Definition of a new Parameter for use in Active Structural Acoustic Control

Dan Hendricks
Brigham Young University

Abstract— A new parameter was recently developed by Jeffery M. Fisher (M.S.) for use in Active Structural Acoustic Control (ASAC) which showed potential for achieving better sound radiation reduction than current parameters. This parameter, known as “Vcomp,” was shown analytically to produce results comparable to control of radiated sound power and volume velocity on a simply supported plate. However, experimental tests were less encouraging. This paper is an overview of current efforts to better understand and improve the Vcomp parameter so that it may be used to effectively control sound radiation from simply supported and clamped structures.

I. INTRODUCTION

The field of Active Noise Control (ANC) has rapidly expanded in recent years with many advances coming in control algorithms and a better understanding of the proper placement of error sensors and secondary sources. However, ANC is limited in its effectiveness due to the physical nature of sound waves and difficulties in matching wave amplitude and phase speeds over large areas. One potential solution which has gained considerable interest in recent years is to control the sound producing object (structure) instead of the resulting sound waves. This structural based approach to active noise cancellation has been termed Active Structural Acoustic Control (ASAC).

Active Structural Acoustic Control is very similar in concept to Active Noise Control in that both seek to sense the amplitude and phase of a propagating wave and then emit another wave of equal amplitude and opposite phase with a secondary source. Ideally these two waves would interfere and perfectly cancel each other out, creating a zone of silence. However, early researchers in the field of ANC discovered that merely sensing and cancelling the phase and amplitude of a sound wave (sound pressure levels) could produce local sound cancelation but not global cancelation. This led to a search for other parameters to see if better global results could be achieved by minimization of these new parameters. Two common parameters now used in ANC situations are volume velocity and energy density. Minimizing these quantities provide much better global attenuation than minimizing pressure levels.

When ASAC first began to be studied, researchers quickly looked for a similar parameter which could be used to achieve global results. Elliot et al investigated controlling volume velocity and achieved modest results for lower frequencies. Sung and Jan minimized the sound radiation power from plates and similarly achieved modest results. Both of these quantities were originally derived for ANC situations and were carried over into ASAC experiments.

In 2010 Jeffery M. Fisher developed a new parameter specific to structures for use in ASAC situations. This new parameter was termed “Vcomp” and was shown analytically to produce results comparable to control by minimizing radiated energy density and volume velocity for a simply supported plate. This paper is an overview of current efforts to better understand the Vcomp parameter and extend its use beyond simply supported plates into clamped plates and other practical applications.

II. DERIVATION OF VCOMP

Fahy and Gardonio show that there are two common methods for determining the total radiated sound power from a plate. One of these is the method of independent radiation modes. These modes radiate independent of structural modes and present a better understanding of how sound is radiating from a plate.
Each mode shape is dependent upon the size of the plate and the frequency of interest but is independent of boundary conditions. This independence from boundary conditions makes controlling these modes desirable for ASAC situations where it will not always be possible to determine how a real plate is bounded. Thus any parameter based off radiation modes could potentially turn into a universally used metric for all situations.

Fisher noted in his work that the first four radiation modes of a simply supported plate shared many similarities with the squared spatial derivatives of a plate,

\[\left(\frac{dw}{dt}\right)^2, \left(\frac{d^2w}{dxdt}\right)^2, \left(\frac{d^2w}{dydt}\right)^2, \left(\frac{d^3w}{dxdydt}\right)^2\]  

which represent the transverse, rocking in x, rocking in y, and twisting velocities. A plot showing the first four radiation modes and the spatial derivatives is shown below in Figure 1.

The first radiation mode can be viewed as a form of a transverse velocity, the second and third as rocking and the fourth as twisting. Thus by measuring these spatial derivatives on a plate it is possible to get an approximation of the first four radiation modes; which four modes contribute the most to sound radiation.

Fisher further noted that by combining all four derivative terms into a single parameter and using scaling factors, it was possible to get a uniform value at all positions on the plate. Thus a measurement of this composite velocity \(V_{\text{comp}}\) could be taken at any point on the plate and one would know the composite velocity at all points on the plate. This is highly desirable for control situations.

The scaling factors \((\alpha, \beta, \gamma, \delta)\) were chosen by taking the derivatives of the deflection equation for a simply supported plate

\[
\sum_{q=1}^{F} \frac{f_q}{\rho_s h} \sum_{m}^{\infty} \sum_{n}^{\infty} W_{mn}(x_q, y_q) W_{mn}(x, y) \left[ \frac{\omega_{mn}^2 - \omega^2}{\omega_{mn}^2 - \omega^2 - i \eta \omega_{mn}^2} \right]
\]

and comparing common multipliers for all equations. The final equation for \(V_{\text{comp}}\) for simply supported plates was

\[
\left( V_{\text{comp}} \right)^2 = \alpha \left( \frac{dw}{dt} \right)^2 + \beta \left( \frac{d^2w}{dxdt} \right)^2 + \gamma \left( \frac{d^2w}{dydt} \right)^2 + \delta \left( \frac{d^3w}{dxdydt} \right)^2
\]

with the scaling factors given below in Table 1.

<table>
<thead>
<tr>
<th>Factor</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\gamma)</th>
<th>(\delta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{L_x}{m\pi})^2</td>
<td>(\frac{L_y}{n\pi})^2</td>
<td>(\frac{L_xL_y}{mn\pi^2})^2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Simply Supported Scaling Factors

Using these results with a filtered-x least mean squares control algorithm Fisher was able to get analytical results similar to those shown in Figure 2.
III. IMPROVING VCOMP

Fisher was able to achieve results comparable to control of volume velocity and of energy density by minimizing the Vcomp parameter. Recent research has sought to improve the definition of Vcomp so that even better results can be achieved and it can be applied to more general cases (i.e. other than simply supported rectangular plates). Many options have been investigated but only two of the more significant will be discussed in this paper; derivation of Vcomp for clamped plates and optimization of weighting functions.

III-A CLAMPED PLATES

Since the radiation modes from which Vcomp was derived are independent of boundary conditions, it was hoped that the formula for Vcomp for a clamped plate should be similar to that of a simply supported plate. Any differences should arise in the definition of the scaling factors used to achieve uniform results on the plate and not in the combination of the four spatial derivative terms. It was thus necessary to look at the equation for deflection of a clamped plate and take the respective derivatives to determine the scaling factors.

Sung and Jan present an equation for deflection of a clamped plate derived using a method of virtual work (see Sung and Jan³ for complete derivation of and definition of terms in this equation). This equation,

\[ W(x, y) = \sum_{m} \sum_{n} D(l_1l_2 + 2l_3l_4 + l_5l_6) - m\alpha^2l_2l_6 \cdot X_m(x)Y_n(y) \]

was differentiated to get the required Vcomp terms and derive the scaling factors for clamped plates. Doing so results in the scaling shown in Table 2, where \( \lambda_i \) satisfies the equation \( \cosh(\lambda_i) \cos(\lambda_i) = 1 \).

<table>
<thead>
<tr>
<th>Factor</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{l_x}{\lambda_m} ), ( \frac{l_y}{\lambda_n} ), ( \frac{l_xl_y}{\lambda_m\lambda_n} )</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2: Clamped Scaling Factors

Comparing the clamped scaling factors to the simply supported scaling factors yielded a surprising result. The two factors had different values but actually represented the same parameters, namely the inverses of the bending wave number in the x and y direction (\( \frac{1}{k_x}, \frac{1}{k_y} \)). This result is intriguing in its potential significance as universal scaling factors. If this is indeed the case then it would be possible to determine the scaling factor for any situation simply by computing the bending wave numbers.

III-B WEIGHTING FUNCTIONS

Glancing at the equations for the scaling functions above shows that each function is actually dependent upon the structural modes. The one-one structural mode will have a certain value for each factor but the two-one mode will have values different from the one-one. Thus control of multiple structural modes necessitates a combination of the scaling factors into a single universal value. When Fisher did his original work he arbitrarily chose to simply average the scaling factors over the first 15 mode shapes and use these in his Vcomp formula. This process was chosen because it was thought that the first 15 modes would be the dominant radiators but no attempt was made to determine an optimal weighting function.

A decision was made to study the method of combining the scaling factors to determine if there was an optimal weighting. Several of the options studied include; averaging more or less than 15 values, using asymptotic values, using a frequency dependent scaling factor, and running an optimization process.

The first method studied was to use either more or less than the 15 values Fisher used. Computer simulations were run using an average over several numbers of modes and the radiated power calculated for each simulation. The number of averages ranged...
When these simulations were completed and compared to one another, the results varied. No particular number of averages appeared to give significantly better broad band results than the others. Some of the simulations worked better than Fisher’s results at certain frequencies, but were worse at other frequencies. This result is both encouraging and discouraging at the same time. Encouraging because it demonstrates that Vcomp performance is relatively independent of the number of modes averaged over, discouraging because it did not produce better results.

Another method studied to produce better results was to look at frequency dependent scaling factors. This was done by dividing the frequency response of the control function into sections centered on each modal frequency. The scaling factors were determined for each section and applied to all frequencies in that frequency band. Computer simulations were then run to determine the radiated sound power from a simply supported plate.

Results were better than Fisher’s but only slightly. A large portion of the error came in the frequencies between the modal frequencies. These often experienced boosts in radiated power instead of attenuations. While the overall result of this method produced better results than Fisher, this method would be difficult to implement in real life applications. It would require a prior knowledge of the structure dimensions, boundary conditions, and modal response in order to be used effectively. This does not fit the goal of this research, which is to determine a parameter which can be used with very little or no prior knowledge of the structure. Since only minimal improvements in performance were made, it was decided to not pursue this method more.

The final method studied to produce better results was to use an optimization routine to find the best scaling factors. Doing so was very computationally time intensive but produced significantly better results. However, like the frequency dependent method, this too was discarded as a viable method because the resulting scaling factors did not have any tie in to physical quantities. This would make it difficult to use in practicality because it would be necessary to model each situation in a computer and run the optimization routine on the model before applying to a real structure. The goal of this research is to determine a

from one (assuming only the first mode radiated sound effectively) to 150 (assuming many of the higher modes radiated effectively). When the scaling factor was averaged over a high number of modes, it was noted that the value seemed to be reaching an asymptote. A true asymptote could not be achieved because it is the average of a quantity which progressively and continuously decreases (goes as 1/\(m^n\)), but the rate at which the scaling factors changed did reach a semi-asymptotic response, as shown below in Figures 3.1 and 3.2 (\(\beta, \gamma, \delta\) shown for two different Lx/Ly ratios).

![Figure 3.1: Scaling Factors for Lx/Ly = 1.0](image)

![Figure 3.2: Scaling Factors for Lx/Ly = 0.4](image)

Thus a simulation was run with the scaling factors calculated at a very high frequency to approximate the semi-asymptotic response.
parameter tied to physical quantities for implementation in actual situations.

IV: FUTURE WORK

While significant progress has been made in understanding and improving the \( V_{\text{comp}} \) parameter, there is still much work to do. Efforts thus far have focused on rectangular plates but not all physical structures are rectangular. It will be necessary therefore to look at circular and cylindrical plates to determine if similar scaling factors exist. Work will also be needed to be done on proving \( V_{\text{comp}} \) is indeed uniform across these plates as well.

Another field in which work has already begun is the optimization of the measurement technique and placement of physical hardware. One of the strong points of \( V_{\text{comp}} \) is that it is theoretically uniform across the entire plate. However, it is impossible to get a perfect measurement of the four spatial derivatives at any single point on a plate. Instead, four accelerometers are currently used to measure the accelerations at four closely spaced locations and then the measurements are combined to form numerical approximations of the transverse, rocking and twisting velocities. A schematic of the experimental set up is shown in Figure 4.

\[
\begin{align*}
\frac{d^2 w}{dxdt} &= \int \frac{1}{2\Delta x} (a_2 + a_4 - a_1 - a_3) dt \\
\frac{d^2 w}{dydt} &= \int \frac{1}{2\Delta y} (a_1 + a_2 - a_3 - a_4) dt \\
\frac{d^2 w}{dxdydt} &= \int \frac{1}{\Delta x \Delta y} (a_2 - a_1 - a_4 + a_3) dt
\end{align*}
\]

This method creates a good approximation of the \( V_{\text{comp}} \) terms, but is limited by the spacing and accuracy of the accelerometers. Work is currently being conducted to determine what the optimal spacing distance is for the accelerometers to achieve the most accurate measurements. Once this has been determined, it will be necessary to find an appropriate method for attaching the accelerometers in an accurate and repeatable manner.

Similarly, all accelerometer (and all physical measurements) will inherently be subject to noise. Noise could create major problems with \( V_{\text{comp}} \) if the noise in the readings is on the same order of magnitude as the actual readings. In actuality, since several of the \( V_{\text{comp}} \) terms are formed by subtracting half of the accelerometer readings from each other, the noise could have a significant effect if it is even on the same order of magnitude as the difference between any two accelerometer readings. This makes the process inherently more susceptible to noise than most processes. Research is being done to determine the maximum acceptable noise levels for \( V_{\text{comp}} \), and hence what quality of accelerometer would be necessary to use for the project. The better quality required, the more expensive the accelerometers.

This research is currently looking at several causes of noise including improper accelerometer amplitude and phase calibrations, incorrect placement of the accelerometers, and random noise generated throughout the process. The eventual success or failure of the \( V_{\text{comp}} \) parameter could be dependent upon an ability to eliminate or compensate for noise.

CONCLUSIONS

The use of a composite velocity parameter for active structural acoustic control situations is a field of research which has great potential. It is a parameter
which is uniform across simply supported and clamped plates and which appears to be valid for all frequencies. While results in actually controlling acoustic radiation using Vcomp have failed to produce significantly better results than current methods, it nonetheless has several advantages. Controlling a plate by minimizing energy density or volume velocity requires a large number of sensing devices, often spaced at significant distances away from the structure itself. This is bulky, expensive and often impractical where space limitations are required to be taken into effect. Vcomp is a parameter which produces similar results and yet only requires four accelerometers and very little space to operate. Thus even if Vcomp cannot eventually be modified to produce better results than energy density or volume velocity, it will still be a valuable resource due its easy implementation and small physical footprint.

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REFERENCES


