in yield due to pasteurization of cheese milk. The Dairy Research Institute in New Zealand found that by flash pasteurization, actual yield increased 0.5% and by holding (vat pasteurization) for 30 minutes at pasteurization temperature, the actual yield increased 2.2% (10). The possible reasons cited for the increased yield are: (a) increased protein due to the denaturing of some whey proteins, (b) an increase in the moisture content, and (c) a decrease in the fat lost in the whey. However, McDowall (10) could not confirm the decrease in fat loss which other researchers found.

In 1967, Mabbitt and Cheeseman published, "The Effect of Concentrating Milk on the Fat Retention Property of the Cheese Curd" (9). They found that by concentrating milk before making it into cheese, a greater amount of fat was retained in the curd. In order to eliminate the possibility of attributing the increased yield of fat to the greater amount of solids, they added water back to the concentrate until the total solids in the milk were the same percentage as before the concentration took place. They concluded that there was some surface phenomena involved in the concentration process which caused the increased yield. They suggested that a possible change took place in the fat globule membrane causing the casein to bind firmly to the globules during the concentration process.

In "Manufacture of Cheddar Cheese From Reconstituted Milk," I. I. Peters and J. D. Williams (13) found that by homogenizing unsalted butter, low heat skim milk powder, and water, they were able to make a good quality Cheddar cheese. The process recovered a greater portion of the total solids than they could obtain by making cheese from whole milk of equal total solids. They recovered 95.12% of the milk fat in whole milk compared to 97.92% milk fat recovery from the reconstituted milk. The solids-not-fat recovery in whole milk was 25.75% and 32.47% in reconstituted milk. No reasons were advanced for this increased yield.

It appears that there definitly is an increase in cheese yield following pasteurizing, concentrating, or drying of milk. The one common factor in these three processes is heat treatment. This gives evidence that at least part of the increased yield is due to the denaturing of some whey proteins. The "surface phenomena" suggested by Mabbitt and Cheeseman (9) probably requires further research for confirmation and explanation.

Sources of error in estimating cheese yield

One of the difficulties in estimating cheese yield is the many possibilities for error. There is no plant where the data do not contain errors in measuring, sampling, weighing cheese, spillage, etc.

Measurement of milk volume is a common area for error. If there were a 10 gallon error in an 800 gallon vat, then the error in estimating the yield of cheese would be 1.25%. If a fat percentage of 4.0% were read 0.1% low and McDowall's (10) total protein formula used for predicting, then the pounds of cheese per 100 pounds of milk would be underestimated by 1.2%.

Errors occur in sampling cheese, even when greatest care is taken to avoid them. Moisture in cheese flows from one area to another area until it reaches an equilibrium throughout the entire block of cheese. Therefore, sampling the cheese at the proper places and time is important (14).

A study by Smythe and Stanton (16) revealed that the sampling of milk at various places and conditions in the plant made a difference. When samples were drawn from a break in the line or the loosening of a nut, the mean deviation of the fat tests was 0.08% with a standard deviation of ± 0.125. When samples were taken from a vat with the agitator in motion, the mean deviation of the fat tests was 0.05% and the standard deviation was ± 0.039. The third samples were drawn from an in-line sampler designed to take the sample from the center of the pipe carrying the milk to the cheese vats. The mean deviation of the fat tests was 0.03% and the standard deviation was ± 0.028. Their conclusion was that there are variations in milk fat tests because of sampling procedures. Drawing the samples by loosening a nut in a line causes substantial variations in the test. Both the sampling of milk while the agitator was moving and in-line sampling were satisfactory and recommended.

McDowall (10) also found there was some variation in cheese yield when milk of the same composition was made under identical conditions. No explanation is given for the variation.

Dye-binding

"In 1956, Dr. D. C. Udy published a formula to calculate the percent protein in milk using Orange G dye, . . ." (7). In 1965, an addendum was issued for acid orange 12 (22) which has since been approved by the A.O.A.C. as a chemical for determining the protein percentage in the milk. As has been previously mentioned, the advantage of the dye binding approach to determining total protein in milk is that it is a rapid low cost procedure that has a high degree of repeatability.

Udy (22) found that 1 unit of milk protein bound with .312 units of dye. A .311 factor was derived at Utah State University which confirms Udy's findings. The amount of dye needed to bind 1 unit of protein is referred to as the dye-binding capacity (DBC) factor for that protein; thus, .312 is the DBC for whole milk. The DBC for the soluble whey proteins becomes .282; and the DBC for the insoluble milk protein (casein) is .321. These are all based on a 6.38 Kjeldahl factor (7).

It has been recommended that a .312 DBC be used universally on all milk and milk products for two reasons, (a) simplicity, and (b) in accounting procedures the fractions (whey proteins and casein) would add up to the whole or total milk proteins. Using just one DBC results in overestimating the soluble proteins and underestimating the insoluble proteins. A third reason for using only one DBC can be implied from the results of N. P. Tarassuk (18). He determined that the 6.38 Kjeldahl factor was just an average of the true factors of each protein fraction. For example, a more accurate factor for β-casein would be 6.53 and for α-lactalbumin 6.30. These are the two extremes, but they give some justification for using only one DBC for milk since the 6.38 Kjeldahl factor is used universally for all milk products.

There have been many factors that reportedly affect the DBC of milk. Udy reported a slightly different DBC for powdered milk than whole milk, which he contributed to a denaturing of some whey proteins (21). Tarassuk and Abe, (19) however, found that there was no apparent effect caused by heating milk up to 90 C for 15 minutes. When heating occurred to the extent that browning of the milk resulted, a lowering of the DBC was observed. Mastitic and colostrum milks were also reported to have adverse effects on the DBC (18, 19). Tarassuk and Abe (19) further concluded that condensing and homogenizing (up to 4,000 pounds per square inch) had no affect on the DBC.

Two important factors affecting the DBC in everyday plant procedures are the temperature of the milk and dye solution; and the preservatives used in producer's composite samples. Udy found that temperature affected the ability of dye to bind with protein. LeBaron and Brog (7) defined this correction to be .005% per degree Fahrenheit, where increased temperatures caused low protein readings, and 77 F was to be used as the basis. The only preservatives that had no significant affect on the DBC were H_2O_2 and $HgCl_2$. H_2O_2 had no affect on the DBC but it preserved milk for only a short period of time. $HgCl_2$ was a good preservative but it caused a very slight lowering of the DBC (18, 19).

Ashworth reported that the DBC of Cheddar cheese decreased with age (1). This was confirmed in a study by Prahlad H. Patel (12). He found that between 15 and 30 days, a slight decrease in the DBC of cheese occurred. This decrease continued to increase over the 255 days of the experiment.

Linear program models

There has been much published on the benefits of linear programming and there are university courses that are so specific as to deal solely with the planning of farm enterprises and crop rotations. Yet, there are very few references in the literature on the use of linear programming in the food processing industry, and only one indirect reference has been found dealing with the cheese industry specifically.

In 1962 Glickstein, Babb, et. al. published "An Application of Simulation Processes for Production Control in a Cheese Plant" (6). They define their simulation procedures as Monte Carlo techniques or the probabilistic nature of milk volume that would be available for plant use. In a later article, French, (5) a co-author, defines the decision making tool for the study as linear programming. The study simulated a cheese manufacturing plant producing Colby cheese. The decision of how much to produce was made on the basis of milk purchasing policies available to them, the plant capacity, the labor available, and the demand and price for the product manufactured.

A time series analysis for the volume of milk during one year was considered to reflect the plant's needs for milk at each period during the year. Eleven policies for purchasing surplus milk over and above what their producers could supply were then considered for purchase at different seasons of the year. Each policy was easily compared with its alternatives in the decision making process by management. For example, when only producer milk was used in the plant, the cost to produce one pound of cheese was 34.12¢. When a 50¢ premium was paid for surplus milk during October 1 through April 14, the cost per pound increased to 34.70¢. When surplus milk was purchased between April 15 and September

30 at a 25¢ premium, the cost per pound of cheese decreased to 33.78¢. When both of these surplus milk purchasing policies were carried out, surplus milk was purchased the entire year, the cost per pound of cheese for that year was 34.32¢. The study also revealed that for that one plant, the efficiency of labor could be increased if surplus milk were purchased.

The yield coefficient used for the product, Colby cheese, was 10.7 pounds per 100 pounds milk. No adjustment was made for difference in yield due to the seasonal fluctuation of the milk solids nor fluctuations in milk fat and protein percentages due to herd differences. This would have an affect on the allocation of resources and the decisions management would make.

This study concluded by recommending that other studies be made to include production scheduling, marketing, and diversification of product line.

Another milk industry related study was of a fluid milk plant where 2,797 pounds of milk fat and 72,795 pounds of skimmilk were allocated among twenty products a particular plant was marketing at that time. Snyder and French (17) knew the amount of labor available and the time available for each machine, as well as the machine capacities. They also knew the market demand and the per unit price of each product. From the plant production records, the linear program matrix was set up to show the amount of each resource required to produce one unit of product. When the matrix was "solved," all the milk fat and skim had been allocated to thirteen of the twenty products, and a marginal cost above net selling price was given for the remaining seven products not produced. The amount of each resource used was given, with a marginal value product

(MVP) for each resource completely used up. The study was concluded by giving the total net revenue per working day.

The model was then manipulated to test the effects of changes in management policy. The areas tested were labor use, procurement practices, storage room additions, machine capacities, product promotion, and pricing.

One might wonder what not producing the seven products during one time period might have on consumer acceptance of the brand. If it is rational for the firm to be concerned about losing customers because all the products are not made available, then it becomes economically rational to produce a minimum amount of those seven products as well as a maximum amount set by the market demand. This could easily be accomplished by "bounding" (forcing) the program to produce a minimum number of units of each of the seven products.

Limitations in the use of linear programming

In an applied linear programming manual for farm planning, Raymond R. Beneke and Ronald Winterboer (3) suggest seven areas where linear programming is limiting. Consideration should be given to them when an analysis is made.

1. The technique has no ability to formulate future prices. Future prices become input data and the output results can be no more accurate than the accuracy achieved in estimating the future prices. However, it should be noted that an error in estimating the future prices is no more serious in planning by use of linear programming than by budgeting, or any other planning procedure.

2. The technique has little ability to derive coefficients for the activities. These must be estimated from past records or some future expectation. The program does have the versatility of making percentage changes in coefficients when the changes are specified in the input data.

3. Risk and uncertainty cannot be considered. Each activity is assumed equally risky.

4. There is often difficulty in predicting the future restraints of a resource, (i.e., the amount of labor, milk, or capital that will be available in six months or one year). This again, however, is no different than other systems.

5. No consideration is given for diminishing marginal returns. The assumption is made that each unit of input will yield the same proportion of output. If diminishing marginal returns to inputs is an important feature to consider, then linear segments of the production function can be included in the program as separate activities. However, this involves a troublesome process.

6. Decreasing cost activities cannot be considered accurately. An example of a decreasing cost activity would be: the labor requirements for a pasteurizer with a capacity of 50,000 pounds of milk per hour would be no greater than for one with a capacity of 20,000 pounds. Therefore, the labor costs per unit of milk would be much less with the pasteurizer with the capacity of 50,000 pounds than the one with 20,000 pounds per hour. Handling such a situation requires two models, one for each pasteurizer, so the incomes can be compared.

7. A using firm or researcher must have access to electronic equipment capable of handling a large number of activities. It is

possible for an individual to solve a linear program by the normal algebraic procedure for solving simultaneous equations, or by the Simplex procedure. However, as the number of activities increase, the only realistic and economical way to solve the program is by utilizing an electronic computer.

Certain other limitations are inherent in the computer routines used in this study. For example, the IBM/360 routine (employed in this thesis) provides only non-integer solutions. A plant cannot make 14.3 vats of one variety of cheese and 5.7 vats of another variety and say that it has the capacity of making 20.0 vats of cheese a day. The non-integer approach is realistic when considering activities such as powder products. The dryer can be changed over after 8.7 hours from producing nonfat dry milk powder (NDM) to the production of whey powder. A mixed-integer (integer solutions for such activities as cheese and butter where only whole units of the activity are realistic, and non-integer solutions for activities that need not be whole units, such as powders) routine is the program that should be used.

Summary of literature review

As this literature review has made clear, there have been many equations derived to predict cheese yield; any one of which might have been used to estimate yield coefficients for this study. However, none were chosen for several reasons: (a) Since acid orange 12 has been approved as a dye-binding chemical for estimating protein in fluid milk, and researchers (7, 15) have shown that it can be used to monitor the movement of protein through a manufacturing plant; it appears to have the potential to become the rapid, inexpensive "protein" counterpart to the Babcock procedure in monitoring milk fat. (b) Previous equations either do not adjust for the milk fat and protein ratios, or they assume that the moisture content of cheese is entirely a function of the moisture binding potential of protein. (c) Due to a lack of reported statistical tests, there is no indication of how well these equations predict. (d) The equations of earlier researchers were limited to Cheddar cheese and, therefore, would not satisfy the needs of the present study.

This study attempts only to predict total cheese solids from the milk fat and protein in cheese milk, and adjusts the moisture content to the desired amount. This is done because no attempt is made to measure the variables that control moisture.

As a consequence of this reasoning, new yield coefficients were obtained by linear regression analysis on actual plant data under normal manufacturing conditions. These yield coefficients, or predicted outputs for a given quantity and composition of milk, become the key to setting up the activities for the linear program models. It has been shown by French and others (6, 17) that once coefficients are obtained for given products, a linear program can aid management in decision making. Although the present study does not contain examples, nor involve activities or physical facilities reported by French (6), the types of decisions which management must make are identical. Some of the areas which these researchers have tested and found amenable to linear programming are:

(a) The optimal allocation of milk to the products which will maximize the plant's profits.

(b) The marginal values for each product which indicate the amount net returns would be reduced if the profits were not maximized.

(c) The most efficient use of plant capacity.

(d) The allocation of available labor.

(e) In adjusting for changes in demand and price of product produced.

(f) The milk purchasing policies to adjust for seasonal fluctuations.

(g) The effect of adding additional storage space.

The effects of each of these decisions can be evaluated prior to actually implementing a change; and therefore, the costs and time involved in a wrong decision may be minimized.

A GRAPHIC EXAMPLE OF LINEAR PROGRAMMING

Linear programming technique rests on three basic assumptions. First, decision making always involves restraints or limitations on the activities. The maximum amount of production for a plant for a given time period is always limited by some factor or resource. This limiting factor may be the capacity of some piece of equipment, the amount of a resource available for production, or the market demand for the product.

The second assumption is that the firm is part of a purely competitive society where input and output prices are assumed to be unaffected by the actions of that one firm. The prices for both buying and selling are the same regardless of how large or small the quantity is.

The third assumption made is that production activities can be described by simple linear relations. One such assumption is that the production functions are homogeneous to the first degree, or that the firm experiences constant returns to scale for homogeneous products. This implies a linear total cost curve for homogeneous inputs and a linear total revenue curve when the selling price of the product is constant. In cases where the relationship between two variables is not linear, linear segments of that relationship can be introduced (8).

A graphic example of a linear program model of two products, Monterey and Cheddar cheese, is shown in Figure 1. These two cheeses compete for two variable inputs: vat space and milk. Line AB represents vat space. The maximum amount of Monterey cheese that could be produced by the plant is OA units, or the maximum amount of Cheddar cheese

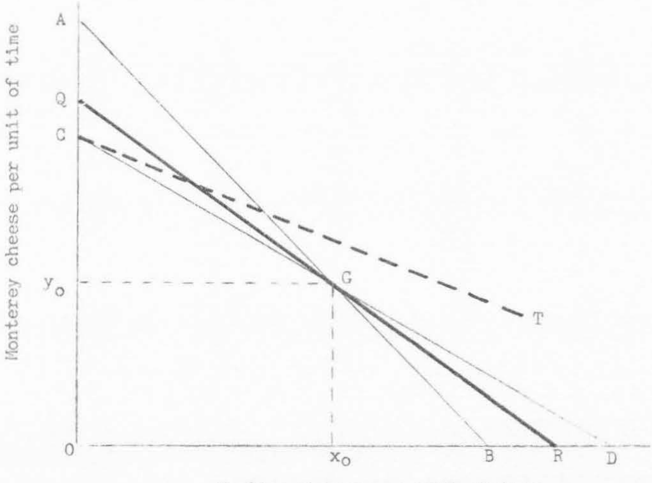

Figure 1. Graphic solution to a linear program model

that could be produced would be OB units, based on the vat space available per unit of time. If management wishes to produce both Monterey and Cheddar cheese then it would produce at some point on AB such that less than OA units of Monterey and less than OB units of Cheddar cheese would be produced. Line CD represents the milk restraint available to the plant for the production of these two cheeses. Based on the milk available, the maximum amount of Monterey cheese that could be produced is OC units and the maximum amount of Cheddar cheese that could be produced is OD per unit of time. Thus, the production possibilities curve (the maximum amount of Monterey and Cheddar cheese the plant could produce per unit of time when both vat space and milk available are considered simultaneously) becomes CGB.

The point on the production possibilities curve that will maximize profits is where a tangency occurs with the budget restraint (the amount of money available for the production of these two cheeses divided by the cost to produce either of the cheeses) line QR. In this case the tangency occurs at point G and profits are maximized by producing x_o units of Cheddar cheese and y_o units of Monterey cheese. If, however, the cost ratio of the two cheeses change, and the slope of the budget line shifts to CT, then profits are maximized by producing OC units of Monterey cheese and no units of Cheddar cheese. In this example there are only three feasible solutions: points C, G, and B.

If more restraints were added to the model then more lines representing the production or resource requirements for each cheese variety would be drawn. This would create more intersections such as G and the production possibilities curve would be the lines that form the area convex to the origin. Each intersection or point where the slope of the production possibilities curve changes is a possible tangency with the budget restraint and therefore, a possible solution.

METHODS AND PROCEDURES[3]

Arrangements were made with Hi-Land Dairyman Association's Richmond Plant, Cache Valley Dairy Association, and Utah State University's Dairy Products Laboratory to use their facilities for sampling purposes. The variable cost figures used in the objective function of the models were also obtained from these three plants, but were altered slightly at their request.

Data collected from these plants consisted of total pounds of milk used for a vat of cheese and the total pounds of cheese obtained from that vat of milk. The milk fat and moisture percentages were determined in part by the laboratory technicians at the Hi-Land and Cache Valley plants. Both the Babcock and Mojonnier (11) procedures were used for testing the milk fat in the milk and cheese. All protein percentage determinations were made using the Udy dye-binding procedure and acid orange 12 dye (7, 20).

The protein tests were made by running duplicate samples, and the weights of all samples were obtained by analytical weighings rather than by the customary calibrated syringe method. Twenty grams of cheese and 80 grams of 0.05 M NaOH were blended together to make a mixture that could be tested by the dye-binding procedure. The .312 DBC figure was the only figure used in this study.

---

[3] Except when noted, all laboratory or field tests were measured by the author.

The effect of using the .312 factor is that the actual percentages of protein in the cheese are higher than reported herein. This however, will not affect the predicted total cheese solids since total cheese solids were found by subtracting moisture from actual cheese weights.

Some difficulty was encountered in obtaining accurate milk and cheese weights. Only one of the four Cheddar cheese vats at Hi-Land's Richmond plant, and none of the vats at Cache Valley Dairy were properly calibrated. It was also observed that hoops of cheese from one vat were being mixed with hoops of cheese from another vat. An attempt was made to correct this error by referring to the production records where the original number of hoops from individual vats were recorded.

In five cases the observed Cheddar cheese weights were altered because the milk fat and protein recovered in the Cheddar cheese was near 100%, which is impossible, given the amounts of these constituents in the whey. Referring to the fat and protein tests from the whey, the amount of cheese was adjusted until the total milk fat and protein from the cheese and whey was less than the total milk fat and protein in the milk. If the five samples were deleted from the data and the linear regression equations recalculated, the change in the cheese predicted, would amount to eight pounds out of a total of 2,850 pounds. Therefore, the adjusted data could have been deleted with no affect on final results.

The actual data and linear regression equations used to obtain the cheese yielding coefficients are presented in the Appendix (Tables 16 through 19).

From the included statistical tests, the linear regression equation for Cheddar cheese total solids appears to be fairly accurate. The R^2

(correlation coefficient) is considered to be more than adequate for predicting purposes. An R^2 1.0 would mean a perfect correlation between the input variables and the output of the product. The "F" test for the equation is highly significant at any level desired. The S_E (standard error of estimate) is a little larger than desired, but when consideration is given to the large volume of milk used, it is acceptable.

The data and regression equation for the high fat Cheddar (Table 17 in Appendix) is statistically the best of all the equations. The R^2, .945, is very high, the "F" is significant at any level desired and a low S_E is reported.

The data for Monterey (Table 18 in Appendix) and Swiss (Table 19 in Appendix) cheese were very difficult to obtain. Although the R^2 for Monterey is high enough for predicting purposes, the "F" test is significant only at the .05 level, and the S_E is higher than expected when compared with the volume of milk used and the S_E of the other cheese varieties. It should be noted that the coefficient for the protein variable is negative. This implies that when the amount of protein in the milk increases, the yield of cheese decreases. This, of course, is not possible, and would not exist if a larger sample size had been taken. But, for the few samples used in deriving this formula, this condition does exist. The equation predicts very well, as long as the pounds of milk fat and protein stay within the range of the data used. Because this study needed to predict the yield of Monterey cheese outside the range of the data, a new formula was derived, based on the recovery of milk fat, protein, and solids other than milk fat and protein. This formula is also found in Table 18 of the Appendix.

The data used for the Swiss cheese are very poor. The vats were not calibrated and the calibration stick was very difficult to read. The R^2 is very low and the regression equation is not significant even at the .05 level. The Swiss cheese total solids equation is used only because the prime objective of this study is to demonstrate the potential use of linear programming in a cheese plant. More and better data are necessary in order to refine this coefficient.

For each variety of cheese, two additional linear regression equations are given; one to predict total pounds of milk fat and the other to predict total pounds of protein to be recovered in the cheese. Only a statistical R^2 is given for some indication of how well the input data correlated with the output or recovery data.

Total pounds of milk fat and total pounds of protein were used to estimate total pounds of cheese solids, because this resulted in a higher R^2 than when the milk fat and protein percentages were used. Therefore, the accuracy in predicting was increased. Since cheese yield is a linear function, the constant (y axis intercept) in the regression formula can be changed proportionately with changes in the volume of milk (i.e., if the volume of milk were reduced from 30,000 pounds to 20,000 pounds, then the constant would also be reduced by one third).

A regression analysis was made to correlate the protein/milk fat ratios with the F.D.M. The formula to predict F.D.M. in Cheddar cheese is:

$\text{F.D.M.} = 0.6618 - 0.1595\ [\text{protein/milk fat}]$.

$R^2 = 0.807$ and $F = 212.2$, both of which are significantly large, which indicates that the protein/milk fat ratio correlated well with the F.D.M.

Actual weights of sweet cream, whey cream, skimmilk, and whey skim were not available. Therefore, the coefficients of the products manufactured from them are derived by theoretically calculating each product based on several assumptions. These assumptions are as follows: the average non-protein solids in skimmilk was 5.46% and the average total solids in whey skim was 6.63%. It is assumed that 99.0% of the fat was recovered in sweet cream butter and 97.2% of the fat was recovered in whey butter. It is also assumed that no spillage or shrinkage occurred in the production processes.

Other assumptions made in this study are as follows: (a) Sweet cream always contains 40% fat while the protein percentage varies. (b) Whey cream contains 25.0% fat and the protein percentage does not vary. (c) All whey is homogeneous regardless of the cheese, (i.e., contains the same percentage of milk fat and protein). (d) The moisture content of Cheddar cheese is 38.0%. (e) The moisture content of high fat Cheddar is 37.0%. (f) The moisture content of Swiss cheese is 39.6%. (g) The moisture content of aged Cheddar is 36.0%. (h) The moisture content of Monterey cheese is 43.0%. (i) The market price for grade A and grade B Swiss cheese is the same, and 85% of all Swiss cheese is graded A or B.

High fat Cheddar cheese is not being commercially marketed at the present time. It is being manufactured and sold by Utah State University's Dairy Products Laboratory and has the potential of being commercially marketed. This cheese has an F.D.M. of 60%, which makes it a more delicate cheese than the standard Cheddar. It is used in this study because it provides an alternative use for sweet cream.

Since the data were collected from three plants, the assumed facilities in this study are somewhat different than any one of them. Figure 2 is a flow chart of the assumed facilities. This shows the flow and alternative uses available to the plant for the different milks as defined by the various models.

Hi-Land Dairy is equipped with the Damrow vat system where there are two vats needed to make one vat of cheese. In the first vat the milk is set, cut, and the curd is cooked. It is then transferred to the second vat where the cheddaring, milling, and hooping processes take place. The advantage of this system is that while the processes of the second vat are going on, another vat of milk can be started in the first. All other systems have only one vat for the entire cheese making process.

This study considered the use of four Damrow vats, which are assigned only to the production of Cheddar and aged Cheddar cheese. Four more vats are available for the production of high fat Cheddar and Monterey cheese. The final four vats are used for the production of Swiss cheese only. Each of the three series of vats are available for a maximum of 72 vat hours, or an 18 hour working day. One churn with a capacity of 3,320 pounds of cream is available for a 16 hour working day. There is also one high temperature short time (HTST) pasteurizer available that can be run 24 hours per day. Since a great volume of whey must be separated, there is a separator available for that purpose only, and it is used on a 24 hour basis. Other pieces of equipment necessary for this study are a condensor and a dryer. They too can be run continuously for the 24 hours of each day.

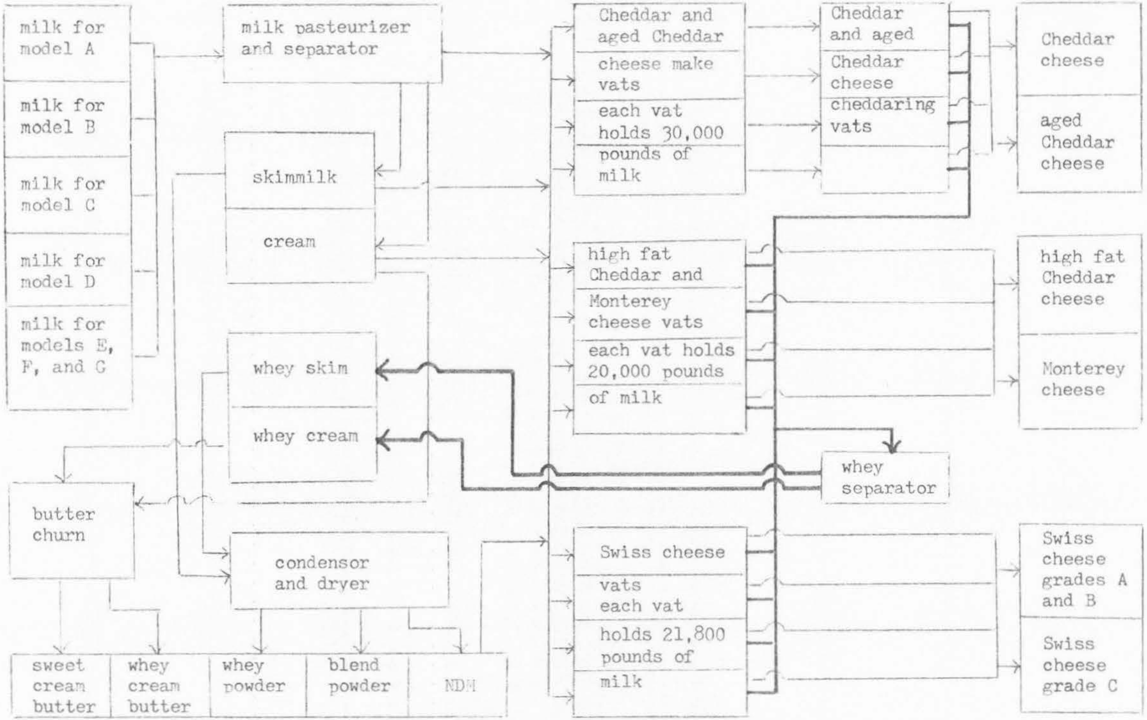

Figure 2. Flow chart of the assumed facilities

Five varieties of cheese (Cheddar, aged Cheddar, high fat Cheddar, Monterey, and Swiss) are considered in this study. The butter products are sweet cream butter and whey cream butter.

There are three powder products: NDM, whey powder, and blend powder (containing 50% skimmilk solids and 50% whey solids). For each of these ten activities there is a corresponding selling activity. Two other activities are provided for: one for separating the whey into whey cream and whey skim, and the other for separating the whole milk into sweet cream and skimmilk.

The models have been analyzed by the IBM MPS/360 routine which uses the Simplex procedure for manipulating and solving the tableau.

RESULTS

Seven linear program models were constructed for handling milk with different percentages of milk fat and protein. Model A considers milk with 3.30% milk fat and 3.14% protein. Model B considers 3.30% milk fat and 3.25% protein. In order to make legal cheese (50% F.D.M.), this model requires that each cheese activity be standardized with cream. The C model is 3.50% milk fat and 3.20% protein. Since the milk for Cheddar, aged Cheddar, and Monterey cheese is not standardized, the resulting cheese has a high F.D.M. percentage. Model D considers 3.50% milk fat and 3.32% protein. Model E considers 3.70% milk fat and 3.35% protein. As with model C, the Cheddar, aged Cheddar, and Monterey have a high F.D.M. because the milk is not standardized. Model F is identical to model E, 3.70% milk fat and 3.35% protein, except that each cheese activity is standardized with NDM to obtain the proper protein/milk fat ratio to yield the desired F.D.M. Model G is also identical to models E and F, with respect to milk composition, except skimmilk is used to make the standardization. Each of the seven models are identical except in the total amount of milk used, pounds of cheese yield, and cream, skimmilk, and NDM needed for standardization purposes.

Table 1 is the restraints or right hand side (RHS) for each of the seven models.[4] There are no restraints on the amount of labor used. However, the cost per hour of labor is varied as shown in Table 2. Table 3 shows the variable cost per unit of activity (i.e., one vat of

---

[4] The restraint on milk is part of the RHS, but not included in Table 1 because it was manipulated throughout the study.

Table 1. Equipment capacity or the RHS (right hand side) restraints in hours

Restraints	Hours
Whey separator	24
Milk pasteurizer and separator	24
Damrow make vats	72
Damrow cheddaring vats	72
High fat Cheddar and Monterey vats	72
Swiss vats	72
Condensor	24
Dryer	24
Churn	16

Table 2. Labor costs per job

Labor	$ per hour
Milk pasteurizer and separator	2.60
Damrow make vats	2.70
Damrow cheddaring vats	2.50
High fat Cheddar and Monterey vats	2.50
Swiss vats	2.70
Wrapping the cheese	2.50
Curing the Swiss cheese	2.60
Condensor and whey separator	2.60
Dryer	2.60
Churn	2.60

Table 3. Variable cost per unit of the activity, and wholesale price per pound of product; labor costs not included

Product	Variable cost	Wholesale price/pound
Cheddar cheese (per vat)	$ 48.00	$.54
High fat Cheddar (per vat)	80.22	.60
Monterey cheese (per vat)	30.00	.60
Swiss cheese (per vat)	83.75	
Grades A and B Swiss cheese		.70
Grade C Swiss cheese		.62
Aged Cheddar (per vat)	233.00	.64
Whey separate (per hour)	0	
Whey butter (per churn)	21.00	.6829
Whey powder (per hour)	125.00	.0575
Milk separate (per hour)	0	
Butter (per churn)	21.00	.6929
NDM (per hour)	125.00	.272
Blend (per hour)	125.00	.22

cheese, one churn of butter, or one hour of drying time). Table 3 also shows the wholesale price per pound for each product produced. The price paid by the cheese plant to the farmers is $4.74 per hundredweight (cwt.) with a $0.07 price differential for milk testing 3.50% milk fat. The labor and equipment requirements for each activity are shown in Table 20 in the Appendix. These requirements are identical for each of the seven models and therefore, are not repeated in Tables 21-27. Tables 21 through 27 (see Appendix) show the flow of whole milk, milk fat, and protein as they are processed. In analyzing Tables 21 through 27, it should be noted that positive numbers are inputs to the activity and negative numbers are outputs. The pounds of milk fat and protein for each input and output are in parentheses. This implies that the parenthetical number has the same positive or negative sign as the non-parenthetical number immediately above. The composition of the whey remains constant in all seven models, and it is recognized that this is one source of error.

The test of a linear program is in its usefulness to management. Although there are many ways to test this effect, this study considers only five. The sections which follow contain descriptions and interpretations of the results.

When considering the results of each of the five tests, model A should be compared to model B; model C should be compared to model D; models E, F, and G should be compared with each other; then simultaneous comparisons may be made of all seven models. The reason for analyzing the results in this manner is because the milk fat percentages are the same, while the protein percentages change, or else milk of identical composition is standardized differently.

Maximum allowable price for producer milk

One concern of management is the price that could be paid producers for milk and still cover short run variable costs. This price can be found by manipulating the MPS/360 routine to create a systematic stepped increase in the price of milk. In this case the price of milk is increased in increments of $0.10 per hundredweight, although it may be increased in any units desired. When the price of milk is raised such that it is no longer economical to produce a given product, milk is no longer allocated to that particular product. The results for this price study are shown in Table 4. The tabulated prices represent the maximum amount that management could pay for milk of each composition (as represented by models A through G), if it wishes to produce any one of the five varieties of cheese.

Table 4. The maximum price per hundredweight a firm could pay for milk if it desired to produce each cheese variety, given the butter and powder activities are not manipulated[a]

Model	A	B	C	D	E	F	G
Milk fat	3.30%	3.30%	3.50%	3.50%	3.70%	3.70%	3.70%
Protein	3.14%	3.25%	3.20%	3.32%	3.35%	3.35%	3.35%
Standardized with		Cream				NDM	Skimmilk
Cheddar	$5.10	$5.20	$5.34	$5.44	$5.68	$5.68	$5.58
High fat Cheddar	5.50	5.60	5.64	5.74	5.98	5.98	5.98
Monterey	5.90	6.00	6.14	6.24	6.48	6.38	6.38
Swiss	5.90	6.00	6.24	6.14	6.38	6.18	6.38
Aged Cheddar	5.20	5.40	5.54	5.64	5.98	5.88	5.88

[a] Base price = $4.74/cwt., 3.5% milk fat, with a $.07/point differential.

In comparing milks of different composition, the milk with the greater amount of protein is the most valuable, except for the price paid for Swiss cheese milk when comparing models C and D. This phenomenon appears in other results and will be discussed in greater detail in the appropriate sections. One observation that may be made in connection with this phenomenon, or paradox, is that the butter and powder activities seem to depend heavily upon the Swiss cheese activity.

Model E, when comparing models E, F, and G, is consistently the milk composition of highest value. (Although for some models, the milk and cheese prices are the same, it should be remembered that the prices are being increased in $0.10 increments. If the price were to be increased in smaller increments, the number of differences would probably be greater.) Since high fat Cheddar cheese is standardized with cream rather than NDM or skimmilk, the maximum price that can be paid to the farmer is the same for models E, F, and G.

The milk of highest value is the composition of model E, when allocated to Monterey cheese.

Minimum acceptable wholesale cheese prices in order to cover variable costs

In this test the linear program routine is again manipulated to determine the minimum price a plant would be willing to receive and still produce some quantity of a given product. The results of this analysis are shown in Table 5. Initially the wholesale prices are set well below the minimum price the plant would be willing to receive and then increased in $0.05 increments until milk is allocated to the production of that cheese variety (i.e., until it becomes profitable to

Table 5. The minimum wholesale price per pound a firm would be willing to receive for its cheese[a]

Model	A	B	C	D	E	F	G
Milk fat	3.30%	3.30%	3.50%	3.50%	3.70%	3.70%	3.70%
Protein	3.14%	3.25%	3.20%	3.32%	3.35%	3.35%	3.35%
Standardized with		Cream				NDM	Skimmilk
Cheddar	$.54	$.49	$.54	$.49	$.49	$.49	$.49
High fat Cheddar	.55	.50	.55	.50	.50	.50	.50
Monterey	.50	.55	.50	.50	.50	.45	.50
Grades A and B Swiss[b]	.50	.50	.45	.45	.45	.50	.45
Grade C Swiss[c]	.42	.42	.37	.37	.37	.42	.37
Aged Cheddar	.59	.59	.59	.59	.59	.59	.59

[a] Selling prices increased in $.05 increments.

[b] Grades A and B Swiss cheese reach maximum amount of production at $.60 per pound.

[c] Grade C Swiss cheese reaches maximum amount of production at $.52 per pound.

produce that product). Here again, a more accurate minimum wholesale price could be determined if the price increments were smaller.

When the price of grades A and B Swiss cheese reach $0.50 per pound and grade C reaches $0.42 per pound, Swiss cheese production begins. But the maximum amount of production of Swiss does not occur until the wholesale prices are $0.60 and $0.52 for grades A and B, and grade C respectively. The other four cheese varieties reach their maximum and minimum production levels at the prices indicated in Table 5.

This is another indication that the butter and powder production activities are somehow tied to the different cheese activities. Another possible explanation of this interaction is that different varieties of cheese are dependent upon each other. For example, high fat Cheddar and

Swiss cheese would seem to be a good combination to produce, since high fat Cheddar requires cream and Swiss cheese requires skimmilk. If, however, only Swiss cheese were produced, the plant would have excess cream, and excess skimmilk, if only high fat Cheddar were produced. Therefore, when excess cream or skimmilk exist in the plant and/or the alternative uses for cream and skimmilk exhibit low profit margins, the average cost of producing Swiss and high fat Cheddar will increase. Therefore the minimum wholesale price and, consequently, the number of vats of Swiss cheese to be produced, is dependent upon the alternative uses for the excess cream required for Swiss production and the alternative uses for its whey.

The fact that one minimum wholesale price "calls" for some production of Swiss cheese while another minimum wholesale price brings in the maximum amount of Swiss cheese the plant is capable of producing, is a good indication that different activities are interdependent. The minimum wholesale price for Monterey cheese in model B is higher than in model A. This is another indication of this same phenomenon.

Optimal product mix for milk of each composition, considered individually

The purpose of this analysis is to determine the profits and optimum product mix that would result if a plant receives milk of only one composition on a given day. In each one of the six examples, Tables 6 through 11, some market variable or plant variable is altered according to each table title. Each composition of milk (as defined by models A-G) is considered independently of all others.

When interpreting Tables 6-11, net profit must be evaluated in the light of the amount of milk the plant is able to utilize before it is

D	E	F	G
3.50%	3.70%	3.70%	3.70%
3.32%	3.35%	3.35%	3.35%
		NDM	Skimmilk
$13,175.57	$13,670.28	$13,767.91	$13,140.80
1,144,068.6	1,130,616.2	1,184,895.9	1,104,461.1
6.08	6.58	6.55	6.59
18,491.7	20,874.3	21,121.4	20,288.2
0	0	0	0
4.57	4.57	4.57	4.57
10,621.7	10,750.6	10,750.6	10,750.6
0	0	0	0
10.0	10.0	10.0	10.0
20,754.0	21,361.0	21,775.0	20,877.0
$104.01	$104.40	$101.97	$94.47
16.0	16.0	16.0	16.0
25,614.4	25,844.8	30,352.0	25,844.8
4,520.0	4,560.0	5,356.8	4,560.0
0	0	0	0
6.0	6.0	6.0	6.0
17,688.0	18,430.2	18,754.8	17,886.6
$58.69	$68.78	$73.36	$61.36
3.82	3.85	3.82	3.86
3,855.7	3,883.4	3,855.7	3,891.5
0	0	0	0
43,379.9	47,025.2	46,564.1	50,483.2
0	0	0	0
4.18	4.15	4.18	4.14
6,866.7	6,821.4	6,866.7	6,808.3
0	0	0	0
0	0	9,095.0	0
-$0.118	-$0.120	-$0.120	-$0.120
22,852.7	17,840.3	11,101.5	13,085.6
0	0	0	0

D	E	F	G
3.50%	3.70%	3.70%	3.70%
3.32%	3.35%	3.35%	3.35%
		NDM	Skimmilk
$12,255.56	$12,700.55	$12,449.59	$12,438.71
1,000,000	1,000,000	1,000,000	1,000,000
.47	1.55	0	2.34
1,433.1	4,915.4	0	7,209.8
0	0	-$0.023	0
4.57	4.57	4.57	4.57
10,621.7	10,750.6	10,750.6	10,750.6
0	0	0	0
10.0	10.0	10.0	10.0
20,754.0	21,361.0	21,775.0	20,877.0
$102.11	$102.31	$89.48	$97.41
16.0	16.0	16.0	16.0
25,614.4	25,844.8	30,352.0	25,844.8
4,520.0	4,560.0	5,356.8	4,560.0
0	0	0	0
6.0	6.0	5.4	6.0
17,688.0	18,430.2	16,842.7	17,886.6
$58.73	$68.91	0	$61.40
3.19	3.28	3.01	3.38
3,215.0	3,310.9	3,039.2	3,406.4
0	0	0	0
36,174.9	40,464.7	38,033.0	44,048.6
0	0	0	0
4.81	4.72	4.99	4.62
7,911.2	7,754.8	8,197.6	7,599.1
0	0	0	0
0	0	8,202.5	0
-$498.62	-$0.119	-$0.119	-$507.13
26,790.1	21,168.8	17,654.9	16,944.8
0	0	0	0

D	E	F	G
3.50%	3.70%	3.70%	3.70%
3.32%	3.35%	3.35%	3.35%
		NDM	Skimmilk
$14,009.08	$14,578.83	$14,617.04	$13,895.76
1,313,711.8	1,286,487.2	1,340,074.2	1,252,989.2
12.0	12.0	12.0	12.0
36,516.0	38,049.6	38,719.2	36,927.6
0	0	0	0
4.57	4.57	4.57	4.57
10,621.7	10,750.6	10,750.6	10,750.6
0	0	0	0
10.0	10.0	10.0	10.0
20,754.0	21,361.0	21,775.0	20,877.0
$113.62	$122.23	$118.00	$108.36
16.0	16.0	16.0	16.0
25,614.4	25,844.8	30,352.0	25,844.8
4,520.0	4,560.0	5,356.8	4,560.0
0	0	0	0
6.0	6.0	6.0	6.0
17,688.0	18,430.2	18,754.8	17,886.6
$58.50	$68.66	$73.15	$61.16
4.04	4.04	4.04	4.04
4,069.7	4,069.7	4,069.7	4,069.7
0	0	0	0
44,336.5	47,812.9	48,090.2	52,198.1
0	0	0	0
3.96	3.96	3.96	3.96
6,517.7	6,517.7	6,517.7	6,517.7
0	0	0	0
0	0	9,817.5	0
-$0.118	-$0.120	-$0.120	-$0.120
21,537.3	16,757.3	8,442.9	10,727.7
0	0	0	0

D	E	F	G
3.50%	3.70%	3.70%	3.70%
3.32%	3.35%	3.35%	3.35%
		NDM	Skimmilk
$14,312.42	$14,882.03	$14,981.98	$14,232.76
1,147,200.0	1,133,404.8	1,187,148.7	1,101,519.4
0	0	0	0
0	0	0	0
-$0.019	-$0.022	-$0.023	-$0.020
0	0	0	0
0	0	0	0
-$0.073	-$0.071	-$0.068	-$0.064
18.0	18.0	18.0	18.0
37,357.2	38,449.8	39,195.0	37,578.6
0	0	0	0
16.0	16.0	16.0	16.0
25,614.4	25,844.8	30,352.0	25,844.8
4,520.0	4,560.0	5,356.8	4,560.0
0	0	0	0
11.2	11.5	11.4	11.5
32,996.1	35,225.2	35,724.4	34,219.4
0	0	0	0
3.99	3.99	3.96	4.0
4,022.1	4,022.1	3,990.0	4,032.1
0	0	0	0
48,374.9	51,195.4	51,197.3	55,409.1
0	0	0	0
4.01	4.01	4.04	4.0
6,595.4	6,594.4	6,647.7	6,579.1
0	0	0	0
0	0	9,647.5	0
-$0.118	-$0.120	-$0.120	-$0.120
15,984.5	12,106.3	4,297.6	6,312.4
0	0	0	0

D	E	F	G
3.50%	3.70%	3.70%	3.70%
3.32%	3.35%	3.35%	3.35%
		NDM	Skimmilk
$13,040.84	$13,541.37	$13,165.45	$13,300.63
1,000,000	1,000,000	1,000,000	1,000,000
0	0	0	0
0	0	0	0
-$0.019	-$0.022	-$0.023	-$0.020
0	0	0	0
0	0	0	0
-$0.071	-$0.068	-$0.067	-$0.067
18.0	18.0	18.0	18.0
37,357.2	38,449.8	39,195.0	37,578.6
0	0	0	0
16.0	16.0	16.0	16.0
25,614.4	25,844.8	30,352.0	25,844.8
4,520.0	4,560.0	5,356.8	4,560.0
0	0	0	0
5.46	6.32	4.18	7.35
16,101.0	19,425.9	13,065.9	21,899.4
0	0	0	0
3.34	3.41	3.14	3.53
3,364.8	3,435.4	3,164.0	3,558.6
0	0	0	0
41,867.6	45,209.1	43,309.0	49,831.2
0	0	0	0
4.66	4.59	4.86	4.47
7,667.0	7,551.8	7,994.2	7,350.9
0	0	0	0
0	0	8,712.5	0
-$0.117	-$0.119	-$0.119	-$0.119
20,024.1	15,520.3	10,950.9	10,075.0
0	0	0	0

D	E	F	G
3.50%	3.70%	3.70%	3.70%
3.32%	3.35%	3.35%	3.35%
		NDM	Skimmilk
$15,620.06	$16,280.19	$16,337.78	$15,398.85
1,349,626.9	1,327,700.5	1,380,609.9	1,286,959.3
0	0	0	0
0	0	0	0
-$0.019	-$0.022	-$0.023	-$0.020
0	0	0	0
0	0	0	0
-$0.086	-$0.074	-$0.071	-$0.064
18.0	18.0	18.0	18.0
37,357.2	38,449.8	39,195.0	37,578.6
0	0	0	0
16.0	16.0	16.0	16.0
25,614.4	25,844.8	30,352.0	25,844.8
4,520.0	4,560.0	5,356.8	4,560.0
0	0	0	0
18.0	18.0	18.0	18.0
53,064.0	55,290.6	56,264.4	53,659.8
0	0	0	0
4.04	4.04	4.04	4.04
4,069.7	4,069.7	4,069.7	4,069.7
0	0	0	0
48,587.7	33,257.9	34,010.1	39,071.0
0	0	0	0
3.96	3.96	3.96	3.96
6,517.7	6,517.7	6,517.7	6,517.7
0	0	0	0
0	0	10,497.5	0
-$0.118	-$0.120	-$0.120	-$0.120
15,691.9	11,832.8	2,155.7	4,499.4
0	0	0	0

constrained by the capacity of some piece of equipment or vat. In addition it should be borne in mind that restraints are placed on vat utilization so that only certain varieties of cheese can be made in each.

Perhaps as important to management as the optimum product mix for a given composition of milk, is the marginal value product (the amount the plant's profits will be increased or decreased if one more unit of a particular activity is produced). The linear program routine provides, or computes, an MVP whenever an activity is "bound" (i.e., the firm can only sell, and therefore, will only produce a certain amount of that product), or no production occurs from that activity. Since there is a selling activity that corresponds with each production activity, the routine has the option to yield an MVP for the selling activity on a per pound basis one time, and an MVP per unit of activity another time. Therefore, it should be noted in Tables 6-11, that when the MVP is less than $1.00, it has reference to the wholesale price of a product on a per pound basis; when the value is greater than $1.00, it has reference to the production of one entire activity (i.e., one vat of cheese, one churn of butter, or one hour of drying time). These values are the opportunity costs, or the values associated with the alternative use for milk.

The most profitable model in all comparisons is model F (Table 11). In this case milk is standardized with NDM, the plant is not restricted in the amount of each product it can sell, more pasteurizer time is added to the model, and the plant can purchase all the milk it desires. Whenever the plant is at full capacity with respect to milk volume, model F is always the most profitable. However, if the plant is not at full capacity, model F is considerably less profitable than other models. This is understandable because there is a high cost involved in converting milk into NDM. Therefore, when the plant is not at full capacity, standardizing with NDM cannot increase total yield and the costs per unit will increase. When the plant is at full capacity, however, total product yield will increase by standardizing with NDM, and although the cost per pound of product must increase, the net profits and volume of milk utilized by the plant will be correspondingly higher. Skimmilk standardization becomes more profitable than the NDM standardization only when the set of conditions exist which are imposed in Table 10.

One other point must be made with respect to the 30 hours of pasteurizer time per day available to the plant in Tables 8 and 11. The initial computer results indicated that the models were being restrained (the maximum volume of milk the plant could utilize) by the capacity of the HTST pasteurizer. Since it is the purpose of this study to analyze the plant with respect to vat capacity, additional pasteurizer time (six hours) was added. This gives some indication of resulting plant conditions if a pasteurizer with greater capacity, or a second pasteurizer were to be installed by the firm. If more accurate results are desired, a new model, reflecting the changes in pasteurizer conditions, must be developed.

However, simply assuming increased pasteurizer availability permits management to make certain decisions or plans. For example, the conditions imposed on the models to obtain the results in Tables 6 and 8 are identical, except in the amount of pasteurizer time available. The profits for each of the seven compositions of milk in Table 8 have increased from \$498.64 to \$928.60, over the seven corresponding compositions of milk in Table 6. Therefore, if the firm had only milk of one composition entering the plant on a given day (the milk being any one of the milks defined by models A-G), the firm could afford to pay an estimated $498.64 to $928.60 a day to lease or buy a larger HTST pasteurizer, given the market and plant conditions assumed in Tables 6 and 8.

A similar comparison can be made between Tables 9 and 11. When the market and plant conditions of these tables are imposed on each model, the increase in profits ranges from $729.06 to $1,398.16, depending on which model or milk composition is being considered. Therefore, management can use estimated profit figures of this sort to decide whether or not to install a new pasteurizer, given that the average milk composition entering the plant approximates one of the milk compositions designated in this analysis.

As the results indicate, the milk with greater percentages of protein (when milk fat percentages are the same) is consistently more profitable than the milk with lower percentages of protein.

Optimum product mix for simultaneous milk receipts of different compositions

Instead of imagining milk of only one composition entering the plant during a given time period, the realistic situation is "simultaneous" receipts of milk of differing compositions. Therefore, management must simultaneously allocate each milk to the product, or products, which yield the greatest profit for the plant. Tables 12 and 13 contain the results of this analysis. In Table 12, 200,000 pounds of each milk, as defined by models A through E, or a total of 1,000,000 pounds

Table 12. 200,000 pounds of each milk, as defined by models A through E, considered simultaneously, making a total of 1,000,000 pounds of milk to be used by the plant. The plant is only able to sell 10 vats of Monterey and 6 vats of aged Cheddar cheese. Net profit is $12,251.92.

Model	A	B	C	D	E	Pounds of product
Milk fat	3.30%	3.30%	3.50%	3.50%	3.70%	
Protein	3.14%	3.25%	3.20%	3.32%	3.35%	
Standardized with		Cream				
Vats of Cheddar	0	0	0	0	.11	363.3
MVP ($)	-17.83	-12.24	-39.99	-14.29	0	
Vats of high fat Cheddar	0	2.63	0	1.94	0	10,476.8
MVP ($)	-6.56	0	-28.63	0	-.21	
Vats of Monterey	1.81	0	0	7.36	.83	20,602.9
MVP ($)	0	-.23	-20.52	0	0	
Vats of Swiss	3.63	2.97	9.40	0	0	29,193.4
MVP ($)	0	0	0	-3.95	-8.75	
Vats of aged Cheddar	0	0	0	0	6.0	18,430.2
MVP ($)	-43.40	-28.81	-54.33	-24.46	0	
Churns of whey butter	.44	.45	.79	.70	.75	3,157.1
MVP ($)	0	0	0	0	0	
Hours of whey powder	0	0	4.91	3.85	4.69	33,631.5
MVP ($)	-34.97	-18.69	0	0	0	
Churns of butter	2.37	1.70	.80	0	0	8,005.5
MVP ($)	0	0	0	-2.87	0	
Hours of NDM	0	0	0	0	0	0
MVP ($)		-27.72	-59.57	-59.45	-91.35	
Hours of blend powder	4.0	4.03	0	.72	0	30,092.5
MVP ($)	0	0	-.05	0	-9.41	

Table 13. 250,000 pounds of each milk, as defined by models A through E, considered simultaneously, making a total of 1,250,000 pounds of milk to be used by the plant. The plant is only able to sell 10 vats of Monterey and 6 vats of aged Cheddar cheese. Net profit is $13,664.62.

Model	A	B	C	D	E	Pounds of product
Milk fat	3.30%	3.30%	3.50%	3.50%	3.70%	
Protein	3.14%	3.25%	3.20%	3.32%	3.35%	
Standardized with		Cream				
Vats of Cheddar	0	3.88	0	3.38	2.33	29,180.8
MVP ($)	-3.64	0	-21.77	0	0	
Vats of high fat Cheddar	0	4.57	0	0	0	10,369.8
MVP ($)	-5.94	0	-27.84	-6.42	-10.44	
Vats of Monterey	2.57	0	0	7.43	0	20,467.4
MVP ($)	0	-1.57	-17.91	0	-9.59	
Vats of Swiss	4.25	0	11.75	0	0	29,121.1
MVP ($)	0	-2.16	0	-2.15	-24.75	
Vats of aged Cheddar	0	0	0	0	6.0	18,430.2
MVP ($)	-29.22	-16.60	-36.09	-10.16	0	
Churns of whey butter	.55	.61	.99	.94	.94	4,069.7
MVP ($)	0	0	0	0	0	
Hours of whey powder	0	0	5.06	5.87	5.86	41,976.9
MVP ($)	-19.73	0	0	0	0	
Churns of butter	2.94	.02	1.0	0	0	6,517.7
MVP ($)	0	0	0	-43.74	0	
Hours of NDM	0	0	0	0	0	0
MVP ($)		-33.59	-82.68	-33.55	-157.95	
Hours of blend powder	5.0	2.21	0	0	0	24,781.8
MVP ($)	0	0	-19.87	0	-46.63	

of milk, are simultaneously allocated to the various products. The daily profit is $12,251.92.

Since the market and plant conditions assumed in Table 7 are identical to those in Table 12, a comparison can be made. For example, suppose the plant receives, in rotation over a five day period, 1,000,000 pounds of each milk composition (A-E). The sum of the five day net profits of models A through E from Table 7 equals $58,925.91. When $12,251.92 (Table 12) is multiplied by 5, the net five day profit equals $61,259.60. The difference between the two is $2,333.69 in favor of the policy represented by Table 12. This is a 4% increase in net profits for the five day period. Therefore, the plant is better off receiving milk of different compositions each day and simultaneously allocating each composition to its most profitable products.

When the volume of milk is increased to 1,250,000 pounds (Table 13) certain changes occur in the allocation patterns of milk. All the high fat Cheddar is produced with milk from composition B rather than B and D, as occurs in Table 12. The milk for Monterey cheese is slightly varied from Table 12. The milk from composition E is transferred to composition A. All 16 vats of Swiss cheese are produced with milk from models A and C, whereas in Table 12, milk from models A, B, and C is used. The milk for aged Cheddar comes from model E in both Tables 12 and 13. The extra 250,000 pounds of milk used in Table 13 is allocated to the production of Cheddar cheese.

Comparing the pounds of product produced reveals that the only variation in cheese production is due to the differences in yield of each milk composition. The pounds of Cheddar cheese increase because that is where the extra 250,000 pounds of milk is allocated in Table 13.

Other changes are: output of whey butter and whey powder increase (the volume of whey increases due to increased production of Cheddar cheese[5]) and butter and blend powder decrease, transferring the milk required for their production to Cheddar cheese.

Numerous other slight alterations of this basic linear program routine might make the results more realistic or adaptable to any existing plant situation. For example, managements can predict fairly accuratly the milk composition of each producer and, consequently, the proportion of each milk composition that will enter the plant during any given day or week. Therefore, they can designate in advance, which cheese should be made from each tank of milk.

Since dye-binding has been approved for determining the percent protein in milk, it is even more realistic to test for protein just as a plant tests for milk fat at the beginning of each day and then allocate the milk to the production activities of that day on the basis of the milk fat and protein tests.

Relative value of milk for different quantities used by the plant

The final area that has interest for management is a comparison of the MVP of milk, for given quantities rather than qualities. While this is not a study of milk pricing, interesting aspects of relative milk values are shown in Tables 14 and 15. Table 14 assumes that all quantities of each variety the plant can produce can be sold. Some market

---

[5] Since the butter churn is being used at full capacity in Table 12, and the whey cream available for whey cream butter production is increased in Table 13, and there is an alternative use available for the milk (Cheddar cheese) from which the sweet cream comes, the transfer of milk from butter and blend powder production to Cheddar cheese production occurs.

Table 14. The MVP[a] per pound of milk for each of the seven models. It is assumed that the firm can sell any and all the varieties of cheese it desires.

Model	A	B	C	D	E	F	G
Milk fat	3.30%	3.30%	3.50%	3.50%	3.70%	3.70%	3.70%
Protein	3.14%	3.25%	3.20%	3.32%	3.35%	3.35%	3.35%
Standardized with		Cream				NDM	Skimmilk
Lbs. milk	$	$	$	$	$	$	$
100,000	.05874	.05959	.06159	.06207	.06388	.06355	.06368
200,000	.05874	.05959	.06159	.06207	.06388	.06355	.06368
300,000	.05874	.05959	.06159	.06207	.06388	.06355	.06368
400,000	.05805	.05929	.06097	.06125	.06304	.06176	.06304
500,000	.05805	.05903	.06097	.06125	.06304	.06176	.06304
600,000	.05805	.05903	.06097	.06125	.06304	.06176	.06304
700,000	.05805	.05903	.06097	.06125	.06304	.06176	.06304
800,000	.05699	.05715	.05850	.05850	.06017	.06176	.06017
900,000	.05182	.05363	.05500	.05625	.05905	.05875	.05818
1,000,000	.05182	.05363	.05500	.05625	.05905	.05875	.05818
1,100,000	.02449	.02110	.00640	.01906	.01609	.01732	.00872
1,200,000	.00326	.00558	.00640	.00756	.01028	.00961	.00872
1,300,000	0	.00558	0	.00353	0	.00404	0
1,400,000	0	0	0	0	0	0	0

---

[a] The MVP of milk is the value of the last pound of milk used in production. This is, in part, a function of the price per pound of milk the plant pays the farm producer.

Table 15. The MVP[a] per pound of milk for each of the seven models. It is assumed that the firm can sell only 10 vats of Monterey cheese and 6 vats of aged Cheddar cheese for each model.

Model	A	B	C	D	E	F	G
Milk fat	3.30%	3.30%	3.50%	3.50%	3.70%	3.70%	3.70%
Protein	3.14%	3.25%	3.20%	3.32%	3.35%	3.35%	3.35%
Standardized with		Cream				NDM	Skimmilk
Lbs. milk	$	$	$	$	$	$	$
100,000	.05874	.05959	.06159	.06207	.06388	.06355	.06368
200,000	.05805	.05959	.06159	.06207	.06388	.06355	.06368
300,000	.05805	.05903	.06159	.06125	.06304	.06176	.06304
400,000	.05805	.05903	.06097	.06125	.06304	.06176	.06304
500,000	.05805	.05903	.06097	.06125	.06304	.06176	.06304
600,000	.05699	.05715	.05850	.05850	.06017	.06176	.06017
700,000	.05699	.05715	.05850	.05850	.05919	.06017	.05919
800,000	.05182	.05523	.05616	.05734	.05919	.05919	.05818
900,000	.05012	.05363	.05500	.05625	.05905	.05875	.05818
1,000,000	.05012	.05363	.05288	.05396	.05639	.05875	.05568
1,100,000	.04977	.05132	.05253	.05362	.05605	.05556	.05533
1,200,000	.04038	.05132	.04090	.04090	.04158	.05556	.04158
1,300,000	.04038	.04055	.04090	.04090	.04158	.04158	.03159
1,400,000	.02906	.04042	.02891	.02891	.02891	.02903	.02891

---

[a] The MVP of milk is the value of the last pound of milk used in production. This is, in part, a function of the price per pound of milk the plant pays the farm producer.

constraints are introduced in Table 15. The pounds of milk are increased in 100,000 pound increments and an MVP (the value of the last pound of milk) is given. It stands to reason that MVP for the 100,000th pound of milk is greater than that for the 1,600,000th pound because it goes into a much more profitable product.

Attention should be directed to models C and D where the milk fat percentages are the same but the protein percentage varies for the two models. When the volume of milk is 800,000 pounds in Table 14 and 600,000 and 700,000 pounds in Table 15, the relative milk values are the same for both models. This occurs because under the plant conditions imposed for this study, the optimal allocation of milk is to blend powder and sweet cream butter. The relative advantage the high protein milk has in cheese production no longer exists and therefore, the values for the two milks are the same. The next 100,000 pounds of milk the plant uses are allocated to cheese production, and once again, the high protein milk has a higher value. This is significant because most cheese plants are involved in butter and powder processes. This further verifies that the phenomenon, or interaction, which has appeared in this study is really a combination of alternative uses for the milk. This condition does not occur for models A and B.

After the plant utilizes 1,000,000 pounds of milk, the relative MVP values in Table 14 decrease much more rapidly than in Table 15. The reason for this is because in Table 14 the model is forced to use all the whey. When the whey is in excess of plant capacity, yet the plant is forced to use the whey, a value for the excess whey is charged against the model. Since the value per pound of whey and the actual pounds of whey in excess are given, the model has created a basis for considering the feasibility of purchasing a larger dryer.

Given the standardization problem of models E, F, and G, it can be observed that the relative value of milk is greatest for no standardization. At lower quantities of milk it is more profitable to standardize with skimmilk than with NDM. However, at larger quantities of milk, where all vats are filled to near capacity, it becomes more profitable to standardize with NDM. This further verifies the results from Tables 6-11, where the model standardized with NDM is the most profitable model when the plant is at full capacity, but less profitable than other models when it is not at full capacity.

SUMMARY AND CONCLUSION

Linear programming has been used extensively in recent years by managers of many industries as a tool for allocating resources. Since cheese production is a function of the amount of milk fat and protein in cheese milk, cheese plants conceptually are suitable for linear programming. There are other activities which managers of cheese plants are concerned with, such as butter, powder, and whey disposal. Each of these augments the need to consider linear programming.

The key to setting up a functional linear program model is in the ability to predict output or yield coefficients. Many equations have been developed to predict cheese yield but were not employed in this study for several reasons: (a) Recently acid orange 12 was approved as a dye-binding chemical for determining protein percentage in fluid milk. This has the potential of becoming the rapid, inexpensive "protein" counterpart to the Babcock procedure in monitoring milk fat. (b) The equations of earlier researchers were limited to Cheddar cheese and, therefore, would not satisfy all the needs of the study. (c) There were no statistical test data reported by other researchers to indicate how well their equations would predict yields.

Operators of Cache Valley Dairy, Hi-Land Dairy's Richmond, Utah plant, and Utah State University's Dairy Products Laboratory were very cooperative in making their plants available for study purposes. From each of these plants, samples of milk, and cheese from that milk, were obtained for milk fat and protein analysis. Yield equations were derived by correlating the total pounds of milk fat and protein with total pounds of cheese solids. Cost data were also obtained from these plants but were altered slightly for the purpose of disguising the cost structure of these plants. Wholesale prices employed were the wholesale prices per pound of product received by the plants during the period under study.

Five varieties of cheese (Cheddar, aged Cheddar, Monterey, Swiss, and high fat Cheddar) were used as activities in the linear program matrix. Other activities employed, or products produced by the plant, were sweet cream butter, whey cream butter, NDM, whey powder, and blend powder (contains 50% skimmilk solids and 50% whey skim solids). Since it was impossible to obtain accurate weights and measurements for the butter and powder activities, it was assumed that 100% of the solids were recovered in the powder activities, 99% of the milk fat in sweet cream was recovered in butter, and 97.2% of the milk fat in whey cream was recovered in whey butter.

There are different amounts of milk fat and protein recovered in the whey from each variety of cheese. However, this study monitored only the whey from Cheddar cheese as it flowed from whey to whey butter, whey powder, and blend powder. Cheddar cheese whey was used because the emphasis on collecting data was on Cheddar, and because the amount of milk fat and protein in the different wheys is relatively constant.

Seven models were developed for this study. Each of the models were identical except for the milk fat and protein percentages of the input milk, or that identical milks were standardized by different means. Each of the seven models, therefore, had different yield coefficients.

Five areas were examined as means of testing the effects of linear programming as an aid to managers for making decisions in a cheese plant.

The five areas (in terms of milk composition), were as follows:

(a) The maximum price management could pay for milk to produce a given cheese.

(b) The minimum wholesale price for which management could sell a pound of cheese and still cover variable costs.

(c) The optimum product mix under various market and plant conditions when each composition of milk was considered individually.

(d) The proper allocation of different compositions of milk simultaneously entering the plant.

(e) The relative value of milk for different quantities used by the plant.

It was found that the hypothetical plant was better off having small amounts of different compositions of milk entering the plant simultaneously, while allocating each milk to the activity it is most suited to, than to have large quantities of each milk composition enter the plant on separate days so that each milk must be individually dominate to each day's production. In fact, with the facilities and conditions imposed on this study, a net income of $2,333.69 could be realized from each 5 million pounds of milk allocated simultaneously per day rather than in rotation by days.

Tables 6 through 11 illustrate that at different quantities of milk and levels of plant capacity, different production policies should be followed. When the plant was at full capacity and excess milk was still available, it was better to standardize the cheese milk with NDM, than to not standardize the milk at all, or to use skimmilk for standardization purposes. However, when the plant was not being utilized to full capacity, it was considerably more profitable to not standardize at all. These same results were repeated in Tables 14 and 15 where the relative values of the different milks were analyzed. Again, when the plant reaches near capacity levels, the relative value of the NDM standardized milk was greater than the relative values of the other milks.

At several places in the Results, it appeared that there were inconsistencies because milk with a low protein percentage was as valuable as milk with a high protein percentage when the milk fat tests were identical. Beginning with Table 4, the firm could afford to pay more for the low protein milk represented by model C than for the high protein percentage milk represented by model D. In Table 5, the minimum wholesale price per pound of Monterey cheese is less for model A than model B. Also in Table 5, Swiss cheese had one minimum wholesale price to begin production and another minimum wholesale price for maximum production. In Tables 14 and 15, there were occasions when the relative values of the milk represented in models C and D were equal.

Given the apparent inconsistent results, there may be some question whether or not there is only one cost in producing an individual product. But it has already been explained that the maximum price payable for Swiss cheese milk varies according to the alternative uses of the excess cream, or the excess skimmilk, in the case of high fat Cheddar.

When a closer examination was made of the computer results, it was observed that in each "inconsistent" case, the production of butter and blend powder was large or increasing. It is, therefore concluded, that the alternative uses for cream, skimmilk, and whey, or more specifically, the combination of alternative uses for cream, skimmilk, and whey have a very significant effect on the value of milk used primarily for the production of cheese.

The same results and conclusions drawn from this study could also be obtained by traditional accounting or budgeting approaches. However, the ability to simultaneously analyze many activities, plus the versatility available in making the analysis with linear programming, is very beneficial to managers in decision making.

This study considered only five areas that should benefit cheese plant managers in decision making. One area that holds great potential is in planning for new equipment and facilities. By creating a linear program for the hypothesized facilities, it becomes possible to determine the profitability of proposed changes and to make certain that the plant capacity is constrained in the proper areas.

Another decision area that could be beneficial to managers is to introduce a time series analysis on the volume of milk which will enter a plant over a period of time. Relative changes in market conditions could also be easily introduced.

LITERATURE CITED

1. Ashworth, U.S. 1966. Determination of protein in dairy products by dye-binding. J. Dairy Sci., 49:33.

2. Babcock, S. M. 1897. The constitution of milk, with special reference to cheese production. Bull. 61. Wisc. Agr. Exp. Sta. 21 p.

3. Beneke, Raymond R., and Ronald Winterboer. 1970. Linear programming applications to farm planning. Iowa State University. 198 p.

4. Davis, J. G. 1965. Cheese. vol. 1. American Elsivier Publishing Co. Inc., New York. 463 p.

5. French, C. E. 1964. Computers for management. Agricultural Engineering, 45(6):324-326.

6. Glickstein, A., E. M. Babb, C. E. French, and J. H. Greene. 1962. Simulation procedures for production control in an Indiana cheese plant. Bull. 757. Purdue University Agr. Exp. Sta., Lafayette, Indiana. 24 p.

7. LeBaron, A., R. Brog and R. Lamb. 1968. An analysis of protein accounting and pricing for milk and its products. Bull. 473. Utah Agr. Exp. Sta., Logan, Utah. 32 p.

8. Leftwich, R. H. 1966. The price system and resource allocation (3rd edition). Holt, Rinehart and Winston, New York. 369 p.

9. Mabbitt, L. A. and G. C. Cheeseman. 1967. The effect of concentrating milk on the fat retention property of the cheese curd. J. Dairy Res., 34(1):73-83.

10. McDowall, F. H. 1936. Cheese yielding capacity of milk. The New Zealand Journal of Science and Technology, 18(3):137-364.

11. Mojonnier Brothers Co. 1925. Instruction manual for setting up and operating Mojonnier milk tester. Bull. 101. Mojonnier Brothers Co., Chicago, Illinois. 71 p.

12. Patel, Prahlad H. 1969. Accounting for milk protein equivalents by dye-binding analysis of cheese and whey. Master thesis, Utah State University, Logan, Utah. 47 p.

13. Peters, I. I. and J. D. Williams. 1963. Manufacture of Cheddar cheese from reconstituted milk. Texas Agricultural Progress, 9:3-4.

14. Price, W. V., W. C. Witer, A. M. Swanson, and H. H. Sommer. 1953. The sampling of Cheddar cheese for routine analysis. Association of Official Agricultural Chemists, 36(2):524.

15. Sherbon, J. W. 1967. Comparison of reproducibilities of the Kjeldahl and dye-binding methods for measuring protein in milk. J. Ass. Offic. Anal. Chemists, 50:559.

16. Smythe, V. R. and N. B. Stanton. 1957. Cheese yield investigations: the sampling of cheese milk. Queensland J. Agr. Sci., 14(4):225-232.

17. Snyder, J. C. and C. E. French. 1958. Selection of product line for a fluid milk plant by activity analysis. Bull. 667. Ind. Agr. Exp. Sta. 35 p.

18. Tarassuk, N. P. 1967. The dye-binding of milk proteins. Tech. Bull. 1369. Agr. Res. Ser., U.S.D.A. in Coop. with Calif. Agr. Exp. Sta. Government Printing Office, Washington, D.C. 44 p.

19. Tarassuk, N. P. and N. Abe. 1963. Factors affecting the dye-binding capacity of milk protein. J. Dairy Sci., 46:612.

20. Udy Analyzer Co. Protein analyzer, Model S, operating instructions. Boulder, Colorado. 12 p.

21. Udy, D. C. 1956. A rapid method for estimating total protein in milk. Nature, 178:314.

22. Udy, D. C. 1965. A rapid method for estimating total protein in milk-addendum. Johnson Publishing Company, Boulder, Colorado. 2 p.

23. Van Slyke, L. L. and W. V. Price. 1949. Cheese. Orange Judd, New York. 522 p.

APPENDIX

Table 16. Cheddar cheese data and regression equations

Pounds milk	% milk fat	Pounds milk fat	% protein	Pounds protein	Pounds cheese	% milk fat	Pounds milk fat	% protein	Pounds protein	% moisture	Pounds[a] solids	$\frac{\text{Protein}}{\text{milk fat}}$	F.D.M.
30,530	3.30	1007.5	3.24	989.2	2927.5	31.40	919.1	25.62	751.0	38.5	1800.4	.983	.510
30,461	3.30	1005.2	3.15	959.5	2954.0	31.27	923.7	24.70	729.6	37.6	1843.9	.955	.500
29,775	3.30	982.6	3.24	964.7	2876.0	32.00	920.3	25.15	723.3	36.0	1840.6	.983	.500
30,392	3.30	1002.9	3.27	993.8	2825.0	31.55	891.3	25.70	726.0	37.5	1767.0	.963	.505
30,014	3.40	1020.5	3.10	930.4	2896.0	32.62	944.7	24.25	702.3	36.7	1834.0	.912	.515
30,048	3.40	1021.6	3.20	961.5	2931.0	32.19	943.5	24.65	722.5	38.0	1818.4	.942	.518
29,707	3.40	1010.0	3.11	923.9	2837.0	32.32	916.9	23.90	678.0	37.0	1787.6	.915	.514
30,254	3.40	1028.6	3.16	956.0	2855.0	31.35	895.0	25.35	723.7	38.0	1769.0	.928	.506
29,808	3.50	1043.3	3.13	933.0	2926.0	32.48	950.4	24.35	712.5	37.0	1842.2	.895	.517
30,667	3.40	1042.7	3.13	959.9	2880.0	32.37	932.3	24.35	701.3	37.0	1814.4	.922	.514
29,775	3.40	1012.4	3.23	961.7	2867.0	32.34	927.2	24.95	715.3	36.9	1809.6	.951	.513
30,117	3.50	1054.1	3.18	957.7	2964.0	32.88	974.6	25.15	745.4	36.7	1877.7	.910	.518
30,117	3.50	1054.1	3.27	984.8	3026.0	31.66	958.0	25.50	771.6	37.4	1894.0	.935	.505
30,383	3.40	1033.0	3.21	975.3	2872.0	32.19	924.5	26.15	751.0	36.3	1830.6	.945	.506
29,707	3.30	980.3	3.17	941.7	2791.0	31.84	888.6	25.10	700.5	37.7	1739.9	.960	.510
30,048	3.35	1006.6	3.11	934.5	2785.0	32.26	898.4	25.30	704.6	37.4	1743.7	.928	.515
29,300	3.45	1010.8	3.13	917.1	2870.0	32.00	918.4	24.95	716.1	37.8	1786.3	.906	.513
29,571	3.35	990.6	3.08	910.8	2746.0	31.99	878.4	25.40	697.5	36.3	1750.6	.918	.502
29,843	3.30	984.8	3.06	913.2	2776.0	31.85	884.2	25.15	698.2	37.3	1740.8	.927	.508
29,911	3.45	1031.9	3.17	948.2	2843.0	32.78	931.9	25.50	725.0	36.8	1797.1	.918	.518
29,605	3.30	977.0	3.13	926.6	2682.0	31.65	848.8	25.70	689.3	37.9	1664.7	.949	.510
30,392	3.35	1018.1	3.13	951.3	2932.0	31.50	923.6	25.35	743.3	38.0	1819.0	.934	.510
29,436	3.40	1000.8	3.11	915.5	2776.0	31.91	885.8	25.60	710.7	37.1	1745.8	.914	.507
29,707	3.35	995.2	3.05	906.1	2789.0	32.08	894.7	25.40	708.4	36.8	1762.4	.910	.507
29,503	3.40	1003.1	3.14	926.4	2784.0	31.50	877.0	25.70	715.5	36.8	1758.4	.924	.500
29,673	3.30	979.2	3.05	905.0	2778.0	31.23	867.6	25.15	698.7	37.7	1731.5	.924	.501
29,469	3.30	972.5	3.04	895.9	2755.0	31.79	875.8	25.10	691.5	38.0	1709.5	.922	.513
31,200	3.29	1026.5	3.32	1035.8	3105.0	30.60	950.0	24.14	749.4	39.7	1872.9	1.010	.507
30,800	3.29	1013.3	3.32	1022.6	3231.0	29.35	948.1	24.02	776.2	41.1	1902.4	1.010	.498
30,200	3.29	993.6	3.32	1002.6	3102.0	28.74	891.2	23.9	741.5	41.3	1820.2	1.010	.490
29,900	3.44	1028.6	3.31	989.7	2957.0	32.34	955.9	25.00	739.5	38.7	1811.5	.960	.528
30,700	3.31	1016.2	3.34	1025.4	3141.0	30.85	969.0	24.10	756.8	39.9	1888.4	1.010	.513

Table 16. Continued

Pounds milk	% milk fat	Pounds milk fat	% protein	Pounds protein	Pounds cheese	% milk fat	Pounds milk fat	% protein	Pounds protein	% moisture	Pounds[a] solids	$\frac{\text{Protein}}{\text{milk fat}}$	F.D.M.
30,100	3.31	996.3	3.34	1005.3	3105.0	30.33	941.7	24.62	764.4	40.1	1859.0	1.010	.507
30,400	3.31	1006.2	3.34	1015.4	3161.0	30.20	954.6	24.35	769.6	41.1	1862.8	1.010	.512
29,800	3.54	1054.9	3.35	998.3	3098.0	31.95	990.0	23.83	738.1	39.7	1869.6	.950	.529
30,100	3.54	1065.5	3.35	1008.4	3130.0	32.14	1005.9	23.36	731.2	39.3	1899.3	.950	.530
31,000	3.48	1078.8	3.35	1038.5	3301.0	31.78	1048.6	23.76	784.3	39.6	1992.8	.960	.526
30,800	3.14	967.1	3.35	1031.8	2998.0	29.66	889.3	25.22	756.3	39.3	1820.4	1.070	.488
31,100	3.14	976.5	3.35	1041.8	3029.0	29.54	894.4	24.84	752.4	39.9	1821.0	1.070	.491
30,600	3.14	960.8	3.35	1025.1	3082.0	28.58	880.8	24.88	766.7	41.2	1813.4	1.070	.486
30,800	3.60	1108.8	3.30	1016.4	3161.0	32.65	1032.1	24.92	787.7	37.2	1985.4	.917	.520
30,400	3.71	1127.8	3.28	997.1	3160.0	33.50	1058.6	25.50	805.8	36.8	1997.1	.884	.530
30,800	3.79	1167.3	3.33	1025.6	3196.3	33.20	1061.2	25.25	807.0	36.8	2020.7	.879	.525
30,200	3.89	1174.8	3.32	1002.6	3137.8	32.78	1028.6	25.30	793.8	38.5	1929.7	.850	.533
30,400	3.83	1164.3	3.33	1012.3	3196.8	32.40	1035.7	25.46	813.9	36.7	2022.9	.869	.512
30,300	3.72	1127.2	3.33	1009.0	3161.7	33.14	1047.8	26.25	829.9	36.2	2018.7	.895	.519
29,700	3.80	1128.6	3.32	986.0	3157.0	33.47	1056.6	26.42	834.1	36.2	1015.7	.874	.524
30,300	3.79	1110.5	3.33	975.7	3168.5	33.40	1058.3	26.65	845.0	35.8	2033.2	.879	.521
30,300	3.61	1093.8	3.30	1000.0	3135.8	33.32	1044.8	27.04	847.9	35.8	2014.1	.914	.519

Total pounds cheese solids = -240.635 + 1.184(milk fat) + 0.888(protein)

$R^2 = 0.849$
df = (2,46)
$F = 129.07^*$
$S_E = 37.59$ pounds

Total pounds milk fat in cheese = -100.62 + 1.0120(milk fat)

$R^2 = 0.835$

Total pounds protein in cheese = 45.36 + 0.7193(protein)

$R^2 = 0.484$

---

[a] Total pounds cheese solids
*Significant at .01 level of significance

Table 17. High fat Cheddar cheese data and regression equations

Pounds milk	% milk fat	Pounds milk fat	% protein	Pounds protein	Pounds cheese	% milk fat	Pounds milk fat	% protein	Pounds protein	% moisture	Pounds[a] solids	$\frac{\text{Protein}}{\text{milk fat}}$	F.D.M.
20,100	4.71	946.7	3.22	647.2	2333.0	37.25	868.6	21.43	500.0	38.0	1446.9	.684	.600
20,200	4.71	951.4	3.22	650.4	2340.0	37.24	871.5	21.05	492.7	38.1	1447.5	.684	.602
19,900	4.71	937.3	3.22	640.8	2355.0	37.20	876.0	20.93	492.9	38.6	1445.0	.684	.606
19,800	4.40	871.2	3.30	653.4	2279.0	35.61	811.7	21.50	490.0	38.7	1396.1	.750	.581
20,000	4.40	880.0	3.30	660.0	2319.0	35.60	825.8	21.02	487.3	39.6	1401.4	.750	.589
21,000	4.40	924.0	3.30	693.0	2463.0	35.43	872.8	21.69	534.3	40.0	1477.1	.750	.591
20,400	4.63	944.5	3.34	681.4	2366.0	36.43	862.2	21.56	510.1	37.2	1487.0	.721	.580
20,800	4.63	963.0	3.34	694.7	2402.0	36.67	880.8	21.93	526.8	36.7	1519.7	.721	.580
20,200	4.63	935.3	3.34	674.7	2333.0	36.73	856.8	22.33	521.0	36.0	1493.0	.721	.574

Total pounds cheese solids = -108.874 + 0.870(milk fat) + 1.138(protein)

$R^2 = 0.945$
df = (2,6)
$F = 51.20^*$
$S_E = 11.24$ pounds

Total pounds milk fat in cheese = 208.94 + 0.6997(milk fat)

$R^2 = 0.869$

Total pounds protein in cheese = -3.48 + 0.7649(protein)

$R^2 = .781$

---

[a] Total pounds cheese solids
*Significant at .01 level of significance

Table 18. Monterey cheese data and equations

Pounds milk	% milk fat	Pounds milk fat	% protein	Pounds protein	Pounds cheese	% milk fat	Pounds milk fat	% protein	Pounds protein	% moisture	Pounds[a] solids	Protein / milk fat	F.D.M.
18,934	3.43	649.0	3.33	630.5	1885.0	29.0	546.6	22.46	423.4	43.1	1072.6	.971	.510
18,828	3.46	651.4	3.30	621.3	1845.0	29.5	544.3	23.60	435.4	42.9	1053.5	.954	.517
19,464	3.53	687.1	3.34	650.1	2063.0	28.0	577.6	22.28	459.6	45.4	1127.4	.946	.512
19,093	3.57	681.6	3.26	622.4	2023.0	29.0	586.7	23.21	469.5	41.4	1185.5	.913	.495
19,146	3.60	689.3	3.30	631.8	2091.0	29.5	592.8	23.05	482.0	40.6	1242.5	.917	.477
19,676	3.50	688.7	3.38	665.0	2076.0	28.5	591.7	22.30	462.9	44.5	1152.6	.966	.513
19,728	3.47	684.6	3.30	651.0	1968.0	29.1	571.8	23.99	472.2	42.2	1137.3	.951	.503
19,500	3.50	682.5	3.32	647.4	1995.0	28.8	574.8	24.32	485.1	42.8	1142.1	1.126	.503

Total pounds cheese solids = -266.730 + 4.051(milk fat) -2.087(protein)

$R^2 = 0.841$
df = (2,5)
F = 13.21*
$S_E = 28.17$ pounds

Total pounds cheese solids = 84.7%(milk fat) + 72.1%(protein)+ 0.986%(total solids other than milk fat and protein)

Total pounds milk fat in cheese = -141.4451 + 1.0561(milk fat)

$R^2 = 0.873$

Total pounds protein in cheese = 145.2710 + 0.4938(protein)

$R^2 = 0.127$

---

[a] Total pounds cheese solids
*Significant only at .05 level of significance

Table 19. Swiss cheese data and regression equations

Pounds milk	% milk fat	Pounds milk fat	% protein	Pounds protein	Pounds cheese	% milk fat
21,824	2.86	624.2	3.16	689.6	1773.5	27.5
21,771	2.91	633.5	3.19	694.5	1852.8	27.5
21,771	2.83	616.1	3.20	696.7	1802.5	27.0
21,771	2.87	624.8	3.20	696.7	1804.8	27.0
21,877	2.86	625.7	3.22	704.4	1848.0	28.0
21,771	2.87	624.8	3.19	694.5	1761.0	28.0
22,299	2.86	637.7	3.15	702.4	1870.5	27.5
22,299	2.92	651.1	3.19	711.3	1827.2	28.0
22,404	2.84	636.3	3.19	714.7	1833.6	27.5
22,352	2.81	628.1	3.10	692.9	1800.0	27.5
22,404	2.91	652.0	3.20	716.9	1844.5	28.0

Total pounds cheese solids = 124.378 + 0.323(milk fat) + 1.103(protein)

$R^2 = 0.384$
df = (2,8)
$F = 2.807^*$
$S_E = 18.24$ pounds

Total pounds milk fat in cheese = -26.1919 + 0.8369(milk fat)

$R^2 = 0.541$

Total pounds protein in cheese = -81.0670 + 0.8398(protein)

$R^2 = 0.554$

---

[a] Total pounds cheese solids
*Not significant at .05 level of significance

Pounds milk fat	% protein	Pounds protein	% moisture	Pounds[a] solids	$\frac{\text{Protein}}{\text{milk fat}}$	F.D.M.
488.0	27.44	486.6	39.8	1067.6	1.105	.457
509.5	27.43	508.2	39.8	1115.4	1.096	.457
486.7	27.75	500.2	39.4	1092.3	1.131	.446
487.3	28.30	510.8	39.6	1090.1	1.115	.447
517.4	27.90	515.6	39.2	1123.6	1.126	.460
493.1	28.15	495.7	39.4	1067.2	1.112	.462
514.4	27.87	521.3	39.6	1129.8	1.101	.455
511.6	28.12	513.8	39.4	1107.3	1.092	.462
504.2	28.02	513.8	39.6	1107.5	1.123	.455
495.0	27.92	502.6	39.4	1090.8	1.103	.454
516.5	28.10	518.3	39.6	1114.1	1.100	.464

Table 20. Labor and equipment requirements per unit of activity in hours

	Cheddar cheese	High fat Cheddar	Monterey cheese	Swiss cheese
LABOR				
Milk pasteurizer and separator	.60	.40	.40	.44
Damrow make vats	2.0			
Damrow cheddaring vats	10.0			
High fat Cheddar and Monterey vats		12.0	9.0	
Swiss vats				3.43
Wrapping the cheese	3.75	2.95	2.63	3.26
Curing the Swiss cheese				1.0
Condenser and whey separator				
Dryer				
Churn				
EQUIPMENT				
Whey separator				
Milk pasteurizer and separator	.60	.40	.40	.44
Damrow make vats	3.0			
Damrow cheddaring vats	4.0			
High fat Cheddar and Monterey vats		7.0	4.0	
Swiss vats				4.5
Condenser				
Dryer				
Churn				

Aged Cheddar	Whey separate	Whey butter	Whey powder	Milk separate	Butter	NDM	Blend powder
.60				1.0			
2.0							
10.0							
3.75							
			.89			1.12	1.06
			1.0			1.0	1.0
		3.0			3.0		
	.68						
.60				1.0			
3.0							
4.0							
			.89			1.12	1.06
			1.0			1.0	1.0
		2.0			2.0		

Table 21. Model A (3.30% milk fat and 3.14% protein) shows the flow of whole milk, milk fat, and protein through the cheese plant. Inputs to the activities are positive and outputs are negative.

	Cheddar cheese	High fat Cheddar	Monterey cheese	Swiss cheese	Aged Cheddar	Whey separate	Whey butter	Whey powder	Milk separate	Butter	NDM	Blend powder
Milk	30,000	19,346	20,000	18,818	30,000				50,000			
(Milk fat)	(990)	(638)	(660)	(621)	(990)				(1650)			
(Protein)	(942)	(608)	(628)	(591)	(942)				(1575)			
Cheese	-2,851	-2,191	-1,964	-1,793	-2,761							
(Milk fat)	(884)	(839)	(559)	(494)	(884)							
(Protein)	(723)	(471)	(453)	(497)	(723)							
Sweet cream		654							-4,125	3,320		
(Milk fat)		(262)							(1650)	(1328)		
(Protein)		(13)							(81)	(65)		
Skimmilk				2,982					-45,875		47,485	19,204
(Protein)				(96.9)					(1,496)		(1548)	(638)
Butter										-1644		
(Milk fat)										(1307)		
NDM											-4,250	
(Protein)											(1548)	
Whey	-27,150	-17,809	-18,036	-20,007	-27,239	27,196						
(Milk fat)	(106)	(61)	(101)	(127)	(106)	(95)						
(Protein)	(219)	(149)	(175)	(191)	(219)	(227)						
Whey cream						-379.6	3,320					
(Milk fat)						(95)	(830)					
(Protein)						(2.3)	(22)					
Whey skim						-26,816		37,708				25,924
(Protein)						(224)		(315)				(217)
Whey butter							-1,009					
(Milk fat)							(807)					
Whey powder								-2,500				
(Protein)								(315)				
Blend powder												-3,438
(Protein)												(854)
Protein/ milk fat	.951	.689	.951	1.111	.951							
F.D.M.	.500	.608	.499	.456	.500							
% Protein								12.6			36.4	24.8

Table 22. Model B (3.30% milk fat, 3.25% protein, and standardized with cream) shows the flow of whole milk, milk fat, and protein through the cheese plant. Inputs to the activities are positive and outputs are negative.

	Cheddar cheese	High fat Cheddar	Monterey cheese	Swiss cheese	Aged Cheddar	Whey separate	Whey butter	Whey powder	Milk separate	Butter	NDM	Blend powder
Milk	29,904	19,267	19,936	19,414	29,904				50,000			
(Milk fat)	(987)	(636)	(658)	(641)	(987)				(1650)			
(Protein)	(972)	(626)	(648)	(631)	(972)				(1625)			
Cheese	-2,963	-2,268	-2,025	-1,846	-2,871							
(Milk fat)	(925)	(859)	(579)	(510)	(925)							
(Protein)	(746)	(487)	(468)	(516)	(746)							
Sweet cream	95.6	732.8	63.7		95.6				-4,125	3,320		
(Milk fat)	(38)	(293)	(25.5)		(38)				(1650)	(1328)		
(Protein)	(2)	(14.8)	(1.3)		(1.9)				(83)			
Skimmilk				2,386.3					-45,875		47,223	19,097
(Protein)				(80)					(1542)		(1587)	(642)
Butter										-1,644		
(Milk fat)										(1307)		
NDM											-4,250	
(Protein)											(1587)	
Whey	-27,037	-17,732	-17,975	-19,954	-27,129	27,196						
(Milk fat)	(101)	(70)	(105)	(131)	(101)	(95)						
(Protein)	(228)	(154)	(181)	(195)	(228)	(227)						
Whey cream						-379.6	3,320					
(Milk fat)						(95)	(830)					
(Protein)						(2.3)	(22)					
Whey skim						-26,816		37,708				25,924
(Protein)						(224)		(315)				(217)
Whey butter							-1,009					
(Milk fat)							(807)					
Whey powder								-2,500				
(Protein)								(315)				
Blend powder												-3,438
(Protein)												(858)
Protein/ milk fat	.950	.690	.950	1.111	.950							
F.D.M.	.503	.601	.501	.457	.503							
% Protein								12.6			37.2	25.0

Table 23. Model C (3.50% milk fat and 3.20% protein) shows the flow of whole milk, milk fat, and protein through the cheese plant. Inputs to the activities are positive and outputs are negative.

	Cheddar cheese	High fat Cheddar	Monterey cheese	Swiss cheese	Aged Cheddar	Whey separate	Whey butter	Whey powder	Milk separate	Butter	NDM	Blend powder
Milk	30,000	19,410	20,000	18,057	30,000				50,000			
(Milk fat)	(1050)	(679)	(700)	(632)	(1050)				(1750)			
(Protein)	(960)	(621)	(640)	(578)	(960)				(1600)			
Cheese	-2,991	-2,235	-2,039	-1,830	-2,898							
(Milk fat)	(953)	(849)	(593)	(503)	(953)							
(Protein)	(736)	(481)	(461)	(511)	(736)							
Sweet cream		590							-4,375	3,320		
(Milk fat)		(236)							(1750)	(1328)		
(Protein)		(11.8)							(87)	(66)		
Skimmilk				2,943					-45,625		47,485	19,204
(Protein)				(127)					(1513)		(1577)	(638)
Butter										-1,644		
(Milk fat)										(1307)		
NDM											-4,250	
(Protein)											(1577)	
Whey	-27,009	-17,765	-17,961	-19,970	-27,102	27,196						
(Milk fat)	(97)	(66)	(107)	(129)	(97)	(95)						
(Protein)	(224)	(118)	(179)	(194)	(224)	(227)						
Whey cream						-379.6	3,320					
(Milk fat)						(95)	(830)					
(Protein)						(2.3)	(22)					
Whey skim						-26,816		37,708				25,924
(Protein)						(224)		(315)				(217)
Whey butter							-1,009					
(Milk fat)							(807)					
Whey powder								-2,500				
(Protein)								(315)				
Blend powder												-3,438
(Protein)												(854)
Protein/ milk fat	.914	.691	.914	1.111	.914							
F.D.M.	.514	.603	.510	.455	.514							
% Protein								12.6			37.1	24.8

Table 24. Model D (3.50% milk fat and 3.32% protein) shows the flow of whole milk, milk fat, and protein through the cheese plant. Inputs to the activities are positive and outputs are negative.

	Cheddar cheese	High fat Cheddar	Monterey cheese	Swiss cheese	Aged Cheddar	Whey separate	Whey butter	Whey powder	Milk separate	Butter	NDM	Blend powder
Milk	30,000	19,315	20,000	18,700	30,000				50,000			
(Milk fat)	(1050)	(676)	(700)	(655)	(1050)				(1750)			
(Protein)	(996)	(641)	(664)	(621)	(996)				(1660)			
Cheese	-3,043	-2,324	-2,072	-1,883	-2,948							
(Milk fat)	(953)	(874)	(598)	(522)	(953)							
(Protein)	(762)	(498)	(473)	(530)	(762)							
Sweet cream		685							-4,375	3,320		
(Milk fat)		(274)							(1750)	(1328)		
(Protein)		(14.1)							(90)	(69)		
Skimmilk				3,100					-45,625		47,485	19,204
(Protein)				(107)					(1570)		(1634)	(661)
Butter										-1,644		
(Milk fat)										(1307)		
NDM											-4,250	
(Protein)											(1634)	
Whey	-26,957	-17,677	-17,925	-20,199	-27,052	27,196						
(Milk fat)	(97)	(76)	(102)	(133)	(97)	(95)						
(Protein)	(234)	(158)	(191)	(198)	(234)	(227)						
Whey cream						-379.6	3,320					
(Milk fat)						(95)	(830)					
(Protein)						(2.3)	(22)					
Whey skim						-26,816		37,708				25,924
(Protein)						(224)		(315)				(217)
Whey butter							-1,009					
(Milk fat)							(807)					
Whey powder								-2,500				
(Protein)								(315)				
Blend powder												-3.438
(Protein)												(877)
Protein/ milk fat	.949	.690	.949	1.111	.949							
F.D.M.	.505	.597	.505	.458	.505							
% Protein								12.6			38.4	25.5

Table 25. Model E (3.70% milk fat and 3.35% protein) shows the flow of whole milk, milk fat, and protein through the cheese plant. Inputs to the activities are positive and outputs are negative.

	Cheddar cheese	High fat Cheddar	Monterey cheese	Swiss cheese	Aged Cheddar	Whey separate	Whey butter	Whey powder	Milk separate	Butter	NDM	Blend powder
Milk	30,000	19,390	20,000	17,838	30,000				50,000			
(Milk fat)	(1110)	(717)	(740)	(660)	(1110)				(1850)			
(Protein)	(1005)	(650)	(670)	(598)	(1005)				(1675)			
Cheese	-3,171	-2,352	-2,136	-1,900	-3,072							
(Milk fat)	(1022)	(882)	(627)	(526)	(1022)							
(Protein)	(768)	(503)	(483)	(536)	(768)							
Sweet cream		610							-4,625	3,320		
(Milk fat)		(244)							(1850)	(1328)		
(Protein)		(12.7)							(96)	(69)		
Skimmilk				3,962					-45,375		47,485	19,097
(Protein)				(137)					(1579)		(1653)	(665)
Butter										-1,644		
(Milk fat)										(1307)		
NDM											-4,250	
(Protein)											(1653)	
Whey	-26,829	-17,648	-17,864	-19,900	-26,928	27,196						
(Milk fat)	(88)	(80)	(113)	(134)	(88)	(95)						
(Protein)	(237)	(159)	(187)	(199)	(237)	(227)						
Whey cream						-379.6	3,320					
(Milk fat)						(95)	(830)					
(Protein)						(2.3)	(22)					
Whey skim						-26,816		37,708				25,924
(Protein)						(224)		(315)				(217)
Whey butter							-1,009					
(Milk fat)							(807)					
Whey powder								-2,500				
(Protein)								(315)				
Blend powder												-3,438
(Protein)												(881)
Protein/ milk fat	.905	.689	.905	1.111	.905							
F.D.M.	.520	.595	.523	.458	.520							
% Protein								12.6			38.8	25.6

Table 26. Model F (3.70% milk fat, 3.35% protein, and standardized with NDM) shows the flow of whole milk, milk fat, and protein through the cheese plant. Inputs to the activities are positive and outputs are negative.

	Cheddar cheese	High fat Cheddar	Monterey cheese	Swiss cheese	Aged Cheddar	Whey separate	Whey butter	Whey powder	Milk separate	Butter	NDM	Blend powder
Milk	29,873	19,390	19,915	21,383	29,873				50,000			
(Milk fat)	(1105)	(717)	(737)	(791)	(1105)				(1850)			
(Protein)	(1001)	(650)	(667)	(716)	(1001)				(1675)			
Cheese	-3,227	-2,352	-2,178	-2,232	-3,126							
(Milk fat)	(1017)	(882)	(624)	(636)	(1017)							
(Protein)	(801)	(503)	(505)	(656)	(801)							
Sweet cream		610							-4,625	3,320		
(Milk fat)		(244)							(1850)	(1328)		
(Protein)		(12.7)							(96)	(69)		
Skimmilk									-45,375		47,485	19,097
(Protein)									(1575)		(1653)	(665)
Butter										-1,644		
(Milk fat)										(1307)		
NDM	127		84.7	417.2	127						-4,250	
(Protein)	(49)		(33)	(162)	(49)						(1653)	
Whey	-26,773	-17,648	-17,823	-19,568	-26,874	27,196						
(Milk fat)	(89)	(80)	(113)	(155)	(89)	(95)						
(Protein)	(249)	(159)	(195)	(222)	(249)	(227)						
Whey cream						-379.6	3,320					
(Milk fat)						(95)	(830)					
(Protein)						(2.3)	(22)					
Whey skim						-26,816		37,708				25,924
(Protein)						(224)		(315)				(217)
Whey butter							-1,009					
(Milk fat)							(807)					
Whey powder								-2,500				
(Protein)								(315)				
Blend powder												-3,438
(Protein)												(881)
Protein/ milk fat	.950	.689	.950	1.111	.950							
F.D.M.	.508	.595	.502	.472	.508							
% Protein								12.6			38.8	25.6

Table 27. Model G (3.70% milk fat, 3.35% protein, and standardized with skimmilk) shows the flow of whole milk, milk fat, and protein through the cheese plant. Inputs to the activities are positive and outputs are negative.

	Cheddar cheese	High fat Cheddar	Monterey cheese	Swiss cheese	Aged Cheddar	Whey separate	Whey butter	Whey powder	Milk separate	Butter	NDM	Blend powder
Milk	28,642	19,390	19,095	17,838	28,642				50,000			
(Milk fat)	(1060)	(717)	(707)	(660)	(1060)				(1850)			
(Protein)	(960)	(650)	(640)	(598)	(960)				(1675)			
Cheese	-3,077	-2,352	-2,088	-1,900	-2,981							
(Milk fat)	(965)	(882)	(598)	(526)	(965)							
(Protein)	(770)	(503)	(484)	(536)	(770)							
Sweet cream		610							-4,625	3,320		
(Milk fat)		(244)							(1850)	(1328)		
(Protein)		(12.7)							(96)	(69)		
Skimmilk	1,358		905.4	3,962	1,358				-45,375		47,485	19,097
(Protein)	(47)		(32)	(137)	(47)				(1579)		(1653)	(665)
Butter										-1,644		
(Milk fat)										(1307)		
NDM											-4,250	
(Protein)											(1653)	
Whey	-26,923	-17,648	-17,912	-19,900	-27,019	27,196						
(Milk fat)	(95)	(80)	(108)	(134)	(95)	(95)						
(Protein)	(237)	(159)	(187)	(199)	(237)	(227)						
Whey cream						-379.6	3,320					
(Milk fat)						(95)	(830)					
(Protein)						(2.3)	(22)					
Whey skim						-26,816		37,708				25,924
(Protein)						(224)		(315)				(217)
Whey butter							-1,009					
(Milk fat)							(807)					
Whey powder								-2,500				
(Protein)								(315)				
Blend powder												-3,438
(Protein)												(881)
Protein/ milk fat	.950	.689	.950	1.111	.950							
F.D.M.	.506	.595	.503	.458	.506							
% Protein								12.6			38.8	25.6