

PROBLEM SET 6

Problem 6.1

Use the method of separation of variables to find a nonzero solution of the 3-dimensional wave equation that vanishes on the faces of a cube. (You can think of this as a mathematical model of sound waves in a room.)

Problem 6.2

Suppose that a function only depends upon the distance from the z -axis:

$$F(x, y, z) = f(\sqrt{x^2 + y^2}).$$

Consider the expression of F in cylindrical coordinates. Show that

$$\frac{\partial F}{\partial \phi} = 0,$$

- (i) directly in cylindrical coordinates (easy!)
- (ii) using the chain rule starting in Cartesian coordinates.

Problem 6.3

Show that spherical polar coordinates are identical to cylindrical coordinates when labeling points in the x - y plane.

Problem 6.4

Suppose that we are considering cylindrically symmetric solutions to the wave equation, $q = q(\rho, t)$. Starting from the wave equation in Cartesian coordinates, use the chain rule to derive the wave equation satisfied by $q(\rho, t)$.

Problem 6.5

If one looks for solutions to the wave equation that do not depend upon time, $q = q(\mathbf{r})$, then one must solve the Laplace equation

$$\nabla^2 q(\mathbf{r}) = 0.$$

Find the general form of the solution to this equation if one assumes

(a) cylindrical symmetry: $q = q(\rho)$

(b) spherical symmetry: $q = q(r)$.

Problem 6.6

Verify that the spherically symmetric functions

$$q_1(r, t) = \cos(kvt) \frac{\sin(kr)}{kr}, \quad q_2(r, t) = \cos(kvt) \frac{\cos(kr)}{kr}$$

solve the (three-dimensional) wave equation. Show that q_1 is well-defined at the origin while q_2 is not.

Problem 6.7

Find a non-zero solution to the wave equation that is spherically symmetric and vanishes on the surface of a sphere (centered at the origin, radius R).

Problem 6.8

Using your favorite mathematical software, write a program to compute the spherical Bessel functions from the formula (13.22). Verify the results shown in (13.21).

Problem 6.9

Using your favorite mathematical software, write a program to display an animation depicting the cylindrically symmetric solution (12.38).