Density of State Models and Temperature Dependence of Radiation Induced Conductivity

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Density of State Models and Temperature Dependence of Radiation Induced Conductivity

Jodie Corbridge Gillespie, JR Dennison, and Alec M. Sim

Abstract—Expressions are developed for radiation induce conductivity (RIC) over an extended temperature range, based on density of states models for highly disordered insulating materials. A general discussion of the DOS of can be given using two simple types of DOS distributions of defect states within the bandgap for disordered materials are considered, one that monotonically decreases within the bandgap and one with a distribution peak within the band gap. Three monotonically decreasing models (exponential, power law, and linear), and two peaked models (Gaussian and delta function) are considered, plus limiting cases with a uniform DOS for each type. Variations using the peaked models are considered, with an effective Fermi level between the conduction mobility edge and the trap DOS, within the peaked trap DOS, and between the trap DOS and the valence band. The models are compared to measured RIC values over broad temperature ranges for two common materials, low density polyethylene (LDPE) and disordered silicon dioxide.

Index Terms— radiation induced conductivity, conductivity, density of states, disordered materials, temperature

I. INTRODUCTION

The conductivity of a material is the primary property for determining charge transport, and hence the dissipation rate, of accumulated charge within a material. A material’s conductivity can be determined through straightforward measurements of current under an applied field, but it can have complex dependence on time, temperature, electric field, magnetic field, and the magnitude and rate of charge deposition [1-3]. Another way to increase conductivity is through the deposition of energy by incident high energy radiation which results in excitation of charge carriers into the conduction band (CB), a process called radiation induced conductivity (RIC). When the incident particle radiation is energetic enough, it penetrates completely through the material, thereby avoiding charge accumulation. The increased number of charge carriers, and hence the magnitude of the enhanced conductivity, is dependent on a number of factors including temperature and the spatial and energy dependence and occupation of the material’s distribution of localized trap states within the band gap—or defect density of states (DOS). Expressions for RIC in terms of the filling of the DOS up to an effective Fermi level were largely developed by Rose [4-5], and were extended by Fowler [6-7], Vissenberg [8], and others. Under these conditions, the enhanced conductivity can be compared to a photoconductivity and is the starting point for understanding the mechanisms involved [4-7,9]. A number of useful reviews of the subject are available [2,4,10-13].

II. DENSITY OF STATES DISTRIBUTIONS

A. Description of DOS Distributions

To obtain a generic model of temperature dependent RIC behavior, one must first develop a general discussion of the distribution of localized defect states (density of states or DOS) in relation to the CB. We consider two types of distributions: one that has a distribution peak within the band gap and one that monotonically decreases within the band gap (i.e., has a peak at energies in the CB). In other words, the two types of distributions have a maximum amplitude at energies of either $E \leq 0$ or $E > 0$.

We consider seven specific DOS modes as shown in Table 1 and Fig 1. These are three monotonically decreasing models (exponential, power law, and linear) and two symmetric peaked models (Gaussian and delta function), plus a limiting case with a uniform DOS for each type. All DOS distributions, $n_A(E)$, are functions of energy $E$ as measured from the CB edge, $E_C$, towards the valence band (VB) edge, $E_V$. Each distribution has an energy width and corresponding effective temperature associated with it of the form $E^0_A \equiv k_BT^0_A$ with: $A = X$ for exponential, $A = P$ for power law, $A = L$ for linear, $A = U$ for uniform, $A = G$ for Gaussian and $A = D$ for delta function distributions (see Table 1, column 3).

All distributions are normalized to the total defect density, $N_t$, by integrating over the entire bandgap (BG):

$$N_t \equiv \int_0^\infty n_A(E) dE.$$  \hspace{1cm} (1)

Each distribution also has an energy centroid (or first moment) associated with it (see Table 1, column 4). This mean energy of all (both occupied and unoccupied) states of the distributions within the BG is

$$E_{centroid} = \frac{1}{N_T} \int_0^\infty E n_A(E) dE.$$ \hspace{1cm} (2)

For the decreasing distributions the centroid can be expressed in terms of the width, while for the symmetric peaked distributions there is an independent centroid, $E^0_A$, at the maximum of the trap distribution.

The mean energy and fraction of all occupied trap states within the BG are

$$\langle E_A(T) \rangle = \frac{1}{N_T} \int_0^\infty E f_{FD}(E) n_A(E) dE.$$ \hspace{1cm} (3)
B. Temperature Dependence of Occupied States

The temperature dependence of $E_0$ is contained in the Fermi-Dirac distribution function, $f_{FD}(E)$, that describes occupation of the trap states:

$$f_A(T) ≡ \frac{n_T(T)}{n_T} = \frac{1}{N_T} \int_0^{E_F} f_{FD}(E, T) n_A(E) dE .$$

(4)

III. EQUATIONS

A. Conductivity Equations

The RIC contribution to the total conductivity, $\sigma_{total}$, adds to the material's dark current conductivity, $\sigma_{DC}$ [17]:

$$\sigma_{total} = \sigma_{DC} + \rho_{RIC}$$
RIC conductivity, \(\sigma_{RIC}\), has been found to follow a simple power law, both theoretically \([4,6,10,12]\) and experimentally \([1,7,9,11,13,15]\)

\[
\sigma_{total} = \sigma_{DC} + \sigma_{RIC}.
\]

To develop a temperature-dependent expression for \(\sigma_{RIC}\) based on material-dependent parameters, we begin with an analogy to a semiconductor system, with dopant states at a single energy, \(E_d\). The effective Fermi level is the energy at which 50% of the states are occupied and may depend on temperature, dose rate, and charge distribution. We assume a reservoir of trapped electrons pinned to the effective Fermi level, \(E_F^{eff}\); that is, with nearly constant excitation energies such that \(E_F^{eff}\) in the semiconductor system is replaced by \(E_F^{eff}\) in highly disordered insulating materials.

The fundamental equation for conductivity

\[
\sigma = \sum_{\text{carrier types}} q_i \mu_i n_i \quad \text{for CB electrons, } q_e \mu_e n_e(T)
\]

requires expressions for the carrier charge, \(q_i\), the carrier mobility, \(\mu_i\), and the density of occupied carrier states, \(n_i\). We restrict the discussion here to electron conduction so \(q_i = q_e\) and \(n_i = n_e\) or \(n_e(T)\) in a single term in the sum of Eq. (10) (Refer to \([2,4,10,12]\) for discussion of other carrier types.) As argued by Rose \([5]\) for volume-excited photocurrents, we also assume that the mobility is constant and determined only by the free (or nearly free) electron mobility, \(\mu_e\). The effective mobility of nearly free electrons in a dielectric medium can be approximated by: (i) replacing the electron mass, \(m_e\), with its effective mass, \(m_e^*\), to model the weak uniform binding potential experienced by electrons traveling in the conduction band, and (ii) including the relative dielectric constant of the material, \(\epsilon_d\), to account for screening or polarization of the trap center charge by the charge background of the medium. That is, \(\mu_e^{NF} = \frac{\mu_e}{\epsilon_d}m_e^*\).

Using these assumptions for fixed values for CB electrons, \(q_e\) and \(\mu_e\) \([14]\), it is only \(n_e(T)\) that controls the temperature dependence of \(\sigma_{RIC}(T)\).

We can now develop an expression for the temperature-dependent density of (nearly) free electrons in the CB, \(n_e(T)\). We assume that there are no interactions between electrons in trap states, or equivalently that the mean spatial separation of defects is larger than their interaction range. Then \(n_e(T)\) is

<table>
<thead>
<tr>
<th>DOS Type</th>
<th>Normalized DOS Function, (n_0(E))</th>
<th>Width, (E_o^{A})</th>
<th>Centroid, (E_0^{C})</th>
<th>Fraction of Occupied Traps at (T\rightarrow 0), (f_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>(n_0(E; E_0^{A}, E_0^{C}) = N_T \left[ \frac{1}{\epsilon} \right] \exp \left( \frac{E_0^{A} - E}{E_0^{C}} \right) \Theta(E) \exp \left( \frac{1}{\epsilon} \cdot \text{width} \right) \right.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power Law</td>
<td>(n_0(E; E_0^{A}, E_0^{C}) = N_T \left[ \frac{1}{\epsilon} \right] \exp \left( \frac{E_0^{A} - E}{E_0^{C}} \right) \Theta(E) \exp \left( \frac{1}{\epsilon} \cdot \text{width} \right) \right.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear (Power Law, (\rho = 1))</td>
<td>(n_0(E; E_0^{A}, E_0^{C}) = N_T \left[ \frac{1}{\epsilon} \right] \exp \left( \frac{E_0^{A} - E}{E_0^{C}} \right) \Theta(E) \exp \left( \frac{1}{\epsilon} \cdot \text{width} \right) \right.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uniform Step (Limit of Top Hat, (E_0^{A} \rightarrow 0)) (Limit of Power Law, (\rho = 0))</td>
<td>(n_0(E; E_0^{A}, E_0^{C}) = N_T \left[ \frac{1}{\epsilon} \right] \exp \left( \frac{E_0^{A} - E}{E_0^{C}} \right) \Theta(E) \exp \left( \frac{1}{\epsilon} \cdot \text{width} \right) \right.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gaussian (Limit of Gaussian, (E_0^{A} \rightarrow \infty))</td>
<td>(n_0(E; E_0^{A}, E_0^{C}) = N_T \left[ \frac{1}{\epsilon} \right] \exp \left( \frac{E_0^{A} - E}{E_0^{C}} \right) \Theta(E) \exp \left( \frac{1}{\epsilon} \cdot \text{width} \right) \right.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Delta Function (Limit of Gaussian, (E_0^{A} \rightarrow \infty))</td>
<td>(n_0(E; E_0^{A}, E_0^{C}) = N_T \left[ \frac{1}{\epsilon} \right] \exp \left( \frac{E_0^{A} - E}{E_0^{C}} \right) \Theta(E) \exp \left( \frac{1}{\epsilon} \cdot \text{width} \right) \right.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uniform Top Hat (Limit of Constant, (E_0^{A} = 0) and (E_0^{C} \rightarrow \infty)) (Limit of Uniform Step, (E_0^{A} \rightarrow 0))</td>
<td>(n_0(E; E_0^{A}, E_0^{C}) = N_T \left[ \frac{1}{\epsilon} \right] \exp \left( \frac{E_0^{A} - E}{E_0^{C}} \right) \Theta(E) \exp \left( \frac{1}{\epsilon} \cdot \text{width} \right) \right.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uniform Top Hat (Limit of Gaussian, (E_0^{A} \rightarrow \infty))</td>
<td>(n_0(E; E_0^{A}, E_0^{C}) = N_T \left[ \frac{1}{\epsilon} \right] \exp \left( \frac{E_0^{A} - E}{E_0^{C}} \right) \Theta(E) \exp \left( \frac{1}{\epsilon} \cdot \text{width} \right) \right.</td>
<td></td>
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</tr>
</tbody>
</table>

\(\Theta(E)\) is a Heaviside step function, equal to 0 at \(E < 0\) and 1 at \(E > 0\).

Error function evaluated at \(E\).

\(\sigma_{total} = \sigma_{DC} + \sigma_{RIC}\) (8)

\(\sigma_{RIC} = k_{RIC}(T) D^{\Delta(T)}\) (9)

\(n_0(E; E_0^{A}, E_0^{C}) = N_T \left[ \frac{1}{\epsilon} \right] \exp \left( \frac{E_0^{A} - E}{E_0^{C}} \right) \Theta(E) \exp \left( \frac{1}{\epsilon} \cdot \text{width} \right) \right. \)

\(\delta(E)\) is the Dirac delta function, equal to infinity at \(E\) and zero elsewhere.

\(\text{erf}(E)\) is the error function evaluated at \(E\).

\(f_0\) Mean energy of trap state within band gap, from Eq. (2).

\(\Delta(T)\) From Eq. (7).
given by Ashcroft and Mermin [16] as

\[ n_e(T) = N_e e^{-E_F^{\text{eff}}(T)/k_BT} + n_e^0, \quad (11) \]

where \( N_e \) is the total density of accessibly energy states that can be thermally excited into the CB (from within a few \( k_BT \) of the CB edge—in this case, electrons in shallow traps such that \( n_e(T) \approx N_e \)); \( n_e^0 \) is the density of free electrons in the CB at \( T = 0 \)—in this case, solely from electrons excited into the CB by the incident high energy radiation; \( T \) is the absolute temperature; and \( E_F^{\text{eff}}(T) \) is the excitation energy from the effective Fermi level to the CB. Under these assumptions, the density of available states has been shown to be [16]

\[ N_e \approx n_e(T) = 2 \left( \frac{m_e k_B T}{2 \pi e^2} \right)^{3/2} \left( \frac{m_e^* m_h^*}{m_e m_h} \right)^{3/4}. \quad (12) \]

\( m_e, m_h, m_e^*, \) and \( m_h^* \) are the electron and hole masses and effective masses, respectively.

We assume that the number of trapped electrons exceeds the number of free electrons (i.e., trap states have relatively long lifetimes); then the density of primary VB positive centers (fixed holes) created by the high energy radiation, \( n_p \), is approximately equal to the density of occupied traps, \( n_e \):

\[ n_p = n_t + n_e \approx n_t. \quad (13) \]

That is, almost all electrons excited from the VB into the CB by high energy radiation as electron-hole pairs spend most time in trapped states and not the CB. (When this assumption is no longer valid, RIC will exhibit saturation effects.) If \( n_t >> n_e \), then even at low \( T \), \( n_t >> n_e^0 \) and Eq. (11) becomes

\[ n_e(T) \approx N_e e^{-E_F^{\text{eff}}(T)/k_BT}. \quad (14) \]

Solving for \( E_F^{\text{eff}}(T) \)

\[ E_F^{\text{eff}}(n_e, T) \approx k_BT \ln \left[ \frac{N_e}{n_e(T)} \right]. \quad (15) \]

This is a variation of the familiar law of mass action for semiconductors dopant defects [16]. For further discussion on these assumptions and their resultant implications for disordered materials see [2,4,7,10,12,14].

At finite temperatures \( E_F^{\text{eff}}(T) \) moves towards \( E_F \) as increased charge is stored in trapped states, the excitation energy is reduced, and more electrons can be thermally excited into the CB. It is this temperature dependence of the resulting balance of trap charge buildup from radiation excitations and thermal depletion of the trapped charge that principally determines the temperature dependence of RIC [2,4,7,10,12]. For the monotonically decreasing DOS and uniform DOS models, thermal depletion moves the effective Fermi level further from the CB, thereby increasing \( E_F^{\text{eff}}(T) \), and we expect a resultant decrease in \( \sigma_{\text{RIC}} \). By contrast, for a delta function DOS, \( E_F^{\text{eff}} \) is pinned to the single trap energy irrespective of the number of trapped electrons that are thermally excited. Hence, we expect no associated temperature decrease in its \( \sigma_{\text{RIC}} \); rather \( \sigma_{\text{RIC}}(T) \) will be driven by the increase in carriers thermally excited into the CB.

In the steady-state condition for RIC, the rate of excitation of VB electrons into the CB by radiation equals the rate of recombination of conduction electrons with primary centers (VB holes),

\[ u_{eh} = v_{he} = \frac{1}{\tau_e} = v_T s_c n_e n_p. \quad (16) \]

Here, \( v_T \) is the thermal velocity of electrons; \( s_c \) is the capture cross section of primary centers for free electrons; and \( u_{eh} \) is the rate of radiation excitations of electrons per unit volume, which is proportional to the dose rate \( D \) (or power deposited per unit mass).

\[ u_{eh} = \dot{D} \rho_m \quad (17) \]

Here \( \rho_m \) is the material mass density and \( E_{eh} \) is the mean energy required for an electron-hole pair to be created by radiation.

We can find an expression for \( v_T \) by setting the thermal energy of a free electron equal to its kinetic energy:

\[ \frac{3}{2} k_BT = \frac{1}{2} m_e v_T^2 \quad \text{or} \quad v_T = \sqrt{\frac{3k_BT}{m_e}} \quad (18) \]

For the nearly free electron case, \( m_e \rightarrow m_e^* \) here. Solving Eq. (16) for \( n_e \), we find

\[ n_e(T) = \frac{v_{eh}}{v_T s_c n_p} \approx \frac{v_{eh}}{v_T s_c n_t} = \frac{v_{eh}}{v_T s_c} \left[ \int_0^\infty f_{FD}(E, T) n_A(E) dE \right]^{-1} \]

\[ \approx C_o \dot{D} T^{-1/2} \left[ \int_0^\infty f_{FD}(E, T) n_A(E) dE \right]^{-1} \]

with \( C_o \equiv \rho_m s_c E_{eh} (\sqrt{3k_BT/m_e})^{-1} \)

(19)

where we have made substitutions for \( v_{eh} \) and \( v_T \) using Eqs. (17) and (18), respectively. Also, from Eq. (13), \( n_p \) is approximately equal to \( n_t \), which in turn is expressed as an integral using Eq. (4).

Using the low temperature Fermi-Dirac function approximation from Eq. (5) and assuming \( E_F^{\text{eff}}(T) \geq 2k_BT \), we can calculate the density of filled trap states, \( n_t \), for the steady-state condition at low \( T \) by integrating an expression for the trap state density as a function of energy over all occupied states, or over all trap states in the distribution \( n_A(E) \):

\[ n_t = \int_0^\infty f_{FD}(E, T) n_A(E) dE \]

\[ \approx \left\{ \begin{array}{l} \int_{E_F^{\text{eff}}(T)}^{\infty} \frac{1}{2} \left[ 1 + \frac{E - E_F^{\text{eff}}(T)}{2k_BT} \right] n_A(E) dE + \int_{E_F^{\text{eff}}(T)}^{\infty} n_A(E) dE \end{array} \right\} \]
where \( E_{F}^{eff}(T) \equiv E_{F}^{eff}(T) \pm 2k_{B}T \).

This expression is the only part of the RIC expression that contains information about the material, at least up to a proportionality constant. The first integral in this expression contains all of the temperature dependence of RIC.

### B. Final Expression for Temperature-Dependent RIC

Inserting Eq. (19) into Eq. (10), we arrive at the final expression for temperature-dependent RIC:

\[
\sigma_{RIC}(T) = q_{e}\mu_{e}n_{C}(T) \\
m \approx q_{e}\mu_{e}C_{0}D^{-1/2}\left[\int_{0}^{\infty} f_{FD}(E,T)f_{A}(E)dE\right]^{-1} \tag{21}
\]

Table 2 column 2 shows expressions for \( n_{c}(T) \) in the low \( T \) approximation from Eqs. (19) and (20), for all DOS listed in Table 1 evaluated with \( E_{F}^{eff}(T) \) below, above, or within the distributions.

For \( E_{F}^{eff}(T) \) between the trap distribution and the valence band \( (E_{F}^{eff}(T) >> E_{D}^{0}) \), \( n_{c}(T) \approx 0 \) as expected, since there are no electrons in the distribution to be excited into the CB. For \( E_{F}^{eff}(T) \) between the CB and the distribution, at energies less than those within the distribution, \( n_{c}(T) = C_{0}DT^{-1/2} \).

Solving for \( n_{C}(T) \) within the distributions in general requires numerical calculations, due to the implicit dependence of \( E_{F}^{eff}(T; n_{c}(T)) \) on \( n_{C}(T) \), as seen in Table 2. Expressions for these distributions have all been solved previously in the zero temperature limit [4,9,14]. In addition, with the restrictions that the effective Fermi level has a small temperature-induced change and is small compared to the distribution energies, \( E_{F}^{eff}(T) - E_{F}^{eff}(T=0) \approx 2k_{B}T \) and \( E_{D}^{0} - E_{F}^{eff}(T) \approx k_{B}T \), approximate expressions for most of the distributions have been determined [4,9,14].

### C. RIC for Exponential Monotonically Decreasing DOS Model

An important case with an explicit solution is the exponential monotonically decreasing DOS with the exponential width, where the expression in Table 2 can be explicitly solved for \( n_{c}(T) \) when \( E_{F}^{eff}(T) \) is at least a few times \( 2k_{B}T \) [4,9,14]:

\[
n_{c}(T) = \begin{cases} \\
[C_{0}DT^{-1/2}] \left[\int_{0}^{\infty} f_{FD}(E,T)f_{A}(E)dE\right]^{-1}\left[\frac{\tau}{\tau + \tau_{S}}\right]\left[\frac{\tau}{\tau + \tau_{S}}\right] \\
[\left\{\frac{2T}{\tau_{c}}\sinh\left(\frac{2T}{\tau_{c}}\right)\right\}]C_{0}DT^{-1/2}\left[\int_{0}^{\infty} f_{FD}(E,T)f_{A}(E)dE\right]^{-1}\left[\frac{\tau}{\tau + \tau_{S}}\right]
\end{cases} \tag{22}
\]

The exponential monotonically decreasing DOS is commonly used to model shallow traps within the bandgap [2,4,14].

### D. RIC for Gaussian Symmetric Peaked DOS Model

Finally, we consider a Gaussian-like distribution of traps, \( n_{c}(E) \), within the bandgap,

\[
n_{c}(E; E_{D}^{0}, E_{G}^{0}) = \frac{N_{F}}{(\sqrt{\pi}e)^{3/2}} \exp\left(-\frac{1}{2}\left(\frac{E_{c} - E_{D}^{0}}{E_{G}^{0}}\right)^{2}\right) \tag{23}
\]

with mean energy \( E_{D}^{0} \) and standard deviation \( E_{G}^{0} \). The Heaviside step function, \( \theta(E) \), truncates occupation within the CB at \( E < E_{C} = 0 \); the error function in the denominator corrects the normalization for this truncation such that \( N_{F} \) is still the total (occupied and unoccupied) trap state density; this correction term approaches unity when the peak is well within the bandgap, i.e., when \( E_{D}^{0} \gg 0 \) or \( E_{D}^{0} \gg E_{G}^{0} \).

For this Gaussian DOS distribution, the density of conduction electrons is

\[
n_{c}(T) = \frac{C_{0}D^{-1/2}T^{-1/2}E_{D}^{0}}{1 + \text{erf}(\frac{E_{D}^{0}}{2E_{G}^{0}})} \tag{24}
\]

By adjusting the width \( E_{G}^{0} \) limiting cases of the uniform top hat and the delta function distributions are obtained (see Fig. 1 and Table 2). When the width of the distribution is large (i.e., \( E_{G}^{0} \gg 0 \)), the limiting case of a uniform distribution is obtained. The first two terms in the square bracket will cancel each other and the exponential terms will disappear faster than the coefficient will blow up. In this case \( n_{c}(T) \propto C_{0}DT^{-1/2} \) as expected for \( E_{F}^{eff}(T) \) within a uniform distribution. Similarly, when the width of the distribution is small (i.e., \( E_{G}^{0} \approx 0 \)), the distribution approaches that of a delta function. \( (R_{\pm})^{2} \gg 1 \) and the exponential term in the square bracket will go to zero much faster than \( E_{G}^{0} \) in the coefficient can blow up. \( n_{c}(T) \approx C_{0}DT^{-1/2} \) as expected for a delta function. Further details of these limiting cases and the effect the relative position of the effective Fermi level, \( E_{F}^{eff} \), on the temperature dependent RIC can be found in [14].

### IV. RESULTS

Figure 3 shows fits to measured RIC values as a function of temperature for two common spacecraft materials, disordered SiO\(_{2}\) and low density polyethylene (LDPE). To obtain data over extended temperature ranges, three distinct data sets were compiled for each material and the data from the different studies were modestly scaled to agree at room temperature. Details about the materials and experiments [17] are found in the respective references.

Figure 3(a) shows the SiO\(_{2}\) data (two data sets from USU [18] and one from Culler [19]) fit with a curve that would be expected for a material with an exponential monotonically decreasing DOS (see Eq. (22)) [9,14]. Data from the USU Data Set 2 shows a smaller decrease in RIC at the lowest temperatures, as predicted by the exponential monotonically decreasing DOS model. RIC for SiO\(_{2}\) increases by only a
Table 2. Conduction band electron expressions for various DOS [14].

<table>
<thead>
<tr>
<th>DOS Type</th>
<th>Density of Conduction Band Electrons, $n_c(T)$</th>
<th>Temperature Dependence</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Monotonically decreasing DOS models with $E_o^t \leq 0$.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exponential $0 &lt; E_F^{eff}$</td>
<td>$n_c = C_o \cdot \hat{D} T^{-1/2} \cdot \left( 1 + \text{erf} \left( \frac{2 E_o^0}{\sqrt{2} E_o^0} \right) \right)$</td>
<td>$T^{-1/2}$ when $</td>
</tr>
<tr>
<td>Power Law $0 &lt; E_F^{eff} &lt; E_o^p$</td>
<td>$n_c = C_o \cdot \hat{D} T^{-1/2} \cdot \left( \frac{E_o^p}{4 k_B T (p+2)} \left( (P_+)^{p+2} - (P_-)^{p+2} \right) \right)^{-1}$</td>
<td>$T^{-1/2}$ when $</td>
</tr>
<tr>
<td>Linear $0 &lt; E_F^{eff} &lt; E_o^L$</td>
<td>$n_c = C_o \cdot \hat{D} T^{-1/2} \cdot \left( \frac{E_o^L - E_F^{eff}(n_c,T)}{E_o^L} \right)^2 + \frac{1}{3} \left( \frac{2 k_B T}{E_o^L} \right)^2$</td>
<td>$T^{-1/2}$ when $</td>
</tr>
<tr>
<td>Uniform Step $0 &lt; E_F^{eff} &lt; E_o^{US}$</td>
<td>$n_c = C_o \cdot \hat{D} T^{-1/2} \cdot \left( \frac{E_o^{US} - E_F^{eff}(n_c,T)}{E_o^{US}} \right)^{-1}$</td>
<td>$T^{-1/2}$ when $</td>
</tr>
<tr>
<td>Power Law, Linear, Uniform Step $0 &lt; (E_o^t + 2 k_B T) &lt; E_F^{eff}$</td>
<td>$n_c = 0$</td>
<td>$T^{-1/2}$ when $E_o^t - E_F^{eff} \geq E_o^t \geq 0$</td>
</tr>
<tr>
<td><strong>Peaked DOS models with $E_o^t &gt; 0$.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gaussian $0 &lt; E_F^{eff}$</td>
<td>$n_c = C_o \cdot \hat{D} T^{-1/2} \cdot \left( 1 + \text{erf} \left( \frac{2 E_o^0}{\sqrt{2} E_o^0} \right) \right)$</td>
<td>$T^{-1/2}$ when $</td>
</tr>
<tr>
<td>Delta Function $0 &lt; E_F^{eff} &lt; E_o^t$</td>
<td>$n_c = C_o \cdot \hat{D} T^{-1/2} \cdot \left( \frac{E_o^t - E_F^{eff}(n_c,T)}{4 k_B T} \right)$</td>
<td>$T^{-1/2}$ when $</td>
</tr>
<tr>
<td>Uniform Top Hat $0 &lt; E_F^{eff} &lt; E_o^T$</td>
<td>$n_c = C_o \cdot \hat{D} T^{-1/2}$</td>
<td>$T^{-1/2}$ when $</td>
</tr>
<tr>
<td>Delta Function $0 &lt; E_F^{eff} &lt; E_o^L$</td>
<td>$n_c = C_o \cdot \hat{D} T^{-1/2} \cdot \left( \frac{E_o^L - E_F^{eff}(n_c,T)}{2 E_o^L} \right)$</td>
<td>$T^{-1/2}$ when $</td>
</tr>
<tr>
<td>Delta Function, Uniform Top Hat $0 &lt; [E_o^t + 1/2 \cdot E_o^T^{width} + 2 k_B T] &lt; (E_F^{eff})$</td>
<td>$n_c = 0$</td>
<td>$T^{-1/2}$ when $</td>
</tr>
</tbody>
</table>

\[ C_o \equiv \frac{p_m}{N_{c0} e h \sqrt{2 \pi k_B m_e}} \]

- **$E_F^{eff}$** between the CB and the DOS distribution.
- **$E_F^{eff}$** between the DOS distribution and the valence band.
factor of ~4 from ~50 K to ~420 K, almost three orders of magnitude less than observed for LDPE over similar temperature ranges. A weak temperature dependence suggests a narrow distribution with a smaller $E_0$, here $E_0 \approx 20$ meV. Cathodoluminescence for these SiO$_2$ materials have suggested the presence of an exponential shallow trap DOS and several fairly narrow (~10-50 meV wide) deep level trap DOS distributions centered between ~2 eV to 4.5 eV within the bandgap [18,20].

Figure 3(b) shows three normalized LDPE data sets (USU [9], Yagahi. [21], and Fowler [7]) fit with a curve predicted for an exponential monotonically decreasing DOS [9,14], with a much higher $E_0 \approx 140$ meV. At temperatures below ~240 K, LDPE data exhibits a modest factor of ~3 increase in RIC. Such an increase at low temperatures is predicted for an exponential monotonically decreasing DOS of the form given by Eq. (22). However, for expected ranges of $E_0$ and $N_T$, these increases are predicted at ~30-80 K (see, e.g., Fig. 3(a)). A better fit below ~240 K is found for a curve proportional to $T^{-1/2}$, as is expected for a peaked distribution (see Table 2).

The behavior observed in LDPE may be related to a LDPE structural phase transition observed at between 250 K and 262 K. This $\beta$ transition is routinely observed in branched polyethylene, and has been associated with conformational changes along polymer chains in the interfacial matrix of disordered polymers between nanocrystalline regions in the bulk. Similar abrupt changes near ~250 K have been seen in prior studies of mechanical and thermodynamic properties and in dark current conductivity [1,23], RIC [19], electrostatic discharge [24], and other electronic properties.

V. CONCLUSION

We have calculated expressions for radiation induced conductivity applicable over an extended temperature range that encompasses most practical applications encountered for highly disordered insulating materials in spacecraft charging. Solutions were found for both monotonically decreasing and symmetric peaked DOS distributions of defect states within the bandgap. Variations were considered, with an effective Fermi level above the trap DOS, within the trap DOS, and below the DOS. Approximations were identified that led to analytic solutions for the distributions for many specific cases of energies and temperatures. When analytic solutions were not found, the general expressions for the occupied trap state distribution as a function of temperature can be found numerically by recursively solving the complex expressions for the occupied trap DOS as a function of temperature.

The derived expressions were used to fit measured RIC values over broad temperature ranges for two common materials, low density polyethylene (LDPE) and disordered silicon dioxide. Both LDPE and SiO2 were fit best with an exponential monotonically decreasing DOS model. Reasonable fits were obtained and the observed temperature dependence of RIC was successfully used to glean information about the nature of the distribution of defect states for the materials.

VI. ACKNOWLEDGEMENT

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REFERENCES


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