Problem Set #8

David Peak
Utah State University, david.peak@usu.edu

Follow this and additional works at: https://digitalcommons.usu.edu/intro_modernphysics_problems

Part of the Physics Commons

Recommended Citation
https://digitalcommons.usu.edu/intro_modernphysics_problems/8

This Course is brought to you for free and open access by the Introductory Modern Physics at DigitalCommons@USU. It has been accepted for inclusion in Problems by an authorized administrator of DigitalCommons@USU. For more information, please contact digitalcommons@usu.edu.
Problem Set #8

A bit of stat mech

Problems 1-3 refer to: $N$ identical, noninteracting, and *distinguishable* spin-1/2 particles (i.e., their separation is much greater than their de Broglie wavelength) are placed in an external magnetic field. Assume the ground state energy of one such particle is 0 and the excited state energy is $\varepsilon$, and the system is in thermal equilibrium at temperature $T$. Note: For distinguishable particles, the chemical potential cancels out of calculation of probabilities, so $P_\sigma = \frac{\exp(-\varepsilon\sigma/k_B T)}{\sum_\sigma \exp(-\varepsilon\sigma'/k_B T)}$. Here the quantum state $\sigma$ only has two values, one with energy 0, one with energy $\varepsilon$.

1. Suppose the probability of finding a particle in its ground state is $a$. Show that the temperature of the system is $T = -\frac{\varepsilon}{k_B} \frac{1}{\ln\left(\frac{1}{a}-1\right)}$.

2. Using the result in 1. above, find a numerical value for the excited state energy if the ground state probability is 3/4 at $T = 1000$ K.

3. Suppose $\varepsilon/k_B = 100$ K. Using the result from problem 1 determine the “temperature” that would be required for the ground state probability to be 1/4. (Note: the excited state is more probable than the ground state. This situation, a “population inversion,” can’t be achieved by thermal excitation, but can be by some other mechanism. Lasers depend on population inversions.)