

Backprojection Analysis of MIMO SAR

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Abstract—Multiple-input multiple-output (MIMO) techniques have brought significant advances to wireless communications. In recent years, researchers have sought to bring similar advances to radar using MIMO. One specific area that has received relatively little attention is MIMO synthetic aperture radar (SAR). The advantages that MIMO might provide to SAR are not well represented in literature. This paper discusses the motivation for MIMO SAR and derives MIMO signal correlation in order to determine what imaging geometries are required for MIMO SAR.

Index Terms—Synthetic aperture radar (SAR), Backprojection, Multiple-input multiple-output (MIMO).

I. INTRODUCTION

The use of signals transmitted from and then received by multiple antennas is called multiple-input and multiple-output (MIMO). In wireless communications, use of MIMO techniques can significantly increase channel capacity and link range. Because of the advances MIMO has brought to communications in recent years, researchers have sought to bring similar advances to radar.

Current research divides MIMO radar into two categories: colocated (or coherent) and distributed (or statistical)[1]. With a colocated MIMO radar, transmit/receive antennas are placed close together, while a distributed MIMO system has antennas separated over a wide area. In both cases, it has been shown that despite the disadvantages of cost and complexity, several potential advantages exist in MIMO surveillance radar over conventional radar[1].

A colocated MIMO surveillance radar has a number of advantages over a traditional single-input and single-output (SISO) radar or even a linear phased array. A MIMO radar array is capable of detecting slower moving targets[1], [2]. More targets can be tracked using a MIMO array than a phased array[3]. It enjoys a higher probability of

detection as long as the signal-to-noise ratio (SNR) is sufficiently high ($> 10\text{dB}$). Also, a MIMO array can have increased angle detection[4].

A distributed MIMO array shares many of the advantages of a colocated MIMO array and includes several more. Because of angular diversity, it enjoys a better probability of detection[5], [6], [7], [8], [9]. Swerling cases 1 and 3 also exhibit increased angle detection[10]. Additionally, it is possible to maintain the same detection threshold as a SISO array while lowering the total radiated power, thus decreasing the probability that a transmitter will be detected by an opponent [3]. As with a colocated array, these advantages come at the cost of a required minimum SNR threshold, below which a linear array performs better.

Unlike real aperture radar, synthetic aperture radar (SAR) utilizes platform motion to obtain much finer resolution in the along-track direction than would otherwise be possible. Because of its fine resolution, SAR is typically used as an imaging radar. Unlike surveillance radar where the normal ground returns would be considered clutter, in imaging radar this clutter is generally the signal of interest. As MIMO techniques lead to advantages in surveillance radar, researchers seek to similarly improve SAR performance.

The advantages, however, of MIMO SAR over traditional SAR are considerably more vague—very few papers have been published on the topic. Of those papers that have explored MIMO SAR, each presents a specific method which utilizes MIMO methods, but none truly motivates the use of the methods, especially given the additional cost and complexity incurred with such a system. For example, one paper discusses MIMO interferometry waveform techniques, but doesn't show or cite any advantages or performance analysis[11]. Another paper shows how to lower the minimum required pulse repetition frequency (PRF)[12], but given the state of current hardware this is probably not a

profound advantage. Several papers discuss using a specific constellation of distributed MIMO SARs with varying incidence angles to obtain higher range resolution than what is given by the transmit bandwidth[13]. This is again perhaps useful, but not profound in itself. Yet another paper claims range resolution improvement via orthogonal frequency-division multiplexing (OFDM)[14], however the final resolution is no better than if a single transmit signal occupying the same total bandwidth were exploited. There are a few other papers that present signal synthesis and hardware design, but once again neglect any treatment of why MIMO SAR would be useful in practice[15], [16], [17], [18].

Our research seeks to characterize MIMO SAR performance in the most general cases. This analysis includes the SNR gain achievable, as well as improvements in interferometry and motion detection. In order to facilitate this analysis, the distinction must be made between colocated and distributed arrays. The type of array determines the extent to which signal returns are correlated, and thus which techniques are feasible.

To analyze signal coherence, we begin by presenting a general expression for a range and azimuth compressed image in the ground plane through the backprojection algorithm. Using these results we then show the correlation of MIMO signals for antennas in various configurations. This allows one to determine when a MIMO array can be considered colocated vs. distributed.

II. BACKPROJECTED SAR IMAGE

In order to facilitate analysis of MIMO SAR, we describe a general SAR signal from basic principles. An equation for a narrowband, bistatic radar's baseband, complex, range compressed signal with a stop-and-go approximation from an isotropic point scatterer for a single pulse, p , is given by

$$s_p(t) = \left(\frac{G_t(\theta, \psi)}{4\pi r_t^2} \right) \left(\frac{G_r(\theta, \psi)}{4\pi r_r^2} \right) \sigma \cdot e^{-jk(r_t+r_r)} R\left(t - \frac{r_t+r_r}{c}\right) \quad (1)$$

where t is fast-time, $G_t(\cdot)$ and $G_r(\cdot)$ are transmit and receive antenna gain functions, r_t , r_r , θ , and ψ are functions of the individual target geometry relative

to the antennas at pulse p , σ is the radar cross-section of the point scatterer, k is the wavenumber of the carrier signal, $R(t)$ is the range compressed radar response including range windowing, and c is the speed of light.

Explicitly identifying variables with a dependence on geometry at pulse p , let

$$\alpha_p = \left(\frac{G_t(\theta, \psi)}{4\pi r_t^2} \right) \left(\frac{G_r(\theta, \psi)}{4\pi r_r^2} \right), \quad (2)$$

$\phi_p = k(r_t + r_r)$, and $\tau_p = \frac{r_t+r_r}{c}$. The signal above can then be rewritten as

$$\begin{aligned} s_p(t) &= \sigma \left[\alpha_p e^{-j\phi_p} R(t - \tau_p) \right] \\ &= \sigma h_p(t - \tau_p). \end{aligned} \quad (3)$$

This utilizes the expression for the *slow-time* (nominally azimuth) matched filter, $h_p(t)$, at each pulse p :

$$h_p(t) = \alpha_p e^{-j\hat{\phi}_p} R(t - \tau_p), \quad (4)$$

where $\hat{\phi}_p$ is the phase of the matched filter due to the target geometry. Ideally, the target geometry is known and the matched filter geometric phase becomes $\hat{\phi}_p = \phi_p$. For the p pulses that contain significant contributions for the given point target, convolving h_p with the original signal s_p yields the matched filtered signal at each pulse:

$$f_p(t) = \int_{-\infty}^{\infty} h_p^*(\tau) s_p(t + \tau) d\tau. \quad (5)$$

In order to focus the target's energy that is spread across many pulses, its contribution from each pulse must be combined. Coherently summing across all pulses and applying a slow-time window of w_p gives the slow-time compressed signal

$$A(t) = \sum_p f_p(t) w_p \quad (6)$$

$$= \sigma \alpha^2 |R(t - \tau_p)|^2 \sum_p w_p \quad (7)$$

where $\alpha = \sum_p \alpha_p^2$. Setting $w = \sum_p w_p$ and assuming the range response only significantly contributes at its peak $t = \tau_p$, the pixel containing the scatterer's compressed response simplifies to

$$A_0 = A(t = \tau_p) = \sigma \alpha^2 |R(0)|^2 w. \quad (8)$$

The case where the range response significantly contributes at more than just its peak is treated below.

Recall that use of this method to focus a target's signature in slow-time requires precise knowledge of its geometric phase at each pulse, and thus its three-dimensional position. For this analysis, we assume that the three-dimensional surface height is known *a priori* via a digital elevation map (DEM).

For convenience sake, we express the signal above in terms of spatial coordinates instead of time. Thus, the function f_p from Eq. (5) is transformed to a domain of distance rather than time ($t_p = \frac{d_p}{c}$), where d_p is the euclidean distance from the antenna(s) to the target. With a sample point located at $\mathbf{v}_0 = [x_0 \ y_0 \ z_0]^T$, for the monostatic case where the antenna is located at $\mathbf{v}_p = [x_p \ y_p \ z_p]^T$ at each pulse, p , the distance is

$$d_{p,m} = 2 \|\mathbf{v}_0 - \mathbf{v}_p\|. \quad (9)$$

Here $\|\cdot\|$ denotes the euclidean norm. For the bistatic case with transmit and receive antennas located at $\mathbf{v}_{p,t}$ and $\mathbf{v}_{p,r}$, respectively,

$$d_{p,b} = \|\mathbf{v}_0 - \mathbf{v}_{p,t}\| + \|\mathbf{v}_0 - \mathbf{v}_{p,r}\|. \quad (10)$$

For a sample point located at \mathbf{v}_0 , Eq. (6) above can now be rewritten as

$$A(x_o, y_o, z_o) = \sum_p f_p(d_p) w_p. \quad (11)$$

This is the backprojection equation for a perfectly matched target.

The slow-time, compressed result of Eq. (8) above assumes that the phase of the matched filter, $\hat{\phi}_p$, perfectly matches the geometric phase of the signal, ϕ_p , at each pulse. For this assumption to hold, the target must be located at precisely the expected position. However, in the case of a signal, $s_{p,u}(t)$, whose target is not located at the position parameterizing the matched filter, the result of filtering is

$$f_{p,u}(t) = \int_{-\infty}^{\infty} h_p^*(\tau) s_{p,u}(t + \tau) d\tau \quad (12)$$

$$= \sigma \alpha_p \alpha_{p,u} e^{j\tilde{\phi}_p} \int_{-\infty}^{\infty} R^*(\tau) R(t + \tau + \delta_p) d\tau. \quad (13)$$

Here, $\tilde{\phi}_p = \phi_p - \hat{\phi}_p$ is the difference in expected vs actual geometric phase and δ_p is likewise the delay corresponding to the difference between the expected and actual position of the target.

For targets sufficiently displaced from the expected range (i.e. large δ_p), the integral vanishes and the filtered signal is approximately zero. However, if the target's displacement from the expected position is sufficiently small, the range response mismatch can be neglected and the important term in slow-time compression is the difference in phase from the expected position (at the pixel center) to the actual scatterer:

$$\begin{aligned} A_{0,u} &= \sum_p f_{p,u}(d_p) w_p \\ &= \sigma \sum_p \underbrace{(\alpha_p \alpha_{p,u} w_p |R(0)|^2)}_{\gamma_p} (e^{j\phi_{p,e}}) \end{aligned} \quad (14)$$

Assuming that γ_p is purely real, the phase of $A_{0,u}$ is determined solely from the geometric phase error. This phase error can be described as $\phi_{p,e} = k\Delta d_p = k(c_p - b_p)$, where Δd_p is the relative displacement from the scatterer to pixel center, b_p is the distance to the center of the pixel, and c_p is the actual distance to the target. If n distinct targets surround the pixel center, the final signal becomes

$$A_{0,u} = \sum_n \sigma_n \sum_p \gamma_{p,n} e^{jk(\Delta d_{p,n,t} + \Delta d_{p,n,r})}, \quad (15)$$

where $\Delta d_{p,n,t}$ is the displacement relative to the transmit antenna for pulse p of target n , and $\Delta d_{p,n,r}$ is like the displacement for the receive antenna.

III. CORRELATION

In order to proceed with analysis of MIMO SAR performance, we need to determine whether a MIMO array can be considered collocated or distributed. This distinction dramatically affects MIMO processing methods because the signals from a collocated array can be considered coherent while those in a distributed array cannot.

As shown previously in [19], [20], [21], traditional SAR image processing leads to geometric baseline decorrelation in the signals received by an antenna pair with different incidence angles to target. This, however, is not the case with ideal backprojection processing. Assuming the same antenna gain at both receive antennas, for a point

target whose height is known, the variation in phase is completely removed as a natural result of backprojection. This is seen in Eq. 8 above.

In the papers previously mentioned, one assumption made in deriving the geometric decorrelation is that the complex backscatter $f(x, y)$ for a given azimuth, x , and range, y , can be represented by

$$f(x, y) = \sigma^0 \delta(x) \delta(y). \quad (16)$$

This may not be the case in general however, where the complex backscatter at a given location will have both azimuth and elevation angle dependence. Therefore, while backprojected signals remain correlated for point targets, in nature there will be decorrelation that is entirely dependent on the nature of the scatterers inside a resolution cell.

In the case of side-looking multistatic radars with roughly similar antenna patterns, parallel flight paths will cover the same angular extent in azimuth. Parallel paths differ only in incidence angle and range to target. The case where the flight paths are not parallel is treated later.

If the surface is rough relative to the wavelength of the radar or with many scatters inside a resolution cell, then the complex backscatter $f(x, y)$ at each cell can be assumed to consist of uniformly distributed, uncorrelated scattering centers

$$f(x, y) = \sigma^0 \delta(x) \delta(y),$$

with x and y the azimuth and range positions of the target, respectively, and $\delta(\cdot)$ is the Dirac delta function. Utilizing this assumption, the derivations of regarding decorrelation of interferometric signals applies to any transmit/receive pair in the multistatic configuration. Assuming a radar response function

$$W(x, y) = \text{sinc}\left(\frac{x}{R_x}\right) \text{sinc}\left(\frac{y}{R_y}\right)$$

with azimuth and range resolutions R_x and R_y , the spatial baseline decorrelation is

$$\rho_{\text{spatial}} = 1 - \frac{2 \cos \theta |\delta\theta| R_y}{\lambda},$$

where θ is the mean incidence angle, $\delta\theta = \theta_1 - \theta_2$ is the angular difference in incidence angle between the two paths. Notice that if the range to target the antennas is different, the spatial correlation is

unaffected (it will only affect the correlation phase [21]). If the flight paths of the transmit/receive antennas differs, then decorrelation due to rotation will also occur.

We examine the equation above for distributed targets modeled as $\sigma^0 \delta(x) \delta(y)$ for a bistatic configuration sharing the same transmitter. As long as the range resolution is relatively fine (i.e. the $R_y : \lambda$ ratio is less than 10), several degrees of angular separation are permitted before significant decorrelation occurs. This result holds for most common SAR imaging geometries. This implies that signals obtained from multiple antennas aboard the same platform will not exhibit significant decorrelation. This means there is most likely not enough decorrelation to utilize MIMO techniques when the MIMO array is all located on the same platform.

IV. SNR

In signal processing, the SNR of a system is a figure of merit. In communications, increased SNR results in increased channel capacity. In surveillance radar, increased SNR leads to increased probability of detection for given a false alarm rate. In imaging radar, increased SNR provides better image quality and allows for more sophisticated, post-image formation algorithms to be performed. Thus, if MIMO techniques applied to SAR lead to higher SNR (assuming equal input/transmit power in both cases), then a case for MIMO SAR may exist.

First, we introduce MIMO techniques applied to communications and detection radar, followed by application to SAR using both coherent and non-coherent summing approaches, and a discussion of each.

A. Channel Capacity and Detection Radar

In MIMO communications, channel capacity increases as SNR increases. However, the amount of increase depends on the rank of the channel matrix. For a line-of-sight MIMO system with M transmitters and N receivers, the capacity for an additive white gaussian noise (AWGN) channel is given by

$$C = \log_2 \left(1 + NM \frac{E}{\sigma_n^2} \right),$$

where E is the signal energy and σ_n^2 is the noise variance. In the case of a channel of diversity such that the channel matrix is full rank (and known), the channel capacity is

$$C = M \log_2 \left(1 + \frac{E}{\sigma_n^2} \right)$$

if $M = N$. This leads to the interesting result that in the line of sight case the capacity increase is the result of power gain and increases only logarithmically, while in the full rank case, channel diversity leads to a linear increase in capacity.

In the case of radar, a similar demonstration of a low rank channel can be seen from Brennan's rule. Following the derivation for the clutter rank of MIMO radar given in [1], let \mathbf{y} be the NML vector containing the linear combination of signals, where M is the number of transmitters, N is the number of receivers, and L is the number of pulses. Then the covariance matrix R is $NML \times NML$.

Using a modification of *Brennan's rule*, the clutter rank is

$$\text{rank}(\mathbf{R}_c) \leq \min \left[N_c, NML, \right. \\ \left. N + \gamma(M - 1) + \beta(L - 1) \right]$$

where N_c is the number of clutter signals, γ is the array spacing ($\gamma = d_T/d_R$), and $\beta = 2vT/d_R$. In general, N_c and NML are much larger than the latter expression. This suggests that in general, the clutter covariance matrix rank will be low.

B. Synthetic Aperture Radar

In detection radar, signals that would generally be considered "clutter" are actually the signal of interest for SAR. As shown above, the clutter covariance matrix in SAR applications will generally be low, mainly due to the line-of-sight nature of SAR imaging. Because the SNR improvement with MIMO communications techniques requires the channel matrix rank to be high, it would be unreasonable to expect the dramatic improvements these techniques bring to communications to also bring dramatic improvements to SAR. However, any technique that improves SNR also leads to improved image quality.

The above discussion of power gain is easily applied to the SNR improvement achievable in a MIMO SAR system via coherent summation.

When N identical receivers are placed such that the incidence angle and backscatter are highly correlated, then the received signals can be summed coherently leading to an SNR improvement of N . Similarly, if M transmitters are used, each with the same output power as transmitter in the single input case but with orthogonal waveforms, a coherent summing yields an SNR improvement of M . Thus, the maximum achievable SNR improvement from such a MIMO system is MN . In practice, some correlation between the orthogonal waveforms may exist, which decreases the actual gain. An additional decrease in gain results from any decorrelation between targets as seen from various imaging geometries.

For some imaging geometries the receivers may be so far separated that the signals are too decorrelated to sum coherently. In the case where coherent summing can not be performed, the maximum achievable improvement in SNR is \sqrt{MN} .

V. CONCLUSION

This paper presents a general derivation of the backprojection equation for a multi-static SAR. This includes analysis of signals resulting from the inability to perfectly know the location of each pixel's scattering centers.

The geometric decorrelation for imaging geometries separated in elevation is given. This allows one to determine when a pair of multi-static signals may be considered correlated enough to perform coherent processing, or when they are decorrelated enough to perform MIMO processing. The SNR improvement possible with coherent and non-coherent combining is given.

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