Frank J. Redd
Student Scholarship Competition

Robust Attitude Control with Fuzzy Momentum Unloading for Satellites Using Reaction Wheels

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Background Information

- University Nanosat Program
  - Provided funds for the design, fabrication, and testing of TEST
  - Fostered opportunities for spacecraft research and education
- TEST Nanosatellite Project
  - TEST = Thunderstorm Effects in Space Technology
  - Intent: To study the correlation of various thunderstorm related phenomena.
- 3-Axis Attitude Control
  - TEST experiment requirements led to initial investigations in 3-axis control
  - Further attitude control studies pursued for master's thesis

Figure 1: TEST

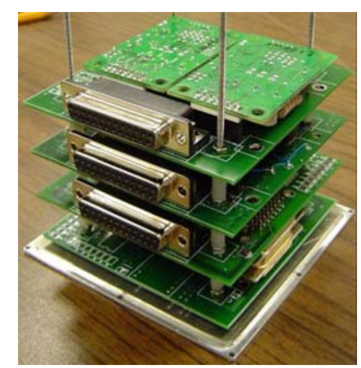


Figure 2: ADCS Electronics

Introduction

- Project Objectives
  - To develop an intuitive and comprehensive spacecraft simulator for satellite attitude control development and testing.
  - To develop a robust reaction wheel attitude control strategy for Earth-pointing low Earth orbit satellites.
  - To develop an adaptable momentum unloading strategy for better power management.
- Industry Relevance
  - Simulation confidence = confidence in mission success
  - Robustness ensures against known model uncertainty
  - Better power management allows for more possibilities

Presentation Overview

- Spacecraft Simulation
  - Coordinate system definitions
  - Modeling the spacecraft orbit
  - Modeling the external disturbances
  - Modeling the attitude response
- Attitude Control
  - Linearization of the dynamic equations of attitude motion
  - Pulling out the inertial uncertainty
  - Controller synthesis and analysis
  - Fuzzy gain-scheduling for magnetic momentum unloading

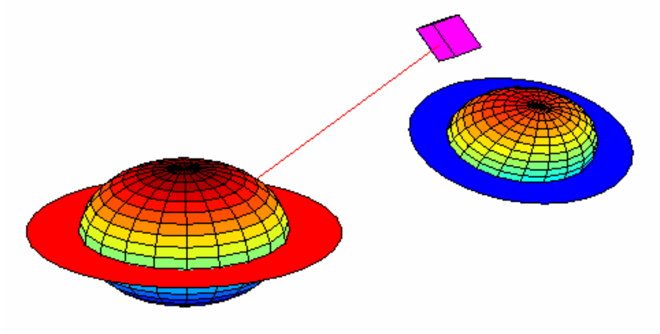


Figure 3: Orbit Simulation

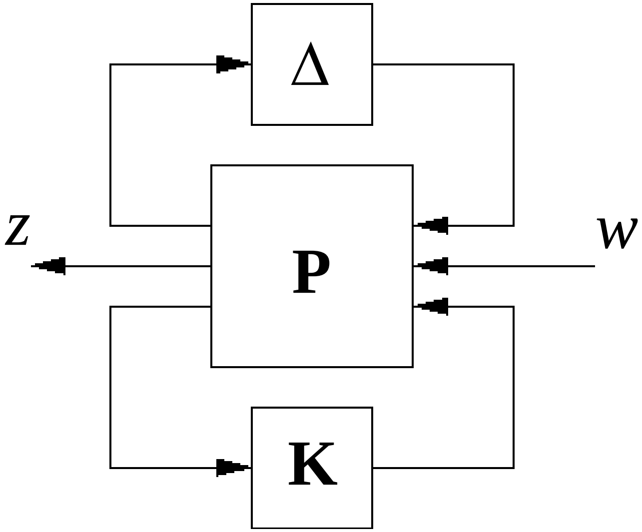


Figure 4: Feedback Diagram

Spacecraft Simulation

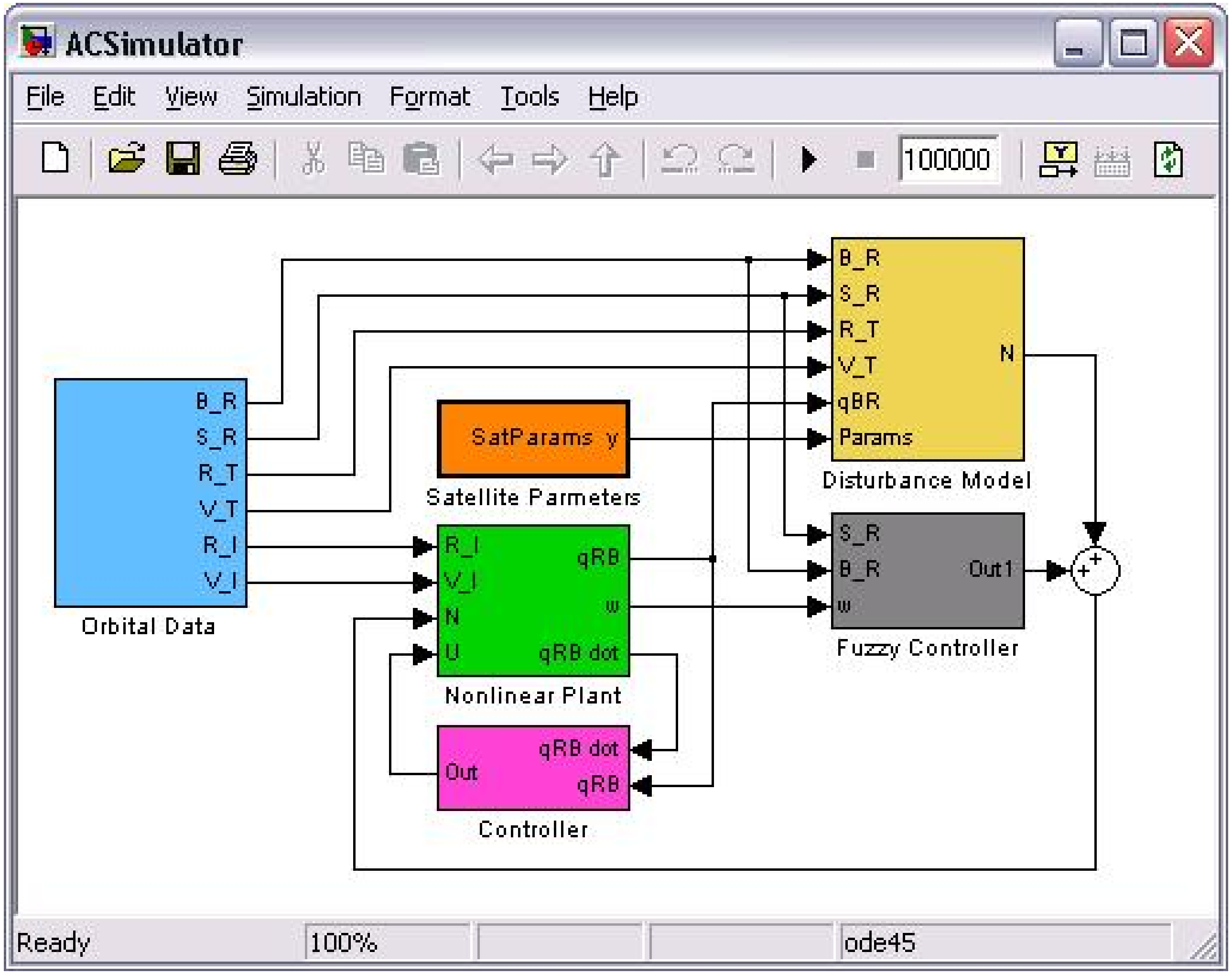


Figure 5: Spacecraft Simulator

Coordinate Systems

- Inertial Coordinate Systems
  - Newton's laws require a fixed inertial frame
  - Geocentric and heliocentric systems are needed for most spacecraft applications
- Terrestrial Coordinate Systems
  - Frames from which satellite observations are made
  - Fixed to the rotating Earth
- Reference Coordinate Systems
  - Satellite-based
  - Frames most often used to describe the satellite attitude

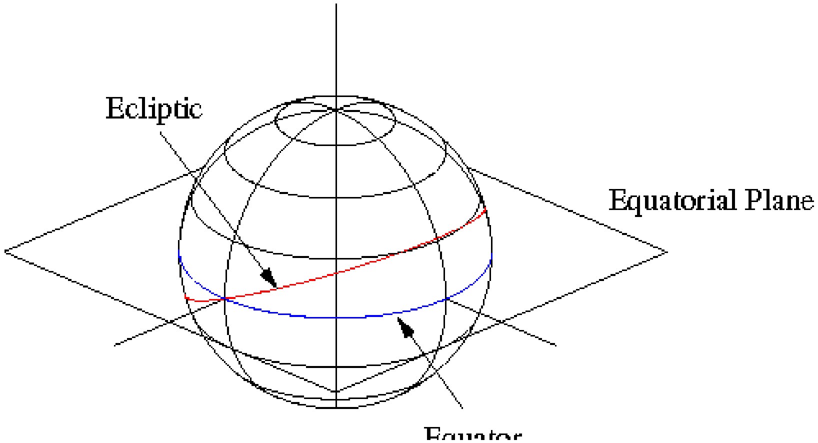


Figure 6: GCRF

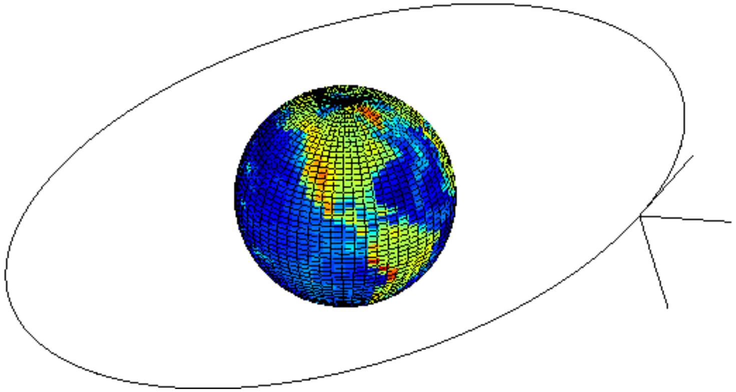


Figure 7: LVLH

Orbit Model

- Orbit model computes the position and velocity of the satellite with respect to the Earth and the Sun
- Based on Newton's law of universal gravitation and Kepler's first law of planetary motion

$$\mathbf{F} = -m\frac{GM}{r^2}\hat{\mathbf{e}}_r \qquad r = \frac{a\left(1-\varepsilon^2\right)}{1+\varepsilon\cos\theta} = \frac{\alpha}{1+\varepsilon\cos\theta}$$

- Cast as a System of First Order Differential Equations

$$\dot{x}_1 = x_2 \qquad x_3(0) = \theta_0$$

$$\dot{x}_2 = x_1 x_4^2 - GM / x_1^2 \qquad x_1(0) = \alpha / \left[1+\varepsilon\cos x_3(0)\right]$$

$$\dot{x}_3 = x_4 \qquad x_4(0) = \sqrt{\alpha GM} / x_1(0)^2$$

$$\dot{x}_4 = -2x_2 x_4 / x_1 \qquad x_2(0) = \alpha\varepsilon x_4(0)\sin x_3(0) / \left[1+\varepsilon\cos x_3(0)\right]^2$$

Orbit Model (Continued)

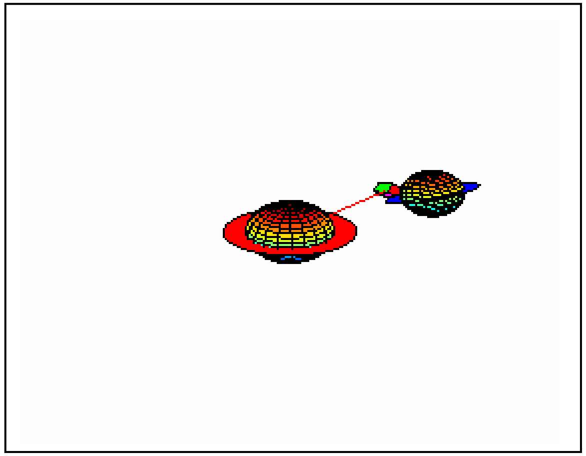


Figure 8: HE Frame

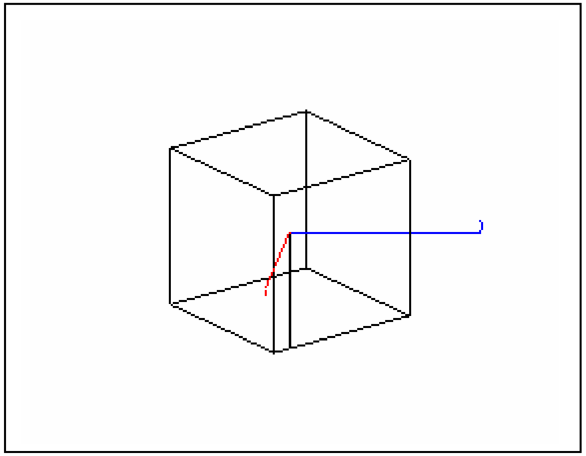


Figure 9: LVLH Frame

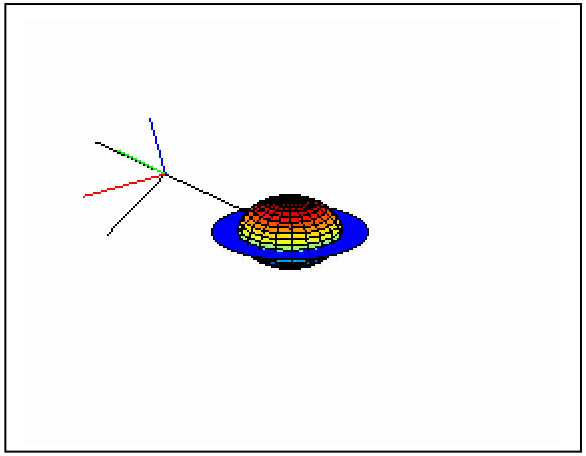


Figure 10: GCRF Frame

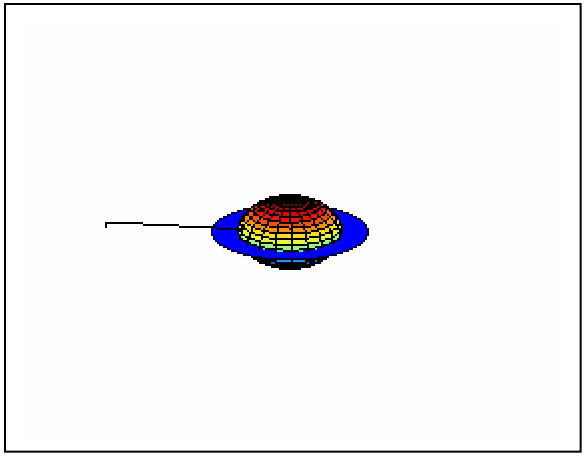


Figure 11: ITRF Frame

Disturbance Model

- Computes the expected environmental disturbance torques for computer simulation
- Gravity Gradient Torque

$$\mathbf{N}_{GG} = \frac{3GM}{R_s^2}\left[\hat{\mathbf{R}}_s \times \left(\mathbf{I}\hat{\mathbf{R}}_s\right)\right]$$

- Solar Radiation Torque

$$\mathbf{N}_{solar} = \int \mathbf{R} \times d\mathbf{F} \qquad d\mathbf{F} = -P\int\left[\left(1 - C_s\right)\hat{\mathbf{S}} + 2\left(C_s \cos\theta + \frac{1}{3}C_d\right)\hat{\mathbf{N}}\right]\cos\theta dA$$

- Aerodynamic Friction Torque

$$\mathbf{N}_{Aero} = \int \mathbf{r}_i \times d\mathbf{F}_{Aero} \qquad d\mathbf{F}_{Aero} = -\frac{1}{2}C_D \rho V^2\left(\hat{\mathbf{N}} \cdot \hat{\mathbf{V}}\right)\hat{\mathbf{V}}dA$$

- Magnetic Dipole Torque

$$\mathbf{N}_{mag} = \mathbf{M} \times \mathbf{B}$$

Attitude Response Model

- Computes satellite attitude response to external disturbances
- Dynamic Equations of Attitude Motion
  - Needed for computer simulations
  - Derived from the general expression of angular momentum
- Kinematic Equations of Motion
  - Integrated to compute the change in attitude over time
  - Differ in form according to the parameterization chosen
- Combined Equations of Attitude Motion
  - Completely describes the rotational motion of the satellite
  - Cast as a system of first order non-linear differential equations

$$\dot{\omega} = \mathbf{I}^{-1}\left[\mathbf{N} - \dot{\mathbf{h}} - \omega \times \left(\mathbf{I}\omega + \mathbf{h}\right)\right]$$

$$\dot{\mathbf{q}} = \frac{1}{2}\Omega\mathbf{q}$$

Linearization

- A linear approximation of the dynamic equations of attitude motion must be found to use linear control techniques

$$\mathbf{N} = \mathbf{I}\frac{d\omega}{dt} + \mathbf{u} + \omega \times \left(\mathbf{I}\omega + \mathbf{h}\right)$$

- Using small angle approximations the angular velocity and gravity gradient torque are expressed in terms of Euler angles
- Substituting these new expressions into the dynamic equation of attitude motion and discarding higher order terms gives

$$\ddot{\phi} = \frac{1}{I_{\hat{\mathbf{x}}\hat{\mathbf{x}}}}\left[\left(\omega_0 h_{\hat{\mathbf{y}}} - a\right)\phi - h_{\hat{\mathbf{z}}}\dot{\theta} + \left(h_{\hat{\mathbf{y}}} + b\right)\dot{\psi} + N_{\hat{\mathbf{x}}} - u_{\hat{\mathbf{x}}} + \omega_0 h_{\hat{\mathbf{z}}}\right]$$

$$\ddot{\theta} = \frac{1}{I_{\hat{\mathbf{y}}\hat{\mathbf{y}}}}\left[-\omega_0 h_{\hat{\mathbf{x}}}\phi + h_{\hat{\mathbf{z}}}\dot{\phi} + c\theta - \omega_0 h_{\mathbf{z}}\psi - h_{\hat{\mathbf{x}}}\dot{\psi} + N_{\hat{\mathbf{y}}} - u_{\hat{\mathbf{y}}}\right]$$

$$\ddot{\psi} = \frac{1}{I_{\hat{\mathbf{z}}\hat{\mathbf{z}}}}\left[-\left(h_{\hat{\mathbf{y}}} + b\right)\dot{\phi} + h_{\hat{\mathbf{x}}}\dot{\theta} + \left(\omega_0 h_{\hat{\mathbf{y}}} + d\right)\psi + N_{\hat{\mathbf{z}}} - u_{\hat{\mathbf{z}}} - \omega_0 h_{\hat{\mathbf{x}}}\right]$$

Parametric Uncertainty

- An LFT framework is sought to handle the plant uncertainty
- Sources of Uncertainty
  - Material property variation
  - Changing environment
  - Unmodeled dynamics
- Parametric Uncertainty
  - Reaction wheel angular momentum is time-varying
  - Inertial properties are difficult to measure/calculate
  - Multiplicative and/or additive uncertainty
- Pulling out the $\delta's$
  - Let the input **w** include the input noise and sensor noise
  - Let the output **z** include the weighted state error **x** and control effort **u**

Parametric Uncertainty (Continued)

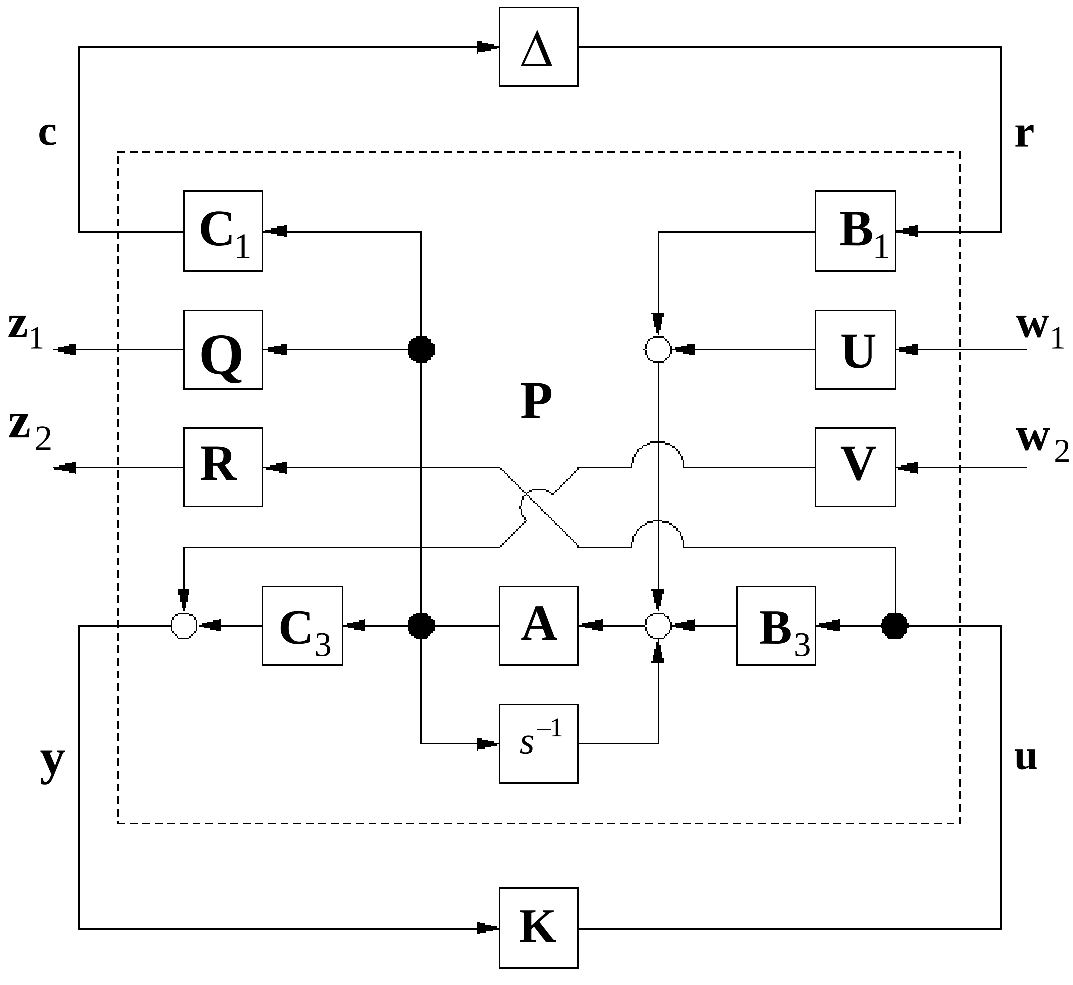


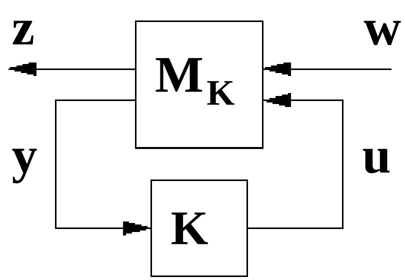
Figure 15: DPK Model

Parametric Uncertainty (Continued)

- State Space Models

$$\Delta = \begin{bmatrix} \delta_1 \mathbf{1}_3 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \delta_2 \mathbf{1}_4 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \delta_3 \mathbf{1}_3 \end{bmatrix} \qquad \mathbf{P} = \left[\begin{array}{c|ccc} \mathbf{A} & \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{B}_3 \\ \hline \mathbf{C}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_2 & \mathbf{0} & \mathbf{0} & \mathbf{D}_{23} \\ \mathbf{C}_3 & \mathbf{0} & \mathbf{D}_{32} & \mathbf{0} \end{array}\right] \qquad \mathbf{K} = \left[\begin{array}{c|c} \mathbf{A_K} & \mathbf{B_K} \\ \hline \mathbf{C_K} & \mathbf{D_K} \end{array}\right]$$

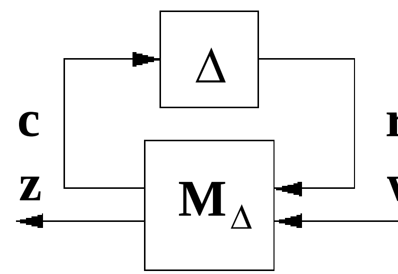
- LFT for Controller Synthesis



$$\mathbf{M_K} = \left[\begin{array}{c|cc} \mathbf{A} & \mathbf{B}_2 & \mathbf{B}_3 \\ \hline \mathbf{C}_2 & \mathbf{0} & \mathbf{D}_{23} \\ \mathbf{C}_3 & \mathbf{D}_{32} & \mathbf{0} \end{array}\right]$$

Figure 16: Lower LFT

- LFT for Stability Analysis



$$\mathbf{M}_\Delta = \left[\begin{array}{c|cc} \mathbf{A} & \mathbf{B}_1 & \mathbf{B}_2 \\ \hline \mathbf{C}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_2 & \mathbf{0} & \mathbf{0} \end{array}\right]$$

Figure 17: Upper LFT

Controller Synthesis

- Standard H_∞ Problem

  Given the dynamical system

$$\mathbf{M} = \left[\begin{array}{c|cc} \mathbf{A} & \mathbf{B}_1 & \mathbf{B}_2 \\ \hline \mathbf{C}_1 & \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{C}_2 & \mathbf{D}_{21} & \mathbf{0} \end{array}\right]$$

  Find the dynamic system **K** that minimizes

$$\|\mathbf{M}\|_\infty = \sup_{\omega \in \Re} \bar{\sigma}\left[\mathbf{M}(j\omega)\right]$$

- Solution to H_∞ Problem
  - Riccati method
  - LMI method
  - Often requires a search

- Remarks on H_∞ Synthesis
  - Minimizes the worst-case effect on the energy of **z** due to the excitation **w**
  - Appropriate when little is known about the spectral characteristics of **w**
- Computing the H_∞ Norm
  - For SISO transfer functions the Bode plot may be used
  - For MIMO state-space systems, the infinity norm is found using the bisection algorithm

Stability Analysis

- Small Gain Theorem

  Assume Δ is a complex ball with bounded norm, then the system given by Figure 17 is well-posed and internally stable for

  $$\|\Delta\|_\infty \leq 1 \; iff \; \|\mathbf{M}(\mathbf{G},\mathbf{K})\|_\infty < 1.$$

- Conservativeness of the Small Gain Theorem

  Small gain theorem ignores any known block diagonal structure of the uncertainty Δ.

- Scaled Small Gain Theorem

  The system $(\mathbf{M},\Delta_a)$ is robustly well-connected (i.e. stable) iff

  $$\mu(\mathbf{M},\Delta_a) = \inf_{\Theta\in\Theta_a} \left\|\Theta\mathbf{M}(\mathbf{G},\mathbf{K})\Theta^{-1}\right\|_\infty < 1,$$

  where Δ_a is some bounded arbitrary block diagonal-structured uncertainty, Θ_a is the commutant of Δ_a, and $\mu(\mathbf{M},\Delta_a)$ is the structured singular value of **M** with respect to Δ_a.

TEST Nanosat Example

- Orbital Angular Velocity

$$\omega_0 = 0.00104 \frac{\text{rad}}{\text{s}}$$

- Satellite Inertial Parameters

$$x_{sat} = 0.30\, m \qquad y_{sat} = 0.30\, m \qquad z_{sat} = 0.45\, m \qquad m_{sat} = 30.0\, kg$$

$$I_{\hat{\mathbf{x}}\hat{\mathbf{x}}} = \frac{1}{12} m_{sat} \left(y_{sat}^2 + z_{sat}^2 \right) = 0.7312\, kg \cdot m^2$$

$$I_{\hat{\mathbf{y}}\hat{\mathbf{y}}} = \frac{1}{12} m_{sat} \left(x_{sat}^2 + z_{sat}^2 \right) = 0.7312\, kg \cdot m^2$$

$$I_{\hat{\mathbf{z}}\hat{\mathbf{z}}} = \frac{1}{12} m_{sat} \left(x_{sat}^2 + y_{sat}^2 \right) = 0.4500\, kg \cdot m^2$$

- Wheel Inertial Parameters

$$r_{wheel} = 0.04\, m \qquad m_{wheel} = 1.00\, kg$$

$$I_{wheel} = \frac{1}{2} m_{wheel} r_{wheel}^2 = 0.0008\, kg \cdot m^2$$

TEST Nanosat Example (Continued)

- Inertial Uncertainty

$$\Delta h_{\hat{\mathbf{x}}} = \delta_1 h_{\max} \qquad \Delta h_{\hat{\mathbf{y}}} = \delta_2 h_{\max} \qquad \Delta h_{\hat{\mathbf{z}}} = \delta_3 h_{\max} \qquad \delta_i \in [-1,1]$$

$$h_{\max} = 0.0008\left(5000 \times \frac{2\pi}{60}\right)\frac{kg \cdot m^2 \cdot rad}{s}$$

$$\Delta I_{\hat{\mathbf{x}}\hat{\mathbf{x}}} = \delta_4 \widetilde{I}_{\hat{\mathbf{x}}\hat{\mathbf{x}}} \qquad \Delta I_{\hat{\mathbf{y}}\hat{\mathbf{y}}} = \delta_5 \widetilde{I}_{\hat{\mathbf{y}}\hat{\mathbf{y}}} \qquad \Delta I_{\hat{\mathbf{z}}\hat{\mathbf{z}}} = \delta_6 \widetilde{I}_{\hat{\mathbf{z}}\hat{\mathbf{z}}} \qquad \delta_i \in [-1,1]$$

- Assume unity weights on all inputs and outputs
- μ-analysis Results using DK-iteration

$$\mu(\mathbf{M}, \Delta)_{H_\infty} < 0.12$$

- Conclusions and Remarks
  - Produces stable results within the specified range of uncertainty
  - Stability and performance cannot be assessed in terms of $\|\mathbf{M}\|_\infty$

Momentum Unloading

- The reaction wheels will saturate if an external torque greater than the sum of the environmental torques is not applied.
- Basic Control Law for Magnetic Momentum Unloading

$$\mathbf{M} = -\frac{k}{B^2}\left(\mathbf{B} \times \Delta\mathbf{h}\right) \qquad \mathbf{T} = -\frac{k}{B^2}\left[B^2 \Delta\mathbf{h} - \mathbf{B}\left(\mathbf{B} \cdot \Delta\mathbf{h}\right)\right]$$

- Determining the Control Gain *k*
  - Control law is time-varying because **B** is time varying
  - Search must be performed to find a feasible solution
- Problems with Constant Gain Solution
  - Does not take advantage of ideal unloading conditions
  - May result in power failure during critical satellite operations
- Advantages of a Fuzzy Logic Gain-scheduler
  - Relatively simple to implements
  - Fairly robust under a changing environment

TEST Nanosat Example

- Assume the same satellite parameters as in the last example
- Step 1: Input Variables and Ranges
  - Let the first input describe the relative orientation of the Sun vector to the solar array normal vectors $\mathbf{N}_k$
  - Let the second input describe the relative orientation of the **B**-field to the momentum error vector $\Delta\mathbf{h}$.
  - Note that this particular choice of inputs is somewhat arbitrary

$$u_1 = \sum_{i=1}^{n} \hat{\mathbf{S}} \cdot \hat{\mathbf{N}}_k \qquad u_2 = \left| \hat{\mathbf{B}} \times \Delta\hat{\mathbf{h}} \right|$$

- Step 2: Output Variables and Ranges

$$k \in [0,1]$$

- Step 3: Fuzzy Membership Functions
  - Assume 3 evenly distributed membership functions for each input
  - Assume 5 evenly distributed membership functions for the output

TEST Nanosat Example (Continued)

- Step 4: Fuzzification
  1. If u1 is low and u2 is low, then k is very low.
  2. If u1 is low and u2 is med, then k is very low.
  3. If u1 is low and u2 is high, then k is med.
  4. If u1 is med and u2 is low, then k is very low.
  5. If u1 is med and u2 is med, then k is very low.
  6. If u1 is med and u2 is high, then k is high.
  7. If u1 is high and u2 is low, then k is med.
  8. If u1 is high and u2 is med, then k is high.
  9. If u1 is high and u2 is high, then k is very high.
- Step 5: Defuzzification

  Assume a centroid deffuzzification with cutoff

$$k = \frac{\int x_i \mu(x_i)}{\int \mu(x_i)}$$

TEST Nanosat Example (Continued)

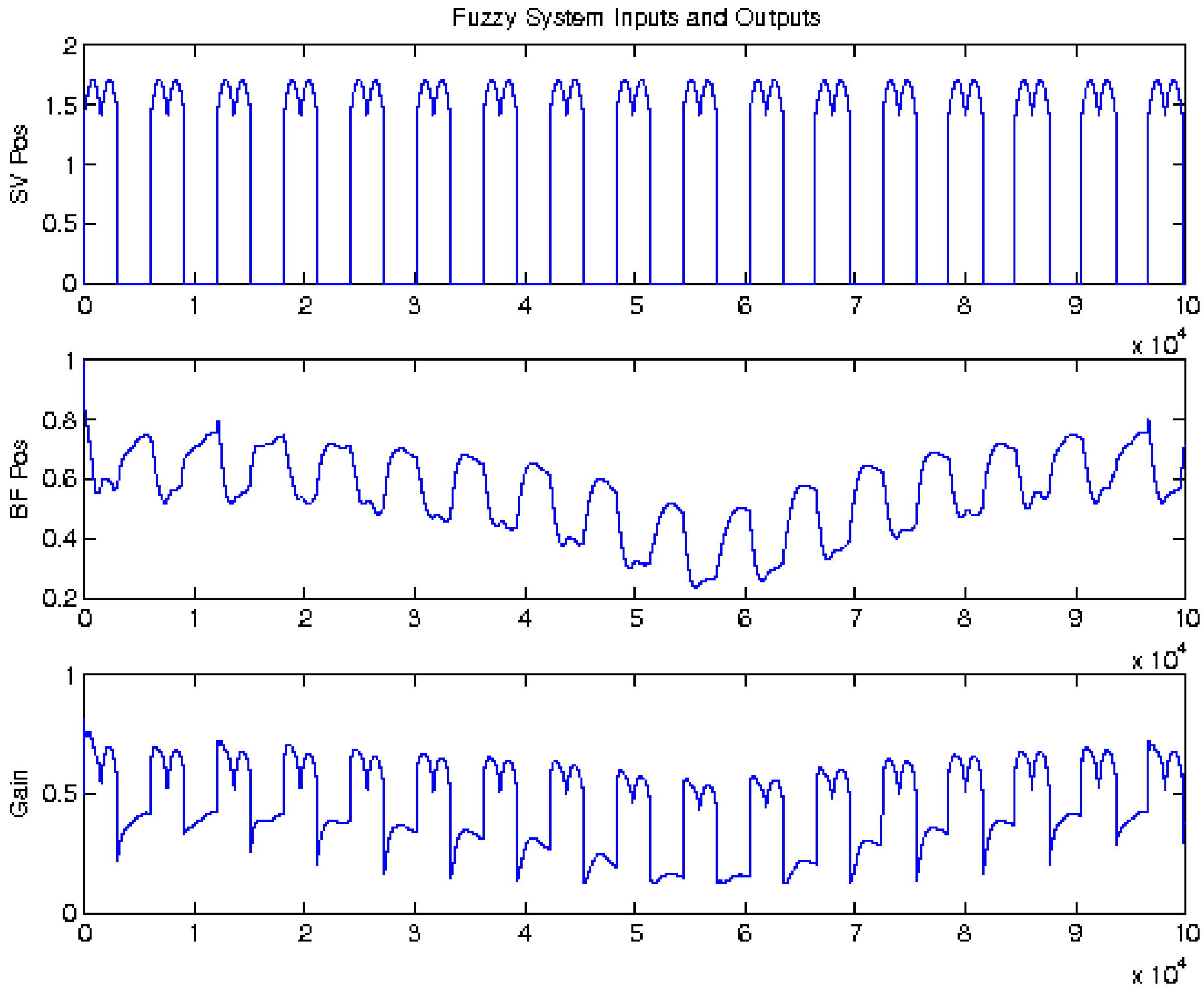


Figure 18: Fuzzy Inputs and Outputs

TEST Nanosat Example (Continued)

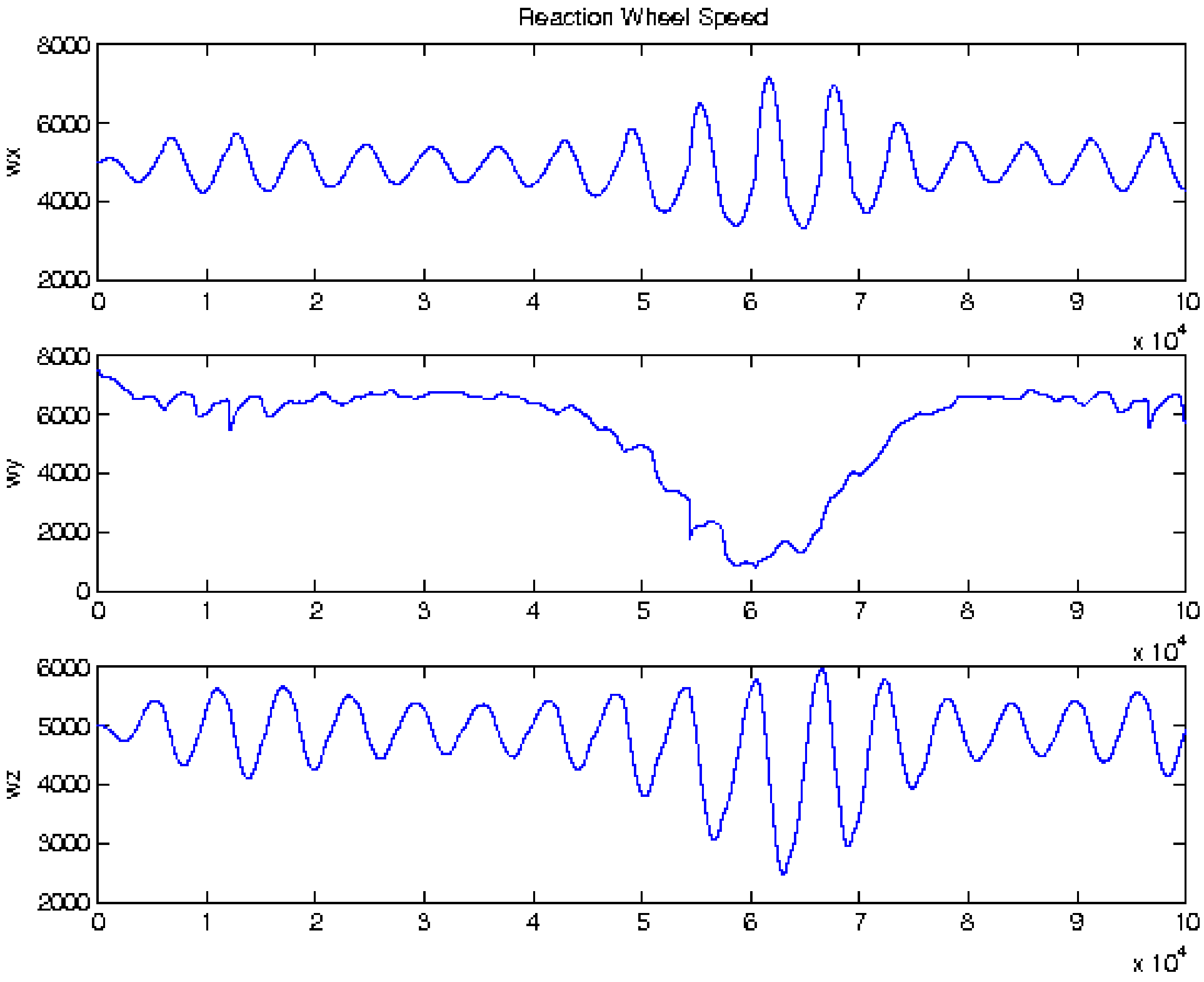


Figure 19: Wheel Speeds

Conclusion

- Summary of Major Themes
  - Practical spacecraft simulation tutorial
    - Coordinate systems
    - Orbit model
    - External disturbance model
    - Attitude response model
  - Robust control in the presence of parametric uncertainty
    - LFT formulation
    - H_∞ synthesis method
    - Stability analysis using the structured singular value
  - Fuzzy gain-scheduling for momentum unloading
    - Basic magnetic control law
    - Power management using fuzzy logic