5-2016

A Counting-Focused Instructional Treatment for Developing Number System Knowledge in Second-Grade: A Mixed Methods Study on Children's Number Sense

Jessica F. Shumway
Utah State University

A COUNTING-FOCUSED INSTRUCTIONAL TREATMENT FOR DEVELOPING NUMBER SYSTEM KNOWLEDGE IN SECOND GRADE: A MIXED METHODS STUDY ON CHILDREN'S NUMBER SENSE

by

Jessica F. Shumway

A dissertation submitted in partial fulfillment
of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Education

Approved:

Patricia S. Moyer-Packenham, Ph.D. Major Professor	Kerry Jordan, Ph.D. Committee Member
James Dorward, Ph.D. Committee Member	Ann M. Berghout Austin, Ph.D. Committee Member
Amy Bingham Brown, Ed.D. Committee Member	Beth L. MacDonald, Ph.D. Committee Member

Mark R. McLellan, Ph.D.
Vice President for Research and
Dean of the School of Graduate Studies

UTAH STATE UNIVERSITY
Logan, Utah

2016

ABSTRACT

A Counting-Focused Instructional Treatment for Developing Number System Knowledge in Second-Grade: A Mixed Methods Study on Children's Number Sense

by

Jessica F. Shumway, Doctor of Philosophy

Utah State University, 2016

Major Professor: Patricia S. Moyer-Packenham, Ph.D.
Department: School of Teacher Education and Leadership

Instruction for developing students' number sense is a critical area of research in mathematics education because of the role number sense plays in early mathematics learning. Specifically, number system knowledge has been identified as a key cognitive mechanism in number sense development. The purpose of this mixed methods study was to explore variations in second-grade students' number sense development as they engaged in a counting-focused instructional treatment, geared towards developing number system knowledge, for differing amounts of time. Sixty second-grade students participated in number sense assessments and two students participated in in-depth, task-based interviews to provide quantitative and qualitative data to investigate the change and development of students' number sense during the instructional treatment.

A generalized estimating equations (GEE) analysis showed an associated average increase in test scores for students participating in 9 weeks of the instructional treatment as compared to students participating in 3 weeks of the instructional treatment. This indicated that the counting-focused instructional treatment influenced and changed students' number sense. An important implication of this result is that it highlights the importance of number sense developing over time with multiple, connected experiences.

The in-depth analyses of two cases showed learning growth from pretest to posttest for a low-achieving and a high-achieving student. However, the two students' number sense developed in different ways and their access of number system knowledge varied. Shifts in learning mainly occurred after 6 weeks of the instructional treatment and depended on the student's existing use of number sense. The implication of this result is that the multiple access points and the high-ceiling of the instructional treatment benefited low- and high-achieving students in this study.

Findings from this study showed that the counting-focused instructional treatment provided number sense learning opportunities for students from a wide range of abilities and backgrounds within the classroom setting. For many teachers, it is difficult to orchestrate differentiated, whole-class mathematics instructional activities due to their students' wide-ranging mathematics abilities. This study identifies a promising instructional practice for elementary mathematics teachers that can facilitate opportunities for students to develop their number sense during whole-class mathematics instruction.

(214 pages)

PUBLIC ABSTRACT

A Counting-Focused Instructional Treatment for Developing Number System Knowledge in Second-Grade: A Mixed Methods Study on Children's Number Sense

Jessica F. Shumway

Instruction for developing students' number sense is a critical area of research in mathematics education because of the role number sense plays in early mathematics learning. Specifically, number system knowledge has been identified as a key cognitive mechanism in number sense development. The purpose of this mixed methods study was to explore variations in second-grade students' number sense development as they engaged in a counting-focused instructional treatment, geared towards developing number system knowledge, for differing amounts of time. Sixty second-grade students participated in number sense assessments and two students participated in in-depth, task-based interviews to provide quantitative and qualitative data to investigate the change and development of students' number sense during the instructional treatment.

A generalized estimating equations (GEE) analysis showed an associated average increase in test scores for students participating in 9 weeks of the instructional treatment as compared to students participating in 3 weeks of the instructional treatment. This indicated that the counting-focused instructional treatment influenced and changed students' number sense. An important implication of this result is that it highlights the importance of number sense developing over time with multiple, connected experiences.

The in-depth analyses of two cases showed learning growth from pretest to

posttest for a low-achieving and a high-achieving student. However, the two students' number sense developed in different ways and their access of number system knowledge varied. Shifts in learning mainly occurred after 6 weeks of the instructional treatment and depended on the student's existing use of number sense. The implication of this result is that the multiple access points and the high-ceiling of the instructional treatment benefited low- and high-achieving students in this study.

Findings from this study showed that the counting-focused instructional treatment provided number sense learning opportunities for students from a wide range of abilities and backgrounds within the classroom setting. For many teachers, it is difficult to orchestrate differentiated, whole-class mathematics instructional activities due to their students' wide-ranging mathematics abilities. This study identifies a promising instructional practice for elementary mathematics teachers that can facilitate opportunities for students to develop their number sense during whole-class mathematics instruction.

ACKNOWLEDGMENTS

I would like to begin by acknowledging and extending my gratitude to my committee members, Drs. Patricia Moyer-Packenham, Ann Austin, Kerry Jordan, Jim Dorward, Amy Brown, and Beth MacDonald, for their support and guidance throughout this project. Thank you for the time you invested in me! My most heartfelt thanks goes to my chair, Dr. Patricia Moyer-Packenham. I truly appreciate your unending support, invaluable mentorship, and depth and breadth of knowledge. It is impossible to quantify or express what I have learned from you over the last 6 years. I will always be grateful to Gwenanne for encouraging me to take the leap and move across the country to study with you! This experience with you, Patricia, has impacted my life in countless ways. Thank you for all you have done for me and my boys over the years.

Thank you Sundee, Ginger, Jenny, Kerry, and Barbara. You were instrumental in bringing this research to life! Sundee, your support and enthusiasm made this study possible. Ginger, Jenny, and Kerry, thank you for your time, flexibility, and willingness to participate in research. Barbara, always a sounding board, always full of excellent ideas and advice! The moment I walked into your office in 2011, we connected mathematically and otherwise, and many wonderful things have developed ever since.

A big thank you to the second-grade students who participated in the study. I learned so much from you! To my Bailey’s students, who are now in high school, this desire to learn more about how children develop number sense all started with you.

To my academic siblings: Arla, Katie, Jennifer, Stephen, Max, HyeKyoung, Marialuisa, Emma, Christina, and Kristy. Our shared experiences have formed a lasting bond. I appreciate each and every one of you in a unique way. Throughout my program, I have been grateful to be surrounded by smart, thoughtful people who continuously push my thinking forward.

My parents, Phillip and Joellen, have always encouraged and supported me to pursue every educational opportunity possible and done everything in their power to make my dreams come true. This pursuit of a Ph.D. in education was no exception! I cannot even begin to list the many ways you made this possible for me and kept me sane! Grandma Betty, without your constant nourishment, both emotionally and with actual delicious food, I never would have made it through. To my sister, Jennifer, thank you for your empathetic ear. You are the best listener I know and compassionate beyond belief.

Finally, a most special thank you to my family! To my best friend, Ronen; thank you for being by my side through every adventure these last 16 years. These last 6 years have been our most eventful yet. Balancing home life and "dissertation land" was challenging. You made it doable and constantly reminded me it was doable (and enjoyable!). I love you and am forever grateful for the ways you push me to excel and be the best I can be. To Benjamin, David, and Nathaniel, this dissertation is for you! I love you more than words can express.

Jessica F. Shumway

CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

> *"Well, I solved it really fast because I knew three 10s is 30 and three 5s makes 15 so that means Ms. Shumway has 45 cookies for the party."*

This was second-grader Sadia's response to the following story problem. *Ms. Shumway has 3 bags of cookies with 15 cookies in each bag. How many cookies does Ms. Shumway have for the party?* Sadia decomposed the 15 into tens and ones (10 and 5) then used these friendlier numbers to solve for 3 bags of 15 cookies. She took the 10 and grouped "three 10s" together to make 30. Then, she grouped the three 5s together to make 15. Finally, Sadia composed the number 45 by combining the 30 and the 15 together. The description of Sadia's cognitive process for solving the problem requires four sentences, yet in action, this process is quick, efficient, and employs deep mathematical understanding. Sadia used her number sense to quickly solve the problem in her head. Big mathematics ideas based in number sense—such as decomposing numbers, place value ideas of tens and ones, unitizing, counting, patterns, and relationships among numbers—are all embedded in Sadia's strategy and explanation for this story problem. Sadia's number sense approach to problem solving highlights a deeper conceptual understanding of mathematics and fluency with numbers and their relationships.

Education research indicates that number sense is a complex construct, and it involves many components including counting, numerical magnitude comparisons, estimation, number patterns, and the combination of amounts (Jordan, Kaplan, Olah, &

Locuniak, 2006; Locuniak & Jordan, 2008). Although number sense is a common term and is known to be important to students' mathematics achievement (Chard et al., 2005; Jordan, Glutting, & Ramineni, 2010), there are few education research studies that examine instructional treatments focused on specific components of number sense. For example, a better understanding of the influence of a counting-focused instructional treatment on children's number sense development could potentially contribute to teasing out the critical skills embedded in the complex definition of number sense. Such insight could contribute to designing early intervention programs, identifying effective instructional practices, and extending the knowledge base of early number sense. Therefore, the purpose of this study was to more closely examine the counting construct of number sense by exploring the variations in second-grade students' number sense development as they engaged in a counting-focused instructional treatment for differing amounts of time.

Background of the Problem

In the opening vignette, Sadia's explanation for solving three groups of 15 gives one snapshot of a student using her number sense to solve a story problem. Unlike the example of Sadia, many children lack the foundational number sense needed to succeed in mathematics. Although some children have strong understandings in mathematics computational procedures, they may not have the foundational number sense to truly understand the meaning behind the procedure. Lack of number sense is a factor in the troubling statistics regarding U.S. students' mathematics achievement on international tests and the difficulties in recruiting students for Science Technology Engineering and

Mathematics (STEM) careers (Geary, Hoard, Nugent, & Bailey, 2013).

Number sense is the critical foundation for not only mathematics achievement in school but also for lifelong *numeracy*. Numeracy involves the ability to use and interpret quantitative information (Maclellan, 2012). Numeracy affects a person's ability to handle the increasing quantitative demands of the modern economy (Hudson, Price, & Gross, 2009). Numeracy is also tied to adults' employability and wages (Parsons & Bynner, 1997). Unfortunately, in the U.S., 22% of adults are functionally innumerate (Geary et al., 2013). With number sense being an important factor in a child's path to numeracy, helping children develop number sense throughout their schooling is vitally important to their futures, both in terms of their jobs and daily life.

The good news is that mathematics education in the U.S. is moving in new directions that emphasize not only mathematics proficiency, but also mathematics understanding. The Common Core State Standards for Mathematics (CCSSM) aim to transform the country's curriculum into a more focused and coherent set of standards (Common Core State Standards Initiative [CCSSI], 2010). As a result, deep and connected understandings of foundational number concepts and organizing principles, such as the structure of the number system, are being emphasized in the elementary mathematics standards (CCSSI, 2010; Confrey & Krupa, 2010). Although the CCSSM set grade-specific standards, the document does not define or specify *how* to teach foundational number concepts and *how* to provide number sense experiences.

Hence, number sense and instruction for developing number sense are critical areas of research in mathematics education because of the key role number sense plays in early mathematics development. Although the construct of number sense has been described and studied since the early 20th century (e.g., Brownell, 1945; Dantzig, 1954), number sense has slowly, yet increasingly gained more attention by researchers and educators, especially in the last two decades (Berch, 2005; Resnick, Lesgold, & Bill, 1990). The field is particularly beginning to focus on studies finding that early development in number sense is critical to students' later mathematics achievement (Chard et al., 2005; Jordan et al., 2006, 2010; Lago & DiPerna, 2010; Locuniak & Jordan, 2008). These important findings have led researchers to begin investigating interventions for early development in number sense.

Researchers are currently developing and testing interventions to improve preschool and kindergarten children's early understanding of number (e.g., Clements, Sarama, Spitler, Lange, & Wolfe, 2011; Dyson, Jordan, & Glutting, 2013; Ramani & Siegler, 2008). While we know that early intervention for developing students' number sense is important and interventions are being tested, there continues to be a lack of research on how various number sense constructs interact and impact children's development of number sense, particularly in the grades beyond kindergarten. The cognitive psychology literature provides findings about mechanisms that facilitate early number learning. In particular, researchers have pinpointed key developments in elementary grades, such as second grade. These findings in the cognitive psychology literature have the potential to inform the development of instructional interventions for elementary mathematics classrooms.

Cognitive psychologists have learned that a key development in early learning of number is the linking between children's innate nonsymbolic number sense with the number words and Arabic numerals that represent quantities (Wilson, Dehaene, Dubois,

& Fayol, 2009). Another key mechanism is knowledge of the systematic relations among Arabic numerals and the skills in using this knowledge to solve arithmetic problems (Geary et al., 2013). First and second grades are a critical time for students to bridge innate understandings of number and symbolic cultural aspects of number that lead them to be successful in mathematics and develop strong numeracy for today's society. Integration of the non-symbolic number sense and symbolic systems of number is a key development in children's understanding of number. The integration of these systems through verbal counting during first and second grades is thought to pave the way for refinement in numerical precision and understanding of the number system (Carey, 2001; Le Corre & Carey, 2007; Lipton & Spelke, 2005). Specifically, second-grade students must use the translation between symbolic and non-symbolic quantity to begin extending their understanding of the base-10 system and develop fluency with addition and subtraction (CCSSI, 2010). This type of number sense knowledge makes formal mathematics learning more accessible. Studies have found that having this knowledge in elementary school predicted better functional mathematical ability in adolescence (Geary et al., 2013).

Therefore, better understanding of number sense and its multiple constructs is important for mathematics education research. Overall, cognitive psychology research provides insights into the mechanisms that facilitate early quantitative learning. Mathematics education studies operationalize number sense, thereby, both providing insight into components of number sense that predict later mathematics achievement and allowing educators to better identify students at-risk for failure in mathematics. Both bodies of research provide a solid foundation for instructional intervention research.

More specifically, researchers suggest instructional intervention research that can reveal the extent to which early instruction in number sense relates to learning formal mathematics (Locuniak & Jordan, 2008). Within the broad construct of number sense, research has identified weaknesses in symbolic number sense as a key to learning difficulties in mathematics and as an area for future research (Geary et al., 2013; Jordan et al., 2010). Geary et al. proposed that intervention programs designed to help students understand systematic relationships between numbers could better prepare students for later employment, help them make wiser economic choices, and improve the future U.S. workforce.

Problem Statement and Research Questions

Cognitive psychology literature provides insights into the mechanisms that facilitate early quantitative learning, such as intuitive number sense, mapping of number words and symbols onto intuitive number sense, and understanding relationships among numbers. Cognitive psychology research on the processes involved in numerical cognition has the potential to inform elementary mathematics instruction. While some instructional intervention research in preschool and kindergarten has been conducted by cognitive psychologists and educationalists, research is needed in elementary grades, such as second grade, when students are learning to link non-symbolic and symbolic understandings of quantity and learning systematic relationships among numbers (i.e., number system knowledge). Furthermore, much of the cognitive psychology intervention research was conducted in one-on-one, laboratory, or small group settings. Classroom-based research is needed in order to bridge understanding between numerical cognition theory and classroom-based practices that provide children with opportunities to develop robust number sense.

Current mathematics education research shows that number sense predicts later mathematics achievement (Jordan et al., 2006, 2010; Lago & DiPerna, 2010; Locuniak & Jordan, 2008). Although number sense is a common term and is known to be important to students' mathematics achievement, there are few studies that examine the role of specific constructs of number sense in instructional interventions for improving children's number sense development. A better understanding of the influence of specific number sense constructs on children's overall number sense development is needed. Research on specific number sense constructs could potentially contribute to teasing out the critical skills embedded in the complex definition of number sense, designing instructional activities and programs, and extending the knowledge base of early number sense.

To address the gap in knowledge in the area of number sense research, this study examined a specific construct of number sense in the context of a quasi-experiment of an instructional treatment. Specifically, the purpose of the study was to investigate the counting construct of number sense and its influence on second-grade students' development of number sense. This research examined the implementation of a counting-focused instructional treatment in second-grade classrooms that involved daily verbal counting and discussions about number system knowledge (i.e., symbolic number patterns and number relationships). Throughout the study, students' number sense achievement was measured by assessments of computational fluency, story problems, and number line estimations.

To understand the counting construct of number sense and its influence on students' development of number sense, this study used an embedded mixed methods approach with quantitative and qualitative data collected before, during, and after treatment instruction of second-grade students. The research questions guiding this study were:

> Overarching Research Question: In what ways does a counting-focused instructional treatment (that focuses on patterns in the number system and relationships among numbers) influence, change, and develop second-grade students' number sense (specifically, computational fluency, strategies for solving story problems, and number line estimation)?
>
> 1. What are the variations in number sense development when students engage in counting interventions for differing amounts of time (3 weeks, 6 weeks, and 9 weeks)?
>
>   a) What are the variations among three intact classes?
>   b) What are the variations for individual students within each class?
>
> 2. What are the variations in number sense development for one low-achieving student and one high-achieving student?

Significance of the Study

Research on number sense has the potential for providing teachers with an understanding of how to build on students' innate number sense while bridging the innate number sense with symbolic systems taught in schools. With a topic as complex and essential as number sense, children's mathematics achievement in the elementary grades will be positively impacted as research on number sense instruction develops and becomes better aligned with classroom practice. The application to teaching and learning in the classroom will be better linked to research as this study occurs in the area of number sense development and is set directly in second-grade classrooms.

Definition of Terms

The following key terms are defined for this study.

N*umber sense* is defined from an educationist's perspective, which takes a broader and more-inclusive approach. *Number sense* entails foundational understandings of quantities, number, relationships among numbers, and the number system. Children are born with intuitive number sense, which develops over time with experiences. This informal number sense develops into more formal number sense knowledge as children learn symbolic cultural tools to represent their informal sense of number. In this study, the term *number sense* encompasses *number system knowledge, number sense access,* and *intuitive number sense* (defined below) and is used in the context of an elementary classroom setting.

Intuitive number sense is the innate, evolutionary understandings of number, which entails the approximate number system and subitizing (defined below). Cognitive psychologists typically call this "the number sense" and define it as the ability to "quickly understand, approximate, and manipulate numerical quantities" (Wilson et al., 2009). I call it *"intuitive number sense"* to distinguish cognitive psychologists' definition of number sense with the broader educationalist definition of number sense. *Intuitive number sense* is measured by non-symbolic quantity tasks that involve viewing, comparing, adding, or subtracting non-symbolic numerosities.

The *Approximate Number System (ANS)*, also known as approximate representation of magnitude, is the foundational system that underlies the ability to nonverbally represent number (Halberda & Feigenson, 2008). It represents number approximately and is thought to provide a foundation for arithmetic computation. It is measured by non-symbolic numerical discrimination tasks. I define it as a part of *intuitive number sense.*

Subitizing is the ability to quickly perceive a non-symbolic set of items less than 4 (Clements, 1999; Kaufman, Lord, Reese, & Volkmann, 1949). Cognitive psychologists refer to *subitizing* as a system for keeping track of small numbers of individual objects (Feigenson, Dehaene, & Spelke, 2004). Along with the ANS, the two systems make up *intuitive number sense*. It is measured by non-symbolic numerical discrimination tasks.

Number sense access is the linking between symbolic representations (e.g., number words and Arabic numerals) with their non-symbolic representation of quantity. *Number sense access* is measured by comparing the results from non-symbolic tests to those from tests involving symbolic stimuli.

Number system knowledge is the processing of Arabic numerals and understanding relationships among numerals (Geary et al., 2013). This includes understanding of relative magnitude of numerals, ordering of numerals, and the composition of numbers. *Number system knowledge* is measured by knowledge of the systematic relations among Arabic numerals (such as number line estimation tasks) and the use of this knowledge to solve arithmetic problems (e.g., computational fluency).

Counting for this study is used conceptually to encompass the rote counting sequence (verbal count words and/or numerals) and enumeration (i.e., counting objects), as well as an understanding of number, number patterns, number relationships, and the number system. Verbal *counting* and the discussions around counting are the foci of the instructional treatment in this study. While I acknowledge that the term *counting* encompasses many components, for the purpose of brevity, I call the intervention a "counting-focused instructional treatment," using the broad term *counting*. The counting-focused instructional treatment will take place in second-grade classrooms, hence, the counting focus of the intervention will be on verbal counting and discussions about the relationships among numbers in these counting sequences.

Computational fluency, within this study, is the basic number combinations of single-digit addition and subtraction items (Baroody, Bajwa, & Eiland, 2009). In this study, it is assumed that fluency with the basic number combinations grows out of number sense (i.e., number sense perspective), and more specifically, number system knowledge (Geary et al., 2013).

CHAPTER II

LITERATURE REVIEW

Researchers are finding that early number sense development is critical to students' later mathematics achievement (e.g., Geary et al., 2013; Jordan et al., 2010). A body of literature exists on number sense constructs, assessments, and instructional interventions. A systematic literature review for this study provided a research foundation for investigating how children learn number, how foundational understandings of number develop and are interconnected, and what teachers can do to support students' early number sense development.

Number sense is a complex construct in mathematics education literature. It is multilayered and multifaceted, therefore, various researchers have different definitions for the term "number sense" (Berch, 2005; Jordan et al., 2006; Lago & DiPerna, 2010; Resnick et al., 1990). Education researchers generally acknowledge constructs of number sense as counting, number knowledge (such as numerical magnitude comparisons), estimation, number patterns, number relationships, and number transformations (such as the combination of amounts; Jordan et al., 2006; Locuniak & Jordan, 2008). Some researchers consider a child's fluidity and flexibility with numbers, mental math, and ability to subitize to be critical components of number sense (Lago & DiPerna, 2010; Resnick et al.,1990). Cognitive psychologists describe humans' and animals' innate abilities of approximating number (discriminating quantities) and subitizing (recognizing and distinguishing small amounts) as "the number sense" (Dehaene, 1997; Halberda & Feigenson, 2008; Schleifer & Landerl, 2011). Both the education and cognitive psychology views of the number sense construct consider it to be foundational in students' learning and development of mathematics.

This literature review is organized into two major sections: (1) Conceptual Framework, and (2) Reviewing the Literature: Number Sense Measures and Interventions. The first section reviews both cognitive psychology and mathematics education literatures in order to develop a theoretical understanding of the mechanisms for number sense and numerical cognition. The second section entails a systematic review of current empirical studies about preschool and elementary students' number sense. The conceptual framework grounds the study theoretically while the systematic review of the literature places the research study within the broader number sense research context.

Conceptual Framework

This study was framed by both cognitive psychologists' and mathematics education researchers' conceptions of number sense. An understanding of students' nonverbal and pre-school knowledge provides an important foundation for research on mathematics instruction that builds on students' intuitive number sense. Bridging the intuitive number sense with symbolic systems taught in schools has important implications for mathematics education research. A conceptual framework (see Figure 1) that emphasizes the relationships of the intuitive number sense (defined and studied by cognitive psychologists) with the broader conception of number sense that is more closely tied to symbolic representations of quantities (mainly explored in mathematics education research) informed the analysis and synthesis of articles included in this

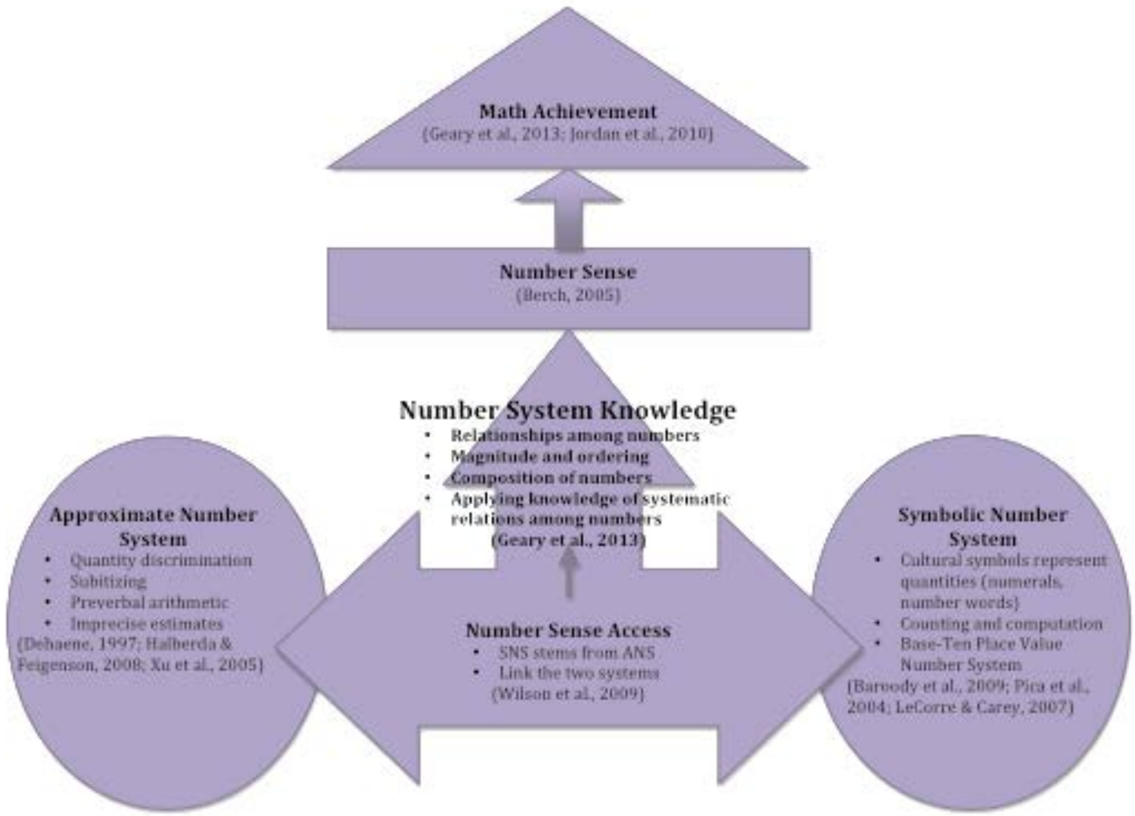


Figure 1. Conceptual framework: Bridging approximate and symbolic number systems for developing number sense.

literature review. The Approximate Number System (ANS) describes the intuitive number sense, which includes nonverbal and pre-symbolic notions of quantity (Dehaene, 1997; Halberda & Feingenson, 2008; Xu, Spelke, & Goddard, 2005). The Symbolic Number System (SNS) includes symbolic notions of number such as cultural symbols that represent quantities such as numerals, verbal counting, and place value concepts (Baroody, Eiland, & Thompson, 2009; Le Corre & Carey, 2007; Pica et al., 2004).

Figure 1 illustrates the conceptual framework for understanding the interactions and relationships between the ANS and SNS, indicating that Number Sense Access plays a key role in bridging the two systems as children develop their number sense (Wilson et al., 2009). Number System Knowledge is developed as students bridge the ANS and SNS leading to more formal and abstract understandings of numbers and their systematic relationships (Geary et al., 2013). Number System Knowledge leads to the broad educationalist term Number Sense (Berch, 2005), which captures the components of number system knowledge and expands that definition to include applications of number sense within mathematics in general (leading to Math Achievement). The researcher for this study hypothesized that the symbiotic relationship and interaction between the ANS and the SNS is particularly important during children's second-grade school year as they develop their number system knowledge, providing a solid foundation for further number sense development and mathematics achievement. Each of these terms within the conceptual framework is described in detail below.

Nonverbal Mathematics: The Approximate Number System

Recent research indicates that humans, and other animal species, are endowed with an innate sense of number (Dehaene, 1997; Feigenson et al., 2004; Xu, Spelke, & Goddard, 2005). Studies showed that infants are able to subitize (recognize exact amounts of up to three objects), discriminate numerosities, and hold expectations about the outcomes of simple arithmetic (e.g., one more or one less). These capabilities are independent of language and are not taught or transmitted through culture (Feigenson et al., 2004; Pica, Lemer, Izard, & Dehaene, 2004). This intuitive number sense is represented as the ANS oval of the graphic in Figure 1. For the purposes of this conceptual framework, the term, Approximate Number System (ANS), refers to the early and intuitive abilities of subitizing, quantity discrimination, magnitude comparison, and preverbal arithmetic. It has been argued that subitizing (i.e., object-file system) is separate from the ANS (Feigenson et al., 2004), however, this debate is beyond the scope of this literature review. For ease of discussion, subitizing is included within the term ANS, although the researcher recognizes that some cognitive psychologists are finding that it is a core system distinct from the ANS.

The preverbal recognition of and discrimination between quantities is imprecise and is ratio-dependent; meaning, for example, that infants can discriminate between 8 and 24 objects, but cannot discriminate between 8 and 16 objects. Additionally, this early numerical knowledge is accessed across multiple modalities of input. For example, infants not only discriminate visual amounts, but also discriminate with the same ratios between amounts presented as sounds.

Cultural Symbols Representing Quantities: The Symbolic Number System

The right side of the conceptual framework graphic (see Figure 1), named the Symbolic Number System (SNS), refers to the part of number sense that is transmitted through cultural symbols of numerals and number systems, based in language, and formally taught in elementary school. While human infants are born with an innate sense of number, it takes children many years to learn verbal counting and the symbolic representations of quantity.

There is evidence that between the ages of 3 and 5, children construct meaning to symbolic notations of quantity (Le Corre & Carey, 2007). They learn to map symbolic numbers (i.e., number words and numerals) onto their pre-existing notions of quantity based in the ANS (Le Corre & Carey, 2007; Pica et al., 2004; Siegler & Booth, 2004). As this mapping of symbolic numbers takes place, the foundation for counting, exact enumeration, and arithmetic is laid. An understanding of the base-ten place-value system, number patterns, and relationships among numbers becomes accessible to children. This is the groundwork for more formal and abstract mathematics.

The Role of Counting: Number Sense, Number Sense Access, and Number System Knowledge

The horizontal arrows linking the ANS and SNS in Figure 1 show a relationship between these two systems. It can be argued that the SNS stems from the ANS, as symbolic representations of quantities are mapped onto the pre-existing conceptions of quantity. However, there is evidence that the two systems function separately while also supporting the development of each other. Research indicates that the ANS continues to sharpen in acuity through the elementary years (Halberda & Feignenson, 2008) simultaneous to the development of the SNS. This sharpening of the ANS results from maturity but also from experiences with symbolic numbers and with symbolic numbers combined with visual representations of quantities. As the ANS becomes more precise, the SNS also develops with improvement in children’s ANS acuity. Hence, the separate ovals represent the ANS and the SNS as two independent systems, while the horizontal arrows represent the symbiotic relationship between the systems.

In addition to highlighting the relationship between the ANS and the SNS, the horizontal arrows also highlight an important process that takes place as the two systems work together, labeled as “number sense access.” Wilson et al. (2009) emphasized that “a key development which must occur during human learning is the association between non-symbolic number sense and the cultural symbols which represent number (e.g., number words and Arabic digits)" (p. 225). They referred to the conversion from nonverbal, nonsymbolic number knowledge to the culturally-based symbolic system for representing numbers as "number sense access." The Wilson et al. study results suggested that many students with "low number sense" have difficulty with number sense access.

Building from the idea of number sense access, "number system knowledge" represents the further development of these culturally-based symbolic number understandings. Number system knowledge is the processing of Arabic numerals and understanding relationships among numerals (Geary et al., 2013). This includes understanding of relative magnitude of numerals, ordering of numerals, and the composition of numbers. Number system knowledge is measured by knowledge of the systematic relations among Arabic numerals (such as number line estimation tasks) and the use of this knowledge to solve arithmetic problems (e.g., computational fluency).

The number system knowledge aspect of the conceptual framework is where this study on the counting construct of number sense is centrally located. Verbal counting is set in the SNS, as it is a symbolic process, and it also accesses students' ANS because counting associates number words and numerals with quantities. A verbal counting-focused instructional treatment has the potential for developing students' number system knowledge, thereby improving students' number sense (as represented in the vertical arrow connected to the number system knowledge horizontal arrows). As students' number system knowledge improves, their number sense is strengthened, and students' mathematics achievement increases (Jordan et al., 2006, 2010; Jordan, Kaplan, Locuniak, & Ramineni, 2007) as indicated by the arrow leading from number sense to mathematics achievement in Figure 1.

A Conceptual Framework as a Lens for Research: Current Understandings of Number Sense and its Constructs

The conceptual framework for this study highlights the complexity of the number sense construct. It emphasizes the relationships of the innate number sense defined and studied by cognitive psychologists with the broader conception of number sense that is more closely tied to symbolic representations of quantities and explored in mathematics education research and practice. Recent discussion on the role of number system knowledge in remedying “low number sense” (Geary et al., 2013; Wilson et al., 2009) is presented in this framework and could be a key element for research on the role of counting in developing students’ number sense. A better understanding of the influence of verbal counting on children’s number sense development (specifically, its influence on children’s understanding of estimation, computation, relationships among numbers, patterns in the number system, and place value) is needed and could potentially contribute to and extend the research base on number sense. The conceptual framework in Figure 1 could serve as a tool for framing such studies, as proposed in this study. The conceptual framework was used to synthesize relevant research on current understandings of number sense and its constructs.

The aim of this next section is to review empirical studies with the primary purpose of understanding the construct of number sense, which mainly comes out of the cognitive psychology research, and more specifically from cognitive neuroscience and/or numerical cognition literature. These studies serve to identify and define the psychological aspects of number sense and its constructs as well as explain how children's schemas for number sense develop as they are introduced to numbers in a more formal school setting. This section is organized around major topics of the conceptual framework (ANS, SNS, number sense access, number system knowledge, and number sense) and the framework is used to understand how the counting construct of number sense could potentially be a key factor in the concept of number system knowledge.

Number Sense in Mathematics Education and Cognitive Psychology

Researchers make a distinction between the conceptual definition of number sense and its operational definitions (Berch, 2005). Being a multifaceted construct, this distinction emphasizes the complexity of number sense (conceptual definition) while providing a framework for researchers to utilize in order to assess number sense (operational definitions). Even within these distinctions, researchers further distinguish number sense as either the broad, educational definition or as the more specific, cognitive psychology definition, both of which have conceptual and operational definitions.

The cognitive psychology conceptualization of number sense is based on the idea that humans and nonhuman animals are born with an ancient and evolutionary notion of number (Dehaene, 1997). This intuitive understanding of quantity involves the ability to quickly perceive small amounts (subitize), approximate numerical magnitudes, and comprehend simple number transformations (such as one more or one less; Dehaene, 1997; Feigenson et al., 2004; Halberda & Feigenson, 2008). This sense of number is nonverbal and nonsymbolic, and it is an innate internal cognitive process.

The mathematics education conceptualization of number sense is typically built upon the nonverbal, nonsymbolic definition of number sense put forward by cognitive psychologists, but tends to also involve the symbolic representations and understandings of number acquired through formal and informal experiences. For example, much of the research on number sense in mathematics education is focused on the formal school experiences that promote counting, more detailed and exact representations of number, quantities tied to symbols, and number system concepts (e.g., Baroody, Eiland, & Thompson, 2009; Chard et al., 2005; Dyson et al., 2013; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Lago & DiPerna, 2010; Locuniak & Jordan, 2008; Malofeeva, Day, Saco, Young, & Ciancio, 2004; Ramani & Siegler, 2008).

Rather than compare the methodologies of these studies, this section of the literature review will focus on synthesizing and interpreting the findings of current research. It is important to note that the methodologies of these studies mainly occur in laboratories, which is not as applicable to the practical applications of mathematics education research. A review of methodologies will be important in the analysis of intervention studies and is discussed in a subsequent section. The following sections are organized around the major components of the conceptual framework for the study: (1) nonverbal number knowledge, (2) symbolic number system, (3) number system knowledge and number sense, and (4) the role of the counting construct of number sense.

Nonverbal number knowledge: Subitizing, Approximate Number System, and early arithmetic. Human infants, children, and adults across all cultures as well as some nonhuman animals possess an innate ability to conceptualize quantities (Xu et al., 2005). One system within this nonverbal number knowledge realm is the ANS, which is the internal cognitive system of magnitude representations. It is a nonverbal representation of quantities (independent of symbols), operates across modalities (e.g., visual and auditory), and tends to hold imprecise approximations of number (often called analog magnitudes; Halberda & Feigenson, 2008; Pica et al., 2004). The imprecise approximations of number distinguish the ANS from counting and the use of symbols (such as number words or numerals) to represent quantities. Studies on the ANS are reviewed here as they have implications for the teaching and learning of number sense. Additionally, these studies lay the foundation for understanding how the ANS and SNS systems are separate yet interact as young children develop their number sense.

Xu et al. (2005) conducted a study on one aspect of this nonverbal number knowledge in infants. Their research investigated 6-month-old infants' sense of approximate numerical magnitudes (i.e., ANS) through four experiments using a preferential looking method as infants were presented with numerosities in various visual-spatial displays. Prior research (Xu & Spelke, 2000) had demonstrated infants' capacities for numerosity discrimination in the ratio of 8:15, hence, this study (Xu et al., 2005) tested infants' abilities to discriminate larger numerosities. The results pointed to infants' abilities to discriminate arrays of 16:32, but not 16:24, which supported previous research that numerosity discrimination depends on the set-size ratio. Additionally, results of the third and fourth experiments in the study showed that infants successfully discriminated the large-number displays (e.g., discriminating 16 from 32 dots), but not the small-number displays (e.g., discriminating one dot from two dots). The results were interpreted to mean that there are separate systems within this nonverbal number knowledge with one representing large numerosities and one representing small numerosities.

While Xu et al.'s (2005) investigation into infants' sense of approximate numerical magnitudes set the stage for understanding children's innate abilities with number, Halberda and Feigenson (2008) extended this line of research to explore the developmental trajectory of children's ANS beyond that of 6-month-old infants. Halberda and Feigenson investigated changes in 3-, 4-, 5-, and 6-year-old children's ANS representational acuity. Their purpose was to find out if the ANS continues to develop and "sharpen" (i.e., become more precise) even as children begin formal schooling. They conducted a cross-sectional design testing 16 participants in each of the four age groups plus a group of 16 adults. The numerical discrimination task involved a quick display of two sets of arrays (66 test trials). Halberda and Feigenson found that 3- to 6-year-old children are still developing the acuity of the ANS. The potential implications of this finding for teaching and learning in the school setting include educating teachers about the ANS development and how to enhance and build on children's innate understandings of quantity (i.e., their number sense). Halberda and Feigenson argued that some of the sharpening of the ANS is likely due to maturation, but that experience also likely affects the development of the ANS. Changes in the ANS acuity impact children's numerical discrimination abilities as well as their estimation of numerical magnitudes. The authors stated that, "the protracted nature of ANS development, spanning the period when symbolic mathematical instruction begins, has implications both for mathematics education and for our understanding of the interplay between individual experience and the 'number sense'" (p. 1464). Because the nonverbal ANS plays a central role in mathematics throughout the human lifespan, understanding its development during children's early school years is important to consider in number sense research.

Furthermore, understanding its development alongside and/or with children's formal symbolic number instruction could positively impact teaching methods in young children's mathematics classrooms.

Symbolic Number System: Number words, numerals, counting, and the number system. Researchers, such as Xu et al. (2005) and Halberda and Feigenson (2008) studied aspects of humans' nonverbal, nonsymbolic understandings of number. Other research (Le Corre & Carey, 2007; Le Corre, Van de Walle, Brannon, & Carey, 2006; Pica et al., 2004) extended these findings on humans' innate number sense abilities that develop through the lifespan and investigated the uniquely human aspects of number knowledge that involve language, symbols, and a counting system. The Le Corre and Carey; Le Corree et al.; and Pica et al. studies further highlight the interactions between the nonverbal and symbolic systems for understanding number and the implications for teaching and learning number sense in schools.

Pica et al. (2004) conducted their numerical cognition study with native speakers of Munduruku, a language used by members of an Amazonian tribe. Munduruku is a language that has number words for numbers 1 through 5, but does not have a count-based representation of number. Through a battery of numerical tests (e.g., magnitude comparisons, estimation tasks, and manipulation of exact numbers), Pica et al. (2004) found that the speakers of Munduruku had numerical approximation competence, as they were able to represent large numbers and understand the concept of relative magnitude. Hence, these results provided evidence that the ANS competencies are common across human cultures and are independent of language. However, Pica et al. also found that language plays a role in the emergence of more exact representations of number as well as exact arithmetic. Munduruku speakers, without a count-based system, allowed approximate use of number words in the range of 3 to 5; whereas, English speakers use the number words and numerals in this range to refer to precise quantities. Additionally, the Munduruku speakers failed on the Manipulation of Exact Numbers task—results that were interpreted to mean that there is a distinction between approximate and exact mental representations of number and that language plays a special role in the emergence of exact arithmetic. Pica et al. concluded that beyond just the language for numbers, the lack of a counting system in the Munduruku language played a role in these results. They conjectured that the counting system in English (i.e., the exact one-to-one pairing of objects with sequence of number words) promotes a conceptual integration of the ANS, discrete object representations, and the verbal code.

Counting and number sense. To explore this role of verbal counting and how it is linked to the nonverbal nonsymbolic number knowledge, Le Corre and Carey (2007) and Le Corre et al. (2006) conducted studies to investigate how verbal counting principles are acquired. Some researchers claim that acquiring the verbal counting principles takes place as numerals in the counting sequence are mapped onto the ANS (e.g., Le Corre et al., 2006), while other researchers purport that innate non-verbal counting principles guide the development of counting skills (Gelman & Gallistel, 1978). In either case, both views recognize the long process and the complexity of learning the verbal counting sequence for young children. Since current empirical studies are testing this first view, known as the principles-after view, this literature review focuses on summarizing these recent empirical studies coming out of the Le Corre cognition laboratory.

In both the Le Corre and Carey (2007) and Le Corre et al. (2006) studies, experiments were conducted with children between the ages of 2- to 5-years-old to investigate their understanding of counting and how the nonverbal, nonsymbolic number knowledge supports the verbal number count and its principles. In both studies, a variety of numerical tasks were presented to the children. Based on initial tasks, children were divided into cardinal-principle-knowers (they know how counting works) and subset-knowers (they do not know how counting works) in order to compare the roles of this knowledge with children's abilities to use verbal counting. Both studies' results were interpreted to mean that the counting system is a new representational format for children, meaning that counting is not an innate skill like the ANS. Le Corre et al. (2006) argued that the verbal count list had a long construction process in human history and this same type of construction process is witnessed in the studies involving children ages two to five as they construct understandings of verbal counting and its principles. Le Corre and Carey's (2007) experiment with 2- and 3-year-old children showed that although many of the participants knew the count list up to ten, none of them could estimate a set of more than four objects. In other words, the children mapped the numerals 1 to 4 onto their innate sense of numerical magnitude, but then used numerals randomly when estimating a set of objects in the range of 5 to 10. The researchers interpreted this to mean that the mapping of large numerals to the analog magnitudes of the ANS occurs after the acquisition of the counting principles. For example, their studies indicated that children develop the verbal count list around the age of three, then around 4½- years of age, they begin to map verbal number words onto the analog magnitudes of the ANS. These findings complement the findings of Pica et al.'s (2004) study, which indicated that the lack of a counting system in the Munduruku language did not impede their ANS, but did prevent the Munduruku speakers from using their number words from 3 to 5 to assign meaning to discrete numbers. Pica et al. stated;

> Around the age of 3, Western children exhibit an abrupt change in number processing as they suddenly realize that each count word refers to a precise quantity. This 'crystallization' of discrete numbers out of an initially approximate continuum of numerical magnitudes does not seem to occur in the Munduruku. (p. 503).

Number sense access, number system knowledge, and number sense. The studies on the ANS, verbal number words, and counting system reviewed here specifically investigated how the nonverbal, nonsymbolic number knowledge interacts or does not interact with the culturally-based symbols for quantity. While the debate on the ontology of the counting system continues to thrive in the cognitive psychology research, the critical aspect that informs the current study is that the two systems, the nonverbal, nonsymbolic system and the culturally based symbolic system, are separate yet support one another. Understanding the ways in which the ANS and SNS are linked is important for educators because students begin their formal schooling just as they begin to understand and use the counting system. Wilson et al. (2009), in their number sense intervention study, highlighted the importance of understanding children's nonsymbolic and symbolic understandings of number both separately and in the ways that they are linked. Their study involved the use of both symbolic and nonsymbolic measures of students' numerical understandings, which led them to consider that the source of children's low "number sense," as the term is used in mathematics education, may actually be difficulty with number sense access, not difficulty with their innate number sense. Number sense access is the linking of symbolic representations to their representation of quantity (Wilson et al., 2009).

As children move beyond the early stages of knowing the verbal count list up to ten and mapping numbers onto larger quantities (LeCorre & Carey, 2007), number sense access continues to be important as they progress through the early school grades. These early years of school involve learning a longer count list (beyond 10), becoming more adept with precision of larger quantities, and beginning to link the counting sequences and counting principles to the larger base-ten place-value number system. Geary et al. (2013) have pinpointed number system knowledge, knowledge of these systematic relations among Arabic numerals and using that knowledge in computation, as a predictor for children's functional numeracy as they progress in their elementary mathematics learning.

The following section summarizes and synthesizes lines of research tied to children's development of counting as they progress to more symbolic representations of number and how these representations are linked to their innate analog magnitudes from the ANS. This development has implications for children's success with more advanced number knowledge, estimation, and number transformations such as arithmetic and mental math. It could be argued that number sense access and number system knowledge are at play in the following studies.

Counting and factors that affect its successful use. The Jordan et al. (2009, 2010) longitudinal correlational studies showed strong and significant relationships between students' number sense and their mathematics achievement. Counting, along with number relationships and basic operations, was found to contribute to students' success or lack of success in later mathematics (Jordan et al., 2010). Current empirical studies on counting, as it is tied to number sense, indicate the importance of counting to students’ success in mathematics as well as factors that account for differences in children’s counting skills. For example, in Aunola, Leskinen, Lerkkanen, and Nurmi’s (2004) longitudinal study, counting ability was found to be a predictor for mathematics performance. Aunola et al. investigated the developmental dynamics of 5- and 6-year-old Finnish children’s mathematics performance as they transitioned from preschool to second grade. In an eye tracking study conducted on German children between ages 8 to 14, Schleifer and Landerl (2011) found that age was more important in performance on the counting tasks than it was in the subitizing tasks. Aunio et al. (2006) examined why students in early grades (ages 4-7) come to school with varying levels of number sense. They examined the influence of nationality, age, and gender on young children’s number sense by administering the Early Numeracy Measure, which measures general numerical skills (relational skills) and specific numerical skills (counting skills) to 130 Chinese children and 203 Finnish children. The researchers reported a difference in the counting skills between Chinese children and Finnish children, and that Finnish children’s number sense developed at a slower pace than that of the Chinese children. Therefore, Aunio et al. concluded that counting skills rely on culturally-based symbolic systems which require systematic and explicit teaching, which are characteristics of Chinese classrooms. The findings also suggested that language may have been another factor influencing the differences in Chinese and Finnish children’s test scores.

These studies suggest that age, language, and methods of teaching potentially impact students’ counting skills, thereby also affecting their number sense development. Each of these are important factors to be considered in mathematics teaching and learning, therefore, future research in these areas is warranted. This review of the current understandings of number sense and its constructs indicate that number sense is a complex construct and that different fields view and conceptualize it differently. The cognitive psychology research on number sense studies the psychological processes of number sense, which makes their literature a fruitful basis for mathematics education research. The next sections summarize, synthesize, and interpret literature on number sense measures and number sense interventions. Many of these studies come out of the mathematics education and special education fields. However, several studies (e.g., Geary, 2011; Siegler & Ramani, 2008; Wilson et al., 2009) illustrate the encouraging integration of cognitive psychology research with classroom research.

Reviewing the Literature: Number Sense Measures and Interventions

The purpose of the "Conceptual Framework" section of the literature review was to review studies that lead to a deeper understanding of the number sense construct and how the constructs are related and interact. The purposes of the sections that follow are to examine, synthesize, and interpret current empirical studies in the field of number sense research and discuss implications based on research conducted during the last 12 years. This section of the literature review presents the current state of knowledge on number sense assessments or measures and number sense interventions. The review is organized in the following sections: (1) Literature Review Objectives, (2) Literature Review Procedures, (3) Number Sense Measures, (4) Number Sense Interventions and Programs, and (5) Building on Current Research. The final section presents the research questions that have emerged from and developed based on the current empirical research in the number sense field.

Literature Review Objectives

A systematic review of the literature (Kennedy, 2007) was conducted in order to lay the foundation for research on number sense and the impact of its various components on children's development of number sense and their mathematics achievement. A search of the literature on the broad topic of number sense was necessary in order to understand the interrelatedness of the multiple components of the number sense construct and locate the areas needing further research. Both mathematics education research (general as well as special education) and cognitive psychology literature were searched in order to understand the teaching and learning implications of number sense research as well as the cognitive learning processes that take place as students develop number sense. This literature review identified, analyzed, evaluated, synthesized, and interpreted empirical studies on the teaching, learning, and cognitive processes of number sense. The objectives of this literature review were: (1) To describe the current state of research on number sense, specifically its constructs, how it is measured, and its impact on student learning and achievement; (2) To discuss the findings, strengths, and weaknesses in previous research, particularly in regard to the designs and methods of the studies; and (3) To draw conclusions based on this information and develop the research questions and methods for this study based on a thorough examination of the literature in the field.

Literature Review Procedures

Databases and Keywords

ERIC via EBSCO, Education Full Text via EBSCO, Academic Search Premier, PsycINFO, and Google Scholar were used in a search to locate empirical studies on number sense between 2002 and 2014. A variety of search terms were used, both singularly and in combination, including: *number sense, achievement, counting, number concepts, mathematics, number system,* and *cognitive processes* (examples of combinations included *number sense* + *achievement* and *number sense* + *counting*). A search of the databases yielded close to 350 results. Approximately 13,000 results were produced in a Google Scholar search for *"number sense,"* while 5,280 results came up with the keywords *"number sense"* + *"counting"* with the custom range of 2002-2014. Reference lists of articles found were also manually searched for further references.

Inclusion Criteria

Current, peer-reviewed, empirical, primary studies were included in this literature review. In order for the analysis of literature to remain focused on current research, only studies published in the last 12 years (2002-2014) were included in the review. Seminal studies prior to 2002 and books (typically secondary sources) were consulted for developing the study's conceptual framework, theoretical underpinnings, and its place in the historical context of number sense in education (e.g., Dantzig, 1954; Gallistel & Gelman, 1992; Markovits & Sowder, 1994).

Once research studies meeting the above-mentioned criteria were located, the following additional inclusion criteria were implemented.

1. The study's main focus was on either examining number sense as a whole construct or examining constructs of number sense; and
2. The study's dependent variable involved a measureable or coded student learning outcome, such as student achievement, student learning, and/or student understanding.

Studies that investigated number sense in children beyond fifth grade, studies that focused on a topic other than number sense (e.g., technology as the main topic with number sense as a secondary topic), and textbook analyses were excluded from the current literature review.

Themes in the Literature

In the process of locating studies and determining whether to include or exclude them in the review, a total of 16 studies met the above-mentioned inclusion criteria in the current review and two major themes emerged: (1) Number sense measures (assessments for determining number sense as a predictor of achievement and for sorting out constructs of number sense), and (2) Number sense interventions. During the 2002-2014 timeframe, the cognitive psychology literature mainly studied approximate quantity discrimination (i.e., analog magnitudes) and its relationship to mathematics symbols that provide access to precise representations of quantity, counting, and the number system. The mathematics education field seemed to move from an initial descriptive research phase to prediction (using correlational designs) to improvement (using experimental designs testing various interventions).

Number Sense Measures: Assessing Number Sense and Its Constructs

Through a systematic search of the number sense literature, ten articles with a focus on number sense measures that met the inclusion criteria for the review were located and analyzed. These ten articles are discussed in the following sections and referred to as “the assessment studies.” All 10 studies were nonexperimental studies that either investigated the reliability and validity of number sense measures and/or examined the correlation between early number sense and later mathematics learning outcomes through the use of number sense measures. Though these were the main purposes of these assessment studies, understandings of the constructs of number sense emerged by operationalizing number sense for the tested measures. Hence, the following sections are organized around these themes within the assessment studies literature: (1) the correlation between number sense and mathematics achievement, (2) the reliability and validity of number sense measures, and (3) the process of operationalizing the number sense construct.

Number Sense and Mathematics Achievement

Of the 10 assessment studies, six studies (Geary, 2011; Jordan et al., 2006, 2007, 2009, 2010; Locuniak & Jordan, 2008) investigated the predictive relationship between early number sense (e.g., number sense competencies in preschool or kindergarten) and later mathematics achievement learning outcomes (e.g., mathematics achievement in third grade). In order to explore the relationships between early mathematics learning and later mathematics learning as well as identify the key predictors of students' growth and learning of mathematics, the six studies used longitudinal panel studies that followed one group of students over three or more years. All 10 assessment studies were based on the theory that the early number sense competencies play a major role in children's mathematics learning, and the identification of children's learning needs can serve to design early interventions that prevent later difficulties in mathematics.

Number sense as an early predictor of mathematics achievement. Five of these six longitudinal studies were part of the Children's Math Project led by Nancy Jordan from the University of Delaware. Jordan et al. (2006) launched the longitudinal panel investigation of children at risk for mathematics difficulties with a group of kindergarten students, assessing the children with the same number sense tasks at four points in their kindergarten school year. In the initial 2006 research, Jordan et al. used a variety of number sense tasks (e.g., counting, number recognition, nonverbal calculation) for assessing and examining kindergarten students' number sense development over the course of the school year. This assessment was used throughout the longitudinal study (Jordan et al., 2007, 2009, 2010) and later named the Number Sense Brief measure (Jordan et al., 2010). In addition to the Number Sense Brief, the researchers began using the Woodcock Johnson III Calculation and Applied Problems subtests with the first grade students to measure mathematics achievement at the end of first grade (then again in subsequent studies through the end of third grade). Their findings indicated that number sense is a powerful predictor of later mathematics outcomes and weak number sense becomes cumulative as students progress through school (Jordan et al., 2009, 2010). Additionally, counting, number relationships, and basic operations emerged as uniquely predictive constructs within number sense for success in mathematics learning (Jordan et al., 2009, 2010; Locuniak & Jordan, 2008).

Similar to the Jordan et al. studies, the purpose of Geary's (2011) research was to identify the quantitative competencies of first grade students that predict the mathematics achievement and growth of students through fifth grade through a predictive longitudinal panel study. Also using a variety of number sense tasks (Number Sets test, violations of counting rules test, number line task, and numerical operations task), findings supported the Jordan and colleagues' results indicating that counting procedures, number knowledge, and basic operations are particularly important in predicting students' mathematics achievement. Further, similar to the number sense tasks and findings in the Jordan and colleagues' studies, Geary also used a Number Sets Test and a number line task, both of which moved beyond assessing students' number recognition and naming and assessed students' fluency in attaching Arabic numerals to small quantities as well as students' knowledge of the number line. Geary's findings suggest that mapping numerals onto quantities and mapping numbers onto the mathematical number line may be critical to early number skills that impact later mathematics achievement. Hence, the findings from Geary's (2011) research supported and extended Jordan et al.'s (2008, 2009, 2010) findings that specific early number sense skills correlated with later mathematics achievement.

Screening for difficulties and early intervention. All six studies discussed in this section indicated a strong and significant relationship between early number sense and later mathematics learning. Hence, early mathematics intervention in kindergarten and first grade has the potential to screen students for mathematics learning difficulties and provide early intervention, thereby, mediating the long-term effects of weak number sense. The potential early mathematics intervention has for students' mathematical development highlights the need for the availability of early number sense screening tools for educators. Jordan et al. (2010) tested the predictive validity of the Number Sense Brief measure, however, other studies (Chard et al., 2005; Clarke & Shinn, 2004; Lago & DiPerna, 2010; Malofeeva, 2004) examined the reliability, validity, and sensitivity of experimental early mathematics measures for the purpose of early identification of mathematics difficulties and/or formative evaluation of students' progress in mathematics.

Reliability and Validity of Number Sense Measures

Four of the 10 assessment studies located for this literature review (Chard et al., 2005; Clarke & Shinn, 2004; Lago & DiPerna, 2010; Malofeeva et al., 2004) focused their research on investigating the reliability and validity of number sense measures. All of these studies framed their purposes within the context of the need for reliable and valid measures for early childhood educators' identification of students who are likely to struggle with later mathematics learning.

The Clarke and Shinn (2004) study built on previous research of the validated and standard mathematics curriculum-based measurement (M-CBM) by adjusting the measure for floor effects (which made the test unusable in kindergarten and first grade) and creating the early mathematics curriculum-based measurement (EM-CBM). This assessment was made up of four measures of number sense: oral counting, number identification, quantity discrimination, and missing number. Clarke and Shinn tested the measure on 52 first-grade students against several criterion measures and found that the four experimental measures had sufficient evidence of reliability, validity, and sensitivity. Chard et al. (2005) replicated the Clarke and Shinn study to support their findings on the EM-CBM while also extending the EM-CBM assessment to include a set of other measures related to number sense including counting, counting on from an identified number, count bys, and number writing. Chard et al.'s results corroborate the reliability and validity findings of the EM-CBM with first grade students. In addition, Chard et al.'s sample included 168 kindergarten students and 207 first grade students, thereby providing evidence that the EM-CBM also holds validity with kindergarten students. While Clarke and Shinn's study was mainly an initial reliability study, Chard et al. also examined the predictive and concurrent validity of the measure, which the results of their study indicated that the EM-CBM could be used to predict achievement in later grades and would be a useful measure for identifying at-risk students.

While the Clarke and Shinn (2004) research tested their measure on first grade students and Chard et al. (2005) confirmed the reliability and validity of the measure with first grade students and also kindergarten students, Malofeeva et al. (2004) tested a measure for preschool students. Malofeeva et al. evaluated the reliability and validity of the Number Sense Test, a measure that assessed six number sense skills that are purported to develop during the preschool years: counting, number identification, number-object correspondence, ordinality, comparison, and addition-subtraction. Similar to other assessment studies, Malofeeva et al. refined and extended previous preschool number sense tests. Their sample consisted of forty 3- to 5-year-old children in a Head Start preschool. An instructional condition was used in the study in order to assess the measure's validity of pre- to posttest improvements in instructed number sense skills. Malofeeva et al.'s results indicated the Number Sense Test's internal consistency and validity as a pre- to posttest measure.

Lago and DiPerna's (2010) investigation of a set of number sense tests also examined internal consistency and test-retest reliability. Their number sense measure was based on the previous studies and was made up of number-related tasks that appeared in at least 20% of the literature they reviewed for the study. Hence, their assessment battery for number sense included counting objects, counting aloud, quantity discrimination, number identification, measurement concepts, nonverbal calculation, and estimation. Some Lago and DiPerna results supported and extended reliability and validity findings of other assessment studies (Chard et al., 2005; Clarke & Shinn, 2004; Jordan et al., 2006). However, counting objects and estimation tasks had weak communalities, possibly because of low variable reliability. Measurement concepts, nonverbal calculation, and estimation had low internal consistency reliabilities. Measurement concepts, number identification, and quantity discrimination had low item ceilings, therefore, the range of difficulty was not adequate for kindergarten students. Hence, the Lago and DiPerna study highlighted the importance of interpreting students' results on these tasks with caution, conducting additional research on these various number sense tasks, and revising some of these tasks to improve internal consistency and reliability.

Lago and DiPerna (2010) indicated that examining the reliability of the number sense tasks was one of two purposes for their study. Their primary purpose was to use the number sense assessment tasks as a means for examining the structure of the number sense construct, specifically whether it is a unitary or multidimensional construct. Although their study was the only study of the ten assessment studies that specifically stated this purpose, patterns of common number sense constructs emerge when analyzing the tasks used for number sense assessments.

Operationalizing Constructs of Number Sense

The 10 assessments studies included in this literature review were chosen due to their explicit discussion of number sense and its constructs. In order to measure the complex construct of number sense, researchers had to identify its components, thereby operationalize constructs of number sense. The researchers of these assessment studies used previous number sense measures (e.g., Chard et al., 2005), revised and built on existing number sense tasks (e.g., Clarke & Shinn, 2004), and/or created new measures to test their operational definition of number sense (e.g., Geary, 2011). Analyzing which constructs of number sense that each study operationalized and tested provides insight into the common operational definitions of number sense as well as the unique and not as common definitions of the construct.

Counting (including verbal counting, counting procedures for solving problems, and/or counting objects), quantity discrimination, and number combinations (combining amounts, addition and subtraction, story problems, and/or use of counting procedures to solve problems) tasks were used in all 10 of the assessment studies of number sense measures. Number identification tasks were present in all studies, except Geary (2011). The EN-CBM (Chard et al., 2005; Clarke & Shinn, 2004) and Geary studies had either a missing number task or violation of counting rules task as a way to assess students' counting skills, in addition to simple oral counting. In six of the 10 assessment studies, estimation tasks were used in the number sense measures. Geary's study was one of those six studies testing estimation tasks, however, unique to his study, a number line was used for the estimation task rather than a set of dots or other objects. Hence, counting, quantity discrimination, number combinations, number identification, and estimation were commonly used in operationalizing number sense and assessing children's number sense.

In addition to these common constructs, Lago and DiPerna (2010) also assessed students on 20 items worth of measurement concepts (i.e., comparing taller, shorter, higher, lower), though their results indicated that these tasks' reliability and validity did not hold up as well as other constructs. Jordan and colleagues (Jordan et al., 2006, 2007, 2009, 2010; Locuniak & Jordan, 2008) used number patterns tasks in their studies, which could provide important information to supplement the tasks on counting.

The Number Sets test (Geary, 2011) was a unique extension of the other counting, number combinations, and number identification constructs. In the Number Sets test, a target sum (either 5 or 9 in the form of an Arabic numeral) was presented to the child along with five rows of domino-like rectangles with different combinations of objects and numerals. The child was asked to "circle any groups that can be put together to make the top number" (p. 1542). According to Geary, this type of measure assesses not only students' subitizing and number combination abilities, but also their ability to map Arabic numerals onto representations of small quantities.

Also distinctive in Geary's (2011) number sense measures was the number line task for estimation. In this task, children were presented with a blank number line and a target number. They were asked, in 24 instances, to mark the line where the target number should be located. Geary explained that children's marks on the number lines may reflect how they represent approximate large numerical magnitudes. Both the Number Sets test and the number line task draw upon students' understandings of the links between their nonverbal number knowledge and the symbols used to represent this knowledge.

Synthesizing the current operational number sense definitions informs the current study's measures for number sense. Counting, quantity discrimination, number combinations, number identification, and estimation are common constructs across multiple measures in a variety of studies. Missing number, violations of counting rules, and patterns tasks should also be considered for future number sense measures. Additionally, Geary's (2011) measures that link the nonsymbolic and symbolic number knowledge could be a critical aspect of measures for future studies in mathematics education. It could be argued that the Number Sets test and the number line task are assessing both students' symbolic understandings of number and their "number sense access" (Wilson et al., 2009).

Wilson et al. (2009) explained that the broad definition of number sense used in mathematics education is based upon and includes cognitive neuroscientists' and numerical cognition researchers' more specific definition of number sense as the nonverbal and nonsymbolic aspects of humans' intuitive understanding of quantities. For this reason, number sense has most often been assessed with symbolic tests. Wilson et al. argued that including nonsymbolic measures on number sense tests would more accurately assess number sense, while symbolic measures and measures that link the nonsymbolic and symbolic representations as in Geary's (2011) study would more accurately assess number sense access. Though nonsymbolic measures are frequently used in cognitive neuroscience studies (Wilson et al., 2009), these new and unique measures within the mathematics education literature could inform researchers and educators about children's understandings of the links between the nonsymbolic and symbolic representations of number.

Number Sense Interventions and Programs

The assessment studies laid important groundwork in the field for exploring and operationalizing components of number sense as well as for providing evidence of reliable and valid measures for assessing these constructs. Additionally, the correlational findings of the longitudinal studies described the critical importance of number sense in students' early mathematics learning. These findings set up the purpose for exploring and improving number sense interventions in elementary classrooms.

A search of the literature led to six empirical studies that met the inclusion criteria for this literature review. For the purposes of the following discussion, these six studies will be referred to as "the intervention studies." Though this review will critically analyze these six studies, it is important to note that there are other number sense curricula, programs, and interventions that have been used and tested in classrooms (e.g., Griffin et al., 1994; Markovits & Sowder, 1994; Yang & Tsai, 2010), though their publications did not meet the criteria for this review.

Five of the six intervention studies were experimental studies based on the pretest-posttest control group design. The Wilson et al. (2009) study was a quasi-experimental study and used the two-period cross-over design. Three of the six studies were number sense program curriculum interventions (e.g., a set of lessons or activities) while the other half were game-based curriculum interventions (e.g., a game was the major component of the curriculum). Of these studies, 83% had students participate in the intervention 2-3 times per week with only one study meeting less frequently. The length of each session was similar across all groups. The length of the intervention varied from less than one month (33% of studies), 1-6 months (50%), and 6-9 months (17%). These specific pieces of implementation of the interventions have the potential to inform future research on number sense interventions.

Program Curriculum

Three studies (Aunio, Hautamaki, & Van Luit, 2005; Dyson, Jordan, & Glutting, 2013; Jordan, Glutting, Dyson, Hassinger-Das, & Irwin, 2012) tested the effects of number sense interventions, in the form of a program (i.e., a set of lessons or activities). Aunio et al.'s participants were 45 preschool students and were administered the intervention in small groups of 5-6 children with approximately 60 sessions. Their intervention was based on two established programs (*Let's Think!* and *Maths!*). Dyson et al. conducted their researcher-developed intervention with 121 kindergarten students in small groups (consisting of 24 lessons). Jordan et al. also implemented a researcher-developed intervention (based on operational definitions of number sense) with 44 kindergarten students in small groups (24 lessons).

The three intervention studies also differed in terms of their number sense focus. Aunio et al. (2005) used two programs that focused on the thinking aspect of number sense. In other words, *Let's Think!* was designed to develop students' general mathematics thinking skills, such as their metacognitive abilities for problem solving while *Maths!* purpose was to stimulate transfer of more specific math thinking skills by using diverse problems. Aunio et al. explained their reasoning for using the programs as a method for developing children's number sense: "We assumed that combining these two programmes would generally accelerate young children's number sense, since the development of general mathematical thinking abilities supports the development of specific mathematical thinking skills and vice versa" (p. 135). To test children on the programs' effects on preschoolers' number sense, Aunio et al. used the Early Numeracy Test, which consisted of a Relational Scale of 20 items (comparison, classification, one-to-one correspondence, and seriation) and a Counting Scale of 20 items (number words, structured counting, resultative counting, and general understanding of numbers). In their article, number-word sequence skills were specifically noted as a critical precursor to counting, and although it was not clear how this was taught to children in the intervention, it does appear as an important component of their number sense measures.

Rather than a main focus on thinking skills, Dyson et al. (2013) and Jordan et al. (2012) targeted counting, comparing, and manipulating sets in their intervention with kindergarten students. Additionally, their articles laid out with specificity the content of the intervention lessons. Hence, it was clear that their number sense measure, the Number Sense Brief (Jordan et al., 2009), assessed the specific number sense constructs that were taught in the intervention (counting, number recognition, number knowledge, nonverbal calculation, story problems, and number combinations).

In all three studies, precursors to counting and counting skills were a key part of their interventions and number sense measures. In comparing the three interventions, it was evident that number sense was conceptualized as embodying a thinking ability in Aunio et al.'s (2005) study, while Dyson et al. (2013) and Jordan et al. (2012) delineated specific skills tied to number sense and operationalized the construct. These two different approaches to number sense interventions and measures have implications for future research. While the thinking aspect of number sense is important (as it influences students' fluency and flexibility with numbers as well as their transfer of knowledge to novel situations), it may be important in future studies to clearly tie the thinking aspect of number sense to the more measurable constructs of number sense such as counting, nonverbal calculation, and problem solving.

Game-Based Curriculum

Researchers of the game-based intervention studies also examined the impact of their interventions on preschool and kindergarten children's number sense. In contrast to the program-based intervention studies, these three game-based intervention studies narrowed their focus to more specific constructs of number sense, although to varying degrees.

Ramani and Siegler (2008) focused their simple linear board game intervention on number line estimation with numerical magnitude comparison, counting, and numeral identification tied to this construct. They tested the intervention with 124 preschool students, and their results indicated that the effect of playing the linear number board game increased students' proficiency on numerical tasks.

Baroody, Eiland, and Thompson's (2009) study focused on the number after construct (e.g., what number comes after 5?) and the prerequisites to mental arithmetic. They implemented the intervention with 80 preschool students in two phases: (1) manipulatives and game-based activities with dice, and (2) discovery-based computer software program for training preschoolers in mental arithmetic. Using the Test of Early Mathematics Ability-3 as a pretest-posttest measure, researchers found that general achievement and fluency with n+0/0+n combinations improved significantly, but n+1/1+n combinations showed improvement if success included slow or counted answers. Baroody, Eiland, and Thompson (2009) interpreted the findings to mean that mental arithmetic training might be better suited to older participants who are more developmentally ready.

Wilson et al.'s (2009) research narrowed in on the concept of number sense access (linking symbolic and nonsymbolic representations of number) with numerical comparisons, number line understandings, and fluency of basic facts as other factors involved in the intervention and measures. Researchers developed an adaptive number sense computer game to improve students' performance on symbolic numerical comparison tasks. They tested the intervention with 53 kindergarten students and assessed their learning with symbolic and nonsymbolic measures for number sense. Results showed improvement on the symbolic numerical comparisons, but no improvement on the nonsymbolic measures. Wilson et al. concluded that the intervention improved students' number sense access, not number sense, meaning the intervention aided students in their linking of symbolic representations to their representations of quantity.

Of the six intervention studies, the three program-based curriculum interventions had multiple components involved in their interventions, which makes it difficult to specify exactly which components had significant impacts on students' number sense development and mathematics understanding and achievement. Although some researchers may argue that the constructs of number sense are so intricately interwoven that interventions and assessments cannot and should not be isolated and tested (Greeno, 1991), others make the case that isolating key instructional factors will better determine the constructs that impact number sense instruction, inform future interventions, and provide educators with more information about students' specific difficulties and strengths (Dyson et al., 2011; Ramani & Siegler, 2008; Wilson et al., 2009). In this review, the three game-based intervention studies entailed a narrower focus on specific number sense constructs, which seemed to lead the researchers to be better equipped to theorize as to why and how a specific intervention was impacting students' number sense development.

A synthesis of the number sense assessments and interventions literature underscores the complexity of the broad term "number sense" and the multiple aspects of the construct whether viewed through the cognitive psychology or mathematics education lenses.

Building on the Current Knowledge in the Field

Placing this Study in the Broader Literature

The first section of this literature review, "Current Understandings of Number Sense and Its Constructs," described a number of cognitive psychology studies that provided insight into the cognitive processes of children's development of number sense. Many of these studies were conducted in a controlled laboratory setting and were descriptive studies of children's numerical cognition (i.e., did not test an instructional intervention), hence, they did not inform the design and methods of this study. The numerical cognition studies did, however, provide the solid basis for the conceptual framework and informed the theoretical basis of this research study. As assessments and interventions are designed and tested, it is critical that they are based on the findings and interpretations of children's number sense and numerical competence development.

The second section of this literature review, "Number Sense Measures," informed this study in two ways: (1) designated reliable and valid measures for assessing students' number sense, and (2) further defined and operationalized number sense. The studies reviewed in this section mainly informed the operational definition of number sense used in this study as well as the instruments used to assess students' number sense.

Comparing the Intervention Studies' Methodologies

The third section of the review, "Number Sense Interventions and Programs," informed the design and methods for this study. As previously noted, a review of the literature, between 2002 to 2014, highlighted the direction of the lines of research in number sense within mathematics education. Mathematics education researchers seem to have moved from an initial descriptive research phase to prediction (using correlational designs) to improvement (using experimental designs testing various interventions). This study builds on the improvement phase of the line of research by designing and testing a verbal counting intervention at the classroom level.

An analysis of the intervention studies met the second objective of this literature review, which was to discuss the strengths and weaknesses in previous research, particularly with regard to the designs and methods of the studies. The six empirical studies in this section of the review were systematically coded for sample characteristics (sample size, grade level, demographics) and research design (design methods, measures, setting, type of intervention, threats to internal validity) in order to recognize strengths and weaknesses of the intervention studies.

Sample characteristics. All of the intervention studies in the review (*n* = 6) had more than 30 participants in their samples. Of the six studies, four were in the 30-100 participants range while two of the six studies had between 100-300 participants. Future intervention studies should continue to include sample sizes above 30 and closer to *n* = 100 in order for findings to be generalizable, especially because studies conducted in the classroom setting naturally involve threats to internal validity.

Of the six intervention studies, three were conducted with preschool students and three with kindergarten students. Researchers are developing a solid base for the early preschool and kindergarten grade levels. It will be critical for future research to build on this base and study number sense interventions for first grade and beyond. This study took a sample from second-grade students in order to meet this need in the field.

Five of the six studies were conducted in low-SES, urban settings. This emphasis on low-SES students was purposeful in the current literature because of the call for closing the achievement gap. However, future studies will need to be conducted within diverse demographics and settings in order to gain a fuller picture of number sense development and in order to generalize results from both assessment and intervention studies. The U.S. education system experiences not only a problem of achievement gaps, but also a problem of overall low achievement in mathematics—both areas need to be addressed. Future assessment studies that are based in schools with all types of income levels will be important in determining the interconnectedness of number sense constructs in general. Additionally, it will be important to study and assess students with well-developed number sense, not just students struggling with mathematics. Using a sample of diverse students (in terms of ability, SES, region, area, race/ethnicity, and language groups) will continue this line of research into the number sense construct and the success of interventions and instructional strategies for all students.

Research design. Five of the six studies were experimental research, while one study (Wilson et al., 2009) was a quasi-experimental two-period crossover design because random assignment was not possible in the specific school setting. The five experimental studies were all based on the pretest-posttest control group design (with three studies using a delayed posttest in addition to the posttest following the intervention). The Wilson et al. quasi-experimental study used a cross-over design that proceeded in the following steps: (1) a pretest was administered to all groups, (2) one of two interventions was provided (e.g., Group 1: math intervention, Group 2: reading intervention), (3) a posttest was administered to all groups (mid-study), (4) the interventions were swapped (e.g., Group 1: reading intervention, Group 2: math intervention), and (5) the posttest was re-administered.

The designs of these studies differed among each other in their interventions more than in their design methods. Three of the six studies were number sense curriculum interventions while the other half were game interventions. Of these studies, 83% had students participate in the intervention 2-3 times per week. Only one study (Wilson et al., 2009) met less frequently, though it was unclear how often and when they met students for the intervention. The length of each session was similar across all groups. The length of the intervention varied from less than one month (33%), 1-6 months (33%), and 6-9 months (33%).

History and maturation were two threats to the internal validity of the study lasting nine months due to the length of the intervention. The Hawthorne Effect was a threat to the internal validity of the Dyson et al. (2013) study because the "business as usual contrast group" had no control treatment, therefore, the results of the effects on the experimental group could be confounded by their special treatment. Instrumentation and regression were common threats to internal validity in all of the intervention studies (100%, $n = 6$) due to the pretest-posttest nature of the experimental design.

While the quasi-experimental Wilson et al. (2009) study did not randomly assign students to groups, the use of the two-period crossover design controlled for threats to internal validity that would have been otherwise present in a pretest-posttest control group design without random assignment because both groups received both treatments, just in a varied order and on two distinct topics (math and reading). Although some controls were in place, the main threat to internal validity for this study was differential selection due to lack of random assignment to treatment groups.

A comparison of research design and methods indicates that there is a need for research into classroom-based (as opposed to small group or one-on-one) instructional interventions. Additionally, with a growing foundation of research on preschool and kindergarten number sense intervention, understanding the impact of number sense interventions on older children is essential. Finally, the pretest-posttest control group experimental design with $n > 30$ participants is the most commonly used design for the intervention studies. While this design's strengths—such as random assignment and pre- to posttest growth—should inform future research, the design does not lend itself well to classroom research. Typically, random assignment of students in classroom-level research often poses a problem. Various solutions are possible. One is to randomly assign students within one classroom to two intervention groups. Another solution could be to use quasi-experimental methods and control for nonequivalent groups by using pretests and matching intact classrooms.

Research Questions

The third objective of the literature review was to draw conclusions based on the review and to develop research questions for the current study. The research base is growing and supports the notion that early number sense is critical to students' later mathematics achievement (Geary et al., 2013; Jordan et al., 2009, 2010). Number sense research has the potential for providing teachers with an understanding of students' nonverbal/nonsymbolic number knowledge, its development, and its interactions with the symbolic number system. Using this research in teaching and learning could inform instruction and make it more responsive to students' learning needs. For example, information about students' number sense could not only serve as a screening tool for intervention, but could also provide teachers with information that could help them build instruction on students' innate number sense while bridging the innate number sense with the symbolic systems traditionally taught in schools.

Several findings within the literature reviewed here point to counting as being an important component of number sense and impacting students' later mathematics outcomes (Aunio et al., 2006; Aunola et al., 2004; Jordan et al., 2010). Additionally, counting tasks were used in all ten of the assessment studies reviewed, indicating that counting is a key construct of number sense. Finally, Wilson et al.'s (2009) study opened a discussion about number sense access, the linking of nonsymbolic and symbolic knowledge. The researcher of this study hypothesized that counting, and specifically verbal counting, plays an important role in number sense access in later grades, such as second grade. Hence, a better understanding of the influence of verbal counting on children's number sense development (specifically its influence on children's number system knowledge) could potentially contribute to specifying the critical skills embedded in the complex definition of number sense, designing early intervention programs and instructional practices, and extending the knowledge base of early number sense.

Therefore, the purpose of this study was to more closely examine the counting construct of number sense and explore the variations in second-grade students' number sense development as they engaged in a counting-focused instructional treatment for differing amounts of time. The research questions guiding this study were:

> Overarching Research Question: In what ways does a counting-focused instructional treatment (that focuses on patterns in the number system and relationships among numbers) influence, change, and develop second-grade students' number sense (specifically, computational fluency, strategies for solving story problems, and number line estimation)?
>
> 1. What are the variations in number sense development when students engage in counting interventions for differing amounts of time (3 weeks, 6 weeks, and 9 weeks)?
>
>   a) What are the variations among three intact classes?
>   b) What are the variations for individual students within each class?
>
> 2. What are the variations in number sense development for one low-achieving student and one high-achieving student?

CHAPTER III

METHODS

The researcher proposed that the counting construct of number sense plays a key role in developing second-grade students' number system knowledge. Hence, a better understanding of the influence of verbal counting on children's number sense development (in particular, its influence on children's number system knowledge) could contribute to specifying the critical skills embedded in the complex definition of number sense. This knowledge is important for designing early intervention programs, identifying effective instructional practices, and extending the knowledge base of early number sense. Therefore, the purpose of this research study was to explore the variations in second-grade students' number sense development as they engaged in a counting-focused instructional treatment for differing amounts of time.

Research Design

To investigate the research questions, the researcher used an embedded mixed methods approach by collecting, analyzing, and mixing quantitative and qualitative data (Creswell & Plano Clark, 2011). Pretest, benchmark, and posttest assessments of 60 second-grade students along with in-depth, task-based interviews with 6 of the 60 students were administered to provide quantitative and qualitative data to investigate the change and development of students' number sense during a counting-focused instructional treatment. Lesson artifacts—including video of the teaching episodes, chart paper with records of class discussions, students' counting journals, field notes, and teaching episodes' lesson plans—were collected and qualitatively analyzed throughout the study to help support, interpret, and extend the quantitative and qualitative analyses.

Table 1 presents an overview of the research questions, data sources used to answer the research questions, and the methods of analysis. To answer research question 1, a quasi-experimental pretest-benchmark1-benchmark2-posttest design was conducted within three intact second-grade classrooms. To answer research question 2, a purposive sample (Bamberger, Rugh, & Mabry, 2012) of six second-grade students (two students from each of the three classes) was selected for participation in task-based interviews to understand students' strategies and number sense and the influences of the intervention

Table 1

Data Analysis Overview

Research questions	Data sources	Data analysis
Overarching Research Question: In what ways does a counting-focused instructional treatment (that focuses on patterns in the number system and relationships among numbers) influence, change, and develop second-grade students' number sense (specifically, computational fluency, strategies for solving story problems, and number line estimation)?		
1. What are the variations in number sense development when students engage in counting interventions for differing amounts of time (3 weeks, 6 weeks, and 9 weeks)? a) What are the variations among three intact classes? b) What are the variations for individual students within each class?	Whole-Class Tests Lesson Artifacts	Descriptive statistics Generalized Estimating Equations Graphs of mean test scores at each measurement point (intact classes and individual students) Qualitative analysis (open and axial coding)
2. What are the variations in number sense development for one low-achieving student and one high-achieving student?	Whole-Class Tests Lesson Artifacts TEMA-3 Videotaped Task-Based Interviews	Graphs of individual students' test scores at each measurement Qualitative analysis (open and axial coding)

on low- and high-achieving mathematics students (Goldin, 2000). Two of the six students (one low-achieving student and one high-achieving student from Class 1) were selected as cases to study in-depth to answer research question 2. The qualitative data were embedded within the larger quasi-experiment at each data collection phase of the study (Creswell & Plano Clark, 2011). The major elements of the study are discussed in detail in subsequent sections.

Experimental Variables

The independent variable in this study was the counting-focused instructional treatment. The dependent variable in this study was number sense development—specifically, students' computational fluency, strategies for solving story problems, and number line estimation—as measured by whole-class tests, individual interviews, and lesson artifacts. Covariates included group (Class 1, 2, or 3), gender, race, socioeconomic status, English Language Learner status, and special education services.

Participants and Setting

Sixty second-grade students from three public school classrooms located in one elementary school in the western U.S. participated in this study. All students (including English Language Learners, students with Individualized Education Plans [IEPs], etc.) in the three classes were invited to participate in the study. Of the 71 students invited to participate, 64 returned permission forms (90% response rate). Seven students did not return permission forms, and therefore were not included in the study. Of the 64 students who returned permission forms, four students were not included in the study: Two students opted not to participate in the study; one student moved before the instructional treatment began; and one student's pretest was lost and he was excluded from the study. Of the 60 participating second-grade students, 52% were male, 48% qualified for free or reduced lunch (indicating low SES), and 85% were white. Eight students (13%) had an IEP for special education services. Three students (5%) were labeled as English Language Learners, meaning they participated in English as a Second Language services.

Table 2 provides demographic information for the three classes disaggregated by group. These groups are labeled as Class 1, Class 2, and Class 3 throughout the study. In terms of demographic characteristics, Class 1 had more students requiring special education services than the other two groups (n = 6; 27% of Class 1). There were gender differences between Class 2 and Class 3: 63% of Class 2 was male while 63% of Class 3 was female. Finally, 68% of Class 2's students qualified for free/reduced lunch, indicating low SES, while 36% of Class 1 and 42% of Class 3 had students from low SES homes.

Participants were assigned to the three classes by the school at the beginning of the school year. It was not possible to randomly assign students to treatment groups. Of the 60 participants, 23 students were in Class 1, 18 students were in Class 2, and 19 students were in Class 3 for the majority of the intervention treatment. Class 1 received the instructional treatment first, followed by Class 2, and finally, Class 3 (this is described in more detail in the Procedures). One student was switched from Class 3 to Class 1 after the first Benchmark test. This student was included in Class 1 for instructional treatments. However, in total, he only received six weeks of instructional treatment while his peers in Class 1 received nine weeks of instructional treatment.

Table 2

*Demographic Characteristics of Participants (*N *= 60)*

	Class 1 (*n* = 22)		Class 2 (*n* = 19)		Class 3 (*n* = 19)	
Characteristic (or Variable)	*n*	%	*n*	%	*n*	%
Gender						
Male	12	55	12	63	7	37
Female	10	46	7	37	12	63
Socioeconomic (SES) Status						
Low SES	8	36	13	68	8	42
Average/high SES	14	64	6	32	11	58
Race						
White	18	82	17	90	16	84
Black	0	0	1	5	1	5
Hispanic	2	9	0	0	1	5
Asian	2	9	1	5	0	0
Pacific Islander	0	0	0	0	1	5
English as a Second Language (ESL) services						
ESL Services	1	5	2	11	0	0
No ESL Services	21	96	17	90	1	100
Special education services						
IEP	6	27	1	5	1	5
No IEP	16	73	18	95	18	95

Note. Total percentages are not 100 for every characteristic because of rounding.

Therefore, this student was included in Class 2 for the quantitative analysis due to time of treatment (6 weeks). For the purposes of descriptive statistics and the Generalized Estimating Equations (GEE) analysis, the number of participants in each group was: Class 1, $n = 22$; Class 2, $n = 19$; and Class 3, $n = 19$.

The study took place during the first half of the school year (September to December) in three different second-grade classrooms in one school. All three classroom teachers used their district-adopted mathematics curriculum *Go Math! Grade 2 Common Core Edition* (Houghton Mifflin Harcourt) as the main source of lesson plans. The three classroom teachers followed a similar schedule of units for instruction during the course of the study, which included Chapter 1 Number Sense in September, Chapter 2 Place Value to 1000 in September and October, Chapter 3 Mental Math Addition and Subtraction in October and November, and Chapter 4 Two-Digit Addition in November and December. The teachers conducted their mathematics lessons as usual throughout the course of the study. The researcher taught the instructional treatment in all three classrooms during the mathematics block of time. The counting-focused instructional treatment took place in the meeting area of each of the classrooms where students sat in a circle on the rug.

Sampling Procedures

Students from three intact second-grade classrooms within one school participated in this study, where random assignment to treatment groups was not possible. Although conducting the study in one school limits generalizability of the study results and does not overcome teacher effects, using only one school overcomes the problem of comparing different populations. The demographics of students vary widely from school to school, even within the same, small district. Similarly, teachers within one school have more similar experiences (e.g., school policies, student populations, curriculum, working under one leadership) than do teachers in different schools. Hence, while many factors in a school and classroom setting cannot be controlled, the researcher selected one school as the site for data collection to overcome the problem of comparing widely different populations of students. A pipeline design, with a staggered instructional treatment, was used in order to have comparison groups to strengthen the research design (Bamberger et al., 2012).

In order to better investigate individual children's cognitive processes that occurred over the course of the study, a purposive range sample of students across the three classes was selected for individual task-based interviews (Bamberger et al., 2012). This sample represented the range of number sense knowledge within each classroom (i.e., low- and high-achieving scores on the pretest). To ensure this range, six students were selected after the completion of the pretest. Students' pretest scores were displayed in a box plot grouped by class, and two students from each of the three classes were selected for task-based interviews based on their score's relationship to the median and other students' scores. Within each class, a student with a pretest score above the median and a student with a pretest score below the median were selected for interviews. In addition to considering where students' scores fell in relation to the median and other students' scores, the researcher sought to have a gender balance (i.e., three males and three females) and took into account parents' permission for students to be videotaped (as stated as an option on the IRB letter for permission to participate). Figure 2 shows each class's box plot of pretest scores and where the students' scores fell in relation to the median and other students' scores.

In Class 1, the "high-achieving" student selected for interviews had a score of .57. His pretest score was situated in the upper quartile, six scores above the median. Two students in his class scored higher than him. The "low-achieving" student selected for interviews in Class 1 had a score of .32. Her score was four away from the median within

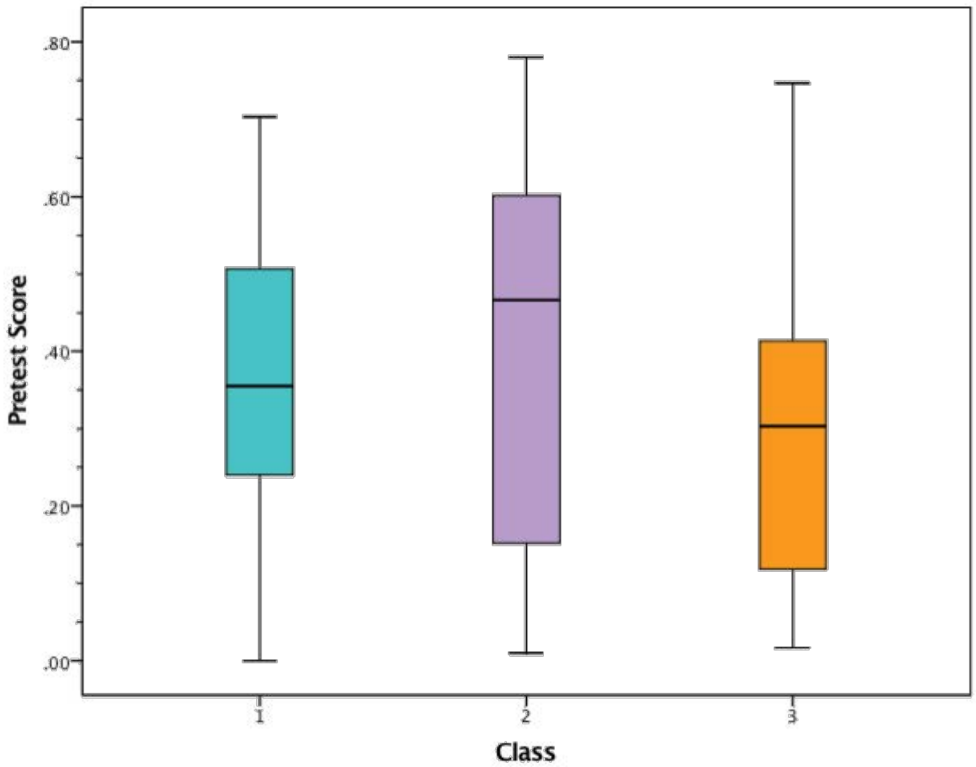


Figure 2. Box plots of student pretest scores by group (Classes 1, 2, and 3).

the third quartile. There were five students in her class that scored lower than this student.

In Class 2, the "high-achieving" student selected for interviews scored .67 on the pretest, which was five scores above the median. Three students in his class scored higher. The "low-achieving" student selected for interviews in Class 2 scored .18 on the pretest, which was four scores away from the median. There were five students in her class that scored lower than her.

The pretest scores in Class 3 were more clustered around the median. The "high-achieving" student selected for interviews scored .43 on the pretest, which was five scores above the median. Four students in her class scored higher. The "low-achieving" student selected for interviews in Class 3 scored .29, which was one away from the median. Despite being closer to the median than the other “low-achieving” students selected for interviews, she was selected for interviewing for several reasons. There were eight students in her class that scored lower than her, and six of those scores were below .15. With six very low pretest scores in the class, the researcher decided to select this student to interview because her score was close to the Class 1 “low-achieving” student’s pretest score.

The other considerations, such as a gender balance (i.e., three males and three females) and permission for students to be videotaped, played a role in the final selection of the six students for task-based interviews. Some of the difficulty in selecting a “low-achieving” student in Class 3 was based on the low pretest scores in the class; another difficulty was that most of these low scores were attached to female students. The researcher sought to include another male in the interviews, however, of the eight students below the selected “low-achieving” student’s score five were female, two of the male students did not give permission to be videotaped, and the other male’s score was very low (.10) and the teacher requested that he not be interviewed for behavior reasons. Hence, another female was selected for interviews and while her pretest score was near the median of her class, it was close to the Class 1 “low-achieving” student selected for interviews.

A purposive sampling technique, rather than a random sample, presents a threat to the statistical conclusion validity (Bamberger et al., 2012). However, the purposive sample technique was selected because the sample size for the interviews was small, and a probability sample could overlook the influence of a counting-focused instructional treatment on students with strong number sense and/or students struggling to develop number sense.

The researcher conducted each task-based interview (24 interviews total). The individual task-based interviews took place in a quiet location (e.g., teacher's lounge, math manipulative room, or hallway) in the school, in order to be free of disruptions for children to have time and space to think and explain their mathematical reasoning.

Instructional Materials

Over the course of a year, the researcher developed and piloted 27 teaching episodes for this study, which are rooted in a constructivist epistemology. The teaching episodes appear in Appendix B. Each teaching episode is based on a number system knowledge focus area (such as magnitude of numbers, estimation, and counting patterns). Each teaching episode consisted of one to three counting sequences and questions to facilitate classroom discussion. Most teaching episodes involved the use of a number grid, an open number line, written counting sequences, and/or other visual materials to highlight key ideas and students' strategies and ideas about patterns and relationships among numbers. The researcher used a large tablet for recording daily sequences and discussions during instruction. Additionally, students had opportunities to write counting sequences in their Counting Journals as a way to individually solidify understanding and/or reflect on the counting sequence and discussions for the day.

While the 27 teaching episodes were developed pre-instruction as a guide for the instructional treatment, and the researcher implemented each teaching episode according to the written lesson plan, the enactment in each classroom (i.e., researcher's questions, students' discussions, highlights of big ideas) contained differences in response to students' interactions and those interactions with the researcher.

Data Sources and Instruments

Data were collected using the following instruments: (a) pretests, benchmark tests, and posttests; (b) task-based interviews and a standardized TEMA-3 assessment; and (c) lesson artifacts including records of instructional activities and field notes. Data were triangulated using these multiple sources. The following sections describe the data sources in further detail.

Pretests, Benchmark Tests, and Posttests

There were three types of whole-class tests administered during the study: a pretest, two benchmark tests, and a posttest. The pretest scores served as baseline data and were administered to all three classes prior to Class 1's instructional treatment. The first benchmark test collected data on how students' learning in Class 1 changed during the first three weeks of the instructional treatment. The first benchmark test also provided comparison data for the other two classes that had not yet received the instructional treatment. The second benchmark test provided information on students' progress in Classes 1 and 2, while providing comparison data on Class 3. The posttest provided data on students' learning progress in all three classes at the conclusion of the instructional treatments.

The pretest, benchmark tests, and posttest included the following sections: (1) The Assessment of Math Fact Fluency, (2) Story Problem Situations, and (3) Number Line Tasks. All of these tests were administered to all students within one second-grade class at the same time (i.e., whole-class assessment). Students in all three classes participated in the test within the same week. A sample of a test format appears in Appendix C.

Assessment of Math Fact Fluency. The Assessment of Math Fact Fluency (Fuchs, Hamlett, & Powell, 2003) is a battery of addition and subtraction problems (sums up to 18 and minuends up to 18) that measure computational fluency. Students have one minute on each addition fluency measure and one minute on each subtraction fluency measure to complete as many problems as they can with a pencil. The coefficient alpha for calculation fluency in third grade, on a tested sample, was equal to or greater than .89 for each subtest (Fuchs et al., 2003; Locuniak & Jordan, 2008).

Story problem situations. The story problem situations section included four different cognitively guided instruction problem types (Carpenter et al., 1999; Hiebert et al., 1997) and CCSSM problem situations (CCSSI, 2010). Story problem situations were used to better understand how students used their number sense foundations to solve problems. Story problems are used in a variety of number sense assessments (e.g., Jordan et al., 2010). Multiplication, part-part-whole, subtraction, and join-change-unknown problem types (Carpenter et al., 1999) were used for all tests in this study. Prior to the study, the researcher tested this instrument in a second-grade classroom during a one-year pilot project, which helped to select the problem types and the number choices for each problem.

Number Line Tasks. The Number Line Tasks came from Geary et al.'s (2013) longitudinal study. The Number Line Tasks assess students' knowledge of the number line and their estimation abilities, specifically their understanding of where numbers fall in relation to one another. In this task, students were presented with a blank number line from 0 to 100 and a target number. They were asked to mark the line where five target numbers should be located. Geary (2011) explained that children's marks on the number lines may reflect how they represent approximate large numerical magnitudes. His findings suggest that mapping numbers onto the mathematical number line may be critical to early number skills that impact later mathematics achievement. The number line task draws upon students' understandings of the links between their nonverbal number knowledge and the symbols used to represent this knowledge.

Task-Based Interviews and TEMA-3 Assessment

Six selected students each participated in four task-based interviews focused on their strategies for solving problems on the whole-class tests. Two TEMA-3 interviews were administered with the six students at pretest and posttest measurement points to yield a standardized, overall ability score.

Task-based interviews. Task-based interviewing is a qualitative method used to observe and interpret mathematical behavior (Goldin, 2000). As exhibited in this study's conceptual framework, developing number sense is complex and is demonstrated by more than just getting the correct solution. Strategies and reasoning involved in solving computation, story problems, and number line tasks are key in developing strong number sense and learning mathematics. The task-based interview method values the complexity of children's thinking and makes it possible to focus on students' processes for solving mathematics problems (Goldin, 2000). Rather than use a think-aloud technique (Ericsson, 2006), in which the student explains her strategy while solving the task during the task-based interviews, the retrospective interview technique was used to elicit students' knowledge, strategies, and processes after the tasks were complete. Retrospective interviews avoid the cognitive workload of solving a task while simultaneously explaining strategies, and instead elicit more information about strategy use (Feldon, 2010; Taylor & Dionne, 2000).

During the task-based interviews, the researcher provided a copy of the student's original assessment (i.e., the pretest, benchmark tests, and posttest) and asked questions about predetermined problems within each subset of the test. The questions (see Appendix D) were designed to elicit the student's account of how he or she solved the problem. From that starting point, the researcher asked each student follow-up or clarification questions. The follow-up questions were deliberately nonstandardized in order for the researcher to better understand individual variation (Ginsburg, 1997). Students were encouraged to explain their thinking, show their thinking (with numbers, drawings, manipulatives, and/or other representations), and/or describe what was difficult or easy for them. Eliciting students' strategies and reasoning for solving story problems, computation problems, and other number sense tasks reveals aspects of students' thinking beyond the correct/incorrect information (Cai, 1995, 2000).

The task-based interviews allowed the researcher to infer students' uses of number sense in the process of solving computation, story problem, and number line tasks. Since each child participated in four interviews, children's responses provided insight into their number sense and thinking processes over time and provided information about how and why students' learning trajectories differed when developing number sense. Each interview was videotaped and segments of the videotaped interviews for one low-achieving and one high-achieving student were transcribed for in-depth analysis.

TEMA-3 assessment. The *Test of Early Mathematics Ability – 3rd Edition* (TEMA-3) was administered individually to the six interviewed students during the pretreatment phase of the study as well as at the conclusion of the treatment. The purpose of this assessment was to gain a fuller understanding of students' learning and progress over the course of the 12-week study. The TEMA-3 (Ginsburg & Baroody, 2003) is designed to measure overall mathematical knowledge and yields an overall ability score. Although the TEMA-3 is not specifically described as a measure of number sense, many of the items on the scale assess skills related to number sense (e.g., counting, quantity discrimination, number combinations). Since the test assesses overall mathematics ability, it was used to examine changes in students' mathematics achievement over the course of the study. The test has strong internal reliability, criterion validity, and content validity (Bliss, 2006).

Lesson Artifacts: Records of Instructional Activities and Field Notes

Throughout the study, the researcher collected lesson artifacts tied to the whole-class counting-focused instructional treatments. Records of instructional activities included the Teaching Episodes lesson plans, records of the in-action activities that took place (i.e., chart paper recording class discussions and video of each Teaching Episode), students' Counting Journals, and the researcher's field notes on what happened during each instructional treatment teaching episode. All whole-class, counting-focused instructional treatment episodes were videotaped with one camera. The camera was positioned to capture the students in the meeting area and directed at the chart paper where the researcher recorded number sequences and students' ideas. The videos were not used to analyze the lessons, rather, the purpose of the videos was to aid the researcher to remember what occurred during specific teaching episodes. In addition, the researcher's field notes were collected to help explain how, why, and when changes in understanding took place in each class during the instructional treatment (Steffe & Thompson, 2000), thereby providing data to answer research question 1.

Software and Hardware

Data collection software for this study included Microsoft Excel for organizing test scores and iPhoto to save video and audio recordings. SPSS was used for quantitative data analysis. The hardware needed for the study included the researcher's personal laptop computer, video recorder, and an external hard drives for saving video data.

Procedures

The study was conducted in three phases: (1) pretreatment, (2) instructional treatment, and (3) posttreatment. Figure 3 provides an overview of the three phases and a visual representation of the pipeline design for the staggered instructional treatments.

Phase 1: Pretreatment

In the pretreatment phase, the researcher met with the district mathematics coordinator to select a school for the study, met with the principal and three second-grade teachers to gauge interest in participating, and obtained appropriate Institutional Review Board (IRB) and district approval (see Appendix A). In May 2015, the researcher met

Study Phases	Instructional Treatment		
Pretreatment	Pretest (Classes 1, 2, & 3) and Interviews (Task-based and TEMA-3 with 6 students)		
Week 1	Class 1 begins instructional treatment		
Week 2			
Week 3			
Week 4	Benchmark #1 (Classes 1, 2, &3) and Interviews (Task-based interviews with 6 students)		
Week 5	Class 1 continues instructional treatment	Class 2 begins instructional treatment	
Week 6			
Week 7			
Week 8	Benchmark #2 (Classes 1, 2, &3) and Interviews (Task-based interviews with 6 students)		
Week 9	Class 1 continues instructional treatment	Class 2 continues instructional treatment	Class 3 begins instructional treatment
Week 10			
Week 11			
Posttreatment	Posttest (Classes 1, 2, & 3) and Interviews (Task-based and TEMA-3 with 6 students)		

Figure 3. Pipeline staggered instructional treatment.

with three classroom teachers who agreed to participate in the study to discuss procedures of the study and randomly select which classrooms would receive nine weeks (Class 1), 6 weeks (Class 2), and three weeks (Class 3) of the counting-focused instructional treatment. Class 2 and 3 teachers understood that while their students would not receive nine weeks of the instructional intervention by the researcher during the study, the teachers would receive all lesson plans at the end of data collection and the researcher was willing to teach those instructional treatments after data collection was complete.

In the pretreatment phase, the researcher administered the pretest to all students in the three classes prior to any instructional treatment. All of the tests were administered to all students at the same time as a whole-class assessment. Students in all three classes participated in the test within the same week. The six students selected for the interviews participated in task-based interviews and the TEMA-3 assessment. All interviews were conducted in one-on-one (interviewer-interviewee) sessions in a quiet, semiprivate location at the school (e.g., the teachers' lounge, the mathematics coordinator's office, or a "pod"—quiet nook off the main hallway—in the school).

Phase 2: Instructional Treatment

In the next phase of the study, instructional treatment, the researcher implemented the counting-focused instructional treatments and administered two benchmark tests and two task-based interviews.

Format of the instructional treatment teaching episodes. Students participated in the counting-focused instructional treatment three days per week during 15-25 minutes of each class's regularly scheduled mathematics block of time. The researcher taught the instructional teaching episodes, while the classroom teachers continued to use their district-adopted curriculum materials for planning and teaching their regular mathematics lessons.

Each teaching episode for the counting-focused instructional treatment followed a fairly standard format using the *Count Around the Circle* number sense routine (Shumway, 2011; Shumway & Kyriopolous, 2013). Count Around the Circle is a routine that involves whole-class participation, with each child saying a number as the class counts around the circle. The researcher used a counting sequence, for example, count by tens starting at 57. One student counted on by ten and said "sixty-seven," the next student in the circle said "seventy-seven," and so on, until students counted all the way around the circle. The researcher facilitated a classroom discussion (Chapin, O'Conner, & Anderson, 2009) with students about the counting sequence, eliciting number system knowledge ideas such as patterns in numbers, place value, decomposing and composing numbers, estimation, computation, and relationships among numbers. Based in a constructivist epistemology, each counting sequence was planned to specifically highlight a topic or big idea that children were anticipated to construct based on the planned verbal counting sequence and questions for facilitating discussion. For example, the counting sequences "count by tens starting at zero" and "count by fives starting at zero" were used to highlight the doubling and halving relationships among numbers in these sequences.

Finally, each teaching episode included some type of symbolic or non-symbolic representation. Some sessions included the open number line while others used lists of numbers in the sequence written in a very specific format to highlight patterns or big ideas about the number system. Continuing with the example of counting by tens starting at zero and counting by fives starting at zero, portions of these counting sequences were written in a vertical list so that students could use the visual representations to highlight important number system knowledge ideas and/or to represent their reasoning about an idea.

Timeline for the instructional treatment and assessment. During weeks 1 to 3 of the study, the researcher taught the first 15-25 minutes of Class 1's mathematics block of time three times a week. The researcher administered a benchmark assessment to all three classes during week 4 and conducted six task-based interviews about the first benchmark assessment. During weeks 5 to 7 of the study, the researcher taught 15-25 minutes of Class 1 and Class 2's mathematics block of time three times a week. The researcher administered the second benchmark assessment to all three classes during week 8 and conducted six individual task-based interviews. During weeks 9 to 11 of the study, the researcher taught 15-25 minutes of the mathematics block of time in all three classes three times a week.

Phase 3: Posttreatment

Week 11 of the study marked the end of the instructional treatments. Following week 11, the researcher administered the posttest to all three classes, conducted six task-based interviews, and administered the TEMA-3 to all six interview students.

Data Analysis

Data were analyzed using a variety of methods including descriptive and inferential statistics (quantitative) and open and axial coding (qualitative). The primary data sources for the quantitative analyses were the whole-class pretests, benchmarks, and posttests. The primary data sources for the qualitative analyses were the task-based interviews and lesson artifacts. Three forms of data analysis included: (1) quantitative analyses of variations in class mean test scores and individual student test scores, (2) qualitative analyses of variations among intact classes and individual students' number sense development, and (3) qualitative and quantitative analyses of variations of one low-achieving and one high-achieving student's interviews and test scores. Finally, data were analyzed holistically to answer the overarching research question.

Quantitative Analyses of Variations in Class and Student Test Scores

The first step of the quantitative analysis was exploratory data analysis of the test scores. To explore the pretest data, the data were organized and cleaned in wide format in Microsoft Excel. A score was entered for each subsection of the pretest; in other words, a score was entered for the one hundred computational fluency problems, a score for the four story problems, and a score for the five number line tasks. These scores were averaged to make up a holistic score for the pretest, giving each subsection an equal weight in the overall pretest score. If students' scores were calculated out of 109 problems, instead of weighted subtotals, the computational fluency problems would potentially overshadow student achievement in the story problems and number lines tasks.

Excel data were then exported to SPSS software to conduct a visual and numerical inspection of the data. Descriptive statistics, including measures of central tendency and indicators of dispersion, on the pretest scores for each class provided an overview of student performance on the pretest. Graphical representations of the pretest data, specifically box plots and histograms, were used to alert the researcher to any outliers or unusual aspects of the data and provided a graphical way to interpret the dispersion and whether or not the data were skewed. These analyses summarized and aided the researcher in making sense of the data (Cohen, 2008). The visual inspection of the data was corroborated by a Shapiro-Wilk test of normality on all three classes' test scores to find out if the pretest scores in each class were normally distributed. These procedures were repeated for the benchmark tests and the posttest. Once the data from all the tests were collected, line graphs were used to plot the mean test scores across time points for each class and for individual students within each class.

Variations among intact classes' test scores. Once the data from all time points were visually and numerically inspected, the researcher used SPSS software to restructure the data from wide format to long format for the purpose of using the Generalized Estimating Equations (GEE) analysis. To answer research question 1, the researcher conducted a GEE analysis for overall performance on the measures to determine variations in test scores among the three groups (Classes 1, 2, and 3). The study involved multiple observations (pretest, benchmark 1, benchmark 2, and posttest) collected from individual students in three different classrooms. This clustered data (by class) with repeated measurements of students' number sense necessitated a statistical analysis framework capable of handling data within clusters that are correlated. The GEE was the most appropriate method for the analysis of this type of clustered data (Hardin, 2005).

While a two-way repeated measures analysis of variance (ANOVA) is typically used in educational research to determine whether there are significant differences between the test score means of three unrelated groups (Class 1, 2, and 3) across measurement points (pretest, benchmarks, posttest), the data in this study violated several assumptions for ANOVA, including random assignment of participants to treatment groups and spherecity (each class's mean pretest scores had different starting points). Due to violations of assumptions for ANOVA, a more sophisticated model was needed to analyze students' test scores within clustered classes across measurement points.

Another analysis option, generalized linear models (GLM) with repeated measures, would provide a more appropriate analysis of this type of data, however, this technique assumes that observations (i.e., each testing point) are independent. In this study, each participant's data is likely not independent at each time point because the observations (pretest, benchmark 1, benchmark 2, and posttest) were close together in terms of time. Furthermore, lack of random assignment may lead to the test scores being impacted by teacher effects and/or students' regular interaction with each other.

An analysis method was needed that could describe changes in groups of students' test scores and explore the associated effect of variables, such as time participating in the instructional treatment, while controlling for non-independent observations. Hence, the GEE analysis, which is based on GLM, is a procedure designed for repeated measures yet controls for a lack of independence and takes into account this possible within-group correlation (Ghisletta & Spini, 2004; Hardin, 2005). While GEEs are not frequently used in educational research, Ghisletta and Spini argued:

> …data naturally organized within hierarchies or from longitudinal and panel studies are very frequent in educational and social sciences. For such data, the application of traditional regression models is not adequate; in particular, the statistical dependence arising from the similarity of observations organized within the same cluster, or stemming from the same participant assessed repeatedly, necessitates analyses that do not assume such dependence to be zero. (p. 431)

To account for the lack of independence, the researcher selected the autoregressive 1 (AR(1)) correlation structure as the Working Correlation Matrix for the data. When running a GEE analysis, SPSS provides five correlation structure options for analyzing the data: Independent, Autoregressive 1, Exchangeable, M-Dependent, and Unstructured. The Independent model is the simplest and assumes that the repeated observations are uncorrelated. The AR(1) assumes a temporal dependence within clusters and the level of correlation depends on distance between the repeated measures. In this study, it was assumed that the measurements taken close together (e.g., pretest and benchmark 1) were more correlated than the measurements further apart (e.g., pretest and posttest). The AR(1) working correlation matrix assumes that as the distance between repeated measures increases, the correlation between them decreases. An initial visual analysis of the line graphs of individual students' mean scores across the repeated measures supported this assumption. Overall, each student's score was somewhat similar to the previous time point, but less similar from pretest to posttest. Hence, the AR(1) was the most theoretically appropriate structure for this study's data (Ghisletta & Spini, 2004; Hardin, 2005).

Variations in individual students' test scores. In addition to line graphs with mean test scores across time points for each class, graphs of individual students' test scores at each measurement point were used to visually show students' variations in number sense development. These line graphs were also used to answer research question 1, this time from the perspective of individual student test scores across time within each of the three classes.

Qualitative Analyses of Variations in Number Sense Development

The researcher used qualitative analyses of the lesson artifacts to support, interpret, and extend the quantitative analyses and to understand the nature of the learning that took place during the counting-focused instructional teaching episodes as a whole. Rather than a systematic approach to analyzing each type of lesson artifact (e.g., field notes, video of the instructional treatments, chart paper recording student discussions, counting journals), the researcher referred to these pieces of evidence as themes emerged during the data collection phase of the study and during the quantitative analyses, graphical analyses, and case study analyses.

Qualitative and Quantitative Analyses of a Low- and a High-Achieving Student's Interviews and Test Scores

To answer research question 2, the researcher conducted a preliminary review of the 24 task-based interviews in order to select two cases (one low-achieving student and one high-achieving student) for video transcription and in-depth analysis. Preliminary analysis involved an examination of the line graphs of individual students' test scores across measurement points and a comparison of each graph with class graphs. An analysis of their subtotal scores was used to more narrowly observe variations in test scores over measurement points. Then, the researcher viewed videos from the task-based interviews and annotated initial descriptions and broad interpretations of the students' verbal explanations, actions, and behaviors. Using both the line graphs and video data, the researcher selected two cases for in-depth analysis.

Iterations of open and axial coding. The in-depth analysis of two cases involved multiple iterations of open and axial coding of both the video data and transcribed video data. The analysis began with open coding of the video data in order to produce overarching concepts and categories that fit the data (Miles & Huberman, 1994; Saldana, 2009; Westbrook, 1994). The researcher noted general variations within each student's interview sequence from pretest to posttest. Next, the researcher viewed the video data again, this time using existing frameworks from the mathematics education literature to code students' strategies.

Existing frameworks for axial coding. The researcher used three different frameworks for each of the three subsections of the assessments: (1) The Assessment of Math Fact Fluency; (2) Story Problem Situations; and (3) Number Line Tasks.

Holistic coding of The Assessment of Math Fact Fluency. The analysis of students' computational fluency was based on the three phases students typically progress through when learning basic number combinations: (Phase 1) Counting strategies, (Phase 2) Reasoning strategies, and (Phase 3) Retrieval (Baroody, Eiland, & Thompson, 2009; Baroody & Rosu, 2006). Using this framework, the researcher coded the overall phase the student was in at each time point. The researcher used these codes identify shifts in phases and if those shifts (based on the task-based interview data) converged or diverged with shifts in the line graphs (based on the test score data).

Holistic coding of the Story Problem Situations. The Cognitively Guided Instruction framework for students' development of problem solving strategies was used to code students' strategies on the four story problem tasks (Carpenter et al., 1999). This framework delineates students' typical strategies for solving problems as direct-modeling strategies, counting strategies, invented strategies (e.g., using counting or known facts), and standard algorithms. A direct-modeling strategy involves using concrete manipulatives or drawings to express each part of the problem. Counting strategies include counting on and counting on from first with or without objects for keeping track (e.g., fingers), or abstract counting (e.g., counting without objects; skip counting). Invented strategies vary, though often involve using known facts to solve a problem. Standard algorithms refer to using an algorithmic procedure to solve the problem. These broad categories provided a starting point for understanding how the student approached each problem at the various time points and how the student's approaches and strategies changed over the study. The researcher used this analysis to identify converging or diverging trends in the line graphs (based on the test score data).

Holistic coding of the Number Line Tasks. Diezman and Lowrie's (2006) structured number line studies led to descriptions of students' responses when they are successful and unsuccessful with number line tasks. The characteristics of these responses were used in the holistic coding of students' responses to the number line tasks in order to understand how they were viewing the number line (as a measurement model or a counting model) and to categorize their responses (e.g., strategies relating to distance; proximity of numbers; counting from zero).

Variations in students' number sense development. The axial coding process led to further sorting and defining themes in each student's number sense development over the course of the study (Miles & Huberman, 1994). As key variations or shifts in learning were noted, the researcher identified specific sections of video to transcribe. Then, open and axial coding of the transcribed video data was used to further explore how concepts and categories were related to discern themes, patterns, and processes (Coffey & Atkinson, 1996).

The in-depth qualitative analysis of two cases involved multiple iterations of coding the videos and transcribed interview data. The coding schemes were maintained in a Microsoft Word document and organized for each subtest category while notes and memos where photocopied and archived.

Holistic Analysis: Embedded Mixed Methods

The researcher used three forms of data analysis: (1) quantitative analyses of variations in class mean test scores and individual student test scores, (2) qualitative analyses of variations among intact classes and individual students' number sense development, and (3) qualitative and quantitative analyses of variations of one low-achieving and one high-achieving student's interviews and test scores. As results emerged from these three forms of analysis, the researcher holistically considered the findings to answer the overarching research question: In what ways does a counting-focused instructional treatment influence, change, and develop second-grade students' number sense?

CHAPTER IV

RESULTS

The purpose of this study was to explore variations in second-grade students' number sense development as they engaged in a counting-focused instructional treatment for differing amounts of time. The researcher used quantitative and qualitative analyses to answer the research questions. The overarching research question was: In what ways does a counting-focused instructional treatment influence, change, and develop second-grade students' number sense? Two subquestions focused on variations in students' test scores and strategies for solving problems: (1) What are the variations in number sense development when students engage in counting-focused instructional treatments for differing amounts of time (3 weeks, 6 weeks, and 9 weeks)? (2) What are the variations in number sense development for one low-achieving student and one high-achieving student? The results presented in the sections that follow are organized around the research questions and the three forms of data analysis used to answer the research questions: (1) the quantitative analyses of variations in the intact classes' mean test scores and individual students' test scores; (2) qualitative analyses of variations among intact classes' and individual students' number sense development; and (3) qualitative and quantitative analyses of variations of one low-achieving and one high-achieving student's test scores and interviews.

Variations in Number Sense Development When Students Engage in Counting-Focused Instructional Treatments for Differing Amounts of Time

Exploratory Data Analysis: Initial Pretest Data

Table 3 presents the means and standard deviations for the pretest scores by class. The initial exploratory analysis of the pretest data showed that the means and standard deviations of pretest scores were relatively similar across the three classes. However, on average, students in Class 2 performed with more correct responses than students in Class 1 and 3. Class 3's students began the study at the lowest starting point in terms of performance.

The graphical representations of the pretest data, specifically the box plots (see Figure 2 from Chapter III), showed that the data in each class were approximately symmetrical, but better explained the variations in the standard deviations and mean scores from the numerical analysis. The spread of the data (in terms of the standard deviation and the visual analysis of the box plots) indicated that Class 2 had a larger dispersion of data, suggesting that more students had high scores as compared to the

Table 3

Mean Pretest Scores (in percentages) and Standard Deviations by Class

Class	*M*	*SD*
1 (*n* = 22)	.36	.19
2 (*n* = 19)	.40	.26
3 (*n* = 19)	.29	.19
Total (*n* = 60)	.35	.21

other two classes while still having students with mid-range and low scores. These high scores in Class 2 increased the mean and median and created a wider spread of scores across this class. Class 1 and Class 3 both had a standard deviation of .19, suggesting less variation in scores on the pretest. A visual inspection of the box plots indicated a more symmetrical dispersion of test scores for Classes 1 and 3 (see Figure 2 from Chapter III). The results of the descriptive analysis of the pretest scores suggest that, while the pretest scores across the three classes are not identical, they are within a similar range as would be expected of one grade level within the same school.

Six individual one-way analysis of variance (ANOVA) models were used to explore the bivariate relationships between demographic variables and students' pretest scores. Table 4 shows that only the ANOVA exploring the differences in pretest scores by IEP was statistically significant, $F(1,58) = 6.43$, $p = .014$. No statistical difference was found between the mean pretest scores for class (i.e., Group 1, 2, or 3), gender, SES, race, or English as a Second Language.

Table 4

One-Way Analysis of Variance Summary Table for Pretest Scores and Six Independent Variables

Variable	*F*	*p*
Class	1.39	.258
Gender	1.70	.198
SES	1.51	.225
Race	0.55	.699
ESL	1.46	.232
IEP	6.43	.014

A Spearman's Rank-Order Correlation was conducted to further assess the relationships among pretest scores and demographic variables. The nonparametric Spearman's Rank-Order Correlation was used instead of the Pearson Correlation because of the ordinal nature of the demographic variables. The variable, race, was not included in the analysis because it contained four levels that were neither continuous nor ordinal. The results of the correlational analyses are presented in Table 5. Two of the 15 correlations were statistically significant. The correlation between pretest score and IEP was moderate. Both the exploratory ANOVA and correlational analyses indicated that students with IEPs scored differently from their peers on the pretest.

An exploratory numerical and visual analysis using the Shapiro-Wilk Test of Normality (Razali & Wah, 2011; Shapiro & Wilk, 1965) was conducted to test the data's distribution. The results of the Shapiro-Wilk analysis indicated that all three classes had *p*-values greater than .05, meaning the data were approximately normally distributed:

Table 5

Spearman Correlations Among Pretest Scores and Demographic Variables (n = 60)

Variable	1	2	3	4	5	6
1. Pretest score	--					
2. Gender	-.17	--				
3. SES	-.15	-.13	--			
4. ESL	-.14	.-08	.24	--		
5. IEP	.32*	.09	-.18	-.09	--	
6. Class	-.13	.14	-.06	.08	.28*	--

Abbreviations and codes: Gender (0 = male, 1 = female); SES = socioeconomic status based on qualifying for free/reduced lunch (0 = yes, 1 = no); ESL = receives English as a Second Language services (0 = yes, 1 = no); IEP = has an Individualized Education Plan (0 = yes, 1 = no); Class = Class 1, 2, or 3 (1 = 9 weeks of intervention, 2 = 6 weeks, 3 = 3 weeks).

* $p < .05$.

Class 1, $p = 0.63$; Class 2, $p = 0.07$; Class 3, $p = 0.34$. Additionally, a visual analysis of the histograms for each class showed an approximately normal curve; the Q Plots for each class's pretest scores showed that the data were approximately distributed along the line; and an analysis of the box plots for each class showed the spread of data were approximately symmetrical.

Variations in Test Scores Across Time Points Among the Three Classes

Once data were collected from all four time points (pretest, benchmark 1, benchmark 2, and posttest), the researcher used descriptive statistics and line graphs to analyze overall trends in the data. Table 6 presents the means and standard deviations for each measurement point by class. The results of the descriptive analysis suggest that the test score means in all three classes followed a similar pattern of improvement in test scores at Benchmark 1, followed by a slight decline at Benchmark 2, and concluded with another improvement at Posttest. The line graph in Figure 4 shows this pattern of test scores visually and further accentuates the striking consistency in terms of one class not

Table 6

Mean Scores (in Percentages) and Standard Deviations at Each Measurement Point by Class

	Pretest		Benchmark 1		Benchmark 2		Posttest	
Class	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
1 (*n* = 22)	.36	.19	.43	.29	.41	.26	.54	.23
2 (*n* = 19)	.40	.26	.53	.28	.51	.25	.60	.24
3 (*n* = 19)	.29	.19	.37	.24	.33	.21	.47	.24
Total (*n* = 60)	.35	.21	.44	.27	.41	.25	.54	.24

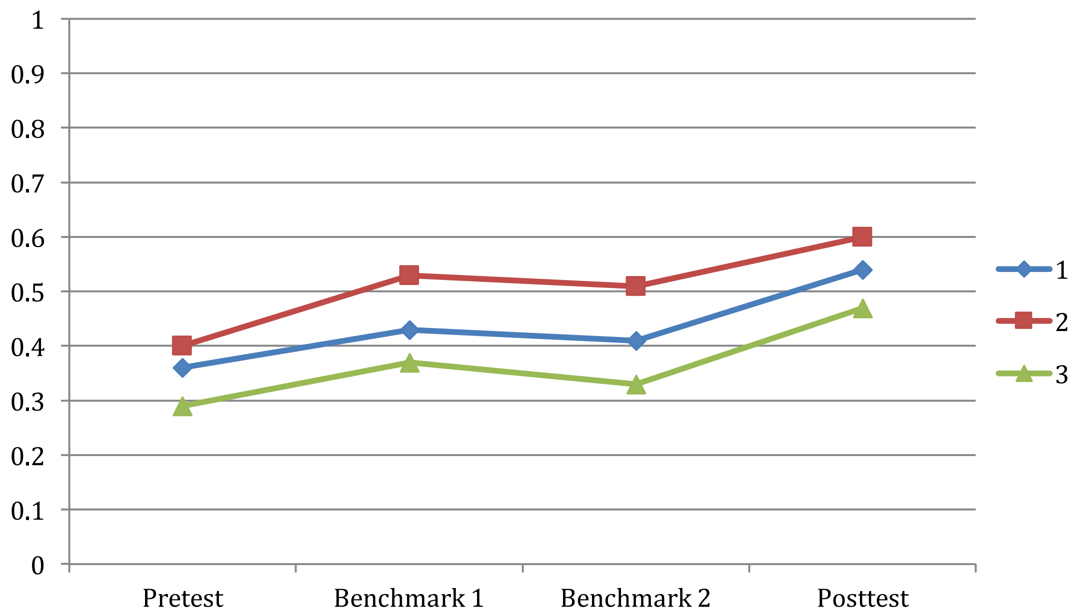


Figure 4. Line graph of mean test scores by class across measurement points.

out-performing another throughout the study. In other words, despite the movement in mean scores across measurement points for each class, the mean scores tended to have the same distance between each other with Class 2 consistently performing with the highest scores and Class 3 with the lowest scores. Class 1's mean scores are almost identical to the total mean across the grade level at each measurement point. The results suggest similar gains (and regressions) for each class throughout the study. One interesting aspect of the data is the change in Class 2's initial dispersion of data. Class 2's pretest scores had a standard deviation of .26, which indicated a wider spread of performance across the class. At each subsequent measurement point, Classes 1 and 3's standard deviations were more similar to the spread in Class 2, which remained relatively close to Class 2's initial standard deviation.

While the line graphs and descriptive statistics provided some information about the variations in each class's test scores across measurement points, the analysis did not provide results that explained to what degree the classes' variations in performance differed from one another. To answer research question 1, the researcher conducted a GEE analysis for overall performance on the measures to determine variations in test scores among the three groups (Classes 1, 2, and 3). The results of the GEE analysis are presented in Table 7. Significant parameters from the GEE analysis included group (i.e., Class 1, 2, or 3), gender, and special education services (IEP). The beta (β) reports a population-averaged parameter representing the averaged effect of a unit change in the predictor for the population, when holding all other variables constant.

Table 7 shows that, when holding all other variables constant, Class 1 had an associated average score of 12.4 percentage points higher than Class 3, which was statistically significant ($\beta = .12$, $p = .054$). Class 2 had an associated average score of 8 percentage points higher than Class 3, which was not statistically significant ($\beta = .08$, $p = .222$). The results from this model suggest that there was an associated increase in test scores when students participated in the counting-focused instructional treatment for longer periods of time (e.g., Class 1 = 9 weeks v. Class 3 = 3 weeks).

In considering other factors that may influence test scores, such as demographic variables, the population-averaged parameters showed that students from low socioeconomic homes scored on average 6.2 percentage points higher than their peers from average/high SES homes when controlling for all other variables ($\beta = .06$, $p = .217$). Though it is not statistically significant, this outcome is atypical of what is generally

Table 7

Generalized Estimating Equations (GEE) Results

Variable	β	SE	95% CI	*p*
Class				
1	.12	.06	-.00, .25	.054*
2	.08	.07	-.05, .22	.222
3	0			
Gender				
Male	.10	.05	.00, .19	.049*
Female	0			
Socioeconomic Status				
Low	.06	.05	-.03, .16	.217
Average/High	0			
Race				
White	-.07	.05	-.17, .03	.179
Black	-.01	.12	-.24, .21	.912
Hispanic	-.20	.13	-.44, .05	.121
Asian	-.16	.11	-.37, .05	.138
Pacific Islander	0			
English as a Second Language				
ESL services	.04	.11	-.17, .26	.7
No ESL services	0			
Special Education Services				
IEP	-.23	.07	-.36, -.10	.001**
No IEP	0			

Dependent Variable: Score.

Model: Class, Gender, SES, Race, ESL, IEP.

* $p < .05$.

** $p < .001$.

expected. This outcome could be a result of low power or it could be noise, however this direction is interesting and possibly with more participants could be important.

The model showed that gender and special education services had significant population-averaged parameters. Table 7 shows that male students had a statistically significant associated average score of 10 percentage points higher than female students when controlling for all other variables (β = .10, p = .049). The results also indicated that students with IEPs (i.e., special education services) had an associated average score of 23 percentage points lower than their peers without IEPs, which was statistically significant (β = -.23, p = .001). These results suggest that the counting-focused instructional treatment may or may not have been as effective for some female students and some students receiving special education services. Hence, the researcher ran the GEE with subsequent models to investigate interaction effects between class (i.e., Class 1, 2, or 3) and the gender and IEP variables. Using the base model, an interaction model with class*gender as an interaction variable, was investigated through another GEE analysis. Similarly, with the base model, an interaction model with class*IEP as an interaction variable was investigated through a third GEE analysis.

The results of the interaction effects analysis showed no significant interactions for gender (Class=1*Gender=0, β = -.092, p = .461; Class=2*Gender=1, β = .179, p = .152). Therefore, the effects of the instructional treatment did not depend on gender.

There were significant interactions for class and IEP (Class=1*IEP=0, β = .38, p = .002; Class=2*IEP=0, β = .27, p = .040). The line graphs in Figure 5 show that the effect of the intervention depended on whether or not a student had an IEP. The graphs show that students with IEPs did better by being in Class 1. According to the graphs, students without IEPs had the same levels of performance in Class 1 and Class 2. Students with IEPs and students without IEPs performed the lowest in Class 3. It is possible that individual teachers had an effect on these results. Nevertheless, there seems to be evidence that for students with IEPs, the more time they engaged with the

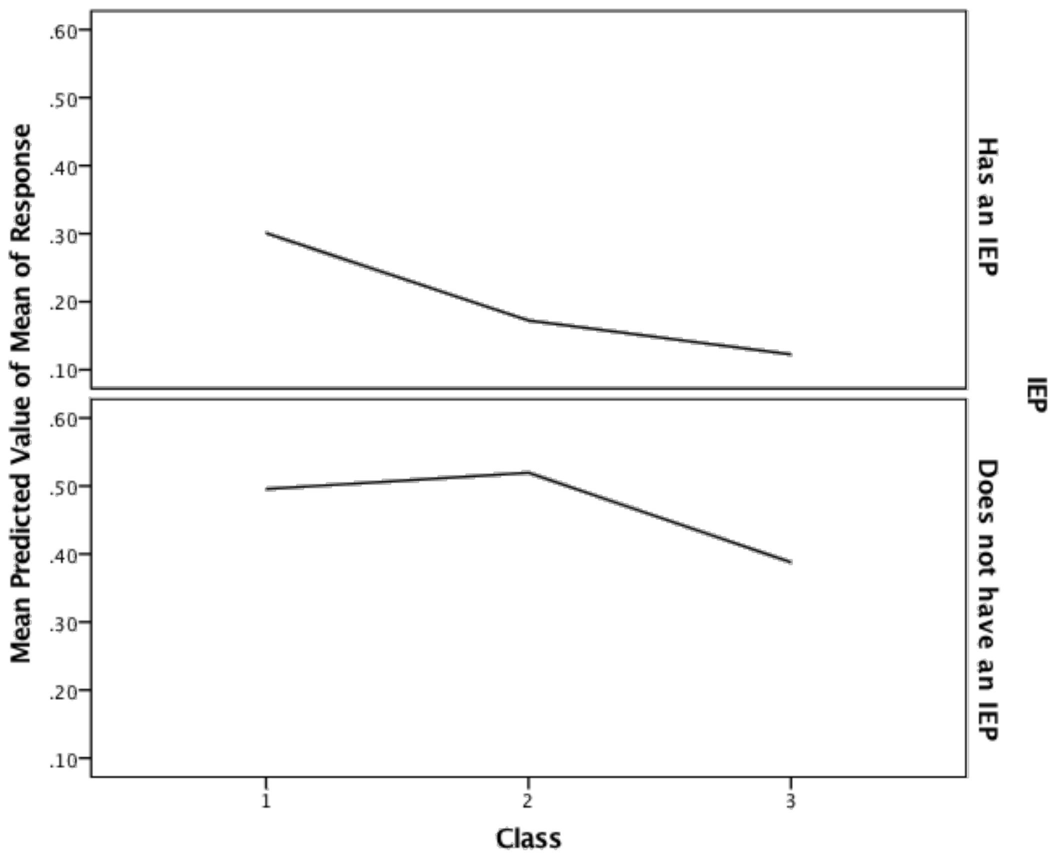


Figure 5. Interaction plot for the Class*IEP interaction.

instructional treatment, the better their outcomes. The line graphs provide some initial evidence that students without IEPs did not need as much time in the intervention to have positive learning outcomes.

Variations in Individual Students' Test Scores within Each Class

Line graphs of individual student test scores at each measurement point were grouped by class and used to visually show students' variations in number sense development. Figure 6 shows the line graphs for Class 1's individual student test scores. Overall, student learning increased from pretest to posttest. In the path from pretest to

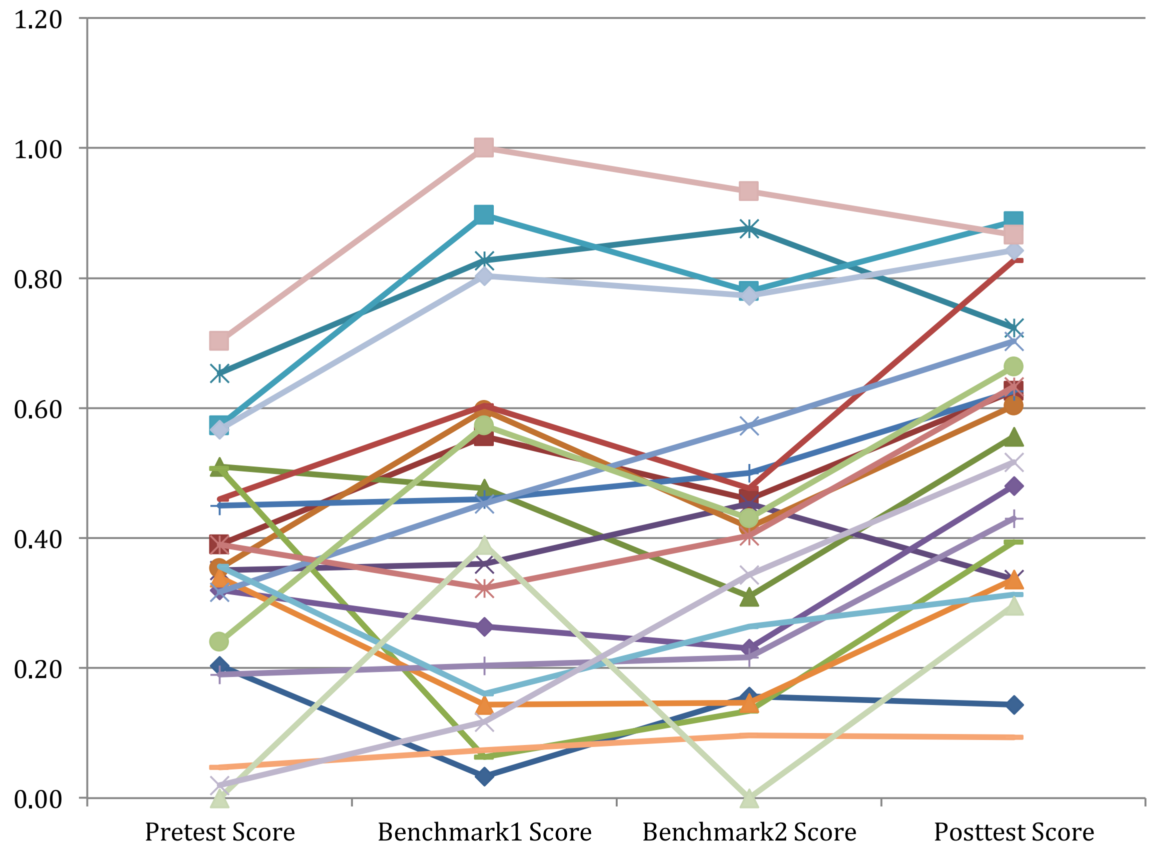


Figure 6. Line graphs of Class 1's individual student test scores across measurement points.

posttest, again there was an increase in students' scores at Benchmark 1 followed by a decrease in scores at Benchmark 2. Most of the students' scores followed this pattern, but not all. Of the four students who scored close to 60% or higher on the pretest, two of those students scored lower on the posttests than they did on the benchmark tests. Students who scored between 30% and 55% on the pretest tended to score higher on the posttest than any of the other tests, which was expected.

Figure 7 shows Class 2's individual student test scores across measurement points. Similar to Class 1's trend, three students who scored highest on the pretest scored

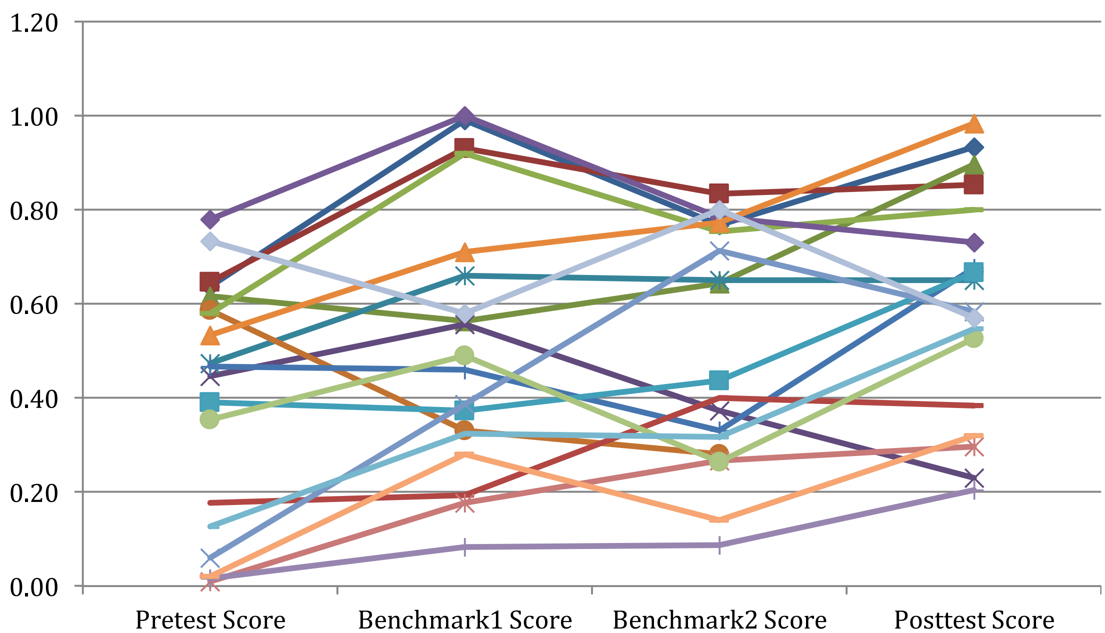


Figure 7. Line graphs of Class 2's individual student test scores across measurement points.

lower on the posttests than they did on the benchmark tests. Also similar to Class 1's trend, students who scored in the mid-range of performance on the pretest tended to follow the more expected pattern of higher scores on the posttest over the other three measurement points, despite a decrease in test scores at Benchmark 2.

Figure 8 visually highlights that no one in Class 3 scored above 60% on the pretest. Interestingly, students who scored the highest on the pretest in Class 3, followed a similar pattern to the other two classes in that several students scored lower on the posttest than they did on benchmark 1. This pattern remained the same despite starting at a lower score on the pretest. Also notable in Class 3's line graphs is that several students' scores went up by more than 20 percentage points from benchmark 2 to posttest (e.g., from 45% to 70%, from 30% to 65%, from 55% to 83%).

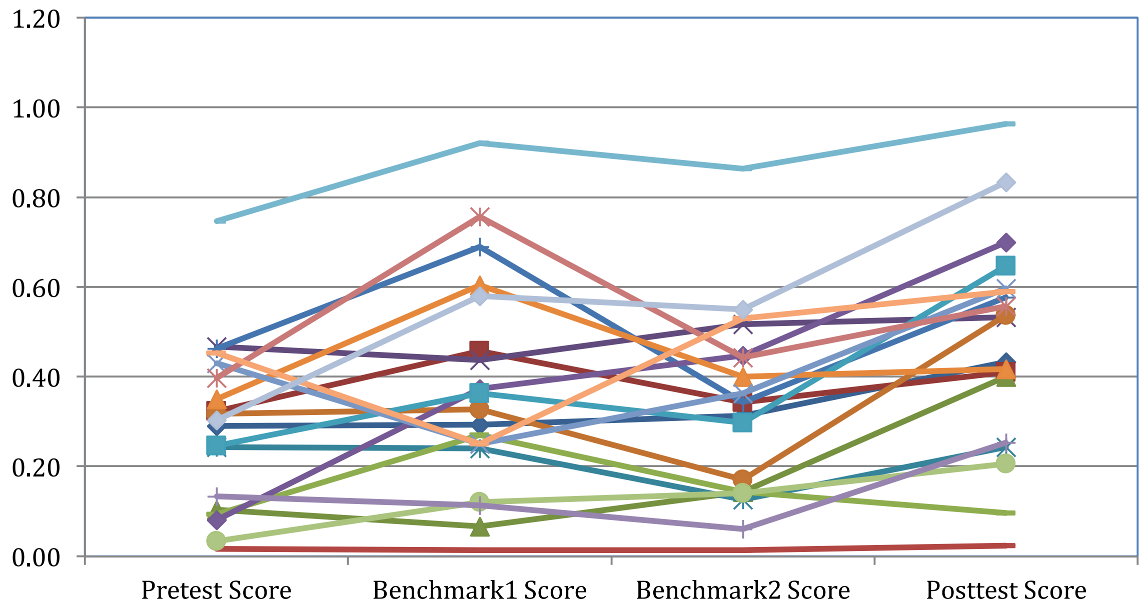


Figure 8. Line graphs of Class 3's individual student test scores across measurement points.

Qualitative Analyses of Variations in Number Sense Development

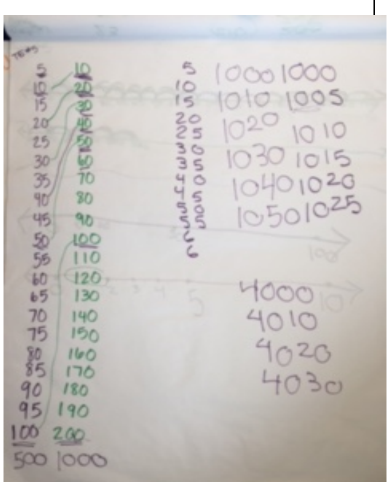
The researcher used qualitative analyses of the lesson artifacts (i.e., field notes, chart paper records of discussions, counting journals, and teaching episode videos) to support, interpret, and extend the quantitative analyses and to understand the nature of the learning that took place during the counting-focused instructional teaching episodes as a whole. Two important themes emerged: (1) a general progression of enthusiasm and use of number system knowledge, regardless of the class, and (2) struggling students' shining moments during weeks 5-9 of the Teaching Episodes.

Progressions of Enthusiasm and Number System Knowledge

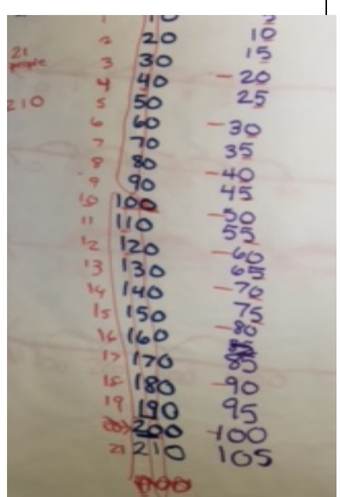
Regardless of the class a student was in, there was a typical progression of enthusiasm during classroom discussions for each class which went from simply counting and participating during the first week to a turning point in enthusiasm during the second week of the intervention, specifically during Teaching Episodes 5 and 6 (see Figure 9). Students demonstrated their enthusiasm about numbers and their relationships through their affect, body language, tone, and sustained interest during the discussions and by students' noticing of patterns and playfulness with numbers.

During Teaching Episodes 5 and 6, the researcher's field notes indicated a shift in participation and more noticing of patterns. The research notes indicated increased enthusiasm about what students were seeing and more willingness to play with numbers (e.g., decomposing numbers, finding relationships among numbers, exploring new ideas about numbers). For example, the researcher's field notes from Teaching Episode 5 with

Class 1



Class 2



Class 3

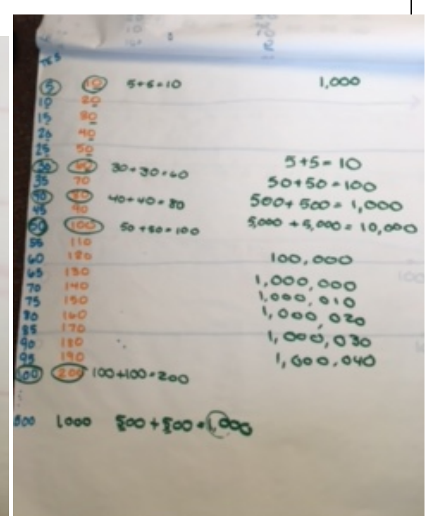


Figure 9. Chart paper records of discussions from Teaching Episode 5.

Class 1 stated, “First time I saw some excitement about numbers. Today they were a little more willing to play!” This note was about students’ playfulness with the patterns and more open disposition toward find interesting relationships. The researcher noted that their discussions went beyond simply noticing patterns related to the ones and tens. As shown in Figure 9, the chart paper records of discussion for Class 1 show students’ noticing of relationships between the counting by 5s sequence and the counting by 10s sequence (e.g., the lines connecting matching numbers between the two sequences). Students discussed where 30 was located in the counting by 5s sequence in relation to where 30 was located in the counting by 10s sequence. They noticed that the distance between matched numbers increased as the numbers became larger (e.g., 30 compared to 100).

Also shown in Figure 9, Class 2’s chart paper illustrated a similar discussion about the relationships between the counting by 5s and 10s sequences. A line next to each 10 in the counting by 5s sequence was the researcher’s illustration of a student making the connection between the 5s and 10s sequences. The students also enjoyed discussing how the pattern changed at each hundred, then the pattern of the tens place repeated (e.g., boxes around decades of numbers in Figure 9). Additionally, the researcher’s field notes about Teaching Episode 5 with Class 2 stated, “They love Counting Journals!!! [Calvin] told me he noticed that there are two 20s, two 30s, etc. when we count by 5s. [Sam] counted from 15 by tens to 925 in his counting journal. He was so into it! [Kali] tried 5, 10, 15 then 10, 20, 30 then 100, 200, 300. This was huge for her. So far she has copied what was on the chart. Today she played around with numbers based on her own thinking. Andrew was trying larger numbers, but incorrectly (10,00 20,00 30,00). He’s thinking outside what we've done so far and shows a desire to generalize the patterns he knows." These examples in the field notes were evidence of students demonstrating a more open disposition to finding interesting patterns and a willingness to play with the numbers.

Figure 9 also shows Class 3's playfulness with the doubling pattern they observed in Teaching Episode 5. Students made statements such as, "30 plus 30 is 60 so that is why there is a 30 in the 5s sequence and a 60 in the 10s sequence." This discussion led them to play with other equations, such as 500 + 500 = 1,000. The field notes for Class 3's Teaching Episode 6 indicated more hands up, greater "buzz" during partner talk, and kids staying by the chart after the conclusion of the episode to explore more patterns. For Class 3, the field notes seemed to indicate an even greater shift to enthusiasm for numbers during Teaching Episode 8. The field notes stated, "[counting by] 10s at 40, 140, and 1,040 was so exciting for [teacher's] class today. Noticing! Playfulness!" In viewing the video of Teaching Episode 8 and comparing it with the previous teaching episodes, the noticing and playfulness was evidenced by students' louder tone during the pair-share discussions about the counting sequences; students' excited affect when sharing what they noticed about the numbers; increased student engagement and desire to participate in the whole group discussion (many hands raised); and students' requests to tell the researcher more about what they noticed after the discussion was over (students staying on the rug near the chart paper with the researcher when it was time to go back to their desks for the rest of the math lesson).

A general progression of students' use of number system knowledge was particularly evident in an analysis of Teaching Episodes 2 (for all classes), 12 (for

Classes 1 and 2), and 20 (for Class 1). The same counting sequence, counting by ones from 34 and tens from 34, was used for each of these teaching episodes. During Teaching Episode 2, all three classes' conversations about the counting sequences focused on the counting itself and about patterns in tens (e.g., "it goes 1, 2, 3, 4, 5") and ones (e.g., "they are all fours"). Class 1 participated in Teaching Episode 2 in September of the school year; Class 2 did so in October; and Class 3 participated in November. Regardless of the time of year students participated in Teaching Episode 2, there were similarities in their conversations.

For Classes 1 and 2, Teaching Episode 12 brought forth more discussion about counting and patterns with the hundreds. There was also discussion about how the patterns changed in the counting sequences. For example, a student in Class 1 showed his classmates that he was counting by 100s down the column. A classmate of his had not noticed that and said, "That's cool, [Kevin]!" In Class 2, a student said she noticed that, from where we had started the counting sequence to where we ended, she could count down twice by tens and over 1. Essentially she was adding 34+10+10+1 to get to 55, explaining that there were 21 students counting in the circle.

Shining Moments in Weeks 5-9

Overall, the researcher's field notes indicated some frustration that attention and focus were issues for struggling students in all three classes, and it was more difficult for them to participate for the full fifteen minutes. The researcher noted that they were not fluent with talking about numbers and did not seem to access some of the conversations, while the instructional treatment seemed fun and interesting for many other students.

Participation and access varied among students and the variation seemed to be the same in each class. However, the researcher noted an important change for several students in Class 1 who initially had difficulty participating in the instructional treatment teaching episodes. Many of these students began having “shining” moments during the whole class discussions during Weeks 5-9 of the instructional treatment. For example, the researcher noticed this change with two students who typically struggled with counting and participating in the conversations. During Week 5, they began using the chart paper to write counting sequences in their Counting Journals. They were engaged in writing these sequences and finding a way to more fully access the content. This was the first instance of these students showing interest and initiative. During Week 6, Teaching Episode 17, some students in Class 1 showed enthusiasm for counting the same sequence three times in a row, trying to beat the time it took the class to get around the circle (i.e., a counting fluency exercise). One struggling student said, “I discovered I said the same number each time,” which seemed to make the counting easier and open her up to hearing patterns in the numbers as we counted fluently around the circle. During Week 7, Teaching Episode 21, another struggling student was the one to notice a counting pattern before anyone else. The researcher-teacher asked students, “What would happen if we counted around the circle again, starting at 334, but Jade doesn’t say her number? We skip her, then…” This struggling student jumped in excitedly and stated, “We’d be counting by 4s!” This same student was able to explain to her peers the difference in the meaning of the numeral 2 in the numbers 1,203; 1,023; and 1,230 and where those numbers belong on the number line during Teaching Episode 23. This was evidence that she was not only participating more in the whole-class instructional treatments, she was also discussing big ideas embedded in number system knowledge. While some students struggled to access the conversations in the beginning of the instructional treatment, the more time they were part of conversations about number system knowledge, the more likely it was that they began to have "shining" moments and participate in conversations about complex ideas.

Variations in Number Sense Development for One Low-Achieving Student and One High-Achieving Student

In this section, the qualitative and quantitative results used to answer research question 2, are organized by case (Anna, low-achieving on pretest and Anthony, high-achieving on pretest). Within each case, the results are presented with the following headings: Overall Test Score Variations, Subtest Variations, and Overall Themes.

Anna

Based on her pretest score of .32, Anna was selected as the "low-achieving" student for interviews from Class 1. Anna is a Caucasian female who does not qualify for free/reduced lunch, ELL services, or special education services.

Overall test score variations. Anna began the study at 7 years, 9 months of age and scored in the 10th percentile on the TEMA-3 Form A pretest. At posttest, Anna was 8 years of age and scored in the 25th percentile on the TEMA-3 Form B. The standardized mathematics achievement scores provided evidence of Anna's mathematics learning growth over the course of the study (14 weeks).

Anna's whole-class test scores also indicated growth from pretest to posttest, as well as provided information about the variations in her learning during the study. For example, the line graph in Figure 10 shows a slight decrease in Anna's test scores from pretest to benchmark 1. Her score decreased from .26 to .23 from benchmark 1 to benchmark 2. The line graph then shows an increase to .48 on the posttest, 16 percentage points higher than her pretest score and 25 percentage points higher than her benchmark 2 score.

Subtest variations. Anna's test scores across measurement points provided an overview of her achievement at the various time points in the study. These results, disaggregated by subtest, provided more nuanced findings about Anna's learning. Figures 11-13 show the line graphs of Anna's achievement by subtest (Computational Fluency, Story Problems, and Number Line Tasks).

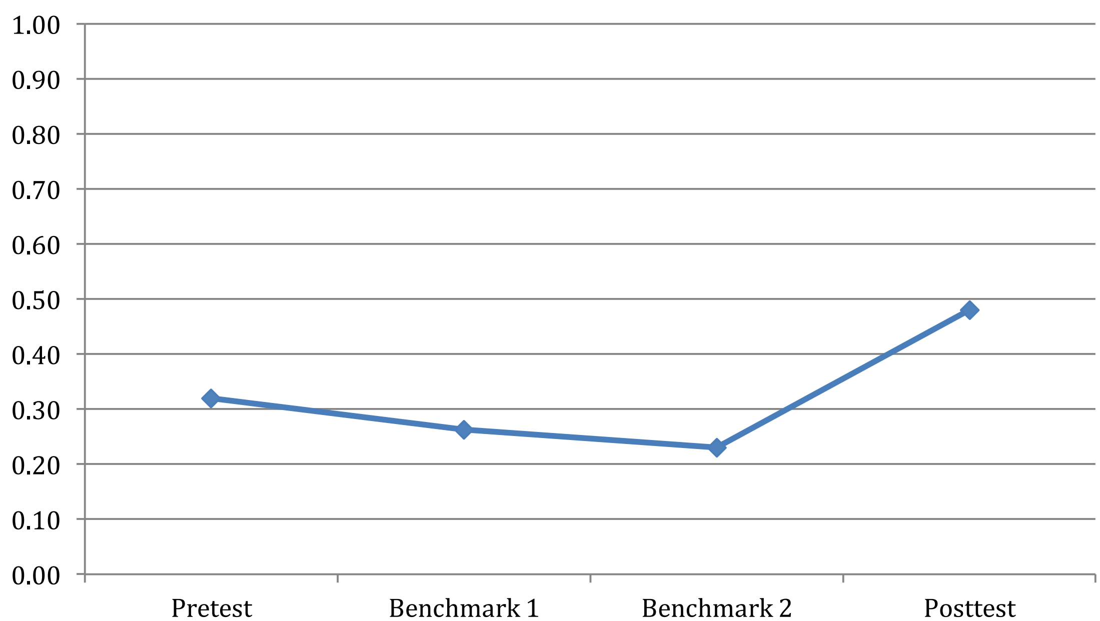


Figure 10. Line graph of Anna's test scores across measurement points.

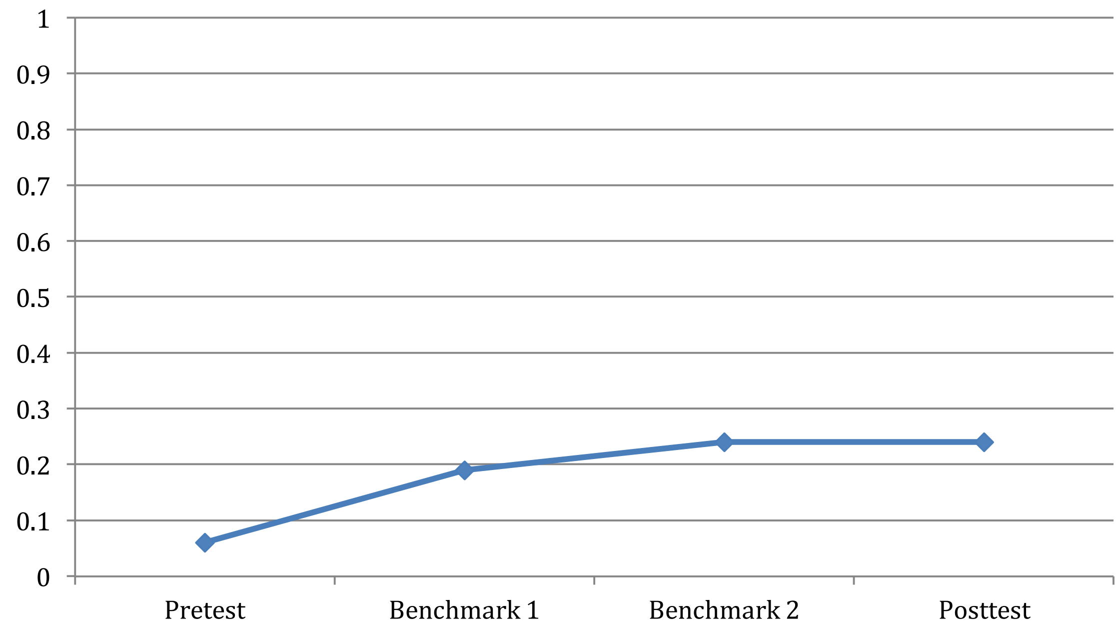


Figure 11. Line graph of Anna's Computational Fluency scores across measurement points.

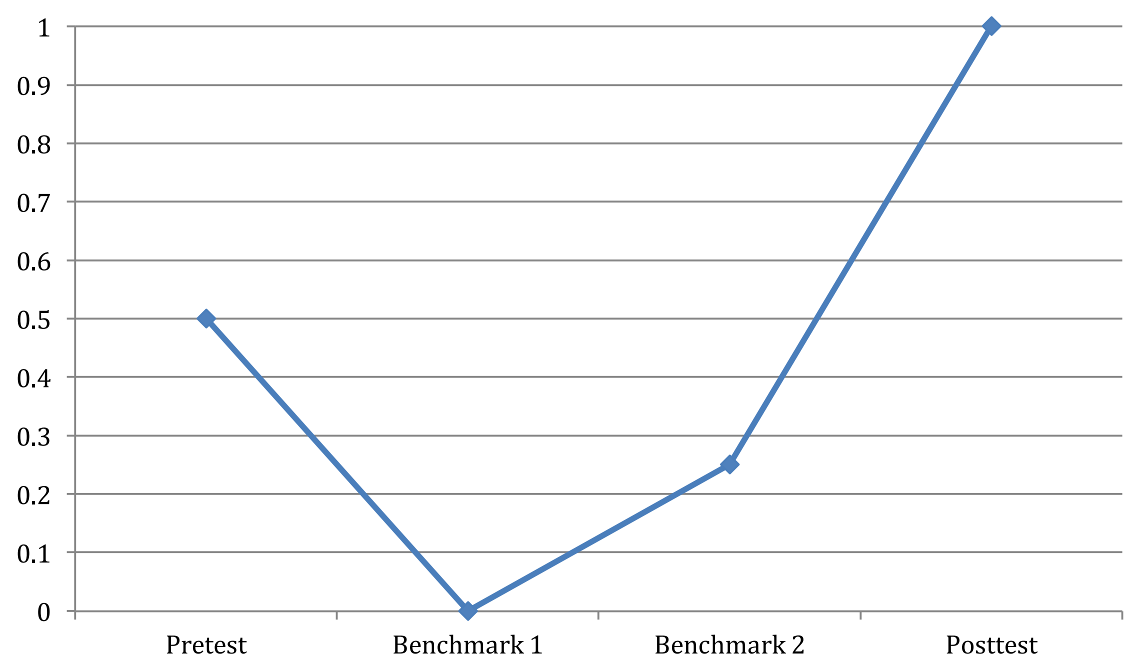


Figure 12. Line graph of Anna's Story Problem scores across measurement points.

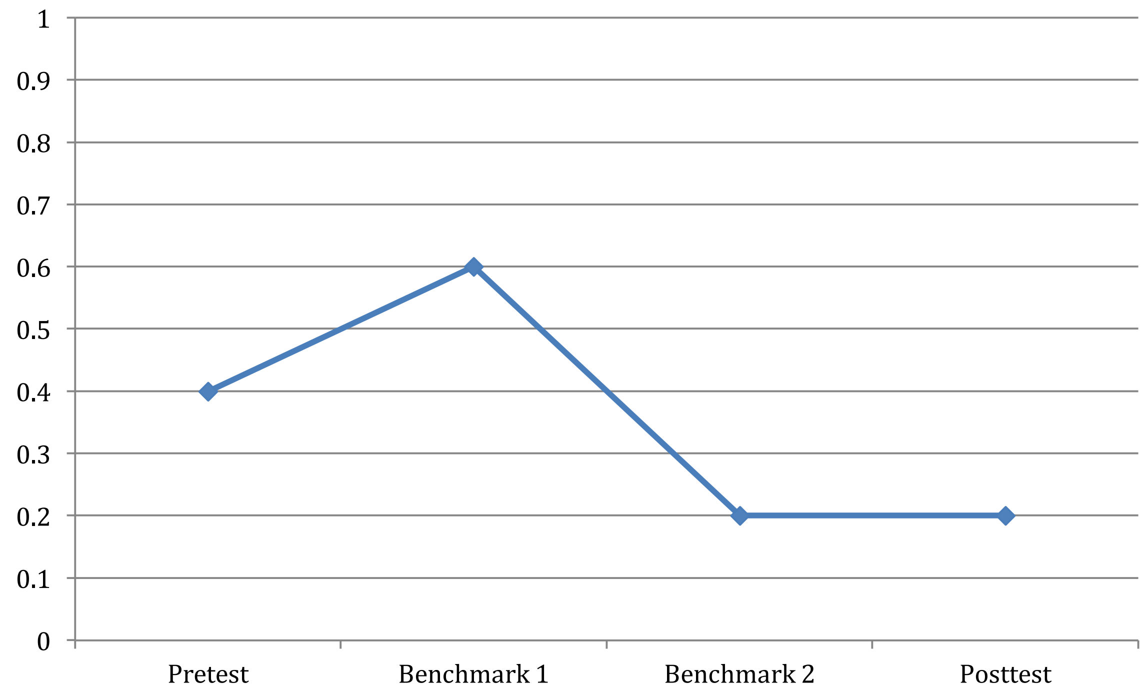


Figure 13. Line graph of Anna's Number Line Task scores across measurement points.

Anna's Computational Fluency scores show a consistent increase from pretest to benchmark 2, followed by a static score of .24 for benchmark 2 and the posttest. The Story Problem scores reflect a similar trend to Anna's overall test score line graph. The Number Lines tasks show a decrease, instead of an increase. The task-based interviews provided evidence explaining why these graphs follow these trends. The following section summarizes the themes in Anna's variations in number sense development based on the qualitative analysis of the task-based interviews. The following sections are organized by subtest. The final section, "Overall themes," will describe these subtest results holistically and tie together concepts and themes.

The Assessment of Fact Fluency. Three themes emerged in coding Anna's computational fluency strategies: (1) finger counting to abstract counting, (2) changes in counting fluency, and (3) a shift in the phases of computational fluency.

Finger counting to abstract counting. When relying on counting strategies to solve or explain the problems on the fact fluency subtest, Anna progressed from reliance on finger counting to abstract counting. During the pretest interview, most of Anna's strategies involved counting with her fingers, unless one of the addends or minuend/ subtrahend was 2. Table 8 shows Anna's strategies for the interview portion of the fact fluency problems at each measurement point. The Phase 1 row of Table 8 shows that 4 of the 6 instances Anna used a counting strategy on the pretest, she used her fingers to keep track. The other two abstract counting instances were with 2+9 and 10-2. During the posttest interview, all of Anna's counting strategies (n = 5) were coded as abstract counting instead of finger counting.

Changes in counting fluency. During the benchmark 1 interview, the subsequent descriptions show Anna's frustration with solving problems with addends larger than 9. Due to her reliance on counting strategies at this point in her computational fluency

Table 8

Anna's Strategies for Solving Fact Fluency Problems at Each Measurement Point

Phase	Strategy	Pretest	Bench 1	Bench 2	Posttest
1	Counting with fingers	4	5	3	--
	Counting abstractly	2	2	--	5
2	Reasoning strategies	--	1	2	--
3	Retrieval (reasoning strategy to explain)	--	--	3	2
	Retrieval (memorized)	2	1	2	3

development, the reason for her difficulty seemed to be a lack of counting fluency, especially with counting backwards. One instance of this during the benchmark 1 interview was when Anna was asked to solve 9 + 9, which she had not solved on the assessment. She stated, “This is hard,” “I am trying to solve it,” and “I know this one I just forgot” (she had actually solved it as a memorized fact on the pretest). When the researcher asked, “If you forgot it, what’s another way you could solve it?” Anna laughed and responded, “I could just sit here and wait, or I could use my fingers and try to figure it out.” The dual activity of counting and keeping track of the counting made the problem “hard.” She struggled with counting on from 9 and keeping track of which fingers she counted. She finally responded with an incorrect guess of “19” because “it is a 9 again with a one on it.” Though incorrect, both in terms of the solution and mathematical reasoning, this statement indicated a shift in her thinking in terms of the inefficiency of her counting strategies and consideration of other ways to solve the problem.

Also during the benchmark 1 interview, Anna was asked to solve 18-9. After a long pause with her hands over her face with moving fingers indicating counting with fingers, Anna said, “this one’s hard.” The researcher asked, “I saw your fingers moving, what were you doing?” Anna replied, “I was trying to count down like 18…” The researcher prompted the next number, “17” and Anna continued, “18, 17, 16, 15, 14, 13…” Again, the researcher intervened and said, “12.” Anna said “12…” but still could not come up with the next number. After a researcher prompt of “11,” Anna finished and indicated her solution of “9”: “11, 10, 9.”

This difficulty with counting fluency took place at benchmark 1 after three weeks of participation in the counting-focused instructional treatment. During those first three weeks (nine teaching episodes) up to benchmark 1, there were no instances of counting backwards as part of the instructional treatment. The first instance of counting backwards was during Week 5 of the instructional treatment, in which the objectives of Teaching Episodes 13, 14, and 15 were focused on counting backwards. Benchmark 2 followed Week 6 of the instructional treatment, and Anna showed an improvement in her skills of counting backwards. One more teaching episode during Week 8 focused on counting backwards by ones and tens, though counting backwards became a common theme of exploration during several teaching episodes after Week 5 of the instructional treatment. The instructional treatment could have played a role in Anna's increased fluency with counting, especially counting back, when solving problems. The following transcript presented in Table 9 provides an example of Anna's improved counting fluency. In particular, accurate and more fluid counting took place at benchmark 2 and the posttest.

Table 9

Anna's Transcript for Solving 12-8 from Pretest to Posttest

Pretest	Benchmark 1	Benchmark 2	Posttest
T: How did you figure that out? S: (long pause) 4 T: How did you know? S: I counted backwards with my fingers like last time. T: Did you start with 12… S: Yes and then 12, 11, to get to … (long pause) T: Did you say 4?	S: (long pause) 3 T: How did you do that? S: I started at 12 and I was starting to count back and then I was like 1, 2, 3, 4, 5, 6, 7, 8 (showing fingers) so it was ummm 3. T: So you counted backwards 12, 11… S: 10, 9, 8 7, 6, 5, 4, 3	S: (pause) 12.. 10, 9, 8, 7, 6… 4. T: How did you figure that out? S: I was counting back and I was trying to use my fingers to make sure.	S: Uhhh T: That's a hard one, right? S: Yea. (pause) 4. T: How did you know? S: Because I was counting back. I start at 12 and count back 8 and land on 4.

The one nuanced difference between the benchmark 2 and posttest instances was that Anna used her fingers to keep track of counting back at benchmark 2 but did not use fingers (abstract counting) during the posttest interview.

Note that it is possible Anna struggled to count backwards during the benchmark 1 interview and instead counted on from 8 to 12 to solve the problem as evidenced by her statement, "I was starting to count back and then I was like 1, 2, 3, 4, 5, 6, 7, 8." The researcher may have only focused on her statement "count back" and led her to agree that she counted backwards. The interview and transcript does not provide enough information to fully understand her strategy at that point. What it does reveal is that Anna did not have a solid strategy and had difficulty explaining how to solve 12-8. Then, in the benchmark 2 interview, she relied on counting back and did so fluently and accurately. In the posttest interview, she counted back without using her fingers.

Shift in the Phases of Computational Fluency. While Anna's computational fluency line graph (Figure 10) shows a static score of .24 from benchmark 2 to posttest, the task-based interviews at those two measurement points revealed an interesting shift in her computational fluency strategies. Anna's strategies indicated a shift from Phase 1: Counting to Phase 2: Reasoning Strategies at the benchmark 2 measurement point. This important shift in computational fluency explains the increase in her scores from pretest to benchmark 1 to benchmark 2. At the posttest interview, Anna's strategies indicated that she was solidifying her strategies in Phase 2: Reasoning Strategies while also showing evidence of moving into Phase 3: Retrieval. It is possible that Anna's score of .24 on benchmark 2 remained the same at posttest because of this shift through phases of computational fluency. She was working to solidify her new knowledge of reasoning strategies. Moving through these phases in a matter of a few weeks was a quick progression through the phases for developing computational fluency. This was reflected in the task-based interview, though not evidenced in her test scores. Table 8 shows Anna's movement from reliance on counting at pretest to evidence of reasoning strategies at posttest. This shift is evident in the total instances of strategies within each phase (e.g., four instances of counting with fingers at pretest and none at posttest; no instances of reasoning strategies at pretest and two at posttest).

The following transcription (see Table 10) provides an example of Anna's progression through the phases for computational fluency. On each of the four

Table 10

Anna's Transcript for Solving 5+6 from Pretest to Posttest

Pretest	Benchmark 1	Benchmark 2	Posttest
S: (long pause) 12 T: How do you know? S: Because I know that there's 6 and then I count…there's 4 (pointing to 4 in 4+5) and there's 2 more spaces from 4 to 6 so that's 2 more spaces from 6 and there's 5, so 10, 20…20…I mean 12. T: So were you counting from 9 to get there? S: Yea.	S: (long pause) 11 T: How did you know? S: Because I counted with my fingers and I started with 5 (puts out one hand) and then I did 1, 2, 3, 4, 5 (on her other hand) and then added one more (shows 1 with the first hand) and that's 6 so that equals 11. T: So when you say that you counted 1, 2, 3, 4, 5 and then added one more is 6 that helped you know to count...so you started at five in your head? S: Yea, I started with 5 in my head and then I counted 6, 7, 8, 9, 10 (showing her five fingers) and then I knew one more was 6 so I counted one more in my head and landed on 11.	S: It's like this (pointing to 4+5) so 5+5 is 10 and one more…11.	S: 11 T: How did you know? S: If that was a 5, I just do 5 and then count one more because that's just one more than 5 so I know that's 11.

assessments, Anna did not solve 5 + 6, and was asked by the researcher to solve it during the task-based interview. In this instance, Anna's strategies for solving 5 + 6 progressed from counting with her fingers at benchmark 1 to using the reasoning strategy of a near double (i.e., 5+5=10 and one more is 11). During the pretest interview, Anna attempted to use 4 + 5 = 9, which was the first problem on the assessment and was already solved, to figure out 5 + 6. She recognized that there was a 5 in both problems and that 6 was two away from 4 so she used the sum of 9 and tried to count on two more. While a plausible strategy that showed an attentiveness to using relationships among numbers and equations, Anna incorrectly answered 12.

During the benchmark 1 interview, Anna used her fingers to keep track of 6 so she could count on from 5. During the benchmark 2 and posttest interviews, Anna used the near double reasoning strategy. The only difference in her approach was to talk out the strategy to reach the solution in benchmark 2 versus a quick response of "11" in the posttest interview followed by an explanation of the reasoning strategy. This could indicate increased fluency leading to future Phase 3: Retrieval strategies.

Story Problems. Three major themes emerged in Anna's progress with the story problem section of the assessments: (1) an overall progression from direct modeling to counting strategies, (2) a shift in the use of counting from benchmark 2 to posttest, and (3) difficulty using number system knowledge.

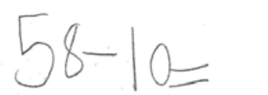
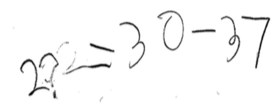
Overall progression from direct modeling strategies to counting strategies. This shift in strategies for two-digit numbers from pretest to posttest was evident in the following transcript. The table below with the transcript also includes the problem for each assessment and Anna's written work on each assessment. In this example of Anna's strategies for solving Separate Result Unknown problems, her pretest highlighted her initial difficulty with subtracting two two-digit numbers: 10 from 58. The researcher presented the same problem type but with subtracting 2 from 10, and Anna was able to successfully solve the problem type. At benchmark 1 she directly modeled the problem using the hundreds chart. At benchmark 2 Anna directly modeled the problem by drawing lines to represent 43 rocks and crossing out 30 of those rocks. At the posttest, there was evidence of Anna's shifting strategies. She drew 58 lines, but then erased all of them and changed her strategy for solving the problem. Instead of directly modeling 58 rocks and crossing out 10, she used a counting backwards strategy (counted back from 58) to correctly solve the problem with a solution of 48. This posttest example highlights Anna's transition to a counting strategy.

The transcript of the Separate Result Unknown example (shown in Table 11) provided evidence of Anna's transition to a counting strategy. Similar to Anna's shifts in computational fluency, these shifts in her strategies for solving story problems took place after 6 weeks of the counting-focused instructional treatment. The Week 5 Teaching Episodes 13, 14, and 15 were focused on counting backwards. Anna's benchmark 2 response to the subtraction problem showed some evidence of considering other ways to solve the problem (when prompted by the researcher) and her posttest response showed more fluency with counting backwards by ones.

Shift in the use of counting from benchmark 2 to posttest. Figure 11 shows Anna's progress from .25 on benchmark 2 to 1.0 on the posttest Story Problem situations. This achievement was also reflected in Anna's posttest interview. Anna's responses to all four story problems showed that counting was still a theme in Anna's mathematical thinking,

Table 11

Anna's Transcript for Solving Separate Result Unknown Problems from Pretest to Posttest

Pretest	Benchmark 1	Benchmark 2	Posttest
Mrs. Nancy had 58 rocks. She gave 10 to Ms. Jessica. How many rocks does Mrs. Nancy have left?	Mrs. Nancy had 37 rocks. She gave 30 to Ms. Jessica. How many rocks does Mrs. Nancy have left?	Mrs. Nancy had 43 rocks. She gave 30 to Ms. Jessica. How many rocks does Mrs. Nancy have left?	Mrs. Nancy had 58 rocks. She gave 10 to Ms. Jessica. How many rocks does Mrs. Nancy have left?
 	 		
T: I see that you wrote 58-10 to solve the problem. Tell me what you were thinking. S: I was thinking…I wrote 58 and then 10…I still haven't figured out… T: How would you figure it out? S: By counting my fingers T: Can you show me? S: 10, 9, 8, 7, 6, 5, 4, 3, 2, 1 (Given the problem again but with 10-2) T: How many rocks do you have now? S: 8 T: How do you know? S: Because I thinked in my head again. T: What did you do in your head again? S: I counted backwards.	S: So 30 to you? T: Mhmm. S: (long pause then gets hundreds chart: points to 37 and counts backwards by ones to land on 7) 7 T: You started here right and you ended here, right? S: Yes. T: What do you notice about that (pointing to 37 and 7)? S: They are both 7s. T: They both have 7s in them, why is that? S: Because this has..this is a row of 7 and they each have a 7 on this side (sliding down across the 7s in that column). And then it goes 1, 2, 3, 4, 5… (pointing to the tens place in 17, 27, 37, 47, 57).	T: Look at how you solved that! So, tell me what you did there. S: I put…I did 30…I did 43 here and I counted and took away 30. So I took away these and then I counted what these were and landed on 13. T: Do you feel pretty confident with that? S: Mhmm. Can you think of another way to solve that might even be faster? S: (refers to previous problem; sifts through pages to look at the problem where she used tens and ones) I can't find it…this one…how you add those two.	T: Tell me about this one, I see lots of lines here. S: I was thinking two ways, I could draw it or I could do it in my mind. I could just count back. T: Show me how you counted back. That's great. S: And then, I just started with 58, 56, 57…I counted forward (laughed)… 58, 57, 56, 55, 54, 53, 52, 51…(pause) 49, 48, 47…(pause). So…I counted backwards wrong. T: Count back again because I think you skipped one number. Try it one more time because you were doing it right.

(*table continues*)

Pretest	Benchmark 1	Benchmark 2	Posttest
T: Can you count backwards for me? Just how you did it? S: 10, 9, 8, 7, 6, 5, 4, 3, 2, 1 T: But how did you count backwards in your head to figure out there were 8? S: I counted by 1 and then by 2 and that makes 8…9, 8.	T: Good noticing. So when you went backwards you passed 27 and then you passed 17 and got to 7. How many did you take away? S: 30 T: 30. So 37 minus 30 to get to 7. S: Yes. T: Hmmm, I wonder if there is an even faster way to count 30? S: I could go up if I wanted but I don't really know how to go up (sliding pen from 37 to 7). T: Let's look at that row and go down, 27, 37, 47, 57…What are we counting by? S: Sevens. T: Not sevens… S: I meant…umm…by 1s. T: Watch, we are counting by 10s.	T: …Is that what you mean, how you looked at the tens and ones in those numbers? How might you do it with this problem? S: (pause) It's time for Samantha's birthday! T: We'll finish up our interview. I think you are on to something. You are saying that you think you can do something with the tens and ones but you are not quite sure what to do with that yet? S: Yes.	S: 58, 57, 56, 55, 54, 53, 52, 51…50! We forgot 50, that's why. 49 and then it went to 48.

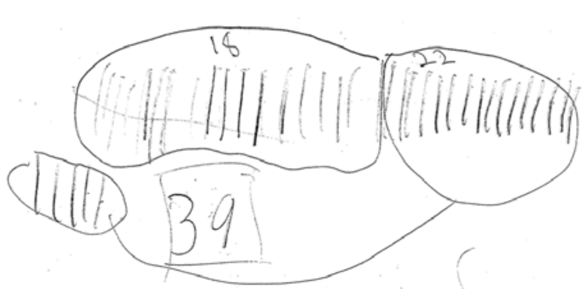
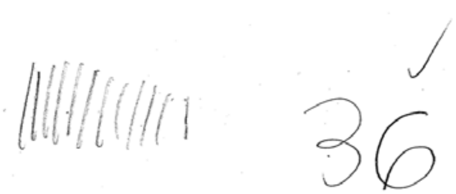
but counting was being used in new ways. For example, she was no longer directly modeling and could skip count (Multiplication problem), count on from the larger number instead of the first number in the problem (Join Result Unknown problem), count backwards by ones (Separate Result Unknown problem), and count up two (Join Change Unknown problem). These were clear changes from Anna's pretest counting strategy for the Join Result Unknown problem when she counted on from the first number instead of the largest number, and her direct modeling strategies on benchmark 1 when she counted all the objects. The following examples of Anna's written work on the benchmark 2 and posttest assessments show this shift in her use of counting when solving the Join Result Unknown problems (see Table 12).

Anna directly modeled the problem on benchmark 2 by first drawing 18 then drawing 22 and finally counting all the lines. On the posttest, she only drew 12 lines to help her count on from 24.

Difficulty with using number system knowledge. During the task-based interviews, the researcher asked Anna questions to find out how she was using her number system knowledge to solve the story problems. To continue with the Join Result Unknown problem from the previous section, once Anna explained how she solved the problem (by directly modeling) to incorrectly get 39, the researcher then asked, "You labeled each of

Table 12

Anna's Written Work for Solving Join Result Unknown Problems on the Benchmark 2 and Posttest Assessments

Benchmark 2	Posttest
Ms. Bobby has 18 red cubes and 22 blue cubes. How many cubes does she have?	Ms. Bobby has 12 red crayons and 24 blue crayons. How many crayons does she have?
	

those groups and then you counted all those to get your answer…now is there a faster way you can figure it out?" The transcript below in Table 13 shows this conversation and illustrates Anna's conflict between a new correct answer and her original incorrect answer. This example highlights that she struggled with using her number system knowledge to solve this problem, but her consideration of a new strategy showed a willingness to think outside her typical direct modeling and counting strategies by attempting to combine tens and ones.

When prompted by the researcher during the task-based interviews, Anna considered combining tens and ones as a strategy. However, when her answer on benchmark 2 conflicted with her original incorrect answer of 39, Anna trusted her counting strategy and believed that the answer was 39. During the posttest interview, the researcher asked questions to elicit explanations for combining tens and ones. Although she combined the tens and the ones, Anna did not use place value language to explain that she combined tens and ones nor could she explain why she combined tens and ones. Anna was still working to solidify her counting strategies and use them more efficiently, so when considering strategies that encouraged her to use her number system knowledge, the evidence shows she had not quite made that transition.

Number Line Tasks. Though Anna had difficulty using number system knowledge to solve problems, Anna's test scores and interviews showed positive shifts in her number sense development both in terms of computational fluency and story problems. Figure 12 shows a different trend in the number line tasks. Benchmark 2 was a turning point for Anna's counting fluency and consideration of strategies beyond counting. However, for the number line tasks, Anna regressed on benchmark 2 in terms

Table 13

Anna's Transcript for Solving Join Result Unknown Problems on the Benchmark 2 and Posttest Assessments

Benchmark 2	Posttest
Ms. Bobby has 18 red cubes and 22 blue cubes. How many cubes does she have?	Ms. Bobby has 12 red crayons and 24 blue crayons. How many crayons does she have?
T: You labeled each of those groups and then you counted all those to get your answer…now is there a faster way you can figure it out? S: I am trying to figure out…(looks at the number) Oh, I have another way! T: Okay. S: You can do…you know how 1 plus 2 equals 3 and there's the three right there and you can add 8 plus 2 equals 9. So you could do that to get your answer. T: So you just broke it into tens and ones? S: Uhhuh. T: So when you say "one" you mean this ten here (pointing to the 18)? S: So the tens spot and the tens spot on this one equals 3 (drawing lines from each number and connecting them writing 3) and it sends it over there (pointing to the 3 in her answer). T: So because that's in the tens spot that becomes a 30? S: Yea, so they're both in the tens spot and this is in the tens spot (pointing to the 3 in 39). And then you add the 2 and the 8 (again drawing lines to connect and writes 9) I just have to take this 2 and this 8 (draws 8 circles and adds two more circles, counts all and gets 10): 1, 2, 3, 4, 5, 6, 7, 8… and then I add two more, 1, 2…1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (looks at the interviewer with a confused face). T: You gave me this look like, wait a minute! What's happening? What are you thinking right now? S: I thought it would equal 9 but it equaled 10. T: So what do you do with it now? S: Oh I know! You can just take one less (pointing to 18) so it will be 17 and then you add 2 more and then it will be 39. T: So you just changed the problem. If it's 17 plus 22 that will make 39. So if its 18 and 22 how's that going to change the answer? S: (pause) T: Instead of 39 what do you think your answer's going to be? S: I'm pretty sure it's going to be 39. I know how I counted so I know it was 39. T: Because you did this strategy (pointing to the drawing) that's convincing you that this is the right answer (pointing to 39). S: Yes.	T: Now if I asked you to solve it all over again, what would be a quick way to solve it? S: Uhhhh…you could do 12 and then plus… (writes 12+24) we do this in real school. We can just do 1 to 2 (draws a line connecting the tens place in the two numbers) and 2 to 4 (draws a line connecting the ones place in the two numbers). 1 and 2 would make 3 and 2 and 4 would make 6 (referring back to her answer of 36). T: Why did you put the 1 and 2 together and the 2 and 4 together? S: Because I thought that would make 3 and that would make 6. So I thought that. T: Well, why not put this 2 with this 2 and that 4 with that 1? S: That would make 5 and that would make 4? T: Why do you think that wouldn't be right? S: Because I don't think that's the answer.

of test score and interview coding. Throughout each of the four interviews, Anna's responses were holistically coded "counting" showing that, overall, she viewed the number line as a counting model instead of a measurement model. Despite the "counting" code at each measurement point, Anna's language showed some evidence of considering relations among number and the magnitude of numbers to place other numbers on the number line. The transcripts below are grouped by similar number (e.g., single digit numbers, numbers close to 50, numbers close to 100) and show these variations in Anna's thinking for solving the number line tasks. The transcripts also indicate whether or not Anna received a "correct" score or "incorrect" score on the test for that particular problem and how many away she was from the accurate placement. Students' responses were scored "correct" if their placement of the number was within 5 hash marks away from the accurate placement.

The transcript below in Table 14 shows Anna's view of the number line as a counting model. Figure 14 shows an example of how Anna often used small tick marks or dots to count up to a single digit number to place that number on the number line. While Anna's strategy was coded as "counting," her explanations show that she considered the whole number line (from 0 to 100) and that she used this strategy for small numbers. She recognized that counting by ones from 0 is not efficient for all numbers.

The next transcript, shown in Table 15, shows that Anna understood that larger numbers such as 84 and 90 would be on the other end of the number line close to 100. The exception was at benchmark 2 when Anna used 64 (from a previous task) instead of 100 as a benchmark. Her estimate for 64 was 41 away from the accurate location, resulting in an even further error of placement for 81 (58 away from the accurate

Table 14

Anna's Transcript for Placing Single-Digit Estimates on the Number Line from Pretest to Posttest

Pretest	Benchmark 1	Benchmark 2	Posttest
8	4	6	8
Incorrect (22 away)	Correct (1 away)	Correct (2 away)	Correct (3 away)
T: …I asked you to show me where 8 belongs. You put it right here. This is a good estimate. How did you know? How did you decide? S: Because I counted on 1, 2, 3, 4, 5, 6, 7, 8 (using finger to show marks on number line).	S: It's the little miniature ones. T: I can see your miniature ones here, is that how you knew that was 4? S: Mhmmm. T: So that's close to 0 not 100. S: (Laughs) No!	S: See you can see the dots again. T: So you counted to 6 and each of those is a one…why did you make the dots so small? S: Because it's a big number (pointing to 100). T: Oh because it goes to 100 and 100's a big number. S: Mhmm.	S: I counted with the dots again! T: That's what you did last time we had a small number, right? S: So I do it with like 33 and stuff, but then I go to the next one like this (39) and then I go like I count from 0 and this is higher (84) so I know it will probably be over here somewhere. T: For 8 and 39 you do those little dots. But you are saying those are less. When it's something like this… S: I go back and see if there's a way to count by tens. T: Why does that work? S: It's easier so it really won't be that hard. T: I really see your thinking. So let me ask you this first, why do you make those so small (the dots). S: 100 is a big number and you have to fit a lot in. It can't be like one big one, one big one…

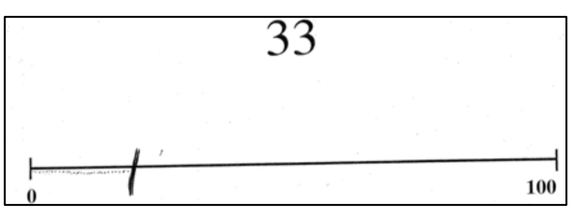


Figure 14. Work sample showing Anna's use of dots to count from 0 to 33 to place 33 on the number line.

Table 15

Anna's Transcript for Placing Estimates Close to 100 on the Number Line from Pretest to Posttest

Pretest	Benchmark 1	Benchmark 2	Posttest
84	90	81	84
Incorrect (16 away)	Correct (4 away)	Incorrect (58 away)	Incorrect (14 away)
T: Why is 84 in a different place from 8? S: Because it's a bigger number so it's farther. T: So you knew it would be further from 8 because it's bigger…how did you know it goes in this place? S: Because I…I pictured one of these [the number line] with lines and I thought it goes right there.	T: How did you know 90 goes there? S: Because it is right by 100, so… T: Why not there? (pointing to the other end) S: That's too far. Because it goes in little things and so it would go 90 and then 100 right here and then it would be a longer number (sliding finger across number line to 100). T: So you are saying 90 and 100 are close to each other? S: Yea…I like to count in little lines so that's how I figured it out.	S: 81…I was thinking like…I was picturing…I looked back at the 60 and it was about right there and I was like it's a little bit more so I'll just put it right here.	T: Tell me a little more about 84 since you didn't use the dots. S: I showed you how I used 39 and its counting by tens. So I knew…and there's another way I can count back and see because 100 and 84 are not that far apart. I still have to go through 90. T: You put your pinky here, is that about where 90 is? S: Yea, and then you have to go up.

location). The posttest transcript shows that Anna considered counting back by tens to get to 90 and then considered where 84 goes in relation to 90 and 100. This strategy led to a more accurate placement of 84, though still not close enough to be scored as a correct answer. The reasoning was improved and was more similar to her reasoning for the placement of 90 on benchmark 1.

The transcript in Table 16 shows Anna's thinking for placing numbers close to 50 on the number line. These numbers were most difficult for Anna, as evidenced by how far away her response was from the accurate location. She did not use 50 as an anchor

Table 16

Anna's Transcript for Placing Estimates Close to 50 on the Number Line from Pretest to Posttest

Pretest	Benchmark 1	Benchmark 2	Posttest
61	42	64	61
Correct (1 away)	Incorrect (10 away)	Incorrect (41 away)	Incorrect (24 away)
T: How did you know 61 goes there? S: Because I did the same thing as the other one. T: So, 84 was here and 8 was about here, why is 61 between those. S: Because it has a 6 and then there's the 8. 8 is more than 6. T: Do you mean the 6 in the tens place? S: Yes. T: …So you were looking at the tens place and that's how you were able to compare those numbers? S: Uhhuh T: Well what about this 8 (pointing to the imagined 8 on the other end of the number line)? S: There's just an 8 not anything else so it's over here just in the ones. T: So you are saying this 8 (teacher writes 8 on that end of the number line) is different from this 8 (in 84; teacher writes 84)? What's the difference? S: This one (pointing to 84) has the number behind it…its tagging along.	T: How did you know 42 belongs there? S: Because I knew…I think I counted in little tiny dots and then I landed right there. T: How come it doesn't go right there? S: Because that's too far. T: Why not there? S: Too short.	T: How did you know 64 belongs here? S: I like to count by small little dots so I was thinking it goes right here. T: So you counted small little dots to get there… S: It's a big number so I had to like fit them all in.	S: I was just guessing. T: Do you have a different estimate or do you think that one's right? S: I think that one's right. T: So on 8 and 39 you used the small dots to help you know where that place is. For 84 you counted by 10s or thought about where it is from 100. 61…that number is a little different from the others. Was that one harder? S: Yea, I don't really count up to it because it's more than a minute. T: It's more than a minute, is that what you said? S: Yea, because 60 is one minute and so it will take a long time so I don't want to count little dots so I just guessed where it was.

number for any of the tasks. Her interviews at benchmark 1 and 2 showed her counting model understanding of the number line in that she used little marks or dots to place 42 and 64 on the number line. Her posttest interview, however, indicated that she guessed because her strategies for small numbers and larger numbers did not work for the number 61.

In solving the number line tasks accurately, Anna had more consistent success with small numbers (e.g., 4, 6, 8) across measurement points. Table 17 shows how far Anna's estimates were from the correct placement of each number. The numbers listed in the first three columns were part of the task-based interviews. The "Other Correct Responses" column were estimates she placed on the number lines correctly, but was not asked about her reasoning during the task-based interviews. The numbers that are highlighted are those that were scored correct on her assessment.

Overall themes. Overall, Anna made progress in her number sense development. Her computational fluency strategies progressed from "Phase 1: Counting" to solidifying

Table 17

Anna's Correct and Incorrect Responses for Solving the Number Line Tasks at Each Measurement Point

Test	Number Line Task Score	Target Number (spaces away from target number)	Target Number (spaces away from target number)	Target Number (spaces away from target number)	Other Correct Responses
Pretest	.4	8 (-22)	84 (-16)	61 (-1)	33 (-5)
Benchmark 1	.6	4 (-1)	90 (-4)	42 (-10)	14 (-3)
Benchmark 2	.2	6 (-2)	81 (-58)	64 (-41)	--
Posttest	.2	8 (-3)	84 (-14)	61 (-24)	--

"Phase 2: Reasoning" and showing evidence of new strategies based in "Phase 3: Retrieval." Anna became more confident and expressive in detailing her strategies over the course of the interviews. Similarly, Anna's strategies for solving story problems progressed from direct modeling strategies to counting strategies. Both shifts in strategy development were evident in the benchmark 2 and posttest interviews after participating in at least 6 weeks of the counting-focused instructional treatments. While the instructional treatments provided opportunities for students to increase their counting fluency, improve their understanding of relative magnitude of numerals and composition of numbers, and understand relationships among numbers, Anna seemed to access the counting fluency objectives and use those to further develop her counting strategies for computation. Hence, her number sense developed over the course of the study, but her use of number system knowledge was just at the beginning stage. There was evidence in the posttest interview that this may be the next developmental phase for Anna.

Anna maintained her view of the number line as a counting model throughout the four task-based interviews. Despite this, Anna's explanations of where numbers close to 100 (e.g., 84, 90) belong on the number line provided some evidence of viewing the number line as a measurement model. She used 100 and 90 as reference points on the posttest. She considered 100 and the relative amount of space between 0 and 100 even when she used counting to place her numbers on the number line. Anna may have focused on the counting aspect of the counting-focused instructional treatments, more so than the discussions about relative magnitude of numerals and the systematic relations among them.

Anthony

Based on his pretest score of .57, Anthony was selected as the "high-achieving" student for interviews from Class 1. Anthony is a Caucasian male who qualified for free/reduced lunch (i.e., low-SES) and did not qualify for ELL or special education services.

Overall test score variations. Anthony began the study at 7 years, 0 months of age (one of the youngest students in the class) and scored in the 96th percentile on the TEMA-3 Form A pretest. At posttest, Anthony was 7 years, 3 months of age and scored in the 99th percentile on the TEMA-3 Form B. The standardized mathematics achievement scores indicated that Anthony was able to complete most of the tasks accurately at pretest and continued to perform in the highest percentiles at posttest.

Figure 15 shows Anthony's whole-class test scores and his learning growth from pretest (.57) to posttest (.84). The line graph shows a steady increase with a 27-percentage point gain overall from pretest to posttest.

Subtest variations. Figures 16-18 show the line graphs of Anthony's test scores disaggregated by subtest (Computational Fluency, Story Problems, and Number Line Tasks).

Anthony's Computational Fluency scores show a consistent increase from pretest to posttest. The Story Problem line graph shows a decrease at benchmarks 1 and 2 followed by an increase back to 100% on the posttest. The Number Lines tasks show an increase from pretest to benchmark 1, a static score at benchmark 2, and a decrease by one problem at the posttest. The task-based interviews provided evidence explaining why the line graphs follow these trends. The following section summarizes the themes in

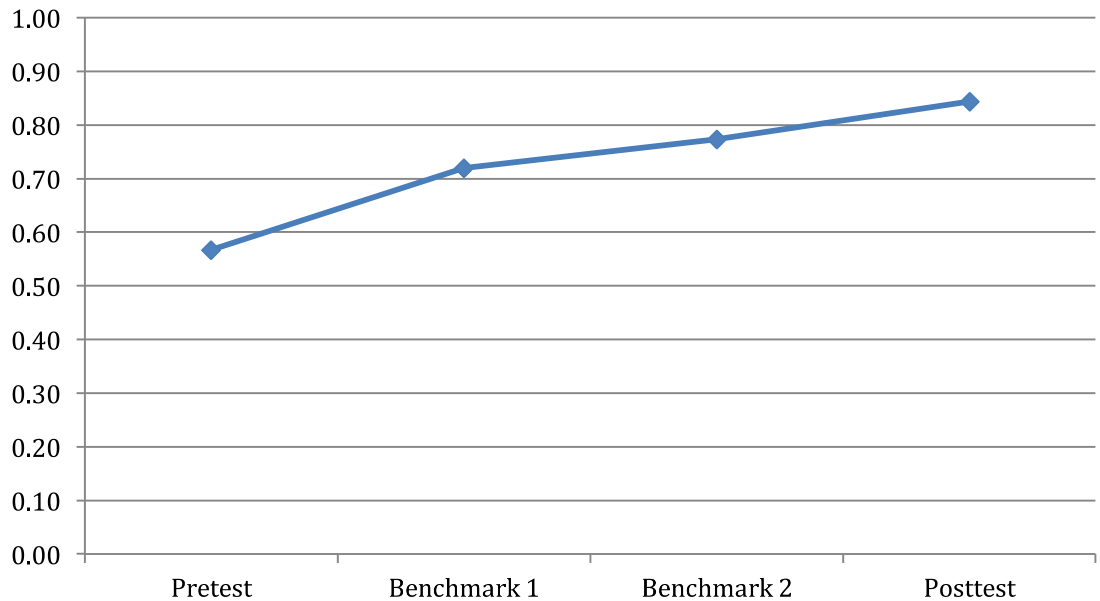


Figure 15. Line graph of Anthony's test scores across measurement points.

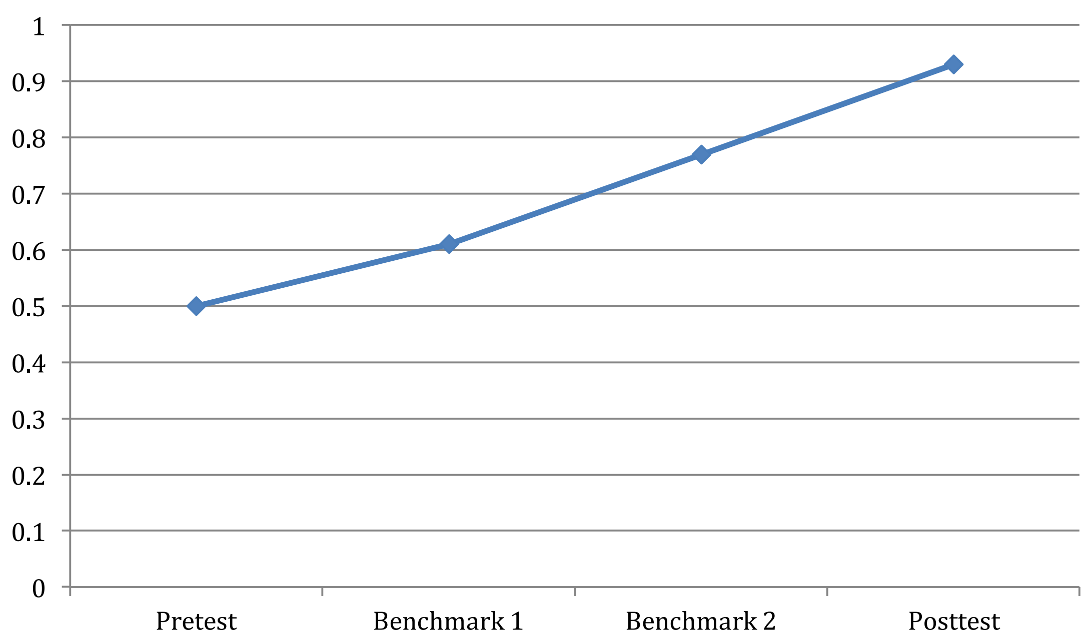


Figure 16. Line graph of Anthony's Computational Fluency scores across measurement points.

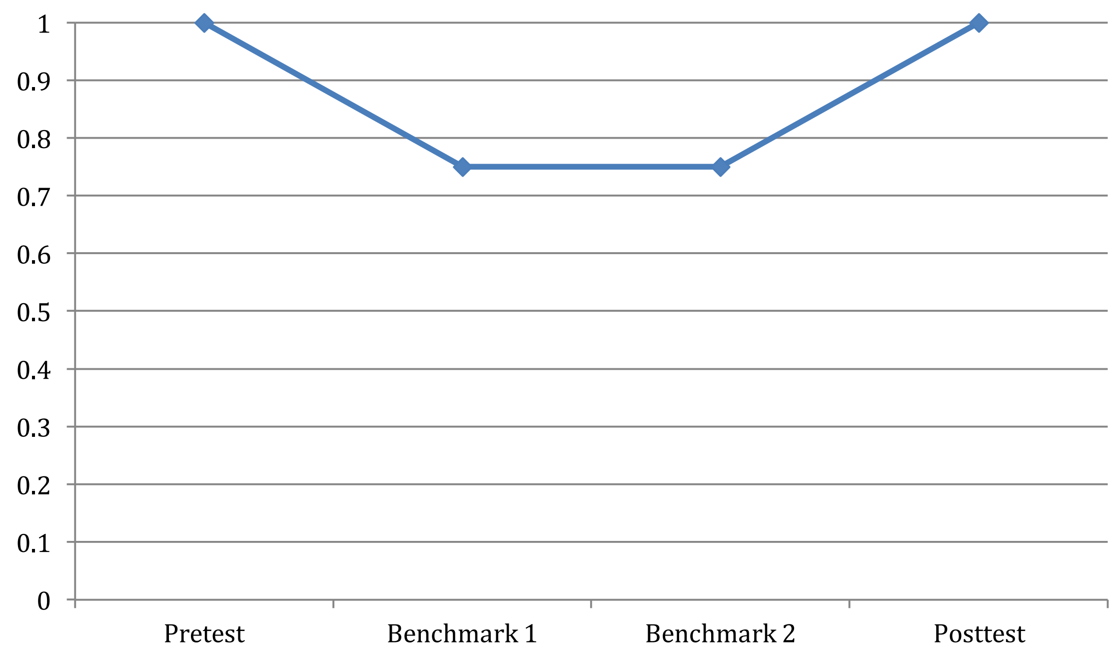


Figure 17. Line graph of Anthony's Story Problem scores across measurement points.

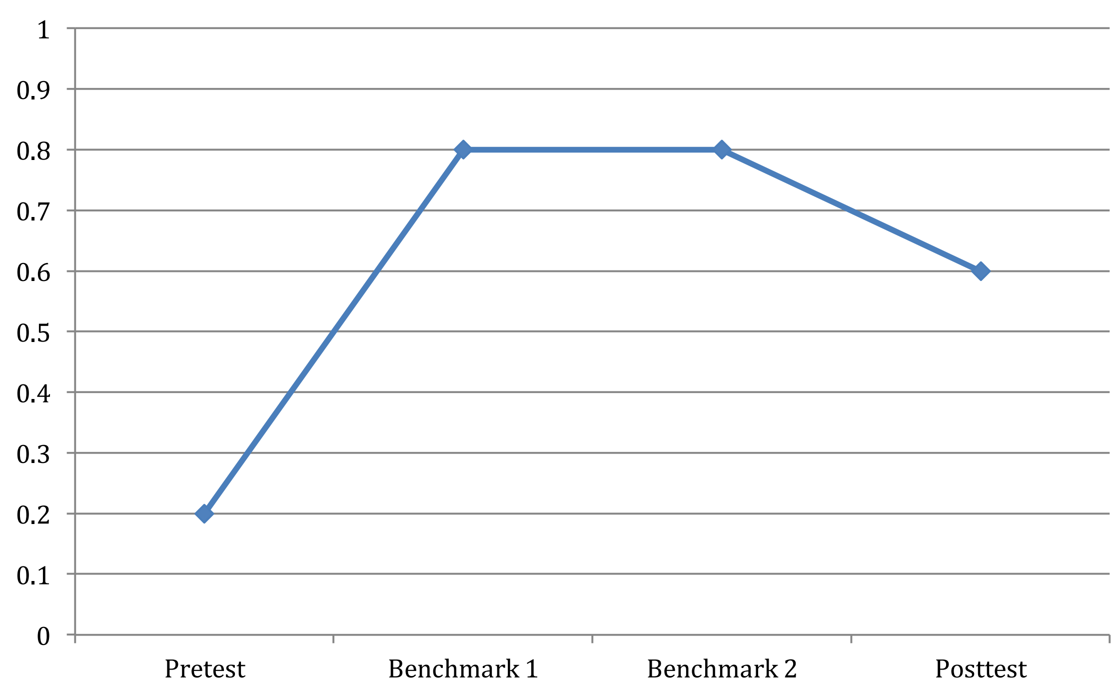


Figure 18. Line graph of Anthony's Number Line Task scores across measurement points.

Anthony's variations in number sense development based on the qualitative analysis of the task-based interviews. The following sections are organized by subtest. The final section, "Overall themes," will describe these subtest results holistically and tie together concepts and themes.

The assessment of fact fluency. The coding schemes for Anthony's computational fluency strategies were fairly consistent across measurement points. For example, a reliance on the number ten emerged as the major theme in coding Anthony's computational fluency strategies at every measurement point. Additionally, his computational fluency strategies were categorized as "Phase 3: Retrieval" on the benchmarks and posttest. While the categories and codes were fairly similar across the measurement points, there was a nuanced shift during Anthony's posttest interview, indicating evidence of flexibility beyond only using 10. The following sections explain the consistent coding across measurement points and the slight nuanced shift in flexibility at the posttest interview.

Phase 3: Retrieval and Anthony's reliance on 10 to explain solutions. During each task-based interview, Anthony's verbal explanations for computing numbers indicated that when "ten" was useful in a problem, such as 5+6 and 2+9, he solved the problem automatically and explained his automatic answer based in the "make a ten" reasoning strategy. For instance, his explanation for 2+9 was consistently that he used 1 (from the 2) to get to 10 and then the other 1 gets him to 11. When use of the ten was not as apparent, such as in 6+8, he found a way to make ten and use ten to solve the problem. In this instance, Anthony solved 6+8 by using 2 from the 6, giving it to the 8 to make 10. Then he had 4 more to add to the 10 to get a solution of 14. Anthony's computation strategies for each assessment were categorized as being in Phase 3: Retrieval, and he used reasoning strategies to explain the solutions. While the most common reasoning strategy was to make 10 and use the 10, he was also able to retrieve doubles facts and consider subtraction-as-addition for the subtraction problems.

Flexibility beyond only using 10. Anthony exhibited a strong sense of confidence on the posttest through his solution explanations. He continued to use 10 to explain his solutions, however, in the posttest interview, there were more instances of using other strategies to explain the solution. One example was Anthony's use of a doubles fact (8+8=16) to solve 6+8. Anthony knew his doubles facts at each interview point (e.g., 9+9=18), but rarely used the near doubles strategies for explaining the computation problems. The transcript in Table 18 shows Anthony's typical explanations across measurement points, with a difference at the posttest when he uses 8+8 to explain the solution for 6+8.

Table 18

Anthony's Transcript for Solving 6+8 from Pretest to Posttest

Pretest	Benchmark 1	Benchmark 2	Posttest
S: 14…wait, wait, let me check for a sec…yep it's 14. T: How do you know? S: Because you are using 2 and you have 4 left over so you get 14. T: Oh you are using 2 from the 6? S: Yea, to get to 10 and then you have 4 extra so you can make 14.	S: That equals 14. I don't know…when you have 8 and you plus 2 and you plus 3 more it equals 14. That's odd, I see my old teacher walk into that other classroom… What's next?	S: 14! T: How do you know? S: You got 8 plus 2 so it equals 14. (He was rushed because of the bell, so the researcher did not push the questioning further.)	S: Because 8+8=16 so 6+8=14. T: It's just… S: Yea, it's just easy!

Based on Anthony's typical responses, the researcher expected him to again make 10 by adding 2 from the 6, then adding 4 to get 14. Instead in the posttest interview, he used a near doubles strategy. This response was even more surprising based on his use of the make-a-ten strategy prior to solving 6+8. The following transcript shows the conversation leading up to the 6+8 explanation.

T: How did you know 9+9?

S: …18-9 equals 9 so 9+9=18.

T: How about 8+9?

S: I got 9 so I plus 8 and I got 7 left because I used one 1 to equal 10 and I got 7 more to plus so I got 17.

T: So you mean you took one off the 8 and gave it to the 9?

S: Yea!

T: How did you know 6+8?

S: Because 8+8=16 so 6+8=14.

T: It's just…

S: Yea, it's just easy!

The other instances in which Anthony did not use the make-a-ten strategy to explain his solutions were on the four subtraction problems. He explained these solutions through the subtraction-as-addition strategy. These appeared in previous interviews, but not as frequently as they did on the posttest.

Story problems. Two major themes emerged in Anthony's approach to the story problem situations: (1) reliance on 10 as a hindrance, and (2) use of other equations to prove his reasoning.

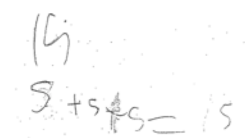
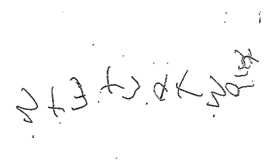
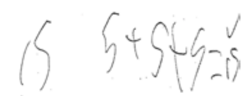
Reliance on 10 as a hindrance. Anthony's understanding of ten as a useful number proved helpful to him throughout the study as he made excellent progress in computational fluency. Understanding ten as an anchor for solving other problems made his reasoning efficient and solutions accurate. However, when Anthony was put in the position to rely on other numbers or keep track of numbers other than ten, he had a bit more difficulty. An example of this was on the multiplication problems. The line graph in Figure 17 shows that Anthony had a score of 100% on both the pretest and posttest for the Story Problems subtest. The number 10 was easily used as an anchor number in these problems by combining the fives to make 10 and adding the other 5 to 10 (3 bags, 5 cookies in each bag). The numbers used in the multiplication problems for the benchmarks were 3x11 and 4x13. Anthony's solutions were scored incorrect for both of these. He particularly struggled with keeping track of the 1s in 11 and the 3s in 13 as is evident in the transcript shown in Table 19. The transcript also shows Anthony's written work for solving the problems.

While the 1 in 11 and the 3 in 13 were more difficult for Anthony to use or keep track of during the benchmark assessments, during the task-based interview, Anthony's language showed that he quickly saw how many 10s his solution should have based on the number of bags in the story problem. Although his answers were scored incorrect on the benchmark tests, the task-based interviews again highlight Anthony's use of number system knowledge to solve arithmetic problems.

Use of other equations to prove his reasoning. The researcher facilitated conversations about number system knowledge during each of the counting-focused instructional treatments. Anthony's task-based interviews, especially at benchmark 2 and the posttest, showed some evidence of this language used during the instructional

Table 19

Anthony's Transcript for Solving Multiplication Problems from Pretest to Posttest

Pretest	Benchmark 1	Benchmark 2	Posttest
Ms. Jessica has 3 bags of cookies. There are 5 cookies in each bag. How many cookies does Ms. Jessica have?	Ms. Jessica has 3 bags of cookies. There are 11 cookies in each bag. How many cookies does Ms. Jessica have?	Ms. Jessica has 4 bags of books. There are 13 books in each bag. How many books does Ms. Jessica have?	Ms. Jessica has 3 bags of cookies. There are 5 cookies in each bag. How many cookies does Ms. Jessica have?
			
S: Because 5+5=10 and 5 more equals 15.	S: 23. T: How did you figure it out? S: Because you have three bags (laughs)…I got it wrong. T: Why, what's the answer? S: 32…uhhhh T: 32 you think now? First you thought it was 23, now you think it's 32… S: Because there's 3 tens. T: 3 tens S: I got mixed up. And two ones. No, three ones. T: Oh, three ones. S: Ahh, 3! T: So how many cookies are there? S: 33 T: 33. I see your thinking, because you are thinking about the 11…there's a 10… S: Yea. First I thought there were only 2. T: So how did you know there are 3 tens and 3 ones? S: Because in 30 there are 3 tens and in 32 there's 2 ones I mean in 33 there's 3…I get mixed up. T: What was it about the problem that told you it would be 33? S: Because you got 3 tens and it comes with 3 ones and you add the 3 ones.	S: 3+3+3+…no +39+30 equals…uhhh…no, it's plus 9, there's supposed to be a plus right there. 3+3+3+9+30=39. T: Oh, so let me ask you this…where did you get 30? Were you getting your 3 from this 13 right here? S: Yea. T: So, tell me what you were doing. S: I kind of forgot. I got like 3 then 3 then 3 I got 6 and I plus…I got 15 plus 30, that's supposed to be a 20 I am pretty sure. T: Let's forget about what you did here. Now, how would you solve it in your head? S: There's four 13s and uh…that doesn't make sense…equals 39. T: Don't worry about that answer you had. You said there were four 13s. If there were four 13s how would you solve it? S: It would equal about…49. 49. T: How do you know 49? S: You got 4 tens and you also got 9 ones so it equals 49. T: So you've got 4 tens, that's the 40. How did you get three…wait, what did you say? 9? Did you say that makes 9? S: Yea. Uh…a few minutes before the bell. Nevermind. Are we still on this one? You might need to write a 4 because it's only one ten away.	S: You got 3 bags and there's 5 cookies in each bag. You plus 5, you plus 5, you plus 5 it equals 15. T: How did you know that 5+5+5=15? S: Because I'm good at math!

treatment. Anthony seemed to access the idea that mathematicians use systematic relationships among numbers and equations to explain their reasoning. This type of reasoning appeared in Anthony's explanations during benchmark 2 and continued into the posttest interview. The following transcripts (Tables 20 and 21) show this type of reasoning.

The examples in the Table 20 transcript show two instances of this occurrence during the benchmark 2 interview. Anthony's written work, especially with the use of the word "so," also provides evidence for this type of reasoning.

The transcript in Table 21 shows two more instances of Anthony using systematic relationships among numbers and equations to explain his reasoning during the posttest interview. This time, the word "so," does not appear in his written work, but does in his explanation for solving the problems.

Number line tasks. Anthony's success with the number line tasks began at benchmark 1, three weeks after starting the counting-focused instructional treatment.

Table 20

Anthony's Transcript for Solving Story Problems on Benchmark 2

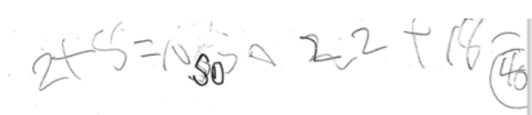
Benchmark 2 Part-Part-Whole Example	Benchmark 2 Separate Result Unknown Example
Ms. Bobby has 18 red cubes and 22 blue cubes. How many cubes does she have?	Ms. Nancy had 43 pencils. She gave 30 to Ms. Jessica. How many pencils does Ms. Nancy have left?
	
S: It equals 46 because you got 3 tens but if you have 2 ones and 10 it equals 40, you get one more 10 so it equals 40.	S: 13. So 43-30=13.

Table 21

Anthony's Transcript for Solving Story Problems on the Posttest

Posttest Separate Result Unknown Example	Posttest Join Change Unknown Example
Ms. Nancy had 58 pencils. She gave 10 to Ms. Jessica. How many pencils does Ms. Nancy have left?	Ms. Jenny had 18 pennies. Ms. Jessica gave her some more. Now Ms. Jenny has 20 pennies. How many pennies did Ms. Jessica give to Ms. Jenny?
58 - 10 = 48	2
S: You minus 10 so it equals 48. T: How did you know that so quick? S: If I have 10 and I give 10 away I have zero. So I am just doing that with 58 so it equals 48. (Physically showing 10 moving away with his body.)	S: She has 18. It's like a sum. What's 18 plus 2, it equals 20. Easy. It's like 8+2=10 so 18+2=20.

Table 22 shows Anthony's number line task test scores for each measurement point and how far his estimates were from the correct placement of each number. The numbers listed in the first three columns were part of the task-based interviews. The Other Correct Responses column were estimates he placed on the number lines correctly, but the researcher did not ask Anthony about his reasoning on that particular task during the task-based interviews. The numbers that are highlighted are those that were scored correct on his assessment.

While Anthony only placed eight correctly on the number line during the pretest, his explanations for why he placed the numbers where he did included phrases such as, "I had a sense it goes there" and "it just came up to me." It seemed that Anthony did not have the language to describe his reasoning and justifications for placing the numbers

Table 22

Anthony's Correct and Incorrect Responses for Solving the Number Line Tasks at Each Measurement Point

Test	Number Line Task Score	Target Number (spaces away from target number)	Target Number (spaces away from target number)	Target Number (spaces away from target number)	Other Correct Responses
Pretest	.2	8 (-3)	84 (-9)	61 (-16)	--
Benchmark 1	.8	4 (-1)	90 (-0)	42 (-1)	14 (-1)
Benchmark 2	.8	6 (-3)	81 (-3)	64 (-4)	14 (-5)
Posttest	.6	8 (-2)	84 (-2)	61 (-3)	--

where he did and could only tell the researcher that "it popped" into his head. He relied on a "sense" of where the numbers belonged and in the instance of 61, he referred to 50 as a benchmark. For these reasons, and because he did not mention counting, his view of the number line was categorized as a measurement model.

There was a shift in Anthony's precision with the number line tasks as well as in his language for justifying his placement of numbers during the benchmark 1 interview. During benchmark 1, Anthony started to use explanations based in the view of the number line as a measurement model. The transcripts below show this shift and the strategies and language tend to remain mostly consistent through the posttest. The differences between the benchmark 1 interviews and the benchmark 2 and posttest interviews were that Anthony began explaining the reference points he drew on the number lines to help him place the target numbers.

The transcripts below are grouped by similar number (e.g., single digit numbers, numbers close to 50, numbers close to 100). The transcripts also indicate whether or not

Anthony received a correct score or incorrect score on the test for that particular problem and how many away he was from the accurate placement.

The transcript in Table 23 presents Anthony's interview with small numbers. Anthony had a correct response for small numbers at all four measurement points.

Table 23

Anthony's Transcript for Placing Single-Digit Estimates on the Number Line from Pretest to Posttest

Pretest	Benchmark 1	Benchmark 2	Posttest
8	4	6	8
Correct (3 away)	Correct (1 away)	Correct (3 away)	Correct (2 away)
T: Look at where you put 8. You were right. S: I was? I didn't think I was going to be right, I was just guessing. T: That was a very good guess, how did you make that guess? S: I was like, 1, 2, 3, 4, 5, 6, 7, 8, I write it really small…It's like, it just came up to me to go right there. I didn't really believe it's going to go there, something just made me want to go there.	S: 4 (laughing) that one's easy! T: Why is that one so easy? S: Because it's only like a half an inch away usually. T: Not a full inch this time? Why? S: Because there's like a half…there's like…I don't know why I did that…but I think it's right there. I was predicting, I was predicting. I was guessing it was right there. Did I get it right? T: That looks about right, don't you think? S: Yea…If I was wrong I think it is somewhere right there. (Points to a space right next to his mark.)	S: Oooh, oooh, that's one's easy because 10's right here 9, 8, 7, 6 is right there.	S: I did 8 little lines by it and landed right there.

Interestingly, his language on the pretest and posttest is based in the counting model view of the number line, while each benchmark interview is based in the measurement model. The benchmark 1 dialogue is an example of Anthony's shift to better precision and use of measurement model language (4 is "half an inch" from 0).

The transcript in Table 24 also shows Anthony's shift to measurement model language (90 is "an inch" from 100). This transcript highlights the difference in Anthony's benchmark 1 interview with the benchmark 2 and posttest interviews. In the

Table 24

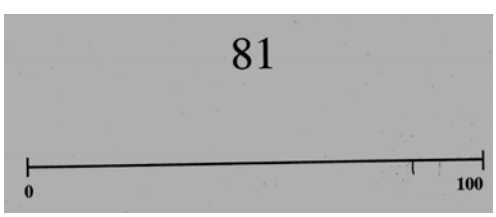
Anthony's Transcript for Placing Estimates Close to 100 on the Number Line from Pretest to Posttest

Pretest	Benchmark 1	Benchmark 2	Posttest
84	90	81	84
Incorrect (9 away)	Correct (0 away)	Correct (3 away)	Correct (2 away)
S: It just came up to me like the other one. T: It did? You just had a sense of where 84 goes? So tell me, how did you decide to put it there? S: It just popped up in my brain. Let's go there a little. T: So tell me why, why did your brain think that it should go there? S: Because I really wanted it to go right but I was pretty sure I was going to be wrong, but I just decided to go there.	S: Because it's right by 100. It's like boom (touching with his knuckle) an inch. It's like you do it by inches away. Boom! (touching his knuckle from 100 to 90 again)	S: I knew it was right there because 80 is right there. T: So these lines that you erased. You thought that was about 80, what's that line you erased? S: I have no idea, like 90 or something. T: So this is 80, this is 90 and 81 would belong here. S: Yea.	S: That took 1, 2 (pointing to the two large hash marks he drew). That was the 90, that was the 80. Then 1, 2, 3, 4 (pointing to the small hash marks past the 80).

benchmark 2 and posttest interviews, Anthony explains the reference points he drew to help him place the target number in the appropriate location on the number lines. Anthony's explanation with using 90 as a reference point and drawing that reference point on the number line (then erasing it) came up in both the benchmark 2 and posttest interviews. His language exuded more assurance in the posttest (benchmark 2 statement, "I have no idea, like 90 or something" versus the posttest statement "that was the 90, that was the 80"), however, it was a similar strategy in both instances. The one difference between the benchmark 2 and posttest solutions was in his written work. Figure 19 shows Anthony's marks on benchmark 2 and the posttest. His posttest written work seemed to aim for more precision where he used longer marks to show 80 and 90 and smaller marks to show the ones leading up to 84.

Similarly, this evidence of slightly more attention to precision also appeared in the following written work in Figure 20 for Anthony's posttest. For 64 and 61, Anthony marked 60 (but not 50 which confused him in his explanation) and then drew 4 more small lines to get to 64. His reference to 50 was not drawn as accurately as it was in the posttest when he placed 61 on the number line. His mark for 50 is better aligned in the

Benchmark 2



Posttest

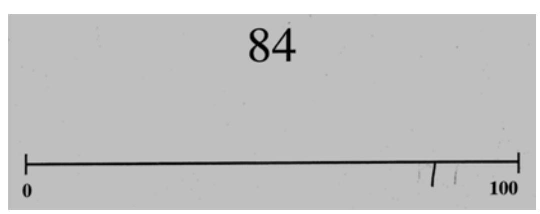
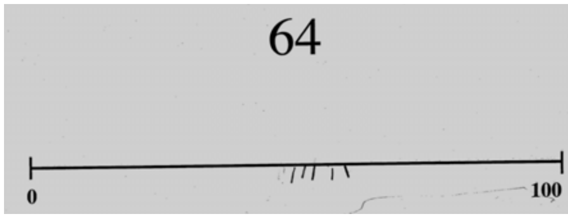


Figure 19. Anthony's written work for 81 and 84 on benchmark 2 and the posttest.

Benchmark 2



Posttest

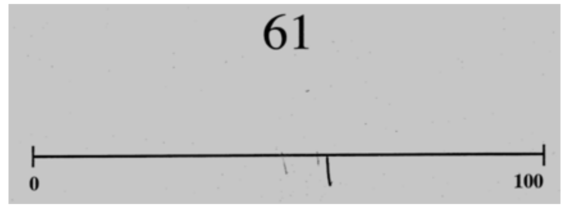


Figure 20. Anthony's written work for 64 and 61 on benchmark 2 and the posttest.

middle of the number line and the mark for 60 is spaced a precise distance from 50. He made a slight mistake here and marked his response one back from 60 instead of one forward from 60. The transcript in Table 25 illustrates Anthony's shift from pretest to posttest in terms of improved precision.

While 24 and 39 were not used in the task-based interview, these written work examples provide another example highlighting the nuanced difference between the benchmark 2 and posttest responses. Figure 20 shows erased marks representing 10 and 20 with four small marks leading up to 24. The posttest example shows marks representing 10, 20, and 30 with nine small marks leading up to 39. The reference marks are more accurately spaced than the 10 and 20 on benchmark 2. The test scores in Table 20 and the line graph in Figure 18 show Anthony's decrease in test score from benchmark 2 (.8) to posttest (.6). On the posttest, Anthony was only 6 away from the correct placement of 39 (which was marked incorrect), as seen in Figure 21, and was 20 away from 33. It is not clear from the written work or the task-based interviews why his estimate for 33 was 20 away from the correct placement of the number.

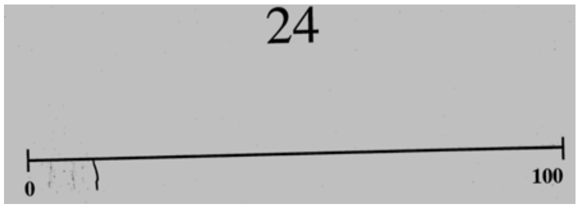
Overall, Anthony had a turning point in terms of accuracy on the tests at

Table 25

Anthony's Transcript for Placing Estimates Close to 50 on the Number Line from Pretest to Posttest

Pretest	Benchmark 1	Benchmark 2	Posttest
61	42	64	61
Incorrect (16 away)	Correct (1 away)	Correct (4 away)	Correct (3 away)
T: Tell me about 61. That is pretty close. This one was a little more off from your other two, but… S: Oh, 50 was right there so 60 was right there. T: Ohhh. S: I just got mixed up at that time. T: Look you used a benchmark, you used 50 to figure out where 61 should go, didn't you? S: Wait, what are benchmarks? T: Benchmarks mean that you knew where 50 goes and you used that to figure our where 61 should go. S: Yea. I just wanted to do a good job with that.	S: Because 50's right here and 40's somewhere right here so 42 should be right there.	S: I was like 50 is right here so 60 is here, 59 is right there and 60 is right there. I forget…I thought 64 was about there. T: So you were trying to think where's 50, where's 60… S: No, 59 is here and about 60, 61, 62, there's supposed to be a 63 there 64 right there.	S: I did 50 and I plussed 10. Which was right there. And then 1.

Benchmark 2 Posttest



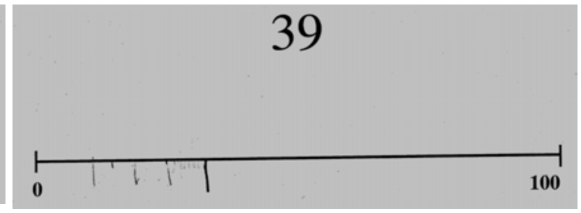


Figure 21. Anthony's written work for 24 and 39 on benchmark 2 and the posttest.

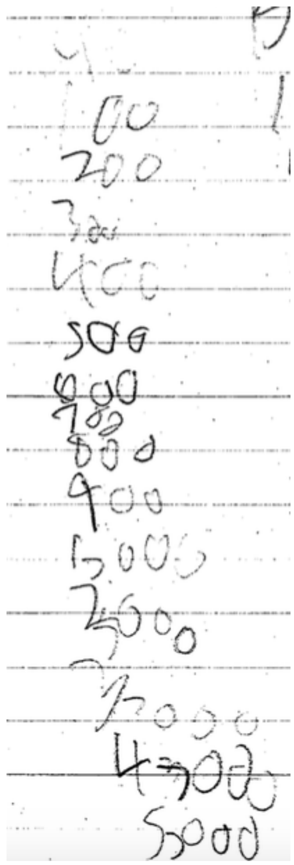
benchmark 1. At benchmark 1 he also had language to describe the distance between numbers and relationships among numbers and was able to express why numbers belong where they do on the number line. The shift in Anthony's strategies from benchmark 1 to benchmark 2 was evident in his marking of reference points on the number line. The posttest written work revealed even more precision with the use of reference points and more confidence in explaining the use of these reference points.

The counting-focused instructional treatments played a role in Anthony's strategies and language with the number line tasks. Open number lines were used as the visual representation during two teaching episodes in the first three weeks of the instructional treatment. Number grids, another format for organizing numbers and highlighting their relationships, were used as the visual representation during four teaching episodes in the first three weeks of the instructional treatment. Number lists, which are treated similar to vertical number lines, were used in three teaching episodes during the first 3 weeks. In conjunction with these visual representations of number relationships, number system knowledge discussions may have helped Anthony have a clearer sense of where numbers belong on the number line which allowed him to develop the language to explain why he placed numbers where he did.

Overall themes. Anthony began the study with a strong sense of number. He made gains in both the standardized TEMA-3 scores and in the overall test scores. The variations in his subtests directed the researcher's attention to various aspects of Anthony's learning. One aspect was his steady increase in computational fluency. A qualitative analysis revealed Anthony's reliance on the number ten as an anchor for solving a variety of problems. This helped him be more efficient and his computational approach was categorized as Phase 3: Retrieval. The posttest interview also uncovered some emerging sense of flexibility, beyond only using ten. Another aspect of Anthony's learning was the decrease in the story problem scores for both benchmarks. The qualitative analysis showed that Anthony's overreliance on ten made it difficult for him to keep track of the multiplication of 11 (in benchmark 1) and 13 (in benchmark 2). Finally, another decrease in Anthony's posttest score for the Number Line Tasks subtests directed the researcher's attention to particular differences between the benchmarks and the posttest. Although Anthony missed one more problem on the posttest, the numbers he placed correctly actually highlighted a more fine-tuned precision to the tasks.

Overall, Anthony seemed to improve his number system knowledge and was able to use it with more confidence and provide more precise explanations by the posttest measurement point. During the posttest interview, the researcher asked Anthony what he learned from Count Around the Circle (the main activity of the instructional treatment). He indicated that he used to think the pattern for counting by 100s in the thousands was "1,000 2,000 3,000, but it's 1,100 1,200 1,300." This growth was also seen in his Counting Journal, as shown in Figure 22. As Anthony participated in the counting-focused instructional treatments, he accessed the number system knowledge concepts and

Early Counting Journal Entry



Later Counting Journal Entry

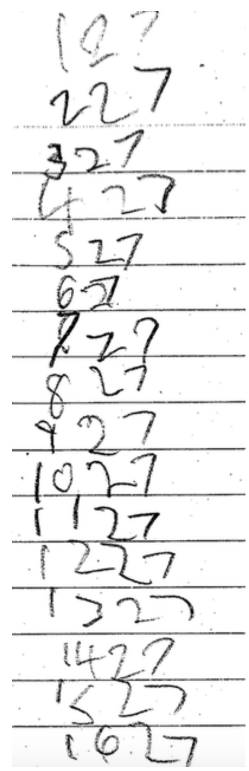


Figure 22. Anthony’s counting journal entries highlight his new learning about counting by 100s as expressed in his posttest interview.

was able to expand those concepts to large numbers while also becoming more precise and flexible with numbers smaller than 100.

Summary: Holistic Analysis

In summary, the results showed that there were variations in number sense development when the second-grade students in this study engaged in counting-focused instructional treatments for differing amounts of time. These variations showed that the counting-focused instructional treatment influenced, changed, and developed these second-grade students' number sense.

Among the three intact classes, the variations showed that there was an associated average increase in test scores when students participated in the counting-focused instructional treatment for longer periods of time (e.g., 9 weeks versus 3 weeks). The population-averaged parameters showed that students from low socioeconomic backgrounds scored on average 6.2 percentage points higher than their peers from average/high socioeconomic backgrounds when controlling for all other variables. Though this finding was not significant, it showed a direction in the data that could be important to investigate in future studies. Students with special education services (IEPs) had significant population-averaged parameters, indicating they scored lower on the tests than their peers. The results of the interaction effects analysis showed significant interactions between Class and IEP, and the line graphs showed that the more time students with IEPs had with the counting-focused instructional treatment, the better their outcomes.

Among the individual students within each class, the variations showed that regardless of which class students were in, many students' test scores followed similar trends across measurement points. Students who scored in the mid-range of performance on the pretest tended to follow the more expected pattern of higher scores on the posttest over the other three measurement points. Interestingly, students who scored the highest on the pretests in all three classes tended to score highest on benchmark 1 as compared to all measurement points, even the posttest. Another interesting variation was that there was a consistent drop in scores at benchmark 2 for all three classes. These variations could be explained by the numbers that were selected to be used in the story problem portions of benchmarks 1 and 2. For example, Anthony's case analysis showed that he struggled with the numbers in the multiplication problem on benchmark 2 because these included groupings of three. This may have been a similar struggle for other students across the three classes. In sum, the nuances of the benchmark tests and number choices for problems on the benchmark tests could have contributed to the nuanced variations in the benchmark outcomes. While the matched pretest and posttest allowed the researcher to easily compare pretest to posttest growth, the different number choices for subtests of the benchmarks helped the researcher gain other perspectives of individual students' number sense variations during the study. Both seemed necessary for the analysis.

Anna and Anthony's variations in number sense development showed that both a low-achieving and a high-achieving student made learning growth from pretest to posttest, though their number sense developed in different ways. Specifically, variations in each student's subtest scores directed the researcher's attention to qualitative variations in different domains of number sense development. Anna seemed to access the counting fluency objectives of the instructional treatment and used those to further develop her counting strategies for computation. These shifts seemed to occur after 6 weeks of the instructional treatment. Anthony seemed to access the number system knowledge concepts and was able to generalize those to larger numbers while also becoming more precise and flexible with numbers less than 100. Some of his shifts in understanding number line estimates occurred after 3 weeks of the instructional treatment and much of his precision began to appear after 6 to 9 weeks. Each student showed important shifts in their number sense development, but these shifts in learning depended on their current level of knowledge and use of number sense.

CHAPTER V

DISCUSSION

The purpose of this study was to explore the variations in second-grade students' number sense development as they engaged in a counting-focused instructional treatment for differing amounts of time. This study represents a link between the cognitive psychology literature and classroom-based instructional practices. The discussion below draws upon the cognitive psychology literature to interpret the findings.

The discussion of the results is organized into five sections. In the first section, the researcher describes the variations in number sense development when students engaged in counting-focused instructional treatments for differing amounts of time. The second section presents a discussion of the variations in number sense development for one low-achieving student and one high-achieving student. The third section outlines this study's implications for educators and researchers. The fourth and fifth sections identify limitations of the study and suggestions for future research.

Participation in Counting-Focused Instructional Treatments and Variations in Number Sense Development

Within the broad construct of number sense, research has identified weaknesses in number system knowledge as a key to learning difficulties in mathematics and as an area for future research (Geary et al., 2013; Jordan et al., 2010). This study tested an instructional treatment that was designed to help second-grade students develop their number system knowledge by verbally counting and discussing systematic relationships among numbers in the counting sequences. The results of the GEE analysis showed an associated average increase in test scores when students in this study participated in the counting-focused instructional treatment for longer periods of time. While the GEE analysis cannot attribute the difference in associated average scores of Class 1 and Class 3 solely to more time with the counting-focused instructional treatment, these results suggest a change in students' outcomes with the counting-focused instructional treatment. Although this study did not control for teacher effects, the evidence suggests that students engaged in 9 weeks of the instructional treatment had differentially better learning outcomes than students engaged in only 3 weeks of the instructional treatment. This means that a larger sample with a rigorous statistical design (e.g., large *n*, control groups, designs controlling for teacher effects) might show that the counting-focused instructional treatment has a positive impact on students' number sense development.

A GEE interaction analysis between group (i.e., Class 1, 2, or 3) and gender was not significant and indicated the instructional treatment worked for males and females alike when holding all the variables constant. An interaction analysis between group (i.e., Class 1, 2, or 3) and IEP was significant and indicated that students with IEPs scored lower on average after accounting for the time students were engaged in the instructional treatment. This finding was not surprising, as prior research has shown that students with learning disabilities or learning difficulties tend to score at the 10^{th} to 25^{th} percentile range in most grades, and these patterns of achievement follow them throughout school (e.g., Geary, Hoard, & Bailey, 2012; Jordan, Hanich, & Kaplan, 2003; Murphy, Mazzocco, Hanich, & Early, 2007). This finding was explained by the nature of the interaction between Class (i.e., time participating in the instructional treatment) and IEP.

The line graphs in Figure 5 showed that students with IEPs, on average, did better in Class 1 when they participated in the instructional treatment for 9 weeks versus in Class 2 or 3 when they participated in the instructional treatment for 6 weeks and 3 weeks, respectively. Again, this result is interpreted with caution because the study did not control for teacher effects. Nevertheless, the longer students with IEPs participated in the counting-focused instructional treatment in this study, the better they performed. This result has important implications for the inclusion of students with special needs in the mainstream classroom and highlights their opportunities for accessing the content in the whole-class setting.

Finally, the GEE analysis revealed that students from low-socioeconomic backgrounds scored on average 6.2 percentage points higher than their peers from higher socioeconomic backgrounds. Although this finding was not significant, the outcome was atypical of what is generally found in the literature (Clements & Sarama, 2008; Jordan & Levine, 2009). Typically, students from low-socioeconomic backgrounds perform worse in mathematics than their peers from higher income families (National Mathematics Advisory Panel, 2008). This result means that the counting-focused instructional treatment could be an important instructional activity for providing opportunities for students from low-socioeconomic backgrounds to develop their number system knowledge. This direction is interesting could be important to investigate in future research for closing achievement gaps in mathematics.

The qualitative analyses of the lesson artifacts revealed that students in all three classes followed a general progression of enthusiasm and use of number system knowledge. This finding suggests that doing the same kinds of rich number sense activities (e.g., Count Around the Circle) with different counting sequences and objectives over time allowed students in this study to focus on the number sense topics as well as make connections among ideas from day to day. As students in this study made these connections over time, their enthusiasm and willingness to search for interesting patterns and relationships in numbers increased. The literature indicates that number sense develops over time and with multiple experiences (Berch, 2005; Feigenson et al., 2004; Pica et al., 2004). The number sense view (Baroody & Rosu, 2006) could provide an explanation for this finding. The number sense view explains that as students construct networks of understanding and deeper understanding of concepts, related skills such as estimation and fact fluency improve. Making meaning of patterns and relationships over time with the instructional treatment could account for increased enthusiasm and changes in whole-class discussions about number system knowledge.

Variations in Number Sense Development for One Low-Achieving Student and One High-Achieving Student

An analysis of one low-achieving and one high-achieving student provided insight into how different students accessed different features of the counting-focused instructional treatment. Students in this study had opportunities to increase their counting fluency, improve their understanding of relative magnitude of numerals and composition of numbers, and understand relationships among numbers. Anna and Anthony’s task-based interviews revealed that they each accessed different opportunities for learning. This access was based on their current number sense and how they were using number sense to solve problems. Additionally, what they accessed from the instructional treatment led to shifts in their learning. Sometimes this was revealed in their test scores. In other instances, their task-based interviews highlighted more nuanced patterns in their number sense development.

Anna seemed to access the counting fluency objectives and use those to further develop her counting strategies for computation. Her shifts in strategy development were evident in the benchmark 2 and posttest interviews, after participating in 6 weeks of the instructional treatment. During weeks 5 to 9 of the instructional treatment, Anna participated in teaching episodes focused on counting backwards and skip counting. These teaching episodes helped Anna become more efficient with counting forwards and backwards by ones, and skip counting. This improved fluency with counting seemed to help Anna's computational fluency strategies shift from Phase 1: Counting to Phase 2: Reasoning. This is a common trajectory, just as Baroody et al.'s (2009) research discovered that, "Computing sums by counting can provide children the opportunity to discover patterns and relations that can serve as the basis for reasoning strategies" (p. 82). Research shows that children who develop mathematics learning difficulties rely on the more basic "count all" finger strategies for extended periods (Jordan & Levine, 2009) just as Anna did at the beginning of the study. The frustration of keeping track of her fingers and what they represented was evident in her pretest transcripts. Just as previous research has shown, Anna's accuracy improved as she learned to use more effective counting procedures, such as counting on from the larger addend (Geary et al., 2012; Jordan & Levine, 2009; Locuniak & Jordan, 2008). This research extends the extant research by showing a potential link between a whole-class instructional treatment and shifts in a struggling student's strategies for solving problems.

Anna's problem solving strategies also shifted from direct modeling to more efficient counting. Again, this is a common trajectory for developing strategies to solve problems (Carpenter et al., 1999; Hiebert et al., 1997). This shift corresponded with the shift in her computational fluency strategies, both of which took place after 6 weeks of the instructional treatment. This means that Anna's participation in the counting-focused instructional treatments influenced her number sense development. A possible explanation for this is that Anna's experiences with symbolic numbers in the context of Count Around the Circle, experience with visual representations of the counting sequences, and discussions about patterns in the numbers and their relationships may have sharpened both her ANS and SNS. Halberda and Feignenson's (2008) research showed that the ANS continues to sharpen in acuity through the elementary years, and especially from experiences with symbolic numbers combined with visual representations (SNS). It is possible that the symbiotic relationship and interaction between Anna's developing ANS and the counting-focused instructional treatments targeting the development of her SNS influenced the shifts in her strategies for solving symbolic number problems. This finding is important for understanding the "interplay between individual experience and the 'number sense'" (Halberda & Feignenson, 2008, p. 1464).

Anna was likely accessing the counting fluency objectives of the counting-focused instructional treatment. Hence, the instructional treatment may have contributed to her regression on number line tasks. This was interesting because at the time of benchmark 1 and benchmark 2, Anna had participated in teaching episodes that focused on making leaps of 10 on the number line and exploring number relationships with each other in terms of tens. Despite these conversations taking place in the classroom, Anna continued to focus on developing her skill with counting forwards and backwards by ones and skip counting by 2s and 4s. As Wilson et al. (2009) described, “a key development which must occur during human learning is the association between non-symbolic number sense and the cultural symbols which represent number (e.g., number words and Arabic digits)” (p. 225). Anna began making these associations, but her counting strategies limited her progress in some areas of number system knowledge, particularly reasoning about the number line as a measurement model instead of a counting model (Diezmann & Lowrie, 2008). Overall, Anna’s number sense developed over the course of the study, but her use of number system knowledge was just at the beginning stage. There was evidence in the posttest interview that this may be the next developmental phase for Anna, if given experiences to continue developing both her ANS and SNS.

Anthony appeared to access number system knowledge concepts and was able to expand those concepts to large numbers while also becoming more precise and flexible with numbers smaller than 100. His shifts in flexibility with small numbers (beyond using only 10) were evident during the posttest interview. Anthony’s shift in using systematic relationships and reasoning to justify his solutions appeared during the benchmark 2 and posttest interviews after 6 weeks of the instructional treatment. Better precision with number line estimates and improved explanations of reference points were evidenced in the task-based interviews at each measurement point. Improvement in language describing the number line estimations began after 3 weeks of the instructional treatment and improvement in his explanations and numerical precision began after 6 weeks of the instructional treatment. These changes in Anthony’s learning could be explained by Pica et al.’s (2004) findings that language plays a role in the emergence of more exact representations of number as well as exact arithmetic. Drawing on Pica et al.'s research that the counting system in English promotes a conceptual integration of the ANS, discrete object representations, and the verbal code, it is possible that the counting-focused instructional treatment influenced some of these changes in Anthony's flexibility with numbers, numerical precision, and ability to explain his reasoning about numbers. In particular, the finding about Anthony's increased precision with the number line estimates is important because of the positive correlation between number-line estimation and mathematics achievement (Siegler & Booth, 2004). Siegler and Booth asserted that exposure to relevant number-line estimation experiences tended to improve estimation accuracy. Much of this research has been conducted in preschool settings. This study extends the findings by providing initial evidence of the influence of a counting-focused instructional treatment on second-grade students' improved number line estimation precision. Overall, these results mean that Anthony was able to access the number system knowledge concepts in the counting-focused instructional treatments and generalize these ideas to his written tests and explanations in task-based interviews.

Implications

This study provides important contributions to classroom-based practice and number sense research. For many teachers, it is difficult to orchestrate differentiated, whole-class mathematics instructional activities due to their students' wide-ranging mathematics abilities. This study identifies a promising instructional practice for elementary teachers facilitating whole-class mathematics instruction.

Patterns and trends emerging from this study indicated that a counting-focused instructional treatment has the potential to influence and change students' number sense development. Findings from this study showed that this type of instructional treatment provided number sense learning opportunities for students from a wide range of abilities and backgrounds within one classroom setting. The box plots in Figure 2 highlighted the wide range of students' pretest scores before the instructional treatment began. The GEE analysis showed that students in this study performed better with 9 weeks of the instructional treatment, students with IEPs had better outcomes with 9 weeks of the instructional treatment, and students from low-socioeconomic backgrounds benefited from the instructional treatment. Anna's case study showed that a low-achieving student who struggled with developing her number sense had important learning shifts due to her improved counting fluency. Anthony's case study showed that a high-achieving student who began the study with robust number sense continued to develop his number sense and accessed number system knowledge concepts.

The findings from this study showed that identifiable shifts in learning often took place after at least 6 weeks of the instructional treatment. Number sense theory (Baroody & Rosu, 2006; Greeno, 1991; Resnick et al., 1990) indicated that number sense cannot be taught as a lesson or unit of study, rather number sense development is ongoing and requires multiple, connected experiences with number sense ideas. This study provides some initial evidence that engagement in at least 6 weeks of connected number sense experiences, at least 3 days per week, can result in important shifts in learning as students develop their number sense.

This study extends the current intervention research by using a mixed methods approach in a whole-class setting to better link research with teaching and learning as it occurs in the general elementary mathematics classroom. If further research on this type of instructional treatment continues to find similar patterns and trends, counting-focused instructional activities, such as the one in this study, hold promise for providing learning opportunities for students with a wide range of abilities and backgrounds.

Limitations

The researcher used an embedded mixed methods design for this classroom-based research in order to capitalize on the strengths of quantitative and qualitative methods. This approach enhanced the research because it brought two lenses to a complex scenario: classroom-based instructional practices and student learning. The variation in data collection enhanced the validity of the study and answered the research questions from several perspectives. While the design was methodologically sound and accounted for the complexity of classroom-based research, the embedded mixed methods approach and the design for this study had limitations.

The quantitative portion of the design did not have a large sample with random assignment and control groups, and therefore, presented limitations to the conclusions and generalizations. The results of the quantitative portion of the study were interpreted with caution and were not interpreted as causal.

To strengthen the statistical conclusion validity, the researcher used psychometrically sound measures (e.g., Fuchs et al., 2003) and codes (e.g., Geary, 2011; Lago & DiPerna, 2010) established by previous research. Data were triangulated through the use of paper-pencil tests, qualitative task-based interviews, and lesson artifacts. Additionally, repeated measurements over time (i.e., pretest, benchmarks, and posttest) compensated for the weaknesses in the statistical design. Since a counterfactual was not used in the study, the pipeline design provided comparison groups.

The sampling technique for the individual interviews was purposive rather than random, which also presented a threat to the statistical conclusion validity. The purposive sample technique was selected because the sample size for the observations and interviews was small, and hence, a probability sample would have overlooked aspects of pretest knowledge and students' mathematics abilities.

In addition to threats to statistical conclusion validity, threats to external validity also presented limitations to the instructional treatment study. The study was conducted at one school. Since it was limited to one context, the results are limited in terms of generalizability to other schools or contexts. Location, history, and local teachers, students, politics, and policies would likely affect the outcomes of the instructional treatment in other settings.

Future Research

The results and implications of this study provide insight for future research on number sense and classroom-based research focused on developing students' number sense. Mathematics education researchers have moved from an initial descriptive research phase in number sense research to prediction (using correlational designs) to improvement (using experimental designs testing various interventions). This study builds on this developing research agenda by designing and testing a counting-focused instructional treatment at the classroom level. Additionally, instructional practices for developing students' number system knowledge had yet to be investigated.

The embedded mixed methods design and analysis allowed the researcher to analyze number sense development variations at both the classroom level and individual student level. This type of mixing of methods allowed the researcher to bridge cognitive science theories with classroom-based instructional practices. The design of the study has implications for research into complex topics such as number sense development. This study showed that a mixed methods approach had the capacity to capture the complexities of student number sense development and the teaching and learning of number sense in the classroom. Further intervention research using a mixed methods approach in a whole-class setting could make research findings directly applicable to teaching and learning. While the mixed methods design for this study was methodologically sound and accounted for the complexity of classroom-based research, further experimental research on number sense interventions is needed in order to generalize to multiple populations. Experimental research on number sense interventions at the school and district levels could also help to determine if the results of this study were unique to these students or if these learning shifts would be common in the larger population.

Anna and Anthony, two students with differing levels of achievement, accessed different concepts from the same instructional treatment. Similarly, the GEE interaction results indicated that students with IEPs had better outcomes if they were in Class 1 with 9 weeks of the instructional treatment. These findings suggest that number sense instructional activities, such as the counting-focused instructional treatment, were effective practices for reaching the needs of students with robust number sense, students struggling in mathematics, and students with learning disabilities or learning difficulties.

Research on how students with IEPs are participating in whole-class practices and what they are accessing would provide educators and researchers with a stronger knowledge base about what types of practices are equitable and effective for all students. Similarly, it will be important to study students, like Anthony, with robust number sense and what their learning paths are as they participate in number sense instructional treatments. One-on-one teaching experiments could lend further insight into the mechanisms for students struggling to develop number sense and the learning paths of students with well-developed number sense.

Finally, this study investigated a newly identified area of the number sense research: number system knowledge. In particular, second- and third-grade students' number system knowledge is an area yet to be fully explored. Future research investigating classroom practices with samples of diverse students, combined with studies investigating individual students' cognitive structures and mechanisms for developing number system knowledge, will be important in continuing this line of research. This research agenda could be translated into practice and result in successful and equitable instructional strategies for all students.

Conclusion

This study's results showed that a counting-focused instructional treatment influenced and changed second-grade students' number sense. Another major finding from this study suggests that the instructional treatment provided number sense learning opportunities for students from a wide range of abilities and backgrounds within the classroom setting. The implication of these results is that an instructional treatment providing multiple, connected number sense experiences over time may be a promising instructional practice for influencing number sense development.

Number system knowledge—knowledge of the systematic relations among numerals and the skills in using this knowledge to solve arithmetic problems—is a key cognitive mechanism in number sense development (Geary et al., 2013). The researcher for this study proposed that counting-focused instructional treatments in second-grade classrooms could pave the way for refinement in students' numerical precision and understanding of the number system (Carey, 2001; Le Corre & Carey, 2007; Lipton & Spelke, 2005). Specifically, second-grade students must use the translation between symbolic and nonsymbolic quantity to begin extending their understanding of the base-ten system and develop fluency with addition and subtraction (CCSSM, 2010). This type of number sense knowledge makes formal mathematics learning more accessible. Studies have found that having this knowledge in elementary school predicted better functional mathematical ability in adolescence (Geary et al., 2013). Classroom-based research that bridges understanding between numerical cognition theory and classroom-based practices was needed to better understand how to provide children with opportunities to develop robust number sense. This study identified a promising instructional practice, based in numerical cognition theory, for elementary mathematics teachers to facilitate opportunities for students to develop their number sense.

REFERENCES

Aunio, P., Ee, J., Lim, S. E. A., Hautamaki, J., & Van Luit, J. E. H. (2004). Young children's number sense in Finland, Hong Kong and Singapore. *International Journal of Early Years Education, 12*(3), 195-216.

Aunio, P., Hautamaki, J. & Van Luit, J. E. H. (2005). Mathematical thinking intervention programmes for preschool children with normal and low number sense. *European Journal of Special Needs Education, 20*(2), 131-146.

Aunio, P., Niemivirta, M., Hautamaki, J., Van Luit, J. E. H., Shi, J., & Zhang, M. (2006). Young children's number sense in China and Finland. *Scandinavian Journal of Educational Research, 50*(5), 483-502.

Aunola, K., Leskinen, E., Lerkkanen, M. K., & Nurmi, J. E. (2004). Developmental dynamics of math performance from preschool to grade 2. *Journal of Educational Psychology*, *96*(4), 699-713.

Bamberger, M., Rugh, J., & Mabry, L. (2012). *Real world evaluation: Working under budget, time, data, and political constraints* (2nd ed.). Thousand Oaks, CA: Sage.

Baroody, A. J., Bajwa, N. P., & Eiland, M. (2009). Why can't Johnny remember the basic facts? *Development Disabilities Research Reviews 15*, 69-79.

Baroody, A. J., Eiland, M., & Thompson, B. (2009). Fostering at-risk preschoolers' number sense. *Early Education & Development*, *20*(1), 80-128.

Baroody, A. J., & Rosu, L. (2006, April). *Adaptive expertise with basic addition and subtraction combinations: The number sense view*. Paper presented in symposium at the annual meeting of the American Educational Research Association, San Francisco, CA.

Berch, D. B. (2005). Making sense of number sense: Implication for children with mathematical disabilities. *Journal of Learning Disabilities, 38*(4), 333-339.

Bliss, S. (2006). Test reviews: Test of Early Mathematics Ability (3rd ed.). *Journal of Psychoeducational Assessment, 24*(1), 85-91.

Brownell, W. A. (1945). When is arithmetic meaningful? *Journal of Educational Research, 38*(3), 481-498.

Cai, J. (1995). A cognitive analysis of U.S. and Chinese students' mathematical performance on tasks involving computation, simple problem solving, and complex problem solving. *Journal for Research in Mathematics Education Monograph Series 7*. Reston, VA: National Council of Teachers of Mathematics.

Cai, J. (2000). Mathematical thinking involved in U.S. and Chinese students' solving of process-constrained and process-open problems. *Mathematical Thinking and Learning, 2*(4), 309-340.

Carey, S. (2001). Cognitive foundations of arithmetic: Evolution and ontogenesis. *Mind and Language, 16*, 37-55.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction.* Portsmouth, NH: Heinemann.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn* (2nd ed.). Sausalito, CA: Math Solutions.

Chard, D. J., Clarke, B., Baker, S., Otterstedt, J., Braun, D., & Katz, R. (2005). Using measures of number sense to screen for difficulties in mathematics: Preliminary findings. *Assessment for Effective Intervention, 30*(2), 3-14.

Clarke, B., & Shinn, M. R. (2004). A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. *School Psychology Review, 33*(2), 234-248.

Clements, D. H. (1999). Subitizing: What is it? Why teach it? *Teaching Children Mathematics, 5*(7), 400-405.

Clements, D.H., & Sarama, J. (2008). Experimental evaluation of the effects of a research-based preschool mathematics curriculum. *American Educational Research Journal, 45*, 443-494.

Clements, D. H., Sarama, J., Spitler, M. E., Lange, A. A., & Wolfe, C. B. (2011.) Mathematics learned by young children in an intervention based on learning trajectories: A large-scale cluster randomized trial. *Journal for Research in Mathematics Education, 42*(2), 127-166.

Coffey, A. & Atkinson, P. (1996). Concepts and coding. In *Making sense of qualitative data* (pp. 26-53). Thousand Oaks, CA: Sage.

Cohen, B. H. (2008). *Explaining psychological statistics* (3rd ed.). Hoboken, NJ: Wiley.

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics*. Retrieved from http://corestandards.org

Confrey, J., & Krupa, E. (2010). *Curriculum design, development, and implementation in an era of common core state standards: Summary report of a conference.* Arlington, VA. Retrieved from http://www.mathcurriculumcenter.org/reports_research.php

Creswell, J. W., & Plano Clark, V. L. (2011). *Designing and conducting mixed methods research* (2nd ed.). Thousand Oaks, CA: Sage.

Dantzig, T. (1954). *Number: The language of science*. New York, NY: MacMillan.

Dehaene, S. (1997). *Number sense: How the mind creates math.* New York: Oxford University Press.

Diezmann, C. M., & Lowrie, T. (2006). Primary students' knowledge of and errors on number lines. In *The 29th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 171-178). Canberra, Australia: MERGA.

Diezmann, C. M., & Lowrie, T. (2008). The role of information graphics in mathematical proficiency. In *The 31st Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 1-4). St. Lucia, Brisbane, Australia: MERGA.

Dyson, N. I., Jordan, N. C., & Glutting, J. (2011). A number sense intervention for low-income kindergartners at risk for mathematics difficulties. *Journal of Learning Disabilities, 46*(2), 166-181.

Dyson, N. I., Jordan, N. C., & Glutting, J. (2013). A number sense intervention for low-income kindergartners at risk for mathematics difficulties. *Journal of Learning Disabilities 46*(2), 166-181.

Ericsson, K. A. (2006). Protocol analysis and expert thought: Concurrent verbalizations of thinking during experts' performance on representative tasks. In K. A. Ericsson, N. Charness, P. J. Feltovich, & R. Hoffman. (Eds.), *Handbook of expertise and expert performance* (pp. 223-241). New York: Cambridge University Press.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, *8*(7), 307-314.

Feldon, D. F. (2010). Do psychology researchers tell it like it is? A microgenetic analysis of research strategies and self-report accuracy along a continuum of expertise. *Instructional Science, 38,* 395-415.

Fuchs, L. S., Hamlett, C. L., & Powell, S. R. (2003). *Grade 3 math battery.* Nashville, TN: Peabody, Vanderbilt University.

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition*, *44*(1-2), 43-74.

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A 5-year longitudinal study. *Developmental Psychology*, *47*(6), 1539-1552.

Geary, D. C., Hoard, M. K., & Bailey, D. H. (2012). Fact retrieval deficits in low achieving children and children with mathematical learning disability. *Journal of Learning Disabilities, 45*(4), 291-307.

Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2013). Adolescents' functional numeracy is predicted by their school entry number system knowledge. *PLoS ONE 8*(1): e54651. doi:10.1371/journal.pone.0054651

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.

Ghisletta, P. & Spini D. (2004). An introduction to Generalized Estimating Equations and an application to assess selectivity effects in a longitudinal study on very old individuals. *Journal of Educational & Behavioral Statistics, 29*(4), 421-437.

Ginsburg, H. P. (1997). *Entering the child's mind: The clinical interview in psychological research and practice.* New York, NY: Cambridge University Press.

Ginsburg, H. & Baroody, A. (2003). *Test of Early Mathematics Ability* (3rd ed.). Austin, TX: Pro-Ed.

Goldin, G. A. (2000). A scientific perspective on structured, task-based interviews in mathematics education research. In A. E. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 307-334). Mahwah, NJ: Erlbaum.

Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education*, *22*, 170-218. doi:10.2307/749074

Griffin, S. (2004). Building number sense with number worlds: A mathematical program for young children. *Early Childhood Research Quarterly, 19*(1), 173-180.

Griffin, S. A., Case, R., & Siegler, R. S. (1994). Rightstart: Providing the central conceptual prerequisites for first formal learning of arithmetic to students at risk for school failure. In K. McGilly (Ed.), *Classroom lessons: Integrating cognitive theory and classroom practice* (pp. 24-49). Cambridge, MA: MIT Press.

Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the "number sense": The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology, 44*(5), 1457-1465.

Hardin, J. W. (2005). Generalized estimating equations. *Encyclopedia of statistics in behavioral science.* Wiley Online Library.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., … Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.

Hudson, C., Price, D., & Gross, J. (2009). *The long-term costs of numeracy difficulties.* London, UK: Every Child a Chance Trust.

Jordan, N. C., Glutting, J., Dyson, N., Hassinger-Das, & Irwin, C. (2012). Building kindergartners' number sense: A randomized controlled study. *Journal of Educational Psychology*. Advance online publication. doi: 10.1037/a0029018.

Jordan, N. C., Glutting, J., & Ramineni, C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning & Individual Differences, 20*, 82-88.

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies of children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development, 74*, 834-850.

Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense trajectories. *Learning Disabilities Research & Practice, 22*(1), 36-46.

Jordan, N. C., Kaplan, D., Olah, L. N., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. *Child Development, 77*(1), 153-175.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2008). Development of number combination skill in the early school years: When do fingers help? *Developmental Science, 11*(5), 662-668.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, *45*(3), 850-867.

Jordan, N. C., & Levine, S. C. (2009). Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities Research Reviews, 15*, 60-68.

Kaufman, E. L., Lord, M. W., Reese, T. W., & Volkmann, J. (1949). The discrimination of visual number. *The American Journal of Psychology, 62*(4), 498-525.

Kennedy, M. M. (2007). Defining a literature. *Educational Researcher, 36*(3), 139-147.

Lago, R. M., & DiPerna, J. C. (2010). Number sense in kindergarten: A factor-analytic study of the construct. *School Psychology Review, 39*(2), 164-180.

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition, 105*, 395-438.

Le Corre, M., Van de Walle, G., Brannon, E. M., & Carey, S. (2006). Re-visiting the competence/performance debate in the acquisition of the counting principles. *Cognitive Psychology*, *52*(2), 130-169.

Lipton, J. S., & Spelke, E. S. (2005). Preschool children's mapping of number words to nonsymbolic numerosities. *Child Development, 76*(5), 978-988.

Locuniak, M. N., & Jordan, N. C. (2008). Using kindergarten number sense to predict calculation fluency in second grade. *Journal of Learning Disabilities, 41*(5), 451-459.

Maclellan, E. (2012.) Number sense: The underpinning understanding for early quantitative literacy. *Numeracy, (5)*2, Article 3.

Malofeeva, E., Day, J., Saco, X. Young, L., & Ciancio, D. (2004). Construction and evaluation of a number sense test with Head Start children. *Journal of Educational Psychology, 96*(4), 648-659.

Markovits, Z., & Sowder, J. (1994). Developing number sense: An intervention study in grade 7. *Journal for Research in Mathematics Education, 25*(1), 4-29.

Miles, M. B., & Huberman, A. M. (1994). *An expanded sourcebook: Qualitative data analysis* (2nd ed.). Thousand Oaks, CA: Sage.

Murphy, M. M., Mazzocco, M. M. M., Hanich, L. B., & Early, M. C. (2007). Cognitive characteristics of children with mathematics learning disability (MLD) vary as a function of the cutoff criterion used to define MLD. *Journal of Learning Disabilities, 40*, 458-478.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

Parsons, S., & Bynner, J. (1997). Numeracy and employment. *Education + Training, 39*(2), 43-51.

Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian Indigene group. *Science, 306*, 499-503.

Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development, 79*, 375-394.

Razali, N. M., & Wah, Y. B. (2011). Power comparisons of Shapiro-Wilk, Kolmogorov-Smirnov, Lilliefors and Anderson-Darling tests. *Journal of Statistical Modeling & Analytics, 2*(1), 21-33.

Resnick, L. B., Lesgold, S., & Bill, V. (1990). From protoquantities to number sense. In G. Booker, P. Cobb, & T.N. de Mendicuti (Eds.), *Proceedings of the Fourteenth PME Conference with the North American Chapter Twelfth PME-NA Conference* (vol. 3, pp. 305-311). Oaxtepec, Mexico: PME-NA.

Saldana, J. (2009). *The coding manual for qualitative researchers*. Thousand Oaks, CA: Sage.

Schleifer, P., & Landerl, K. (2011). Subitizing and counting in typical and atypical development. *Developmental Science*, *14*(2), 280-291.

Shumway, J. F. (2011). *Number sense routines: Building numerical literacy every day in grades K-3*. Portland, ME: Stenhouse.

Shumway, J. F., & Kyriopolous, J. (2013). Mastery multiplied. *Educational Leadership, 71*(4), 73-76.

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development, 75*(2), 428-444.

Siegler, R. S., & Ramani, G. (2008). Playing linear numerical board games promotes low-income children's numerical development. *Developmental Science, 11*(5), 655-661.

Steffe, L. P., & Thompson, P. W. (2000). Teaching experiment methodology: Underlying principles and essential elements. In R. Lesh & A. E. Kelly (Eds.), *Research design in mathematics and science education* (pp. 267-307). Hillsdale, NJ: Erlbaum.

Taylor, K. L., & Dionne, J. P. (2000). Accessing problem solving strategy knowledge: The complementary use of concurrent verbal protocols and retrospective debriefing. *Journal of Educational Psychology, 92*(3), 413-425.

Westbrook, L. (1994). Qualitative research methods: A review of major stages, data analysis techniques, and quality controls. *Library & Information Science Research, 16*(3), 241-254.

Wilson, A. J., Dehaene, S., Dubois, O., & Fayol, M. (2009). Effects of an adaptive game intervention on accessing number sense in low-socioeconomic-status kindergarten children. *Mind, Brain, & Education, 3*(4), 224-234.

Xu, F. & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition, 74*, B1-B11.

Xu, F., Spelke, E. S., & Goddard, S. (2005). Number sense in human infants. *Developmental Science*, *8*(1), 88-101.

Yang, D. C., & Tsai, Y. F. (2010). Promoting sixth graders' number sense and learning attitudes via technology-based environment. *Journal of Educational Technology & Society, 13*(4), 112-125.

APPENDICES

Appendix A

Institutional Review Board (IRB) Certificate and Continuation Approval Document

Institutional Review Board

USU Assurance: FWA#00003308

UtahState University
OFFICE OF RESEARCH AND GRADUATE STUDIES

Expedite #7

Letter of Approval

FROM:	
	Melanie Domenech Rodriguez, IRB Chair
	True M. Rubal, IRB Administrator
To:	Patricia Moyer-Packenham, Jessica Shumway
Date:	July 03, 2014
Protocol #:	5938
Title:	A Counting-Focused Instructional Treatment For Developing Number System Knowledge In Second-Grade: A Mixed Methods Study On Children'S Number Sense
Risk:	Minimal risk

Your proposal has been reviewed by the Institutional Review Board and is approved under expedite procedure #7 (based on the Department of Health and Human Services (DHHS) regulations for the protection of human research subjects, 45 CFR Part 46, as amended to include provisions of the Federal Policy for the Protection of Human Subjects, November 9, 1998):

> Research on individual or group characteristics or behavior (including, but not limited to, research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices, and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies.This approval applies only to the proposal currently on file for the period of one year. If your study extends beyond this approval period, you must contact this office to request an annual review of this research. Any change affecting human subjects must be approved by the Board prior to implementation. Injuries or any unanticipated problems involving risk to subjects or to others must be reported immediately to the Chair of the Institutional Review Board.

This approval applies only to the proposal currently on file for the period of one year. If your study extends beyond this approval period, you must contact this office to request an annual review of this research. Any change affecting human subjects must be approved by the Board prior to implementation. Injuries or any unanticipated problems involving risk to subjects or to others must be reported immediately to the Chair of the Institutional Review Board.

Prior to involving human subjects, properly executed informed consent must be obtained from each subject or from an authorized representative, and documentation of informed consent must be kept on file for at least three years after the project ends. Each subject must be furnished with a copy of the informed consent document for their personal records.

4460 Old Main Hill | Logan, UT 84322-4460 | PH: (435) 797-1821 | Fax: (435) 797-3769 | WEB: irb.usu.edu | EMAIL: irb@usu.edu

UtahState University
OFFICE OF RESEARCH AND GRADUATE STUDIES

Institutional Review Board

USU Assurance: FWA#00003308

Continuation Approval

From: Melanie Domenech Rodriguez, IRB Chair
Nicole Vouvalis, IRB Director

Nicole Vouvalis

To: ***Patricia Moyer-Packenham***

Date: ***06-17-2015***

Protocol #: ***5938***

Title: ***A Counting-Focused Instructional Treatment for Developing Number System Knowledge in Second-Grade: A Mixed Methods Study on Children's Number Sense***

This approval applies only to the proposal currently on file. Any change affecting participants must be approved by the IRB prior to implementation. The Institutional Review Board originally approved your protocol on **July 3, 2014**. As required for yearly continuation review, you have received another year's approval up to the day prior to the anniversary date. All approved protocols are subject to continuing review on a random basis annually, which may include the examination of records connected with the project. Injuries or any unanticipated problems involving risk to subjects or to others must be reported immediately to the IRB Office (797-1821).

Prior to involving participants (if applicable), properly executed informed consent must be obtained from each participant or from an authorized representative, and documentation of informed consent must be kept on file for at least three years after the project ends. Each participant must be furnished with a copy of the informed consent document for their personal records.

Please note Data cannot be used for another study or an extension of the current study without IRB approval either through modification (addendum) or a new application.

4460 Old Main Hill Logan, UT 84322-4460 PH: (435) 797-1821 FAX: (435) 797-3769 WEB: irb.usu.edu EMAIL: irb@usu.edu

Appendix B

Instructional Treatment Teaching Episodes

A Counting-Focused Instructional Treatment:
Second-Grade Teaching Episodes
September – December 2015

Pre-Intervention Week: Pretest & Introductions

Pretest prior to TE#1: Introduction session

Week 1: Talking About Patterns

TE 1

Introducing "Count Around the Circle":
~ How it works (Count by 1s from 0)
~ Think time (Count by 10s from 0)
~ Listen to each other
~ Mistakes are often where the interesting math is

Counting Sequences: Count by 1s from 0, count by 10s from 0
Visual: Pre-write number grid 1 to 110
Counting Journal: Count by 10s or write the number grid
Focus: Introduction and math talk norms
Focus: Number grid and mental picture of numbers
Focus: Talk around the mathematics

TE 2*

Counting Sequences: Count by 1s from 34, count by 10s from 34
Visual: Write number grid as students count

			34	35	36	37	38	39	40
41	42	43	44	45	46	47…			

and

		34	44	54	64	74	84	94	104
114	124	134	144	154…					

Counting Journal: What did you learn?
Focus: Patterns (what do you notice?); If we keep going, what number would go here? How do you know?
(within each of these counting cycles, facilitate discussion about estimation and relationships among numbers; focus on one more/one less, ten more/ten less)

TE 3

Counting Sequences: Count by 1s from 92, Count by 10s from 92
Visual: introduce open number line

92 93 94 95 96 97 98 99 100…

and

92 102 112 122 132 142 152 162 172 182
192 202 212 222 232 242 252…

Counting Journal: Draw an open number line and use it to count.

Focus: Patterns (what do you notice?)
Focus: Leaps on the open number line and the relationships among numbers

Week 2: Estimation and Number Relationships

TE 4
Counting Sequences: Count by 10s starting at 0; Count by 5s starting at 0
Visual: Lists of numbers side-by-side to compare doubling halving and to facilitate estimates (or calculation!)
Counting Journal: Write the sequences in your Counting Journal.
Focus: Estimation
Focus: Number Relationships

TE 5
Start with discussion about yesterday's patterns in 10s and 5s (based on Counting Journals)
Counting Sequences: Count by 10s forward and back on the open number line
Visual: Open number line
Counting Journal: Count by 10s forward and back on the open number line.
Focus: Estimation
Focus: Number Relationships

TE 6
Counting Sequence: Count by 2s starting at 40
Visual: number grid
40 42 44 46 48
50 52 54 56 58
60 62…
Counting Journal: Count by 2s in your Counting Journal
Focus: Patterns in numbers

Week 3: Place Value

TE 7
Counting Sequences: Count by 10s starting at 20, then 23
Visual: Long list to compare the two sequences

20	23
30	33
40…	43…

Counting Journals: Count by 10s starting at one of these numbers: #1: 44; #2: 144, or #3: 1,144.
Focus: Place value system and "ten-ness"

TE 8
Counting Sequences: Count by 10s starting at 40, then 140, then 1,040

Visual: Long list to compare sequences
Counting Journal: Write the counting sequence by tens starting at 1,040. Watch out for the "tricky leaps."
Focus: Place value system

TE 9
Counting Sequences: Count by 10s starting at 80, then 84
Visual: Long list to compare the two sequences
Counting Journal: Write one of the counting sequences in your counting journal.
Focus: Place value system and "ten-ness"

Week 4: Place Value

TE 10
Counting Sequences: Count by 10s starting at 0; 100s starting at 0; 1,000 at 0
Visual: Open number lines (showing addition of 10, 100, or 1000)
Where does 59 belong? 95? 905? 950? 590? 9,500? How do you know?
Focus: Place value system

TE 11
Counting Sequences: Count by 10s starting at 0; 100s starting at 0; 1,000 at 0
Visual: Long lists in columns to compare the sequences
Counting Journal: Write the counting sequences in your counting journal.
Focus: Place value system and "ten-ness"

TE 12*
Counting Sequences: Count by 1s from 34, count by 10s from 34
Visual: Write number grid as students count

			34	35	36	37	38	39	40
41	42	43	44	45	46	47…			

and

		34	44	54	64	74	84	94	104
114	124	134	144	154…					

If time, now count by 100s using the second number grid (write 100s in different color as students count.
Also leave blanks in bottom rows for students to figure out what number will go there if we keep counting.
Counting Journal: Write these in your counting journal.
Focus: Patterns (what do you notice?); If we keep going, what number would go here? How do you know?

Week 5: Counting Along the Open Number Line

TE 13

Counting Sequences: Count backwards by 1s from 10, 20, and 60
Visual: Open number line
Counting Journals: Count backwards in your Counting Journal
Focus: Estimation
Focus: Moving up and down the open number line

TE 14
Counting Sequences: Count backwards by 1s from 110; by 10s from 110 (using same column)
Visual: Pre-write long column of numbers starting at 110
Counting Journal: Write this sequence in your Counting Journal.
Focus: Moving down the column of numbers (vertical number line)

TE 15
Use yesterday's list and count by 10s, then 1s.
Counting Sequences: Start at 60, count back by 2 tens and 6 ones. What did we subtract from 60? Start at 102 and count back by three ones then 8 tens.
Visual: Open number lines
Counting Journal: Try it with any numbers you'd like!
Focus: Moving backwards by different leaps on the number line.

Week 6: Counting (Adding) Fluently with Tens

TE 16
Similar to TE 15, but counting forwards.
Counting Sequences: Start at 12 and add 40; start at 34 and add 60; start at 100 and add 70
Visual: Open number lines (showing leaps of ten) and write equations that match the counting sequence (12+40=52; 34+60=94; 100+70=170)
Focus: Moving forward on the number line in leaps of ten.

TE 17
Counting Sequences: Count by 10s starting at 27. Timer fluency (3x). Then, if time, adding +100 down the column to count by 100s. What do you notice? What helps you know what comes next?
Visual: Long list of numbers in a column.
Counting Journals: Write one or both of the sequences in your Counting Journal. What did you learn today?
Focus: Counting fluently by 10s. Adding 100s.

TE 18
Counting Sequences: Count by 100s starting at 27. Count by 1s starting at 27.
Visual: Columns of numbers.
Counting Journals: Write one or both of the sequences in your Counting Journal. What

did you learn today?
Focus: Counting fluently and looking for patterns.

Week 7: Patterns and Estimation when Counting by 2s

TE 19
Counting Sequences: Count by 2s starting at 114
Visual: Horizontal open number line
Counting Journals: Start at any number between 100 and 1,000 and count by 2s.
Focus: Estimation and patterns

TE 20*
Counting Sequences: Count by 1s from 34, count by 10s from 34
Visual: Write number grid as students count

			34	35	36	37	38	39	40
41	42	43	44	45	46	47…			

and

		34	44	54	64	74	84	94	104
114	124	134	144	154…					

If time, now count by 100s using the second number grid (write 100s in different color as students count.
Also leave blanks in bottom rows for students to figure out what number will go there if we keep counting.
Counting Journal: Write these in your counting journal.
Focus: Patterns (what do you notice?); If we keep going, what number would go here? How do you know?

TE 21
Counting Sequences: Count by 2s starting at 334 (will we make it to the 400s?)
Visual: Vertical column, but start with 334 at the bottom of chart paper and go up.
Counting Journals: Using our counting by 2s pattern, see if you can count by 4s!
Focus: Estimation and patterns

Week 8: Magnitude of Numbers

TE 22
Counting Sequences: Count by 10s and 20s starting at 0, 10, and 20
Visual: Blood pressure visual
Counting Journals: Write the counting sequence in the "Blood Pressure" format. What do you notice about the numbers?
Focus: Relationships among numbers

TE 23
Counting Sequences: On a number line from 1,000 to 1,500, where does 1,203 belong?

1,023? 1,230? How do you know? Count by 10s (or 20s) from 1,000.
Visual: Open number line from 1,000 to 1,500
Counting Journals: Count by 10s on your number line starting from 1,000
Focus: Magnitude of numbers

TE 24
Counting Sequences: On number lines from 0 to 100, count by 10s, count back by 1s, count forward by 1s
Visual: Open number line from 0 to 100; Writing equations representing leaps on the open number line
Counting Journals: Count by 10s on your number line from any number
Focus: Magnitude of numbers and their relationships

Week 9: Using Tens and Ones to Count – Generalizing Number System Knowledge

TE 25
Counting Sequences: Count by 15s starting at 15
Visual: Number grid

15	30	45	60
75	90	105	120…

Counting Journals: Count by 15s
Focus: Decomposing/composing numbers and grouping ideas

TE 26
Counting Sequences: Count by 12s starting at 12
Visual: Number grid

12	24	36	48	60
72…				

Counting Journals: Count by 12s
Focus: Decomposing/composing numbers and grouping ideas

TE 27
Counting Sequences: Count by .25 in the context of money
Visual: Number grid

.25	.50	.75	$1.00
1.25	1.50	1.75	$2.00…

Counting Journals: Count by .25
Focus: Context for counting; grouping ideas

Appendix C

Sample Test Format

Part 1: The Assessment of Math Fact Fluency

Name ____________________________

Add.

6 + 5	8 + 3	4 + 9	8 + 2	6 + 7
7 + 1	4 + 8	0 + 6	8 + 7	9 + 9
6 + 4	5 + 8	9 + 5	6 + 9	3 + 6
4 + 7	6 + 8	4 + 3	5 + 4	7 + 5
3 + 7	9 + 7	9 + 3	8 + 9	3 + 5

Posttest
Addition 0-18

Name ______________________________

Subtract.

12	8	10	13	15
- 8	- 6	- 7	- 8	- 7
16	10	9	12	8
- 7	- 2	- 4	- 3	- 3
18	7	14	13	11
- 9	- 4	- 5	- 4	- 7
17	6	9	14	15
- 8	- 4	- 6	- 8	- 7
10	8	16	5	12
- 4	- 0	- 9	- 1	- 5

Part 2: Story Problem Situations

Ms. Jessica has 6 bags of cookies.
There are 5 cookies in each bag.
How many cookies does Ms. Jessica have?

Ms. Bonnie has 12 red cubes and 24 blue cubes.
How many cubes does she have?

Ms. Nancy had 58 rocks.
She gave 10 to Ms. Jessica.
How many rocks does Ms. Nancy have left?

Ms. Jenny had 18 pennies.
Ms. Jessica gave her some more.
Now Ms. Jenny has 20 pennies.
How many pennies did Ms. Jessica give to Ms. Jenny?

Part 3: Number Line Tasks

(Note: The Number Line Tasks were printed from PowerPoint slides and contained precise measurements.)

8

0 **100**

33

0 **100**

39

0 **100**

61

0 **100**

84

0 **100**

Appendix D

Task-Based Interview Question Guide

Task-Based Interview Question Guide

Initial questions

- How did you solve the problem?
- How did you know it was ____?
- Tell me what you were thinking as you figured it out.

Probing questions based on what the child says or does

- What number did you start with? Then what did you do?
- Can you tell me how you counted?
- Why did you start with that number when you counted?
- Why did that work well for you?
- Why do you like to use that strategy?

Follow-up questions/further probing

- Can you do this problem a different way? (or interviewer give different numbers)
- What if you did this without tools and did it in your head? What would you do?
- What do you notice?

CURRICULUM VITAE

JESSICA F. SHUMWAY

Utah State University
College of Education & Human Services
2805 Old Main Hill
Logan, Utah 84321
jessica.shumway@aggiemail.usu.edu

EDUCATION

Ph.D. May 2016
Education, Utah State University
Specialization: Curriculum and Instruction
Concentration: Mathematics Education and Leadership
Dissertation Title: Shumway, J. F. (2016). *A Counting-Focused Instructional Treatment for Developing Number System Knowledge in Second-Grade: A Mixed Methods Study on Children's Number Sense.*

M.Ed. August 2003
Elementary Education, The George Washington University
DC Public Schools Standard Teaching License, Elementary 1-6 (2008)
Texas Educator Certificate, 1-8 and English As a Second Language (2009)
Commonwealth of Virginia Postgraduate Professional License, Elementary PreK-6 & ESL PreK-12 (2010)

B.A. December 2002
International Affairs, The George Washington University
Minor in Spanish Language and Literature

EMPLOYMENT HISTORY

UTAH STATE UNIVERSITY

University Instructor (2010-present)
School of Teacher Education and Leadership
College of Education and Human Services, Utah State University, Logan, UT
Responsibilities include teaching graduate and undergraduate courses in the elementary education program, collaborating with other instructors, collaborating with the university laboratory school teachers, supervising and mentoring students in their practicum experience, developing course syllabi, and revising courses based on course evaluation feedback.

Early Childhood Mathematics Curriculum and Instruction Consultant (2015-present)
Dolores Doré Eccles Center for Early Care and Education, Utah State University, Logan, UT
Responsibilities include developing and establishing a mathematics curriculum, coaching preschool teachers in their mathematics planning and instruction, and leading professional development in a variety of formats: embedded job development, demonstration lessons, book study, workshops, and action research.

Graduate Research Assistant (2010-present)
School of Teacher Education and Leadership
College of Education and Human Services, Utah State University, Logan, UT
Responsibilities include assisting professors on various research projects in mathematics education such as conducting participant interviews; collecting, organizing, and coding data; conducting quantitative and qualitative analyses; participating in group papers and presentations; and leading a research paper.

PUBLIC SCHOOL TEACHING EXPERIENCE

Elementary School Teacher, Grades 3 & 4, All subjects (2008-2010)
Fairfax County Public Schools, Falls Church, Virginia
Taught in a Title I public magnet school for the arts and sciences. Trained in Cognitively Guided Instruction, Literacy Collaborative, Changing Education Through the Arts, and Responsive Classroom.

Title I Mathematics Teacher/Coach, Grades PreK-5 (2005-2008)
Fairfax County Public Schools, Falls Church, Virginia
Co-created the Math Collaborative (MC) at Bailey's Elementary, a professional development model that included a 40-hour mathematics instruction course and one-on-one coaching. Coached teachers on bridging theory and practice, planning lessons, analyzing student work, reflecting on instructional practices, and assessing students' progress. In addition to MC, developed and facilitated various professional development at the district, school, and team levels. Trained in Developing Mathematical Ideas, Cognitively Guided Instruction, and Cognitive Coaching.

Elementary School Teacher, Grade 2, All subjects (2003-2005)
Round Rock Independent School District, Round Rock, Texas
Taught in a bilingual, Title I public school. Trained in ESL, Love & Logic, Writing Traits, Guided Reading, Cognitively Guided Instruction, and Baldrige Continuous Improvement.

AWARDS & PROFESSIONAL RECOGNITION

Lawson Fellowship Award (2015), Emma Eccles Jones College of Education and Human Services, Utah State University

Graduate Enhancement Award (2014), Student Involvement Office, Utah State University

Fredrick Q. Lawson Fellowship (2013), Emma Eccles Jones College of Education and Human Services, Utah State University

Graduate Research Assistant of the Year Award (2012), School of Teacher Education and Leadership, Utah State University

RESEARCH

RESEARCH INTERESTS

Number sense
Elementary children's mathematics learning trajectories
Pre-service and in-service mathematics teacher learning and development
Preschool and elementary mathematics education curriculum
Representations in the teaching and learning of mathematics

PUBLICATIONS

Books

Shumway, J. F. (2011). *Number sense routines: Building numerical literacy every day in grades K - 3*. Portland, ME: Stenhouse Publishers.

Professional Development Videos

Shumway, J. F. (2014). *Go figure! Number sense routines that build mathematical understanding*. Portland, ME: Stenhouse Publishers.

Book Chapters (Invited)

Shumway, J. F. (in press; online publication date is Fall 2015). Teaching notes for "Building bridges to spatial reasoning." In D. Thiessen (Ed.), *Exploring mathematics through literature: Activities and lessons for prekindergarten through grade 8*. Reston, VA: NCTM.

Journal Articles (Refereed)

Shumway, J. F., Baker, J. M., Moyer-Packenham, P.S., Westenskow, A., Anderson, K., Tucker, S., Boyer-Thurgood, J., & Jordan, K. (2016). Using open-response fraction items to explore the relationship between instructional modalities and students' solution strategies. *International Journal of Education in Mathematics, Science, and Technology, 4*(2), 112-132.

MacDonald, B. L. & **Shumway, J. F.** (2016). Subitizing games: Assessing preschool children's number understanding. *Teaching Children Mathematics, (22)*6, 340-348.

Tucker, S. I., Moyer-Packenham, P. S., **Shumway, J. F.**, & Jordan, K. (2016). Zooming in on students' thinking: How a virtual manipulative app revealed, concealed, and developed students' number understanding. *Australian Primary Mathematics Classroom, 21*(1), 23-28.

Moyer-Packenham, P. S., Bullock, E., **Shumway, J. F.**, Tucker, S. I., Watts, C., Westenskow, A., Anderson-Pence, K. L., Maahs-Fladung, C., Boyer-Thurgood, J., Gulkilik, H., & Jordan, K. E. (2016). The role of affordances in children's learning performance and efficiency when using virtual manipulative mathematics touch-screen apps. *Mathematics Education Research Journal, 28*(1), 79-105.

Moyer-Packenham, P. S., **Shumway, J. F.**, Bullock, E., Tucker, S. I., Anderson-Pence, K. L., Westenskow, A., Boyer-Thurgood, J., Maahs-Fladung, C., Symanzik, J., Mahamane, S., MacDonald, B., & Jordan, K. (2015). Young children's learning performance and efficiency when using virtual manipulative mathematics iPad apps. *Journal of Computers in Mathematics and Science Teaching, 34*(1), 41-69.

Anderson-Pence, K. L., Moyer-Packenham, P. S. Westenskow, A., **Shumway, J. F.**, & Jordan, K. (2014). Relationships between visual static models and students' written solutions to fraction tasks. *International Journal for Mathematics Teaching and Learning, 15*, 1-18.

Moyer-Packenham, P. S., Baker, J. M., Westenskow, A., Anderson-Pence, K., **Shumway, J. F.**, & Jordan, K. E. (2014). Predictors of achievement when virtual manipulatives are used for mathematics instruction. *Journal of Research in Mathematics Education* (*REDIMAT), 3*(2), 121-150.

Westenskow, A., Moyer-Packenham, P. S., Anderson-Pence, K. L., **Shumway, J. F.**, & Jordan, K. (2014). Cute Drawings? The disconnect between students' pictorial representations and their mathematics responses to fraction questions. *International Journal for Research in Mathematics Education, 1*(1), 81-105.

Shumway, J. F., & Kyriopolous, J. (2013/2014). Mastery multiplied. *Educational Leadership, 71*(4), 73-76.

Shumway, J. F. (2013). Building bridges to spatial reasoning. *Teaching Children Mathematics, 20*(1), 44-51.

Moyer-Packenham, P., Baker, J., Westenskow, A., Anderson, K., **Shumway, J**., Rodzon, K., & Jordan, K., The Virtual Manipulatives Research Group at Utah State University. (2013). A study comparing virtual manipulatives with other instructional treatments in third- and fourth-grade classrooms. *Journal of Education, 193*(2), 25-39.

Conference Papers and Proceedings (Refereed)

Bullock, E. P., Moyer-Packenham, P. S., **Shumway, J. F**., Watts, C., & MacDonald, B. (2015). Effective teaching with technology: Managing affordances in iPad apps to promote young children's mathematics learning. Paper presented at the Society for Information Technology and Teacher Education (SITE) Conference, Las Vegas, Nevada.

Moyer-Packenham, P. S., Westenskow, A., **Shumway, J. F.**, Bullock, E., Tucker, S. I., Anderson-Pence, K. L., Boyer-Thurgood, J., Maahs-Fladung, C., Symanzik, J., Mahamane, S., MacDonald, B., & Jordan, K., The Virtual Manipulatives Research Group at Utah State University. (2014). The effects of different virtual manipulatives for second graders' mathematics learning in the touch-screen environment. *Proceedings of the 12th*

International Conference of the Mathematics Education into the 21st Century Project, (Vol. 1, p. 1-6). Herceg Novi, Montenegro.

Westenskow, A., Moyer-Packenham, P. S., Anderson-Pence, K. L., **Shumway, J. F.,** & Jordan, K., The Virtual Manipulatives Research Group at Utah State University. (2014). Cute Drawings? What Students' Fractional Representations Reveal About Their Whole Number Bias. *Proceedings of the 12th International Conference of the Mathematics Education into the 21st Century Project*, (Vol. 1, p. 1-6). Herceg Novi, Montenegro.

Moyer-Packenham, P. S., **Shumway, J. F.**, Bullock, E., Tucker, S. I., Anderson-Pence, K. L., Westenskow, A., Boyer-Thurgood, J., Maahs-Fladung, C., Symanzik, J., Mahamane, S., MacDonald, B., & Jordan, K., The Virtual Manipulatives Research Group at Utah State University. (2014, April). Young children's learning performance and efficiency when using virtual manipulative mathematics iPad apps. Paper presented at the annual National Council of Teachers of Mathematics Research Conference, (NCTM), New Orleans, LA.

Moyer-Packenham, P. S., Anderson, K. L., **Shumway, J. F**., Tucker, S., Westenskow, A., Boyer-Thurgood, J., Bullock, E., Mahamane, S., Baker, J., Gulkilik, H., Maahs-Fladung, C., Symanzik, J., & Jordan, K., The Virtual Manipulatives Research Group at Utah State University. (2014, January). Developing research tools for young children's interactions with mathematics apps on the iPad. *Proceedings of the 12th Annual Hawaii International Conference on Education* (HICE), (pp. 1685-1694), Honolulu, Hawaii, ISSN# 1541-5880.

Tucker, S. I., Moyer-Packenham, P. S., Boyer-Thurgood, J. M., Anderson, K. L., **Shumway, J. F.**, Westenskow, A., & Bullock, E., The Virtual Manipulatives Research Group at Utah State University. (2014, January). Literature supporting investigations of the nexus of mathematics, strategy, and technology in children's interactions with iPad-based virtual manipulatives. *Proceedings of the 12th Annual Hawaii International Conference on Education* (HICE), (pp. 2338-2346), Honolulu, Hawaii, ISSN# 1541-5880.

Moyer-Packenham, P. S., Jordan, K., Baker, J., Westenskow, A., Rodzon, K., Anderson, K., & **Shumway, J. F.** (2013, April). Hidden predictors of achievement: The equalizing effect of virtual manipulatives for mathematics instruction. Paper, Annual Meeting of the American Educational Research Association (AERA), San Francisco, California.

Ng, D., **Shumway, J. F**., & Chedister, M. (2011, October). Teacher educators' discourse moves in supporting preservice elementary mathematics teachers' learning. In T. Lamberg & L. Wiest (Eds.), *Proceedings of the 33rd Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA)*. Reno, Nevada: University of Nevada, Reno.

Journal Articles – Under Review

Shumway, J. F., Westenskow, A., & Moyer-Packenham, P. S. (under review). An exploratory study of a story problem assessment: Task-based interviews for understanding children's number sense. *International Journal for Mathematics and Learning* (IJMTL).

Baker, J. M., Moyer-Packenham, P. S., Tucker, S. I., **Shumway, J. F**., Jordan, K. E., & Gillam, R. (under review). Towards an understanding of the brain's response to math apps: An fNIRS investigation of children's cortical responses to interactions with virtual manipulatives. *Mind, Brain, and Education.*

Journal Articles – Under Development

Shumway, J. F. (under development). Number sense literature review: Constructs, assessments, and interventions.

Shumway, J. F. & Jordan, K. E. (under development). Understanding second-grade students' number sense: A mixed methods pilot study.

Unpublished Manuscripts

Shumway, J. F. (submitted December 2011). Focused professional development: Number sense routines. Unpublished manuscript.

Ng, D., **Shumway, J. F**., Chedister, M., & Chapin, S. (submitted November 2011). Questioning strategies to develop preservice elementary teachers' mathematical explanation and justification ability. Unpublished manuscript.

RESEARCH ACTIVITIES

A Counting-Focused Instructional Treatment for Developing Number System Knowledge in Second-Grade: A Mixed Methods Study on Children's Number Sense (2015-2016). Dissertation study. Utah State University (with chair/PI Patricia Moyer-Packenham).

Accessing Children's Number Sense: An Exploratory Study of Number Sense Assessment Instruments (2013-2014). Created a battery of number sense assessment instruments, piloted a combination of number sense assessment instruments, conducted task-based interviews with participants, collected and coded data, and analyzed data. Project has resulted in refining research tools and a paper submitted for publication. Utah State University (with PI Kerry Jordan).

Captivated! Young Children's Learning Interactions with iPad Mathematics Apps (2012-2015). Pilot tested iPad-based interview protocols, conducted iPad-based interviews with participants, observed and coded participant actions, collected and coded data, and analyzed data. Project has resulted in multiple papers and presentations at the Hawaii International Conference on Education (2014). Utah State University (with PI Dr. Patricia Moyer-Packenham and the Virtual Manipulatives Research Group).

Grades 3-4 Fractions and Virtual Manipulatives Mathematics Project (2010-2012). Taught third- and fourth-grade mathematics fraction units; collected, coded, and analyzed data. Project has resulted in multiple papers and presentations at the NCTM Research Presession (2013), AERA Conference (2013), and the SSMA Conference (2011). Utah State University (with PI Dr. Patricia Moyer-Packenham and the Virtual Manipulatives Research Group).

The Elementary Mathematics Preservice Teachers Project (2010-2011). Taught preservice teachers using the EMP Tasks (fractions unit and area unit), collected data, analyzed and coded transcripts, and conducted library and document searches on the areas of MKT, cognitive demand of tasks, and discourse. Project resulted in a paper and presentation at the PME-NA Conference (2011). Utah State University and EMP Research Group at Boston University with PI Suzanne Chapin (with Dr. Dicky Ng).

Comparing Fractions: Selection and Sequence of Tasks (2011). Collecting, coding, and analyzing data on preservice teachers' choice of examples and sequence of tasks for lessons on comparing fractions. Utah State University (with Dr. Dicky Ng).

Quadrilaterals Study (2010). Taught and assessed preservice teachers using the quadrilaterals lesson, collected data, and analyzed data using SPSS. Project resulted in revisions to a lesson used in the undergraduate mathematics methods course. Utah State University (with Dr. Dicky Ng).

Comparing Fractions Research Study (2010). Collected, coded, and analyzed data on preservice teachers' strategies for comparing fractions. Utah State University (with Dr. Dicky Ng).

GRANTS FUNDED

USUSA Research and Projects Grant. ($1,000). (2015). Utah State University, Student Involvement Office. Funding awarded to support dissertation research costs.

Graduate Student Professional Conference Award. ($750). (2014). Utah State University, Office of Research and Graduate Studies and College of Education. Travel funding awarded for presentation at the NCTM Annual Conference in New Orleans, Louisiana.

Graduate Student Professional Conference Award. ($1,115). (2014). Utah State University, Center for Women and Gender and College of Education. Travel funding awarded for presentations at the Hawaii International Conference on Education in Honolulu, Hawaii.

Graduate Research Assistant ($20,000). *Captivated! Young Children's Learning Interactions with iPad Mathematics Apps.* (2013-2014). Utah State University, Vice President for Research RC Funding. Lead PI – Patricia Moyer-Packenham, Co-PI – Cathy Maahs-Fladung, and the Virtual Manipulatives Research Group. Project goal: build theory and knowledge about the nature of young children's ways of thinking and interacting with virtual manipulatives using touch-screen mathematics apps on the iPad. My role: pilot test interview protocols, conduct iPad-based interviews with participants, observe and code participant actions, collect and code video data, analyze data using SPSS, participate in team meetings, and collaborate on publications and presentations focusing on young children's interactions with iPad apps.

Graduate Student Professional Conference Award. ($550). (2013). Utah State University, Center for Women and Gender and College of Education. Travel funding awarded for presentation at the AMTE Annual Conference in Orlando, Florida.

Graduate Student Professional Conference Award. ($500). (2013). Utah State University, Office of Research and Graduate Studies and College of Education. Travel funding awarded for presentation at the NCTM Annual Conference in Denver, Colorado.

Graduate Research Assistant ($35,000). *Virtual Manipulatives Research Group: Effects of*

Multiple Visual Modalities of Representation on Rational Number Competence. (2011-12). Utah State University, Vice President for Research SPARC Funding. Lead PI – Patricia Moyer-Packenham; Collaborating Faculty - Kerry Jordan, Dicky Ng, and Kady Schneiter. My role: teach experimental lessons in third- and fourth-grade classrooms in local schools, conduct data collection and analysis, participate in research team meetings, collaborate on publications and presentations focusing on using virtual manipulatives to teach rational number concepts.

Graduate Student Professional Conference Award. ($700). (2011). Utah State University, Graduate Student Senate and College of Education. Travel funding awarded for presentation at the NCTM Annual Conference in Indianapolis, Indiana.

TEACHING

UNIVERSITY TEACHING

Utah State University, Logan, Utah (2010-present)
College of Education

TEAL 6521/TEPD 5524 – Mathematics for Teaching K-8: Numbers and Operations (Fall 2015)
Graduate course. Elementary Mathematics Endorsement course. Designed for K-8 teachers to explore the content of Number and Operations to develop a comprehensive understanding of our number system and relate its structure to computation, arithmetic, algebra, and problem solving. Special attention given to how children learn and connect the fundamental concepts of number and operations. Hybrid face-to-face and online; face-to-face includes interactive broadcast (distance education).

ELED 4060 – Teaching Mathematics & Practicum Level III (Fall 2010, Spring 2011, Fall 2012, Fall 2013, Spring 2014)
Undergraduate course. Relevant mathematics instruction in the elementary and middle-level curriculum; methods of instruction, evaluation, remediation, and enrichment. Includes supervision of students in a field experience practicum.

ELED 5150 – Student Teaching Seminar & Supervision (Spring 2012)
Undergraduate field experience and seminar. Supervision of student teachers in primary- (1-3) and upper-elementary (4-6) classrooms. Observed lessons and facilitated triad conferences for ten student teachers. Created and conducted eight seminars based on student teachers' lessons, lesson plans, journal reflections, and questions.

CURRICULUM DEVELOPMENT

Utah State University, Logan, Utah (2013-present)
College of Education

TEAL 6521/TEPD 5524 – Mathematics for Teaching K-8: Numbers and Operations (2015)
Graduate course. Elementary Mathematics Endorsement course. Designed for K-8 teachers to explore the content of Number and Operations to develop a comprehensive understanding of our number system and relate its structure to computation, arithmetic, algebra, and problem solving.

Materials developed included readings, video lectures, application assignments, and assessments for online course delivery. Developed nine modules as the equivalent of a 16-week course.

Elementary Mathematics Teacher Academy – Developed course materials for master's level courses for Utah State University's Elementary Mathematics Teacher Academy (EMTA). Courses designed to develop teachers' mathematical knowledge for teaching aligned with the Common Core State Standards for Mathematics. Materials developed included readings, video lectures, application assignments, and assessments for online course delivery. Developed the following 30 second- and third-grade curriculum modules:

2.G	Big Idea: Foundations for Area, Volume, Congruence, Similarity, and Symmetry (2015)
2.G.2	Composing and Decomposing Shapes: Partitioning Rectangles (2015)
2.G.3	Composing and Decomposing Shapes: Partitioning Equal Shares (2015)
2.NBT.3	Base-Ten Numerals, Number Names, & Expanded Form (2015)
2.NBT.4	Using Place Value: Comparing Three-Digit Numbers (2015)
3.G.2	Partitioning Shapes: Geometry and Unit Fractions (2015)
3.MD.4a,b	Geometric Measurement: Area & Multiplication Operations (2015)
3.MD.4c,d	Geometric Measurement: Decomposing Area & the Distributive Property (2015)
3.MD.5&6	Geometric Measurement: Area (2015)
3.MD.8	Geometric Measurement: Perimeter (2015)
2.OA.1	Addition & Subtraction Word Problems Situations (Part 1) (2013)
2.OA.1	Addition & Subtraction Word Problems Situations (Part 2) (2013)
2.OA.1	Addition & Subtraction Word Problems Situations (Part 3) (2013)
2.OA.2	Add & Subtract Within 20: Developing Fact Fluency (2013)
2.NBT	Big Idea: Properties of Operations (2013)
2.NBT.2	Understanding Place Value: Counting &Patterns in the Number System (2013)
2.NBT.5&9	Addition & Subtraction within 100 (Part 1) (2013)
2.NBT.5,6&9	Addition & Subtraction within 100 (Part 2) (2013)
2.NBT.7&9	Addition & Subtraction within 1000 (Part 1) (2013)
2.NBT.7,8&9	Addition & Subtraction within 1000 (Part 2) (2013)
3.OA.3	Multiplication & Division Word Problem Situations (Part 1) (2013)
3.OA.3	Multiplication & Division Word Problem Situations (Part 2) (2013)
3.OA.3	Multiplication & Division Word Problem Situations (Part 3) (2013)
3.OA.5&6	Properties of Multiplication (2013)
3.OA.7	Multiply and Divide within 100: Developing Fact Fluency (2013)
3.NF.1	Meaning of Fractions (2013)
3.NF.2a,b	Fractions on the Number Line (2013)
3.NF.3a-c	Fractions: Equivalence (Part 1) (2013)
3.NF.3a-c	Fractions: Equivalence (Part 2) (2013)
3.NF.3d	Fractions: Comparing and Reasoning (2013)

SERVICE

PRESENTATIONS

Invited Presentations

Shumway, J. F. (2014, April). *Playing with Numbers: Developing Flexible Computation Strategies*. Presentation for PreK-2 Teachers, 92nd Annual Meeting of the National Council of Teachers of Mathematics (NCTM), New Orleans, LA. (Invited Speaker for the Focus Strand).

Shumway, J. F. (2012, July). *Number Sense Routines: Building Numerical Literacy Every Day.* SDE National Conference on Singapore Math Strategies, Las Vegas, NV. (Invited Presenter).

International Presentations

Shumway, J. F., Westenskow, A., & Moyer-Packenham, P. S. (2014, January). *A Story Problem Assessment: Task-Based Interviews for Understanding Children's Number Sense*. Research Presentation, 12th Annual Hawaii International Conference on Education (HICE), Honolulu, Hawaii.

Moyer-Packenham, P. S., **Shumway, J**., Westenskow, A., Tucker, S., Anderson, K., Boyer-Thurgood, J., & Bullock, E. (2014, January). *Young Children's Mathematics Interactions with Virtual Manipulatives on iPads*. Research Presentation, 12th Annual Hawaii International Conference on Education (HICE), Honolulu, Hawaii.

Boyer-Thurgood, J., Moyer-Packenham, P. S., **Shumway, J**., Westenskow, A., Tucker, S., Anderson, K., & Bullock, E. (2014, January). *Kindergartener's Strategy Development during Combining Tasks on the iPad*. Research Presentation, 12th Annual Hawaii International Conference on Education (HICE), Honolulu, Hawaii.

Tucker, S. I., Moyer-Packenham, P. S., Boyer-Thurgood, J. M., Anderson, K. L., **Shumway, J.**, Westenskow, A., & Bullock, E. (2014, January). *The Nexus of Mathematics, Strategy, and Technology in Second-Graders' Interactions with an iPad-Based Virtual Manipulative*. Paper Session, 12th Annual Hawaii International Conference on Education (HICE), Honolulu, Hawaii.

National Presentations

Shumway, J. F., Kelley, J., Webb, C., & Child, B. (2016, April). *Jumps and Leaps: Number Lines, Number Sense, and Solving Problems*. Presentation for K-5 Teachers, 93rd Annual Meeting of the National Council of Teachers of Mathematics (NCTM), San Francisco, California.

Shumway, J. F. & Everett, J. (2015, February). *Building Number Sense Across the District: Mathematics Professional Development for Elementary Teachers*. Presentation for School Leaders, 2015 National Title I Conference, Salt Lake City, Utah.

Shumway, J. F. (2014, April). *Playing With Numbers: Developing Flexible Computation Strategies*. Presentation for K-5 Teachers, 92nd Annual Meeting of the National Council of Teachers of Mathematics (NCTM), New Orleans, Louisiana.

Moyer-Packenham, P. S., **Shumway, J.,** Tucker, S., Boyer-Thurgood, J., Westenskow, A., Hunt, J., & Bullock, E. (2014, April). *Children's Mathematics Interactions with Virtual Manipulatives on iPads*. Paper Presentation, National Council of Teachers of Mathematics (NCTM) Research Conference, New Orleans, Louisiana.

Moyer-Packenham, P.S., Jordan, K., Baker, J., Westenskow, A., Rodzon, K., Anderson, K., & **Shumway, J.** (2013, April). *Hidden Predictors of Achievement: The Equalizing Effects of Virtual Manipulatives for Mathematics Instruction.* Paper Presentation, Annual Meeting of the American Educational Research Association (AERA), San Francisco, California.

Shumway, J. F., Kyriopolous, J., & Granados, M. (2013, April). *Manipulatives, Models, and Symbols: The Role of Representations in Building Students' Number Sense.* Presentation for K-5 Teachers, 91st Annual Meeting of the National Council of Teachers of Mathematics (NCTM), Denver, Colorado.

Shumway, J. F., Bostwick, A., Anderson, K., & Tucker, S. (2013, January). *Building Partnerships: A Collaborative Lesson-Study Experience in a Preservice Mathematics Methods Course.* Seventeenth Annual Conference of the Association of Mathematics Teacher Educators, Orlando, Florida.

Shumway, J. F. (2012, April). *Fostering Place Value Understandings Through Number Sense Routines.* Presentation for K-5 Teachers, 90th Annual Meeting of the National Council of Teachers of Mathematics (NCTM), Philadelphia, Pennsylvania.

Moyer-Packenham, P.S., Jordan, K., Ng, D., Anderson, K., Baker, J., Rodzon, K., **Shumway, J. F.**, & Westenskow, A. (2011, November). *School Mathematics Research on Virtual Manipulatives: A Collaborative Team Approach.* Panel Presentation, School Science and Mathematics Association Convention, Colorado Springs, Colorado.

Shumway, J. F. (2011, April). *Building Number Sense Through Counting Routines.* Presentation for K-5 Teachers, 89th Annual Meeting of the National Council of Teachers of Mathematics (NCTM), Indianapolis, Indiana.

Shumway, J. F. (2010, April). *Responsive Routines for Early Number Sense.* Presentation for K-3 Teachers, 88th Annual Meeting of the National Council of Teachers of Mathematics (NCTM), San Diego, California.

Shumway, J. & Granados, M. (2009, April). *Math Collaborative: A Journey Through the Complexities of Coaching.* Presentation for Math Coaches and Teachers, 87th Annual Meeting of the National Council of Teachers of Mathematics (NCTM), Washington, DC.

State and District Presentations

Utah

Shumway, J. F., Pace, L., & Christensen, H. (2016, March). *Meaningful Mathematics: Tapping into Preschoolers Natural Strategies for Solving Problems*. Presentation for Early Childhood Educators, 41st Annual Utah Early Childhood Conference, Salt Lake City, Utah.

Shumway, J. F., Kelley, J., Webb, C., & Child, B. (2015, November). *Jumps and Leaps: Number Lines, Number Sense, and Solving Problems*. Presentation for K-6 Teachers, Utah Council of Teachers of Mathematics (UCTM) Conference, Lehi, Utah.

Shumway, J., Ermer, C., Kelley, J., & Webb, C. (2013, November). *Building Students' Math Foundations: Number Sense.* Presentation for K-6 Teachers, Utah Council of Teachers of Mathematics (UCTM) Conference, Salt Lake City, Utah.

Shumway, J. (2011, November). *Fostering Place Value Understandings Through Number Sense Routines.* Presentation for K-5 Teachers, Utah Council of Teachers of Mathematics (UCTM) Conference, Magna, Utah.

Shumway, J. (2010, November). *Building Number Sense Through Counting Routines.* Presentation for K-5 Teachers, Utah Council of Teachers of Mathematics (UCTM) Conference, Bountiful, Utah.

Virginia

Shumway, J. (2008, May). *Math Collaborative: An Embedded Professional Development Model.* Presentation, Fairfax County Teachers As Researchers Annual Conference, Fairfax, Virginia.

Shumway, J. (2008, May). *Math Collaborative: Using a Study Group in Conjunction with Coaching.* Presentation, Fairfax County Title I Mathematics Resource Teachers Workshop, Annandale, Virginia.

Shumway, J. & Granados, M. (2008, January). *Math Talk: Discourse in the Elementary Mathematics Classroom*. Presentation, Fairfax County Math Matters Conference, Lorton, Virginia.

Shumway, J. (2007, January). *Everyday Mathematics and the Math Workshop*. Presentation, Fairfax County Math Mini-Conference, Fairfax, Virginia.

Texas

O'Neil, S., **Shumway, J.**, & Kaynes, S. (2004, February). *Guided Reading Workshop.* TAIR Conference, Austin, Texas.

O'Neil, S. & **Shumway, J.** (2004, January). *Guided Reading Workshop.* Presentation for Elementary Teachers, Texas Council of Teachers of English Language Arts Convention, Austin, Texas.

NATIONAL LEADERSHIP & SERVICE

Reviewer (2014-present)
Research Conference proposals, National Council of Teachers of Mathematics.

Reviewer (2011-present)
Stenhouse Publishers, under the direction of Toby Gordon, Senior Editor. Provide recommendations and feedback on book proposals and manuscripts. Consult for Pembroke Publishers, a Stenhouse sister company.

Reviewer (2011-present)
Teaching Children Mathematics, National Council of Teachers of Mathematics.

INSTITUTIONAL LEADERSHIP & SERVICE – UTAH STATE UNIVERSITY

Reviewer, early childhood mathematics article proposals, Utah State University "Tips on Parenting" newsletter (April 2014).

Invited Presenter (2011-present)
TEAL 7551 Mathematics Education Research (for Dr. Beth MacDonald) (April 2015)
TEAL 7551 Mathematics Education Research (for Dr. Patricia Moyer-Packenham) (April 2013)
ELED 4060 Teaching Mathematics & Practicum (for Katie Anderson) (January 2013)
EDUC 4480 K-3 Methods & Strategies (for Dr. Barbara DeBoer) (February 2012, 2013, and 2014)
TEAL 6521 Number & Operations (for Dr. Amy Brown) (November 2011)
TEAL 6521 Number & Operations (for Janiece Edgington) (November 2011)
ELED 4060 Teaching Mathematics & Practicum (for Dr. Dicky Ng & Arla Westenskow) (October 2011)

Invited Presenter, Panel Presentation for the TEAL Doctoral Program: *Developing Mentor Relationships that Work* with panelists Ronda Bicmore, Steven Camicia, HyeKyoung Lee, Sherry Marx, Patricia Moyer-Packenham, and Amy Alexandra Wilson. (March 2013).

Reviewer, Expert Group for Utah State University Tutoring Intervention & Mathematics Enrichment (TIME) Clinic Assessments led by TIME Clinic Director, Dr. Arla Westenskow. (September 2012).

Invited Contributor, intervention programs in the USU TIME Clinic. Observed three students participating in TIME Clinic services, contributed advice on next instructional steps and interventions for students, and met with the students' parents to provide ideas for working on number concepts at home. (July and August 2012).

STATE SERVICE – OUTREACH FOR PUBLIC SCHOOLS

Utah
Dolores Doré Eccles Center for Early Care and Education, Logan, Utah. *Preschool Mathematics: Solving Problems.* (August 2015). Invited by the Preschool Director, Janet Wahlquist, and Preschool Teachers to conduct a professional development session about using story problems in the preschool classroom.

Dolores Doré Eccles Center for Early Care and Education, Logan, Utah. *Developing Preschoolers' Number Sense.* (April 2014). Invited by the Preschool Director, Maegan Lokteff, to conduct a professional development session for preschool teachers about number sense learning trajectories.

Edith Bowen Laboratory School, Logan, Utah. *Understanding Number!* (March 2014). Invited by the Assistant Principal, Julie Moeller, to conduct a workshop on instructional strategies for helping students develop deeper understandings of whole numbers and fractions.

Logan City School District, Logan, Utah. *Developing Math Elementary Math Teacher Leaders Through a Video Project.* (May 2013 – May 2014). Conducted workshops including video analyses of instructional strategies, article study, and reflective discussions throughout the year for a group of mathematics teacher leaders.

Edith Bowen Laboratory School, Logan, Utah. *A Lesson-Study Partnership*. (May 2012 – December 2013). Led a partnership between the Utah State University Math Methods Instructors and the Edith Bowen Second-Grade Teachers to facilitate preservice teacher learning through an adapted lesson study approach.

Bridger Elementary School, Logan City School District, Logan, Utah. *Developing Counting Routines*. (March 2013). Taught 2nd grade mathematics warm-ups as part of a school-based research project for developing teaching episodes geared toward improving students' number sense.

Edith Bowen Laboratory School, Logan, Utah. *Math Story Problems: Problem Types, Strategies, and Students' Thinking*. (March 2013). Invited by the principal, Dan Johnson, to lead a PLC session for the faculty on CCSSM problem situations and choosing numbers based on students' work. Presented with Andrea Bostwick, Marianne Christian, and Katie Anderson.

Logan City School District, Logan, Utah. *District Elementary Math Training: Math Tasks and Routines* (January 2013). Invited by district math coach, Barbara Child, and math leadership team to lead a professional development session with Grades 1 – 5 teachers on implementing instructional methods for building students' number sense.

Edith Bowen Laboratory School, Logan, Utah. *Science Kits Workshop: Integrating Mathematics and Science.* (September 2012). Invited by Dr. Kimberly Lott to serve as a mathematics teacher resource for Kindergarten, Grade 1, and Grade 2 teachers during their "Integrating Mathematics and Science" session.

Bridger Elementary School, Logan City Schools, Logan, Utah. *Lunch & Learning: Building Number Sense with Counting Around the Circle*. (January 2012). Invited by the principal, David Long, and math coach, Barbara Child, to lead an instructional strategies professional development session with Kindergarten – Grade 5 teachers.

Edith Bowen Laboratory School, Logan, Utah. *Focused Professional Development: Using Counting Routines to Develop 4th and 5th Grade Students' Number Sense.* (October 2011 – November 2011). Lead instructor for a four-session workshop for the two Grade 5 teachers to use counting routines as a means for developing students' number sense.

Nibley Elementary School, Cache County Public Schools, Logan, Utah. *Grades 3-4 Fractions and Virtual Manipulatives Mathematics Project*. (January 2011 – February 2011). Taught third- and fourth-grade mathematics during a fraction unit as part of a school-based research project on the uses of virtual manipulatives.

Ellis Elementary, Logan City Schools, Logan, Utah. *Grades 3-4 Fractions and Virtual Manipulatives Mathematics Project*. (March 2011 – April 2011). Taught third- and fourth-grade mathematics during a fraction unit as part of a school-based research project on the uses of virtual manipulatives.

Nebraska

Kearney Public Schools, Kearney, Nebraska. *Developing Students' Number Sense.* (June 2013 – present). Hired by district superintendent, Dick Meyer, and learning coach, Julie Everett, to provide consulting services for a three-year professional development focus on mathematics teaching and learning. Planned embedded, sustained professional development for elementary teachers, provided Skype presentations, and conducted on-site workshops each year.

Virginia

Bailey's Elementary School for the Arts and Sciences, Fairfax County Public Schools, Falls Church, VA. *Bailey's Math Collaborative Course*. (2007-2008). Developed and implemented a 40-hour course involving analyzing student work, facilitating discussion about assigned course readings and pedagogical math content concepts, and lesson study. Developed and taught the course with mathematics coach Mimi Granados.

Bailey's Elementary School for the Arts and Sciences, Fairfax County Public Schools, Falls Church, VA. *Instructional Assistant Professional Development Series*. (2006-2008). Initiated, developed, and implemented the first school-based professional development for 12 Instructional Assistants at Bailey's Elementary. 6 sessions over the course of a school year.

Bailey's Elementary School for the Arts and Sciences, Fairfax County Public Schools, Falls Church, VA. *Family Math Workshops.* (2007-2008). Designed, coordinated, and co-implemented a series of workshops for Bailey's Elementary families to learn more about how we teach math and how to help their children at home (6 sessions over the course of a school year). Also planned and conducted math workshops for Head Start parents in conjunction with the Head Start district coordinators (4 sessions).

Bailey's Elementary School for the Arts and Sciences, Fairfax County Public Schools, Falls Church, VA. Numerous Grade-level Team Workshops and Turn-Around Training for Bailey's Elementary educators including Mathematics Reasoning Assessments pilot and focus groups, Cognitively Guided Instruction trainings, Juanita Copley Early Childhood Math trainings for Pre-K and Kindergarten Teachers, *Everyday Mathematics* trainings, How to Use *Investigations* trainings, Lesson Study, and Teacher As Reader Study Groups on *Young Mathematicians At Work* by Fosnot & Dolk. (2005-2008).

Fairfax County Public Schools, Falls Church, VA. *Everyday Mathematics and the Math Workshop*. Provided trainings for schools in Fairfax County for teachers new to the curriculum: Sunrise Valley Elementary (2006), Hunters Woods Elementary (August 13, 14, & 15, 2007), and Bailey's Elementary (August 2007).

Texas

Bluebonnet Elementary, Round Rock Independent School District, Round Rock, TX. Created and conducted Guided Reading Workshop for K-2nd grade teachers. (February 2005). Presented overview of guided reading based on current research, facilitated group discussions, and assisted grade-level teams in creating lesson plans.

Bluebonnet Elementary, Round Rock Independent School District, Round Rock, TX. Appointed by principal to implement Embedded Staff Development district initiative. (2004-05) Designed and managed our campus program, which empowered teachers to guide their learning plans and promoted teacher collaboration for student success.

Round Rock Independent School District. *Baldrige Continuous Improvement.* (2004-05). Served as a district trainer and co-presented Baldrige training and continuous improvement systems.

CONTINUOUS LEARNING & SELF-DEVELOPMENT

PROFESSIONAL MEMBERSHIPS

National Council of Supervisors of Mathematics (since 2015)
American Educational Research Association (since 2011)
Association for Mathematics Teacher Educators (since 2011)
National Council of Teachers of Mathematics (since 2005)
School Science and Math Association (since 2011)
Utah Council of Teachers of Mathematics (since 2010)
North American Chapter of the International Group for the Psychology of Mathematics Education (2011)

SELF-SELECTED PROFESSIONAL DEVELOPMENT, 2010-present

July 2015	Camp Completion: Dissertation Writing Workshop, Utah State University, Logan, Utah
April 2013	Research Week presentations, Utah State University, Logan, Utah
April 2012	Grant Proposal Writing Workshop, Utah State University, Logan, Utah
April 2011	Research Week presentations, Utah State University, Logan, Utah
2010-2015	Attended a variety of presentations and Brown Bag Lectures across departments and colleges, such as the 2011 mathematics presentation by Zalman Usiskin, “Performance with Fractions: A Demonstration of Cultural Differences within the United States and Overseas” and the 2014 instructional technology Brown Bag by Brian Stewart on the “Tractor Math” mobile app to teach math to children with autism.

LANGUAGE FLUENCY

Proficient in Spanish: Studied Spanish Language and Literature at The George Washington University (1998-2002) and at Universidad Autónoma in Madrid, Spain (Fall 2000). Further developed and improved my oral communication in Spanish at Instituto Chac-Mool in Cuernavaca, Mexico (June 2008).