Structure of matter, 6

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Quantum Flavor Dynamics (QFD), 1

That each generation of the quark and lepton periodic tables (i.e., electron, muon, and tau—see SM 1, p.1) has two members (a neutrino and a charged particle) that can be flipped into one another by emission or absorption of W bosons is reminiscent of how the angular momentum spin-1/2 component of a charged particle can be flipped between “up” and “down” orientations along some direction (z) in space by emission or absorption of photons. This analogy is made more graphic by labeling flavor rows by a new kind of “spin” (completely unrelated to angular momentum), namely, weak isospin component, \( I \). The neutrino flavors are assigned a weak isospin component value of +1/2 (“up”), and the charged lepton flavors, –1/2 (“down”). Similarly, the \( uct \) quarks are assigned +1/2 weak isospin components, and the \( dsb \) quarks, –1/2.

The local gauge theory story for the origin of the electromagnetic force of QED emerges from the indifference of nature to the exact value of the complex phase of the Dirac field describing a charged particle. Similarly, the local gauge theory story for the color force of QCD emerges from the indifference of nature to the exact color of the Dirac field describing a quark. That is, the “world” appears to be symmetric under continuous phase and color transformations. Requiring these transformations to be (local) functions of position and time leads to the necessity of having electromagnetic and color potential fields. The particles of these are spin-1 bosons that respectively “carry” the electromagnetic and color interactions. In other words, electromagnetic and color forces can be understood as arising from symmetry. It is natural to wonder if the world is similarly symmetric under (local) weak isospin–flavor changing–transformations. Of course, it can’t exactly be because changing a muon into a muon neutrino or a \( d \) quark into a \( u \) quark produces change in mass. But, to start, let’s ignore this potentially embarrassing fact.

What would a local gauge theory of flavor changing interactions look like? To start, leptons and quarks both have angular momentum spin-1/2. Their fields, \( \psi \), therefore obey the Dirac Equation. Each part of \( \psi \) has two additional components corresponding to “up” and “down” weak isospin components. (Leptons don’t carry color, so there are no color components as there were for quarks). Conservation of weak isospin requires that the density of the lepton field, \( |\psi|^2 \), be invariant under isospin transformations represented by 2x2 matrices, \( S : \psi = S \psi \). (These are called SU(2) isospin transformations.) In the spirit of local gauge theory, these transformations can be functions of space and time—i.e., \( S = S(\vec{r},t) \). Such local transformations produce unwanted derivatives-of-\( S \) in the Dirac Equation. To cancel these terms, additional (potential) fields, \( \phi, A \), are required which transform simultaneously with the lepton fields. The components of the potential fields are also 2x2 matrices. Because they obey Maxwell-like equations, their associated particles would be massless and have angular momentum spin = 1, just like photons and gluons.

When discussing the color force, we saw that the force carriers, the gluons, were 3x3 color matrices whose indices could be viewed as a color and an anti-color. In analogy with gluons, the weak isospin changing matrices introduced above can be
thought of as having rows that are isospin values and columns that are anti-isospin values. In other words, the quanta of the isospin potential fields, generically called $W$, can be envisioned as having indices $W^i_{\tau \bar{\tau}}$.

Before proceeding, it is useful to consider how angular momenta are added in quantum mechanics. If the projection of spin-1/2 angular momentum is measured along a ("z") direction in space the result is $\pm 1/2$ ("up" or "down") along that direction. A nice pictorial way of representing "z-component" of isospin $+1/2$ is $\uparrow$ and $-1/2$ is $\downarrow$. Now, two spin-1/2 components can be combined to make an angular momentum that has spin-1 components. There are four possible $+1/2$ and $-1/2$ combinations: $\uparrow\uparrow, \downarrow\downarrow, \uparrow\downarrow, \downarrow\uparrow$. If the components of a total spin-1 angular momentum is measured along a (z) direction in space the result is $+1, 0, -1$ along that direction. By analogy, the isospin component $+1$ corresponds to the picture $\uparrow\uparrow$, $-1$ to $\downarrow\downarrow$. According to quantum mechanics the two up/down pictures can also be combined into $\uparrow\downarrow + \downarrow\uparrow$ and $\uparrow\downarrow - \downarrow\uparrow$ combinations, both with z-component equal to zero. The difference between the two is that when the two arrows in each term are switched, the first combination doesn’t change, while the second gets a negative sign. Note that when the two arrows in the up/up and down/down pictures are switched they also don’t look any different. In other words, the set $\uparrow\uparrow, \downarrow\downarrow, \uparrow\downarrow + \downarrow\uparrow$ forms a “triplet” of total isospin = 1 possibilities, while the $\uparrow\downarrow - \downarrow\uparrow$ combination is a “singlet” combination corresponding to a total isospin = 0, z-component = 0 state. Thus we can think of the $W$ fields as having four possibilities $W_i = W_{i\uparrow\uparrow}$, $W_2 = W_{\downarrow\downarrow}$, $W_3 = W_{\uparrow\downarrow\downarrow}$, and $W_4 = W_{\downarrow\uparrow\uparrow}$.

What do these fields do when they interact with a lepton? The right hand “index” of the $W$ is an “anti-isospin.” If that index is the same as the isospin carried by the lepton then the lepton remains unchanged after the interaction. If it is different, then the lepton’s isospin is annihilated and replaced by the W’s left index: i.e., it changes flavor. The two W’s with both up and down indices annihilate the lepton’s isospin, but then “put it back again”; they do not change flavor.

Gell-Mann’s early organization of the lightest baryons and mesons (SM 3, p.2) employed two hypothetical properties called the z-component of strong isospin ($I_z$) and strong hypercharge ($Y_z$), which were connected to electric charge, $Q$ (in units of electron charge), by the equally hypothetical Gell-Mann-Nishijima formula: $Q = I_z + \frac{1}{3}Y_z$. The triumph of the quark model has rendered these hypothetical properties obsolete, but if weak isospin is a valid property of matter, perhaps so also is weak hypercharge, $Y$, and perhaps also $Q = I_z + \frac{1}{3}Y$. To get the neutral electric charge using this relation of the neutrinos and the negative electric charge of the other leptons requires they carry weak hypercharge = −1, then. In addition, the quarks must carry weak hypercharge = +1/3 (do you see why?). For now, let’s assume all of the seemingly unmotivated ideas for the weak interactions are true.

Assuming the Gell-Mann-Nishijima formula in weak form applies to the $W$ fields, and that their hypercharge is zero, the particles of $W_1$ should have electric charge = +1 and the quanta of $W_2$ should have electric charge = −1; that sounds a lot like the putative $W^\pm$ bosons discussed in SM 5 (p.2-3). So maybe this crazy idea of deriving the
weak interaction from isospin switching symmetry (which, because of the mass difference between flavors in a generation, we know can’t be right) has some merit after all. And if it does, the prediction is that there should be electrically neutral weak exchange bosons as well.

From a more sophisticated version of the model outlined above, Sheldon Glashow, Steven Weinberg, and Abdus Salam in the 1960s predicted a neutral weak-force carrying boson, $Z^0$ (in addition to the $W^\pm$ bosons) and indirect evidence for it was obtained in 1973. GWS were awarded the 1979 Nobel Prize for their prediction, even though direct experimental evidence for the $W^\pm$ and $Z^0$ didn’t come until four years later. These particles were inferred from the analysis of collisions involving beams of protons and anti-protons at CERN in Switzerland, in 1983. In high-energy $p,\bar{p}$ collisions many things can happen, among them $p + \bar{p} \rightarrow W^\pm + e^\pm \nu_e$, $p + \bar{p} \rightarrow Z^0 \rightarrow e^- + e^+$. In the first process the $W^+$ is accompanied by an electron and an electron anti-neutrino, while the $W^-$ is accompanied by a positron and an electron neutrino. To infer the existence of the $W^\pm$ requires looking for events in which high-energy electrons or positrons emerge and not much else. The second process can be confused with the much more likely $p + \bar{p} \rightarrow \gamma \rightarrow e^- + e^+$. The probability of the latter is calculable using QED and this has to be subtracted from the measured yield to look for residual events not explainable by electrodynamics. In any case, billions of events were examined and about ten corresponding to each process were found. That was enough to convince the Nobel committee to award their Prize to Carlo Rubbia and Simon van der Meer. The masses of the $W^\pm$ are both about 80 MeV (the two particles are antiparticles of each other), while that of the $Z^0$ is about 90 GeV. As a consequence, if these particles are responsible for carrying the weak force, the associated range would have to be about $10^{-3}$ nm—about 1/1000 times the size of a nucleus. The weak force is not weak because its intrinsic strength (i.e., $\alpha_w$) is small (it’s actually, about four times stronger than the electromagnetic strength, $\alpha_e$), but because particles have to be so close to interact via $W^\pm$ or $Z^0$ exchange. (Incidentally, the detection of a neutrino by the Cherenkov radiation produced when the neutrino kicks an electron out of an atom (see SM 5) is due to $Z^0$ exchange.)

**Weak interactions violate parity symmetry**

Before worrying about how mass screws up the flavor-changing symmetry, another important aspect of the weak interactions has to be reckoned with. Until 1956, it was common wisdom that the laws of physics worked equally well in the real world or in a mirror reflection of the real world. More precisely, it was believed that physical processes would be identical under the position-vector reflection, or “parity,” transformation, $\vec{r} \rightarrow -\vec{r}$ . Such a transformation has several implications: velocity switches direction, $\vec{v} \rightarrow -\vec{v}$; acceleration switches direction, $\vec{a} \rightarrow -\vec{a}$; because of Newton’s Second Law and the fact that mass is independent of $\vec{r}$, force switches direction, $\vec{F} \rightarrow -\vec{F}$. Not all of the objects we traditionally call vectors transform this way. For example, angular momentum, $\vec{L} = \vec{r} \times m\vec{v}$, does not switch direction under parity transformation because $-\vec{r} \times m(-\vec{v}) = \vec{r} \times m\vec{v}$. Magnetic field, $\vec{B}$, is another example; the magnetic force, $\vec{F} = q\vec{v} \times \vec{B}$, changes sign when $\vec{r} \rightarrow -\vec{r}$ and so does $\vec{v}$, thus, $\vec{B}$ does not. (Objects like angular momentum and magnetic field are more properly called
pseudovectors.) So what does “physical processes are identical under parity transformation” mean? It means that if something is conserved in a process for \( \vec{r} \), it will also be conserved in that process for \(-\vec{r}\).

It seems so obvious that physics should be invariant under parity transformation that it’s a wonder anybody would have suggested otherwise. In a 1956 paper, however, C.N. Yang and T.D. Lee pointed out that though electromagnetic and strong forces had been experimentally demonstrated to be insensitive to parity transformation, no similar experiments had yet been done involving weak interactions. They suggested several possible experiments to test this, including measuring the rate, in different directions, of decay products when spin-polarized \(^{60}\text{Co}\) nuclei undergo beta decay:

\[ ^{60}\text{Co} \rightarrow ^{60}\text{Ni} + e^- + \bar{\nu}_e. \]

In this process, a neutron in the cobalt nucleus becomes a proton in the nickel nucleus while the electron and antineutrino come out. The electrons are easily detected. The experiment consists of placing a sample of \(^{60}\text{Co}\) in a strong magnetic field (to orient the nuclear spin) and measuring electron rates along the magnetic field direction and opposite it. The nuclear spin of \(^{60}\text{Co}\) is \(5\hbar\) “up” (i.e., in the direction of the external magnetic field) and the nuclear spin of the (excited state of) \(^{60}\text{Ni}\) it decays into is \(4\hbar\) (up). Thus, the electron and antineutrino have to emerge from the decay with opposite velocities to conserve momentum and with their \(1/2\) spins both up to conserve angular momentum. Thus, because for a free fermion velocity and spin are either parallel or antiparallel, when a \(^{60}\text{Co}\) nucleus decays the emitted electron emerges either in the direction of the nuclear spin or opposite it.

To test parity invariance for this process, simply count the number of electrons emerging in the up direction versus the number emerging down. In a parity-transformed world, magnetic field does not change direction; nuclear spin (an angular momentum) does not change direction, but velocity does change direction. For parity transformation symmetry there must be equal numbers of electrons going up as going down. There’s a technical problem to doing this experiment, though. To keep the nuclear spins aligned requires low temperature—about 0.003K! In a triumph of experimental design and execution (“Madame”) C.S. Wu accomplished this on December 27, 1956, and found that, amazingly, the decay rates are different in different directions. In fact, we now (with better experimental resolution) know that electrons only come out in the down direction. Beta decay is not symmetric under parity transformation. (This astonishing result yielded Yang and Lee the Nobel Prize. Unfortunately, like several of our previous stories of great work going unrewarded [i.e., Leavitt, Hubble, Alpher, Bell, Rubin], Madame Wu—who was the genius behind making the beta decay experiment work—did not share the glory.) Today, parity asymmetry has been established in all weak processes that involve neutrinos.

As mentioned above, the electron and anti-neutrino must have their \(1/2\) spins aligned in the same direction as the original nuclear spin when they are emitted. Since electrons only come out the south pole of the decaying nucleus, anti-neutrinos must only come out the north pole; in beta decay, the electron spin direction must always be opposite to its momentum, and the anti-neutrino spin must always be in the same direction as its momentum. A particle whose spin is in the same direction as its momentum is called “right-handed,” and “left-handed” otherwise. So, in beta decay only right-handed anti-neutrinos (and left-handed electrons) are produced. A similar decay experiment can be performed with \(^{58}\text{Co}\), which decays by “inverse beta decay” (involving...
a positron and a real neutrino). The directions of the emerging particle and antiparticle are reversed in this experiment indicating that only left-handed neutrinos (and right-handed positrons) are involved. Overall, it is understood that in weak interactions, neutrinos are always left-handed and antineutrinos are always right-handed.