1 Introduction

Imagine, if you will, that you are taken captive by an “evil genius” (AKA your teacher). This genius truly is evil, and has quite a diabolical plan for you.

“I have a container of liquid.” says the Evil Genius. “If you are to make it out of here alive you must tell me how long it will take for the liquid to drain out of my container. After you have made your guess we will start the flow of the liquid and see if you will survive. Are you up to the challenge?”

In an attempt to survive you will be allowed to work with fellow captives in an initial “testing” phase where you will measure data from a basic experiment before you go up against the Evil Genius. It is up to you to ensure that you have plans to measure all the parameters needed in your model. This may involve different levels of ingenuity, flexibility, and special equipment from the instructors, depending on the models used. The Evil Genius has agreed to play by a few rules. Holes on more than one level will not be used, however multiple holes may be used. The shape and size of the holes will also be freely adjusted. Can you survive?

2 Procedure

With your group you will measure the height of a liquid draining out of a container as a function of time. You will then create a model to describe the situation.

2.1 Materials

The following materials are needed:

- A plastic bottle (2 liter soda bottles work well)
- Water
- Bucket to drain the water into
- Drill, scissors, or other sharp object to make a hole in the bottle
- Ruler
- Stopwatch
- Optional: Marker
- Optional: Tape
- Optional: Funnel

2.2 Setup

1. Record the cross-sectional area of your container.

2. Make a hole in your container and record its cross-sectional area.

   You may also want to record the location of the hole for modeling purposes.

3. Prepare the container for measuring when the liquid is at different heights. You may choose to attach the ruler to the side of the container, or mark increments on the container with a marker.
2.3 Data Collection

To run the experiment, assign one group member to be your “Timekeeper,” one member to be the “Water Watcher,” and one member to be the “Data Recorder.”

1. Fill the container with water, covering the hole with tape or your finger

2. Uncover the hole and start your stopwatch. The “Water Watcher” will call out each time the liquid reaches a height increment and the “Timekeeper” will hit the “lap” button on their stopwatch.

3. After the container is empty, the “Data Recorder” will record the heights and the time it took to reach them.

3 Modeling

3.1 Torricelli’s Law

Torricelli’s Law is the classic starting place for relating the speed of fluid flowing out of an opening to the height of the fluid above the opening

\[
\frac{dh}{dt} = -\frac{a}{A} \sqrt{2gh}
\]  

(1)

where \(a\) is the cross-sectional area of the opening, \(A\) is the cross-sectional area of the container, \(h\) is the height of the fluid above the opening as a function of time, and \(g\) is the acceleration due to gravity. In class we solved this equation for \(h\) giving

\[
h(t) = \left(\sqrt{h_0} - \frac{a}{\sqrt{2A} \sqrt{gt}}\right)^2
\]  

(2)

It is up to your group to parameterize Torricelli’s Law to fit your data. You will report your findings so you should discuss:

- Is this model a good fit?
- What are the error sources?
  Is there anything poorly accounted for by the model?
- What would happen if the cross-sectional area of the container was not constant?

3.2 Alternative Model

Now it is up to your group to come up with a model that is significantly different than Torricelli’s Law. You may have found that Torricelli’s Law is not a very good model of the situation, and so strive to make your alternative model better match the situation. You will report your findings so you should discuss:

- Is this model a good fit?
- What are the error sources?
  Is there anything poorly accounted for by the model?
• What do the parameters in the model mean?

When you have finished your modeling you will be given a container by the Evil Genius and asked to use your alternative model to find how long it will take for the container to empty. This container may be drastically different from the containers we did our initial experiment with so in your modeling process you may find it helpful to try the experiment again with different conditions. For example, you may choose to change

• The size of the exit hole
• The shape of the exit hole
• The size of your container
• The shape of your container
• You may also try having a non-constant cross-sectional area with your container

The group with the model that most closely matches the Evil Genius’ container will be declared the survivors.

4 Report

4.1 General Writeup Guidelines

Each writeup we will do in this class should have the following format:

• Introduction - Contains a general discussion of the problem

• Methods - Contains a summary of lab setup, how data was collected, description of model, and parameterization

• Results - Contains an analysis of the model

• Discussion and Conclusion - Contains an evaluation of results
4.2 Specific Lab Items

For this specific lab your writeup should include the following items in the indicated sections:

Methods -

- Estimation of parameters (and their units) for Torricelli’s Law using your data
- A model that is significantly different than Torricelli’s Law to predict the height of the fluid as a function of time. Be sure to include what your parameters mean and why you chose them
- Estimation of parameters (and their units) for your alternative model as well as the procedure used to estimate them
- What assumptions did you make? What variables/mechanisms did your group focus on with your model?

Results -

- Plot of your data and the parameterized Torricelli model
- Plot of your data and your parameterized alternative model

Discussion and Conclusion -

- Which model better reflects the data, and why?
- What did you learn from this experience?