Comparison of Pressure Distribution in 2D and 3D Jet-Driven Scour Processes

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Comparison of Pressure Distribution in 2D and 3D Jet-Driven Scour Processes

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Abstract: Water jets impacting on a non-cohesive granular bed cause a dynamic pressure distribution on the surface of the scour hole. Such distribution greatly differs from the hydrostatic one, especially in proximity of the impact zone, where the kinetic energy of the jet is dissipates due to the shear stresses acting on the scour surface. Here, the excess of shear stress causes the erosion of the bed material and can lead to the failure of hydraulic structures. Therefore, a detailed analysis of the forces acting on the scour surface is fundamental to understand the erosion mechanisms. Experimental tests were conducted under 3D and 2D conditions. Under 3D condition, the channel was wide enough to allow the full lateral development of the scour hole, whereas it was narrowed by means of vertical plates to achieve the 2D condition. Tests were conducted by monitoring the pressure distributions along the axial profile of the scour hole under both static and dynamic equilibrium conditions. The most influential parameters governing the scour process have been identified and varied, including the water discharge, the jet angle and the tailwater depth. Finally, the pressure distributions pertaining to 2D and 3D cases were compared, suggesting the dependence of the relative pressure on the tailwater and on the densimetric Froude number, and two empirical equations predicting the maximum relative pressure in case of low and high values of tailwater have been derived for design purposes. The proposed analysis represents a contribution to the development of theoretical methods for jet-driven scour processes.

Keywords: scour hole; plunging jets; pressure fluctuations.

1. INTRODUCTION

The erosion phenomena due to an impacting water jet on a non-cohesive granular bed is of great interest for the design of hydraulic structures. There are multiple practical situations concerning a jet plunging into a pool of water with granular material at its bottom, such as potholes occurring downstream of dam spillways. The jet action results in a dynamic pressure distribution at the surface of the scour hole that may greatly differ from the hydrostatic one, especially in the impact region (Palermo et al 2020). During this process, the kinetic energy of the jet is dissipated due to the effects of the shear stresses which cause the movement of the granular material. If the shear stress exceeds the critical counterpart, then erosion takes place, leading to a possible failure of the hydraulic structure (Bombardelli & Gioia, 2005). The evaluation of the shear stresses due to a jet impinging on a granular bed material represents a challenging problem. A detailed knowledge of the shear stress distribution and magnitude can furnish a new insight on the dynamics of three-phase flows. Numerous studies have been conducted using laboratory models, and recently, an analytical formulation of the phenomenon has been proposed as well (see Di Nardi et al 2021). Empirical approaches have been widely used in hydraulic engineering for design purposes. Among others, significant contributions in this regard are due to Mason and Arumugam (1985), Breusers & Raudkivi (1991). In these studies, the following equation has been identified and calibrated:

\[ h_0 + z_m = K q^{e_q} H^{e_H} g^{e_g} d^{e_d} \left( \frac{\rho}{\rho_s - \rho} \right)^{e_p} \]  

where \( e_q, e_H, e_g, e_d \) and \( e_p \) are independent exponents, \( K \) is a multiplicative coefficient, \( \rho \) and \( \rho_s \) are the
density of water and sediments, respectively, \( g \) is the gravitational acceleration, \( d \) is the sediment diameter, \( H \) is the falling height of the jet and \( q \) is the unit or total discharge, depending on the 2D or 3D equilibrium morphology conditions. More recently, Gioia & Bombardelli (2005) has provided the values of the abovementioned coefficients by revising equations from different authors (e.g., Schoklitsch 1932; Mueller & Eggenberger 1944; Kotoulas 1967).

An additional empirical approach was developed by Pagliara et al (2006) and then extended by Pagliara et al (2008a, b) and Pagliara & Palermo (2008). For the first time, two different equilibrium conditions have been distinguished, i.e., dynamic and static configurations, highlighting the important role played by the suspended material during the scour process. They evidenced that the scour geometry (i.e., maximum scour, length and height of the dune) mostly depends on the densimetric Froude number, the jet angle, bed sediment gradation and the air concentration in the jet. In addition, predicting relationships of the maximum scour depth for both static and dynamic equilibrium conditions have been proposed under both 2D and 3D conditions. Finally, Pagliara et al (2008a) provided a quantitative criterion to distinguish 2D and 3D cases. Namely, they introduced a three-dimensionality parameter \( \lambda = b_m/B \), where \( B \) is the channel width and \( b_m \) is the extrapolated scour hole width. These authors also identified the fundamental parameters involved in the scour mechanism, i.e., the jet discharge \( Q \), the jet velocity \( V_s \), the jet angle \( \alpha \), the duration \( T \) of the jet action, the bed material characteristics (e.g., sediment density \( \rho_s \), average diameter \( d_{50} \) and sediment non-uniformity parameter \( \sigma \)) and the water depth above the original sediment bed level \( h_0 \).

Likewise, semi-theoretical methods have been also developed. Among others, Stein & Julien (1993, 1994) focused on the scour evolution concluding that the equilibrium characteristics depend on the diffusion length. Hoffmans (1998) proposed an analysis based on Newton’s second law, highlighting the shortcomings of empirical approaches. Finally, an attempt to use a fully theoretical approach was due to Bombardelli & Gioia (2005, 2006) who based their analysis on the phenomenological theory of turbulence.

Despite the large number of studies dealing with the topic, there are still open questions regarding the estimation of the shear stresses acting on the surface of the scour hole due to an impinging jet, such as their relationship with the pressure distribution at the bottom of the scour hole. The present study aims at highlighting the differences between the pressure distributions along the axial cross section of the scour hole under 2D and 3D conditions.

2. EXPERIMENTAL SETUP AND METHODOLOGY

2.1. Three-dimensional Plunge Pool Scour

Two different models for investigating the evolution of three-dimensional and two-dimensional scour holes due to an impinging jet have been built at the Hydraulic Laboratory of the University of Pisa, Italy. The experimental channel was 6 m long, 0.8 m wide and 0.9 m deep. The equilibrium morphology was always 3D under the tested ranges of discharge and tailwater depth.

Namely, a circular pipe with diameter \( D_p = 0.0215 \) m was used to simulate the jet. The pipe was inserted in a frame allowing for an angle variation between 0° and 90°. Two different jet inclinations were tested, i.e., \( \alpha = 45^\circ \) and \( 60^\circ \). The jet discharge \( Q \) ranged between 0.00115 and 0.00165 m³/s and the distance between the jet pipe and the water surface was kept at a distance approximately equal to \( 2D_p \) (Pagliara et al 2020). The experimental apparatus is schematically represented in Figure 1. One uniform granular material was adopted for the movable bed. This was characterized by a density \( \rho_s = 2144 \) kg/m³, a mean diameter \( d_{50} = 0.00225 \) m and a non-uniformity parameter \( \sigma = 1.22 \). The bed material was levelled at the beginning of each test. The densimetric Froude number \( F_{d50} \) is a non-dimensional parameter that plays an important role for scour phenomena, and it is defined as \( F_{d50} = V_s/(g' d_{50})^{1/2} \), where \( g' = g(\rho_s/\rho) \).

Under three-dimensional scour condition, a preliminary test (termed “Test 0”) was carried out to verify the consistency of half-model results with those obtained with full-model arrangement (see Pagliara et al 2008 for details). Test 0 was performed by positioning the jet along the axis of the channel, with a discharge \( Q = 0.003 \) m³/s, water depth \( h_0 = 0.11 \) m, jet angle \( \alpha = 45^\circ \) and \( D_p = 0.028 \) m. After 300
seconds from the beginning of the test, the scour hole profile was surveyed under dynamic conditions by means of a special point gauge (Pagliara et al. 2006). Measurements were taken in selected points. After the complete scour development (i.e., \( t^* = 4200 \text{ sec} = 70 \text{ min} \)) the water jet was stopped, the channel was slowly dried, and the survey of the scour hole was repeated under static condition. The full model test was repeated by locating the jet close to the glassed wall of the flume for \( Q = 0.0015 \text{ m}^3/\text{s} \) and a pipe diameter \( D_p = 0.0215 \text{ m} \) in order to preserve the same hydraulic conditions of the full model arrangement (i.e., same densimetric Froude number and nondimensional tailwater depth). The scour profiles and maximum scour depths obtained using the two different arrangements are consistent, thus confirming the findings of Pagliara et al. (2008a). Therefore, all the other tests were conducted in the half-model setup, allowing us to monitor the scour evolution.

A total of 18 experimental tests were conducted under black-water conditions, i.e., no air was present in the water jet. A first series of 9 tests, namely “reference tests”, allowed to establish the reference dynamic equilibrium configurations. At dynamic condition, the maximum scour depth \( z_m \) was measured by a point gauge at fixed time intervals. Once that the equilibrium configuration was reached, the profile of the scour hole was surveyed using a 1 mm precise point gauge. Measurements were taken along 3 longitudinal sections, with the first one located 0.02 m apart from the glass wall (i.e., axial section).

![Diagram sketch of the experimental setup](image)

Figure 1 - Diagram sketch of the experimental setup: a) side view; b) top view and 3D condition; c) top view and 2D conditions
For the second series of experiments, the 9 tests were repeated by locating a pressure transducer and seven piezometers approximately 1 cm below the equilibrium scour surface pertaining to the corresponding reference tests. The pressure transducer was a SENSIT Type A-BIVAA-001 (Sensit, Woking, UK), characterized by a precision of ±1 % and an acquisition frequency of 10 Hz. It was located in correspondence with the jet impinging area, just below the point of maximum scour depth. Likewise, piezometers were located at other selected points below the axial equilibrium profile. The adopted instrumentation allowed to measure the distribution of the dynamic ($P_d$) and static ($P_s$) pressures along the axial profile at dynamic equilibrium condition.

Finally, the jet was stopped, resulting in the deposition of the suspended material and the complete drainage of the water inside the flume, and the static equilibrium scour profile was surveyed by means of the point gauge previously used.

### 2.2. Two-dimensional Plunge Pool Scour

The same experimental setup described before was adopted by Palermo et al (2020) for the investigation of the evolution of a two-dimensional scour hole due to an impinging jet. In this case, the channel width was $B = 0.2$ m. It is worth remarking that Pagliara et al (2008) experimentally found that a 3D scour hole occurs for $\lambda < 1.5$, while a 2D scour hole is characterized by values of $\lambda$ larger than 3.0 (see Figure 1b and c; see Paragraph 1 for details about $\lambda$).

Two series of 12 experimental tests were conducted by Palermo et al (2020) and used herein for comparison. The ranges of the parameters tested under both 2D and 3D conditions are reported in Table 1, where $D_{eq}$ indicate the equivalent jet diameter in case of 3D test condition performed with half scour and $T_w$ is the tailwater defined as $T_w = h_0/D_{eq}$. For the 2D case, other details can be found in Palermo et al (2020).

<table>
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<tr>
<th>Test #</th>
<th>Cond. 2D/3D</th>
<th>$Q$ [m$^3$/s]</th>
<th>$h_0$ [m]</th>
<th>$\alpha$ [$^\circ$]</th>
<th>$D_p$ [m]</th>
<th>$D_{eq}$ [m]</th>
<th>$V_w$ [m/s]</th>
<th>$F_{d50}$ [-]</th>
<th>$T_w$ [-]</th>
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<td>0.020</td>
<td>60</td>
<td>0.0215</td>
<td>0.0215</td>
<td>4.54</td>
<td>27.76</td>
<td>0.93</td>
</tr>
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<td>0.0215</td>
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<td>27.76</td>
<td>5.12</td>
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<td>60</td>
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<td>0.0215</td>
<td>4.54</td>
<td>27.76</td>
<td>6.98</td>
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<tr>
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<td>0.020</td>
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<td>0.0215</td>
<td>3.17</td>
<td>19.35</td>
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<td>0.110</td>
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<td>0.0215</td>
<td>3.17</td>
<td>19.35</td>
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<tr>
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<td>19.35</td>
<td>6.98</td>
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<td>0.0215</td>
<td>3.17</td>
<td>19.35</td>
<td>0.93</td>
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<td>3.17</td>
<td>19.35</td>
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<tr>
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<td>3.17</td>
<td>19.35</td>
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<td>0.0215</td>
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<td>25.24</td>
<td>4.93</td>
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<td>0.0304</td>
<td>2.75</td>
<td>16.83</td>
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<td>45</td>
<td>0.0215</td>
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<td>2.75</td>
<td>16.83</td>
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<td>4.13</td>
<td>25.24</td>
<td>4.93</td>
</tr>
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</table>
3. RESULTS AND DISCUSSIONS

3.1. Scour Hole Evolution

The scour evolution can be subdivided into two phases, i.e., developing and developed phases. The developing phase represents the initial enlargement of the scour hole and it is followed by the developed phase, when, for both 2D and 3D cases, a homothetic expansion of the scour hole occurs. The dynamic pressure increases with the scour depth until reaching an asymptotic value at equilibrium condition. In our tests, the equilibrium condition has always been reached for \( t^* < 70 \) minutes after the beginning of the test. In Figure 2, we show the pressure measured by the transducer as function of time \( t \). It can be observed that in the first instants (i.e., developing phase), there is a significant increment of the pressure, followed by a quasi-stationary condition corresponding to the developed phase.

Furthermore, for both 2D and 3D conditions, the difference between dynamic and static pressure distribution is significant, because of the material rotating within the scour hole. Note that when the jet action ceases, the suspended material falls back into the scour hole, thus partially replenishing it; to this end, Figure 3 shows the non-dimensional scour depth \( Z = z/z_m \) versus the non-dimensional longitudinal coordinate \( X = x/L \) for the axial cross section of the scour hole of Test 14, where \( L \) is the length of the scour hole at dynamic equilibrium condition. This behaviour applies to both 2D and 3D cases. However, scour features under 3D condition are much more complex than the 2D counterpart. Namely, for the 3D case, a radial flow occurs within the scour hole; conversely, for the 2D case, the flow dynamics is characterized by a macro-vortex and a sediment transport directed only downstream. These differences reflect on the dynamic pressure distribution along the axial cross section the scour, as highlighted in the next section.

![Figure 2 - Measured dynamic pressure \( P_d \) in Test 16 (Sensit pressure transducer).](image)

![Figure 3 - \( Z \) vs \( X \) for the axial cross section of the scour hole created in Test 14.](image)
3.2. Distribution of Pressures at Equilibrium Condition

The main objective of the present study is to investigate the differences and similitudes between the pressure distributions in correspondence with the axial cross section of the scour hole, under similar hydraulic conditions. To highlight the effects of the most influential parameters, the experimental tests have been grouped according to similar values of $T_w$ and $F_{d50}$. In the following, “low tailwater” (symbol $T_w \uparrow$) indicate tests conducted for $T_w = 0.93$ and 0.99; whereas “high tailwater” (symbol $T_w \downarrow$) indicates tests for $T_w = 5.12$ and 4.93. Likewise, with “low densimetric Froude number” (symbol $F_{d50} \downarrow$) we mean those tests with $F_{d50} = 19.35$ and 16.83; and with “high densimetric Froude number” (symbol $F_{d50} \uparrow$), tests for which $F_{d50} = 27.76$ and 25.24 (see Table 1).

First, the relative pressures $P_d/P_s$ were plotted against the non-dimensional longitudinal coordinate $X$ in Figure 4. For $\alpha=60^\circ$ and higher values of $F_{d50}$, the relative pressure $P_d/P_s$ pertaining to 2D tests are always larger that the 3D counterpart, regardless of the tailwater depth $T_w$ (Figure 4a and b, respectively). An opposite behaviour occurs for $\alpha=45^\circ$ and for low values of $T_w$ and $F_{d50}$ (Figure 4c). Conversely, a significant similitude of pressure distribution occurs for $\alpha=45^\circ$, low values of $F_{d50}$ and high values of $T_w$.

\begin{align*}
\text{Figure 4} - P_d/P_s \text{ vs } X \text{ for: a) } \alpha = 60^\circ, \text{ high } F_{d50} \text{ and low } T_w; \text{ b) } \alpha = 60^\circ, \text{ high } F_{d50} \text{ and high } T_w; \text{ c) } \alpha = 45^\circ, \text{ low } F_{d50} \text{ and low } T_w; \text{ d) } \alpha = 45^\circ, \text{ low } F_{d50} \text{ and high } T_w.
\end{align*}

Then, the relationship between the maximum values of relative pressure $(P_d/P_s)_{\text{max}}$ and the densimetric Froude number $F_{d50}$ has been investigated. For low values of $T_w$, the behaviour of the 2D and 3D cases is consistent and $(P_d/P_s)_{\text{max}}$ increases with $\alpha$ (Figure 5a) because of the increase of the vertical component of momentum flux. Conversely, if the diffusion length increases (high values of $T_w$), a clear distinction cannot be pointed out (Figure 5b). Furthermore, the ratio $(P_d/P_s)_{\text{max}}$ of the two cases is consistent and it decreases with the tailwater depth (Figures 5c and d). Overall, for design purposes, the following two equation can be used to estimate the values of $(P_d/P_s)_{\text{max}}$ (see Figure 6):

\begin{align*}
(P_d/P_s)_{\text{max}} &= 0.0074 \cdot F_{d50} + 1.345 \quad (2) \\
\text{valid for } 16.83 \leq F_{d50} \leq 27.76, 0.93 \leq T_w < 0.99, 45^\circ \leq \alpha \leq 60^\circ, \text{ and}
\end{align*}

\begin{align*}
(P_d/P_s)_{\text{max}} &= 0.0074 \cdot F_{d50} + 1.096 \quad (3) \\
\text{valid for } 16.83 \leq F_{d50} \leq 27.76, 4.93 \leq T_w \leq 5.12, 45^\circ \leq \alpha \leq 60^\circ.
\end{align*}
Figure 5 - \((P_d/P_s)_{max} vs F_{d50}\) for: a) low values of \(T_w\); b) high values of \(T_w\); c) jet angle \(\alpha = 45^\circ\); d) jet angle \(\alpha = 60^\circ\).

Figure 6 - \((P_d/P_s)_{max} vs F_{d50}\) for all tests, together with the predicting equations in case of low \(T_w\) (red line) and high \(T_w\) (blue line).

4. CONCLUSIONS

The scour process due to a water jet impinging on a granular bed is of great interest in hydraulic engineering since a dynamic pressure distribution greater than the hydrostatic one develops on the surface of the scour hole. Furthermore, the scour morphology and the pressure distribution differ according to the 2D or 3D condition of the scour itself. In the present study, differences between the dynamic pressure distribution along the axial section of the scour hole due to an impinging jet have been investigated for both 2D and 3D conditions. Several tests have been carried out for different hydraulic conditions: two inclinations of water jet angle \(\alpha\) (45° and 60°), four values of water discharge \(Q\) (ranging between 0.00115 and 0.00165 m³/s) and four values of water depth \(h_0\) (ranging between 0.02 and 0.15 m). Qualitative differences between 2D and 3D conditions have been assessed: scour features under 3D condition are more complex since a radial flow occurs within the scour hole itself, while the 2D case is characterized by a macro-vortex causing the sediment transport to be directed in the downstream direction only. These differences reflect on the dynamic pressure distributions: for \(\alpha=60^\circ\) and high values
of \( F_{d50} \), \( P_d/P_s \) under 2D condition are always larger that the 3D counterpart, regardless of the \( T_w \), while a significant similitude of the pressure distribution occurs for \( \alpha = 45^\circ \), suggesting a marked dependency on \( \alpha \). Further results show that the relative pressures \( P_d/P_s \) greatly depends on the tailwater \( T_w \) and on the densimetric Froude number \( F_{d50} \), suggesting that \( P_d/P_s \) is larger for low values of \( T_w \) and large \( F_{d50} \) within the tested range (i.e., \( 16.83 \leq F_{d50} \leq 27.76 \)). Finally, the relationship between \( (P_d/P_s)_{\text{max}} \) and \( F_{d50} \) was studied and two equations predicting the maximum relative pressure in case of low and high values of \( T_w \) were derived for design purposes. These may help to predict the ratio between the maximum dynamic and static pressures occurring at the point of maximum scour, and they are valid within the ranges \( 16.83 \leq F_{d50} \leq 27.76 \) and \( 45^\circ \leq \alpha \leq 60^\circ \).

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