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Understanding Pyrotechnic Shock Dynamics and Response Attenuation Over Distance

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UNDERSTANDING PYROTECHNIC SHOCK DYNAMICS AND RESPONSE
ATTENUATION OVER DISTANCE

by

Richard J. Ott

A dissertation submitted in partial fulfillment
of the requirements for the degree

of

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in

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ABSTRACT

Understanding Pyrotechnic Shock Dynamics and Response Attenuation Over Distance

by

Richard J. Ott, Doctor of Philosophy
Utah State University, 2016

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Department: Mechanical and Aerospace Engineering

Pyrotechnic shock events used during stage separation on rocket vehicles produce high amplitude short duration structural response that can lead to malfunction or degradation of electronic components, cracks and fractures in brittle materials, local plastic deformation, and can cause materials to experience accelerated fatigue life. These transient loads propagate as waves through the structural media losing energy as they travel outward from the source. This work assessed available test data in an effort to better understand attenuation characteristics associated with wave propagation and attempted to update a historical standard defined by the Martin Marietta Corporation in the late 1960's using out of date data acquisition systems. Two data sets were available for consideration. The first data set came from a test that used a flight like cylinder used in NASA's Ares I-X program, and the second from a test conducted with a flat plate. Both data sets suggested that the historical standard was not a conservative estimate of shock attenuation with distance, however, the variation in the test data did not lend to recommending an update to the standard.

Beyond considering attenuation with distance an effort was made to model the flat plate configuration using finite element analysis. The available flat plate data consisted of three groups of tests, each with a unique charge density linear shape charge (LSC) used to cut an aluminum plate. The model was tuned to a representative test using the
lowest charge density LSC as input. The correlated model was then used to predict the other two cases by linearly scaling the input load based on the relative difference in charge density. The resulting model predictions were then compared with available empirical data. Aside from differences in amplitude due to nonlinearities associated with scaling the charge density of the LSC, the model predictions matched the available test data reasonably well. Finally, modeling best practices were recommended when using industry standard software to predict shock response on structures. As part of the best practices documented, a frequency dependent damping schedule that can be used in model development when no data is available is provided.
PUBLIC ABSTRACT

Understanding Pyrotechnic Shock Dynamics and Response Attenuation Over Distance
Richard J. Ott

Component requirements govern design and production in the aerospace industry. One such potential requirement for a component is the survival and continued function upon exposure to shock environments. A shock event is a high amplitude, short duration traveling wave that induces large loads on components in its path that can cause degradation of electronic components, cracks and fractures in brittle materials, local plastic deformation, and materials to experience accelerated fatigue life. In defining these environments for new structures, industry experts rely mainly on empirical data. Measurements from similar structures are scaled and enveloped to create a predicted bounding case. This enveloping process is often times conservative which leads to increased design and risk reduction costs. This work focuses on two ways to aid in the reduction of shock environments. First, attenuation with distance is considered. Second, the development and correlation of a model to empirical data was conducted in order to establish modeling best practices and provide a frequency dependent damping schedule that could be used in modeling efforts where data is not available in an effort to reduce model uncertainty in predicting shock response.

Attenuation with distance is considered in an attempt at validating or updating a historical standard that provides knock down factors that can be used to aid in the reduction of high shock environments defined for components that may not feel the full effects of the environment due to their spatial separation from the source of the shock event. To assess this, two sets of test data were analyzed using two methods for which the results were compared to the historical standard developed in the 1960’s based on data collected on technologically obsolete data acquisition systems. The data sets assessed were composed of measurements from two different structural configurations. The first set of data was from
a full scale flight like structure simulating a portion of NASA’s Ares I-X vehicle, while the
second set comes from a series of tests performed on a flat plate and conducted as part
of the same NASA program. The first analysis method followed the historical standard
approach such that a direct comparison with the standard could be completed. Here a
ratio of the peak shock response spectrum as a function of distance from the source was
used. The second approach ratioed an approximate energy calculation in a similar fashion.
Both approaches resulted in similar results, with the approximate energy method better
aligning with the historical standard. Though there was some agreement with the historical
standard, the data generally suggested that the historical standard is not a conservative
estimate of attenuation with distance. The variation in the test results, however, was large
enough to require further testing to provide an updated standard.

In developing and correlating the model, there were several goals. First, as part of the
development and correlation process, any best practices associated with modeling shock re-
sponse were documented. This included defining a frequency dependent damping schedule
that can be used as a basis when modeling other structures for which no data is available.
Second, use a wavelet transform in the correlation process. Rather than relying on time
history and spectral density comparisons only, a different tool, the harmonic wavelet trans-
form, was used to correlate the model from a time-frequency stand point. Third, the model
was used to extrapolate the same structural configuration with different loading conditions
to determine how well the correlated damping schedule and overall response characteristics
could be simulated using the identified best practices. Finally, an investigation into a split
peak response characteristic that was discovered as part of the flat plate data quality review
was conducted.

In model development, several best practices could be determined. First, plate elements
provided the quickest solution time combined with overall good response characteristics
making them the recommend element type. Second, when the load on the structure is due
to a linear shaped charge, not including the finite detonation velocity of the charge was
determined to alter the response, leading to the recommendation that this be included in
future modeling efforts. Third, in defining the applied load, it is best to keep as short as a
duration load application as feasible and to use a forcing function shape with a finite rise
and fall rate. Lastly a frequency dependent damping schedule was provided for use in other
modeling efforts where correlation data may not be available.

The spit peak response characteristic of the flat plate data was determined to be wave
reflection from the edge of the plate. This was determined by investigation with the har-
monic wavelet, and the realization that the traveling wave speed was not driven by the
elastic modulus, but rather the shear modulus.

Three general conclusions can be taken away from this work. First, based on comp-
parison with the available test data, the historical standard should be further investigated.
Second, when modeling structural shock response it is important to use as short of a du-
ration forcing function as feasible, incorporate the appropriate detonation velocity of the
explosive input when appropriate, and to use a forcing function with a finite rise and fall
rate. Furthermore, plate elements provide the best balance of solution time and good re-
sponse characteristics. Lastly, the harmonic wavelet transform provides a better tool for
shock characterization, model correlation, and data investigation than the shock response
spectrum. The transform requires no damping assumptions, and it maintains the connec-
tion between the time and frequency domains of the forcing function, both of which are not
captured when using a shock response spectrum.
To my parents, without whom my success would not be possible;
To my wife, for her unwavering love, support, belief, and understanding;
To my son, that your dreams come true.
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Lastly, I want to acknowledge the contribution of my wife, Bethany. Without her selflessness and support, this work would not have been possible. She is a great wife and a better mother.
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CHAPTER 1

Introduction

As in any industry, component requirements govern design and production in the aerospace industry. In order to show compliance to these requirements, components must be qualified. Qualification is the process of demonstrating positive margins for different loading scenarios either through analysis or by test for a component.

Dynamic loads qualification is one subset of the qualification process and dynamic environments directly feed these loads. Self-induced random vibration and shock (ignition and separation) are two primary environments that induce loads on components. Scaling methodologies and mass loading techniques can be used to predict random vibration environments; however, predicting shock environments is much more difficult. Modeling and extrapolation methods do exist for shock prediction; however they often produce unrealistic results when generating levels for a new design [1]. As a result, shock levels from several similar motors are typically enveloped during environment development on new programs. This leads to added conservatism which leads to increased costs in risk reduction tasks prior to qualification testing; the standard for shock qualification.

With the added conservatisms in the environment development, attenuation for distance, joints, and other structural features becomes important. Current industry standards are based on data collected nearly 50 years ago on archaic data acquisition systems and processed using the shock response spectrum (SRS) - an analysis tool that has major limitations. Advancements in computing, improvements in testing technology, and newly developed processing tools warrant the need to validate current standards, however, any validation effort done by industry professionals is likely not wide spread due to proprietary concerns.
CHAPTER 2

Literature Survey

Shock is a local transient mechanical loading characterized by its short duration, high amplitude, and high frequency response. Events such as the impact of a falling body, earthquake, near miss of an underwater explosion, or a pyroshock from an explosive device can all be characterized as a shock event. The time scale of such events can be as long as several minutes or as short as a few microseconds. The focus of this work will be on pyroshock events in aerospace applications. Fig. 2.1 shows an example of an expected time series response to one of these events. The structural environment developed in response to shock loading can induce major failure resulting in total or partial loss of equipment. On a component basis, malfunction or degradation of electronic components can occur. On a larger more global scale, cracks and fractures in brittle materials can develop, local plastic deformation can occur, or materials can experience an accelerated fatigue life.

One way of representing this type of load is the shock response spectrum or SRS. The SRS was first developed by Maurice Biot in 1932 to describe the response of a building during an earthquake [3]. This analysis tool is still widely used in dynamic subfields such as earthquake engineering or the aerospace industry today. The SRS is a graphical representation of an array of single degree of freedom (sdoF) systems each having a unique natural frequency. It is calculated by determining the maximum response of each sdoF system in the array, to the same transient base excitation. In order to calculate an SRS, an assumption about damping has to be made. The dynamic amplification, Q, is essentially a measure of damping and is defined as the ratio of the critical damping to twice the system damping of a sdoF system and is often used in SRS algorithms. Fig. 2.2 visualizes one such bank of sdoF systems.

Focusing on solid rocket applications, SRS curves are used to characterize ignition and pyrotechnic device transients that occur as part of a system level launch vehicles. Examples
Fig. 2.1: **Expected Shock Transient Time History Shape.** Notice the smooth exponential decay as time goes on and the symmetry of the response, these are two key aspects that help distinguish good from suspect data. [2]

of these pyrotechnic devices include explosive bolts and the linear shaped charge (LSC). An example of an explosive bolt application was the connection between the solid rocket boosters and the core stage of NASA’s Space Shuttle program. An LSC is a linear chevron shaped plastic explosive device that is used in several applications. Two such examples are thrust termination and during staging. As part of thrust termination the LSC stretches along the case of the motor and is used to cut the case releasing pressure in the event the flight needs to be terminated. As part of staging, the LSC is wrapped circumferentially around an interstage and is used to cut the case separating adjacent stages of the rocket. This work will focus on the response of structural systems where an LSC is used.

In practice, several sources, including an ignition transient at the start of burn for a solid rocket may be present in any one system. Although each source may be characterized individually, they are often enveloped to determine a bounding environment. The
Fig. 2.2: A bank of sdof systems used in an SRS generation.

enveloping procedure, however, can result in an overly conservative environment, especially when predicting environments for new programs. This bounding case is then used to define load factors or as an input for dynamic qualification testing. The industry standard used to characterize shock environments is the SRS. Though it is the standard, it does have its limitations. Its biggest limitation is that it fails to address the temporal nature of a transient event. For every SRS generated, there are an infinite number of potential inputs. The example in Fig. 2.3 shows five radically different time series, that when passed through the SRS algorithm, generate the same shock response. The lower right corner of the figure shows the resulting outputs of these different time series. This indicates the loss of temporal information about the transient event in the calculation of the SRS. Ideally, when defining a bounding environment for structural loads determination or qualification testing, the environment should capture both spectral and temporal aspects of the flight environment. There are two additional limitations of the SRS. The first, as previously mentioned, is that the SRS requires an assumption about damping. Secondly, is does not account for
any modal interactions or coupling that can occur in physical systems. This is a result of treating each frequency as its own independent sdof system.

Fig. 2.3: SRS input comparison. Five radically different time histories are passed into the SRS algorithm, symbolized in the top right by the bank of sdof systems, that generate the same SRS. The figure is taken directly from [4].

With the dissipative mechanisms inherent to physical systems, it is easy to conclude a traveling wave such as a shock wave, would undergo energy loss as it propagates. Combining the inherent high-level structural response that shock events cause with the conservatisms associated with the enveloping process, it is easy to see the need for a reduction factor that can be used to reduce loads the further a component is from the source. Or more appropriately, it points to the need to understand the attenuation characteristics of these waves as they propagate through open media and joints. In March of 1970 The Martin Marietta Corporation, under a contract with NASA’s Goddard Space Flight Center published a report entitled Aerospace Systems Pyrotechnic Shock Data (Ground Test and Flight) [5]. In this report the authors present attenuation guidelines for structures and equipment being designed to pyrotechnic environments. They consider attenuation for distance and several types of jointed interfaces. These guidelines have since been the industry standard for shock attenuation. Fifteen years later in 1985, a survey of pyroshock prediction methods was done.
by Van Ert [6]. His survey primarily focused on prediction methodologies; however, a subsection on attenuation references the Martin Marietta report. Further work was completed by Spann et al. looking at shock attenuation through joints in [7–9]. In 2001 and 2011 the latest versions of the NASA Handbook 7005: Dynamic Environmental Criteria (2001), [1], and NASA Technical Standard 7003: Pyroshock Test Criteria (2011), [10], still cite the work completed by Martin Marietta. In addition, the same two NASA technical documents provide two scaling relationships. The first accounts for source energy differences,

\[
SRS_n(x) = SRS_r(x)\frac{E_n}{E_r},
\]  

(2.1)

where \(E_n\) and \(E_r\) is the total explosive energy of the new and reference structure respectively, and \(x\) is the distance from the source. The second provides an alternative for the distance attenuation of a point source on a complex structure,

\[
SRS(D_2, f_n) = SRS(D_1, f_n)e^{(-8\times10^{-4}f_n^2+4f_n^{0.105})(D_2-D_1)},
\]  

(2.2)

where \(D_1\) and \(D_2\) are the distances from the pyrotechnic source to the reference and new locations respectively, and \(f_n\) is the \(n^{th}\) natural frequency. Both Eqn. 2.1 and Eqn. 2.2 share the limitation of using the SRS in their definition, and furthermore, Eqn. 2.2 was developed using a point source at ground level and may not be representative of other sources in different conditions such as in the upper atmosphere or in space. Additionally, a point source is only applicable in limited situations such as an explosive bolt or hammer strike [1].

Focusing on the attenuation over distance, the Martin Marietta document discusses several structural configurations ranging from a cylindrical shell with no stringers (vertical stiffeners), rings, or frames, to a honeycomb structure. For each configuration, the documented attenuation curve was calculated using the SRS peak spectrum response over the frequency range of interest. The curves were normalized to a factor of one at the measurement nearest the source. In most cases, the nearest measurement available was ap-
proximately 5 in. from the source. Thus, the attenuation curves presented in the document are arbitrarily normalized to a distance of 5 in. Fig. 2.4 summarizes the findings for the cylindrical shell. This structural configuration is of most interest in the solid rocket motor structures. Two primary concerns arise from the plotted data: first the curve was generated with limited data (as is self-stated within the report) using old data acquisition technology, and second, the SRS was used to determine the spectrum peaks; and the short comings of this analysis technique have already been pointed out.

![Figure 2.4](image)

Fig. 2.4: Martin Marietta Cylindrical Shell Shock Distance Attenuation Curve. Figure 3.1 in [5].

As recent as 2009 and 2011 two sets of LSC separation development tests were conducted by Orbital ATK as part of NASA’s ARES I and ARES I-X programs. Fig. 2.5 gives an overview of the differences between the two vehicles [11]. In 2009, a full scale separation
test plan, [2], was executed and documented in [12] to demonstrate the functionality of the ARES I-X LSC severance system. The test consisted of an ARES I-X full scale flight-like cylindrical segment that jointed the frustum to the forward skirt. The test was conducted in two parts, the forward skirt extension (FSE) to forward skirt joint was cut followed by a cut of the FSE to frustum joint. During each event accelerometers recorded the response on the cylinder at various locations; the data will be discussed in depth in a later chapter. Figs. 2.6 and 2.7 show the various structural components of the vehicle and give an indication of where the separation events occur [13, 14]. Figs. 2.8 and 2.9 show the test article prior to ordinance detonation and just after detonation of the FSE - frustum LSC [15, 16]. A high speed video of the FSE - frustum separation test can be found in [16].

Fig. 2.5: Comparison of the ARES I and ARES I-X Configurations. Fig. taken from [11].

In 2011, a series of flat plate LSC development tests were conducted by Orbital ATK. In these tests a large flat aluminum plate was used to demonstrate cutting abilities of different LSC strengths. In addition, the tests were used to help characterize source shock levels for different staging events that were specific to the program. The test plan and final report [17, 18] discuss the test description and results.

Though both the 2009 and 2011 sets of data are ideal for determining attenuation as
a function of distance, no analysis was done to verify the findings in the Martin Marietta document. In both tests, the data were characterized at a top level by the time history and the SRS. The only dynamics interest in the data was to estimate the environment produced by such events, not to investigate the underlying physics of the traveling wave.

![Ares Partial Stack Description](image)

Fig. 2.6: **Ares Partial Stack Description.** Fig. taken from [13].

Though the shock response spectrum is still widely used as an analysis tool, and is the basis for characterizing the events in the previously discussed tests, time-frequency methods incorporate both temporal and spectral aspects of a transient signal. These methods are signal processing techniques that present amplitude and phase as a function of time and frequency simultaneously. They provide a way to characterize the time evolving spectral nature of a transient event. Cohen provides an excellent discussion as to why these techniques are needed in his book [19].

Recently, the use of wavelets has increased in the time-frequency analysis of vibration and shock characterization [4, 20–26]. Wavelets, developed in the 1980’s, are used for numerous applications throughout the literature. Bettella et al. used a wavelet transform to
investigate wave propagation on thin aluminum plate and all aluminum honeycomb sandwich panels due to high velocity impacts [20]. Gendelman et al. used them to characterize passive targeted energy transfer in strongly nonlinear mechanical oscillators [21], and Phu Le and Argoul uses a similar transform for modal identification purposes [22]. These works show how widely applicable wavelets are.

One specific wavelet, the harmonic wavelet, has the unique property of preserving signal strength in the frequency domain. Newland created this wavelet in 1993 [23] with multi-resolution analysis in mind. One of his first published applications of the new wavelet was approved in 1994 and discusses the extension of harmonic wavelets into music and the applications it has in that discipline [24]. He extends the applications in 1999 where he discussed four nonrelated topics: transmission of bending waves in a beam subjected
to an impact, pressure fluctuations in acoustic waveguides, ground response analysis for vibration recorded near an underground train in London, and computing the time varying cross-spectra for multi-channel measurements of soil vibration in a centrifuge [25]. More recently, Edwards looked at correcting accelerometer data when zero shift occurred, [26], and Hacker proposed a replacement for the SRS in shock characterization, [4]. In this work, the harmonic wavelet transform will be used to characterize shock attenuation as a function of distance from the source. A more complete history and development will be provided shortly.

To aid in better understanding the intricacies of the type of short time, high energy event that pyroshock is, finite element analysis (FEA) modeling techniques can be employed to best simulate available test data when unexpected response characteristics are present in measured data. Although FEA provides a flexible way to analyze complex structures, assumptions about the forcing function and damping must be made when considering pyroshock events. Standard approaches for direct and modal transient solutions that include damping can be found in the documentation accompanying the industry standard pack-
Fig. 2.9: Ares I-X Flight FSE Separation Test. Image taken from [16].

...ages [27]. However, both of these methods have limitations in damping control that prevent one from accurately modeling a transient event. In addition, file size often becomes an issue when fine time steps and mesh densities are used. Even with these limitations, FEA provides a quick way to gain a qualitative understanding of the response of simple systems and will be used as tool to help explore unexpected response characteristics in this work. The models that will be discussed should in no way be taken as state of the art in shock modeling; rather the models are used merely as a tool used to gain insight in the response of these simplified systems.

The literature is full of different approaches to modeling impulse response on structures. In Ugural’s book, [28], he formulates a closed form solution using a series expansion for rectangular plates with different loading conditions. His approach however follows the typical pattern: resolve the associated eigenvalue problem and then form the solution by modal superposition. This type of approach works well for simple boundary conditions, however when other, more difficult boundary conditions apply, approximate methods must
be used. Pavic uses a boundary forming approach where the analytical response of an infinite plate is used to obtain a solution by superposing two excitations, the original and a secondary one which has a continuously distributed outside contour bounding the original plate [29]. The secondary solution has the role of creating the proper boundary conditions for the original plate. Botta and Cerri use Reissner-Mindlin theory to investigate a plate under an impulse load and influence of the impulse (rise time and function) along with geometric parameters play on the response [30]. They correlate their work using the SRS, however, a seemingly incomplete way to correlate a model. Various other techniques for modeling simple and orthotropic plates as well as cylindrical shells have been developed [31–36]. More recent analytical modeling procedures involving Hydrocodes have surfaced. These codes model the time history details of the explosive or propellant ignition and burning process and use the results to excite a nonlinear structural deformation or separation model that generate propagating structural waves [37–39]. Unfortunately, the implementation of these types of analyses is generally expensive and produce predictions that are often poor [1,10].

An alternative to Hydrocodes is Transient Statistical Energy Analysis (SEA). SEA methods are typically applied to steady state vibration problems, however when used with transient excitations the steady-state power balance equation is replaced with the corresponding transient equation that incorporates time dependent power and a term describing the time varying dynamic modal energies. When the input power involves a nonlinear process, which is often the case for local structural responses with a pyrotechnic excitation, this type of analysis fails due to its modal nature. Examples of this analysis are found in [40–48]. One common numerical implantation of this technique is called TRANSTAR for Transient Analysis Storage and Retrieval. Further information on this can be found in [49–51].

Related to Transient SEA is Virtual Mode Synthesis and Simulation (VMSS). The process estimates the steady state frequency response magnitude at a selected location due to an input excitation at a different location. Then at high frequencies it is assumed that the frequency response envelope can be represented as the peak response from a collection
of localized modes spaced according to the estimated modal density of the local structure. Virtual mode coefficients are then generated that can be used to simulate a time response of the dynamical system by convolving a measurement or simulated transient excitation with the output. VMSS has several advantages over Transient SEA. In VMSS, there are no quasi-stationary requirements for the excitation and if the structure is reasonably linear, the near-field response estimate may be estimated. VMSS also provides a time domain solution and an SRS (if desired) as opposed to the peak value and spectral envelope estimate that transient SEA provides. More information about VMSS can be found in [52–55].

With numerous sources in the literature describing a variety of ways to model such transient events, it is clear that modeling such events is quite difficult. Each method has strengths and weakness and the application of a method should be chosen to be in line with the desired results of a defined analysis. Regardless of the method, this collection of work show the need for better modeling methods of pyrotechnic excitation of complex structures.
The signal energy for an arbitrary input function, \( f(t) \), is defined as

\[
E_s = \int_{t_1}^{t_2} |f(t)|^2 dt,
\]

where the total signal energy, \( E_s \), can be found by integrating over all time [56]. From Eqn. 3.1 it is easy to see the units of signal energy are \( U^2s \), where \( U \) are the engineering units of the specific application. In the electrical world, \( f(t) \) is typically a voltage (V); giving the signal energy units of \( V^2s \). In this case, the signal energy is proportional to the physical energy through the impedance of the electrical system. The impedance is a measure of the effective resistance of a circuit and is reported in units of Ohm’s (Ω) [57]. The physical energy is the ratio of the signal energy to the system impedance, producing a value with units of energy, Joules (J). It is important to note, that the signal energy is proportional to the physical energy, so a loss in one, results in a loss in the other, regardless of the time dependent form of the impedance. Take the case of resonance for example, there may be significantly higher system response at a particular frequency, but the overall energy output is still less than the input. Resonance and dynamic amplification does not imply an increase in total system energy, but rather a particular energy density shape [19].

This same approach can be extended to mechanical systems. Voltage can be thought of as analogous to force. Both provide an input to their respective systems; voltage is associated with current, while a force is associated with acceleration. Masses and springs store energy in both kinetic and potential forms, just as capacitors and inductors, while dampers dissipate energy just as resistors [57]. Using this analogy, the signal energy of a measured acceleration signal can be converted to the physical energy by multiplying by the mass squared and dividing by the mechanical impedance. For a single degree of freedom
system, the mechanical impedance is defined as

\[ Z_{\text{mech}} = \frac{K}{i\omega} + C + i\omega M, \]  

(3.2)

where \( \omega \) is the natural frequency, \( K \) the spring constant, \( C \) the damping coefficient, and \( M \) the mass [58]. In the mechanical world, impedance can be thought of as a generalized inertia, or as a measure of the resistance of motion. To illustrate the analogy, consider the units: (in SI) acceleration is meters per second squared, mass is kilograms, and the impedance is kilograms per second. An acceleration signal produces a signal energy having units of meters squared per second cubed that when multiplied by the mass squared and divided by the mechanical impedance results in units of Joules, just as before, energy units are obtained. Notice again that the signal energy goes as the physical energy.

Obviously, the analogy does not translate exactly, as the acceleration varies spatially as well as temporally and in general would require integration over space; whereas a voltage can be measured at a point in a circuit and treated as only varying in time. Even with these limitations, acceleration is often treated in the same manner as voltage so that vibration engineers can extract modal response information from measured data. The same approach will be taken here; in the coming chapters the relationship between the signal energy and physical energy is assumed and the signal energy is treated and referred to as a measure of the system physical energy, even though the units do not explicitly agree with the treatment.

In vibration analysis, and in other fields, it is often the case that signals are analyzed using their frequency response. This requires a transformation between the time and frequency domains. This transformation should have no effect on the signal or system energy. Parseval’s theorem demonstrates this.

**Theorem 1. Parseval’s Theorem.** Let \( x(t) \) be a complex valued function that is sufficiently smooth, that decays sufficiently quickly near infinity so that its integrals exist, and has Fourier transform

\[ X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t)e^{-i\omega t} dt, \]  

(3.3)
where $\omega = 2\pi f$, and inverse Fourier transform

$$x(t) = \int_{-\infty}^{\infty} X(\omega)e^{i\omega t} d\omega. \quad (3.4)$$

Then,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega. \quad (3.5)$$

Proof. Let $\delta(t)$ be the well-known Dirac delta function. Its Fourier transform is then

$$\Delta(\omega) = \int_{-\infty}^{\infty} \delta(t)e^{-i\omega t} dt = 1 \quad (3.6)$$

and it’s inverse Fourier transform is

$$\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} d\omega. \quad (3.7)$$

The proof of Eqn. 3.5 starts by rewriting the left hand side of Eqn. 3.5 using the inverse Fourier transform

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} \left( \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\chi)e^{i\chi t} d\chi \right) \left( \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{X}(\xi)e^{i\xi t} d\xi \right) dt, \quad (3.8)$$

where the over bar denotes the complex conjugate. Rearranging the order of integration in Eqn. 3.8 gives,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} X(\chi) \left( \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{X}(\xi) e^{i(\chi-\xi)t} d\xi \delta(\chi-\xi) \right) d\chi, \quad (3.9)$$

and using Eqn. 3.7 the integral over $t$ is evaluated leaving

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} X(\chi) \left( \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{X}(\xi) \delta(\chi-\xi) d\xi \right) d\chi \quad (3.10)$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\chi) \bar{X}(\chi) d\chi, \quad (3.11)$$
or, just as in Eqn. 3.5

\[
\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega.
\]  

(3.12)
CHAPTER 4
General Harmonic Wavelet Theory

4.1 Background and General Harmonic Wavelet Definition

Summarizing the work done by Gao in [59], the first reference of wavelets goes back to the early twentieth century when Alfred Haar wrote his dissertation in 1909. His work on orthogonal systems of functions led to the development of a set of rectangular basis functions which later led to the entire Haar wavelet family. The Haar wavelet family is the simplest family of wavelets and consists of a short positive pulse followed by a short negative pulse. This wavelet family has been used for a variety of applications. Initially developed to illustrate a countable orthonormal system for the space of square-integrable functions on the real line, they were later used in image compression and to investigate Brownian motion. Since Haar’s initial work, numerous individuals have contributed to advancing the field. Major contributions came from Jean Morlet in the mid 1970’s. Morlet developed and implemented a technique to scale and shift the analysis window functions in analyzing acoustic echoes while working for an oil company. This work was the foundation of what later became the Morlet wavelet family. Later, around 1988, Ingrid Daubechies created her own family of wavelets with multi-resolution analysis in mind. Since then, wavelet applications have become more widespread ranging from image and speech processing to signal analysis in manufacturing.

At their core, all wavelets are based on integral transforms, similar to the well-known Fourier transform. These transforms expand an arbitrary function with respect to some set of chosen basis functions. The Fourier transform uses sines and cosines as the basis set, where wavelets use other functions. Wavelets and wavelet families can be continuous or discrete, real or complex, but in general the choice of a basis set used to define the wavelet or wavelet family is arbitrary; ideally, however, the chosen basis has some form of
physical meaning. The sines and cosines used as the Fourier basis, for example, are related to frequency making them useful in spectral analysis. Furthermore, it is well known that choosing a set of basis functions that are normalizable, invertible, orthogonal, and form a complete set is advantageous when defining a wavelet transform.

In time-frequency analysis, a subset of multi-resolution analysis, a signal is decomposed in time and frequency simultaneously. At the heart of this transformation, the input signal is decomposed into sub-signals or different size resolution levels. The term level refers to one of these sub-signals. There can be as many levels as the number of times the input signal can be divided by two \[60\]. The series expansion of such a transformation requires an expansion in two variables. A general series expansion and wavelet transform of this form is

\[
x(t) = \sum_m \sum_n \alpha_{m,n} \phi_{m,n}(t)
\]

\[
\alpha_{m,n} = \frac{1}{\sqrt{m}} \int x(t) \phi_{m,n} \left( \frac{t - n}{m} \right) dt,
\]

where \(\phi_{m,n}(t)\) are the basis functions related to time \(n\) and frequency \(m\) respectively. A general wavelet transform of a function, \(x(t)\), is a transform of the function from \(t\) into a function of \(m\) and \(n\), where \(m\) is a scaling parameter and is related to frequency and \(n\) is a translation parameter and is related to time. Scaling makes the basis function more localized in time shortening the duration of the wavelet, which effectively increases its frequency. This allows for correlation with events that occur at higher frequencies. Translation shifts the basis function such that it correlates with events that occur later in time [4].

The general harmonic wavelet is a unique wavelet developed by David Newland in 1993, and will be referred to as the harmonic wavelet in this work. Newland developed the transform with multi-resolution analysis of vibration problems in mind. His goal was to develop a wavelet whose spectrum is confined exactly to an octave band. By doing so he eliminated all spectral leakage. Spectral leakage is the smearing of energy across a frequency spectrum that occurs when a signal is not periodic in the sample interval \[61\]. An
example of spectral leakage can be seen in Fig. 4.1. A wavelet having a spectrum confined
to an octave band allows the level of a signal’s multi-resolution analysis to be interchanged
with its frequency band. This lead Newland to look for a wavelet whose Fourier transform
is compact and which could be constructed from simple functions [23]. Along with its
compactness\(^1\), the wavelet is orthogonal, normalizable, and invertible [23].

Another benefit of the harmonic wavelet is that it captures both signal dispersion and
dissipation without making any assumptions about the system damping [4]. Dispersion
is a conservative mechanism by which energy in a traveling wave spreads out over time.
Dispersion occurs when the group velocity is different than the phase velocity of the traveling
wave. Generally, the group and phase velocities are frequency dependent, and when they are
different, waves at different frequencies travel at different speeds. The velocity differences
spread the waves out and the peak power of the traveling wave group decreases. Hacker
demonstrates this application in his work, [4]. Dissipation is a decrease in peak power as well;
however, it stems from the conversion of mechanical energy into heat and is non-conservative
in nature. As will be discussed shortly, the harmonic wavelet satisfies Parseval’s theorem,
and therefore, the transform inherently captures any dissipation present in the data being
analyzed.

Following Newland [23], we will develop the harmonic wavelet from its infancy. Certain
details such as the normalization and orthogonality will be assumed. The proof of these
properties can be found in Newland’s work, [23]. Consider the functions \(w_e(x)\) and \(w_o(x)\)
whose Fourier transforms are defined by

\[
W_e(\omega) = \begin{cases} 
\frac{1}{4\pi} & \text{for } -4\pi \leq \omega < -2\pi \text{ and } 2\pi \leq \omega < 4\pi \\
0 & \text{otherwise,}
\end{cases} \tag{4.3}
\]

\(^1\)Compactness is a mathematical property that generalizes the notion of a subset of Euclidean space
being closed and bounded. This means the space contains all its limit points and those points all lie within
some fixed distance of each other. Where a limit point, \(z \in Z\), is a point of set \(S\), if every neighborhood of
\(z\) contains at least one point of \(S\) different from \(z\) itself. [62]
Fig. 4.1: **Spectral Leakage of a Sine Wave.** The plot of the left shows a 7 Hz sine wave and the plot on the right shows the fast Fourier transform of that same sine wave. Image taken from [63].

and

\[
W_o(\omega) = \begin{cases} 
\frac{i}{4\pi} & \text{for } -4\pi \leq \omega < -2\pi, \\
-\frac{i}{4\pi} & \text{for } 2\pi \leq \omega < 4\pi, \\
0 & \text{otherwise},
\end{cases}
\]  

(4.4)

where the subscript \(e\) denotes an even function and \(o\) denotes an odd function. The inverse Fourier transform,

\[
f(t) = \int_{-\infty}^{\infty} F(\omega)e^{i\omega t} d\omega,
\]  

(4.5)

of \(W_e(\omega)\) and \(W_o(\omega)\) are

\[
w_e(t) = \frac{1}{2\pi t} \left( \sin(4\pi t) - \sin(2\pi t) \right)
\]  

(4.6)

and

\[
w_o(t) = -\frac{1}{2\pi t} \left( \cos(4\pi t) - \cos(2\pi t) \right),
\]  

(4.7)

respectively. Combining Eqns. 4.6 and 4.7 into a single complex function as

\[
w(t) = w_e(t) + i \; w_o(t),
\]  

(4.8)
we obtain
\[ w(t) = \frac{1}{i2\pi t}e^{i4\pi t} - e^{i2\pi t}, \] (4.9)

where \( w(t) \) defines the harmonic wavelet. Fig. 4.2 shows the real and imaginary parts of the harmonic wavelet as functions of time. Based on its definition, the Fourier transform, \( W(\omega) \), of the harmonic wavelet is given by

\[ W(\omega) = W_e(\omega) + iW_o(\omega). \] (4.10)

Using Eqns. 4.3 and 4.4 we have

\[ W(\omega) = \begin{cases} \frac{1}{2\pi} & \text{for } 2\pi \leq \omega < 4\pi, \\ 0 & \text{otherwise.} \end{cases} \] (4.11)

To incorporate scaling and translation, as discussed in Eqn. 4.2, we need to replace \( t \) with \( (2^j t - k) \) in Eqn. 4.9:

\[ w(2^j t - k) = \frac{1}{i2\pi (2^j t - k)}\left( e^{i4\pi (2^j t - k)} - e^{i2\pi (2^j t - k)} \right), \] (4.12)

where \( j \) and \( k \) are integers. The shape of the harmonic wavelet described in Eqn. 4.12 does not change, it is simply scaled or translated. Its horizontal scale is compressed by a factor of \( 2^j \), and its position is translated by \( k \) units at a new scale of \( k/2^j \) units of the original scale. \( j \) determines the level of the wavelet. At level \( j = 0 \), the wavelet’s Fourier transform occupies the bandwidth between \( 2\pi \) and \( 4\pi \); at level \( j = J \), it occupies the bandwidth between \( 2\pi 2^J \) and \( 4\pi 2^J \), which is \( J \) octaves higher up the frequency scale. \( j \) and \( k \) act in the same manner as \( m \) and \( n \) in Eqn. 4.2, they simply scale and translate the harmonic wavelet. With the simplicity of the wavelet’s Fourier transform, orthogonality can easily be shown, but will be left to Newland [23].

As the octave band widens, successive levels of the harmonic wavelet decrease in proportion to this increasing bandwidth. Fig. 4.3 shows this relationship for \( j \geq 0 \). For those
Fig. 4.2: Harmonic Wavelet. The two figures show the real and imaginary parts of the harmonic wavelet.

octave bands or levels defined by $j < 0$, the same sequence as in Fig. 4.3 is maintained. These levels see $\omega \to -\infty$. As part of multi-resolution analysis, all frequencies can be rolled together into a single wavelet level for this case. Following the standard terminology, this level is referred to as level -1, and it serves to cover the residual frequency band between 0 and $2\pi$ in Fig. 4.3. The function that describes this frequency band is called the scaling function in wavelet theory, and the inverse Fourier transform of the even scaling function is

$$
\Phi_e(\omega) = \begin{cases} 
\frac{1}{\pi} & \text{for } -2\pi \leq \omega < 2\pi, \\
0 & \text{otherwise},
\end{cases} 
$$

(4.13)
Taking the inverse transform of Eqn. 4.13 gives

\[ \phi_e(t) = \frac{\sin(2\pi t)}{2\pi t}. \]  

(4.14)

Similarly, the odd function defined by

\[ \Phi_o(\omega) = \begin{cases} 
\frac{i}{4\pi} & \text{for } -2\pi \leq \omega < 0, \\
-\frac{i}{4\pi} & \text{for } 0 \leq \omega < 2\pi, \\
0 & \text{otherwise,}
\end{cases} \]  

(4.15)

gives an odd scaling function,

\[ \phi_o(t) = \frac{-(\cos(2\pi t) - 1)}{2\pi t}. \]  

(4.16)

Combining Eqns. 4.14 and 4.16 in the same manner as Eqn. 4.8, the complex scaling function is defined,

\[ \phi(t) = \frac{e^{i2\pi t} - 1}{i2\pi t}, \]  

(4.17)
and its Fourier transform, from Eqn. 4.13 and 4.15, is

\[
\Phi(\omega) = \begin{cases} 
\frac{1}{2\pi} & \text{for } 0 \leq \omega < 2\pi, \\
0 & \text{otherwise.}
\end{cases}
\]  

(4.18)

Fig. 4.4 shows the real and imaginary part of \( \phi(t) \) as functions of time. Just as in the case of the other levels, the scale function \( \phi(t) \) can be shown to be orthogonal [23].

As was mentioned previously the harmonic wavelet can be normalized, the details of which will be left to Newland, [23]. The result used in normalization of \( w(t) \) and \( \phi(t) \) in Eqns. 4.9 and 4.17 is

\[
\int_{-\infty}^{\infty} |w(2^j t - k)|^2 dt = \frac{1}{2^j},
\]  

(4.19)

and

\[
\int_{-\infty}^{\infty} |\phi(t - k)|^2 dt = 1.
\]  

(4.20)

### 4.2 Multi-Resolution Analysis

The general expansion formulas of wavelet theory shown in Eqn. 4.1 and 4.2 applied to a function, \( f(t) \), using the harmonic wavelet and keeping all levels is written as

\[
f(t) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} a_{j,k} w(2^j t - k).
\]  

(4.21)

If the negative levels are replaced by the scale function, Eqn. 4.21 becomes

\[
f(t) = \sum_{k=-\infty}^{\infty} a_{\phi,k} \phi(t - k) + \sum_{j=0}^{\infty} \sum_{k=-\infty}^{\infty} a_{j,k} w(2^j t - k).
\]  

(4.22)

Both Eqn. 4.21 and 4.22 assume that the wavelets expanding \( f(t) \) are derived from the solution of two-scale dilation equations\(^2\) with real coefficients, and that there is only one wavelet for each \( j,k \) pair. Recall though that the harmonic wavelet is composed of both an

\(^2\)A two parameter dilation equation is an equation that includes two parameters to shift and scale the independent variable and requires solution for coefficients. [64]
Fig. 4.4: **Harmonic Wavelet Scale Function.** The two figures show the real and imaginary parts of the harmonic wavelet scale function.

even wavelet, \( w_e(2^j t - k) \), and an odd wavelet, \( w_o(2^j t - k) \). In combining these two into a single complex wavelet, the wavelet coefficients, \( a_{j,k} \), are forced to be complex. The pair of complex wavelet coefficients is defined as

\[
 a_{j,k} = 2^j \int_{-\infty}^{\infty} f(t) \bar{w}(2^j t - k) \, dt, \\
 \text{and} \\
 \tilde{a}_{j,k} = 2^j \int_{-\infty}^{\infty} f(t) w(2^j t - k) \, dt, \\
\]

(4.23)
where the over bar indicates the complex conjugate. Similarly, the corresponding pair of complex coefficients for the scaling function are

\[ a_{\phi,k} = \int_{-\infty}^{\infty} f(t) \bar{\phi}(t - k) \, dt, \]

and

\[ \tilde{a}_{\phi,k} = \int_{-\infty}^{\infty} f(t) \phi(t - k) \, dt. \]  

(4.24)

When \( f(t) \) is real, \( \tilde{a}_{j,k} = \bar{a}_{j,k} \), thus \( \tilde{a}_{j,k} \) is not a new coefficient. Allowing \( f(t) \) to be complex requires that \( \tilde{a}_{j,k} \) be distinguished separately from \( \bar{a}_{j,k} \). The contribution of a single complex wavelet to the expansion of function, \( f(t) \), is

\[ a_{j,k} \, w(2^j t - k) + \tilde{a}_{j,k} \, \bar{w}(2^j t - k). \]  

(4.25)

As a result, the expansion formulas given in Eqn. 4.21 and 4.22 become

\[ f(t) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \left\{ a_{j,k} \, w(2^j t - k) + \tilde{a}_{j,k} \, \bar{w}(2^j t - k) \right\}, \]

(4.26)

and

\[ f(t) = \sum_{k=-\infty}^{\infty} \left\{ a_{j,k} \, \phi(2^j t - k) + \tilde{a}_{j,k} \, \bar{\phi}(2^j t - k) \right\} + \sum_{j=0}^{\infty} \sum_{k=-\infty}^{\infty} \left\{ a_{j,k} \, w(2^j t - k) + \tilde{a}_{j,k} \, \bar{w}(2^j t - k) \right\}. \]

(4.27)

Using the band-limited structure of the Fourier transform of \( w(2^j t - k) \), it can be shown (see Appendix B) that Parseval’s theorem is satisfied:

\[ \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} 2^{-j} \left( |a_{j,k}|^2 + |\tilde{a}_{j,k}|^2 \right) = \int_{-\infty}^{\infty} |f(t)|^2 \, dt. \]

(4.28)

This, in the terminology of wavelet theory, states that the set of functions, \( \{ w(2^j t - k) \} \), forms a tight frame. Or that the band-limited harmonic wavelet defined in Eqn. 4.12 provides a complete set of basis functions for the expansion of an arbitrary function, \( f(t) \),
provided that \( f(t) \) decays to zero as \( t \to \pm \infty \), which implies

\[
\int_{-\infty}^{\infty} |f(t)|^2 \, dt < \infty, \tag{4.29}
\]

or that the arbitrary function has finite energy.

### 4.3 Numerical Implementation of the Transform

An algorithm to compute \( w(2^j t - k) \) and \( \phi(t - k) \) from their defining Fourier transforms and the coefficient integrals can be set up in a straightforward way. There is, however, a more numerically efficient way these functions can be calculated [23]. This alternative method starts by considering a real, discretely sampled function, \( f(t) \), represented by the sequence

\[
f_r, \quad r = 0, 1, 2, \cdots, N - 1, \tag{4.30}
\]

where \( N = 2^n \). Using the discrete Fourier transform, the corresponding Fourier coefficients of \( f_r \) are

\[
F_m = \frac{1}{N} \sum_{r=0}^{N-1} f_r e^{-\frac{2\pi imr}{N}}, \quad m = 0, 1, 2, \cdots, N - 1, \tag{4.31}
\]

where \( F_{N-m} = \bar{F}_m \), and all the \( \{F_m\} \) are generally complex with the exception of \( F_0 \) and \( F_{N/2} \), which are always real. Now suppose that the corresponding complex wavelet coefficients are known and are represented by,

\[
a_s, \quad s = 0, 1, 2, \cdots, N - 1, \tag{4.32}
\]

The Fourier coefficients repeat themselves in the second half of the sequence so that \( a_N - s = \bar{a}_s \), and they are generally complex, except for \( a_0 \) and \( a_{N/2} \) which are both always real. The coefficients can be arranged in octave bands or synonymously wavelet levels, as follows:
<table>
<thead>
<tr>
<th>Wavelet Level</th>
<th>Coefficient in Wavelet Transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>$a_0$</td>
</tr>
<tr>
<td>0</td>
<td>$a_1$</td>
</tr>
<tr>
<td>1</td>
<td>$a_2, a_3$</td>
</tr>
<tr>
<td>2</td>
<td>$a_4, a_5, a_6, a_7$</td>
</tr>
<tr>
<td>3</td>
<td>$a_8, a_9$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$j$</td>
<td>$a_{2^j}$ to $a_{2^{j+1}-1}$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$n-2$</td>
<td>$a_{\frac{N}{2}}$ to $a_{\frac{N}{2}-1}$</td>
</tr>
<tr>
<td>$n-1$</td>
<td>$a_{\frac{N}{2}}$</td>
</tr>
</tbody>
</table>

where level $j$ has $2^j$ coefficients, $a_{2^j+k}$, and $k = 0, 1, 2, \cdots, 2^j - 1$. Each of these coefficients defines the complex amplitude of a wavelet whose Fourier transform is described by the Fourier coefficients in the band. For $k = 0, 1, 2, \cdots, 2^j - 1$, the coefficients are $F_{2^j+k}$.

The first wavelet in the sequence has a constant spectral density of relative level $\frac{1}{2^j}$ with amplitude $a_{2^j}$; this implies it contributes $\frac{a_{2^j}}{2^j}$ to the general coefficient, $F_m$, where $2^j \leq m < 2^{j+1}$. The second wavelet at level $j$ has amplitude $a_{2^j+1}$ and is translated by $\frac{1}{2^j}$ with respect to its neighbor. This results in a rotation of its Fourier transform by $e^{-\frac{i\omega k}{2^j}}$ when $k = 1$. $F_m$ is the Fourier coefficient at frequency $\omega = 2\pi m$, so each $F_m$ has a contribution of $\frac{a_{2^j+1}}{2^j} e^{-\frac{i\omega k}{2^j}}$. Combining all contributions from $k = 0$ to $k = 2^j - 1$ we get

$$F_m = 2^{-j} \sum_{k=0}^{2^j-1} a_{2^j+k} e^{-\frac{i2\pi mk}{2^j}}. \quad (4.33)$$
As an example, if the first nine terms of a 16 term sequence are written out in matrix form, the pattern is

\[
\begin{bmatrix}
F_0 \\
F_1 \\
F_2 \\
F_3 \\
F_4 \\
F_5 \\
F_6 \\
F_7 \\
F_8
\end{bmatrix} =
\begin{bmatrix}
1 & Level - 1 \\
1 & Level 0 \\
1 & W_2 \\
1 & W_3 \\
\frac{1}{4} & 1 & W_4 & W_4^2 & W_4^3 \\
1 & W_5 & W_5^2 & W_5^3 \\
1 & W_6 & W_6^2 & W_6^3 \\
1 & W_7 & W_7^2 & W_7^3 \\
1
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
a_2 \\
a_3 \\
a_4 \\
a_5 \\
a_6 \\
a_7 \\
a_8
\end{bmatrix},
\]

(4.34)

where \( W_m = e^{-\frac{im\pi}{2^j}} \). If terms \( F_9 \) to \( F_{16} \) and \( a_9 \) to \( a_{16} \) were included, the diagonal blocks would continue in reverse order with their elements replaced by their element complex conjugates and their rows in reverse order. One advantage of \( F_{N-m} = \bar{F}_m \) and \( a_{N-s} = \bar{a}_s \) is, for a real input, the additional blocks do not need to be computed; only computations up to term \( \frac{N}{2} + 1 \) need to be shown. Furthermore, each block of wavelet coefficients is transformed by a symmetric unitary matrix after multiplying \( 2^j \), where the order of the matrix is \( 2^j \times 2^j \). This symmetry is demonstrated for level \( j = 2 \) by comparing \( W_7^2 \) and \( W_6^3 \); in general, \( e^{iN\pi} \), where \( N \) is an integer is equal to unity for even \( N \) and negative unity for odd \( N \):

\[
W_7^2 = \left( e^{\frac{i\pi}{2^j-1}} \right)^2 = e^{i7\pi} = -1,
\]

and

\[
W_6^3 = \left( e^{\frac{i\pi}{2^j-1}} \right)^3 = e^{i9\pi} = -1.
\]
Thus, the inverse of the matrix is just its complex conjugate. This results in the wavelet coefficients being easily calculated using the Fourier coefficients. In practice, all the coefficients other than those that are unity in Eqn. 4.34, can easily be calculated using the fast Fourier transform (FFT). As such, Eqn. 4.33 can be rewritten in the standard form of the discrete Fourier transform after substituting \( n = 2^j \) and \( s = m - n \),

\[
F_{2^j+k} = \frac{1}{n} \sum_{k=0}^{n-1} a_{2^j+k} e^{-\frac{i2\pi k}{n}},
\]

since \( e^{-2\pi k} = 1 \) for all \( k \). Aside from those coefficients which are unity (and will be considered next), the wavelet coefficients can be obtained by computing the inverse discrete Fourier transform of successive blocks of the Fourier coefficients of the signal being decomposed.

The first unitary block in Eqn. 4.34 is related to the band of frequencies between zero and \( 2\pi \) and is the contribution of the scaling function. Obtaining unity for this block requires an assumption about the input signal. In the continuous case, the function characterizing the input is considered to overlap the unit interval. That the function is assumed to wrap around the interval appearing at the opposite side. Because of this wrap around effect, the scaling function is replaced by unity in the discrete case and therefore \( a_0 = F_0 \), the mean value of \( f_r \). This is shown in Appendix A. The second unit block in Eqn. 4.34 comes from the frequencies in level 0 and is the FFT of a single term sequence. The last block in Eqn. 4.34 indicates \( F_{N/2} = a_{N/2} \); this term must be included to pick-up the Nyquist frequency component of the signal.

For a discrete transform of sequence length \( N = 2^n \), only wavelet levels up to \( n - 2 \) can be included, because both coefficients, \( \tilde{a}_{j,k} \) and \( a_{j,k} \) (Eqns. 4.26 and 4.27), must be computed. As a result, there is no space for a level \( n - 1 \) wavelet and only one harmonic can be fitted with the Nyquist frequency, \( N\pi \). This defines the wavelet

\[
f_r = \sum_{k=0}^{N-1} F_r e^{\frac{i2\pi k}{N}}
\]

\[
= F_N e^{i\pi r},
\]

(4.36)
for \( r = 0, 1, 2, \cdots, N - 1 \). Since all other \( F_k \) are zero, this produces the saw-tooth sequence

\[
f = [1 \ -1 \ 1 \ -1 \ 1 \ -1 \cdots],
\]

(4.37)

when \( F_{N/2} = 1 \). The saw-tooth wavelet can be shown to be orthogonal to the other discrete sequences that describe the harmonic wavelets [23]. This last term, \( a_{N/2} \), is important because it serves to pick up any additional energy at the Nyquist frequency ensuring Parseval’s theorem is satisfied, just as in Eqn. 4.28:

\[
\frac{1}{N} \sum_{r=0}^{N-1} |f_r|^2 = |a_0|^2 + \sum_{j=0}^{n-2} \frac{1}{2^j} \sum_{k=0}^{2^j-1} \left(|a_{2^j+k}|^2 + |a_{N-2^j-k}|^2\right) + |a_{N/2}|^2,
\]

(4.38)

where \( N = 2^n \). The algorithm is described graphically in Fig. 4.5. When the input sequence, \( f_r \), has complex elements, the wavelet sequence \( a_s \) no longer satisfies \( a_{N-s} = \overline{a}_s \), but rather \( a_{N-s} = \tilde{a}_s \). As a result, the second half of the sequence can no longer be written across as shown in Fig. 4.5. In this case, the block pattern in Eqn. 4.34 must continue to fill the complete \( N \times N \) matrix, where each block of order \( 2^j \times 2^j \) in the lower half of the matrix is the complex conjugate of the block of the same order in the upper half of the matrix, with the rows in reverse order. The corresponding version of the algorithm for a complex input shown is shown in Fig. 4.6. Additionally, Newland provides an alternative to these algorithms in [65], which uses an interpolation technique that reduces the processing time required to provide the wavelet coefficients.
Fig. 4.5: **Harmonic Wavelet Transform Algorithm For Real Inputs.** The image shows a graphical description of the algorithm used to calculate the harmonic wavelet transform for real inputs. Image taken from [23].

Fig. 4.6: **Harmonic Wavelet Transform Algorithm For Complex Inputs.** The image shows a graphical description of the algorithm used to calculate the harmonic wavelet transform for real inputs. Image taken from [23].
CHAPTER 5
Shock Attenuation with Distance on a Cylindrical Shell

5.1 Background

In March of 1970, The Martin Marietta Corporation, under a contract with NASA’s Goddard Space Flight Center, published a report entitled *Aerospace Systems Pyrotechnic Shock Data (Ground Test and Flight)* [5]. In this report, the authors present attenuation guidelines for structures and equipment being designed to pyroshock environments. Their work discusses several structural configurations, each having a specific attenuation curve. The attenuation curves they generated were based on an omni-directional envelope of the directional specific ratio of SRS peak spectrum responses over the frequency range of interest at various distances away from the source shock, normalized to the measurement nearest the source.

Of particular interest are the attenuation characteristics of a cylindrical shell. This type of structure is common place in the rocket industry and knowing the attenuation along the cylinder is useful in defining design loads for components and structures attached to a cylindrical case. Fig. 2.4 summarizes the findings of the work conducted in the late 1960’s. As has been previously stated, there are two primary concerns that arise when reviewing this figure. First, the curve was generated based on limited data (as is self-stated within the report) using old data acquisition technology, and second, the SRS was used to determine the spectrum peaks.

In 2009, Orbital ATK conducted a full scale separation test as part of NASA’s ARES I and ARES I-X programs [2, 12]. The test objectives were to demonstrate the functionality of the ARES I-X Linear Shaped Charge (LSC) severance system and to capture data to help understand the structural dynamic response of the detonation. The test used LSCs to separate the forward skirt to forward skirt extension (FSE) joint and the FSE to frustum
joint. The first test conducted separated the FSE from the forward skirt, while the second separated the frustum from the FSE. Fig. 2.6 shows these joints and where they fall with respect to the larger system. The nominal core design of the LSCs used an explosive nitroamine, RDX [12], with a detonation velocity of approximately 345,000 in/s [66]. Though the data set was used to help characterize the structural response, attenuation with distance was never considered. As the data is assessed, keep in mind the limitations associated with using an archived data set - changes in test configuration to verify or highlight findings is not possible. Furthermore, the type of data used in this assessment is expensive to collect and of limited availability in many archives. The costs associated with this type of test often prohibit similar tests from being performed. The data available is the best available to the author for this type of assessment at the current time.

Each test consisted of three basic pieces of hardware, an LSC severance system, the FSE, and a mass simulator used to represent either the forward skirt or frustum depending on the test. In each test, the LSC severance system was bolted onto the FSE, which was in turn bolted onto the appropriate mass simulator. The LSC severance system consisted of the LSC, which sat inside an aluminum retaining ring (the gold ring in Fig. 2.8), an initiator manifold that the flexible confined detonating cord (FCDC) ran into, and a set of aluminum plates that acted as debris shields. During the actual test, the LSC severance system was where the severance actually occurred. By designing the test in this fashion the FSE was preserved and available for use in both test configurations.

In addition, the FSE contained flight like features such as the main parachute support system (MPSS). The MPSS was the anchor location for the drogue parachute used as part of atmospheric reentry and consisted of interior arms that attached to the outer shell of the cylinder along with a plate that covered the aft end of the cylindrical segment. Fig. 5.1(a) shows these interior arms and plate and Fig. 5.1(b) shows the azimuthal attachment locations of the arms. The separation events that were duplicated during the ground test were conducted in the opposite order as they occurred in flight. During flight, following the upper stage separation between the frustum and an interstage segment, the frustum
was severed from the FSE. On top of the FSE, a nose cap contained the pilot and drogue parachutes. Once the nose cap was jettisoned, the pilot parachute served to deploy the drogue chute. Then, when conditions were appropriate, the FSE to forward skirt was severed and the main parachute system would deploy slowing the booster until water impact sometime later [67]. A pictorial representation of the flight sequence leading to water impact is provided in Appendix D. During both ground tests, the MPSS was located on the side of the FSE near the ground [12]. Despite the order of separation events being backwards, and the hardware oriented with the aft end of the separation joints pointing up, the hardware was consistent with the flight configuration. Fig. 5.2 shows an image describing where the MPSS was located during both ground tests.

Accelerometers were located at several locations on the shell outer radius at varied distances from the cut plane of the LSC line in each test. Each measurement location collected the structural response in three mutually perpendicular axes. These axes were defined using a cylindrical system with the origin located at the center of the shell on the plane defined by FSE to frustum joint. Fig. 2.7 shows the relative location of the FSE to frustum joint. The station of a measurement refers to the axial distance away from the origin, while the azimuth of a measurement refers to the angular distance from the zero reference point. Fig. 5.3 shows the top view of the azimuthal coordinate system relative to the incoming FCDC initiation lines connecting the LSCs. Figs. 5.4(a) and 5.4(b) show station locations of interest for the FSE to forward skirt test and FSE to frustum test respectively. The separation plane for the FSE to forward skirt test was at station -4 in, while the separation plane for the FSE to frustum test was at station 72.6 in. Table 5.1 summarizes the accelerometer locations on the shell, along with the axial distance away from the separation plane. Though the origin was located at the same location on the hardware in both tests, the FSE to frustum test configuration dictated that the test article was roughly 8 in. higher off the ground than during the FSE to forward skirt separation test.
Fig. 5.1: **Main Parachute Support System Description.** (a) The image shows the main parachute support system arms and plate that tie into the cylindrical shell or what would the outer case in flight. [12] (b) The image shows a pictorial representation of the main parachute support system arms that tie into the cylindrical shell. The azimuthal location of the accelerometers used this assessment is represented by the red star. Based on a diagram in [12].

All measurements, except channel A109A, used an Endevco model 7270A-200K accelerometer, while channel A109A used a PCB model 350C02 accelerometer. All channels
were ranged to ±100,000 g’s, and the sample rate varied based on the test and location. Generally, the sample rate ranged between 1,000,000 and 25,000,000 samples per second. Again channel A109A was the anomaly; here the sample rate was 102,400 samples per second. Regardless of the sample rate, the data available for analysis was decimated as part of posttest procedures to expedite the analysis. The FSE to forward skirt separation data was decimated to a 100 kHz sample rate and the FSE to frustum separation test was decimated to a 200 kHz sample rate. In either case, the decimated data sample rates are sufficiently high to characterize the frequency range between 10 and 10,000 Hz\(^1\).

Channel A008T had a severe mean offset present throughout the recorded time history. Looking at the noise level prior to the shock transient for this channel, it was clear the offset was roughly 86,000 g. As a result, a mean of 86,000 g was assumed and subtracted over the complete time history. Doing so brought the channel in family with all other data. There were no other channels that exhibited obvious data quality issues. All measurements were located on the FSE during both tests aside from channel A109A. As part of the FSE to frustum separation test, channel A109A was located at station -2.1 in, putting it on the forward skirt tooling ring. The tooling ring is an adapter that simulated the attachment of the forward skirt and was used as the support point of the structure during the test. Based

\(^1\)The frequency band used in the calculation of the SRS is consistent with the approach used in [5].
Table 5.1: **Accelerometer Location By Test.** This table shows the locations of the accelerometers of interest to this assessment. [12]

<table>
<thead>
<tr>
<th>Test</th>
<th>Channel Name</th>
<th>Azimuth (°)</th>
<th>Station (in.)</th>
<th>Radius (in.)</th>
<th>Axially Distance to Separation Plane (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FSE to Forward Skirt Test</strong></td>
<td>A001A</td>
<td>0.0</td>
<td>18.0</td>
<td>70.3</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>A002T</td>
<td>0.0</td>
<td>18.0</td>
<td>70.3</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>A003R</td>
<td>0.0</td>
<td>18.9</td>
<td>70.3</td>
<td>22.9</td>
</tr>
<tr>
<td></td>
<td>A004A</td>
<td>0.0</td>
<td>23.0</td>
<td>70.3</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>A005T</td>
<td>0.0</td>
<td>23.0</td>
<td>70.3</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>A006R</td>
<td>0.0</td>
<td>34.5</td>
<td>70.3</td>
<td>28.5</td>
</tr>
<tr>
<td></td>
<td>A007A</td>
<td>0.0</td>
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On the location of channel A109A, the shock pulse had to travel through an additional bolted joint to reach the measurement location. The effects of this bolted joint are not considered separately from the effects of distance attenuation in this analysis. The data for channel A109A is shown alongside the other data, but should be taken for reference only because of this difference. It is well known that bolted joints have significant influence on damping and attenuation characteristics in shock response.

As has been mentioned, the measurements in the current standard were normalized to the measurement nearest the source (approximately 5 in. away) in a direction specific format. The same normalization process is followed here. For example, the axial measure-
Fig. 5.3: **Azimuthal Coordinate System.** This image shows a cartoon depiction of the top view of the azimuthal coordinate system on the shell and the detonation path followed by the pyrotechnic devices used to sever the FSE from the forward skirt and FSE from the frustum joints. Image based on work done in [12].

Measurements for a particular test are normalized based on the peak SRS or approximate energy of the axial measurement nearest the source.

The relative distance to the source of the heritage measurements is different than that of the measurements nearest the source given in Table 5.1. As a result, the measurements in the data set were shifted to account for the difference, and just as in the case of the amplitude, this was performed in a direction specific fashion. The relative distance between measurement locations was determined by subtracting the axial station of the measurement nearest the source from each successive location (including the measurement nearest the source). This translated the data points to the left, positioning the measurement nearest the source as if it were located directly above the pyrotechnic charge, or having an axial difference of zero. Finally, to standardize the current data set with the historical standard, the measurement locations were translated right by adding the station location of the nearest point measured in the historical standard (roughly 5 in. away). This allows for a direct
Fig. 5.4: **Axial Station Locations of Interest.** (a) The image shows the relative station locations of key locations on the test article. The separation plane is located at station -4 in. [12] (b) The image shows the relative station locations of key locations on the test article. The separation plane is located at station 72.6 in. [12]

comparison with the historically accepted standard. The standardized distance, or the translated distance to source, are used in the work that follows. As an example, consider channel A003R, here the axial distance to the source, 22.9 in., was subtracted from itself, placing it at 0 in., and then the 6.93 in. accounting for the historical standard was added. Thus the standardized distance from the source is 6.93 in. Table 5.2 shows the standardized
distance compared with actual distance to the separation plane for all accelerometers.

There is a significant draw back to normalizing the measurements in this fashion, in doing so, the relationship between the directional responses is lost. In order to provide an understanding of the relative differences between response directions, the peak acceleration from each measurement was normalized relative to the peak axial measurement nearest the source for each test. That is, for the FSE to forward skirt test, all measurements were normalized relative to channel A001A and for the frustum to FSE test; the channels were normalized relative to A021A. This ratio is presented alongside the axial distance to the separation plane and the standardized distance to the separation plane in Table 5.2. Based on the data in Table 5.2, there is a significant difference in the relative peak response with distance between the FSE to forward skirt test and FSE to frustum test. The most significant difference between these two tests is the MPSS location relative to the separation plane. In the FSE to forward skirt test, the MPSS was located on the opposite end of the FSE from the separation plane, while the FSE to frustum test separation plane was on the same end of the FSE as the MPSS. The data suggest that the response dynamics are significantly different for an excitation near the MPSS compared to one further away.

Figs. 5.1(a) and 5.1(b) show the relative location of the accelerometers being used in the assessment compared to the attachment points of the MPSS. The measurements used in the assessment are roughly $15^\circ$ away from the attachment point of one of the arms, and although the separation from the measurement locations is reasonably far, their presences affects the global stiffness of the structure, and locally, they act as an artificial boundary condition to the response characteristics nearby. As a result, the unexpected peak response characteristics in Table 5.2 could be due to the presence of the MPSS proximity to the separation plane. Based on this, it is not unreasonable to assume difference between the attenuation characteristics in the two tests will exist.

The remainder of this chapter looks at the available data from the two tests documented in [12] and compares the attenuation with distance exhibited in the data with the historical standard in [5]. The attenuation with distance in the data sets is characterized in two ways:
Table 5.2: **Accelerometer Distance to Source Standardization.** This table shows the locations of the accelerometers relative to the separation plane and the standardized location used for comparison with historical standard. [12] Additionally, it shows the magnitude of the ratio of peaks for all channels relative to the axially measurement nearest the separation plane.

| Test          | Channel Name | Axial Distance to Separation Plane (in.) | Standardized Distance to Separation Plane (in.) | |Normalized Peak Time History|
|---------------|--------------|------------------------------------------|-----------------------------------------------| |----------------------------|
|                | A001A        | 22                                       | 6.93                                          | 1.00                                      |
| FSE to Forward | A002T        | 22                                       | 6.93                                          | 0.69                                      |
| Skirt Test     | A003R        | 22.9                                     | 6.93                                          | 0.69                                      |
|                | A004A        | 27                                       | 11.93                                         | 0.62                                      |
|                | A005T        | 27                                       | 11.93                                         | 0.63                                      |
|                | A006R        | 28.5                                     | 12.53                                         | 0.79                                      |
|                | A007A        | 40                                       | 24.93                                         | 1.07                                      |
|                | A008T        | 40                                       | 24.93                                         | 1.01                                      |
|                | A009R        | 40.8                                     | 24.83                                         | 0.54                                      |
|                | A010A        | 51.3                                     | 36.23                                         | 0.95                                      |
|                | A011T        | 51.3                                     | 36.23                                         | 0.62                                      |
|                | A012R        | 53                                       | 37.03                                         | 0.55                                      |
| Frustum to    | A021A        | 18.8                                     | 6.93                                          | 1.00                                      |
| FSE Test       | A022T        | 18.8                                     | 6.93                                          | 0.89                                      |
|                | A023R        | 17.1                                     | 12.03                                         | 1.60                                      |
|                | A024A        | 25.1                                     | 13.23                                         | 2.21                                      |
|                | A025T        | 25.1                                     | 13.23                                         | 1.54                                      |
|                | A026R        | 17.1                                     | 18.03                                         | 2.59                                      |
|                | A027A        | 36.8                                     | 24.93                                         | 1.36                                      |
|                | A028T        | 36.8                                     | 24.93                                         | 1.12                                      |
|                | A029R        | 35.1                                     | 81.99                                         | 1.70                                      |
|                | A030A        | 48.6                                     | 36.73                                         | 2.25                                      |
|                | A031T        | 48.6                                     | 36.73                                         | 1.34                                      |
|                | A032R        | 47.2                                     | 42.13                                         | 1.98                                      |
|                | A109A        | 74.4                                     | 62.83                                         | 1.98                                      |
|                | A206R        | 12                                       | 6.93                                          | 2.21                                      |

First, following the historical standard, the peak SRS response over the frequency band between 10 and 10,000 Hz is used, secondly, an alternative method using the approximate energy of the time history (Eqn. 3.1) is used. In both cases, keeping with the historical standard, measurements are ratioed with the directionally consistent measurement nearest the source to determine the percent of the shock remaining. In order to compare the attenuation characteristics of the ARES I-X data with the historical standard, the Martin Marietta attenuation with distance curve, Fig. 2.4, had to be digitized from the available
Fig. 5.5 shows the result. The red curve is a curve fit based on a four parameter fit to the sum of two exponential functions having general form: \( y = a_1 e^{a_2 x} + a_3 e^{a_4 x} \), where \( a_i \) are the fit parameters. The curve fit was developed based on the black solid points, which were the digitized points from a screen capture of Fig. 2.4.

![Digitized Martin Marietta Attenuation for Cylindrical Shell](image)

**Fig. 5.5: Digitized Martin Marietta Standard.** The figure shows a digitized version of Fig. 2.4.

The primary response direction of interest for the available test data is the axial direction. Although the first wave to arrive at a given measurement location is not from the LSC detonation directly in line with the measurement, the test configuration used in the FSE to forward skirt and FSE to frustum separation tests lends itself to primarily an axial excitation at each measurement point. Fig. 5.6 shows a pictorial representation of the test set up. Point A is some azimuthal distance away from the projection of the measurement location onto the separation plane (point B). There are two paths from which waves arrive at the measurement location early in time. They follow either path one, which follows the
detonation path of the LSC up to the point B and then travels down the shell to the measurement location, or they follow path two, which is the wave traveling through the shell associated with the detonation of the LSC an angle $\phi$ away from the measurement location. To demonstrate the excitation is essentially axial in nature, we need to show the angle $\phi$ is sufficiently small such that the contribution of the waves arriving before the waves following path one is small. That is, we need to show, $\phi \sim 0$. The time required to travel from point A to the measurement location along path one is

$$t_1 = \frac{R\phi}{v_d} + \frac{z}{v_m},$$

(5.1)

where $R$ is the radius of the cylinder, $z$ is the vertical distance to the measurement location, $v_d$ is the detonation velocity of the LSC, $v_m$ is the wave velocity in the material$^2$, and $\phi$ is the angle of interest. Similarly, the time required to travel along path two is

$$t_2 = \frac{\sqrt{(R\phi)^2 + z^2}}{v_m}.$$  

(5.2)

Setting Eqns. 5.1 and 5.2 equal and solving for the nontrivial solution we obtain

$$\phi = -\frac{2z(\frac{v_m}{v_d})}{R\left[\left(\frac{v_m}{v_d}\right)^2 - 1\right]}.$$  

(5.3)

Since the velocities are fixed, Eqn. 5.3, is driven by the ratio, $\frac{z}{R}$. Thus, the closer to the separation plane the measurement is, the more axial the excitation is. In fact, for the measurement locations of interest, the measurement with the most non-axial excitation is channel A109A. A109A is located roughly 75 in. axially away from the separation plane. This translates to an approximate 1.8° angular excitation relative to the vertical line from which the measurements are taken. Thus assuming the detonation velocity is sufficiently fast as to consider the measured response free from energy contributed by waves originating at locations other than axially in line with the measurement locations, is reasonable. This

$^2v_m = \sqrt{\frac{E}{\rho}}$, where $E$ is the modulus of the material, and $\rho$ the density.
makes the axial response the most relevant data set to compare to the historical standard.

Fig. 5.6: **Wave Paths Following to Measurement Locations.** The image shows the possible paths that can be taken by the wave front once the LSC is detonated.

### 5.2 SRS Approach

The Martin Marietta document does not explicitly specify the length of time used in the attenuation work, as a result two assessments using two time periods were completed using the peak SRS approach. Based on the fact that the SRS was first developed as a tool to capture a peak design load, the full time history of the impulse will be kept for the first assessment. This ensures all energy due to the excitation is accounted for and captured in data processing. The second time period considers trying to characterize the traveling wave only and as a result is significantly shorter than the first. A review of the time histories indicated the peak response measured by the accelerometer furthest from the source in either test occurred before $t = 0.0025$ s. In actuality, both of these time periods should produce similar results. The basis of the SRS is to determine the peak response due to some basis excitation at a particular frequency. Theoretically, that means finding the peak response of a single degree of freedom system at each frequency of interest. If the peak input acceleration is captured in the input, the peak response for a particular natural
frequency will be captured as well. This is basically a statement of the conservation of energy, the only energy sink or source after the initial excitation are the dissipative effects modeled as a viscous damper in the SRS determination. This implies an exponential decay in the response over time. Therefore, regardless of the time varying nature of the input, if the peak input base excitation is captured in the time period used as the input to the SRS, the response spectrum should very similar for different time periods. This just serves to illustrate a known short coming of the SRS; the SRS does not account for the temporal nature of the excitation.

Figs. 5.7 and 5.8 show the attenuation with distance for both time periods for the FSE to forward skirt and FSE to frustum separation tests respectively. As expected, the results for the time periods are similar, and although the plots show results for the three directions, the axial direction is of primary interest based on the essentially axial nature of the excitation demonstrated in Eqn. 5.3. Consider the axial data for the FSE to forward skirt test and FSE to frustum test, the FSE to forward skirt data set suggest the historical standard over estimates the attenuation with distance, while the FSE to frustum test tends to increase with distance, an unexpected result. This behavior is likely due to the proximity of the MPSS to the separation plane, similar to what was seen in Table 5.2. The radial and tangential attenuation characteristics are peculiar as well. The odd behavior in the radial and tangential directions could also be an implication of the MPSS; however, there is no conclusive evidence upon which to tie the behavior too. Keep in mind, channels A109A and A008T deserve special consideration, either due to their location, or extra processing needed. Also note that some data points in Figs. 5.7 and 5.8 may fall directly on top of one another, causing it to look like data has been excluded; this however, is not the case.

The results of both data sets do not lend themselves to making strong conclusions about the historically accepted Martin Marietta attenuation standards. There are three primary sources of concern that could be sources of the anomalous results produced. First, as was

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3Note: Some data points overlap for both time periods, causing it to seem as if data were omitted; this however, is not the case. A good example of this is at the location nearest the source, here, all directions and time periods overlap one another, causing it to seem as if only one data point is plotted, when in reality, all directions and time periods are at this point.
previously discussed, the test article used in this assessment was not a true cylindrical shell, whereas, the Martin Marietta test configuration was a true cylindrical shell. Any change in configuration can result in a significant response differences which likely influence attenuation with distance. Second, many decimation schemes use filtering as part of the process. The “decimate.m” function in Matlab, for example, uses an eighth order Chebyshev type I low pass filter by default. If this were the function used in the decimation process, it is likely there would be little effect over the frequency range used in this assessment, however, without knowing the decimation process followed in the post processing of the data, it is hard to completely rule out filtering effects on the data set. Lastly, there is always the possibility of data quality concerns that stem from instrumentation related issues such as misaligned accelerometers or accelerometer cross axis bleed that contribute to uncertainty in the analysis of any experimental data; however, quantizing these effects is difficult when data is pulled from an archive and the analyst is not involved in the data collection process.

5.3 Approximate Energy Approach

Two assessments using two time periods were completed following the approximate energy approach. As a comparison to the SRS approach, the first time period keeps the portion of the response between, $t = 0$ to $0.0025$ s. The second time period was based on the understanding of how the approximate energy changes in time. As longer periods of time are kept, the more likely the modal standing response is to influence the attenuation characteristics. If a measurement happens to be located on a node of a high energy mode, the approximate energy of the response can be skewed. As a result, trying to correlate wave propagation attenuation over some distance becomes unreliable. Measurements further away from the source maybe located away from a node of a high energy mode, while measurements more near the source may be located on one, causing the measurement further from the source to appear as if it has more energy, artificially skewing the wave propagation attenuation estimate. The next logical idea is to keep the time period prior to the modal response being active. Isolating the traveling wave from the modal response requires knowing exactly when the modal response of the system is actively starting to influence the
energy, which is extremely difficult. The modal response is inherently tied to any excitation and determining an adequate cut off time may be nearly impossible. Working under the assumption that the previously analyzed time period is too long and incorporated portions of the modal response, the data was reprocessed over two distinct time periods, one specific to each test. The time periods were determined by selecting the portion of time starting just before the first wave arrival at the measurement nearest the source and ending after the time required for the response at the measurement furthest from the source to complete roughly one full period. By choosing this time period, the noise floor contribution prior to wave arrival is eliminated, but more importantly, it ensures the modal response contribution is negligible. The time period focuses solely on the first pulse of the shock that is exciting the system. The time period used in the case of the FSE to forward skirt test was between \( t = 0.0008 \) to \( 0.00104 \) s and the time period used in the case of the FSE to frustum test was between \( t = 0.001 \) to \( 0.00122 \) s. Before any conclusions are drawn, the reader should be reminded again about subtle differences associated with channels A008T and A109A. Figs. 5.9 and 5.10 show the attenuation with distance for both time periods for the FSE to forward skirt and FSE to frustum separation tests respectively. Looking only at the axial data, the shorter time period better represents the expected behavior for sets of test data. For the FSE to forward skirt test, and depending on which time period is considered, the data points to two opposing conclusions. The longer time period data suggest that the historical standard over estimates the attenuation with distance; a similar result as was in the case of the SRS approach, while the shorter time period suggests the exact opposite conclusion. Here the data suggest the historical standard is overly conservative and the attenuation with distance could be increased. Additionally of note, there is evidence that the modal standing response may have started to skew the longer time period data even for the relatively short duration that it is. Channel A007A increases relative to A004A, which could be due to nodal effects of the modal response.

The results are similar for the FSE to frustum test. The shorter time period data here again suggests the historical standard is overly conservative and the attenuation with
distance could be increased; where as in the case of the longer time period, the potential
effects of the modal response and proximity of the MPSS to the separation plane make it
difficult to draw any conclusions. Attempting to determine the relative impact of the nodal
effects and the MPSS proximity to the separation plane for the longer time period data
would be very difficult; however, based on the slight differences between the longer and
short time periods in the FSE to forward skirt test and the similar differences seen between
the two tests’ attenuation characteristics using the SRS approach and shown in Table 5.2,
it is likely that the MPSS proximity to the separation plane plays a larger role than the
modal effects.

5.4 General Conclusions

Though the results do not lend themselves to being definitive, several general conclu-
sions can be established.

• First, the time period used in the SRS approach is of little importance in the deter-
mination of the expected attenuation with distance, so long as the peak acceleration
of the system is captured and input into the SRS algorithm.

• Second, processing the data with either approach, clear differences between the FSE
to forward skirt and FSE to frustum tests can be seen. These differences are likely
due to the proximity of the MPSS to the separation plane and the corresponding
structural response differences.

• Third, based on the comparison of the standard to the test data, and the inconclusive
nature of the results, further testing should be conducted to either validate or update
the attenuation with distance standards.

• Fourth, the two methods used in the analysis, the peak SRS response and approximate
energy approach, both produce results that are similar. When using a short time
period, the approximate energy method generally produces results indicating there is
more attenuation then the historical standard. Though both methods follow similar
trends, the results do not lend themselves to one particular method. The benefit of the approximate energy method however, is that it requires no assumptions about system damping, or a frequency band to implement.

- Lastly, the need for a method that isolates a traveling shock wave from a generic response is highlighted here. In the tests considered, the excitation is due only to the impulse from the pyroshock event, however in practice, responses often have other superimposed excitations, such as random or sinusoidal excitation, making it even more difficult to distinguish between a modal response and the shock response.
Fig. 5.7: **FSE to Forward Skirt Separation Test Attenuation.** The plots show the attenuation with distance for the measurements taken as part of the FSE to frustum separation test using the peak SRS approach for (a) all directions and (b) for the axial direction.
Fig. 5.8: **FSE to Frustum Separation Test Attenuation.** The plots show the attenuation with distance for the measurements taken as part of the FSE to forward skirt separation test using the peak SRS approach for (a) all directions and (b) for the axial direction. Channel A008T required additional processing to bring it in family with the other data’s time histories, and channel A109A’s location required the traveling wave to pass through a bolted joint prior to reaching the measurement location.
Fig. 5.9: **FSE to Forward Skirt Separation Test Attenuation.** The plots show the attenuation with distance for the measurements taken as part of the FSE to frustum separation test using the peak SRS approach for (a) all directions and (b) for the axial direction.
Fig. 5.10: **FSE to Frustum Separation Test Attenuation.** The plots show the attenuation with distance for the measurements taken as part of the FSE to forward skirt separation test using the peak SRS approach for (a) all directions and (b) for the axial direction. Channel A008T required additional processing to bring it in family with the other data’s time histories, and channel A109A’s location required the traveling wave to pass through a bolted joint prior to reaching the measurement location.
CHAPTER 6
Flat Plate Shock Response Modeling and Attenuation Characteristics

6.1 Background

The Ares I launch vehicle had several separation events using linear shaped charges (LSC) that occurred throughout flight. These type events inherently create large shock levels which can affect avionics and other sensitive electrical components nearby. In order to better understand the impact of these events, a series of pyrotechnic shock tests were performed by Orbital ATK in October of 2010. The primary purposes of the test series was to demonstrate cutting capability of the LSC units to the designed thickness of the case, and to measure and characterize the shock environment produced from such events. The test series saw three different linear shape charges used, however, due to International Traffic in Arms Regulatory (ITAR) restrictions the specific values of these cannot be released. As a result, the baseline LSC is ‘normalized’ and labeled as having a charge density of 1 grain/foot (gr./ft.). The two additional LSC configurations were more densely concentrated and are designated as 1.125 gr./ft. and 4.125 gr./ft. The relative charge density relationship between the three LSC configurations is preserved in the definitions here, that is to say the third configuration had 4.125 times more charge per length than the baseline configuration did.

The flat plate test article consisted of three primary pieces. A sacrificial or scar plate, the LSC severance system, and the primary aluminum plate that is joined to the scar plate using a butt joint interface. The LSC was attached to the scar plate; by attaching the LSC on a separate scar plate, the primary test article was able to be reused indefinitely. The thicknesses of the scar plate varied based on the cut depth goals of the test plan and were thicker relative to the main plate thickness. The main plate was 4 ft. wide, 8 ft. long, and 1 in. thick and although the test configuration was designed to simulate free
boundary conditions, four cables supported the primary plate in a test stand. A cartoon
description of the test set up is shown in Fig. 6.1. Though the test set up does not provide
the desired free boundary conditions, the presences of the cables minimally influences the
collected data. The shock response of the primary plate is driven by the high frequency
transient produced by the blast, while the boundary condition affects typically affect the
low frequency response ($<\sim 100$ Hz) of any system.

![Cartoon Depiction of Test Article and Stand](image)

Fig. 6.1: **Cartoon Depiction of Test Article and Stand.** The red star points out
the location of the origin used with the accelerometers in the full test configuration. This
location is consistent with the location shown in Fig. 6.2. Image based on figures in [18].

There were 24 tests performed during the test series, each with a designation between
T1 and T24, and all having the initial LSC detonation near the origin, which is designated on
Fig. 6.1. Upon initiation, the LSC detonation propagated up the scar plate ending at the full
width of the plate. In each case, the accelerometers used to measure the plate response were
placed in the same locations and were configured the same in the data acquisition system.
Fig. 6.2 shows the placement of each accelerometer along with the channel name associated
with it and the reference location designated as the origin. The accelerometer type and
relative location to the origin are shown in Table 6.1. All accelerometer measurements were normal to the plate, were ranged at ±50,000 G, and were sampled at 1,000,000 samples per second.

Fig. 6.2: Accelerometer Locations. The image shows the accelerometer locations relative to the selected origin. All measurements are normal to the plate. Table 6.1 provides the actual distances from the origin. Image based on figures in [12].

Table 6.1: Accelerometer Location. This table shows the locations of the accelerometers of interest to this assessment. $X_{rel}$ and $Y_{rel}$ are defined relative to channel A04N, the measurement that experiences the shock pulse first when the LSC is detonated. Note: Channels A11N and A12N were not present in many of the data sets used in this analysis, as a result they were not assessed. [12]

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<td>28</td>
<td>Endevco 7270A-60K M6</td>
</tr>
<tr>
<td>A02N</td>
<td>18</td>
<td>36</td>
<td>0</td>
<td>28</td>
<td>28</td>
<td>PCB 350C02</td>
</tr>
<tr>
<td>A03N</td>
<td>18</td>
<td>12</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>PCB 350C02</td>
</tr>
<tr>
<td>A04N</td>
<td>18</td>
<td>10</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>Endevco 7270A-60K M6</td>
</tr>
<tr>
<td>A05N</td>
<td>55</td>
<td>38</td>
<td>37</td>
<td>28</td>
<td>46.4</td>
<td>PCB 350C02</td>
</tr>
<tr>
<td>A06N</td>
<td>55</td>
<td>10</td>
<td>37</td>
<td>0</td>
<td>66.85</td>
<td>PCB 350C02</td>
</tr>
<tr>
<td>A07N</td>
<td>59</td>
<td>38</td>
<td>41</td>
<td>28</td>
<td>59.84</td>
<td>PCB 350C02</td>
</tr>
<tr>
<td>A08N</td>
<td>59</td>
<td>10</td>
<td>41</td>
<td>0</td>
<td>70.18</td>
<td>PCB 350C02</td>
</tr>
<tr>
<td>A09N</td>
<td>71.5</td>
<td>38</td>
<td>53.5</td>
<td>28</td>
<td>60.38</td>
<td>Endevco 7270A-60K M6</td>
</tr>
<tr>
<td>A10N</td>
<td>71.5</td>
<td>10</td>
<td>53.5</td>
<td>0</td>
<td>80.97</td>
<td>PCB 350C02</td>
</tr>
</tbody>
</table>
As part of the data review, individual channels were assessed to determine validity and ensure the data reliability. In assessing the data, there were two key characteristics that were of interest in the quality review. The first was to identify any accelerometer saturation. This occurs when the natural frequency of the accelerometer is excited causing a voltage overload that saturates the internal amplifier of the sensor and results in unreliable response data and anomalous looking discontinuities in the time history. The second characteristic of interest was the magnitude of the primary peak. The goal was to identify any peak clipping that occurred. Peak clipping refers to the actual response being larger than the ranged value, and is identified by several points in time that measure the same amplitude and are higher than the system ranged value. Peak clipping results in unreliable data early in the time history. Later in the ring down the data is reliable; however, that portion of the time period isn’t of interest in this assessment. A summary of the data quality check, along with the LSC charge density used in each test, is provided in Appendix E.

In reviewing the time history data for quality issues, an old phenomenon was noticed that seemed to be a characteristic inherent to all responses for all tests. The transient response, at all measured locations on the plate, exhibits a split peak phenomenon as compared to the expected ring down in Fig. 2.1. This split peak phenomena is shown in Fig. 6.3. The further the measurement was from the source the harder the secondary peak was to identify, but it was present in all data. Based on the time history data alone, there is no obvious answer as to why the response measured across the plate would have such a characteristic. This odd response characteristic is discussed further in section 6.2.3.

6.2 Modeling and The Split Peak Phenomena

The goals of this section are outlined below:

- Develop and correlate a finite element model to test T3 data, which was arbitrarily chosen, using the time history, harmonic wavelet transform, and the frequency response.
- Use the correlated model to predict the response of the plate for an increased charge density LSC load case and comment on the reliability of such prediction capability.

- Identify any best practices associated with the use of industry standard software in modeling pyroshock structural response.

- Use finite element modeling and data processing techniques to investigate and understand the source of the split peak phenomena shown in Fig. 6.3.

![Peak Normalized Time History](image)

**Fig. 6.3: Example Flat Plate Time History Response.** The two arrows illustrate the two peaks that are atypical of the expected response. [18]

### 6.2.1 Model Development and Correlation

As in the case with building any finite element model, simplifications are made to approximate reality. In this case, three primary simplifying assumptions exist between the model and the test set up. First, the model does not include a sacrificial attachment plate and butt joint. Joints, being inherently difficult to model, only add complexity to a model whose purpose is for shock response prediction and not joint modeling. Additionally, the
lack of data across the joint precludes any correlation of the joint that could be done. The second simplification was to model the plate using free boundary conditions. As was previously stated, the boundary conditions in the test influence the low frequency portion of the response and will have minimal impact on the high frequency shock response due to the LSC. The third and final simplifying assumption is the damping schedule dependence. In general, damping is a function of amplitude, time, frequency, and space. The work here correlates the model response amplitude using damping, but the limitations of standard software preclude being able to capture all dependencies associated with damping.

In transient simulation using industry standard software, there are two solution types: a direct transient solution and a modal transient solution. These solution types each have a different way of implementing damping, however, both have limitations in their representation of reality. If a direct transient solution is used, damping is typically modeled using a structural approach. This works by specifying an equivalent viscous damping at a specified frequency generating a damping matrix that is proportional to the stiffness matrix. Direct integration of the system of equations then provides the selected damping. Because the damping is only specified at a single frequency, this often results in the higher frequency range being overly damped. The alternative modal transient solution applies damping as a function of frequency allowing for a more control of the system damping. The solution processes uses a sum of normal modes to estimate the total response as opposed to directly integrating for the solution like the direct transient solution. The damping associated with the shock transient response is amplitude, time, and frequency dependent in general. As a result, the standard damping simplifications often do not provide as desirable prediction capabilities as one might like. This is one of the primary difficulties in predicting shock response for structures.

Before tuning the model to better match the test data, a mesh density study must be performed. The goal in completing this to determine what element size is needed to achieve the desired fidelity output. In this case, the goal is to identify the cutoff frequency of the mesh, which will be below the Nyquist frequency defined by the sample rate of the test data.
This type of study can be performed using either of the methods discussed previously. When using a modal solution, the number of modes included in the solution determines the high frequency limit of the solution, and is independent of the time step used. If however, the time step used is not short enough in duration, higher order modes may not be visible in the plotted output. Here the time step used in the model development is consistent with the test data. The data was sampled at 1,000,000 samples a second, which defines the Nyquist frequency at 500,000 Hz, and sets the time step to $1.0 \times 10^{-6}$ s. In this case, the limit of the mesh fidelity will likely be well below that of the Nyquist frequency. Understanding this cutoff point is the goal in performing a mesh density study. Using the modal solution method, the model time history will converge to the same output once the frequency cutoff limit of the mesh density has been reached.

A direct transient solution mesh density study can be performed in a similar manner. No cutoff frequency variation is available here though. The direct solution format integrates the equations of motion and by doing so, attempts to capture all frequency content of the response. In general, a direct solution mesh density study is dependent on both the time step used and the mesh fidelity in the model. Here a sufficiently small time step is assumed in order to match the sample rate of the test data. This implies the mesh density sets the frequency limit of the solution. As a result, to identify the frequency limitations associated with a particular mesh density, looking at the spectral density of the model output will provide a clear cutoff point. If the highest frequency of the mesh density being considered is under what is desired, the mesh can be refined until the desired fidelity is achieved. As the mesh is refined, it is important to keep in mind the dimensional dependence of the mesh. That is refining one direction might provide quicker convergence than the other. As an example, the brick elements used as part of this work, required more refinement in one direction than the other to achieve convergence.

Fig. 6.4 shows the spectral density of channel A04N of test T3. From the figure it is clear that there is a significant drop in contribution to overall response above 55,000 Hz. As a result, the final mesh density, regardless of other modeling parameters, should be able
to characterize frequency content up to roughly 60,000 Hz to provide some margin. Both linear solid brick elements and plate (or shell) elements were initially used to determine if one better replicated the frequency response than the other. These two element types were chosen for two distinct reasons. The plate elements were chosen because their expedited solution speed, while the brick elements were chosen based on the applied load. It is well known that brick elements handle shear load well, and being that the load is essentially shear in nature, these were a natural choice for comparison sake.

In the mesh density study performed the convergence criteria required a converged time history and frequency content up to 60,000 Hz. All initial studies were performed for both types of elements using a direct solution approach with a triangular load shape\(^1\), and no damping. Table 6.2 shows the approximate frequency cutoff for each mesh density and element type modeled. Fig. 6.5 shows the time histories produced for each of the modeled

\(^1\)Load shape will be further evaluated shortly.
mesh densities. The plate elements had converged time histories and the desired frequency content when using a 0.5 in. by 0.5 in. density and the 1 in. thickness property. The solid elements produced much higher frequency content; however, it took several attempts to get an approximate time history convergence. A mesh density of 0.5 in. by 0.25 in. by 0.33 in. was needed to achieve the desired frequency content and a converged time history using the solid brick elements.

Table 6.2: **Mesh Density Frequency Content.** This table shows the approximate cutoff frequency for each element type and mesh density modeled.

<table>
<thead>
<tr>
<th>Element Type</th>
<th>X Mesh Width (in.)</th>
<th>Y Mesh Width (in.)</th>
<th>Z Mesh Width (in.)</th>
<th>Approximate Cutoff Frequency (Hz.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate</td>
<td>1</td>
<td>1</td>
<td>N/A - 1 in. Property Used</td>
<td>30,800</td>
</tr>
<tr>
<td>Plate</td>
<td>0.5</td>
<td>0.5</td>
<td>N/A - 1 in. Property Used</td>
<td>66,100</td>
</tr>
<tr>
<td>Solid Brick</td>
<td>1</td>
<td>1</td>
<td>0.33</td>
<td>185,000</td>
</tr>
<tr>
<td>Solid Brick</td>
<td>1</td>
<td>0.5</td>
<td>0.33</td>
<td>185,000</td>
</tr>
<tr>
<td>Solid Brick</td>
<td>1</td>
<td>0.25</td>
<td>0.33</td>
<td>170,000</td>
</tr>
<tr>
<td>Solid Brick</td>
<td>0.5</td>
<td>0.5</td>
<td>0.33</td>
<td>180,000</td>
</tr>
<tr>
<td>Solid Brick</td>
<td>0.5</td>
<td>0.25</td>
<td>0.25</td>
<td>220,000</td>
</tr>
</tbody>
</table>

Fig. 6.6 compares the time histories of the solid and plate elements to channel A04N of test T3 over the first three milliseconds of the response. Fig. 6.7 compares the spectral density of the solid and plate elements to the spectral density of the same data, over the same time period. The choice of three milliseconds over which to compare the responses was chosen for quick solution time coupled with a duration long enough to account for the split peak response characteristic seen in the test data. Neither the solid elements nor the plate elements respond significantly different when comparing the model time histories to the test data (Fig. 6.6).

Now consider the frequency domain. In using this short time period, it does limit the low frequency response that can be compared. Using the 0.003 s time period set the low frequency cut off at about 300 Hz. Although this limits the comparison range, shock events like this are typically much higher in frequency, so there isn’t much loss in this assumption. In looking at Fig. 6.7, the plate elements seem to better characterize the frequency response near 20,000 Hz than the solid elements. Additionally, the solid elements seem to respond with more energy in the higher frequency range (~80,000 Hz) than is desired.
(a) Peak Normalized Plate Element Mesh Density Convergence.

(b) Peak Normalized Solid Element Mesh Density Convergence.

Fig. 6.5: **Mesh Density Study.** Peak Normalized Model Time History for (a) Plate Elements (b) Solid Brick Elements. The responses shown in these figures are normalized using the peak model response.
From a run time standpoint, the plate element model for the chosen time segment and frequency content completed in roughly eight minutes, while the solid element model required nearly 12 times longer, or roughly an hour and forty minutes. The significant time differences are primarily due to the significant difference in the number of elements required convergence between the two models. Where the plate elements are capable of accounting for rotational components of the response with a single layer of elements, the solid elements require a minimum of three elements through the thickness in order to accurately characterize any rotational response. Combining the significantly longer run time and relative inaccuracy of the energy distribution in the frequency domain of the solid elements, it seems that plate elements are a stronger choice for use in further sensitivity studies and the final model correlation. As a result, unless otherwise explicitly stated, when a model is discussed,
Fig. 6.7: **Element Type FFT Comparison.** (a) Plate and (b) solid element spectral density at the A04N location are compared with test data spectral density to aid in determining which element is more appropriate for use. The amplitude was normalized relative to the peak model response in this case.

It is safe to assume the 0.5 in. by 0.5 in. plate element model was used in the analysis. Fig. 6.8 shows an image of this model for reference.
Fig. 6.8: 0.5 in. by 0.5 in. Plate Element Model.

Fig. 6.9: **Input Shape Sensitivity Study.** The three input shapes shown here were used to provide a better understanding of how the input shape affected the impulse response of the plate.

With element choice and mesh density concerns addressed, the next important aspect of any model is determining how to best represent the load. With the destructive nature of a pyroshock event, it is extremely difficult to capture the response resulting from the
detonation at or near the point of detonation. As a result, the applied load considered here was tuned based on qualitative aspects of the response prior to any tuning of the amplitude. The load shape was the first aspect considered. The goal in varying the load shape was to attempt to match the split peak response characteristic as best as possible. Three load shapes were considered; a triangular pulse, a unit step, and a half sine pulse. Fig. 6.9 shows the three input functions that were used to generate responses at the nodal location describing channel A04N in test. Before going further, these three input functions were not normalized based on the input energy like a more rigorous quantitative study would have done. The goal here is to qualitatively compare the general response characteristics of the model to the test data using the three different input shapes. Using the solid brick element model with a mesh density of 0.5 in. by 0.25 in. by 0.33 in. as an example, the responses of these three inputs are shown in Fig. 6.10. In comparing the general response characteristics of the three inputs it is immediately obvious that the step function misses important characteristics of the response near $t = 0.005$ s and can be rejected as an option. Focusing on the half sine and triangular inputs, it is difficult to see any significant difference in their responses compared to the test data, however, there is a slight phase difference near $t = 0.005$ s between the two input cases. The triangular input seems to better follow the test data in this region. This slightly better response, in conjunction with the ease of adjustment from an input deck stand point, lead to the adoption of the triangular input load shape as the basis for further model correlation and prediction efforts.
(a) Peak Normalized Time History: Test T3 - Channel A04N vs. Triangular Input.

(b) Peak Normalized Time History: Test T3 - Channel A04N vs. Half Sine Input.

(c) Peak Normalized Time History: Test T3 - Channel A04N vs. Step Input.

Fig. 6.10: **Mesh Density Study.** Peak normalized time history comparing test T3, channel A04N response to the solid brick element model response for a (a) triangular shaped load (b) half sine shaped load, and (c) step shaped load.
Initially, it was assumed the detonation velocity of the LSC was infinite. This assumption meant the load was modeled as a uniform shear load applied at the nodes along the edge of the plate in the location the butt joint was in test. In reality, the detonation of the RDX explosive used in the LSC is approximately 345,000 in/s [66]. In comparing the response of the model with and without the detonation velocity included, it was clear the detonation velocity needed to be accounted for. Fig. 6.11 shows the spectral density if the model with and without the detonation velocity included in the model. The model solution only includes the first three milliseconds of time, which was done to reduce run time, however, this plot clearly shows that higher frequency portions of the response, near $\sim 40,000$ Hz, are missed when the detonation velocity is not included in the model.

![Plate Element Shear to Propagating Load Comparison - Spectral Density](image)

Fig. 6.11: Detonation Velocity Inclusion Study. The three spectral density plots were normalized relative to the peak response of test T3 cable A04N. From this it is clear that the detonation velocity must be included in the model.

To model the load in NASTRAN, the TLOAD1 card was used. The benefit of using this
card lies in the incorporation of the detonation velocity. The TLOAD1 card has an optional
delay feature in its definition. This feature only requires knowing the time delay at the load
application site. To obtain this, the time required for detonation to propagate across one
element is determined by multiplying the element width and the detonation velocity. This
is then multiplied by the number of elements out to each successive nodal application site
giving the needed delay at a particular node. The delay is obviously a function of the mesh
density; an important reason for element type choice and obtaining response convergence
prior to any other model development. In the case of the 0.5 in. by 0.5 in. plate element
model, there was a $1.4 \times 10^{-6}$ s delay between each successive nodal load application.

![Figure 6.12: Peak Normalized Input Load.](image)

During the process of including the detonation velocity in the load definition, the model
was discovered to be sensitive to the load application duration. The model response seemed
to exhibit a lead-follow effect that varied based on the load duration. As an example,
consider the similar shape of the response in Fig. 6.5 to that of the triangular input shape
used in the model, shown in Fig. 6.12. Notice the sharp rise and fall in the response
that is similar to the load shape. A sensitivity study was performed using a 1 in. by 1 in. plate element model to reduce solution time, the results of which are shown in Fig. 6.13. The effect is worse for longer applied load duration. As the load duration decreased, the sharpness of the primary peak in the response increases. When the load duration is sufficiently small, the plate is unable to respond quickly enough to follow the load shape itself, producing a result more in line with the test data.

Fig. 6.13: **Load Duration Study.** The time histories here have been normalized by their peak values.

The last two aspects to consider in model development are the amplitude of the response and the model damping. Both are intimately tied together. Damping is energy loss, which inherently decreases amplitude. Until now, damping has been neglected in all modeling decisions to eliminate a parameter and qualitatively assess general response characteristics first. It is well known that typical damping values for metal structures range between 0.5% to 3%, depending on the metal type and preparation. In considering the test configuration,
it seems reasonable to assume the plate (and its restraints) are a lightly damped system, which implies using a value near the lower end of the typical range. Additionally, to provide better control over damping application, a modal transient solution is now used. The solution uses a 60,000 Hz cutoff frequency and initially assumed 1% damping across the spectrum. After numerous runs, the damping was tuned so to best represent the frequency content of the test data response using the harmonic wavelet transform and the spectral density. The resulting damping schedule is provided in Table 6.3.

Now that damping has been accounted for, the amplitude of the time history can be adjusted to match the test data. To do this, the peak response of the test data was ratioed with the peak response of the model so the model loading amplitude could be adjusted linearly by the appropriate percentage. Fig. 6.14 shows the final result of this adjustment process. The primary peak is well correlated with the test data, however, the secondary peak and the decaying ring down seem to miss higher frequency components of the response which likely contribute to the slight differences between test and model response amplitude over the later time ranges.

Table 6.3: Correlated Damping Schedule. This table shows the damping schedule used in the final correlated model. Between data points NASTRAN linearly interpolates to provide continuity for the solver.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Modal Damping (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>10,000.0</td>
<td>1.0</td>
</tr>
<tr>
<td>10,000.01</td>
<td>0.5</td>
</tr>
<tr>
<td>20,000.0</td>
<td>0.5</td>
</tr>
<tr>
<td>20,000.01</td>
<td>0.2</td>
</tr>
<tr>
<td>40,000.0</td>
<td>0.2</td>
</tr>
<tr>
<td>40,000.01</td>
<td>0.8</td>
</tr>
<tr>
<td>60,000.0</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Figs. 6.15 and 6.16 show the spectral density and harmonic wavelet transform respectively. These figures compare the model output to the test data using the correlated damping schedule. Though the model spectral density aligns well with the test data in both amplitude and spectral content, there are clear deficiencies in the wavelet transform. The model misses some of the higher frequency response near $t = 0.002$ s and seems to
Fig. 6.14: **Damped Time History Comparison and Correlation, t = 0 to 0.01 s.** The time history shown here has been normalized relative to the peak response of test T3 at channel A04N and shows the effect of the applied damping given in Table 6.3 on model response.

miss the amplitude spreading out in time beyond $t = 0.002$ s. The misalignment in the higher frequencies later in time $t > \sim 0.002$ is consistent with the misalignment with the time history in same time range. Additionally, the overall amplitude of the transform is lower than that of the test. This misalignment in the high frequencies, seen in both the time history and the harmonic wavelet, along with the missed amplitude of the wavelet transform might be the contribution by the frequencies beyond 60,000 Hz that are not captured in the model, but clearly present in the frequency response of the test data. Based on the wavelet transform of the test data, however, it is more likely from the limitations of modeling the damping. Using a modal solution, only the frequency dependence of the damping can be controlled. The solution method assumes an exponential decay for each individual modal contribution, when in reality, modal coupling and nonlinear response can cause deviations
from the assumed single degree of freedom damping model used in the solution process. Additionally, the initial transient seems to have more energy higher in frequency than the test data. Even with these deficiencies, the model generally aligns well with the test data.

Fig. 6.15: Damped Spectral Density Comparison and Correlation. The spectral density shown here has been normalized relative to the peak response of test T3 at channel A04N and shows the effect of the applied damping given in Table 6.3 on the spectral content of the model.
Fig. 6.16: **Damped Model Harmonic Wavelet Transform Comparison and Correlation.** The wavelet transforms shown here have been normalized relative to channel A04N of test T3 and show the effects of the applied damping on the system response over the time period $t = 0$ to $0.01$ s for (a) test T3 channel A04N and (b) the modal nodal response representing channel A04N’s location.
There is one final correlation step left to complete; a comparison between the normal modes of the plate determine as part of a modal survey on the test article and a normal modes run on the model can be performed. Ideally, a modal assurance criterion (MAC) would be used to compare the mode shapes of the model and test data, allowing for a decisive conclusion about frequency alignment. Unfortunately, however, the required data files were not available from the test to do this. The available test data did provide frequency response functions, however. Using these frequency response functions the primary response peaks were identified and compared with the model output assuming the mode shapes for corresponding mode numbers were similar. Table 6.4 summarizes the comparison between the empirically collected modal survey data and the normal modes model results generated using NASTRAN. The fifth predicted mode is low compared to the test data; however, the sixth predicted mode occurred at 103.54 Hz and better aligns with the empirically collected mode. Likely, the predicted fifth mode is a different mode shape than that of the empirical mode, however, a MAC comparison is needed to determine this hypothesis. Even with this slight discrepancy, the overall accuracy of the model modes prediction aligns well with the collected test data, providing confidence in the model.

Table 6.4: Normal Modes Correlation. This table compares the first five experimentally obtained normal modes to the normal mode model prediction.

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Experimental Normal Mode (Hz)</th>
<th>Model Prediction (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23.75</td>
<td>22.15</td>
</tr>
<tr>
<td>2</td>
<td>28.75</td>
<td>26.97</td>
</tr>
<tr>
<td>3</td>
<td>60.00</td>
<td>59.49</td>
</tr>
<tr>
<td>4</td>
<td>64.38</td>
<td>61.53</td>
</tr>
<tr>
<td>5</td>
<td>102.50</td>
<td>91.18*</td>
</tr>
</tbody>
</table>

To provide further confidence in the model, a comparison of the model response to test data at a different spatial location was considered. Using the same test, T3, channel A06N is compared with the equivalent nodal response from the model. Figs. 6.17 and 6.18 show the time history and harmonic wavelet transform comparisons. Both figures are normalized relative to the peak response of channel A04N to provide an understanding of the relative

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*Model predicts a 103.54 Hz mode that might better align with this particular frequency.
amplitude difference further from the source. Fig. 6.17 shows a slight difference in arrival time between the test and model responses. Additionally, the primary peak amplitude of the model misses the test response. The amplitude differences are likely due to the limitations associated with modeling the damping of the system. In general, however, the model response represents the test data reasonably well. Comparing the model to test wavelet transform in Fig. 6.18, there are similar issues as seen in Fig. 6.16. The model seems to miss the higher frequency portion of the response. Additionally, near $t = 0.002$ s, the test data shows higher amplitude contours compared to the model. Again, the model transform amplitude does not align well with that of the test data. In general, however, the model does reasonably well at predicting the response at a far field location that was not directly correlated to the test data. This provides additional confidence in the model and confidence that any additional predictions will provide reasonable results.

Fig. 6.17: Model Nodal Time History Response Compared to Test T3 Channel A06N, $t = 0$ to 0.01 s. The time histories shown here have been normalized relative to the peak response of test T3 at channel A04N and provide a point of comparison for the model response away from the correlation point.
Fig. 6.18: **Comparison of Model Nodal Output for A06N to Test.** The wavelet transforms shown here have been normalized relative to channel A04N of test T3 and provide a comparison for the model away from the correlation point. The transforms are shown both for (a) the test data and (b) the model output over the time period between $t = 0$ to $0.01$ s.
6.2.2 Model Extrapolation

The primary purpose of model correlation is to use the correlated model to extrapolate and predict response on a structure that may have slight configuration differences, or different load cases. Using the correlated 0.5 in. by 0.5 in. mesh density plate model, with a triangular pulse applied as a shear load having a duration of $5.0 \times 10^{-7}$ s on each applied node and accounting for the detonation velocity of the LSC, the load amplitude was scaled up by 12.5% and 412.5% to attempt to extrapolate and predict the response of the other collected load cases during the test series. The scaling method assumes the LSC charge density scales linearly with the impulsive output in the structure. Test T2 and T10 are taken as a basis for comparison to the 1.125 and 4.125 gr./ft. charge density LSC test sets, respectively. The test data and model response were normalized in a test consistent fashion, relative to the test data measured by channel A04N. The time histories, spectral densities, and harmonic wavelet transform of the different cases are compared and commented on.

Extrapolation to 1.125 gr./ft. Charge Density LSC Input - Test T2 Comparison

Figs. 6.19, 6.20, and 6.21 show the time history, spectral density, and harmonic wavelet comparisons between the test T2 channel A04N and a model whose load has been linearly scaled for charge density, respectively. Comparing the time history between test and model in Fig. 6.19, the peak amplitudes align well with one another. The model slightly over predicts the max response, but the overall model time history represents the test data well. Similarly, the spectral density, shown in Fig. 6.20, agrees well with the test data. In general, the amplitude of the model spectral density aligns well with the test data. Additionally, within the bounds of the 60,000 Hz limit of the model, the predicted spectral content is similar to that of the test data. Comparing the wavelet transforms of the test data and model data in Fig. 6.21 shows the model transform under predicts the amplitude and spreads the energy out in lower frequency bands that are not present in the test data. The agreement here could be better, however, it does seem to capture characteristics associated with the split peak response characteristic of the test data well.

Overall there are amplitude alignment concerns between test and model prediction,
however, the time history response and spectral density content align well between test and model. The wavelet transform comparison highlights some of the shortcomings of the model, but the model does reasonably well in predicting the structural response for a load case that is similar in magnitude to the correlated model.

Fig. 6.19: Model Nodal Time History Response Compared to Test T2 Channel A04N, $t = 0$ to 0.01 s. The time histories shown here have been normalized relative to the peak response of test T2 at channel A04N and provide a point of comparison for the model response when extrapolating for a load case similar to the correlation load.
Fig. 6.20: Model Nodal Spectral Density Compared to Test T2 Channel A04N

The spectral densities shown here have been normalized relative to the peak response of test T2 at channel A04N and provide a point of comparison for the model response when extrapolating for a load case similar to the correlation load.
Fig. 6.21: **Comparison of Model Nodal Output for A04N to Test.** The wavelet transforms shown here have been normalized relative to channel A04N of test T2 and provide a comparison for the model when extrapolating for a load case similar to the correlation load. The transforms are shown both for (a) the test data and (b) the model output over the time period between $t = 0$ to 0.01 s.
Extrapolation to 4.125 gr./ft. Charge Density LSC Input - Test T10 Comparison

Comparing the time history between test and model in Fig. 6.19, the peak amplitudes align well with one another. The model slightly over predicts the max response, but the overall model time history represents the test data well. Similarly, the spectral density, shown in Fig. 6.20, agrees well with the test data. In general, the amplitude of the model spectral density aligns well with the test data. Additionally, within the bounds of the 60,000 Hz limit of the model, the predicted spectral content is similar to that of the test data. Comparing the wavelet transforms of the test data and model data in Fig. 6.21 shows the model transform under predicts the amplitude and spreads the energy out in lower frequency bands that are not present in the test data. The agreement here could be better, however, it does seem to capture characteristics associated with the split peak response characteristic of the test data well.

Overall there are amplitude alignment concerns between test and model prediction, however, the time history response and spectral density content align well between test and model. The wavelet transform comparison highlights some of the short comings of the model, but the model does reasonably well in predicting the structural response for a load case that is similar in magnitude to the correlated model.

Figs. 6.22, 6.23, and 6.24 show the time history, spectral density, and harmonic wavelet comparisons between the test T10 channel A04N and a model whose load has been linearly scaled for charge density, respectively. Comparing the time history between test and model in Fig. 6.22, the peak amplitude of the model prediction does not align well with the test data. From this, it is clear there is not a linear relationship between the charge density and impulsive output of the LSC. The model prediction does capture the general response characteristics of the test data well, outside the amplitude alignment issues. A similar result is present in Fig. 6.23. Here the frequency response matches reasonably well with the test data, with the exception of the amplitude. The higher frequencies, >∼ 35,000 Hz, show signs of a difference in the damping between test and model prediction, however, the general shape of the spectral density is similar to the test data. The suspected damping
differences between test and model are further exhibited in the wavelet transforms. Here the amplitudes do not align well, and the wavelet transform of the model response shows far more low frequency response than the test data.

Using a linear scaling methodology to define the mode load condition does not result in reliable response prediction for load cases significantly different than those of the correlated model. In general, the model captures many of the qualitative response features of the test data. If the input loads are more appropriately scaled for larger charge density differences of the LSC used, the model will likely perform better than has been demonstrated here. A model’s reliability is often restricted to the reliability of its inputs.

Fig. 6.22: Model Nodal Time History Response Compared to Test T10 Channel A04N, $t = 0$ to 0.01 s. The time histories shown here have been normalized relative to the peak response of test T10 at channel A04N and provide a point of comparison for the model response when extrapolating for a load case significantly different than the correlation load.
Fig. 6.23: Model Nodal Spectral Density Compared to Test T10 Channel A04N

The spectral densities shown here have been normalized relative to the peak response of test T10 at channel A04N and provide a point of comparison for the model response when extrapolating for a load case significantly different than the correlation load.
Fig. 6.24: **Comparison of Model Nodal Output for A04N to Test.** The wavelet transforms shown here have been normalized relative to channel A04N of test T10 and provide a comparison for the model when extrapolating for a load case significantly different than the correlation load. The transforms are shown both for (a) the test data and (b) the model output over the time period between $t = 0$ to 0.01 s.
6.2.3 The Split Peak Phenomena

Fig. 6.3 shows an example of the split peak phenomena. Throughout model development different aspects of the test setup were simulated and adjusted in order to obtain a correlated model. In the end, the model accurately predicted the same response characteristic. As part of the model development process, two noticeable sensitivities affecting the split peak response characteristic were identified. First, when the detonation velocity of the LSC was incorporated the high frequency response of the model more accurately matched the test data. Fig. 6.11 illustrates this. Second, when damping was introduced, a drastic decrease in the amplitude of the secondary peak in the model response could be seen. Fig. 6.14 shows this. Based on the behavior of the model, and considering the parameters that affect the split peak response characteristic the greatest, no definitive conclusions could be drawn about the response characteristics source. As a result, a deeper look at the test data was required to better understand the origin of the unexpected response.

The harmonic wavelet transform was used to decompose the response measured by channel A04N of test T3, and is shown in Fig. 6.16(a). Here distinct high amplitude response bands can be seen at specific periods of time. Furthermore, these bands align with the timing of the primary and secondary peaks occurring in the time history response (Fig. 6.14). Initially one might expect the secondary peak to be a reflection of the traveling wave off the opposite end of the plate. For channel A04N, the wave must travel the 18 in. to the accelerometer, down another 78 in. to the end of the plate and then the 78 in. back to the accelerometer location. In total the wave travels roughly 174 in. The speed at which the wave propagates can be estimated using the speed of sound in aluminum. Using the square root of the ratio of the elastic modulus to the density can be used to provide a theoretical estimate. Using a modulus of $1.0 \times 10^7$ psi and a density of $2.52 \times 10^{-4}$ lbf-s$^2$/in.$^4$ the theoretical wave speed is roughly 199,000 in./s. This gives an estimated arrival time for the secondary peak relative to the primary peak of approximately $\Delta t = 0.000874$ s. Comparing this to the approximate arrival time shown in Fig. 6.25, it’s clear that the arrival of the peak occurs after the predicted estimate by roughly one-half millisecond.
In considering the harmonic wavelet transform, the pockets of energy at successive periods in time, with each successive pocket spreading out over a longer period of time and having a reduction in amplitude suggests the secondary peak is indeed a reflection of the initial traveling wave. Waves propagating in dispersive media experience two related mechanisms of energy transfer: dissipation and dispersion. Dissipation is the conversion of energy into heat and is equivalent to damping. Dispersion is the mechanism that causes the wave to spread in time, and unlike dissipation conserves energy. Physically this is caused by a difference in phase velocity and group velocity of the wave packet and stems from the fact that different frequency waves travel at different rates. A wave packet refers to the superposition of all the individual frequency contributions and is where the propagation starts once the LSC is detonated. In order to better understand this, consider a unidirectional harmonic solution to the wave equation: $Ae^{i(kx-\omega t)}$, where $A$ is an arbitrary coefficient, $k$ is
the wave vector and is related to the wavelength, and \( \omega \) is the angular frequency \( (\omega = 2\pi f) \) of the traveling wave. Based on this wave form, it is clear that the propagation speed of the wave is dependent on \( k \) and \( \omega \) in general. When the phase between these two pieces is constant, \( kx - \omega t = \text{const.} \), there is no phase difference between the spatial and temporal components of the wave; hence the name ‘phase velocity’. If however, \( \omega \) depends on \( k \), the group velocity arises. The group velocity describes the speed of the wave packet as a function of frequency. The wave velocity dependence on frequency is what causes the wave packet to spread in time \([68]\). This spreading effect can be seen in Fig. 6.16(a) at the secondary and each successive pocket of energy. In comparing the test data with the model output, it’s clear that the misalignments mentioned in the correlation portion of this chapter are likely due to nonlinearities associate the dispersion and damping, which manifest themselves by the elimination of the reflected waves earlier in time in the model wavelet transform when compared to the test data.

Based on the evidence from the wavelet transform, the secondary peak seems to be the reflection of the traveling wave, however, the theoretical arrival time doesn’t align well with the test data. The misalignment in time could be due to an imperfect dispersion relationship in which there are actual losses in reality, however, it’s difficult to be believe this would cause such a significant difference in arrival time. A closer look at the system, however, yields a pointed fact; the applied load is essentially a shear load. This indicates that the wave might be a shear wave in nature and the propagation speed might be better predicted using a shear modulus as opposed to the elastic modulus. The shear modulus for aluminum is roughly \( 3.77\times10^6 \) psi. When accounting for this, the rough propagation speed of the wave is 122,000 in./s, resulting in a predicted relative arrival time of \( \Delta t = 0.00142 \) s after the primary peak. Comparing this to the test data, it aligns well with the arrival time of the secondary peak. Based on this, the secondary peak is in fact a reflection of the traveling wave off the plate boundaries, but it is not an in plane wave like initially thought.

More interestingly, however, is the amplitude of the secondary peak. Fig. 6.25 shows that the secondary peak exhibits roughly a 40% reduction compared to the primary peak.
in the time history. This is in direct conflict with the Martin Marietta standard, which suggests that beyond 110 inches the amplitude of the traveling wave should be lower than 10% of its original peak. Here, however, the wave has traveled approximately 18 inches to the location of channel A04N, another approximately 78 inches to the edge of the plate, reflected, and then traveled another 78 inches back to the measurement location and still has an amplitude of roughly 60% of its peak. This further highlights the need to reassess the Martin Marietta distance attenuation standard.

The conclusions here suggest a similar response may be present in the full scale cylinder data. The configuration differences between the two tests, however, may make it difficult to identify. Additionally, intervening structures of the test article will surely cause more reduction and change the path of the traveling waves. Even with these differences, there might still be evidence of this event in the data. Figs. 6.26 and 6.27 show the time history and harmonic wavelet transform for both sets of full scale test data for channels A001A and A021A between \( t = 0 \) and \( t = 0.01 \) s respectively. In comparing the wavelet transform of the two tests, there is a significant difference in the measurement nearest the source. The FSE to forward skirt test exhibits evidence of the secondary peak near \( t = 0.003 \) s, while the frustum to FSE test data provides little evidence of this feature. The differences highlighted here are likely due to the internal structure in the test article and its relative position to the detonation plane. The frustum to FSE test positioned the detonation plane more near the internal structure of the test article than the FSE to forward skirt test. In comparing the flat plate and full scale test data responses, the relative prominence of this effect between the two configurations is quite interesting. It provides an ideal example of how a change in geometry can drastically affect the structural response and corresponding wave propagation of shock events.
(a) FSE to Forward Skirt Test, Channel A001A.

(b) Frustum to FSE Test, Channel A021A.

Fig. 6.26: Full Scale Separation Test Data Investigation - Time History. The images show the response time histories for measurements on the (a) FSE to forward skirt test and (b) frustum to FSE test.
Fig. 6.27: **Full Scale Separation Test Data Investigation - Harmonic Wavelet Transform.** The images show the response harmonic wavelet transform for measurements on the (a) FSE to forward skirt test and (b) frustum to FSE test.
6.3 Attenuation with Distance

Following the work completed in chapter 5, the flat plate data was evaluated to characterize attenuation with distance. This test data provides two distinct differences outside the obvious configuration change. First, only the normal response direction is measured at the accelerometer locations. Having the full tri-axial response at each location would be better; however, this is all that is available. Second, is the availability of 24 individual tests that can be grouped into three different sets based on the charge density of the linear shaped charge used in the individual test. This offers the opportunity to identify any trends that might be present and provides an opportunity to distinguish out of family behavior.

For the flat plate configuration, channels A04N, A06N, A08N, and A10N are of most interest. They present the locations that capture the initial pulse at locations that will be the least influenced by reflections of the wave off the edges of the plate. Just as in the case of the full scale data, the distance from the source was standardized to align with the Martin Marietta standard for easier comparison, and the peak response was normalized. Peak normalization was done in a test consistent manner relative to the measurement nearest the initial detonation source, or channel A04N. Table 6.1 summarizes the distance standardization to the source. Relative peak differences are not provided in this case, because all measurements are normal to the plate and the relative differences are inherent in the attenuation with distance comparison.

Here again the Martin Marietta standard for a cylindrical shell is used as a basis for comparison. Though the historical report does not provide a standard for a flat plate, the cylindrical shell provides a reasonable configuration for comparison because of its lack of intervening structures. Many of the other configurations in the documentation involve bolted joints or more complex structures. Here again, the peak SRS response and approximate energy methods were used in the comparison to the standard. The time period used for both comparison methods was $t = 0$ to 0.001 s, and was long enough to capture the primary peak only. Keeping only a shortened period of time attempts to isolate the traveling shock wave from the modal response developed as a result of the excitation. Furthermore,
to be consistent with the Martin Marietta standard, the frequency band between 10 and 10,000 Hz is used in the SRS development, even though spectral analysis of this data shows frequency response well above the 10,000 Hz cutoff.

Figs. 6.28 to 6.30 show the attenuation with distance using the peak SRS method in a charge density consistent format. Similarly, Figs. 6.31 to 6.33 show the attenuation with distance using the energy method. In general, both methods behave as one would expect; the larger the separation from the source, the lower the amplitude response. There are a few outliers indicating an increase in energy or peak response, which is likely due to any number of measurement or data collection issues. In comparing with the cylindrical Martin Marietta standard, the 1.0 and 1.125 gr./ft. test sets seem to align better with the standard than the 4.125 gr./ft. test set, for both methods. In general, the data do not align as well as one might like. This disagreement could be in part to the configuration differences between the plate and cylindrical shell used in the standard in addition to the circular load application on the cylindrical shell. The full scale cylinder LSC applied a shear load which followed the circumference of the cut plane, giving the load an angular component; where the load case applied to the flat plate was essentially completely shearing in nature. The investigation into the split peak response characteristic tends to support this claim. In looking for a similar response characteristic in the full scale test data there were significant response differences between the configuration differences.

In looking at the three subdivided data sets from the flat plate test series, there are three primary conclusions that can be made. First, the 1.0 and 1.125 gr./ft. test sets seem to align better with the standard than the 4.125 gr./ft. test set, however, it is difficult to make any definitive conclusions based on the variation in the results. Second, although only normal measurements were available in this configuration, in determining the source of the split peak characteristic of the response, it provides confidence that only considering the normal response direction is appropriate in the case of the flat plate. Lastly, the disagreement present in comparing the test data to the Martin Marietta standard suggests further testing and analysis should be done to either validate or update the current standard. Furthermore,
the test data suggests the historical Martin Marietta standard is not a conservative estimate of the attenuation with distance of shock waves in structures.

Fig. 6.28: Attenuation With Distance, SRS Method, LSC Charge Density: 1.0 gr./ft. \((Q = 50)\).
Fig. 6.29: Attenuation With Distance, SRS Method, LSC Charge Density: 1.125 gr./ft. (Q = 50).
Fig. 6.30: Attenuation With Distance, SRS Method, LSC Charge Density: 4.125 gr./ft. ($Q = 50$).
Fig. 6.31: Attenuation With Distance, Energy Method, LSC Charge Density: 1.0 gr./ft.,
Fig. 6.32: Attenuation With Distance, Energy Method, LSC Charge Density: 1.125 gr./ft.
Fig. 6.33: Attenuation With Distance, Energy Method, LSC Charge Density: 4.125 gr./ft.
6.4 Modeling Best Practices and Conclusions

Modeling shock propagation is inherently difficult. The time and frequency dependencies associated with the damping and wave propagation characteristics are aspects of a model that must often just be accepted as a best effort. There are however, some best practices that can be gleaned from this work. First, plate elements provide adequate response, even to an essentially shear load, and solve much faster than solid brick elements when modeling transient events. Second, when there is a lack of data for model correlation, the damping schedule provided in Table 6.3 can be used as an assumed damping profile. Though there are certainly instances where geometry affects damping and wave propagation, this provides a reasonable set of values over a broad frequency range. Moreover, the damping correlation effort here suggests higher frequency components of the response, are better modeled with less damping. Lastly, the harmonic wavelet transform provides better insight about the model response than just a time history and spectral density plot. It allows for investigation and potentially correlation via comparing both time and frequency content of a signal simultaneously.

The model correlation effort provided a model that was used to extrapolate to two other cases in which a higher charge density LSC was used as part of the flat plate test series. Using the correlated model as a basis, and scaling for the increased charge density of the LSC, model predictions were compared with available test data to aid in trying to better understand the predictability of pyroshock response. The results made it clear that a nonlinear relationship between the charge density in an LSC and the impulsive load it provides exists. Better understanding this relationship would likely better align the model predicted response with available test data, and would provide better reliability for future modeling efforts. However, understating the relationship between the charge density of an LSC and the impulse load it provides is out of scope for this work; as such the task is taken as forward work. In general, the model predictions did reasonably well in representing the two additional load cases. There was evidence that the assumed damping schedule (defined in the correlated model) does not capture all the subtleties associated with this type of
scenario, however, the damping schedule did provide reasonable results in all modeled cases, providing confidence to its use in general practice when there is a lack of better data.

The split peak phenomenon was determined to reflection of the shear wave off the plate boundaries. In general, the loss in overall amplitude of the secondary peak in the test data is due to both dispersion and dissipation. Dispersion causes a spread in the time domain stemming from a traveling waves’ velocity frequency dependence, while dissipation converts energy into heat and is equivalent to damping. When investigating the full scale data for similar response characteristics, evidence was found in the FSE to forward skirt test, but not in the frustum to FSE test. This difference between the two full scale tests is contributed to the relative location differences of the internal structure to the separation plane of the shell.

In looking at attenuation with distance for the flat plate configuration, both the peak SRS and approximate energy methods produce similar results. In general, the 1.0 and 1.125 gr./ft. test sets seem to align better with the Martin Marietta standard than the 4.125 gr./ft. test set. Though some individual tests were agreed better with the standard than others, in general the data did not align well with the standard. Furthermore, the data suggested the historical standard was not a conservative estimate of the attenuation with distance for shock events. Thus further testing should be conducted to validate or update the current standards.
CHAPTER 7

Conclusions

Component requirements govern design and production in the aerospace industry. In order to show compliance to requirements components are qualified. Qualification is the process of demonstrating positive margins for different loading conditions either through analysis or test. Self-induced random vibration and shock are two primary environments that induce load on components. Scaling methodologies and mass loading techniques can be used to predict random vibration environments, however, shock environments are much more difficult to predict. Modeling and extrapolation methods do exist for shock prediction; however, they often produce unrealistic results when generating levels for new designs [1]. Because of the deficiencies associated with these techniques, shock environments are often defined by scaling structural response from several similar structures and enveloping these predictions. This leads to conservative environments which increases cost by adding additional risk reduction tasks prior to qualification testing; the standard for shock qualification.

With the added conservatisms of the enveloping process, understanding the attenuation characteristics associated with the traveling shock wave over distance and through joints becomes an important way to reduce these environments. Current industry standards are based on data collected nearly 50 years ago on data acquisition systems that are technologically obsolete and based on the SRS - an analysis tool that has major limitations. With the improvement of today’s technology, verification of these standard knock down factors seems a valid concern, however, any validation efforts done by industry professionals are likely not wide spread due to proprietary concerns.

The test data required for this type of standards validation or model development effort is difficult to come by, but sources do exist. Two tests governed by two separate tests plans were made available for assessment in this work. Both data sets were collected as part of the Ares I-X program by Orbital ATK and were used to aid in shock environment
development for the launch vehicle. The first set of test data was collected on a full scale, flight-like, cylindrical structure and provided two sets of response data. The second set of data was collected as part of an LSC severance capability investigation. This test provided 24 individual sets of data that could be grouped together into three functional groups based on the charge density of the LSC used in each test.

The available test data was used to aid in satisfying the primary goals of this work, which are: First, investigate the attenuation with distance characteristics associated with a cylindrical shell using the same process the historical standard was defined with, and by using an alternative approximate energy approach based on the signal energy of the response. Second, investigate the attenuation with distance characteristics associated with the flat plate configuration using the same processing methods as the cylindrical shell. Third, build and correlate a model based on the flat plate configuration, which can be used to predict response for different load cases. Forth, as part of model generation and correlation, identify any best practices associated with shock response prediction using industry standard software. Lastly, using the model and other processing tools attempt to better understand the unexpected split peak response identified during preliminary review of the flat plate data. An example of this split peak response is shown in Fig. 6.3.

As mentioned, both sets of full scale test data were analyzed using two different processing techniques. As part of the investigation, different time periods were used to try and isolate the traveling wave response from the developing modal response of the structure. In the end, the peak SRS approach was found to produce similar results regardless of the time period kept, which is in line with the assumptions of how the processing method is defined. The approximate energy approach highlighted the need for a method to better isolate the modal standing contribution from the transient traveling wave contribution to the structural response. Using shorter time segments when processing the data with the approximate energy approach provided results that were more in line with the Martin Marietta standard.

Comparing the two sets of data available to the Martin Marietta standard, there were clear differences between the two configurations tested. The FSE to forward skirt test
initiated the separation further away from an internal structure designed to carry the loads associated with parachute deployment as part of reentry, while the FSE to frustum test initiated the separation more near this structure. The differences were contributed to the near field structural differences altering the propagation path of the traveling wave. In either case, the peak SRS and approximately energy method produced similar trends, for which the results do not lend themselves to one particular method being better than the other. The benefit of the approximate energy method, however, is that it requires no assumptions about the damping of the system to calculate. The results using both processing methods were generally inconclusive, which could be related to the internal structure of the MPSS. This, however, points to the need to conduct further testing in an effort to validate or update the historical Martin Marietta standard.

The flat plate test series was evaluated in a similar fashion, however, only normal response measurements were available in this case. The test series was broken into three subsets each having a unique charge density LSC that was used as the severance charge. Both the peak SRS and approximate energy methods were used to assess attenuation with distance. In general, the 1.0 and 1.125 gr./ft. test sets seem to align better with the Martin Marietta standard than the 4.125 gr./ft. test set, however, none of the test data aligned well with the standard using either processing method. Based on the relative disagreement between the test data and standard, further testing should be undertaken to either validate or update the current standards. The test data analyzed in this case suggests the historical Martin Marietta standard is not a conservative estimate of the attenuation with distance characteristics of similar shock events.

Using the same flat plate configuration, the model development and correlation effort started by first investigating the effects of the mesh density on the model. The time step was assumed to be small enough that the mesh density of the model was the driving factor in the frequency content produced in the output. The mesh density study was performed for both solid brick elements and plate elements, and based on solution time and minor frequency content differences, the plate elements both solve quicker, and better represent
the available test data. The forcing function was the next item to consider. Here the shape, duration, and detonation velocity were investigated to determine response sensitivities in the model. Model response was extremely sensitive to the load application duration, and mildly sensitive to the forcing shape. In addition, including the detonation velocity better accounted for the high frequency response of the test data. In general, it is good practice to include the detonation velocity in modeling shock response, as well as to use as short of application load duration as feasible, and either a half sine pulse, or triangular shaped input. Lastly, the damping and amplitude of the model was tuned based on the harmonic wavelet transform and the peak time history response. Though the amplitude of the wavelet transform of the model output and test data did not align well and the transform of the test data indicated underlying pieces of physics related to damping were missing, the model proved to be well correlated.

Using the correlated model with a linearly scaled input, the model was used to predict the response of the same plate under higher loads due to charge density increases in the input LSC. In the case that the scaled input was similar in magnitude to the correlated load, the model performed well in predicting the structural response when compared with the available test data. In the case the load was scaled by a significant amount, the model over predicted the response amplitudes, but did provide a reasonable estimate of the frequency content and the spectral density. For both extrapolated cases, the damping schedule developed in the model correlation did seem to provide decent results in appropriately weighting the spectral content of other similar tests. Just as in the case with the correlated model, the wavelet transform of the model data compared to test in both extrapolation cases saw misalignment of the transform amplitudes. Furthermore, the transform demonstrated similar physics related to damping that was missed by the extrapolated models. Even with these deficiencies, the extrapolated models did reasonably well at predicting the spectral content of both load cases along with the general characteristics of the time histories. These results suggest that the defined damping schedule in Table 6.3 can be used as a baseline assumption for work when no data is available.
Lastly, the split peak phenomenon was determined to be reflection of the shear wave off the edge of the plate. In general, the loss in overall amplitude of the secondary peak in the test data is due to both dispersion and dissipation. Dispersion causes a spread in the time domain stemming from the traveling wave’s velocity dependence on frequency, while dissipation converts energy into heat and is equivalent to damping. When investigating the full scale data for similar response characteristics, evidence could be found in the FSE to forward skirt test, but not in the frustum to FSE test. The difference between the two full scale tests is contributed to the relative location of the internal intervening structure to the separation plane.

Three general conclusions can be drawn from the contributions put forth here. First, based on the available data, the Martin Marietta attenuation with distance standard should be further investigated. Both the full scale separation test data and series of flat plate tests suggest that the Martin Marietta distance attenuation standard is not a conservative estimate of the attenuation with distance for these types of pyrotechnic excitations. This points to the need for further testing to update the standard’s attenuation with distance scale factors, and points to the need to investigate the other defined scale factors for bolted joints and intervening structure. Second, several modeling best practices have been set forth. When modeling the structural response to shock events of this nature, it is important to use as short of a load duration as feasible, incorporate the detonation velocity of the explosive input when appropriate, and use a forcing function with a finite rise and fall rate (like that of a half sine or triangular shaped pulse). Furthermore, plate (or shell) elements provide good response prediction and solve significantly faster than solid elements. Lastly, the harmonic wavelet transform provides a better tool for shock characterization, model correlation, and data investigation than the SRS. It’s use in model correlation here provides an example of the transforms usefulness compared to the SRS. The wavelet transform allowed for a better understanding of the fundamental damping characteristics of the test data, highlighting the need for a finite element scheme that incorporates both time and frequency dependence in damping definition at a minimum.
Throughout the course of this work, several additional points of interest came to light. First, based on the understanding that the split peak characteristic of the response in the flat plate data is wave reflection, it suggests a similar response characteristic should be present in the response of a truly cylindrical shell. Using the available flat plate data and with additional testing on a truly cylindrical shell, it would be interesting to establish a method that isolates the primary peak from the secondary peak and considers the energy loss relative to one another. This approach would provide not only a better definition for the attenuation with distance, but also might provide insight about the damping of the system. Further testing should simulate a free boundary condition case by supporting a cylindrical shell via cabling and should include tri-axial acceleration measurements at five to ten inch increments away from the source separation plane; with the final measurement being at least 110 inches from the separation plane. By configuring the test in this fashion it allows for a more well-defined attenuation with distance curve for comparison with and update of the standard. Second, though the correlated model performed reasonably well in predicting response to different load cases, it would be beneficial to have a solution method which could account for amplitude, time, and frequency dependence of damping in transient analysis. As such, it would be interesting to try and better understand these dependencies analytically, in order to facilitate the development of a finite element approach that attempts to incorporate all the dependencies of damping. This study would start by considering a simple geometry such as the plate, and develop the full equations of motion using shell theory. The theory would contain several terms responsible for damping. These include a viscous damping term to simulate modal damping of the structure, a drag term to account for the atmospheric drag on the structure from the air, a term describing the transfer of energy from the propagating wave into heat, and a term attempting to account for the energy loss at the boundary conditions, in the case of the flat plate, the cable attachment points would be treated as energy sinks and modeled as discrete attachments to the primary plate. By including these additional terms, it allows to a deeper understand of the relative importance between the different damping mechanisms and potentially allows for an
effective damping definition which simplifies the contribution of all sources to a single term. Lastly, based on the poor amplitude alignment when comparing the extrapolated models to available test data, having a better scaling method to better account for input differences corresponding with charge density in LSC would provide better prediction capability for future modeling efforts. This final effort would use the available flat plate test series as a basis, but would require additional testing with different charge density LSCs. The additional testing should have at a minimum ten repetitions per charge density. Additionally, the testing should vary only the charge density of the LSC, items such as standoff distance should not be varied. The additional testing would provide more data points to develop a better understanding of the nonlinear relationship between the charge density and peak amplitude recorded on the structure and would allow for a more complete scaling practice to be developed. Having multiple tests within each charge density subset of the test would allow for better characterization of uncertainty in the test series, and it would allow for the use of normal tolerance techniques to define a statistically predicted peak response that could be used to help bound the final recommendations on scaling.
Bibliography


APPENDICES
Appendix A

Continuous - Discrete Comparison: Circular Scaling and Wavelet Functions

Again following Newland in \[23\], comparing the continuous and discrete wavelet transform assumes the discrete transform covers a unit interval of \( t \), and that the function being analyzed is periodic in \( t \) with a period of unity. When a continuous function overlaps the interval, they are considered to be wrapped around and to reappear at the opposite side of the interval. Each continuous function that overlaps the interval sides must be replaced by a corresponding circular continuous function. In the case of the scale function, \( \phi(t) \), the circular scaling function, \( \phi_c(t) \), is defined as

\[
\phi_c(t) = \sum_{k=-\infty}^{\infty} \phi(t - k). \tag{A.1}
\]

Substituting \( \phi(t - k) \) with its Fourier transform we obtain

\[
\phi_c(t) = \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi(\omega)e^{i\omega(t-k)} d\omega. \tag{A.2}
\]

Then according to Poisson's summation formula\(^1\) \[69\],

\[
\sum_{k=-\infty}^{\infty} e^{-i\omega k} = 2\pi \sum_{m=-\infty}^{\infty} \delta(\omega - 2\pi m). \tag{A.3}
\]

Since the scaling function is defined as zero outside the frequency band \([0, 2\pi]\), only the \( m = 0 \) term must be retained in Eqn. A.3, giving

\[
\phi_c(t) = 2\pi \int_{-\infty}^{\infty} \Phi(\omega)e^{i\omega(t)} \delta(\omega) d\omega, \tag{A.4}
\]

\(^1\)Poisson's summation formula relates the Fourier series coefficients of a periodic summation of a function to values of the function's continuous Fourier transform.
which, with the definition of $\Phi(\omega)$ in Eqn. 4.18, gives

$$\phi_c(t) = 1.$$  \hspace{1cm} (A.5)

When $\phi(t - k)$ is replaced by its complex conjugate in Eqn. A.1, the same argument can be used, yielding the same result.

In the discrete case, the expansion of a circular function, $f_c(t)$, by the discrete transform has only one term derived from the contribution of the scaling function. From Eqn. 4.22 the term can be seen to be $a_0 = a_\phi \phi_c(t) + \bar{a}_\phi \bar{\phi}_c(t)$, which, from A.5 is $a_0 = a_\phi + \bar{a}_\phi$, a single constant. In the case of a general wavelet, $w(2^j t - k)$, its circular equivalent is

$$w_{c,j}(2^j t - k) = \sum_{m=-\infty}^{\infty} w(2^j(t-m)-k)
= \sum_{m=-\infty}^{\infty} 2^{-j} \int_{-\infty}^{\infty} e^{-i\omega k} e^{i\omega(t-m)} W(2^{-j}\omega) \, d\omega,$$  \hspace{1cm} (A.6)

for $k = 0, 1, 2, \cdots, 2^j - 1$

where $W(2^{-j}\omega)$ is of the same functional form as in Eqn. 4.18. Since $W(2^{-j}\omega) = \frac{1}{2\pi}$ over the domain $2\pi 2^j \leq \omega < 2\pi 2^{j+1}$, and zero elsewhere, the only terms required to be considered are $m = 2^j$ to $2^{j+1} - 1$. Substituting Eqn. A.3 into Eqn. A.6 and integrating gives

$$w_{c,j}(2^j t - k) = 2^{-j} \sum_{m=2^j}^{2^{j+1}-1} e^{i2\pi m(t-\frac{k}{2^j})},$$  \hspace{1cm} (A.7)

for $k = 0, 1, 2, \cdots, 2^j - 1$.

When $j = 0$ in Eqn. A.7 and only one wavelet per unit interval is kept we have

$$w_{c,0}(t) = e^{i2\pi t}.$$  \hspace{1cm} (A.8)

Similarly, the two circular wavelets at level 1 are given by

$$w_{c,1}(2t - k) = \frac{1}{2} \left[ e^{i2\pi(2t-k)} + e^{i6\pi(t-\frac{k}{2})} \right],$$  \hspace{1cm} (A.9)
for $k = 0,1$ and so on for higher levels. Based on the form of Eqn. A.7, it is clear that the circular wavelet of level $j$ has $2^j$ discrete harmonics whose frequencies are $2\pi 2^j, 2\pi (2^j + 1), 2\pi (2^j + 2), \cdots, 2\pi (2^{j+1} - 1)$. 
Appendix B

Proof of Parseval’s Theorem for the Generalized Harmonic Wavelet Transform

Prove Eqn. 4.28:

\[ \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} 2^{-j} \left( |a_{j,k}|^2 + |\tilde{a}_{j,k}|^2 \right) = \int_{-\infty}^{\infty} |f(t)|^2 dt \]  

(B.1)

Proof. Following Newland [23], we start by defining some useful quantities. Let \( W(\omega) \) and \( V(\omega) \) be the Fourier transform of Eqns. 4.9 and 4.12 respectively. Next, we make a coordinate transformation so \( V(\omega) \) can be written in a slightly different way, let \( z = 2^j t - k \), so that, \( dz = 2^j dt \), this gives

\[
V(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} w(z) e^{-i\omega (2^{-j} z - k) 2^{-j}} 2^{-j} dz
= 2^{-j} e^{-i\omega k 2^{-j}} W\left( \frac{\omega}{2^j} \right),
\]

so that \( w(2^j t - k) \) can be written as:

\[
w(2^j t - k) = 2^{-j} \int_{-\infty}^{\infty} W\left( \frac{\omega}{2^j} \right) e^{-i\omega (2^{-j} k + t)} d\omega.
\]

(B.3)

The proof starts by making no assumptions about the wavelet expansion in Eqn. 4.21. Using the coefficient definitions in Eqn. 4.23, writing \( f(t) \) as the inverse Fourier transform, \( F(\omega) \), and using the result in Eqn. B.3 it can be shown the coefficients (Eqn. 4.23) can be written as

\[
a_{j,k} = 2\pi \int_{-\infty}^{\infty} F(\omega) W\left( \frac{\omega}{2^j} \right) e^{i\omega k} d\omega,
\]

and

\[
\tilde{a}_{j,k} = 2\pi \int_{-\infty}^{\infty} F(-\omega) W\left( \frac{\omega}{2^j} \right) e^{-i\omega k} d\omega,
\]

(B.4)
The Fourier transform of the harmonic wavelet, Eqn. 4.11, is zero except over 2π and its complex conjugate. According to Poisson’s summation where

\[ F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} \, dt, \quad \text{and} \quad F(-\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{i\omega t} \, dt. \]

The modulus squared of the coefficients in Eqn. B.4 is

\[ |a_{j,k}|^2 = \int_{-\infty}^{\infty} 2\pi F(\omega_1) \overline{W(\frac{\omega_1}{2^j})} \omega_1 \int_{-\infty}^{\infty} 2\pi F(\omega_2) W(\frac{\omega_2}{2^j}) e^{i(\omega_1 - \omega_2)k} \omega_2 \, d\omega_1, \]

and

\[ |\tilde{a}_{j,k}|^2 = \int_{-\infty}^{\infty} 2\pi F(-\omega_1) \overline{W(\frac{\omega_1}{2^j})} \omega_1 \int_{-\infty}^{\infty} 2\pi F(-\omega_2) \overline{W(\frac{\omega_2}{2^j})} e^{-i(\omega_1 - \omega_2)k} \omega_2. \]  

(B.5)

We now sum over all \( j \) and \( k \). The summation over \( k \) involves only two terms, \( \sum_{k=-\infty}^{\infty} e^{i(\omega_2 - \omega_1)k} \), and its complex conjugate. According to Poisson’s summation\(^1\) [69],

\[ \frac{1}{2L} \sum_{k=-\infty}^{\infty} e^{-\frac{i\omega k}{L}} = 2\pi \sum_{m=-\infty}^{\infty} \delta(x - 2mL). \]

(B.6)

If, from Eqn. B.6, one takes \( x = \omega_2 - \omega_1 \) and \( L = 2^j \pi \), they find

\[ \sum_{k=-\infty}^{\infty} e^{i(\omega_2 - \omega_1)k} = 2\pi 2^j \sum_{m=-\infty}^{\infty} \delta(\omega_2 - \omega_1 - 2m\pi 2^j). \]

(B.7)

The summation over \( k \) in Eqn. B.5 is of the form in Eqn. B.7 and is multiplied by the product \( \overline{W(\frac{\omega_1}{2^j})} W(\frac{\omega_2}{2^j}) \), or its complex conjugate, which because of the band limited spectrum of the harmonic nature (its orthogonality), is zero unless \( \omega_1 \) and \( \omega_2 \) fall within an interval \( 2\pi 2^j \leq \omega < 4\pi 2^j \). It then follows that \( W(\frac{\omega_1}{2^j}) \overline{W(\frac{\omega_2}{2^j})} \) will be zero for all values of \( m \) in the summation in Eqn. B.7, except for \( m = 0 \). This result implies

\[ \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} 2^{-j} |a_{j,k}|^2 = \sum_{j=-\infty}^{\infty} (2\pi)^3 \int_{-\infty}^{\infty} F(\omega) \overline{F(\omega)} \overline{W(\frac{\omega}{2^j})} W(\frac{\omega}{2^j}) \omega \, d\omega, \]

and

\[ \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} 2^{-j} |\tilde{a}_{j,k}|^2 = \sum_{j=-\infty}^{\infty} (2\pi)^3 \int_{-\infty}^{\infty} F(-\omega) \overline{F(-\omega)} \overline{W(\frac{\omega}{2^j})} W(\frac{\omega}{2^j}) \omega. \]

(B.8)

The Fourier transform of the harmonic wavelet, Eqn. 4.11, is zero except over \( 2\pi \leq \omega < 4\pi \), where, \( W(\omega) = \frac{1}{2\pi} \). Thus, \( W(\frac{\omega}{2^j}) \overline{W(\frac{\omega}{2^j})} = \frac{1}{(2\pi)^2} \) over \( 2\pi 2^j \leq \omega < 4\pi 2^j \), and its summation overall \( j \) is the constant, \( \frac{1}{(2\pi)^2} \), for \( 0 \leq \omega < \infty \). Using this result, and adding the expressions

\(^1\)Poisson’s summation formula relates the Fourier series coefficients of a periodic summation of a function to values of the function’s continuous Fourier transform.
in Eqn. B.8, we have
\[
\sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} 2^{-j} \left( |a_{j,k}|^2 + |\tilde{a}_{j,k}|^2 \right) = 2\pi \int_{0}^{\infty} \left( F(\omega) \bar{F}(\omega) + F(-\omega) \bar{F}(\omega) \right) d\omega \\
= 2\pi \int_{-\infty}^{\infty} F(\omega) \bar{F}(\omega) d\omega. \tag{B.9}
\]

Finally, using the result from Chapter 3, Eqn. 3.12, we have proven Eqn. B.1:
\[
\sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} 2^{-j} \left( |a_{j,k}|^2 + |\tilde{a}_{j,k}|^2 \right) = \int_{-\infty}^{\infty} |f(t)|^2 dt. \tag{B.10}
\]
Appendix C
Algorithm Reliability Study

The discussion in the coming sections speaks to inputting signals into the both the SRS and generalized harmonic wavelet transform algorithms that produce an expected output. By inputting a signal with a known frequency or time-frequency response the reliability of the algorithms in use can be assessed.

C.1 Shock Response Spectrum Algorithm Reliability Study

In 1981, Smallwood presented a filtering method for calculating the SRS [70]. His approach has sense become the aerospace industry standard. In following with standard practice, the algorithm developed for this analysis is based on the Smallwood approach. The method uses a ramp invariant filter to approximate the system response at each frequency of interest. The filtering technique greatly improves computation time, but being an approximation, the algorithm is expected to produce slightly different output when compared to an analytically exact solution.

Using the work done by Harris and Crede in [71] as a basis, a check of the algorithm in use is simple. Inputting a step function with a constant rise time,

\[
\xi(t) = \begin{cases} 
\xi_0 \frac{t}{\tau} & 0 \leq t \leq \tau, \\
\xi_0 & \tau \leq t,
\end{cases}
\]  

(C.1)

the expected output is described by the normalized relationship

\[
\frac{\nu}{\xi_0} = 1 + \left| \frac{T}{\pi \tau} \sin \frac{\pi \tau}{T} \right|, 
\]  

(C.2)

where \( T \) is the period, and \( \tau \) is the rise time. Setting \( \tau = 0.01 \text{ s} \), \( \xi_0 = 5000 \text{ g} \), and \( T = \frac{1}{f} \), where \( f \) is the frequency of interest, Eqn. C.1 is shown graphically in Fig. C.1. Fig. C.2 compares the analytically expected result given in Eqn. C.2, to the algorithm
generated SRS for several different processing bandwidths. Dividing the spectrum into bandwidths essentially bins the spectrum; resulting in the output only being calculated at evenly spaced\(^1\) discrete points over each band. The industry standard bandwidth is \(\frac{1}{6}\)th octave. This results in six response points being calculated for each octave bandwidth. As expected, the algorithm produces slightly different results than the analytically exact solution given in Eqn. C.2, however, the results are reasonable and generally reproduce the expected output.

---

**Fig. C.1: SRS Input.** The plot shows the graphical representation of Eqn. C.1, for \(\tau = 0.01\) s, \(\xi_0 = 5000\) g, and \(T = \frac{1}{f}\), where \(f\) is the frequency of interest.

\(^1\) The points are only evenly spaced using a log scale.
Fig. C.2: **SRS Input.** The plot shows the graphical representation of Eqn. C.2 compared to the algorithm generated SRS for several different processing, for $\tau = 0.01\ s$, $\xi_0 = 5000\ g$, and $T = \frac{1}{f}$, where $f$ is the frequency of interest.
C.2 Generalized Harmonic Wavelet Transform Algorithm Reliability Study

Since there are no closed form results to compare the wavelet transform algorithm to, a slightly different approach will be used here. Two sinusoidal based inputs will be input and the results compared against what would be expected. For a sinusoidal input of the form, sin(2πft), the output in the time-frequency domain is expected to only have a frequency component, f, throughout the period of the time in question.

Starting with a sinusoidal input of the form

\[ \xi(t) = \xi_1 \sin(\omega_1 t) + \xi_2 \sin(\omega_2 t), \]

where \( \omega_i = 2\pi f_i \), we expect to see two linear bands in time-frequency space, one for each frequency component. Fig. C.3 shows the input sinusoid as a function of time when \( \xi_1 = \xi_2 = 50 \text{ g}, \omega_1 = 2\pi 200, \text{ and } \omega_2 = 2\pi 550 \). Fig. C.4 shows the generalized harmonic wavelet transform of the sinusoidal input for the same values using several processing bandwidths. As expected, there are two bands in the output. The bandwidth differences serve only to localize the response - that is for smaller processing bandwidths, the response energy is spread over fewer frequencies in each band.
Fig. C.3: Sinusoidal Input. The plot shows the first half second of data for a 35 s long sinusoidal input described in Eqn. C.3 with parameters defined as $\xi_1 = \xi_2 = 50$ g, $\omega_1 = 2\pi200$, and $\omega_2 = 2\pi550$. 
Fig. C.4: Harmonic Wavelet Transform of Sinusoidal Input. The plot shows the harmonic wavelet transform for a sinusoidal input of the form given in Eqn. C.3 with the parameters defined as $\xi_1 = \xi_2 = 50 \text{ g}$, $\omega_1 = 2\pi 200$, and $\omega_2 = 2\pi 550$, for different processing bandwidths. Fig. C.3 shows the input.
The same check can be done for a chirp input. A chirp input is sinusoidal in nature, with the frequency in the sinusoid being time dependent. The definition of a chirp goes as \( \sin(2\pi t^2) \). It is essentially a sine sweep over some time period. In the time-frequency domain, the output is expected to be quadratic in time. Fig. C.5 shows the first second of a chirp function that was used as an input. Fig. C.6 shows the harmonic wavelet transform of the chirp shown in Fig. C.5. As expected, the frequency response is quadratic in time. Here again, several bandwidths are shown.

![Chirp Input](image)

Fig. C.5: **Chirp Input.** The plot shows the first second of data for a 4 s long chirp (sine sweep) input.
Fig. C.6: **Harmonic Wavelet Transform of Chirp Input.** The plot shows the harmonic wavelet transform for a chirp input for different processing bandwidths. Fig. C.5 shows the input.
Appendix D

Trajectory Summary of Ares I-X Booster After Operation.
Table D.1: Trajectory Summary of ARES I-X Booster After Operation. This image shows an artist rendition of the sequence of events the booster experienced after operation. The FSE to frustum separation and the FSE to forward skirt separation were duplicated as part of [2]. Image taken from [67].
Appendix E

Flat Plate Data Quality Summary
Table E.1: **Data Quality and Charge Density By Test.** KEY: B - Bad Channel, S - Saturated, P - Peak Outside Range, No Clipping, Blank - Indicates Reliable Data

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CURRICULUM VITAE

Richard J. Ott

Professional Experience

• Senior Loads and Environments Engineer
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Published Journal Articles


Published Conference Papers

