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## Black Scholes Delta Hedge in Imperfect Markets

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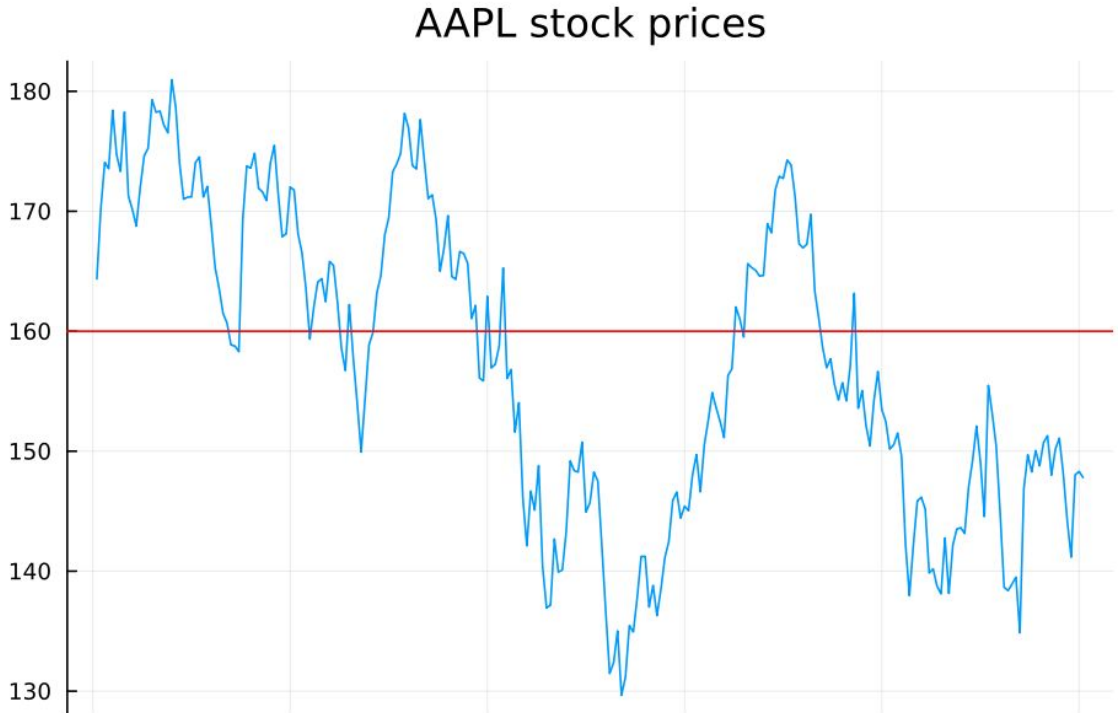
# Black Scholes Delta Hedge in Imperfect Markets

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How stock option market makers manage their risk

# Call Options

- Financial contract
- Lets holder buy stock at a set price
- Option to buy - not an obligation to buy



# Market makers



# Black Scholes model

## Assumptions:

- No arbitrage opportunity
- Can borrow/ lend any amount at the risk-free interest rate
- No dividends
- Ability to buy and sell any amount (even fractional) of the stock
- No transaction costs
- **Log normal-distributed stock returns**

$$C(S_t, t) = pN(d_1) - se^{-rt}N(d_2)$$

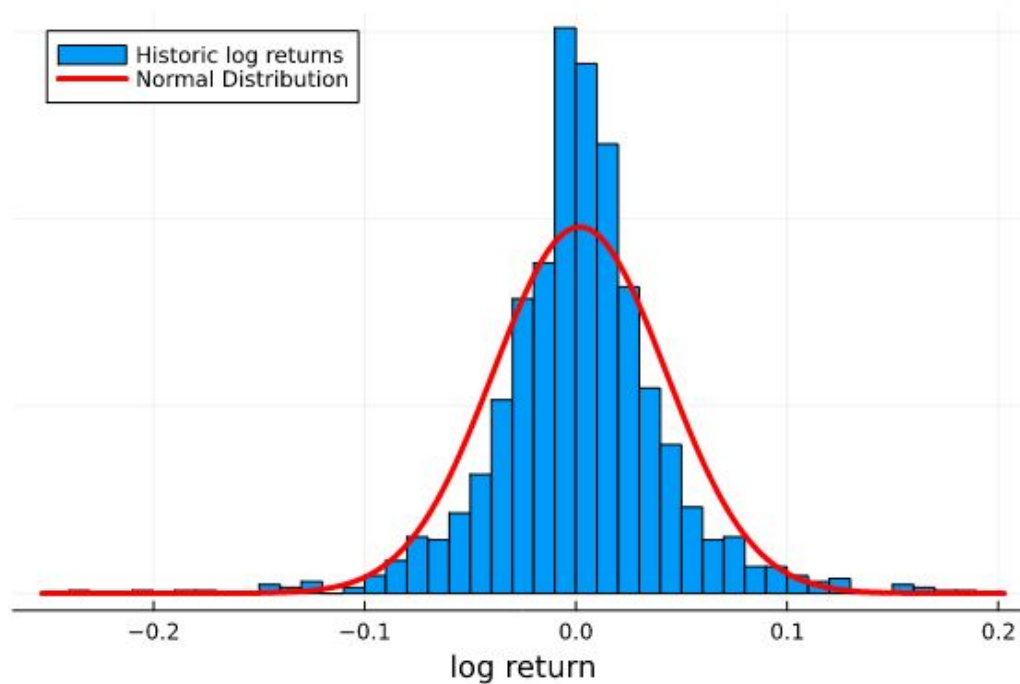
where

$$d_1 = \frac{\ln\left(\frac{p}{s}\right) + \left(r + \frac{v^2}{2}\right)t}{v\sqrt{t}}$$

$$d_2 = d_1 - v\sqrt{t}.$$

# Log-normal returns?

TSLA stock daily log returns



Is Black Scholes delta hedge a viable  
hedging strategy given market  
'imperfections'?

# Methods

## Bruno Julia package

### Monte Carlo simulations

- Sample size: 50,000
- Geometric Brownian Motion
  - Parametric
- Stationary Bootstrap of historic returns
  - Non-parametric

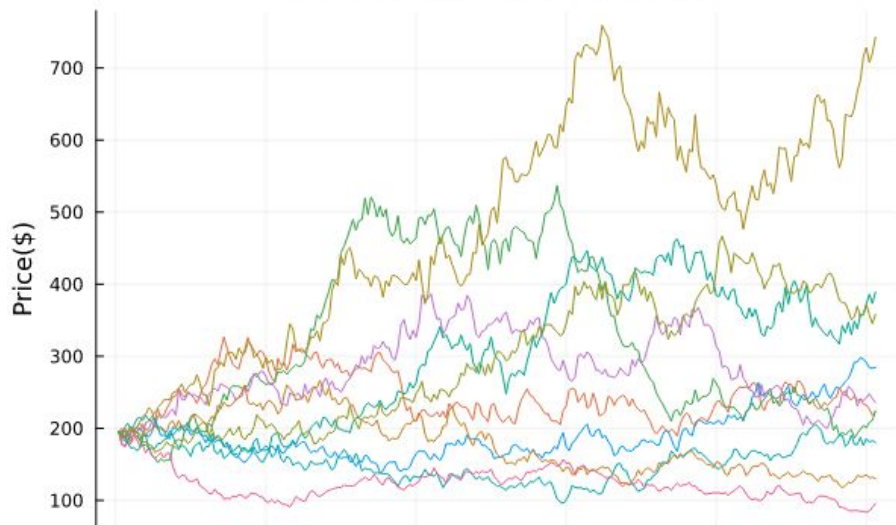




# How are GBM and Stationary Bootstrap different?



GBM simulated stock paths



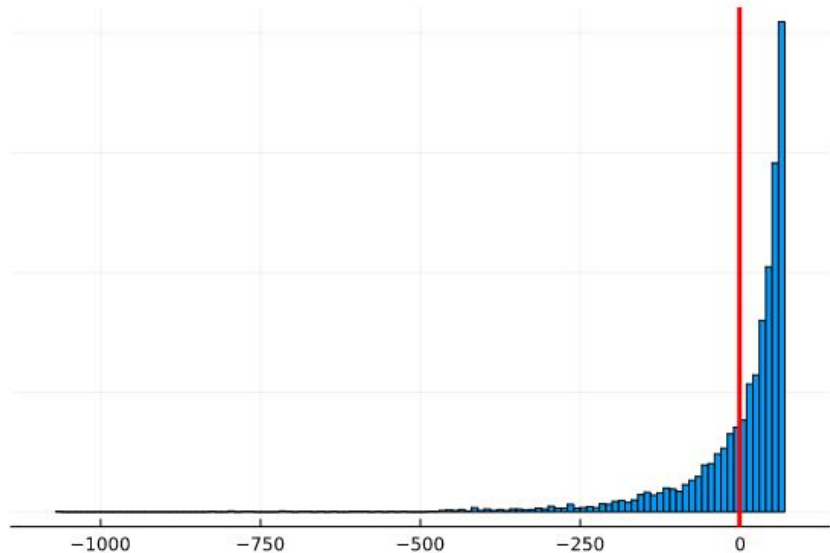
Stationary bootstrap simulated stock paths



# Naked Strategy (No hedging)

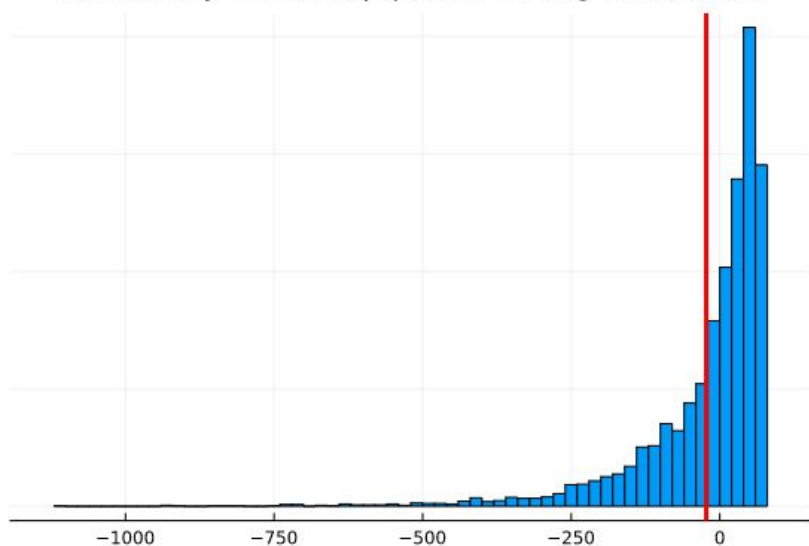
Mean: -0.977

GBM prices selling call naked



Mean: -22.903

Stationary Bootstrap prices selling call naked



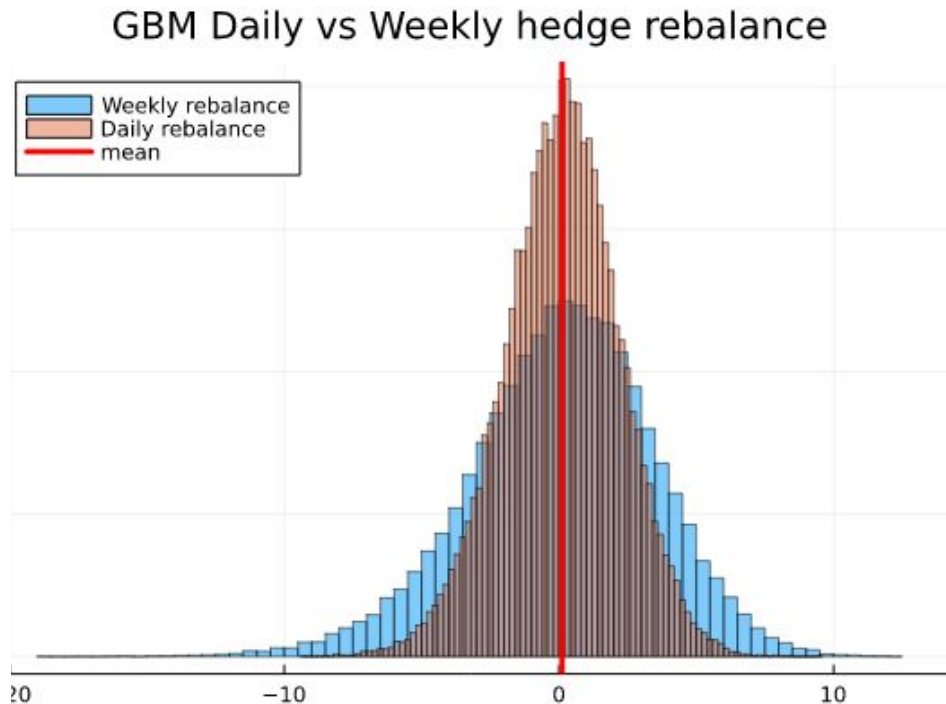
# Geometric Brownian Motion (parametric model)

Weekly rebalance:

- mean = 0.094
- std = 3.389

Daily rebalance:

- mean = 0.115
- std = 2.1001



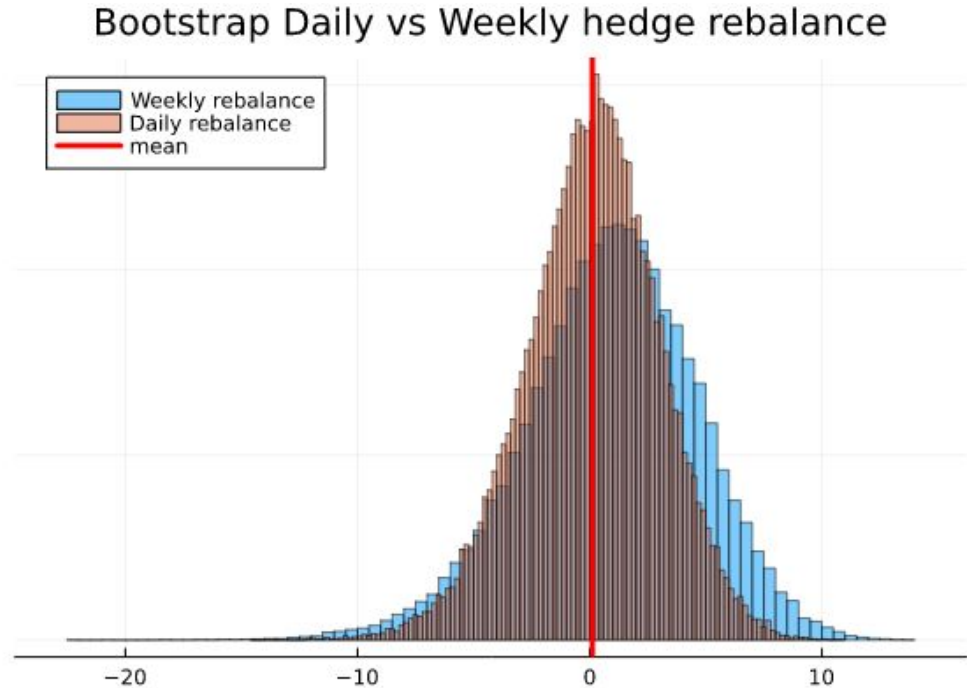
# Stationary Bootstrap

Weekly rebalance:

- mean = 0.903
- std = 3.759

Daily rebalance:

- mean = 0.114
- std = 2.891



# Conclusions

- Black Scholes delta hedging is a viable strategy for market makers
- Variance decreases as time between hedge decreases

**Bruno.jl documentation**



# Works Cited

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