APPLYING THE SUBMERGED JET EROSION TEST TO EMBANKMENT DAM BREACH MODELING

TONY L. WAHL[1] AND BLAKE ARMSTRONG[2]

[1] U.S. Dept. of the Interior, Bureau of Reclamation
Hydraulics Laboratory
PO Box 25007, Mail Code 86-68560
Denver, Colorado, USA
e-mail: twahl@usbr.gov

[2] U.S. Dept. of the Interior, Bureau of Reclamation
Geotechnical Laboratory and Field Support
PO Box 25007, Mail Code 86-68550
Denver, Colorado, USA

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Abstract. The submerged jet erosion test (JET) is one of several methods available for quantifying erodibility of cohesive soils, a crucial input for modeling erosion and breach of embankment dams with models such as WinDAM, EMBREA (Morris 2011), and DL BREACH (Wu 2013; 2016a; 2016b). The JET was initially developed in the late 1980s as a relatively large-scale device with a 13-mm diameter nozzle creating an impinging jet that erodes a soil sample submerged in a 61-cm diameter tank. This first device was documented in an ASTM standard (D5852-1995), but the standard was withdrawn in 2016. In the last 15 to 20 years, smaller scale devices have become common, including one described as the "original JET" (6.4-mm nozzle operating within a 30.5-cm tank) (Hanson and Cook 2004) and the newer "mini-JET" (3 mm nozzle and 10-cm tank) (Al-Madhhachi et al. 2013b). New methods for analyzing the scour-vs.-time data obtained from the JET have also been proposed, some (Daly et al. 2013) based on the traditional linear excess stress equation describing the soil detachment process, and others (Al-Madhhachi et al. 2013a) based on nonlinear soil erosion models such as the mechanistic Wilson model (Wilson 1993a, 1993b). This paper provides an overview of JET history and development and then summarizes a recent study (Wahl 2021) comparing JET data analysis methods. Fifty-two JET experiments on four different soil types (lean clay, silty clay, clayey sand, and silty sand) were analyzed by nine different methods. The results give indications of the practical value of linear vs. nonlinear soil erosion models and the effectiveness of different curve-fitting methods for estimating erodibility parameters. Suggestions for the use of the JET in dam breach modeling are offered.

1 INTRODUCTION

Models that simulate erosion processes leading to the breach of embankment dams have advanced markedly in the last three decades, largely as a result of improvements in the ability to quantify erodibility of embankment soils. These improvements have also provided benefits in other fields, such as the modeling of stream bank erosion, bridge scour, earthen spillway erosion, and rill erosion of agricultural soils. Three classes of erosion test are prominent for these applications: jet erosion tests, internal erosion tests, and flume-type erosion tests. The development of downhole methods for assessing erodibility during geotechnical drilling operations is also an active area of research but will not be discussed further here.

Jet erosion tests utilize a hydraulic jet impinging normally on an exposed soil surface, while internal erosion tests utilize pressurized flow through a pre-formed slot or hole in a soil specimen and flume-type tests utilize flow parallel to a soil surface. In all cases, rates of erosion are observed directly or inferred from related measurements, applied stresses are estimated, and soil erodibility parameters are determined that correlate applied stress to observed erosion through selected erosion modeling equations. The parameters of these relations define the erodibility of the soil in a numerical way. Specific examples of these devices include the submerged jet erosion test (JET) (Hanson and Cook 2004; ASTM D5852), the Hole Erosion Test (HET) (Wan and Fell 2004), the Erosion Function Apparatus (EFA) (Briaud 2001), and the Sedflume (McNeil et al. 1996).

2 JET EROSION TEST DEVELOPMENT

Efforts to develop devices for evaluating the erodibility of cohesive soils date back to at least 1959 (Hanson 1990). To describe soil erodibility numerically, a mathematical model or erosion law must be selected, and devices for applying stress and measuring erosion must be devised. A commonly adopted mathematical model is the excess stress equation:

$$\varepsilon_r = k_d(\tau - \tau_c)^a \tag{1}$$

where ε_r is the volume of material removed per unit surface area per unit time (units of velocity), τ is the applied shear stress, τ_c is the critical shear stress needed to initiate sediment detachment, and k_d is a detachment rate coefficient (units of length per time per stress). The exponent a is typically assumed to have a value of 1, and the model is described as the linear excess stress equation; when a has any other value this becomes the nonlinear excess stress equation. Even when the linear model is used, it is recognized that soil behavior might be nonlinear, so it is recommended that erosion tests should be conducted at stress ranges comparable to those expected in the full-scale event that is being modeled.

2.1 Early Jet Tests

Much early erosion research focused on assessing the critical shear stress, τ_c. However, in applications where applied stresses are much greater than the critical stress, it is more important to know the value of the detachment rate coefficient, k_d. A forerunner of today's submerged jet test and a device focused on erosion rate was the relatively large 0.46-m (1.5-ft) diameter in situ jet test apparatus developed at the U.S. Department of Agriculture (USDA) Agricultural Research Service Hydraulics Laboratory, Stillwater, Oklahoma (Hanson 1990), today known as the Hydraulic Engineering Research Unit (HERU). A pin profiler was used to map a cross section of the scour hole produced by a 13-mm (½-inch) diameter hydraulic jet driven by water pressure produced from a head tank located nearby. Typical applied pressure heads were about 500 to 2400 mm (1.5 to 8 ft). Typical test durations ranged from a few minutes to a few hours, depending on the erosion resistance of the soil, and typical scour depths ranged up to several cm. Analysis of the data yielded a detachment rate coefficient (k_d) that was compared to erosion rate tests conducted in large open channel flumes. Subsequently, the analysis method was modified to determine a dimensionless jet index parameter (Hanson 1991) that could be related back to k_d. In both analysis methods the critical stress was assumed to be negligible. ASTM standard D5852 for this device was first published in 1995, and the standard described how to determine the jet index parameter and the detachment rate coefficient.

2.2 The Blaisdell Solution Method

The jet erosion test became a valuable tool in 1990s-era studies of earthen spillway headcut erosion (Temple & Hanson 1994), and later studies of embankment dam breaching (Hanson et al. 2005) at the HERU. Although k_d was the most important parameter in these applications, there was broader interest in data analysis methods that could yield both k_d and τ_c. Hanson and Cook (1997) explored three solution methods:

- A nonlinear curve fitting routine to simultaneously estimate both parameters.
- A two-step method that first fits logarithms of dimensionless scour and jet velocity-time parameters to a hyperbolic function (Blaisdell et al. 1981) found to fit scour progression data from plunge pools below cantilevered spillway outlet and culvert pipes. This yields an estimate of the equilibrium depth of scour that should occur after infinite time, and the stress that would be applied by the jet at this distance is adopted as τ_c. Once τ_c has been determined, k_d is estimated by fitting dimensionless scour depths and times to a model predicting the evolution of the scour depth during the testing period.
- A method that first estimated τ_c based on particle size via Shield's diagram and then determined k_d similarly to the second method.

The first method was found to be unstable, with results varying unpredictably based on initial guesses of the parameter values. (Later researchers have had more success with simultaneous solutions.) The second and third methods both provided consistent, useful results and the Blaisdell method (the name commonly used in recent literature for the second method) became widely adopted. Notably, all three methods utilized the scour depth measured along the jet axis, which served as a suitable index and avoided the need to use the pin profiler to measure details of the scour hole. A spreadsheet implementing the Blaisdell method was described by Hanson and Cook (2004). Although it was never incorporated into the ASTM standard, the Blaisdell method has been one of the most common forms of jet data analysis since the late 1990s.

2.3 The "Original JET" and "Mini-JET" Devices

To make jet erosion testing more practical for HERU's field and laboratory use, the jet device was reduced in size to use a 0.3-m (12-inch) diameter submergence tank and 6.35-mm (¼-inch) nozzle in a configuration that could also be readily installed on inclined surfaces such as embankment slopes and streambanks (Hanson et al. 2002). This configuration has come to be described in recent literature as the "original JET" (Figure 1).

Figure 1: Submerged jet erosion test device for laboratory and in situ use, and a schematic diagram with the stress profile applied to the soil boundary (Hanson and Cook 2004).

Further downsizing of the device took place in the late 2000s and led to the adoption of the "mini-JET" for many field applications (especially streambank erosion studies). This device uses a 3-mm (⅛-inch) nozzle and 100-mm submergence tank. A primary advantage for field use is the light weight of the device and a very low flow rate requirement. One limitation of the mini-JET vs. the original JET and other predecessors is a fixed starting distance between the nozzle and the soil surface; other JET devices allow the initial shear stress to be controlled in two ways, by adjusting the head pressure and the nozzle distance.

2.4 New Data Analysis Methods

The JET was first developed to investigate soil erodibility in the context of headcut erosion in earthen spillways and embankment breach, and the linear excess stress equation using JET-determined parameters is an integral component of the SITES and WinDAM models developed by USDA. Hanson and Simon (2001) used the JET to evaluate erodibility of cohesive stream bed soils, establishing in the process that there was an inverse relation between τ_c and k_d. Soils that erode rapidly (large k_d) initiate erosion at low values of τ_c. Several subsequent investigators have proposed variations of this relation.

Coincident with the development of the mini-JET, an interest developed in the early 2010s in application of JET-derived erodibility parameters over wide ranges of applied stress, especially in the arena of streambank erosion modeling. This led to new data analysis methods based on the linear excess stress equation (Daly et al. 2013) and nonlinear models (Al-Madhhachi et al. 2013a) such as the Wilson model (Wilson 1993a, 1993b).

Although the Blaisdell method has been the dominant approach to analyzing JET data since the late 1990s, motivations for development of new methods included:

- The inherent variability of soil erosion makes it challenging to obtain consistent results, so outlying results are not uncommon.
- Conducting tests in one stress range and then extrapolating to larger or smaller stresses during modeling efforts sometimes leads to poor outcomes that prompts questioning of erosion models and parameter values.
- The Blaisdell method often estimates large values of the equilibrium scour depth and hence low values of the critical shear stress, τ_c, especially when there are inconsistent variations of scour rate in the later stages of a test.
- Low estimates of τ_c naturally lead to high estimates of k_d, which become especially problematic if results must be extrapolated to higher stresses than actually applied during a test.

2.5 The Wilson Model

The model developed by Wilson (1993a; 1993b) makes the erosion rate a function of the square root of the applied stress, modified at low stresses by combined exponential functions that yield a three-phase nonlinear behavior. The full model (Wilson 1993a) includes 16 mechanistic parameters, but a simplified 2-parameter model (Wilson 1993b) is most commonly applied with the JET:

$$\varepsilon_r = \frac{b_0\sqrt{\tau}}{\rho_d}\left(1 - e^{-e^{3-\frac{b_1}{\tau}}}\right) \tag{2}$$

in which ρ_d is the soil dry density and b_0 and b_1 are soil erodibility parameters. The three regions of erosion behavior are illustrated in Figure 2: an initial region with a low but exponentially increasing erosion rate as stress is increased; a linear region; and the final region in which erosion rate increases with the square root of applied shear stress. The parameters b_0 and b_1 are analogous to k_d and τ_c, respectively—a rate coefficient and a shear stress threshold. Al-Madhhachi et al. (2013a) first demonstrated the use of the model for JET analysis.

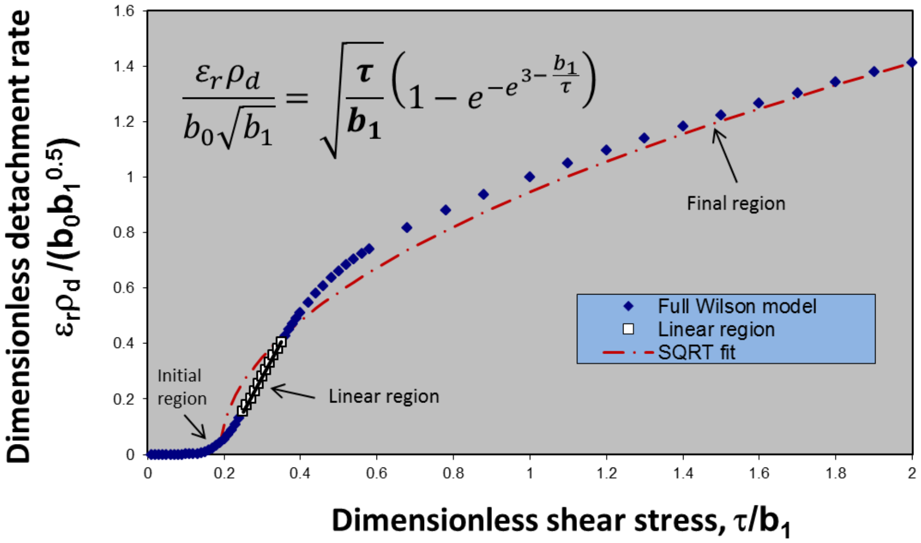

Figure 2: Dimensionless erosion rates vs. shear stress predicted by the Wilson model, exhibiting three stages of erosion behavior.

2.6 Summary of JET Data Analysis Methods

Table 1 summarizes several alternative, common methods of JET data analysis. Differences in the methods include the erosion models, optimization objectives, and constraints placed on the parameters.

Table 1: JET data analysis methods.

Method	Erosion Model	Details
Blaisdell method (Hanson and Cook 2004)	Linear excess stress	1. Predicts τ_c based on estimate of equilibrium scour at t=∞. (Asymptote of hyperbolic scour-time curve) 2. Adjusts k_d with Excel Solver to minimize sum of squared errors in **predicted times** to reach measured scour depths. Data-fitting uses dimensional times, although data are plotted nondimensionally.
Scour depth method (Daly et al. 2013)		Adjusts k_d and τ_c simultaneously with objective of minimizing sum of squared errors in **predicted scour depths** (dimensional) at specific times.
Al-Madhhachi et al. 2013a	Wilson model	Adjusts b_0 and b_1 simultaneously to minimize sum of squared errors in **predicted erosion rates**. Optimizing to minimize errors in predicted **scour depths** has been adopted for more recent work (personal communication with Al-Madhhachi).

3 EVALUATIONS OF NONLINEAR EROSION MODELS

Interest in nonlinear erosion models has increased in recent years with greater recognition that erosion tests cannot always be performed across the broad range of stresses that may occur in a field-scale application. Extrapolation increases the chance for error whenever the general form of the relationship is not consistent in the testing and application environments. Several investigators have recently suggested that erosion is an inherently nonlinear process. For example, Walder (2016) analyzed data from a broad array of domains and concluded that erosion rate is proportional to the 1.75 power of the excess stress. This work also questioned relationships developed between k_d and τ_c and the validity of the k_d parameter itself as an indicator of erodibility.

Khanal et al. (2016) applied the Wilson model to JET, HET, and rill erosion data sets and showed that the Wilson model provided good fits to some nonlinear erosion behavior observed in HET data sets collected by Wahl et al. (2008). However, entrance and exit losses (Luthi 2011; Riha & Jandora 2015) that were not considered in those HETs create uncertainty about this conclusion.

Wardinski et al. (2018) performed laboratory mini-JETs on four soil mixes ranging from clay to sandy loam and applied statistical tests to evaluate whether the data from individual trials met linearity criteria. Most trials did not, while some were inconclusive. Khanal et al. (2020) performed mini-JETs on clay loam and sandy loam soils compacted at different water contents to evaluate how soil moisture affected results obtained from the linearly based scour depth method (Daly et al. 2013) and nonlinear Wilson model. The linear model was more appropriate for the sandy loam, while the nonlinear model was better for the clay loam.

3.1 Evaluating Linear vs. Nonlinear JET Analysis Methods in Individual Tests

The first author has considered the question of linear vs. nonlinear erosion models in recent research. Wahl (2016) applied the Blaisdell, scour depth, and Wilson model methods to individual original JETs performed on several cohesive soils. Data fits to the Wilson model were performed in two ways: seeking a best fit to observed scour rates and a best fit to observed scour depths. The results from each analysis method were used to consider how well erosion rates and scour depths were modeled, both during the test and in an extrapolation of the test to a longer duration.

One example illustrates some pitfalls of both linear and nonlinear analysis approaches (Figures 3-5). The soil was a sandy lean clay s(CL) with 31% sand, 19% clay, 50% silt, and a plasticity index (PI) of 9, compacted near optimum water content with standard Proctor effort (Wahl & Erdogan 2008). Figure 3 shows the observed erosion rates versus applied stresses over a testing time of about 2 hours. (The test duration was selected based on practical considerations, with the objective of obtaining enough data to support curve fitting efforts during data analysis.) The general trends of the Wilson model are well represented, and the model can be optimized to fit the scour rate data very well (solid black line). However, when the resulting erodibility parameters are used to predict scour depths vs. time (black dashed line in Figure 4) they produce poor predictions in the later stages of the test. In contrast, when the model is optimized to fit scour depths (solid black line in Figure 4), the fit to the scour rate data is poor (Figure 3, black dashed line). The Blaisdell method fits the erosion rate data very poorly at the start of the test and quite well near the end of the test period. (In Figure 3 the test proceeds from high stress to low stress, left-to-right). The Blaisdell method predicts scour depths adequately, but underpredicts early scour (Figure 4). The scour-depth method provides a mediocre fit to the nonlinear erosion rate data but predicts the scour depths well (Figure 4). This jet test lasted about 2 hours, but Figure 5 extends the prediction time of each model out past 5 hours to demonstrate the ability of each solution to predict longer-term scour depths. The Wilson model and scour-depth solutions (Daly et al. 2013) predict almost no continuing scour, but the Blaisdell model predicts scour that appears to follow the trend of the observations in the latter part of the test.

This single example illustrates that the “best” model depends greatly on the perspective used to analyze the data and that visual examination of the data and how the models are reacting to it is important during analysis. Many data sets exhibit characteristics that call for judgment in the analysis process. Conclusions from Wahl (2016) include:

- The Wilson model appears to describe real soil behavior in some tests, but not others.
- Nonuniformity of erosion resistance within a tested specimen can confound attempts to define parameters of the Wilson model. Example data sets illustrate that many specimens exhibit a high degree of random variation of erosion rates, and the Wilson model may fit these data no better than a linear or simple square-root model. Fluctuating erosion rates (noise) may lead to misidentification of the initial and final regions of the Wilson model erosion curve.
- Defining all three regions of the Wilson model in a single test is difficult and rare because a wide stress range must be covered but erosion rates at the start of the test cannot be so large that the sample is fully eroded before lower rates are also observed.

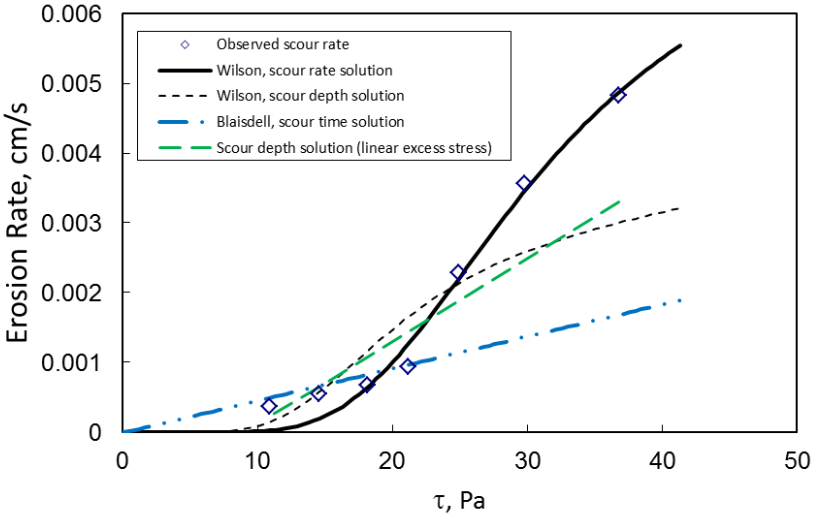

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Figure 3: Jet erosion rate observations analyzed with several predictive models.

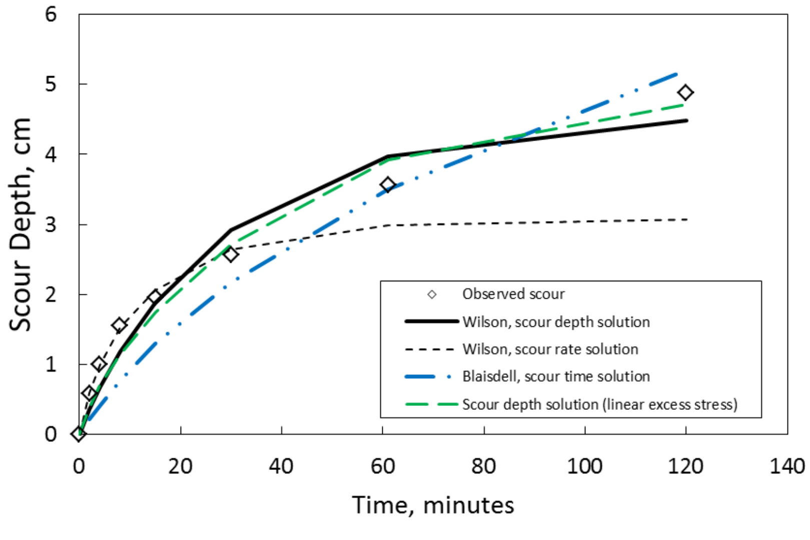

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Figure 4: Jet test scour depth observations analyzed with several predictive models.

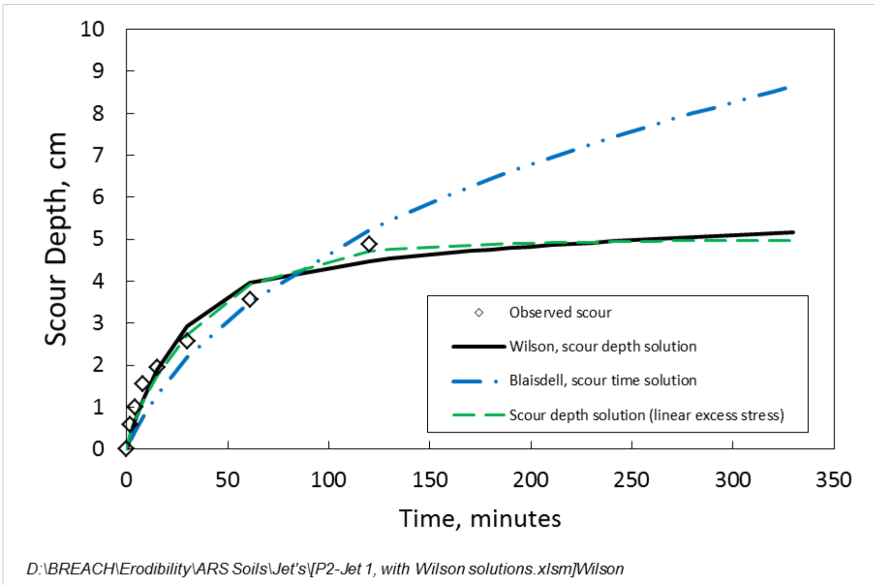

Figure 5: Jet test scour depth predictions by several predictive models at extended times.

3.2 Evaluating Linear vs. Nonlinear Methods Across Large Data Sets

Wahl (2021) reported the results of applying multiple analysis methods to a large set of original JETs comprising 52 specimens of lean clay CL, silty clay CL-ML, clayey sand SC, and silty sand SM, compacted with standard Proctor effort over a range of moisture conditions. Observations similar to those of Wahl (2016) were made regarding individual tests, and the Wilson model and other nonlinear models were found to be capable of providing very good fits to a few tests, just as linear models fit a few individual tests quite well. No consistent pattern or soil characteristic could be identified to explain why certain tests were better fit by nonlinear or linear models. Future research could consider whether other soil parameters such as the coefficient of uniformity, coefficient of curvature, friction angle, cohesion, particle angularity, or clay mineralogy might be correlated with the tendency for nonlinear or linear behavior.

Attempts were made in the Wahl (2021) study to perform tests that would demonstrate erosion behavior in all 3 regions of the Wilson model. Despite using a range of starting stresses, no test ever clearly demonstrated the existence of the “final region” in which erosion rate would be consistently approach proportionality with the square root of the applied stress. Single data points (first observations of a given test) would sometimes appear consistent with the Wilson model’s final region, but multiple data points fitting the final region curve better than a linear model were never observed. More significantly, changes in the initial stress often yielded dramatically different values of the erodibility parameters b_0 and b_1. Multiple tests did demonstrate behavior consistent with the initial and linear regions. This prompted the development of an exponential-linear model that was able to successfully represent the behaviors seen in some of these tests, with slow exponential increase of erosion rate at low stresses, gradually transitioning to a linear relation at larger stresses.

To evaluate which erosion model could best be applied for general use, the full set of 52 tests was used to examine relations between the erosion rate and erosion threshold parameters (e.g., k_d and τ_c for linear models, b_0 and b_1 for Wilson models). In this comparison the simplest analysis method tested was superior—direct linear regression of average scour rate vs. average applied stress. This method, which like the scour depth method (Daly et al. 2013) produces generally larger τ_c and k_d values than the Blaisdell method, provided the best correlation of these key parameters. This was believed to indicate that the linear regression of scour rates method gives the most consistent quantitative measure of the erodibility of soils.

4 CONCLUSIONS

The submerged jet erosion test has been one of the most successful devices used to quantify erodibility of embankment dam soils and has also had a significant impact on other engineering problems related to soil erosion. The recent development of new solution methods—some based on nonlinear erosion models—offers potential for better understanding of soil erosion processes, but there is also a great need to fully understand the limitations of these methods and continue to exercise care in the analysis of data related to highly variable processes.

Most spillway erosion and dam breach models (e.g., SITES, WinDAM) presently use linear erosion models based on the excess stress equation. Thus, it is necessary to use corresponding linearly based methods to analyze JET data. Even if nonlinear erosion equations are incorporated into future application models, there is significant research that indicates linear models to be superior to nonlinear models in their ability to provide consistent results, without excess sensitivity to random variabilities in soil behavior.

Practical points to keep in mind for the analysis and use of JET data include:

- One must consider the stress ranges used in the jet test vs. those experienced in the application environment. Whenever possible, erosion tests should be performed in the stress range that will be experienced in the field.
- Nonlinear solution methods can produce good fits to data obtained from individual erosion tests, but consistency of the resulting erodibility parameters across multiple tests was poor because nonlinear models apparently overfit themselves to noise in the data sets.
- Tests conducted in different ranges of applied shear stress have been unable to demonstrate the “final region” erosion behavior predicted by the Wilson model. An exponential-linear erosion model was developed in Wahl (2021) was also able to provide good fits to individual JETs but produced inconsistent correlation of results from multiple tests.
- Determining the k_d and τ_c parameters of the linear excess stress equation using a simple linear regression of average scour rate versus average applied stress offers a good combination of simplicity and consistency of results. This method provides the best correlation of k_d and τ_c, which has significant value for ranking and classifying the erodibility of soils.
- When large ranges of shear stress are expected to occur in a specific application, the potential for nonlinear erosion behavior still makes it advisable to conduct JETs in the expected stress range to avoid the need to extrapolate erosion rates at untested stresses.
- Manual evaluation of collected JET data by visualization in charts and graphs should be a common practice for ensuring high-quality erodibility parameters.
- The linear regression of scour rates method suggested by Wahl (2021) tends to yield larger τ_c and k_d values than the Blaisdell method. Because application models were developed, tested, and validated using data analyzed by the Blaisdell method, there may be systematic biases introduced by the use of newer methods. For this reason, the Blaisdell method may necessarily remain in use for consistency with past work.
- Previous investigators have proposed various inverse power curve relations between k_d and τ_c and have defined ranges of k_d and τ_c values that define descriptive erodibility categories spanning about five orders of magnitude of k_d (e.g. Hanson and Simon, 2001). Many of these studies relied on the Blaisdell solution method or other methods that tend

to produce significantly different results from the newer methods described here. Development of new category definitions based on new solution methods would be a beneficial future research topic.

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