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Problem Set #8

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Problems 1-3 refer to: N identical, noninteracting, and *distinguishable* spin-1/2 particles (i.e., their separation is much greater than their de Broglie wavelength) are placed in an external magnetic field. Assume the ground state energy of one such particle is 0 and the excited state energy is ε, and the system is in thermal equilibrium at temperature T. Note: For distinguishable particles, the chemical potential cancels out of calculation of probabilities, so $P_\sigma = \exp\left(-\varepsilon_\sigma / k_B T\right) \Big/ \sum_{\sigma'} \exp\left(-\varepsilon_{\sigma'} / k_B T\right)$. Here the quantum state σ only has two values, one with energy 0, one with energy ε.

1. Suppose the probability of finding a particle in its ground state is a. Show that the temperature of the system is $T = -\dfrac{\varepsilon}{k_B} \dfrac{1}{\ln\left(1/a - 1\right)}$.

2. Using the result in 1. above, find a numerical value for the excited state energy if the ground state probability is 3/4 at T = 1000 K.

3. Suppose ε / k_B = 100 K. Using the result from problem 1 determine the "temperature" that would be required for the ground state probability to be 1/4. (Note: the excited state is more probable than the ground state. This situation, a "population inversion," can't be achieved by thermal excitation, but can be by some other mechanism. Lasers depend on population inversions.)