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Viscous and Induced Heating in Plasma Focus Plasmoids

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Abstract

Recently, Abolhasani et al. proposed that the high ion energies observed in plasmoids formed in the plasma focus could be explained by viscous heating. We here elaborate this proposal, demonstrating that during plasmoid formation, ion motion along magnetic field lines can be rapidly converted, at least in part, to thermal energy through viscous diffusion. This effect is strongly enhanced by higher-z ions. We compare the theoretical predictions with the recent observation by Lerner et al., of trapped ion energies of 160 keV.

In addition, we propose a second source of heating. The mildly relativistic electron beam emitted by the plasmoid, generates an induced current within the plasmoid comparable to the beam current and confined to approximately the same region. The induced current electrons, with drift velocity $v_{de} << v_b$ of the beam electrons, are thus far more effective than the beam itself in ohmically heating the plasmoid. We show that both these mechanisms are capable of generating ion energies of tens to hundreds of keV for a wide variety of plasmoid conditions.
Plasma focus devices

- A Plasma focus device is a pulsed power source capable to produce dense and hot plasma.

- It produces short bursts of energetic electrons, ion beams, electromagnetic radiations, and fusion reactions.

- It consists of two cylindrical electrodes nested inside each other. The electrodes are enclosed in a vacuum chamber with a low-pressure gas filling the space between them.
We have detected trapped ions with mean energies up to 160 keV [1]. It is well known that this range of ion temperatures cannot be adequately explained in terms of classical heat transfer from the electron population.

It was recently proposed recently that these high-energy ions observed in the plasma focus could be explained by viscous heating [2].

In fact, as the ions temperature raises the ion parallel viscosity effects will be important for fast plasmoid heating.

Our calculations show that for 1 keV plasmoid the viscosity coefficient is around 120 poise which means that the plasmoid is highly viscous.
In a strong magnetized plasma ($\omega_{ci}\tau_i$ and $\omega_{ce}\tau_e \gg 1$) where the gyrocyclotron frequency is much more than collision frequency between ions and electrons, the viscous stress tensor is given by [3]:

$$
\pi_{ij} = 3\eta_0 \left( \frac{\delta_{ij}}{3} - \frac{B_i B_j}{B^2} \right) \left( \frac{B \cdot (B \cdot \nabla) u}{B^{-2}} - \frac{\nabla \cdot u}{3} \right)
$$

The presence of magnetic field leads to significant differences between momentum transfer along the magnetic field and across the field.
The ion viscosity coefficient parallel to magnetic field is given by:

$$\eta_0 = 0.96n_i T_i \tau_i$$

Since the ion viscosity exceeds the electron viscosity by the square root of mass ratio, the plasma viscosity is almost determined by the ion velocity. All the quantities are in cgs units except temperature expressed in eV.
The mean time between momentum changing due to ions collisions in all possible interacting angles with a Maxwillian distribution is given by [4]:

\[
\tau_i = 2.09 \times 10^7 \frac{\mu^{1/2} T_i^{3/2}}{z_{eff}^4 n_i \ln \Lambda}
\]

Here, \( \mu = m_i/m_p \) and \( \ln \Lambda \) the Coulomb logarithm.
Effects of ionized impurity particles

In the case of mixed plasma with $N$ ion species, the effective charge on ions comes from:

$$Z_{eff} = \left( \sum_{i,j=1}^{N} \frac{n_i n_j z_i^2 z_j^2 \zeta_{ij}}{n_{tot}^2} \right)^{1/4}$$

Where $z_i$ the charge on $i$ ion species, $\zeta_{ij}$ is 1 when $i=j$ and $1/2$ in the other cases, and $n_{tot}$ the total density of plasma.
Viscous heating

- Viscosity arises from the shear velocity between the fluid layers \( \frac{\partial u_i}{\partial x_j} \) that ultimately opposes any applied force and transforms kinetic energy of macroscopic motion into heat energy.

- It play an important role in the dynamics of fluids with strongly temperature-dependent viscosity coefficients because of the coupling between the energy and momentum equations.
The volumetric heating rate due to viscosity is [4]:

\[ Q_v = - \sum_{i,j} \pi_{ij} \frac{\partial u_i}{\partial x_j} \]

Consider the coordinate system with \( z \) axis parallel to the magnetic field \( \mathbf{B}=B_\mathbf{z} \), then:

\[ Q_v = \frac{\eta_0}{3} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} - 2 \frac{\partial u_z}{\partial z} \right)^2 \]
In the case of kinking to a helical coil for force-free magnetic field plasma, the plasma expansion occurs with, we believe, Alfven velocity along the magnetic field.

In the case of Lundquist number $L_u < 2$, the wave energy will be converted to ion thermal energy in a time around $\lambda/V_A$ where $\lambda$ is the wavelength of waves that are damped [5]. Therefore:

$$\frac{\partial u_z}{\partial z} \approx \frac{V_A}{\sqrt{3} \lambda} \quad \text{and} \quad \lambda = \pi v_{th}^2 \tau_i / 3V_A$$
Therefore, the volumetric heating rate due to viscosity is:

\[ Q_v = 0.41p_i v_i \beta^2 \]

Where \( p_i \) is the plasma pressure, \( v_i \) the collision frequency, and \( \beta \) the kinetic to magnetic pressure. Viscosity transforms kinetic energy of motion into thermal energy and so causes ions temperature increment. Therefore, the ion power density can be:

\[ Q_i = -\frac{3}{2} n \frac{\partial T_i}{\partial t} \]
Finally, we conclude:

\[ \frac{\partial T_i}{\partial t} = 3.22 \times 10^{13} \frac{z_{eff} \ln \Lambda B^4}{\mu^{1/2} n_i T_i^{5/2}} \]

So the ion temperature should be:

\[ T_i = 1.03 \times 10^4 \frac{z_{eff}^{8/7} \ln \Lambda^{2/7} B^{8/7}}{\mu^{1/7} n_i^{2/7}} \tau^{2/7} \]
However, if $\lambda > L_{\text{max}}$, where $L_{\text{max}}$ is longest path length in the plasmoid, then all waves are damped and heating ceases, unless bulk motion continues. So for $\lambda \leq L_{\text{max}}$ we have:

$$T_i \leq 6.2 \times 10^{-4} z_{\text{eff}}^{1.6} n_i^{0.2} (\ln \Lambda L_p B)^{0.4}$$
Comparison to FF-1 experiment

- We observed plasmoids with, Length $L_p=0.15$ (cm), $n_i=3\times10^{19}$ (cm$^{-3}$), and $B=4\times10^8$ (Gauss).

- Also, our theory of DPF predicts $L_{max}$ around helical filament as $L_{max} = 9.7 (\mu z)^{1/3}L_p$ which with pure deuterium ($z=1$) should be 1.8 cm [6].

- Therefore, the predicted $T_i$ with pure deuterium is 53 keV, less than observed max of 160 keV.
Comparison to FF-1 experiment

However, ion temperature depends on $z_{\text{eff}}$ and so considering the plasmoid contaminated with 0.5% Cu by number, we have $z_{\text{eff}} = 2.08$. In this case, the ion temperature increases to $T_i = 170$ keV.
Viscosity should produce change with heavy gas mix

Comparison with previous experiments in 2010 with Dena (Tehran), Fillipov-type DPF with 0.8 MA peak current [7].
Induced Return Current

- Relativistic e-beam electrons have too small *x-section* to heat by ohmic collisions but long known that e-beams induce return current in plasma if \( n_p \gg n_b, \omega_{pe} \gg \omega_b \) and \( r_b/(c/\omega_{pe}) \gg 1 \).

- In fact, the beam current introduces the magnetic field to plasma and this return current acts to neutralize the beam-induced magnetic field.

- Finally, the collision between the induced high-density and velocity electrons with background ions in thermal energy will introduce the ohmic heating.
Gerwin finds that the power loss fraction $P$ is [8]:

$$ P = 8 \left( \frac{I}{I_A(\gamma - 1)} \right) f \left( \left( \frac{L}{r_b} \right) \left( \frac{v}{\omega_p} \right) \left( \frac{c}{\omega_p r_b} \right) \right) = 8 \left( \frac{I}{I_A(\gamma - 1)} \right) f(x) $$

Where $P$ is fraction of beam power going to ohmic heating of plasma electrons and since for our plasmoids $x \ll 1$ so $F(x) = x/2$. $I_A$ is the Alfven current 17 kA, and $I$ the beam current. Substituting in standard values:

$$ P = 8.7 \times 10^{-9} \frac{z_{eff}^2 ln\Lambda LI}{Z(\gamma - 1)r_b^2 T_e^{3/2}} $$

Where $Z$ is her the ratio of electron density to ion density.
Our theory and observation, show total e-beam power per plasmoid electron to be $P_{\text{tot}} = 7.2 \times 10^5 / \tau_p$ (eV) so:

$$\frac{\partial T_e}{\partial t} = 6.3 \times 10^{-9} \frac{z_{\text{eff}}^2 \ln \Lambda LI}{Z(\gamma - 1)r_b^2 T_e^{3/2}}$$

Integrating and setting $t = \tau_p$ we have:

$$T_e = 0.19(z_{\text{eff}} r_b)^{0.8} \left(\frac{\ln \Lambda LI}{Z(\gamma - 1)}\right)^{0.4}$$

For example when $r_b = 8$ m and $I = 200$ kA, we have $T_e = 51$ keV
Lower hybrid waves, too

- Calculations now underway indicate Low Hybrid Drift Instability heating will be highly relevant for denser plasmoid and will channel more heat into ions rather than electrons.

- This happens when the electron drift velocity is equal or greater than ion thermal speed. Then the effective total resistivity of plasma will increase due to the generation of LH waves and their transfer of energy to the plasma ions.
Conclusions

- Viscous heating of DPF plasmoids can explain high trapped-ion energies, noble gas mix results
- Induced currents from electron beams can heat plasmoid electrons significantly
References


