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Optimization of Fuel Consumption with Respect to Orbital Requirements for High Resolution Remote Sensing Satellite Constellations

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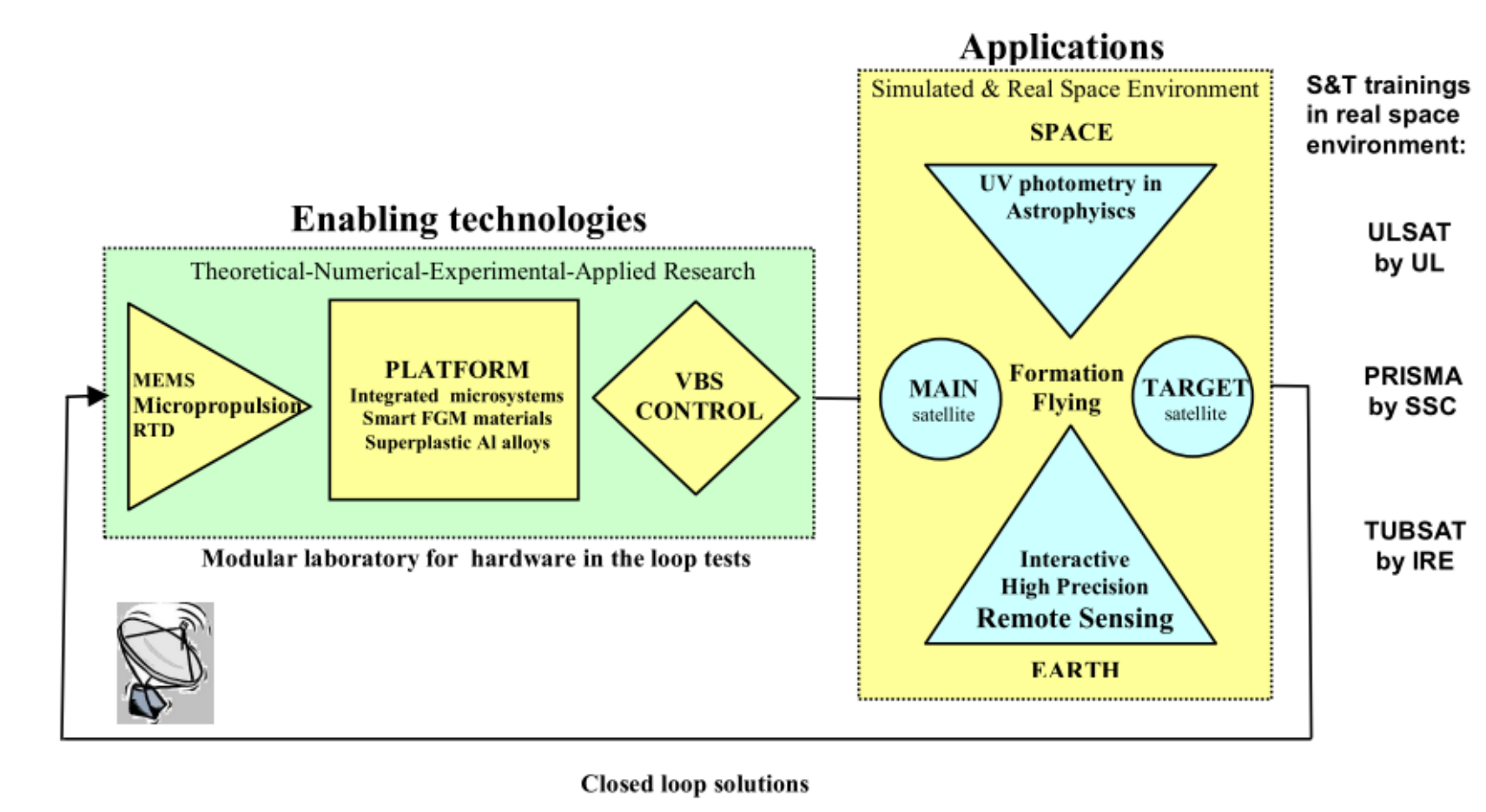
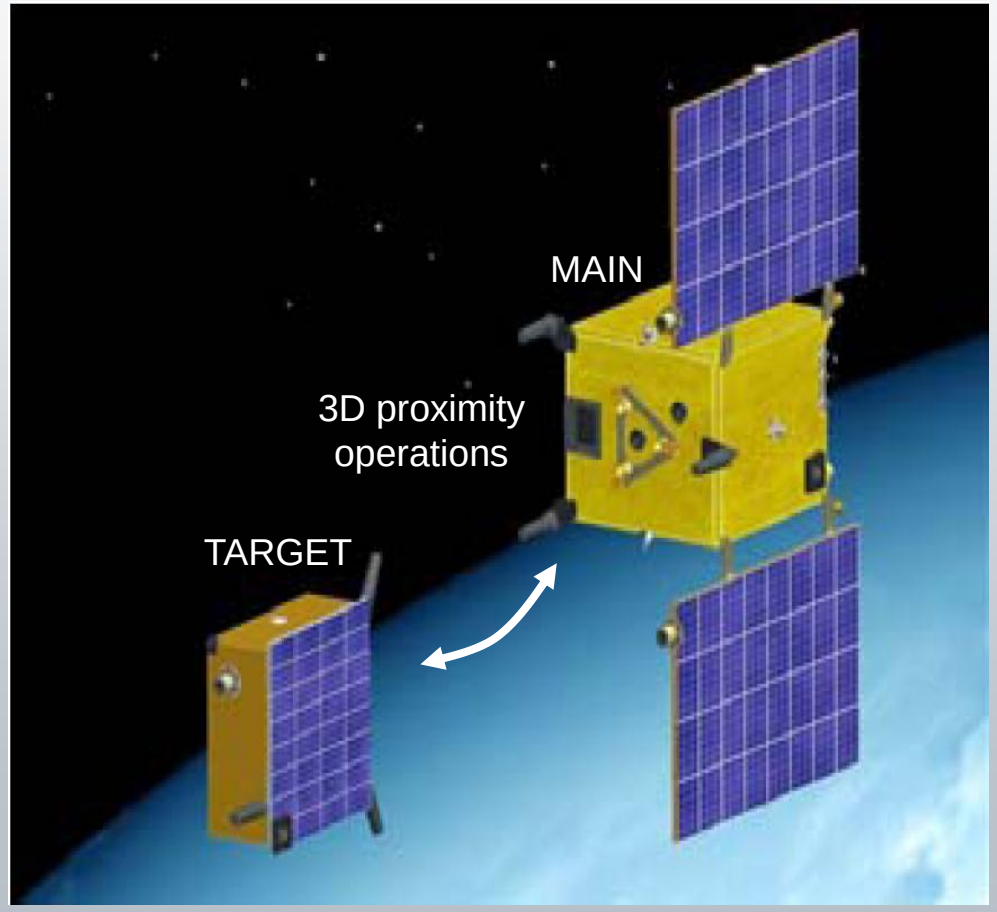
1. INTRODUCTION

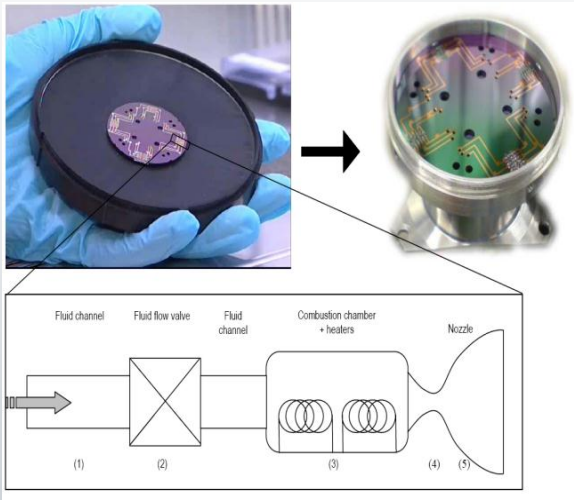
- Centre of Excellence for
- Space Sciences and Technologies
- Established in 2010

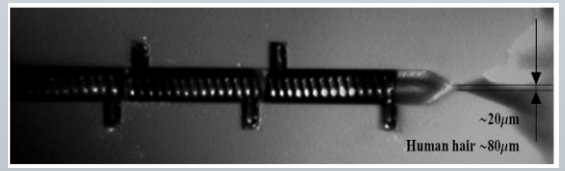
- **Modular laboratory for multidisciplinary hardware in the loop tests**

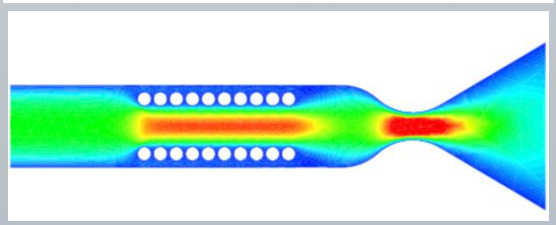


M5 modelling MEMS micropropulsion by NanoSpace









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2. MANOEUVRES

- **Parallel flying – In-track displacement**
- **Changing the In-track displacement**
- ***Changing the Radial position***
- ***Transition to the nadir observation constellation***
- ***Keeping the nadir observation constellation***
- ***Changing and keeping the Cross-track position***
- **Circumvolution of the target in the x-y plane**
- **Encirculation of the target**

3. FORMATION FLYING MODELS

- The most common way to describe formation flying are the nonlinear Hill-Clohessy-Wiltshire (HCW) equations:

$$\ddot{x} - 2\dot{\varphi}_R \dot{y} - \ddot{\varphi}_R y - \dot{\varphi}_R^2 x = -\frac{\mu(R+x)}{((R+x)^2 + y^2 + z^2)^{\frac{3}{2}}} + \frac{\mu}{R^2}$$

$$\ddot{y} + 2\dot{\varphi}_R \dot{x} + \ddot{\varphi}_R x - \dot{\varphi}_R^2 y = -\frac{\mu y}{((R+x)^2 + y^2 + z^2)^{\frac{3}{2}}}$$

$$\ddot{z} = -\frac{\mu z}{((R+x)^2 + y^2 + z^2)^{\frac{3}{2}}}$$

x, y and z are the coordinates of the target satellite in the local vertical/local horizontal (LVLH) co-ordinate system

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The movement of the first satellite is described by:

$$\ddot{R} = R\dot{\varphi}_R^2 - \frac{\mu}{R^2}$$

$$\ddot{\varphi}_R = -\frac{2\dot{R}\dot{\varphi}_R}{R}.$$

- R – distance from the Earth mass centre
- φ_R - true anomaly
- μ - Earth gravity constant

- For close formation flying and small eccent*ricitie*s these equations can be linearized. If the first satellite has a circular orbit, its angular acceleration is zero*, and its mean motion can be* expressed by:

$$\ddot{\varphi}_R = 0 \qquad \dot{\varphi}_R = n = \sqrt{\frac{\mu}{a^3}}.$$

Linear system of equations:

$$\ddot{x} - 2n\dot{y} - 3n^2 x = a_x$$
$$\ddot{y} + 2n\dot{x} = a_y$$
$$\ddot{z} + n^2 z = a_z.$$

ax, ay, az are accelerations in the *x, y* and *z* directions respectively*.*

For constant accelerations a_x, a_y, and a_z, this system of equations has the solution:

$$x(t) = \left[x(t_0) + \frac{1}{3n^2} a_x \right] (4 - 3\cos n(t - t_0)) + \frac{\left[\dot{x}(t_0) - 2a_y \right]}{n} \sin n(t - t_0) +$$
$$+ \frac{2\dot{y}(t_0)}{n} (1 - \cos n(t - t_0)) - \frac{1}{3n^2} a_x + 2a_y (t - t_0)$$

$$y(t) = y(t_0) + \dot{y}(t_0) \left[\frac{4}{n} \sin n(t - t_0) - 3(t - t_0) \right] + 6 \left[x(t_0) + \frac{1}{3n^2} a_x \right] \left[\sin n(t - t_0) - n(t - t_0) \right] +$$
$$+ \frac{2\left[\dot{x}(t_0) - 2a_y \right]}{n} \cdot (\cos n(t - t_0) - 1) - \frac{3n}{2} a_y (t - t_0)^2$$

$$z(t) = \left[z(t_0) - \frac{1}{n^2} a_z \right] \cos n(t - t_0) + \frac{\dot{z}(t_0)}{n} \sin n(t - t_0) + \frac{1}{n^2} a_z$$

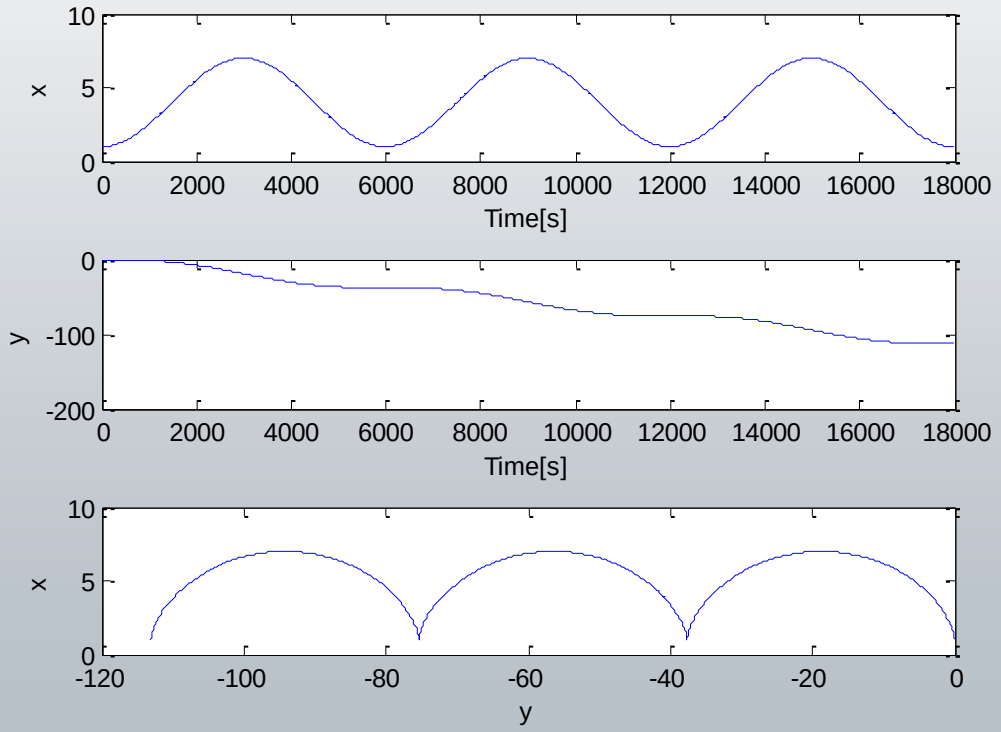
4. THE INFLUENCE OF INITIAL CONDITIONS ON THE RELATIVE ORBIT

- ***Influence of x_0 – Initial Radial displacement***

$$x(t) = x(t_0)(4 - 3\cos n(t - t_0))$$

$$y(t) = 6x(t_0)\left[\sin n(t - t_0) - n(t - t_0)\right]$$

$$z(t) = 0$$

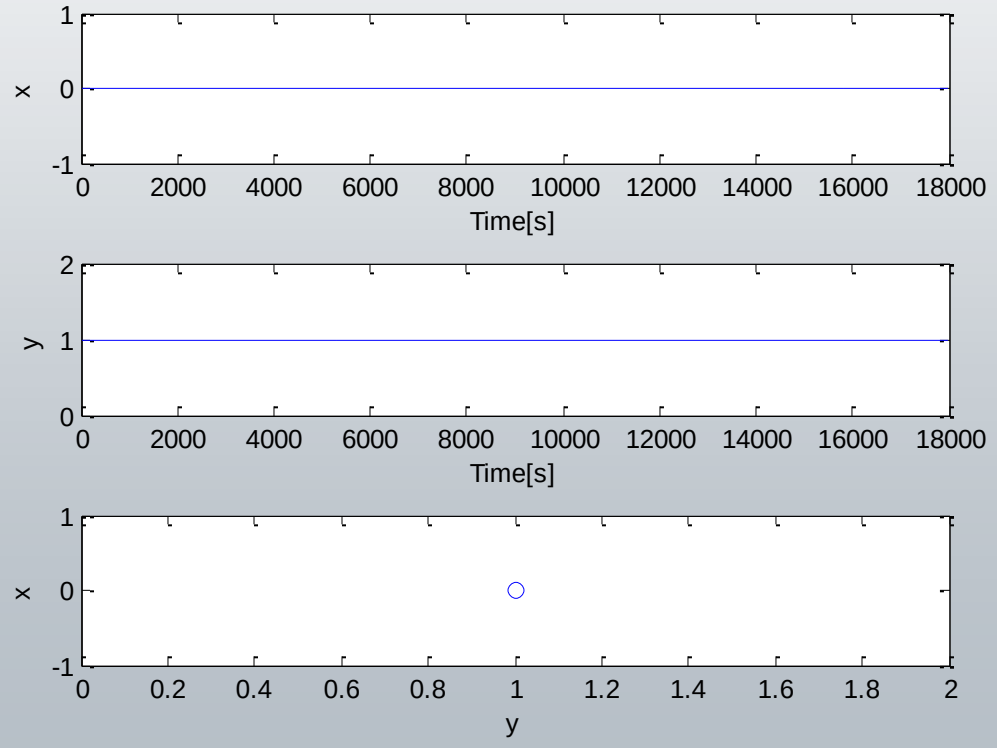


- ***Influence of y_0 – Initial In-track displacement***

$$x(t) = 0$$

$$y(t) = y(t_0)$$

$$z(t) = 0$$



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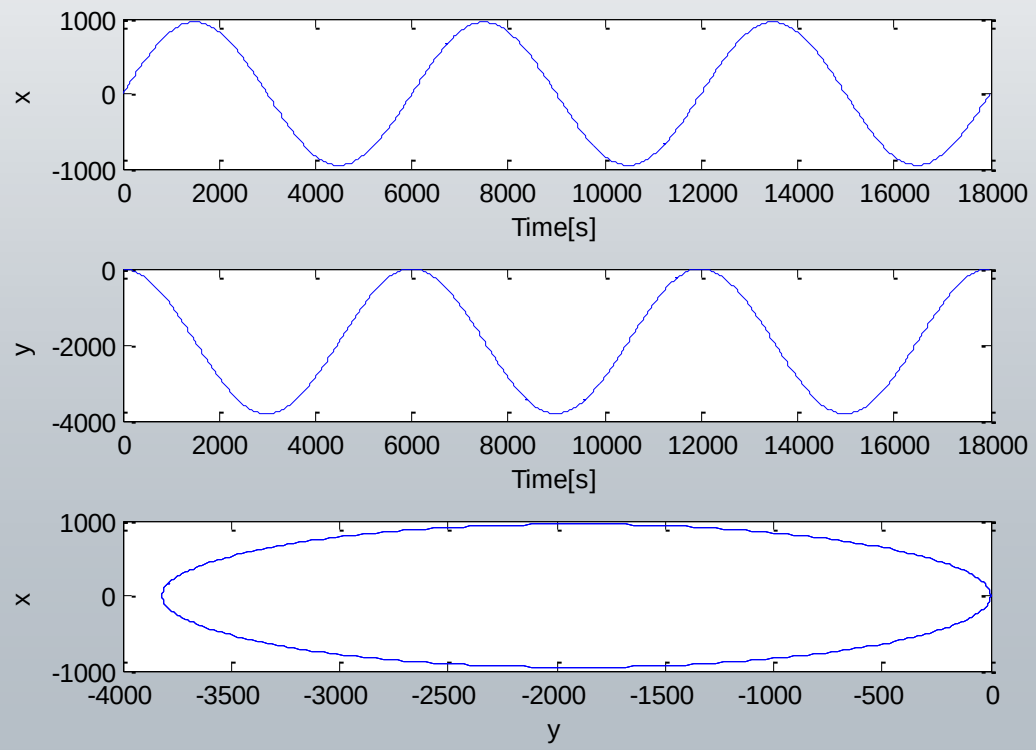
- ***Influence of*** $\dot{x}(t_0)$ ***– Initial Radial velocity***

$$x(t) = \frac{\dot{x}(t_0)}{n} \sin n(t - t_0)$$

$$y(t) = \frac{2\dot{x}(t_0)}{n} (\cos n(t - t_0) - 1)$$

$$z(t) = 0$$

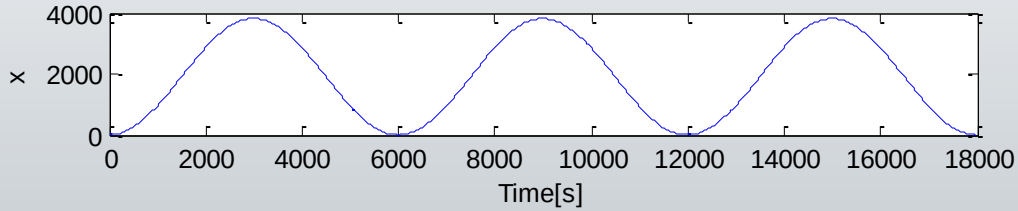


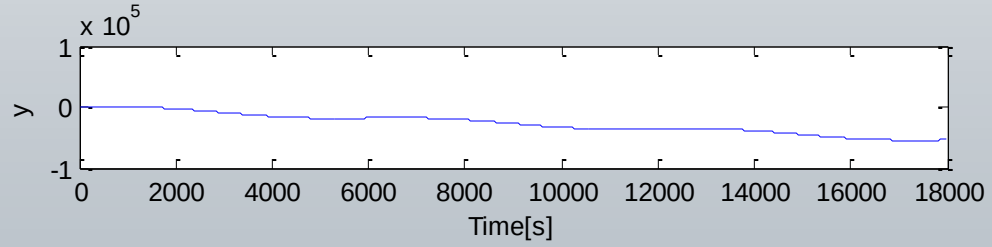
- ***Influence of*** $\dot{y}(t_0)$ ***– Initial In-track velocity***

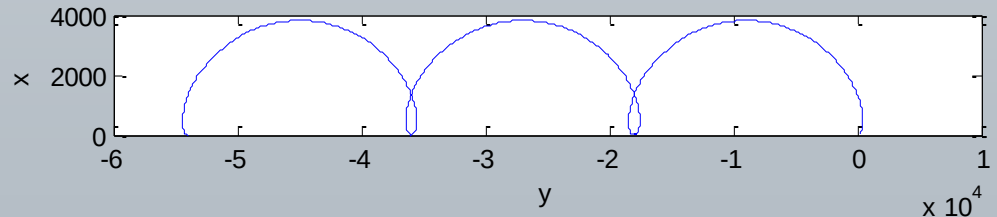
$$x(t) = \frac{2}{n}\dot{y}(t_0)(1-\cos n(t-t_0))$$

$$y(t) = \dot{y}(t_0)(\frac{4}{n}\sin n(t-t_0) - 3(t-t_0))$$

$$z(t) = 0$$







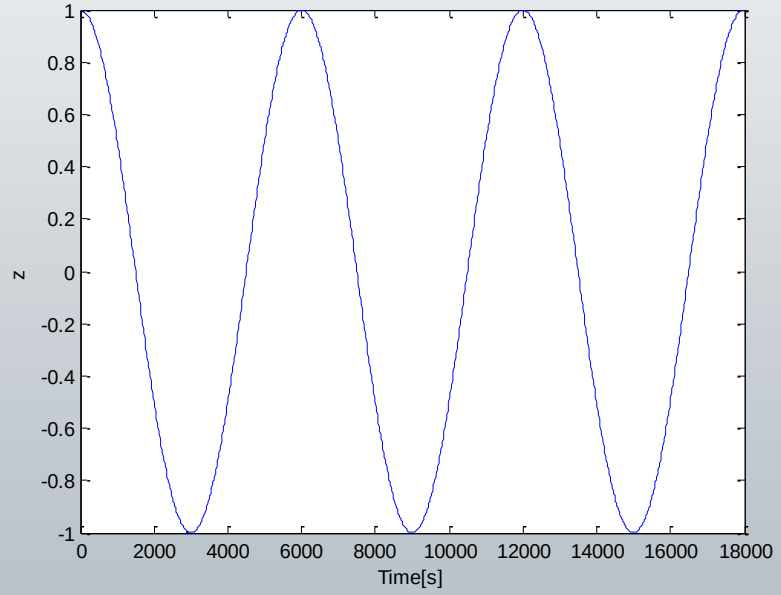
- ***Influence of*** $z(t_0)$ ***and*** $\dot{z}(t_0)$ ***- Initial Cross track disp. & Velovity***

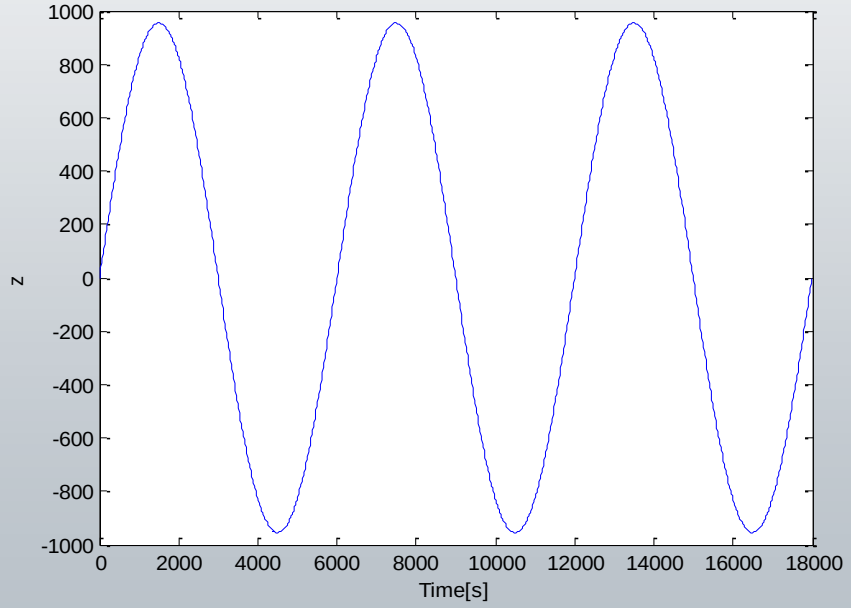
$$x(t) = y(t) = 0$$

$$z(t) = z(t_0)\cos n(t - t_0)$$

$$x(t) = y(t) = 0$$

$$z(t) = \frac{\dot{z}(t_0)}{n}\sin n(t - t_0)$$





5. APPLICATION OF THE LINEAR MODEL TO DIFFERENT MANOEUVRES

- **Parallel flying – In-track displacement (*Along track flying – trailing formation)***

$$x(t_0) = 0 \qquad y(t_0) = D \qquad \dot{x}(t_0) = 0 \qquad \dot{y}(t_0) = 0 \qquad FC = 0$$

- **Changing the In-track displacement**

$$\begin{bmatrix} x(t_0) \\ y(t_0) \\ z(t_0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} x(t_f) \\ y(t_f) \\ z(t_f) \end{bmatrix} = \begin{bmatrix} 0 \\ Y \\ 0 \end{bmatrix}$$

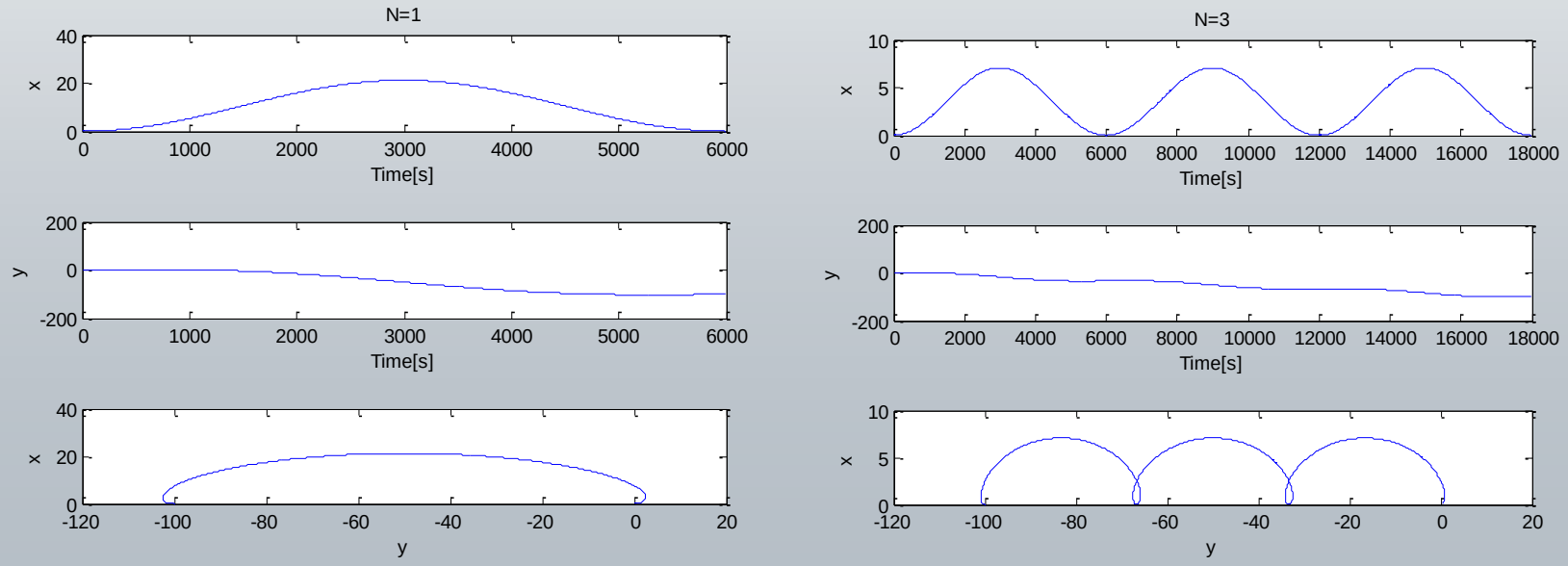
$$t_f = t_0 + N\frac{2\pi}{n} \qquad N = 1, 2, 3, \ldots$$

$$\dot{y}(t_0) = -\frac{nY}{6N\pi} = -\frac{Y}{3TN}$$

$$x(t_f) = \frac{2}{n}\dot{y}(t_0)(1 - \cos 2N\pi) = 0$$

$$y(t_f) = \dot{y}(t_0)\left(\frac{4}{n}\sin 2N\pi - \frac{6N\pi}{n}\right) = -\frac{6N\pi}{n}\dot{y}(t_0) = Y$$

$$FC = \frac{2Y}{3TN}$$



• *Changing the Radial position*

$$\begin{bmatrix} x(t_0) \\ y(t_0) \\ z(t_0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} x(t_f) \\ y(t_f) \\ z(t_f) \end{bmatrix} = \begin{bmatrix} X \\ arbitrary \\ 0 \end{bmatrix}$$

Applying the initial velocity in the x (Radial) direction

$$TFC = 4nX$$

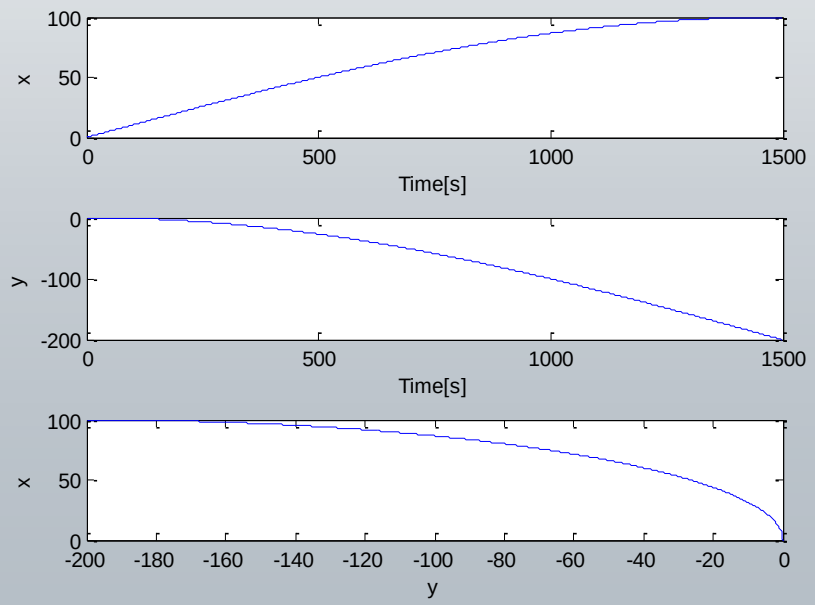
$$x(t_f) = \frac{\dot{x}(t_0)}{n} \sin n(t_f - t_0) = X$$

$$y(t_f) = \frac{2\dot{x}(0)}{n} (\cos n(t_f - t_0) - 1)$$

$$t_f = t_0 + \frac{T}{4}$$

$$x(t_f) = X \Rightarrow \dot{x}(t_0) = nX$$

$$y(t_f) = -2X$$



Applying the initial velocity in the y (In-track) direction

$$x(t_{f1}) = \frac{2}{n}\dot{y}(t_0)(1-\cos n(t_f - t_0)) = X$$

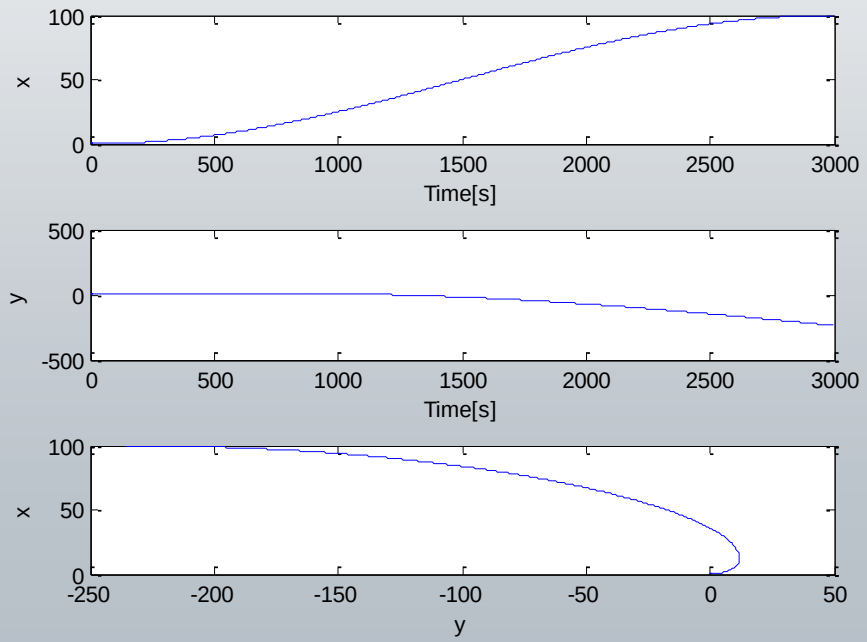
$$y(t_{f1}) = \dot{y}(t_0)(\frac{4}{n}\sin n(t_f - t_0) - 3(t - t_0))$$

$$t_{f1} = t_0 + \frac{T}{2}$$

$$x(t_{f1}) = X \Rightarrow \dot{y}(t_0) = \frac{n}{4}X$$

$$y(t_f) = \frac{-3\pi}{4}X$$

$$TFC = 2nX$$



. The starting and final points are

- ***Transition to the nadir observation constellation***

,

$$\begin{bmatrix} x(t_0) \\ y(t_0) \\ z(t_0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} x(t_f) \\ y(t_f) \\ z(t_f) \end{bmatrix} = \begin{bmatrix} X \\ 0 \\ 0 \end{bmatrix}$$

$$TFC = nX\left(2 + \frac{1}{4N}\right)$$

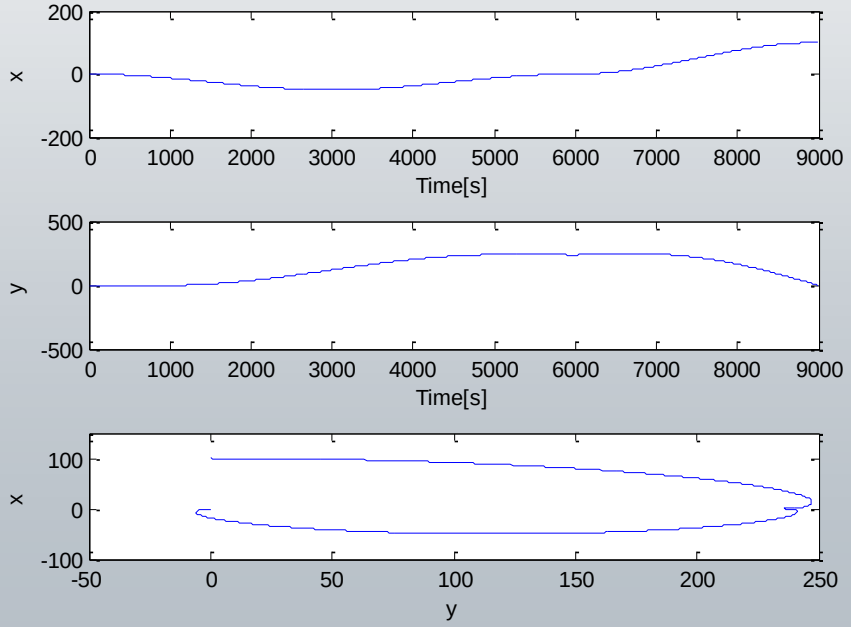
1.step:

$$\dot{y}(t_0) = -\frac{nX}{8N}$$

2.step:

$$x(t_i) = 0 \qquad y(t_i) = 2X$$

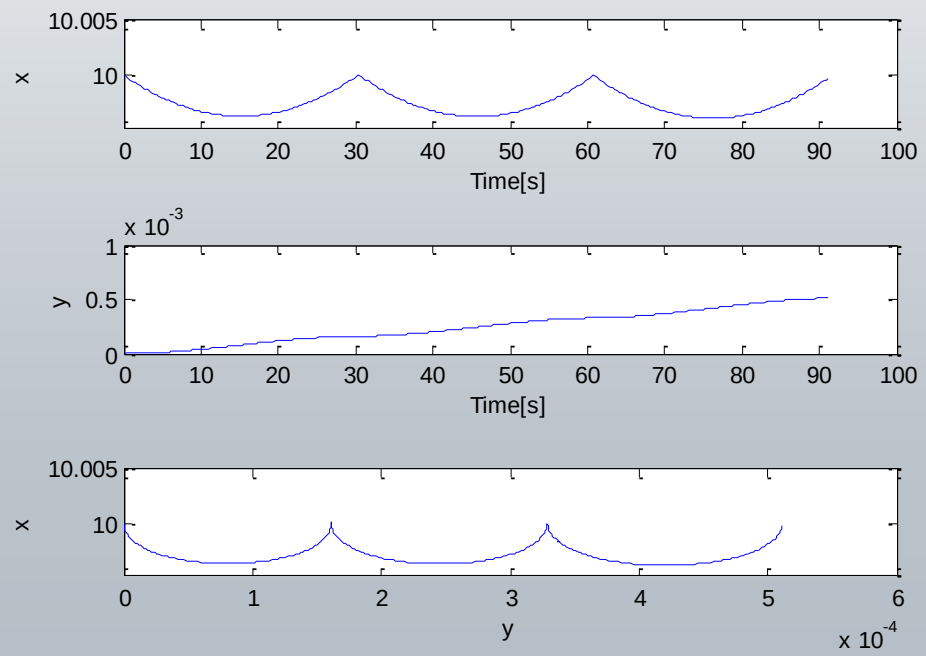
$$\dot{x}(t_i) = nX \qquad \dot{y}(t_i) = 0$$



- ***Keeping the nadir observation constellation***

$$a_x = -3n^2 x(t_0)$$ X_0 = 10 m $$a_x = -0.0329 mm/s^2$$

Using impulsive cold gas thruster propulsion with typical $\Delta v = 1$ mm/s² impulse every 30.4 s:



ANIMATION

- ***Changing and keeping the Cross-track position***

$$\begin{bmatrix} x(t_0) \\ y(t_0) \\ z(t_0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} x(t_f) \\ y(t_f) \\ z(t_f) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ Z \end{bmatrix}$$

Cross track is decoupled from Radial and In-track

$$t_f = t_0 + \frac{T}{4} \qquad z(t_f) = Z = \frac{\dot{z}(t_0)}{n} \Rightarrow \dot{z}(t_0) = nZ$$

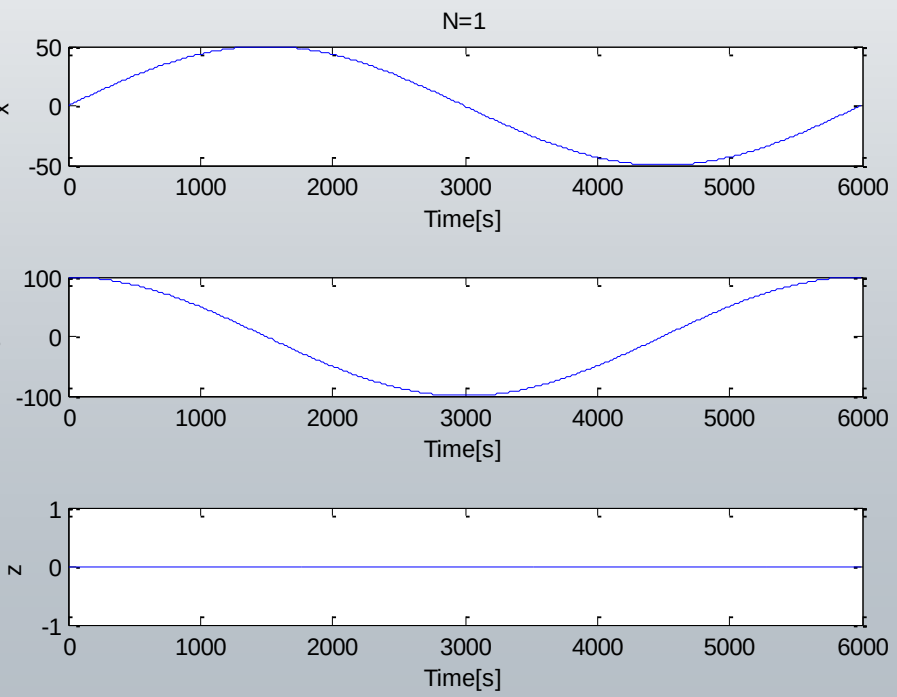
Keeeping: $$a_z = n^2 Z$$

- **Circumvolution of the target in the x-y plane**

$$x(t_0) = 0 \qquad \dot{x}(t_0) = \frac{An}{2}$$

$$y(t_0) = A \qquad \dot{y}(t_0) = 0$$

$$z(t_0) = 0 \qquad \dot{z}(t_0) = 0$$



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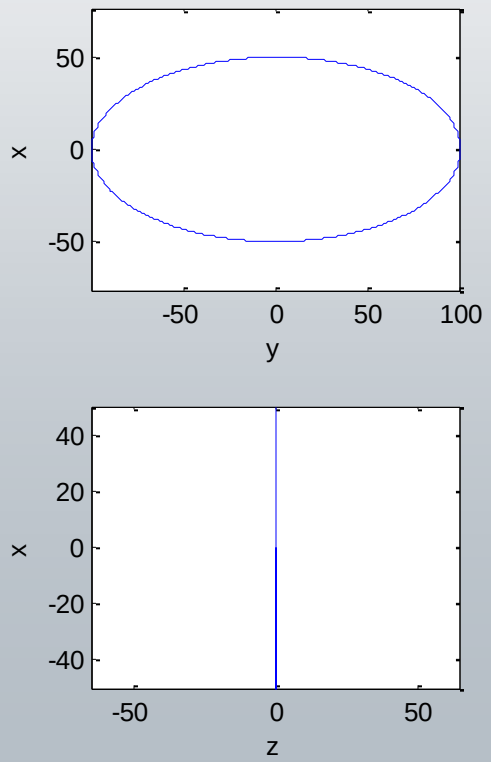


• Encirculation of the target

$$x(t_0) = 0$$

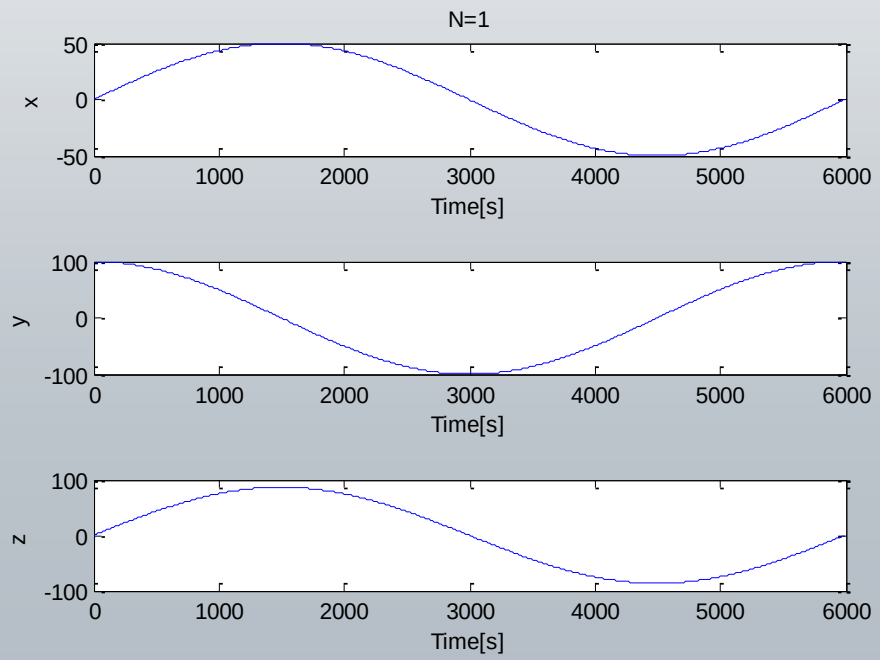
$$y(t_0) = A$$

$$z(t_0) = 0$$

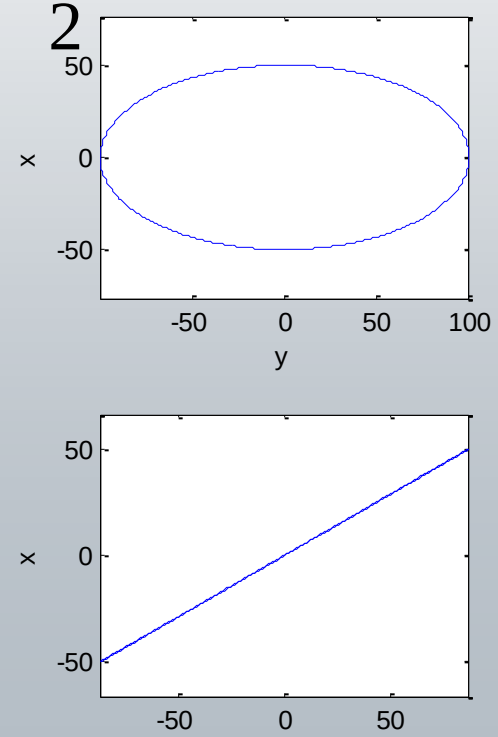
$$\dot{x}(t_0) = \frac{An}{2}$$

$$\dot{y}(t_0) = 0$$

$$\dot{z}(t_0) = \sqrt{3}\frac{An}{2}$$



ANIMATION



10. CONCLUSION

- Various manoeuvres were studied
- Investigated by mathematical models