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The Selective Modulation Interferometric Spectrometer

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Abstract

This paper reviews the theory and the practical implementation of the selective modulation interferometric spectrometer (the SIMS). This spectrometer has an extremely large optical throughput, and it can scan the spectrum in real time while requiring no more signal processing than a chopped radiometer. Equations are presented which describe the relationship between design parameters and spectrometer performance. Practical design problems are identified, and some solutions to these problems are given.

Introduction

Optical engineers consistently are being required to design more sensitive spectrometers. To achieve this increased sensitivity, the designer must often resort to spectrometer configurations which are complex or delicate or large and bulky, or which require extensive data processing to recover the spectrum. In this paper the basic principles of a sensitive yet simple spectrometer which yields the spectrum directly are reviewed. The rudiments of this spectrometer were proposed in papers by R. Prat.^{1, 2, 3} Later Fortunato and Maréchal^{4, 5} developed a spectrometer configuration very similar to Prat's, for which they coined the acronym SIMS to describe their "spectromètre interférentiel à modulation sélective" (selective modulation interferometric spectrometer).

The sensitivity of the SIMS results from its extremely large optical throughput. However, the SIMS is not a multiplex spectrometer; it uses energy from only one spectral element at a time. The SIMS uses the optical power within the bandpass of the selected spectral element by time modulating it while leaving the optical power in all the other spectral elements unmodulated. Thus, the SIMS belongs to a class of spectrometers which can be referred to as "selective modulation spectrometers." These spectrometers scan the spectrum by selectively modulating each spectral element in turn. Selective modulation spectrometers have been constructed using both interferometric and dispersive techniques. For example, the SISAM, introduced by Connes,⁶ combines both interferometric and dispersive techniques, while the Girard⁷ grill spectrometer is based exclusively upon the dispersive technique. The SIMS, on the other hand, is a selective modulation spectrometer based exclusively on the interferometric technique.

The SIMS and the Girard grill spectrometer share many properties in common even though they are interferometric and dispersive, respectively. Both spectrometers form transforms of the spectrum along a geometrical plane; the magnitude of each component function corresponds to the optical power of a particular spectral element. Both spectrometers measure the optical power in a particular spectral element (recover the magnitude of the component function) by taking the difference between the opti-

cal radiation passed through complementary grills placed in the transform plane. However, the two spectrometers form different transforms: the Girard grill spectrometers, as pointed out by Mertz,⁸ forms a convolution type transform while the SIMS forms a Fourier transform.

Since the SIMS forms a Fourier transform of the spectrum, it shares many properties with conventional Fourier spectrometers. However, the SIMS forms the Fourier transform in space whereas conventional Fourier spectrometers form the Fourier transform in time. Furthermore, the SIMS uses an electro-mechanical means to invert the transform whereas conventional Fourier spectrometers commonly use a digital computer for this purpose.

The electromechanical inversion method of the SIMS results in simplicity and real time output, but it precludes the multiplex advantage since it uses the energy in only one spectral element at a time. Theoretically, other inversion methods could be used which would yield the multiplex advantage, but no practical method has yet been proposed.

In passing, it is well to point out that some types of data processing of a spectrum can be done directly with a slightly modified SIMS. For example, Fortunato and Maréchal⁹ have proposed modifications to the SIMS which make the following measurements directly: the derivative of a spectrum, the correlation of a spectrum with a reference spectrum, and the correlation of the derivative of a spectrum with the derivative of a reference spectrum. These modifications exploit the properties of Fourier transforms and the fact that the SIMS displays the Fourier transform of the spectrum in space.

The Principle of Operation

The principle of operation of the SIMS will now be explained by deriving and discussing an equation for the optical power, as a function of wavelength and instrument parameters, which impinges upon the detector.

A schematic drawing of the SIMS is given in Figure 1. An optical system depicted as a box in Figure 1 forms two laterally separated images of the source with a separation distance T . Thus, a typical point on the source M is imaged as the two points M_1 and M_2 . Since M_1 and M_2 are images of the same point, the optical energy radiating from them is coherent and will interfere when superimposed. The two optical paths from the points M_1 and M_2 to a point in the focal plane are shown in Figure 1. These paths are determined by the properties of the ideal lens L_1 . The optical path difference s between these two paths is given by

$$s = T \sin \theta \approx Tx/F, \quad (1)$$

where θ is the angle between the parallel portion of the two paths and the optical axis, x is the distance the point in the focal plane is above the optical axis, and F is the focal length of lens L_1 .

Figure 1. Schematic drawing of the SIMS.

In order to facilitate the explanation, the source will first be considered to be monochromatic with wavelength λ . In a well-designed SIMS the transmittance is the same for the two optical paths from M_1 and M_2 to a point in the focal plane of lens L_1 . Therefore, from the theory of interference for two monochromatic waves,¹⁰ the intensity at a point in the focal plane of lens L_1 due to the superposition of the two waves from M_1 and M_2 is given by

$$I = 2I_1 (1 + \cos 2\pi s/\lambda) \quad (2)$$

where I_1 is the intensity which would result if only one of the waves were allowed to reach the focal plane, and s is the optical path difference between the paths of the two waves. From Eqs. (1) and (2)

$$I \approx 2I_1 (1 + \cos 2\pi Tx/F\lambda) \quad (3)$$

Thus, radiation from the two image points M_1 and M_2 forms biased cosine fringes in the focal plane of lens L_1 . The period of these fringes is

$$q = F\lambda/T \quad (4)$$

Since the two images formed of each source point are separated by the same distance T , the fringe patterns of all pairs of image points coincide in the focal plane of lens L_1 . This coincidence of fringe patterns yields high visibility fringes even when an extended source is used. Thus, Eq. (3) describes the fringes formed in the focal plane of lens L_1 by an extended monochromatic source if I_1 is redefined as the intensity at a point in the focal plane which would result if radiation from only one of the two extended source images reached the focal plane. The high throughput capability of the SIMS results from the ability of the SIMS to form high visibility fringes with a monochromatic extended source.

The source will now be considered to be polychromatic. In this case, radiation at each wavelength present forms its own biased cosine fringe pattern. The period of each fringe pattern is linearly related to the wavelength as specified by Eq. (4), and its amplitude is linearly related to the power spectral density at that wavelength. Thus, the superposition of all these fringe patterns in the focal plane of lens L_1 yields the Fourier transform of the source spectrum plus a constant.

The transmission grill which is located in the x direction. A stationary stop or an image of a stationary stop is usually also located in this focal plane in order to truncate the fringe pattern in a well-defined fashion. (This stop is not shown in Figure 1.) The optical power at wavelength λ transmitted through this stop-grill combination and collected by lens L_2 onto the detector is given by

$$P = \int_{\text{stop}} t I dA \quad (5)$$

where t is the transmittance of the grill, I is given by Eq. (3) with I_1 redefined as the intensity of the optical radiation of wavelength λ which would result if radiation from only one of the two extended source images reached the focal plane of lens L_1 , and the integral is over the area of this stop. In the general case, the transmittance of the grill is described by a one-dimensional Fourier series and the shape of this stop is not unique. However, the principle of operation of the SIMS can be explained by considering the following specific case: 1) The transmittance of the grill at a distance x above the optical axis is given by

$$t = \frac{1}{2} [1 + \cos 2\pi(x-x_0)/p] \quad (6)$$

where p is the period of the grill, and x_0 is the distance the grill is translated above its symmetrical position about the optical axis. 2) The stop or stop image which truncates the fringe pattern is located symmetrically about the optical axis with length L in the plane of Figure 1 and width W perpendicular to this plane. For this specific case Eq. (5) can be written as

$$P = I_1 W \int_{-L/2}^{L/2} [1 + \cos 2\pi(x-x_0)/p] [1 + \cos 2\pi Tx/F\lambda] dx \quad (7)$$

Upon integration, Eq. (7) becomes

$$P = I_1 W \left\{ L + \frac{q}{\pi} \sin \frac{\pi L}{q} + \cos \frac{2\pi x_0}{p} \left[\frac{p}{\pi} \sin \frac{\pi L}{p} + \frac{L}{2} \frac{\sin \pi \left(\frac{1}{q} - \frac{1}{p} \right) L}{\pi \left(\frac{1}{q} - \frac{1}{p} \right) L} + \frac{L}{2} \frac{\sin \pi \left(\frac{1}{q} + \frac{1}{p} \right) L}{\pi \left(\frac{1}{q} + \frac{1}{p} \right) L} \right] \right\} \quad (8)$$

where q is defined by Eq. (4). Equation (8) is the equation whose derivation was promised in the first paragraph of this section. Since I_1 and q are functions of λ , Eq. (8) describes the optical power of wavelength λ which impinges upon the detector.

The explanation of the principle of operation of the SIMS will now be completed by discussing Eq. (8). The first two terms of Eq. (8) are independent of the grill translation x_0 , but the terms within the square brackets are multiplied by a cosine factor which has x_0 as an argument. Thus, the optical power of wavelength λ collected onto the detector varies from a maximum to a minimum if x_0 varies from zero to $p/2$. The amplitude of this power variation depends on the magnitude of the entity within the square brackets in Eq. (8). However, since q is an argument in both the second and third terms of this entity, the magnitude of this entity depends on the wavelength λ . Hence, the amplitude of this power variation depends on the wavelength λ .

It will be shown now that the entity within the square brackets of Eq. (8) is maximum for the particular wavelength

$$\lambda_m = Tp/F \quad (9)$$

This will be done by examining the three terms which constitute the entity within the square brackets of Eq. (8). The first term is independent of wavelength, and its magnitude is equal to or less than p/π . The second term is a sinc function multiplied by $L/2$. Consequently, it has its maximum value $(L/2)$ when the fringe period q equals the grill period p . From Eqs. (4) and (9) this occurs when the wavelength λ equals λ_m . However,

$$L \approx Np \quad (10)$$

where N is the number of complete grill periods in the length L .

Therefore, when λ equals λ_m the value of the second term is approximately equal to $Np/2$. The third term can be written in the form

$$\frac{1}{2\pi} \left(\frac{p}{1 + \frac{p}{q}} \right) \sin \pi \left(\frac{1}{q} + \frac{1}{p} \right) L \quad (11)$$

Due to the fact that both p and q must be positive for a physically realizable SIMS, Eq. (11) implies that the magnitude of the third term is less than $p/2\pi$ for all wavelengths. These magnitude limits for the first and third terms indicate that the magnitude of the sum of the first and third terms must be less than $3p/2\pi$ for all wavelengths. Therefore, the ratio of the maximum value of the second term to the maximum magnitude of the sum of the first and third terms must be greater than $N\pi/3$. Inasmuch as N is usually greater than 1,000, this ratio is usually greater than 1,000. Consequently, the maximum value of the entity within the square brackets in Eq. (8) occurs essentially when the maximum value of the second term occurs. As shown previously, the second term has its maximum value when the wavelength equals λ_m .

It follows from this discussion of Eq. (8) that the optical power of wavelength λ_m impinging on the detector is closely approximated by the expression

$$I_1 WL \left(1 + \frac{1}{2} \cos 2\pi x_0/p \right) \quad (12)$$

Consequently, if x_0 is the triangular function of time shown in Figure 2(a), then the power of wavelength λ_m impinging on the

Figure 2. Relationship between grill displacement and optical power modulation. (a) The grill displacement plotted against time. (b) The power of wavelength λ_m impinging on the detector plotted against time.

detector is the biased cosine function of time illustrated in Figure 2(b). Thus, the optical power of wavelength λ_m is effectively modulated by the grill motion depicted in Figure 2(a). For a fixed grill period p , Eq. (9) indicates that the value of λ_m can be selected by the proper choice of the image separation distance T and the focal length F . As will be shown in a following section

on practical implementation, the value of λ_m is usually varied by varying T .

Throughout a band of wavelengths centered on λ_m the entity within the square brackets of Eq. (8) has a value comparable to its value at λ_m . Hence, the optical power with wavelengths contained within this band is modulated by grill motion, while power with wavelengths outside this band is essentially unmodulated. Thus, the SIMS selectively modulates the optical power in one spectral element while leaving the optical power in all other spectral elements unmodulated. The optical power in the modulated spectral element is measured by measuring the amplitude of the electrical signal from the detector (i.e., demodulating).

Resolving Power

The resolving power for the SIMS design described by Eq. (8) will now be ascertained. The band of modulated wavelengths is determined essentially by the second term within the square brackets of Eq. (8) as revealed by examining the relative magnitudes of the three terms enclosed by these square brackets. As shown in the previous section, the magnitude of the sum of these first and third terms is less than $3p/2\pi$ for all wavelengths. The magnitudes of the first four side-lobes of this second term are equal to or greater than $.07Np/2$. Inasmuch as N is commonly greater than 1,000 and the ratio of $.07Np/2$ to $3p/2\pi$ is greater than $.07N$, the band of modulated wavelengths is essentially determined by this second term.

From Eqs. (4) and (10) and the definition of wavenumber, $\sigma \triangleq 1/\lambda$,

$$\sigma \triangleq 1/\lambda \quad (13)$$

this second term can be written as

$$\frac{Np}{2} \left[\frac{\sin \pi \left(\frac{\sigma T}{F} - \frac{1}{p} \right) L}{\pi \left(\frac{\sigma T}{F} - \frac{1}{p} \right) L} \right] \quad (14)$$

In Expression (14) the entity within the square brackets is a sinc function. This sinc function, which is plotted in Figure 3, is the instrumental profile of this SIMS design. This instrumental profile has a maximum at the wavenumber

Figure 3. Instrumental profile when the fringe pattern is truncated by a rectangular stop.

$$\sigma_m = F/Tp \quad (15)$$

and the wavenumber change in going from this maximum to the first zero is

$$\Delta\sigma = F/TL \quad (16)$$

The resolving power can be defined as

$$R \triangleq \sigma_m / \Delta\sigma \quad (17)$$

Hence, from Eqs. (10), (15), (16) and (17) the resolving power for a SIMS with a rectangular stop in the plane of the grill is given by

$$R \approx N \quad (18)$$

Thus, for this SIMS design, the resolving power is approximately equal to the number of complete grill periods within the stop truncating the fringes.

The instrumental profile can be slightly modified by using a stop with a shape other than a rectangle to truncate the fringes. This dependence on stop shape is analogous to the dependence on data apodization in conventional Fourier spectroscopy.

Grill Modulation Efficiency

Since the total power striking the grill is $2LWI_1$, 100 percent modulation corresponds to a peak-to-peak power variation equal to $2LWI_1$. However, as shown in Figure 2(b), the peak-to-peak power variation of radiation at wavelength λ_m is LWI_1 when the transmittance of the grill is given by Eq. (6). Accordingly, for this grill transmittance the modulation efficiency is given by

$$M.E. = LWI_1/2LWI_1 = 1/2 \quad (19)$$

Throughput

The extremely large throughput of the SIMS is its most significant characteristic.

If there is no vignetting, the angular extent of the pencil of rays from each point on the source within the field of view is determined by the stop which truncates the fringe pattern. Hence, this stop is the aperture stop of the SIMS. This stop or its image is located in the plane of the grill.

In order to ensure a sharply defined field of view, a stop should be located at the source or in an image plane of the source. If this field of view stop is located in the plane of the two laterally separated images of the source, it will crop the top of one image and the bottom of the other image. For example, the image cropping resulting from a circular stop is illustrated in Figure 4. In

Figure 4, two images, A_1 and A_2 , are formed of the source point A , but only one image, B_2 , of source point B and one image, C_1 , of source point C are formed. Thus, radiation from point A on the source contributes to the fringe pattern, but radiation from points B and C does not contribute. Hence, point A must be included in a throughput computation and points B and C must be excluded from this computation. Henceforth, the area of the source imaged in both images will be referred to as the effective area of the source.

It follows from the discussion of the last two paragraphs that the throughput (etendue) of a well-designed SIMS is given by

$$E = A_s \Omega \approx \frac{A_s A_g}{Z^2}, \quad (20)$$

where A_s is the area of either one of the two laterally separated images of the effective area of the source as seen from the plane of the grill, Ω is the solid angle subtended at the plane containing A_s by the stop or stop image located in the plane of the grill, A_g is the area of the stop or stop image located in the plane of the grill, and Z is the distance between the plane containing A_s and the plane of the grill.

As explained in a later section, lens L_1 has the most stringent aberration requirements of any lens in Figure 1. Hence, it is good design practice to minimize the required diameter of this lens. The required diameter of lens L_1 is minimized if the two laterally separated images of the source are formed within the entrance pupil of lens L_1 as illustrated in Figure 5. Since entrance and exit pupils are conjugates, the two laterally separated images of the source as seen from the plane of the grill are located with-

Figure 5. Design which minimizes the required diameter of lens L_1 .

in the exit pupil in Figure 5. If

$$m \triangleq \frac{\text{exit pupil diameter}}{\text{entrance pupil diameter}}, \quad (21)$$

the lateral separation of the images within the entrance and exit pupils of Figure 5 is, respectively, T and mT . Throughout the remainder of this section on throughput, it will be assumed that the two laterally separated images are formed within the entrance pupil of L_1 as shown in Figure 5.

If the field of view is determined by the exit pupil of lens L_1 , the effective image area is the cross-hatched area in Figure 6. This area is given by

$$A_s = \frac{D^2 \pi}{4} - \frac{mT}{2} \sqrt{D^2 - (mT)^2} - \frac{D^2}{2} \sin^{-1} \frac{mT}{D}, \quad (22)$$

where D is the diameter of the exit pupil and

$$T = F \lambda_m / p \quad (23)$$

as required by Eqs. (13) and (15). It follows from Eqs. (20),

Figure 4. Example of image cropping resulting from a circular stop in the plane of the two laterally separated images.

Figure 6. Effective image area when the field of view is determined by the exit pupil of lens L_1 .

(22) and (23) that if the field of view is determined by the exit pupil of lens L_1 , the throughput is a function of λ_m , the central wavelength of the modulated spectral element. Hence, for this SIMS design the throughput varies as the spectrum is scanned.

If this throughput variation with a spectral scan cannot be tolerated or if a differently shaped field of view is required, a stop should be placed at the source (or a conjugate of the source before the lateral shearing occurs). For example, when the field of view is determined by a circular stop at the source, the effective image area is the cross-hatched area in Figure 7. The diam-

Figure 7. Effective image area when the field of view is determined by a circular stop at the source.

eter of this stop must be chosen such that at the maximum image separation distance $T_{\max}$ required in the spectral scan both laterally separated images of this stop are contained within the exit pupil of lens L_1 . For this stop choice, the effective image area is a constant throughout the spectral scan. This effective image area is given by

$$A_s = \frac{\pi}{4} (D - mT_{\max})^2, \quad (24)$$

where $T_{\max}$ is found by substituting in Eq. (23) the largest value of λ_m within the spectral scan.

In order to give an indication of the throughput which can be achieved with a practical SIMS, the throughput of a SIMS with the following practical parameters will now be computed and compared to that of a grating monochromator with comparably sized optics. Let the parameters of lens L_1 be: D equals 16 cm; F and Z both equal 50 cm; m equals unity. Let the remaining parameters be: p equals $1/325$ cm; W and L both equal to 5 cm; $\lambda_{\max}$ equals $0.8 \mu\text{m}$. This grill period p is more than five times the diameter of the diffraction-limit circle of this L_1 lens. It is practical to correct this L_1 lens over the 4° half-angle field required by these W and L values. Using these parameters in Eqs. (10), (18) and (23), one finds that the resolving power R is 1625 and the image separation distance T is 1.3 cm. If these parameters and this value of T are substituted in Eqs. (22) and (24), the effective image area A_s is 180.3 and 169.7 cm^2 , respectively. The area of the stop in the plane of the grill A_g is 25 cm^2 . Substituting these values in Eq. (20), one finds that the throughput is 1.80 and $1.70 \text{ cm}^2\text{-sr}$, respectively.

The throughput advantage of the SIMS is clearly demonstrated if the above throughput values are compared with that of a conventional grating monochromator. The throughput of a grating monochromator is given approximately by

$$E_G \approx \frac{\ell A_G}{FR}, \quad (25)$$

where R is the resolving power, A_G is the projected area of the grating, ℓ is the entrance slit length, and F is the focal length of the collimating optics. The maximum practical value of the ratio ℓ/F is about $1/50$. From Eq. (25), the throughput of a grating monochromator with R equal to 1625 and A_G equal to $64\pi \text{ cm}^2$ is $2.47 \times 10^{-3} \text{ cm}^2\text{-sr}$. (The area of the exit pupil of the lens L_1 used in the computations of the preceding paragraph is $64\pi \text{ cm}^2$.) The SIMS throughput values computed in the preceding paragraph are 729 and 688 times larger, respectively, than the throughput of this grating monochromator.

Signal-to-Noise Ratio

In order to compare the signal-to-noise ratio of the SIMS with a conventional monochromator, it is useful to define the following throughput gain parameter

$$g \triangleq \frac{\text{throughput of SIMS}}{\text{throughput of conventional monochromator}}. \quad (26)$$

The throughput comparison between a SIMS and a grating monochromator made in the previous section illustrates that in practice g can have a value of several hundred.

Since the optical power in all the spectral elements is collected onto the detector even though only one spectral element at a time is measured, the optical power in all spectral elements contributes to the photon noise. Therefore, in practice, an optical passband filter should be used to limit the number of spectral elements striking the detector.

In the following comparison between the SIMS and a conventional monochromator, it is assumed that the transmission coefficients of both instruments have the same value. The signal power is a factor of g larger for the SIMS than for a conventional monochromator. If the field of view of the SIMS is determined by a stop at the source or its conjugate before the lateral shearing occurs, the photon noise for a uniform spectrum and passband filter which passes K spectral elements is a factor of $\sqrt{Kg}$ larger for the SIMS than for a conventional monochromator.

Hence, when photon noise is the dominant type of noise,

$$\frac{(\text{SNR})_{\text{SIMS}}}{(\text{SNR})_{\text{monochromator}}} = \frac{g}{\sqrt{Kg}} = \sqrt{\frac{g}{K}}, \quad (27)$$

where SNR denotes the signal-to-noise ratio. Thus, for photon-noise-limited conditions with a uniform spectrum, if the throughput gain is larger than the number of spectral elements in the op-

tical filter passband, then the signal-to-noise ratio will be larger for the SIMS than for the monochromator. If the spectrum is not uniform but rather consists of only a few lines, the signal-to-noise ratio advantage of the SIMS at these line peaks will be even larger than that given by Eq. (27).

When detector noise is the dominant type of noise, the noise will be the same for both the SIMS and the monochromator if the same detector is used for both spectrometers. This equality of noise together with the fact that the signal in the SIMS is a factor of g larger for the SIMS than for a conventional monochromator, implies that

$$\frac{(\text{SNR})_{\text{SIMS}}}{(\text{SNR})_{\text{monochromator}}} = g \quad (28)$$

when detector noise is the dominant type of noise.

Formation of the Two Laterally Separated Images

Practical optical configurations for forming the two laterally separated images required by the SIMS can be categorized as: 1) cyclic, 2) birefringent, and 3) other. These categories will now be discussed in order.

Fortunato and Maréchal^{4, 5} have used the cyclic design illustrated in Figure 8. The image separation distance T for this configuration is given by

$$T = 2e \quad (29)$$

Figure 8. Cyclic configuration with the beamsplitter at 30° for forming two laterally separated images.

where e , as shown in Figure 8, is the distance the scanning mirror is translated from its symmetrical position. If Eq. (29) is substituted in Eq. (23) and the resulting equation is solved for the central wavelength λ_m of the modulated spectral element, then

$$\lambda_m = 2pe/F \quad (30)$$

It follows from Eq. (30) that if the scanning mirror is translated a distance Δe , then the change in λ_m is given by

$$\Delta\lambda_m = \left[\frac{2p}{F} \right] \Delta e \quad (31)$$

Thus, the spectrum is scanned linearly in wavelength if the scanning mirror is moved linearly.

By the definition of resolving power R , the required wavelength change $\delta\lambda_m$ to scan through one spectral element is given by

$$\delta\lambda_m = \lambda_m/R \quad (32)$$

It can be seen from Eqs. (10), (18), (31) and (32) that the required mirror motion δe to scan through one spectral element is given by

$$\delta e = F\lambda_m/2L \quad (33)$$

Thus, the mirror motion required to scan one spectral element is linearly related to the wavelength of the spectral element. In order to maximize throughput, the ratio F/L should be made as small as possible. For a practical lens, the minimum value of this ratio, for which the required aberration correction of lens L_1 can be achieved, is on the order of ten. (Aberration tolerances are discussed in a subsequent section of this paper.) Therefore, the required scanning mirror motion δe to scan through one spectral element is on the order of five times the wavelength.

For small angular rotations of either mirror about an axis perpendicular to the plane of Figure 8, the image separation distance T changes by an amount linearly related to the angle of rotation. The change in T is also directly proportional to the S dimension in Figure 8. Thus, a narrow spectral region can be scanned rapidly by using a piezoelectric crystal to rotate one of the mirrors.

Figure 9. Cyclic configuration with the beamsplitter at 45° for forming two laterally separated images.

Another cyclic configuration is shown in Figure 9. In this configuration, the angle between the optical beams and the beamsplitter is 45°. Due to the fact that the corresponding angle for the configuration shown in Figure 8 is 30°, the configuration shown in Figure 9 requires a smaller diameter beamsplitter than that required by the configuration shown in Figure 8. In addition, the 90° angle between the incident and exit beams for the configuration shown in Figure 9 allows the collector lens and the lens L_1 to be mounted closer to the beamsplitter than that allowed by the corresponding 60° angle for the configuration shown in Figure 8.

For the configuration shown in Figure 9, the image separation distance is given by

$$T = \sqrt{2} e^* \quad (34)$$

where e^* , as shown in Figure 9, is the distance the scanning mirror is translated from its symmetrical position. For this configuration, the central wavelength of the modulated spectral element is given by

$$\lambda_m = \sqrt{2} pe^*/F \quad (35)$$

and the change in λ_m produced when the scanning mirror is translated a distance Δe^* is given by

$$\Delta\lambda_m = \left[\frac{\sqrt{2} p}{F} \right] \Delta e^* \quad (36)$$

Since for these cyclic configurations both of the two laterally separated images are formed by the same optical components, these cyclic configurations are relatively rugged and they provide a means of scanning the spectrum which requires motion of only one optical component. However, for both these cyclic configurations, half the optical power is directed back towards the source. Consequently, the transmittance of these configurations is less than fifty percent. Also, if the beamsplitting surface is on a substrate, an optical compensator of the same thickness as this substrate must be used if the SIMS is to have a large throughput.

The second category of optical configurations for forming the two laterally separated images is birefringent. Maréchal¹¹ has reported the use of the birefringent configuration illustrated in Figure 10. For this birefringent configuration the spectrum is

Figure 12. Sabater's configuration for forming two laterally separated images.

Figure 10. A birefringent SIMS configuration.

scanned by varying the separation distance between the birefringent prisms, and the fringes are translated across a stationary grill by means of a rotating polarizer. Since for this configuration the optical beams are sheared within the solid birefringent prisms, this configuration is particularly rugged. It should be possible to use field-widened Wollaston prisms¹² as the birefringent prisms in order to maximize the throughput of this configuration.

Configurations other than cyclic and birefringent ones which form two laterally separated images constitute the third category. Examples of such configurations are shown in Figures 11 and 12.

Figure 11. Prat's configuration for forming two laterally separated images.

The configurations of Figures 11 and 12 have been used by Prat² and Sabater, respectively. These configurations have the disadvantage that two optical components must be simultaneously moved to alter the image separation distance T . However, large values of T can be achieved with these configurations.

Modulation and Demodulation

As has been explained previously, amplitude modulation of the selected spectral element can be produced by generating relative motion between the fringe pattern and the grill. The time-varying portion of the detector signal is proportional to this optical power modulation. Due to the fact that it is very difficult to maintain the frequency of this mechanically generated modulation at a precise constant value, synchronous demodulation is usually required to achieve a narrow electrical noise bandwidth. However, if synchronous demodulation is used, a reference waveform containing information about the relative motion between the grill and the fringe pattern must also be generated.

Equation (12) indicates that the normalized time-varying portion of the detector signal is given by

$$v = \cos 2\pi x_0/p \quad (37)$$

Hence, v will vary between its maximum and minimum values, if x_0 varies between zero and an integral multiple of $p/2$. In a practical SIMS it is advantageous to make the variation of x_0 small rather than large, because then it is easier to implement synchronous demodulation and the method of aberration neutralization discussed in the next section is more effective.

The reason it is easier to implement synchronous demodulation for small rather than large variations of x_0 stems from the fact that it is much easier to control accurately small rather than large mechanical motions. If x_0 does not vary between exactly zero and an exact multiple of $p/2$, then from Eq. (37) the phase of v changes abruptly when the mechanical motion reverses directions. If the phase of v changes abruptly, then the phase of the fundamental of the reference signal must also change abruptly. If this abrupt phase change in the fundamental of the reference signal is large, a practical synchronous demodulator may lose its synchronization with the reference signal and require a finite time to relock onto the reference signal.

If x_0 varies between zero and $p/2$ in the triangular fashion illustrated in Figure 2(a), then v is a pure cosine function of time. However, since x_0 is generated by a mechanical motion, higher modulation frequencies can be achieved if x_0 varies in a cosinusoidal fashion rather than a triangular fashion. If the cosinusoidal function

$$x_0(t) = \frac{p}{4} (1 - \cos \omega t) \quad (38)$$

is substituted in Eq. (37), then

$$v = 2J_1(\pi/2) \cos \omega t - 2J_3(\pi/2) \cos 3\omega t + \dots \quad (39)$$

where $J_1, J_3, \dots, J_k$ are Bessel's functions of the first kind of order k . Since $J_1(\pi/2)$ is equal to 0.567,

$$v = 1.13 \cos \omega t \quad (40)$$

if the higher harmonics are eliminated by electronic filtering.

Figure 13. SIMS configuration which eliminates the need for relative motion between the grill and the fringe pattern.

The configuration shown in Figure 13 eliminates the need for relative motion between the grill and the fringe pattern. This configuration, which was suggested by Fortunato and Maréchal,⁹ is based on the same basic principle as the statistical configuration of Girard's grill spectrometer. If one grill is located with x_0 equal to zero and the other grill is located with x_0 equal to $p/2$, then Eq. (37) indicates that the difference between the signals from the two detectors behind these grills is the required measurement. Since this configuration eliminates the need for relative motion between the grill and the fringe pattern, it should prove useful for measuring rapidly changing sources.

Optical Aberrations

The optical quality of the collector lens shown in Figures 8 through 12 is not critical because the optical beam is sheared after passing through this lens. The condenser lens L_2 need not

be highly corrected either, because its only function is to direct the optical radiation onto the detector. The optical quality of the remaining optical components is more critical since they determine the path differences upon which the fringe formation depends.

Lens L_1 must be corrected for an object at infinity. The transverse aberration of this lens for meridional ray fans must be significantly less than the grill period. However, the transverse aberration of this lens for sagittal ray fans can be significantly larger than the grill period. The minimum grill period must be significantly greater than the diffraction limit of lens L_1 . Distortion in lens L_1 causes the fringes to be curved rather than straight and it also causes a gradual change across the focal plane of L_1 in the fringe period. However, some distortion in L_1 can be neutralized by using a photograph of a monochromatic fringe pattern formed by the SIMS as the grill.

The quality of the optical components which shear the optical beam must be such that the path difference errors which they introduce are less than $\lambda/2$.

In practice it is commonly necessary to neutralize residual distortion by the photographic technique just described. But since the photographic process is limited to the visible and near infrared spectral regions, new methods of producing grills to match the shape of the fringes need to be developed. Three possible methods are suggested here. First, a reflective SIMS configuration might be developed. Since such a configuration would have no chromatic aberrations, a grill for use in the infrared spectral region could be produced by photographing visible light fringes. Second, a SIMS might be designed to have the same shape fringes at one particular visible wavelength as it does in the infrared spectral region. Then a photograph of the fringes formed at that wavelength could be used as the grill in the infrared. Third, the optical prescriptions of the components might be used in a computer software program which computes the fringe pattern, and a computer-generated grill could be produced for use in the infrared.

Figure 14. Utah State University's SIMS prototype.

Utah State University's SIMS Prototype

SIMS experimentation is currently being conducted by the author with the laboratory prototype shown in Figure 14. This prototype is based on the cyclic configuration of Figure 9. Lens L_1 is a five-element lens with a focal length of 41.2 cm, an entrance pupil of 7.8 cm, and a half angle field of view of 2.6° . Its wave-front aberration is on the order of $\lambda/4$ from .63 to $1.20\ \mu\text{m}$ if slight adjustments are made in the location of the focal plane. The elements of lens L_1 are mounted in two cells with a piezo-electrically rotated plane mirror mounted between the two cells. The fringe pattern is translated across a stationary grill by small rotations of this plane mirror.

Conclusions

The most advantageous properties of the SIMS are its large throughput capability and its minimal signal processing requirement. The throughput of a SIMS can be made hundreds of times larger than that of a grating monochromator which has optics of a size comparable to that of the SIMS. The signal processing required by the SIMS is comparable with that required by a chopped radiometer. As a result of this minimal signal processing requirement, the SIMS can provide spectral information in real time with a minimum of signal processing equipment.

There are other significant, advantageous properties of the SIMS which are dependent on the optical configuration used to form the two laterally separated images of the source. All such optical configurations discussed in this paper yield SIMS designs which scan the spectrum linearly in wavelength when the tuning components are linearly translated. Each of the cyclic and birefringent configurations has a single tuning component; the other configurations have two tuning components. SIMS designs which are based on any of the cyclic or birefringent configurations are relatively rugged.

The most important practical techniques for improving the performance of the SIMS are the following: 1) constructing the grill to match the shape of the fringes formed by the SIMS optics in order to neutralize residual distortion, 2) forming the two laterally separated images of the source in the entrance pupil of the only lens which requires significant aberration correction in order to minimize the required diameter of this lens, 3) using an optical bandpass filter to limit the number of spectral elements which reach the detector in order to reduce the photon noise, 4) making the relative displacement between the grill and the fringe pattern a sinusoidal function of time in order to increase

the modulation frequency.

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