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## 10 Why "Plane" Waves?

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## 10. Why “plane” waves?

Let us now pause to explain in more detail why we called the elementary solutions (9.9) and (9.26) *plane waves*. The reason is that the displacement  $q(\mathbf{r}, t)$  has the symmetry of a plane. To see this, fix a time  $t$  (take a “snapshot” of the wave) and pick a location  $\mathbf{r}$ . Examine the wave displacement  $q$  (at the fixed time) at all points in a plane that is (i) perpendicular to  $\mathbf{k}$ , and (2) passes through  $\mathbf{r}$ . The wave displacement will be the same at each point of this plane. To understand this most easily, simply choose, say, the  $x$ -axis to be along the vector  $\mathbf{k}$ . The planes perpendicular to  $\mathbf{k}$  are parallel to the  $y$ - $z$  plane. In these new coordinates the wave (9.26) takes the simple form (*exercise*)

$$q(\mathbf{r}, t) = A \cos(kx - \omega t + \phi). \quad (10.1)$$

Clearly, at a fixed  $t$  and  $x$ ,  $q(\mathbf{r}, t)$  is the same anywhere on the plane obtained by varying  $y$  and  $z$ .

A more formal — and perhaps more instructive — way to see the plane wave symmetry of (9.26) is to fix a time  $t$  and ask for the locus of points upon which the wave displacement is constant. At a fixed time, the wave displacement  $q$  is a function of 3 variables. As you know, the locus of points where a function takes the same values defines a surface. Well, with  $t = \text{constant}$ , the surfaces of fixed  $q$  arise when  $\mathbf{k} \cdot \mathbf{r} = \text{constant}$  (*exercise*). But the equation  $k_x x + k_y y + k_z z = \text{constant}$  (with each of  $k_x, k_y, k_z$  a constant) is the equation for a plane (*exercise*)! This plane is everywhere orthogonal to the wave vector  $\mathbf{k}$ , which can be viewed as a constant vector field (*i.e.*, a vector field whose Cartesian components are the same everywhere). To see this, we recall from our discussion in §9 that the gradient of a function is always perpendicular to the surfaces upon which the function is constant. We just saw that the plane of symmetry (where  $q$  doesn’t change its value) arises when the function  $\mathbf{k} \cdot \mathbf{r}$  is constant. Thus, the plane of symmetry for the plane wave is orthogonal to the (constant) vector field

$$\nabla(\mathbf{k} \cdot \mathbf{r}) = \mathbf{k}. \quad (10.2)$$

The wave vector is thus normal to the planes of symmetry of a plane wave. As time evolves, the displacement profile on a given plane of symmetry moves along  $\mathbf{k}$ . In this way  $\mathbf{k}$  determines the propagation direction of the plane wave. The wave vector thus determines the wavelength and direction of motion of a plane wave.