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Problem Set #9

David Peak
Utah State University, david.peak@usu.edu

Problems 1-3 refer to: N identical, noninteracting, and *distinguishable* quantum harmonic oscillators (i.e., their separation is much greater than their de Broglie wavelength) are in thermal equilibrium at temperature T. The energy of each oscillator can be expressed as $\varepsilon_n = n\varepsilon$, where ε is the level spacing and n = 0, 1, 2, … . Note: For distinguishable particles, the chemical potential cancels out of calculation of probabilities, so $P_\sigma = \exp\left(-\varepsilon_\sigma / k_B T\right) \Big/ \sum_{\sigma'} \exp\left(-\varepsilon_{\sigma'} / k_B T\right)$. Here each quantum state σ corresponds to the integer n only.

1. Show that the probability of finding an oscillator in state n is $P_n = \left[1 - \exp\left(-\varepsilon / k_B T\right)\right] \exp\left(-n\varepsilon / k_B T\right)$. (Hint: To do this, you have to evaluate the denominator in the probability expression above. You will also need to recall that the sum of a geometric series is $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$, provided x < 1.)

2. Using the result in problem 1 above, show that the probability an oscillator is in an excited state goes to zero as $T \to 0$.

3. Suppose ε / k_B = 100 K. What is the probability of finding an oscillator in the ground state at room temperature (T = 300 K)?