Orbit/Attitude Determination and Control for the UMR SAT Mission

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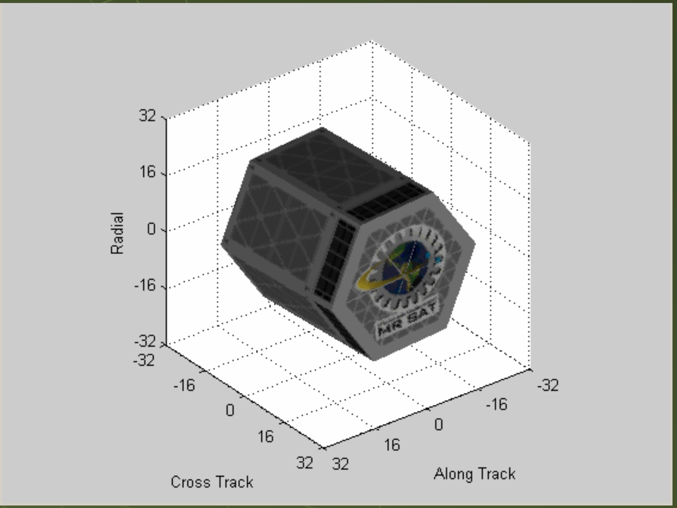

Outline

- UMR SAT Mission
- Real-Time Attitude Determination
  - Magnetometer-Only Measurements
  - AD Simulation Results
- Orbit Determination
  - θ-D Technique
  - OD Simulation Results
- Orbit/Attitude Control
- Conclusion

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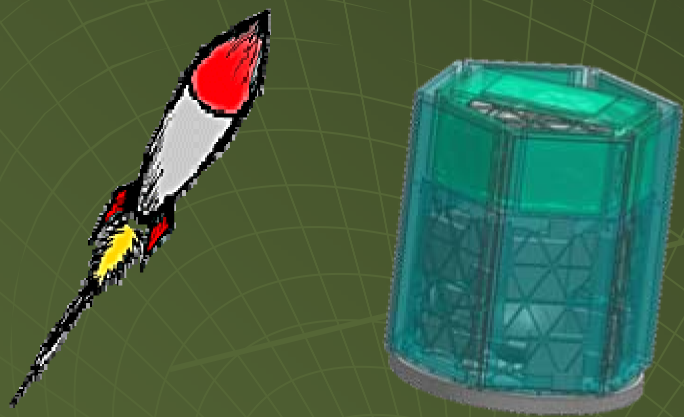
UMR SAT Overview

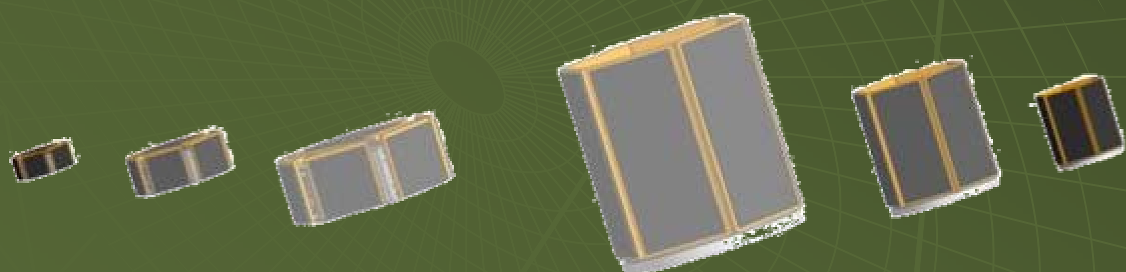
- Multi-disciplinary team
  - Design satellite pair
- Formation flight evaluation
  - Fifty-meter separation
- Low-cost technology demonstration
- University Nanosat-4 participant
  - Final Competition Review: March 2007
  - Achieved 3rd Place and Most Improved

UMR SAT Overview



Phase I: Launch/Orbit Insertion

Phase II: Free Formation Flight



Phase III: Extended Operations

Limitations

"We've got to make this, fit in the hole for this, using nothing but that!" – Apollo 13

- Propulsion System
  - Max Thrust: 25 mN (44 s Isp)
  - Total ΔV: 1.05 m/s @ 100 psi
- Magnetic Coil
  - Max Torque: 10^{-4} N*m
- Power: ~ 50 Watts
- Volume: 47.5 cm x 47.5 cm Cylinder
- Mass: < 30 kg
- Budget: ~ $150,000

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Magnetometer-Only Determination

- *Benefits*
  - Low Cost and Mass
  - No Additional Sensors Needed
  - Reliable
- Challenges
  - Two Axis Resolution w/ Mag. Field Alone
  - Need Additional Vector for Remaining Axis
    - Magnetic Field Derivative (Natanson, 1990)

Linearly Independent Vectors

- Inertial Frame
  - Satellite Position From Orbit Determination
  - NASA Magnetic Field Model
  - Derivatives Found Using Finite Differencing
- Satellite Body Frame
  - Magnetometer Measures Magnetic Field
  - Derivatives via Filtered Measurements
    - Third Order Markov Process

Attitude Calculation

- *TRIAD Method (Shuster, 1981)*
  - Two Linearly Independent Vectors
  - Coordinates Known in Two Frames
  - Calculate Rotation Matrix Between Frames
- Additional Calculations
  - Satellite Angular Velocity Appears in Mag. Field Derivative Equation
  - Second Derivative Used to Resolve Rotation Rates

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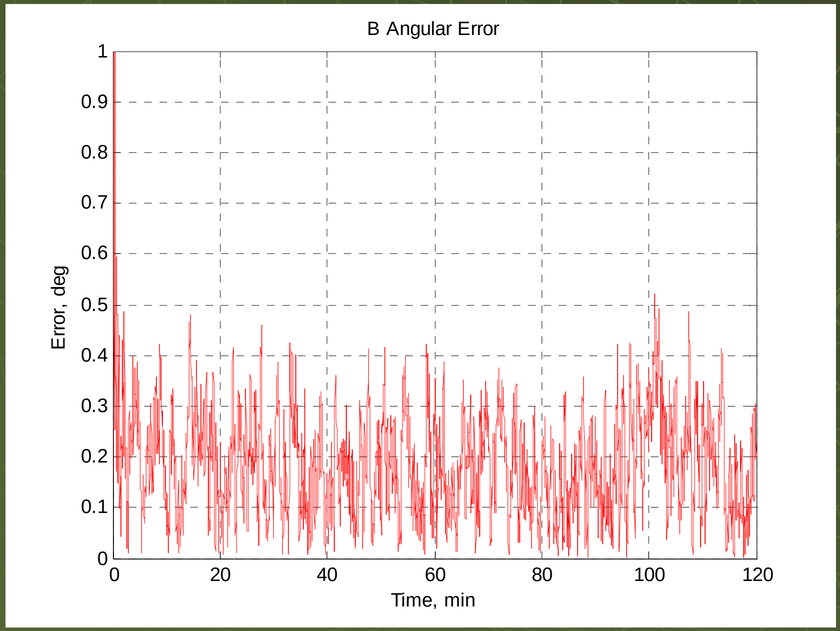
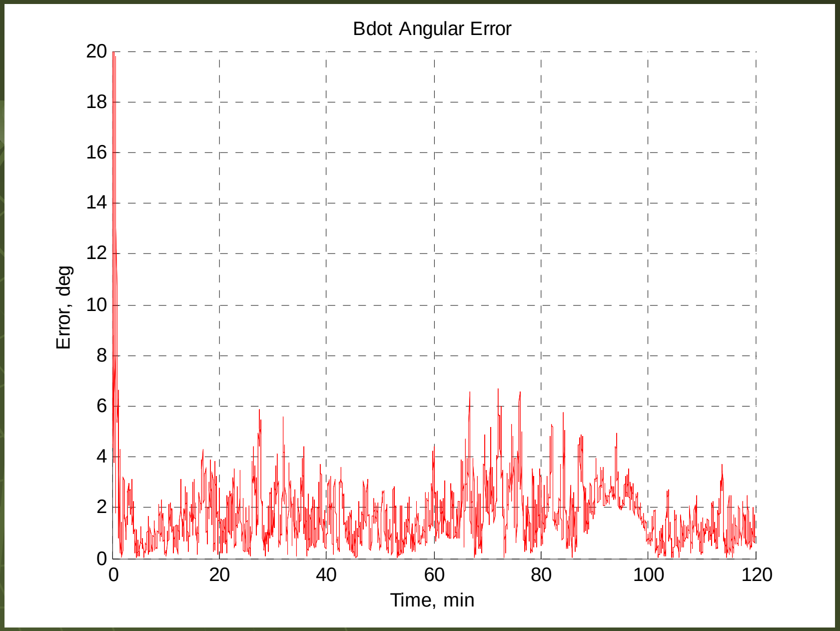
Results



Magnetic Field Vector vs. Time



Derivative vs. Time

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Orbit Determination

- Challenges
  - Limited Computer Processing Power
  - Low Budget Sensors
    - "You get what you paid for!"
- Benefits of θ-D Technique
  - Computationally Efficient
  - Improved Accuracy
    - Fully Nonlinear Formulation

θ-D* Filter Formulation

- Based on State Dependent Riccati Equation technique (SDRE)
  - Cloutier, 1996
- Approximate solution to SDRE (θ part)
- Add disturbance terms to SDRE (D part)
  - Guarantees series convergence
- Suboptimal filter

*Balakrishnan, 2002

θ-D Modifications

θ approximation:

$$\dot{\hat{x}} = F(\hat{x})\hat{x} + K(\hat{x})\left[z - H(\hat{x})\hat{x}\right]$$

$$\dot{\hat{x}} = \left[A_0 + \frac{A(\hat{x})}{\theta}\theta\right]\hat{x} + K(\hat{x})\left\{z - \left[C_0 + \frac{C(\hat{x})}{\theta}\theta\right]\hat{x}\right\}$$

Power series solution:

$$P(\hat{x}) = \sum_{i=0}^{\infty} T_i(\hat{x},\theta)\theta^i$$

Solved recursively!

Disturbance terms:

$$0 = \Gamma W \Gamma^T + P(\hat{x})F^T(\hat{x}) + F(\hat{x})P(\hat{x}) - P(\hat{x})H^T(\hat{x})V^{-1}H(\hat{x})P(\hat{x})$$

$$0 = \left[\Gamma W \Gamma^T + \sum_{i=1}^{\infty} D_i(\hat{x},\theta)\theta^i\right] + P(\hat{x})F^T(\hat{x}) + F(\hat{x})P(\hat{x}) - P(\hat{x})H^T(\hat{x})V^{-1}H(\hat{x})P(\hat{x})$$

$$\left\| \sum_{i=1}^{\infty} D_i(\hat{x},\theta)\theta^i \right\| \ll \left\| \Gamma W \Gamma^T \right\|$$

Ensures near optimal solution!

θ-D Solution

θ^0 :
$$0 = \Gamma W \Gamma^T + T_0 A_0^T + A_0 T_0 - T_0 C_0^T V^{-1} C_0 T_0$$

Need only solve once!!

θ^1 :
$$T_1\left(A_0 - T_0 C_0^T V^{-1} C_0\right)^T + \left(A_0 - T_0 C_0^T V^{-1} C_0\right) T_1 = -\frac{T_0\left[A(\hat{x}) - T_0 C_0^T V^{-1} C(\hat{x})\right]^T}{\theta} - \frac{\left[A(\hat{x}) - T_0 C_0^T V^{-1} C(\hat{x})\right] T_0}{\theta} - D_1$$

Choose D_1 to cancel RHS*

$\vdots$ $\qquad$ $\vdots$

θ^n :
$$T_n\left(A_0 - T_0 C_0^T V^{-1} C_0\right)^T + \left(A_0 - T_0 C_0^T V^{-1} C_0\right) T_n = -\frac{T_{n-1}\left[A(\hat{x}) - T_0 C_0^T V^{-1} C(\hat{x})\right]^T}{\theta} - \frac{\left[A(\hat{x}) - T_0 C_0^T V^{-1} C(\hat{x})\right] T_{n-1}}{\theta}$$
$$+\sum_{j=1}^{n-1}\left(C_0 T_j + \frac{C(\hat{x})}{\theta} T_{j-1}\right)^T V^{-1}\left(C_0 T_{n-j} + \frac{C(\hat{x})}{\theta} T_{n-j-1}\right) - D_n$$

Linear equations!!

Choose D_n to cancel RHS*

$$^* D_n = k_n e^{-l_n t}\left(RHS\right)$$

Disturbance term decay

Filter Implementation

- Continuous Filter Issues
  - Discrete Time Measurements
  - Solution to Filter Dynamics
- Filter Dynamics Numerically Integrated
  - Runge-Kutta 4th Order
  - Staggered Filter Concept

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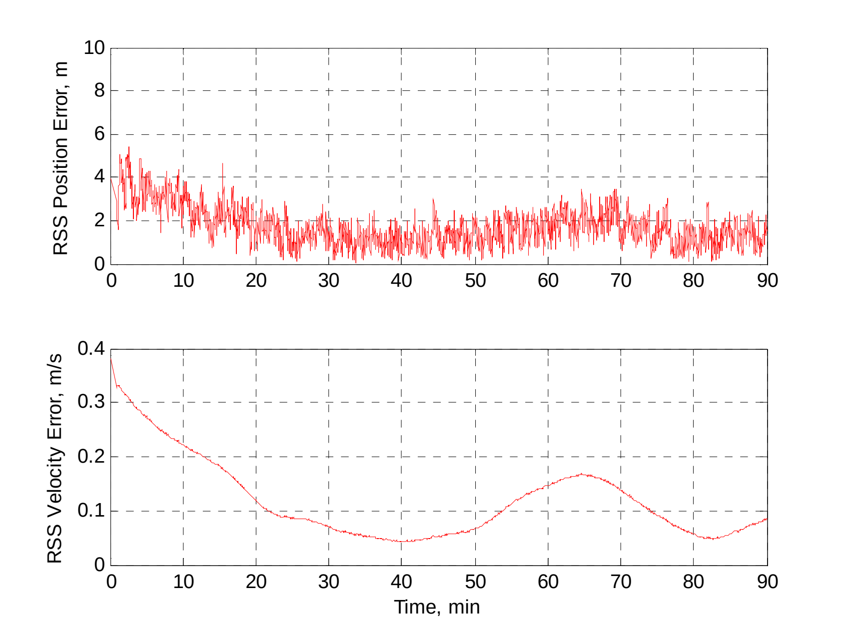
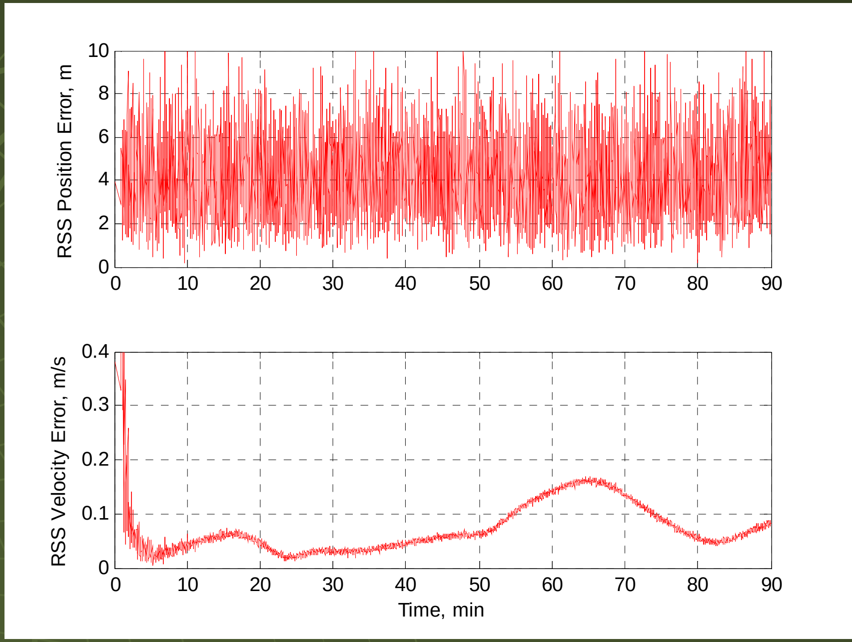
Filter Comparison





θ-D w/ Staggered Filter

Extended Kalman Filter

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Orbit and Attitude Control

- Orbit Control
  - Uses Orbit Determination Model
  - Application of θ-D
- Attitude Control
  - Quaternion Dynamics
  - Application of θ-D
- In Progress
  - Integrated Orbit/Attitude Control

Actuator Logic

- Actuator Selection
- Magnetic Coil Logic
  - Currents via Lagrange Multipliers
  - Minimize Power
- Cold-Gas Thrusters
  - Throttle via Linear Programming
  - Minimize Fuel Consumption Rate
  - Switching Logic (Throttle Filter)

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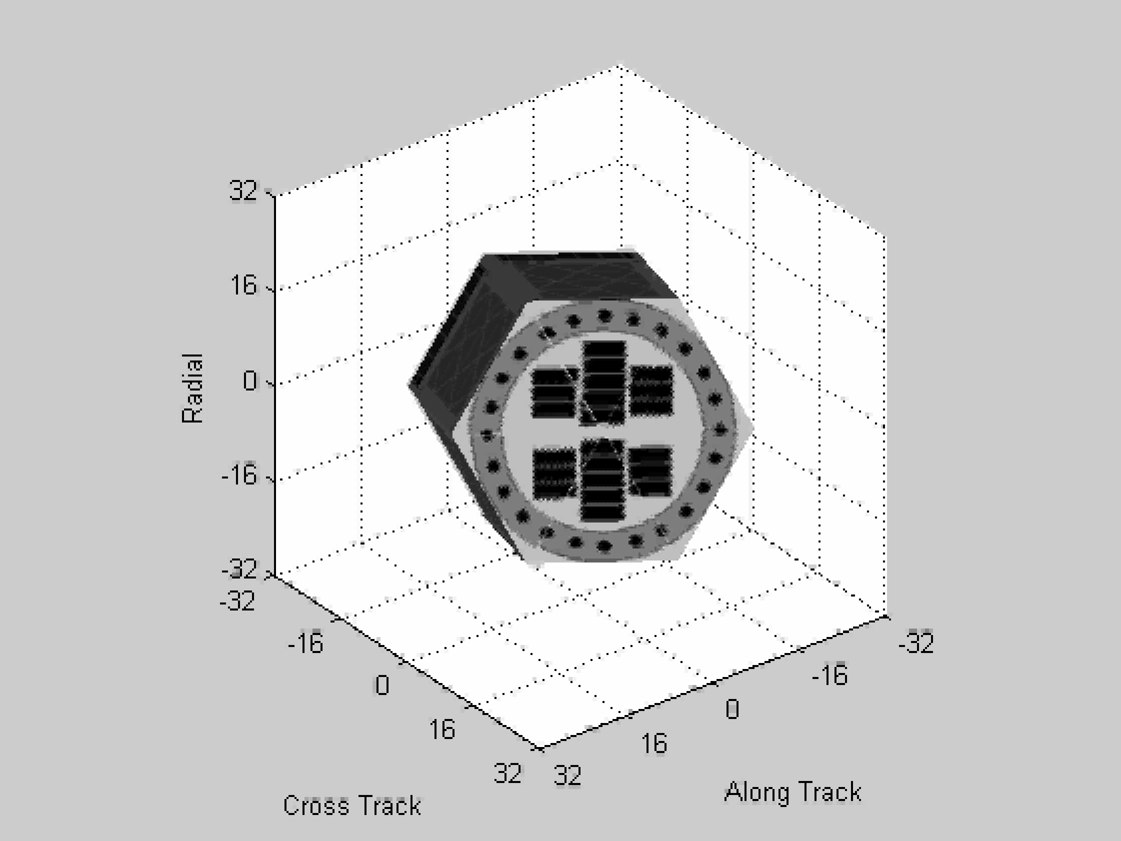
Conclusion

- UMR SAT Design Team
  - Need for Novel ADAC Design
- Real-Time, Magnetometer-Only Attitude Determination
- Revolutionary Orbit Determination
  - Nonlinear θ-D Technique
  - Staggered Filter Implementation
- Orbit/Attitude Control with θ-D

Questions?



32
16
0
-16
-32
Radial
-32
-16
0
16
32
Cross Track
32
16
0
-16
-32
Along Track

Backup Slides

SDRE Filter

Actual system dynamics:

$$\dot{x} = F(x)x + \Gamma w$$

$$z = H(x)x + v$$

Linear-like structure!

$$E\left[w(t)w(\tau)^T\right] = W\delta(t-\tau)$$

$$E\left[v(t)v(\tau)^T\right] = V\delta(t-\tau)$$

Noise spectral densities

Filter dynamics:

$$\dot{\hat{x}} = F(\hat{x})\hat{x} + K(\hat{x})\left[z - H(\hat{x})\hat{x}\right]$$

Measurement error

Filter gain:

$$K(\hat{x}) = P(\hat{x})H^T(\hat{x})V^{-1}$$

SDRE solution

State dependent Riccati equation:

$$0 = \Gamma W \Gamma^T + P(\hat{x})F^T(\hat{x}) + F(\hat{x})P(\hat{x}) - P(\hat{x})H^T(\hat{x})V^{-1}H(\hat{x})P(\hat{x})$$

Filter Implementation

- Continuous filter issues
  - Discrete time measurements
  - Solution to filter dynamics
- Couple filter with numerical integration
  - Runge-Kutta 2^{nd} order
  - Runge-Kutta 4^{th} order
- Staggered filter concept

Numerical Integration

General filter dynamics:

$$\dot{x} = f\left(x,t\right) = g\left[x,t,z\left(t\right)\right]$$

Measurement needed at *t*!

Runge-Kutta 2nd order:

$$k_1 = f\left(x_n, t_n\right)h$$

$$k_2 = f\left(x_n + k_1, t_n + h\right)h$$

$$x_{n+1} = x_n + \left(k_1 + k_2\right)/2$$

Two measurements needed!

Runge-Kutta 4th order:

$$k_1 = f\left(x_n, t_n\right)h$$

$$k_2 = f\left(x_n + k_1/2, t_n + h/2\right)h$$

$$k_3 = f\left(x_n + k_2/2, t_n + h/2\right)h$$

$$k_3 = f\left(x_n + k_3, t_n + h\right)h$$

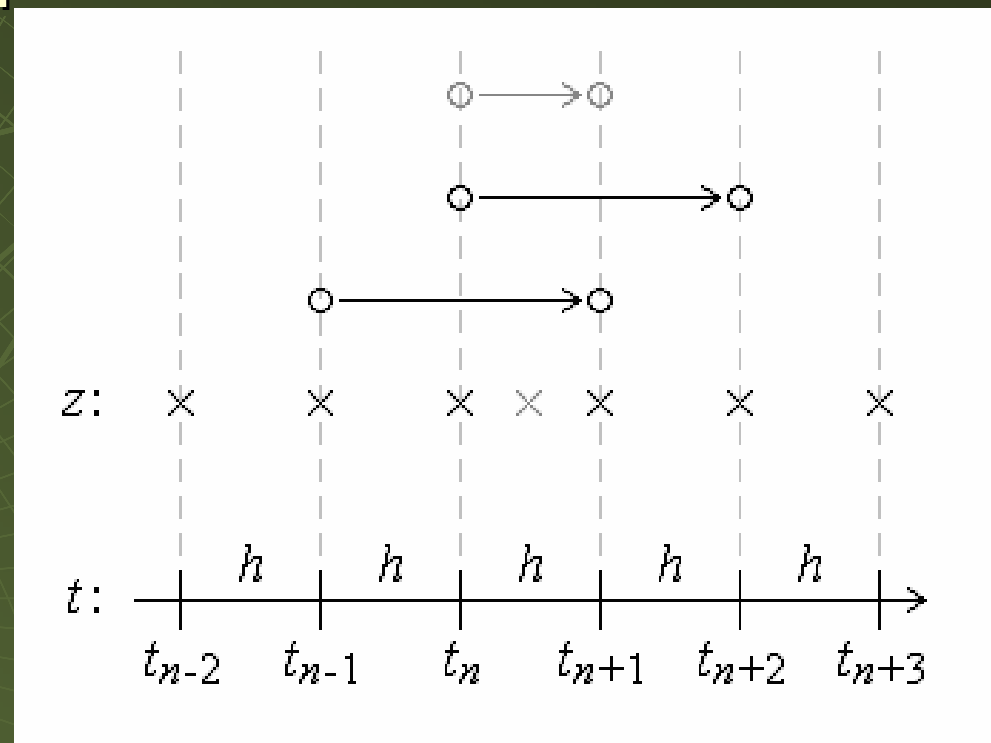
$$x_{n+1} = x_n + \left(k_1 + 2k_2 + 2k_3 + k_4\right)/6$$

Three measurements needed!

Staggered Filter

- Propagate between measurements
- RK2 Integrator
  - Measurements available
- RK4 Integrator
  - Measurement at midpoint needed

- Propagate two measurements intervals
- Measurements available
- Lose state estimate
  - Add second line of estimates



Staggered Filter

- Two filters
  - Alternating
- 4th order accuracy
- Discrete filter
  - Discrete estimates at discrete times
  - Approximates continuous filter