1 Introduction

Lake warming due to climate change can change the mixing dynamics of deep lakes. In the summer months, lakes stratify with warm water layers on top and cold layers on the bottom. The surface water becomes separated from the deep water through a density gradient. The thermocline is a layer of rapid transition in temperatures separating warm surface water from deep cold water. In the fall, the surface water cools down, destroying the gradient. As a result the layers mix and algae are washed out. Winter mixing oxygenate the deep water and brings nutrient-rich water to the surface. Climate warming can prevent winter mixing, affecting water quality and local ecosystems.

Another natural layering phenomenon are winter inversions, a condition in which a layer of cold air is trapped under a layer of warmer air, leading to the accumulation of pollutants close to the ground. A picture of the phenomena can be seen in figure 1.

In groups you will create your own thermoclines, collect data on the temperature diffusion between the two liquids, and then create a model to describe what you observed.

Figure 1: Examples of layering phenomena in nature. The left image is summer lake stratification where warmer layers on top, cooler layers on the bottom, and separated by a transition layer, the thermocline. The right image is winter inversion where in a valley cool air is trapped under a layer of warmer air.

What variables/parameters may be important when modeling this phenomenon?
2 Procedure

You will form small groups of 3-4 students. With your group you will measure the temperature gradient between two liquids (coffee and milk) that are at different initial temperatures. You will then create a model to describe the thermoclines.

2.1 Materials

- Hot coffee or tea
- Milk
- Clear, straight sided glass
- Ruler
- Thermometer
- A small funnel attached to a straw or thin turkey baster
- Stopwatch or other time keeping device

2.2 Setup

1. Prepare the glass for measuring the liquids’ temperature at different heights. You may choose to attach the ruler to the side of the glass, or mark 1 cm increments on the glass with a wet-erase marker.

2. Pour about 6 cm of coffee into the glass.

3. Place the funnel at the bottom of the glass and SLOWLY pour 6 cm of milk into the bottom of the glass. You will see the coffee begin to rise as a layer of milk forms underneath. Be careful not to mix the two layers as you pull out the funnel.

2.3 Data Collection

1. Measure the temperature of the liquid in 1 cm increments and record your data. Move your thermometer slowly so as not to stir the liquids. The thermometer may take a moment to accurately measure the temperature so be sure to give the thermometer enough time before moving to the next height.

2. Record the temperature of each 1 cm increment every 5 minutes for at least 20 minutes.
You may use the table to record your data

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3 Modeling

It is up to your group to explore different ways to describe the thermocline you observed. You will report your findings so you should discuss:

- What do your parameters mean?
- What assumptions did you make?
- What would happen if we changed:
  - the initial temperatures?
  - the amount of initial liquids?
  - the surrounding temperature?
- How does this confirm/invalidate any assumptions that you made for your alternate model?
- At what point is the rate of change of the temperature with respect to time the greatest?
- At what point is the rate of change of the temperature with respect to height the greatest?
- What do you expect to happen after a long time, or mathematically speaking, when time goes to infinity?
- How many variables does a function have that models the temperature at any height and time?

3.1 Comparison to a plane

One basic surface is a plane. Recall that the general equation for a plane is

\[ ax + by + cz = d \]  

With your group discuss what values of \( a \), \( b \), \( c \), and \( d \) make a plane that resembles your data. Be sure to graph your data in Matlab along with your proposed plane. How good is the fit? What might the parameters mean?

3.2 Linear Interpolation

Another method to fit your data is linear interpolation. With your group graph the initial temperature of your data with respect to height. Then graph your final temperature with respect to height. Next linearly interpolate from your initial temperatures to your end temperature. How good is the fit?

3.3 Fun with Functions

Now take this opportunity to explore other options for modeling the thermocline. Have you seen other functions with the shape of your data? Compare and contrast this method with the other methods your group tried.
4 Report

4.1 General Writeup Guidelines

Each writeup we will do in this class should have the following format:

- **Introduction** - Contains a general discussion of the problem
- **Methods** - Contains a summary of lab setup, how data was collected, description of model, and parameterization
- **Results** - Contains an analysis of the model
- **Discussion and Conclusion** - Contains an evaluation of results
4.2 Specific Lab Items

For this specific lab your writeup should include the following items in the indicated sections:

**Introduction -**

- A description of the lab.
- A description of the phenomenon you are trying to model, including 1-2 paragraphs containing background information about winter mixing in deep lakes.

**Methods -**

- Explanations of all of your data fits.
- Estimation of parameters (and their units) as well as the procedure used to estimate them.
- What assumptions did you make?

**Results -**

- A table with your raw data.
- Plots of your simulation data and your solutions to modeling it.

**Discussion and Conclusion -**

- What are the strengths and weaknesses of your model?
- What changes you might make to improve its fit to the data?
- Generally speaking, in what ways would a model for winter mixing be useful to scientists studying winter mixing in lakes under changing climate conditions?
- Address the discussion topics in section 3.
- What did you learn from this experience?