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Wormholes: Gates to the Stars?

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Abstract: One of the most consistently fascinating results of Albert Einstein's theory of general relativity is the prediction of wormholes – astronomical objects which are, among other things, capable of serving as a connection between two distant regions of space. The simplest class of wormholes are Schwarzschild wormholes – wormholes that behave as non-rotating, non-charged black holes, except that the event horizon serves as a connection to another wormhole elsewhere, instead of a point of no return.

This research presentation analyzes the attributes that make a Schwarzschild wormhole unsuitable for human travel, and examines the conditions that would have to hold for human travel through a wormhole to be possible. Following the work of Morris and Thorne, we examine the constraints that these conditions place on the metric and on the stress-energy tensor. It is shown that these constraints require a configuration of matter that violates accepted energy conditions, and is therefore likely to be non-physical. Avenues for further research with the potential to minimize these violations are outlined.

In 1935, a paper published by Albert Einstein and Nathan Rosen attempted to provide the framework for a general description of matter at the atomic level. Although Einstein had been extremely successful in describing macroscopic processes with his general theory of relativity, the theory had proved unable to account for the atomic structure of matter. Pursuing this elusive prize, Einstein and Rosen proposed a novel modification of the solution developed by Karl Schwarzschild in 1915[1], describing the gravitational field surrounding a spherically symmetric distribution of matter.

For them, this solution had a single, overwhelming flaw - it contained a singularity at radius $r = 2m$. Though later mathematical work would show this singularity to be a simple artifact of the coordinates, and not a true singularity of the manifold, its existence nevertheless was distressing for these physicists. At these points, the singularity brought "so much arbitrariness into the theory that it actually [nullified] its laws."[2] To remove this singularity, Einstein and Rosen introduced a new radial coordinate $u^2 = r - 2m$, which went to zero at the site of the previous

[1] K. Schwarzschild, Sitzungsber. Preuss. Akad. Wiss., Phys. Math, 1916
[2] A. Einstein and N. Rosen, Phys. Rev. **48** (1), 73, 1935

singularity. So this coordinate ran from positive infinity down to $r = 2m$, and ran back out to positive infinity again. The solution, under this coordinate transformation, was singularity free.

Attempting to interpret this solution in terms of the normal 4-dimensional space of general relativity yielded a fascinating result. The spacetime was "described mathematically by two congruent parts or 'sheets,' corresponding to $u > 0$ and $u < 0$, which are joined by a hyperplane $u = 2m$...we shall call such a connection between the two sheets a bridge."[3] After redoing the calculation, this time involving electromagnetic fields as well as gravitational fields, Einstein and Rosen asserted that these mathematical bridges were potential candidates for describing particles in general relativity.

New advances in quantum mechanics, including the discovery of two new forces wholly unaccounted for by Einstein and Rosen's model, eventually put an end to this idea as a solution to the particle problem. But the mathematical construction was still valid, two congruent sheets with a bridge between them. And, if one envisioned the construct as being embedded in a higher-dimensional space, there was no reason that the two sheets described there could not be two regions of the universe far distant from each other, or even two different universes.

Although the American physicist John Archibald Wheeler coined the evocative term 'wormhole' to describe these bridges in 1957, these objects did not enter the public consciousness until 1985, when Carl Sagan published the science fiction novel *Contact*. Based on research done by Logan native Kip Thorne and his graduate student Michael Morris as a personal favor to Sagan, *Contact* featured a wormhole as a means of space travel, allowing his characters to travel quickly across the unimaginable distances between the stars without any of the mind-bending effects required by Einstein's special theory of relativity. Since then, the wormhole has become a staple of science fiction in a wide variety of media; most recently, the 2014 film *Interstellar* features wormhole travel as one of the central plot points.

In 1988, Morris and Thorne published a landmark paper on wormholes[4]. Though it was framed as an exploration into novel teaching methods in general relativity, the bulk of the paper outlined the potential use of wormholes as a faster-than-light travel system - conclusions which had grown out of the research they did for Sagan on *Contact*.

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[3] A. Einstein and N. Rosen, Phys. Rev. **48** (1), 75, 1935
[4] M. Morris and K. Thorne, *Am. J. Phys*, **56** (5), 395, 1988

The wormhole described in Einstein and Rosen's paper, often called either a Schwarzschild wormhole or an Einstein-Rosen bridge, was found to possess a number of fatal flaws that precluded it from being used as an actual means of travel. The most glaring flaw is its instability; though the static description of the wormhole is straightforward, the system is actually dynamic. The time evolution of the system - studied by John Archibald Wheeler and others - shows that the Schwarzschild wormhole collapses so rapidly that even an object moving at the speed of light could pass through the throat fast enough to avoid being destroyed by intense gravitational gradients[5].

Even setting aside this insurmountable obstacle, a Schwarzschild wormhole has a number of other features that make it unsuitable for interstellar travel, tied to the fact that the curvature of spacetime in the vicinity of the throat is indistinguishable from the curvature induced by a Schwarzschild black hole - in fact, the most common way to visualize a Schwarzschild wormhole is by pasting two such black holes together at the event horizon. This brings with it a host of concomitant problems, the most notable being the intense tidal gravity.

These can be clearly illustrated by a thought experiment. Let us place a Schwarzschild wormhole in our solar system - for example, near Saturn, and examine the effects on a traveler. The throat of a Schwarzschild wormhole is the same radius as the event horizon of a Schwarzschild black hole: $2GM/c^2$, where G is the universal gravitational constant, M is the mass of the wormhole, and c is the speed of light in a vacuum. We would like the wormhole to be large enough for a traveler to fit through - ideally large enough for even a rather large spaceship to fit through easily, without requiring exceptionally tricky maneuvering to line up the shot. So let us assume a wormhole with a radius of 500m. A wormhole of this size would have a mass of about 3.367×10^{29} kg, or about 100,000 times the mass of the Earth. In comparison, this is about a 1/10 the mass of the Sun!

The gravitational field due to this mass at any point in space can be easily calculated. Since gravitational fields fall off with distance, parts of an object that are closer to the source of a gravitational field feel a stronger force than parts of the object that are further away. This difference in field strength is called a 'tidal force,' because it is the mechanism responsible for the Earth's tides. Although for objects like the Earth acting on something small like a human, these tidal forces are small, as the gravitational field strength increases, so does the gradient of the gravitational field. For objects as massive - and as small - as our wormhole, these can become very large indeed.

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[5] M. Morris and K. Thorne, *Am. J. Phys*, **56** (5), 396, 1988

To first order, these tidal forces are given by the equation $F = \Delta r * [2GM/R^3]$, where R is the distance of some object from the center of gravity of the wormhole, and Δr is the length of that object. As you can see from the equation, the closer you get to the center of gravity, the more intense the difference in gravitational forces becomes. At the throat of the wormhole, where the distance from that center is smallest, this force becomes $F = \Delta r * [c^6/(4*G^2*M^2)]$. Using standard SI units for c and G, this becomes $\Delta r * 4.092 \times 10^{70}/M^2$.

Using the mass we calculated above, we see that the tidal forces on a 1m long object at the throat of the wormhole are 3.6×10^{11} N, with the force growing as the object becomes longer! This is more than sufficient to kill a human traveller and rip apart their spaceship. If we are interested in comfortable travel for our human and their spaceship, we must minimize this force. Luckily enough, the tidal force decreases as the mass of the wormhole increases. This is because the Schwarzschild radius increases faster than the strength of the gravitational field at that radius. Such a wormhole would be much, much larger than our initial prospect, but that is the price we must pay to make it work. If we multiplied the mass of our wormhole by 100,000, we would end up with a tidal force of 36 N at the throat, an easily survivable - even comfortable - force.

Our wormhole now has a radius 100,000 times larger than it originally was - on the order of 50,000 kilometers. This is almost large enough for the planet Saturn to pass through. The mass is also now 10,000 times the mass of the Sun. As we might imagine, placing an object so large near Saturn would be extremely disruptive to the orbital mechanics of the solar system. For the system to survive such a large wormhole, it would have to be placed much farther away.

To determine how far away, we will need to examine the Hill sphere of the Sun. The Hill sphere is the region in which a small body can maintain satellites in the face of a much stronger gravitational field. For example, the Moon is able to maintain a stable orbit around the Earth because its orbit lies within the Earth's Hill sphere - the region where the gravitational field of the Earth dominates the motion of objects - even though the Sun is much more massive and therefore has a much stronger gravitational field.

For a near-spherical orbit, the edge of the Hill sphere can be found using the following equation: $r \approx a\sqrt[3]{(m/3M)}$, where r is the radius of the Hill sphere, a is the radius of the smaller body's orbit around the larger, and m and M are the masses of the smaller and larger bodies, respectively. Stable orbits tend to remain within 1/2 the Hill sphere - orbits larger than that can easily be progressively perturbed, and over long times may end up orbiting the larger body.

To keep the Solar system intact, we would like the orbit of Neptune to fall within this region of stability, roughly half the radius of the Hill sphere. Beyond the orbit of Neptune is the Kuiper belt, and beyond that is the Oort cloud. Although perturbations to these regions caused by a massive wormhole could potentially be catastrophic to life on earth, due to the changes in orbits of various comets and other small bodies, such calculations are beyond the scope of this study. For the purposes of our present inquiry, we will keep Neptune safely orbiting the Sun, which should prevent radical changes to the orbits of other planets.

At its furthest part of its orbit, Neptune lies about 30.33 AU away from the Sun. So, we want the Sun's Hill sphere to extend to roughly 60 AU. Given the mass of the Sun and the mass of the wormhole above, we find that the radius of the Sun's orbit around this massive wormhole must be roughly 30 times the radius of the Hill sphere. So the wormhole must be placed somewhere on the order of 1800 AU away from the Sun, or 2.6x10^11 miles. This is quite a distance, though still nowhere near so vast as the distances between stars.

However, given the fact that these objects are unstable, and that even if they weren't, they would be incredibly large, this kind of wormhole is clearly unsuitable for our purposes. In their 1988 paper, Morris and Thorne explored ways to construct a wormhole to overcome many of the problems facing wormhole travel[6].

In order to adequately explain the results that Morris and Thorne obtained, a succinct overview of their method is expedient. Though the math is complicated, the approach is straightforward. According Einstein's general theory of relativity, matter warps the fabric of spacetime. This warping, or curvature, gives rise to what we measure as the gravitational field. Essentially, all gravity is the result of geometry. The Einstein equation relates the geometry of spacetime - the ways it curves and bends and connects together - to a mathematical object called the stress-energy tensor, which describes the configuration of matter that causes that curvature. So this equation can be used either to find out what the curvature of space due to any given set of matter looks like - a solar system, a nebula, or a local group of galaxies, just to name a few - or to determine how to arrange matter in order to get a desired geometry. This second method is the approach taken by Morris and Thorne.

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[6]M. Morris and K. Thorne, *Am. J. Phys*, **56** (5), 400, 1988

After constructing the basic geometry of a spherical wormhole - the most mathematically tractable kind - they imposed three constraints on the functions. The first required that there be no horizons in the geometry, or regions that were cut off from the rest of spacetime similar to the event horizon of a black hole. This constraint guaranteed that the wormhole could be used for two-way transit. The second and third placed constraints on the gravitational field generated by the wormhole's mass, requiring that tidal forces not exceed the strength of Earth's gravity and that the elapsed time for a traveller passing through be less than one Earth year as measured both by the traveller and by stationary observers. These last requirements are not essential for a wormhole to be traversable, but were added in order to make passage through such a wormhole both comfortable and convenient for the travellers.

These constraints on the wormhole functions led to very strict constraints on the stress-energy tensor. Of these, the most concerning was a constraint laid on the tension. At the throat, they found that the tension on the material threading the wormhole was greater than the mass-energy of that material. For observers traveling sufficiently close to the speed of light, this meant that they would see matter with a negative energy density[7]. This is a violation of important energy conditions, and any material with this characteristic has been labelled exotic. Although there are some indications that quantum effects do create material or regions of spacetime with these conditions, whether these effects are sufficient to hold open a wormhole is unknown.

In the decades since Morris and Thorne's paper, there has been a great deal of research into wormholes. Morris and Thorne went on to examine the possibility of using wormholes as time travel devices, and other researchers have examined a number of different geometries, including regular polygons[8]. In 2003, Matt Visser and his colleagues demonstrated that the amount of exotic material required could be made arbitrarily small[9].

One of the most interesting developments, however, has come from another avenue of research into faster-than-light travel methods. In 1994, Miguel Alcubierre described a distinct configuration of spacetime that allowed a region to be accelerated[10]. By contracting spacetime in front and producing an expansion behind a spaceship, Alcubierre showed that the speed of light could be exceeded. However, as with the creation of a wormhole, to do so required exotic matter. However, in 2013, Gabriele Varieschi and Zily Burstein published a paper examining Alcubierre's geometry in conformal gravity, an extension of Einstein's general theory of relativity

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[7] M. Morris and K. Thorne, *Am. J. Phys*, **56** (5), 405, 1988
[8] M. Visser, *Phys. Rev. D*, **39** (10), 3182, 1989
[9] M. Visser *et al. Phys. Rev. Letters* **90** (20), 201102, 2003
[10] M. Alcubierre, *Class. Quantum Grav.*, **11** (5), L73, 1994

that proposes symmetry under a special class of transformations called conformal transformations[11]. In this formulation, they were able to eliminate the need for exotic matter entirely. Further research will be necessary to see if the same results can be obtained for wormholes.

[11] G. Varieschi and Z. Burstein, *ISRN Astron. Astrophys.* 2013

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