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Third-Graders Learn About Fractions Using Virtual Manipulatives: A Classroom Study

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With recent advances in computer technology, it is no surprise that the manipulation of objects in mathematics classrooms now includes the manipulation of objects on the computer screen. These objects, referred to as *virtual manipulatives*, are essentially replicas of physical manipulatives placed on the World Wide Web in the form of computer applets with additional advantageous features. The purpose of this project was to explore the effects of using several virtual manipulative computer applets for instruction during a fraction unit in a third-grade classroom. The participants in this study were 19 third-grade students. During a two-week unit on fractions, students interacted with several virtual manipulative applets in a computer lab. Data sources in the project included a pre and posttest of students' conceptual knowledge, a pre and posttest of students' procedural computation skills, student interviews, and a student attitudes survey. The results indicated a statistically significant improvement in students' posttest scores on a test of conceptual knowledge, and a significant relationship between students' scores on the posttests of conceptual knowledge and procedural knowledge. Student interviews and attitude surveys indicated that the virtual manipulatives (1) helped students in this class learn more about

> fractions by providing immediate and specific feedback, (2) were easier and faster to use than paper-and-pencil methods, and (3) enhanced students' enjoyment while learning mathematics.

"Tools of some kind are unavoidable and essential for doing mathematics" (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Human, 1997, p. 2). Mathematical tools can build a foundation for children to understand concepts, which can then initialize an abstract understanding (Hiebert et al, 1997). Mathematics educators and cognitive theorists have supported a theory of concept development based on a progression from physical objects (or mathematical tools) to representational forms and abstract thought (Bruner, 1960, 1986; Piaget, 1952). These theories are evident in school classrooms where physical manipulatives have become popular for mathematics instruction, and where teachers are now using computer or *virtual manipulatives*. Although the availability of computer technology in elementary schools has increased rapidly in recent years, national surveys of teaching practices show that a small percentage of elementary-school teachers use computers to teach concepts during mathematics instruction (Weiss, 2000). The purpose of this project was to explore the use of several virtual manipulative computer applets for instruction during a fraction unit in a third-grade classroom.

What are Virtual Manipulatives?

Virtual manipulatives are essentially replicas of physical manipulatives placed on the World Wide Web in the form of computer applets with additional advantageous features. Moyer, Bolyard, and Spikell (2002) define a virtual manipulative as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge." Although the use of physical manipulatives in the teaching and learning of school mathematics has produced positive results with students (Parham, 1983; Raphael & Wahlstrom, 1989; Sowell, 1989; Suydam, 1985, 1986), teachers have often questioned how to translate students' interactions with manipulatives to proficiency with abstract symbols. Researchers and educators have argued that the mere use of manipulatives does not guarantee that students will understand concepts and procedures and be able to connect these concepts to abstract symbols without teachers making these connections explicit (Ball, 1992; Baroody, 1989; Meira, 1998; Moyer, 2001;

Thompson & Thompson, 1990). In other words, simply using the manipulatives does not insure learning.

One feature that makes virtual manipulative applets advantageous for mathematics instruction is their capability to connect dynamic visual images with abstract symbols – one limitation of physical manipulatives. Unlike physical manipulatives, electronic tools use graphics, numbers, and words on the computer screen to connect the iconic with the symbolic mode (Kaput, 1992). For example, a base-10 blocks applet may show both the visual representations of the blocks and, at the same time, show the number of hundreds, tens, and ones in a place-value notation. These pedagogical notations provide support that bridges students' manipulations of the virtual blocks with the formal symbolic system, and this connection holds promise for improving student learning.

Recent publications have described how elementary teachers can use virtual manipulatives to teach fraction concepts (Suh, Moyer, & Heo, 2003) and how middle school teachers can use virtual pattern blocks, virtual platonic solids, and virtual geoboards to investigate concepts in geometry (Moyer & Bolyard, 2002). These virtual manipulative applets are available on the World Wide Web on web sites such as the National Library of Virtual Manipulatives (http://matti.usu.edu/nlvm) or the National Council of Teachers of Mathematics electronic resources (http://www.nctm.org). Elementary teachers have begun to discover these free resources and to develop lessons for their classrooms using these applets.

Limited Research on Virtual Manipulatives

It is a challenge to find research that specifically describes and documents work in classrooms with virtual manipulatives. One of the reasons for this lack of information may be the specialized knowledge required of teachers who wish to use virtual manipulatives for instruction. To use virtual manipulatives, teachers must have an understanding of how to use representations for mathematics instruction as well as an understanding of how to structure a mathematics lesson where students use technology. Research has shown that it is a challenge for teachers to transform mathematical ideas into representations (Ball, 1990; Orton, 1988). Teachers must also be comfortable with technology and be prepared for situations where computers may not be reliable or Internet connections are not working properly. These factors may deter teachers and researchers from designing lessons with virtual manipulatives.

Previous research on related technology sheds some light on the possibilities of virtual manipulatives. Studies on computer-based manipulatives have produced inconclusive results, due to a variety of design and sampling characteristics that may affect student achievement results. A variety of studies in which “computer manipulative programs” were examined for their ability to support student learning in mathematics confirmed positive results in student achievement and attitudes (Char, 1989; Clements & Battista, 1989; Kieran & Hillel, 1990; Thompson, 1992). For example, students’ ideas about shapes were more mathematical and precise after using the computer program Logo (Clements & Battista, 1989). Studies where computer manipulatives were used in combination with physical manipulatives also showed positive gains in increasing students’ conceptual understanding (Ball, 1988; Terry, 1996). Yet other studies using computer-based attribute blocks (Kim, 1993), pegboards and color cubes (Berlin & White, 1986), geometric shapes (Nute, 1997), and transformation geometry concepts (Pleet, 1990) produced results that indicated no significant improvement in student achievement. Although these results are mixed, the amount of research on high-quality dynamic virtual manipulatives is so limited that a judgment about their potential uses in mathematics instruction is entirely speculative.

Research, descriptive information, and classroom projects involving virtual manipulatives are beginning to appear in print (Cannon, Heal, & Wellman, 2000: Dorward & Heal, 1999; Drickey, 2000; Moyer, Bolyard, & Spikell, 2002; Nute, 1997; Terry, 1996). In a recent study, kindergarten children created a variety of patterns using virtual pattern blocks, concrete pattern blocks and drawings. Researchers compared the number, type, complexity, and creativity of the patterns the children created during the project (Moyer, Niezgoda, & Stanley, in press). This project found that the children created a greater number of patterns, used more elements in their pattern stems, and exhibited more creative behaviors when they were using the virtual pattern blocks. The virtual manipulatives gave second language learners a way to express their thinking and understanding of patterns through the manipulation of the blocks.

In another classroom project using virtual manipulatives, the teacher focused on second graders’ abilities to use virtual base-ten blocks to demonstrate regrouping (Moyer, Niezgoda, & Stanley, in press). Researchers were interested in determining how students’ interactions with the virtual base-10 blocks would impact their abilities to create a pictorial representation of the addition regrouping process. In this project, students were better able to express a conceptual understanding of the regrouping process after using the virtual base-10 blocks. Researchers also concluded that the virtual manipulatives and drawings helped the second language learners express their conceptual understanding of the regrouping process when they were unable to verbalize this process.

The Current Classroom Project

This project developed as a classroom teacher's personal action research project. Because action research is an organized approach to classroom inquiry, the teacher chose to collaborate with a university researcher to make the process of inquiry more formalized and structured. The teacher was particularly interested in examining how the use of the virtual manipulatives might enhance students' learning of fractions beyond what students had learned through the teacher's use of physical manipulatives and other instructional strategies. Many teachers are not using virtual manipulatives for mathematics instruction and it was challenging to find resources and research on which to base the design of this project. For this reason, the researchers chose to create teacher-made handouts and assessments to guide and evaluate the students and their use of several virtual fraction manipulatives during the study. The project was conducted in a third-grade classroom and focused on several fraction concepts.

The research focused specifically on the following questions: (1) What impact do virtual fraction manipulatives have on students' conceptual and procedural understanding of fractions? and (2) What are students' attitudes about using virtual fraction manipulatives during the learning of fractions? Our hypothesis for the first question was that there would be some impact on students' conceptual or procedural knowledge following their work with the virtual fraction manipulatives. Our hypothesis for the second question was that students would have positive attitudes about using the virtual fraction manipulatives to learn fraction concepts.

Although research generally supports the use of physical manipulatives and computer technologies, this action research project provides a glimpse into a third grade classroom showing how virtual manipulatives impacted teaching and learning in this setting. The project serves as an impetus for teachers and researchers to use virtual manipulatives in teaching mathematics in other classroom projects.

METHODS

Participants

The participants in this study were 19 of 25 third-grade students. The school where the project was conducted is approximately 30 minutes from the Washington DC Metro area and has a diverse student population. The demographics of the 25 children in the class included Caucasian, Hispanic, African-American, Asian, and Middle-Eastern students. There were several special needs students in the class including four autistic children, three ESOL children, three children with a variety of learning disabilities, and four children receiving various gifted-and-talented services. There was a full-time instructional assistant in the classroom when the autistic children were present. The four autistic children were not included in the data reported here because they attended mathematics classes in a self-contained classroom. Two other students were also not included in the data reported because they were absent for more than three days during the project. The 19 children discussed in the following results include ten Caucasian, two Hispanic, one African-American, three Asian, and three Middle-Eastern students.

Procedures

Environments. Part of this project was conducted in the classroom. The physical environment of the classroom was organized for cooperative learning with desks placed in groups of five and students of different abilities at each table. The classroom contained three computers, a TV, laser disc, classroom library, listening stations, dictionaries, thesauruses, and alpha smarts for the autistic students. During the part of the project when the students were in the classroom, the teacher introduced the virtual fraction manipulatives using a computer connected to a TV monitor. This allowed the teacher to focus students on features of the virtual fraction manipulatives that would be used during each lesson.

Students interacted with the virtual manipulatives in the computer lab on four consecutive days. The computer lab was conveniently located next to the teacher's classroom, which provided a smooth transition from the classroom to the computer lab. There were 35 computers available in the computer lab, and a large screen where the teacher could display examples for the whole class to view. Student worked independently at their own computers during the virtual manipulative activities. Interviewers were present in the computer lab during these sessions.

The instructional program in the classroom was based on standards of learning outlined by the State Department. The teacher used these standards as a guideline to design the fraction tasks used in the computer lab and the pre and post assessments. The fraction standards addressed during the lessons included parts of a whole, parts of a group, comparing fractions, and equivalent fractions.

Design. The project occurred during a two-week time frame during regular school hours. Students participated in the activities in the project during their regularly scheduled mathematics class sessions. During the first week of the project, students completed a pretest that assessed conceptual knowledge of fractions and a pretest that assessed procedural computation skills with fractions. These were both teacher-made tests. During the first week students were also introduced to the virtual manipulative applets. The teacher chose to introduce students to the virtual manipulatives by using the base-10 blocks applet. The purpose of allowing students to use this virtual manipulative prior to the study was to give students the opportunity to become familiar with computer applets. The virtual base-10 blocks contain a place value mat with ones, tens, and hundreds block representations. Students can manipulate the blocks and use them to model addition and subtraction. The teacher chose this particular applet so that students would be familiar with the virtual manipulatives, but they would not be practicing fraction skills using this applet.

During the second week of the project the teacher taught a unit on fractions in the computer lab. The first author of this article, a third-grade teacher, led instruction and discussions with the students. Fraction concepts in this unit included: parts of a whole, parts of a group, equivalent fractions, and comparing fractions. Students worked in the computer lab for four days using the virtual manipulatives, with a one-hour lesson on each day. Each lesson began with an introduction to the virtual manipulative applet that would be used that day and several mathematical tasks for the students. On each day in the computer lab, students were given a teacher-made worksheet that provided instructions for using the virtual manipulatives and prompted them to complete several mathematical tasks. We believed these directions helped students to focus on the mathematical tasks during the lessons. The teacher reviewed the instructions with the class and modeled how to use the virtual manipulatives before students worked independently on the activities.

Students used several applets from the Grades 3-5 strand located on the National Library of Virtual Manipulatives website (http://matti.usu.edu). On

Day 1 in the computer lab, students used the "Fractions – Parts of a Whole" virtual manipulative applet under the Numbers and Operations strand. On Day 2, students explored parts of a group using the "Pattern Blocks" applet under the Algebra strand. On Days 3 and 4, students used the "Equivalent Fractions" and "Comparing Fractions" applets under Number and Operations.

To control for the effect of the series of tasks with the virtual manipulatives, students learned the same fraction concepts (parts of a whole, parts of a group, equivalent fractions, and comparing fractions) using physical manipulatives and visual/pictorial models prior to this project. The teacher's purpose for teaching students these concepts using other methods prior to the use of the virtual manipulatives was to determine whether or not changes in students' test scores (either favorable or unfavorable) could be attributed to the virtual manipulatives.

Data Sources

Several sources of data were collected during the two-week project including a pre and posttest of students' conceptual knowledge, a pre and posttest of students' procedural computation knowledge, student interaction interviews, and student attitude surveys. These sources were used to triangulate the data collected during this project.

Pretests and Posttests. The teacher created four teacher-made tests to assess students' understanding of fractions. These tests included a pre and posttest of *conceptual knowledge* and a pre and posttest of *procedural knowledge*. These tests allowed the teacher to determine students' understanding of the concepts and skills using two different formats for assessment. Students completed the two pretest assessments prior to their investigations in the computer lab with the virtual manipulatives. They completed the two posttests after participating in the unit on fractions in the computer lab.

The pre and posttests of *conceptual knowledge* included four questions and students were asked to explain their thinking using writings and drawings. For example, one question on this assessment asked students to draw a picture representation of the fraction one-fourth and explain their drawing. Students' written responses on this assessment allowed for a richer analysis of their understanding of fractions. The teacher graded student papers using a rubric and assigning two points to each solution and two points to each written explanation. Therefore, students could receive a point value of zero, one, or two for each portion of each problem on this assessment. This gave a total of 16 points for the pretest and 16 points for the posttest on conceptual knowledge. This instrument used a process scoring rubric system because the tasks were open-ended involving students' writings and drawings. This scoring procedure allowed for a range of incorrect, partially correct, and entirely correct values to be assigned to student work.

On the pre and posttests of *procedural knowledge*, students responded to computation questions on various fraction topics. There were 14 mathematics exercises focusing on fractions and each question was assigned one point for a possible total of 14 points on the test. These questions required students to demonstrate fraction knowledge using computation and symbols only. For example, questions on this assessment asked students to compare fractions such as 4/8 and 1/4 or to name a fraction equivalent to 1/2. The pre and posttests of procedural knowledge included questions on each test with different number amounts that were similar in difficulty. The teacher graded student papers by scoring each numerical answer either correct or incorrect. Therefore, students could receive a maximum point value of 14 points for the pretest on procedural knowledge and 14 points for the posttest on procedural knowledge. This instrument used a summative scoring process because there was only one correct answer for each test item, so that answers on this test were either correct or incorrect.

On the pretest version of this assessment, physical manipulatives were made available to students. Students were familiar with these manipulatives because they had recently used these materials to learn fraction concepts. On the posttest version of the assessment, virtual manipulatives, and *not* physical manipulatives, were made available for students. We chose to do this because we wanted to examine if the virtual manipulatives *alone* would impact student scores in any way on the posttest. We were also curious whether or not the students would choose to use the virtual manipulatives as a tool to support them in completing the posttest.

Interviews. During student activities in the computer lab with the virtual manipulatives, interviewers interacted with students while the students were interacting with the virtual manipulatives. The interviewers asked three general questions of each student, while allowing students to select what to talk about. Every student was interviewed on each of the four days. Both interviewers spoke with every student on different days of the project asking the same questions. Having different interviewers ask the same questions of the same students on different days allowed us to check students' responses with different interviewers to determine if individual children's responses were consistent on different days and with different interviewers. These transcripts also provided a more complete picture of each child's response during the project. The two interviewers agreed on a small set of three general questions to ask each student on each day. Students were asked the following questions: (1) Tell me what you're doing, (2) Can you explain to me how you are using the virtual manipulative, and (3) How is this applet helping you learn about fractions? These conversations were transcribed so that a written record of students' direct quotes during their work with the virtual manipulatives could be recorded. We analyzed these comments using a narrative analysis procedure to identify dominant themes in students' experiences (Shank, 2002).

Attitude Survey. A secondary form of data collected was a closed questionnaire. Students provided their opinions about the virtual manipulatives on a Likert scale (questionnaire adapted from Kemmis and McTaggart, 1988). The choices on the survey were a happy face for yes (which represented a positive response), a straight face for maybe (which represented a neutral response), and a sad face for no (which represented a negative response). The questionnaire was administered in the classroom three days after students used the virtual manipulatives for the fraction unit. The survey was anonymous; therefore, students did not include their names on the surveys.

RESULTS

Conceptual Knowledge Assessment

To test the hypothesis that there would be some impact on students' conceptual knowledge following their work with the virtual fraction manipulatives, our first analysis used student test scores of fraction knowledge as measured by the conceptual knowledge assessment. We compared students' scores on the pretest and posttest of conceptual knowledge using a paired *t*-test. These results are shown in Table 1. This analysis indicated that students scored significantly higher on the posttest assessment of conceptual knowledge ($M = 11.0$, $SD = 3.61$) than they scored on the pretest ($M = 9.58$, $SD = 4.53$), $t(18) = 2.05$, $p < .05$. The class averaged a score of 60% on the pretest of conceptual knowledge and 69% on the posttest. These results supported the hypothesis that the virtual fraction manipulatives would impact students' conceptual knowledge.

Table 1
Summary Results of Paired t-tests on Assessment Measures of Students' Conceptual and Procedural Knowledge

Assessment Type		Pretest	Posttest	t	p
Conceptual Knowledge	M	9.58	11.00	2.05	.05*
	SD	4.53	3.61		
Procedural Knowledge	M	12.63	12.74	0.32	.75
	SD	1.34	1.10		

Note: Maximum score on Conceptual Knowledge Assessment = 16; Maximum score on Procedural Knowledge Assessment = 14.
N = 19
*p < .05

To provide a detailed analysis, each student was assigned a number that would stay consistent throughout all of the data analysis. We examined changes in individual students' scores between the pretest and posttest to determine those scores that increased, those that decreased, and those that stayed the same. The data from individual students' scores indicated that 10 out of 19 students improved their scores between the pretest and the posttest, 5 of 19 students' scores decreased, and 4 of 19 students had scores that stayed the same between the two assessments. Nine of ten students whose scores improved showed gains of at least two and as much as eight points between the pretest and posttest. Three of the four students whose scores stayed the same had extremely high scores on the pretest (94%, 100%, and 100%), which allowed little or no room for improvement of these students' scores on the posttest. A majority of the students improved their scores on this assessment after using the virtual fraction manipulatives. These results indicate that the virtual manipulatives helped over half of the students (53%) in this class improve their conceptual understanding of fractions, while four students showed no change (21%) and five students' scores decreased (26%).

Procedural Knowledge Assessment

To test the hypothesis that the virtual fraction manipulatives would impact students' procedural knowledge, we examined student test scores of fraction knowledge as measured by the procedural knowledge assessment. We used a paired *t*-test to compare students' scores on the pretest and posttest of procedural knowledge. These results are shown in Table 1. This analysis indicated no significant differences between students' scores on the pretest of procedural knowledge (M = 12.63, SD = 1.34) and the posttest (M = 12.74, SD = 1.10), $t(18) = 0.32$, $p = .75$. The class average score on this assessment was 90% on the pretest and 96% on the posttest. Because the pretest scores on this assessment were so high, this allowed very little room for improvement in students' scores.

On this assessment, we also examined students' scores for changes between the pretest and posttest to determine those scores that increased, decreased, and stayed the same. The data from individual students' scores indicates 7 out of 19 (37%) students improved their scores between the pretest and the posttest, 5 of 19 (26%) students' scores decreased, and 7 of 19 (37%) students had scores that stayed the same. One of the limitations of these data is that students' scores were already very high on the pretest (i.e., an average of 90%). However, it is also important to note that although the scores were already very high, 74% of the students had scores that stayed consistent or increased on the posttest.

Comparisons Between the Two Assessments

We assigned the same number to each student on each assessment so that we could make further comparisons between the two assessments. We analyzed the relationship between students' scores on both posttests by using a Pearson correlation. This analysis determined a significant positive correlation between students' scores on the posttest of conceptual knowledge and students' scores on the posttest of procedural knowledge ($r = .45$, $p < .05$). It is important to note that these results were obtained with a relatively small sample size. We also compared each individual student's score performance on each assessment. There were 11 of 19 students (58%) whose scores were consistent between the two different assessments; that is, they earned scores on both assessments that improved or stayed the same. However, 8 of 19 students (42%) had scores that increased on one of the assessments and decreased on the other assessment, showing that their performance was not consistent between the two assessments.

Student Attitudes Questionnaire

On the student attitudes questionnaire, students were asked to evaluate their experiences with the virtual manipulatives. A summary of the questions and student responses for this survey are presented in Table 2.

The majority of responses from students on their experiences with the virtual manipulatives was positive. A tabulation of the results shows 59% positive responses, 23% neutral, and 18% negative. The two questions that received the most positive responses addressed how students felt about using the computer to work with fractions, and the ease of using the virtual manipulatives. The majority of students in this class had positive attitudes about using the virtual manipulatives, although many of them did not view the virtual manipulatives as tools to help them answer questions on their assessments.

Table 2

Summary Results of the Student Attitudes Survey

Question	Positive Response	Neutral Response	Negative Response
#1. Did you like using the computer to work on fractions?	15	4	0
#2. Did the virtual manipulatives help you understand fractions?	9	7	3
#3. Do you think the virtual manipulatives were easy to use?	15	4	0
#4. Would you like to use the virtual manipulatives to learn other math concepts?	11	6	2
#5. Did you like using the computer to learn about fractions more than you like the activities we did in the classroom?	12	4	3
#6. Did the virtual manipulatives help you answer the questions on your tests?	5	1	13

Note: N = 19

Interaction Interviews

During computer sessions, we interviewed the students as they interacted with the virtual manipulatives. These interviews were transcribed and coded using a narrative analysis procedure (Shank, 2002). The researchers did not have any specific themes in mind during the initial reading of the data. Themes evolved during the coding process. Common ideas were coded and translated into a few generalized themes. After the themes were chosen and coded, the researchers reread the data and listed some specific examples from the interviews that matched each theme to demonstrate how the data collected in the interviews supported the theme.

There were four consistent themes throughout the interviews. One theme was that the virtual manipulatives were helping students learn about fractions. Some examples of this theme were comments such as: "It's like a computer game that helps me learn" (Student #18); "It shows me the pictures, which helps" (Student #3); and, "I learned how to make it [fraction] easier. It [virtual manipulatives] helps me to understand more"(Student #1).

Another theme was that the students liked the immediate feedback they received on the virtual manipulative applets. On these applets, the computer indicated when students' responses were correct or when they needed to be revised. Some applets also told students specifically which aspect needed to be changed. Students' comments that reflected their appreciation of this feedback were: "If you make a mistake, the computer tells you. On a regular test you have to wait until the teacher grades it, but the computer can tell you right away" (Student #8); "If I was wrong, it [the virtual manipulatives applet] would tell me the information. Like if the denominator was wrong, then I would switch that" (Student #1); and, "If I get it wrong, I can fix the problem because it [the virtual manipulatives applet] tells me" (Student #14).

Another consistent theme in students' comments was that the virtual manipulatives were easier and faster to use than paper-and-pencil. These ideas centered on the notion that the student was able to move quickly through the mathematical tasks the teacher assigned. Students reported the following information to interviewers: "It [the virtual manipulatives applet] helped me more than the paper because it's easier with the mouse than the pen-and-pencil. It's hard for me with pencil-and-paper. I can't draw so good" (Student #9); "It [the virtual manipulatives applet] helps me learn more because it's faster on the computer than when I do it on paper. I can do more problems than when you have to write" (Student #14); and, "It's easier on the computer because it's harder to draw them [fractions] on paper" (Student #5).

The fourth consistent theme was enjoyment. This theme was indicated by comments that showed students had a positive experience while working with the virtual manipulative. Students commented: "This [virtual manipulatives] is cool!" (Student #2); "Cool! This is fun!" (Student #4): and "It [virtual manipulatives] was fun because you could name them, the fractions, yourself" (Student #14). Overall, these four themes were positive and indicated students felt successful during their mathematics experiences. Students indicated that the virtual manipulatives were helping them learn, they were easy to use, they gave specific feedback, and they were enjoyable to use.

Limitations

The results in this project are certainly limited because they report results from only one classroom. We cannot make a generalization about the effects virtual manipulatives will have on other third-grade students. However, this project does provide information that can be used by other teachers and researchers as a resource when developing classroom research projects where virtual manipulatives are used. One of our biases was that we have both had classroom teaching experience where we used manipulatives for mathematics instruction. Because we believe that a variety of mathematical tools can enhance instruction, we began the project with positive attitudes ourselves about using the virtual manipulatives to teach fractions with these third-grade students. To determine the extent to which virtual manipulatives may affect third graders' learning of fractions, many more studies will need to be conducted. We can only conclude that, under these instructional conditions, the virtual manipulatives had a significant positive effect on students' conceptual knowledge in this classroom.

We also acknowledge limitations in our data collection techniques. The assessments used were teacher-made tests, and therefore, not standardized. Although the pre and posttests were similar, there may have been discrepancies in the levels of difficulty on each assessment. In addition, results on the attitudes questionnaire may have been influenced by students' perceptions of the anonymity of the survey. Although this survey was anonymous, students still may have been reluctant to be truthful about their responses because the researcher was also their teacher. This may have caused students to choose responses they thought their teacher would like to hear. This may have also been true of responses during the interviews on the days when their teacher was their interviewer.

DISCUSSION

With these limitations taken into account, we still believe that this project represents important information worth sharing with the education community. The conceptual knowledge assessment showed significant gains in this relatively small sample of students, with over half of the students in this classroom improving their scores after using the virtual manipulatives. One reason students' scores on this assessment may have been significant is that the virtual manipulatives the students used were dynamic visual images of fraction amounts; the pre and posttests included questions where students were required to draw and explain a fraction amount using a pictorial representation. Practicing with the visual computer images could have enhanced students' abilities to explain and represent their thinking using pictorial models. The virtual manipulatives also provided opportunities to practice using a visual model that could be changed and manipulated. Students do not have this opportunity for practice with dynamic visual representations when they view pictorial images on textbook pages or worksheets.

There was much more room for students to demonstrate improvement on the conceptual assessment than there was on the procedural assessment. After learning about fractions using the teacher's current instructional strategies, students' scores on the procedural assessment were already at an average of 90% for the class. These scores may indicate that students had a strong grasp of the procedures for working with fractions, but had not mastered the ability to draw pictures and write explanations about these ideas. Although students are familiar with accurate procedures for solving fraction problems, they may not understand or be able to explain the thinking behind these procedures. Improvement in students' scores on the conceptual assessment after using the virtual manipulatives may indicate that working with these dynamic visual images supported their growth in this area.

On both assessments, student improvement may have been attributed to the immediate and specific feedback students received while using the virtual manipulatives. These specific instances of feedback in written form on the computer may have served the function of correcting or highlighting students' errors, making students more aware of their own misconceptions. This feedback served as a model for students that indicated how to write fraction notations accurately using numbers and words. For example, the feedback on some virtual manipulatives included words on the screen that served as a model for explaining concepts by using mathematical terminology. Students may have learned this terminology while using the virtual manipulatives and then used it accurately in their own written explanations on the assessments.

The virtual manipulatives also allowed for accommodations and differentiation of the different ability levels of the learners in this group of students. Students were able to work at their own pace; therefore, more able students completed many more tasks than students who did not work at this rapid pace. This kept the advanced students interested and engaged. The multiple representations on these applets also supported students with learning disabilities. The virtual manipulatives often included representations in the form of visual objects, written words, and numerical symbols. All of these representations provided support and scaffolded learning for the less able students in the group. These instances of individual feedback, multiple representations for support, and a variable pace for completing tasks may have been an important aspect in the differentiation of instruction during these lessons in the computer lab. The teacher noted that the students who generally perform poorly on mathematics assessments performed better than the teacher expected after using the virtual manipulatives. Some of the students who typically demonstrated lower achievement in this classroom showed improvement in their scores. Other research has reported that when technology is used in mathematics, it enables students to concentrate on the conceptual knowledge in the task because they are not focused only on the computations of the task (Enderson, 1997; Kaput, 1992).

Students were permitted to use the manipulatives and virtual manipulatives on the procedural knowledge assessment, but they did not use them on the conceptual knowledge assessment. Having access to these mathematical tools on this assessment may account for the relatively high scores on the procedural knowledge tests. These mathematics tools may have provided the necessary support for students who needed them to complete the tests successfully. Although this assessment did not indicate significant changes in students' scores, it also indicates that using the virtual manipulatives did not negatively affect students' knowledge.

Based on the results of the pretest, students possessed a good understanding of procedural knowledge on these concepts prior to using the virtual manipulatives. As the two pretests indicate, students started the project with higher scores on the pretest of procedural knowledge than they did on the pretest of conceptual knowledge. One reason for this difference was the type of scoring used for these two assessments, with the procedural knowledge test using one point for each item and the conceptual knowledge test scored with a rubric. Therefore, the rubric scores do not equate to the same percentage scores for this assessment.

Students believed that the virtual manipulatives helped them to be more successful when learning about fractions. Although many students did not use the virtual manipulatives as tools for taking a test, they did believe that they were helpful to their learning prior to taking the test. Their experiences with the virtual manipulatives during the unit may have given them confidence and a belief that they did not need to rely on them for the test. Many students had positive attitudes regarding their virtual manipulative experience, and were engaged in the activities during class sessions. The themes evident in the interviews and responses to the questionnaires support this conclusion.

The teacher-researcher reflected that her knowledge of the virtual manipulatives was essential when using them as an instructional tool. She felt proficient in using physical manipulatives and experimented with several virtual manipulatives prior to planning and teaching the lessons, making her feel more comfortable with these tools. Her own knowledge of how to use these tools effectively may have been a factor in obtaining positive results in her classroom. Similar research on physical manipulatives has indicated that teachers who are more experienced and knowledgeable about how to use manipulatives will produce more positive results in their classroom (Raphael & Wahlstrom, 1989). Because the common themes in the interviews were enjoyment, helpfulness, immediate feedback, and ease of use, the virtual manipulatives supported the learning environment for this class. Student engagement and enjoyment are attributes conducive to a successful mathematics lesson.

Final Thoughts

In mathematics it is beneficial for students to use a variety of tools to help them understand concepts. The use of multiple representations can enhance the development of students' abilities to think flexibly about mathematics topics. The significant results of the conceptual knowledge assessment, where students drew pictorial models and wrote descriptions of their thinking, may point to an important feature of using virtual manipulatives in teaching and learning mathematics – the use of multiple representations in a simultaneous and dynamic visual format. The virtual manipulatives used in this project included visual images of manipulatives, numbers, and words that worked together to respond to students' actions on the computer screen. These visual images, in combination with each other, may have provided important instruction and support while students were interacting with and manipulating the images. This use of dynamic visual models with multiple representations is worthy of further study to determine its impact on students' learning and understanding of mathematical concepts.

Virtual manipulatives are an innovative and useful way to enhance mathematics teaching. Because the virtual manipulatives in this study proved to be effective for these students, the classroom teacher has added this teaching tool to her repertoire of instructional resources. This classroom project highlights some of the advantages of using virtual manipulatives for teaching fraction concepts and prompts researchers to examine the effectiveness of dynamic visual models on developing students' flexibility and knowledge of various representational forms.

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