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Anisotropic Model-Based SAR Processing

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ABSTRACT

Synthetic aperture radar (SAR) collections that integrate over a wide range of aspect angles hold the potential for improved resolution and fosters improved scene interpretability and target detection. However, in practice it is difficult to realize the potential due to the anisotropic scattering of objects in the scene. The radar cross section (RCS) of most objects changes as a function of aspect angle. The isotropic assumption is tacitly made for most common image formation algorithms (IFA). For wide aspect scenarios one way to account for anistropy would be to employ a piecewise linear model. This paper focuses on such a model but it incorporates aspect and spatial magnitude filters in the image formation process. This is advantageous when prior knowledge is available regarding the desired targets’ RCS signature spatially and in aspect. The appropriate filters can be incorporated into the image formation processing so that specific targets are emphasized while other targets are suppressed. This is demonstrated on the Air Force Research Laboratory (AFRL) GOTCHA\textsuperscript{1} data set to demonstrate the utility of the proposed approach.

Keywords: Anisotropic model, SAR, sparse reconstruction, regularized, model-based image formation

1. INTRODUCTION

There has been interest recently in regularized models for SAR processing. This is in part due to the potential compressive sensing applications,\textsuperscript{2,3} and for the inherent ability to provide super-resolution for point targets.\textsuperscript{4} One of the drawbacks of the model-based approach is the intrinsic underdetermined nature. This is due to the fact that radar can be viewed as the convolution of a beam with a 3D scene.\textsuperscript{5} And as with many convolution problems there exists many potential solutions for the measured data that are consistent, or in other words the system is underdetermined. This is particularly true for 3D scenes since the vertical aperture is almost always sparse, even when multiple elevation aspects have been collected. While it is not as grave in 2D scenes, there still exists ambiguity regarding the true 3D data representation on only a 2D surface. And with real SAR data the signal of interest is often buried in what is effectively multiplicative noise. Since many solutions are possible it is advantageous to steer the solution towards desirable characteristics. If prior information is available regarding the targets of interest, this could be incorporated into the image formation process upfront. Fortunately it is often the case that scenes have been imaged previously or targets of interest are known. For example it might be that the glint features are of particular interest for autonomous target recognition (ATR) application, or the mean RCS of a rural scene is the desired objective. In ground moving target indicator (GMTI) data there is a distinct response spatially and in aspect compared to static targets.\textsuperscript{6} In any of the previous mentioned scenarios it is possible to influence the final solution in the image formation process by incorporating target enhancing constraints. These target enhancing constraints can suppress non-target features or clutter. The two natural constraints that are considered are aspect and spatial constraints. These constraints could be combined into a single aspect and spatial filter similar to what is done in space-time adaptive processing (STAP)\textsuperscript{7} or applied independently. Special care is taken to formulate the problem so that global optimality is guaranteed and the problem is tractable for modest scene sizes on a standard computer in a reasonable amount of time. In order to achieve those objectives, in this paper the problem was formulated as a second order cone program (SOCP)\textsuperscript{8} that took advantage of sparse representations where applicable. This paper will explain the approach taken and

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focus on several examples that demonstrate the general utility of such an approach using real SAR data from
the GOTCHA collection by AFRL. The results will be examined and contrasted against imagery formed by the
back-projection algorithm. And the same imagery processed with different spatial and aspect filters will be
contrasted in order to demonstrate how scene characteristics can be accentuated or suppressed.

2. MODEL

SAR is a remote sensing technology that can provide unique capabilities for military applications or remote
sensing. A SAR sensor is generally used on an aircraft or satellite. The SAR sensor transmits a pulse in
a radial direction that is often referred to as the range direction. The transmitted pulse is generally a linear
frequency modulated (LFM) signal that traverses through frequency and is often called a chirp. The LFM chirp
is analogous to an audio chirp. The pulse travels in the radial or range direction until it hits an object and
is reflected. Part of the energy is reflected back towards the sensor. The sensor records the returns. This is
the standard radar use of radio retection and ranging (RADAR). The platform moves forward and the same
operation is repeated. The direction of the platform propagation or velocity is often referred to as the cross
range or azimuth direction. The coherent combination or processing of all the pulses creates a large synthetic
antenna or aperture. This enables high cross range resolution much finer than the beam width.

2.1 Signal Model

The returned LFM signal \( s_n \) has multiple factors and is given by

\[
s_n = A_0(\theta, \phi)w_r(\tau - \frac{R(\eta)}{c})w_a(\eta)e^{j2\pi \frac{R(\eta)}{c}}e^{j2\pi(\tau - \frac{R(\eta)}{c})^2}
\]

(1)

The first factor \( A_0(\phi, \theta) \) is a complex reflection term dependent on the azimuth and elevation angles \( \theta \) and \( \phi \)
due to the collection geometry. The geometrical dependence of the target \( A_0(\theta, \phi) \) is often approximated as a
constant term \( A_0 \). The second factor \( w_r(\tau - \frac{R(\eta)}{c}) \) represents a windowing function in range. It is nonzero
for a specified transmit pulse length \( t_p \) during the interval \( \left[ \frac{-t_p}{2}, \frac{t_p}{2} \right] \) relative to the two-way range \( R(\eta) \) of a given
target for the \( \eta \)-th pulse. The third factor \( w_a(\eta) \) represents the antenna gain and phase due to the geometry of
the \( \eta \)-th pulse. The fourth factor \( e^{j2\pi \frac{R(\eta)}{c}} \) represents the Doppler phase. It captures the phase change from pulse
\( \eta_i \) to pulse \( \eta_{i+1} \). The change in the two-way distance \( R(\eta) \) depends on the collection geometry. Finally the fifth
factor \( e^{j\pi K_r(\tau - \frac{R(\eta)}{c})^2} \) is a delayed replica of the transmitted signal.

The coherent combination of the pulses is what distinguishes SAR from other radar applications. This is
done via the image formation processing, which can be viewed as a single matrix operator \( C(\gamma) \). The radar
pulse samples are appropriately scanned into the vector \( y \). The image formation could be applied to a single
collection or any given combinations of collections by appropriately concatenating the model matrix \( C \) and data
vector \( b \). The \( \gamma \) variable represents all the parameters (geometry, attitude, hardware, etc.) required in the image
formation algorithm. The vector \( b_{img} \) is the resultant image formed by the image formation matrix \( C \) applied
to the raw SAR pulse samples \( y \) as seen in (2). This results in

\[
b_{img} = C(\gamma)y
\]

(2)

and is shown graphically in Figure 1 for the back-projection algorithm.

2.2 Processing Model

SAR images are modeled as the composition of many isotropic ideal point responses. The image domain is
the location where point targets have a natural sparse representation as seen in Figure 1(c). This provides
a natural basis for SAR images. The impulse pulse response (IPR) is infinite in theory due to the sinc-like
nature of the side lobes. But in practice it can be approximated to any desired accuracy by only keeping a few
neighboring pixels that contribute above a selected threshold (e.g., -40 dB). The truncation effect is presumed
negligible due to the inherent noise, quantization, and non-ideal implications (antenna pattern modeling errors,
aircraft positional errors, etc.) found in real SAR data. The back-projection algorithm is ideal for complicated
collection geometries and sparse sampling scenarios because it is a time domain algorithm and it can naturally accommodate these situations. Even though the back-projection algorithm is computationally expensive, it will be the image formation algorithm assumed for the rest of this paper, but any appropriate image formation algorithm could be substituted with appropriate considerations.

Imaging algorithms such as back-projection have residual processing artifacts. The model developed below tries to account for those artifacts and then estimate the true image. The assumed model is

\[ b_{\text{img}} = A(\gamma)x + \eta \]  

(3)

where the vector \( b_{\text{img}} \) represents a vectorized 2D plain or 3D cube. The focus will be on a 2D scene but extension to 3D is straightforward. The region of interest (ROI) is first selected. This is the spatial area of interest. Any data or pixel that contributes to the ROI, as defined by the IPR approximation being used, is referred to as the data area in Figure 2 and is represented as \( b_{\text{img}} \) in (3). The data \( b_{\text{img}} \) is the vectorized image that was formed by processing the raw SAR pulse samples \( y \) in (2). The matrix \( A \) represents the model and in effect tries to fit or explain the data \( b_{\text{img}} \). This requires \( A \) to model all consequential pixel contributions to \( b_{\text{img}} \) which is defined as the model area in Figure 2 and represents \( x \) in (3). The \( \eta \) term represents the model mismatch and noise.

The matrix \( A \) contains the IPRs for all \( x \). The \( x(i) \)th pixel IPR is found in the \( i \)th column of \( A \). The \( \gamma \) term representing the image formation parameters is inherently used in the construction of the \( A \) matrix when generating the IPRs. The generation of the \( A \) matrix entails computing a synthesized response for every \( b_{\text{img}}(i) \)th pixel for every pulse location. By incorporating the actual geometry this allows the IPR to be accurately computed for any sparse sampling scenario and diverse flight geometries. Ideally any modeled hardware characteristics (antenna patterns, etc.) would be incorporated in the synthesis of the \( A \) matrix if available. This is computationally intensive, but it naturally lends itself to sparse sampling scenarios such as limited vertical apertures. And it also allows incorporating flight geometries and diverse waveforms that maximize a desired IPR.

![Figure 1](http://proceedings.spiedigitallibrary.org/)

Figure 1. (a) Raw SAR pulse samples. (b) Matrix back-projection representation. (c) Resultant image.

![Figure 2](http://proceedings.spiedigitallibrary.org/)

Figure 2. 2D scene size impulse response.
The actual generation of the $A$ matrix is an ideal fit for GPU implementations due to the parallel nature with effectively no boundary conditions. This helps make the computational burden tolerable. It is also useful since it is difficult in closed form to generate IPRs when diverse geometries, waveforms and sparse sampling patterns are utilized. Understanding the conglomerate IPR is critical in order to perform theoretical performance analysis. This type of information is intrinsically captured in the $A$ matrix and can be analyzed by examining features like the rank and conditioning, which directly relates to resolution and solution robustness.

The underlying assumption is that the scene RCS is constant or isotropic. The isotropic assumption is required for coherent processing and is generally a valid assumption for small angles. But it is not valid over a wide range of aspects. A natural solution would be to break the diverse aspects into smaller pieces where the isotropic assumption is valid. This changes the variables found in (3) to be updated for $K$ coherent subimages as

$$
\begin{align*}
  b_{\text{img}} &= \begin{bmatrix} b_{\text{img}1} \\ \vdots \\ b_{\text{img}K} \end{bmatrix}, \\
  A &= \begin{bmatrix} A_1 & \cdots & \cdots & \cdots \\ \vdots & \ddots & \vdots & \vdots \\ \cdots & \vdots & \ddots & \vdots \\ \cdots & \vdots & \vdots & A_K \end{bmatrix}, \\
  x &= \begin{bmatrix} x_1 \\ \vdots \\ x_K \end{bmatrix}.
\end{align*}
$$

The $A$ matrix has a block diagonal structure where each block is now a coherent image where the isotropic assumption is valid. The block diagonal structure in $A$ can be seen in Figure 4(a) for 4 subimages.

### 3. ASPECT AND SPATIAL FILTERS

A key realization occurs when the phase and magnitude de-correlation of the RCS are examined independently. Heuristic evaluation of actual SAR data reveals that there often is a RCS magnitude correlation in aspect that exists beyond the coherent integration limits. This is demonstrated for seven pixels selected from the GOTCHA data collect seen in Figure 3(a). The pixels selected have been marked with a star in order to help identify them. The 360 degrees of azimuth integration were divided into four-degree coherent subimages and processed using the back-projection algorithm. Figure 3(b) shows the correlation of the magnitude RCS and Figure 3(c) shows the normalized complex RCS correlation. The selected pixels demonstrate that the magnitude RCS is smoother than the complex RCS in general and implies that the RCS phase often de-correlates at a different rate than the RCS magnitude. The RCS magnitude de-correlation is target specific. For example a rural scene might be fairly consistent regarding the RCS magnitude for a wide range of aspects. On the other hand an urban scene with man made features could exhibit glint phenomenology and de-correlate quickly due to the strong geometrical RCS dependence. The key observation is that the RCS magnitude in aspect can be used to help distinguish specific targets if it is known. The spatial RCS magnitude correlation is clearly visible in most SAR images especially if multi-looking has occurred as shown in Figure 3(a). In a very insightful paper by Munson, he shows that the phase from pixel to pixel for most SAR scenes is uncorrelated. This implies that little, if anything, can be gained from the phase of pixels in close proximity. But the magnitude of the surrounding pixels does provide additional information in most cases that can be beneficial.

![Figure 3](image-url)

(a) Scene with 7 pixels selected (white Stars). (b) Magnitude correlation (dB). (c) Normalized complex correlation. (dB)
The focus of this paper will be on independent spatial and aspect filters in order to simplify the final analysis. But there are some applications like GMTI where it would be advantageous to combine them into a single space-aspect or space-time filter. Let the spatial filter be denoted as $G$ and the aspect filter as $F$. The term $x(i)_k$ denotes the $i^{th}$ spatial pixel in the $k^{th}$ image block. The spatial filter operates only on the $k^{th}$ image block. The aspect filter operates only on the $i^{th}$ pixel. The restriction of $F$ to aspect and $G$ to spatial filters induces specific structure in the matrices $F$ and $G$. The spatial filter $G$ has a diagonal structure since it only ever operates on an individual subimage as seen in Figure 4(c). The aspect filter operates on a single pixel but across subimages. This creates the diagonal banding structure as seen in Figure 4(b). All the matrices are sparse and have distinct diagonal patterns.

![Figure 4](image-url)  
(a) A matrix example. (b) F matrix example. (c) G matrix example.

Spatial filters have been used in traditional image processing extensively.\textsuperscript{15} Cetin proposed a spatial filter in the image formation processing as opposed to traditional post processing in order to segment SAR images.\textsuperscript{16} The underlying idea of aspect filters is leveraged in many algorithms such as the autofocus algorithm called map drift\textsuperscript{17} and GMTI processing. Stojanovic\textsuperscript{18} was one of the first to propose an aspect filter along with a spatial filter in the image formation processing. Depending on the filter composition smooth features (low pass) or glint features (high or band pass) can be targeted during the image formation process. This has important implications since the system is generally underdetermined so there is inherent flexibility to influence solutions while maintaining an appropriate data fit.

4. PROBLEM FORMULATION

The final problem formulation can now be written as

$$J(t, \phi) = \|A(t, \phi)x_c - b_{img}\|_2 + w_1\|Fx_m\|_{p_1} + w_2\|Gx_m\|_{p_2} + w_3\|Dx_m\|_1$$

s.t. $x_m(i) = |x_c(i)|$ for all $i$  \hspace{1cm} (5)

This incorporates the aspect filter $\|Gx_m\|$ and spatial filter $\|Fx_M\|$ on the RCS magnitude. The data fit term $\|A(t, \phi)x_c - b_{img}\|_2$ requires the complex RCS to fit the model. It is no surprise that an accurate model can provide superior results. But as will be shown with real GOTCHA data, where no information about the sensor is available, robust solutions are still attainable when ideal assumptions are employed. There term $\|Dx_m\|_1$ promotes sparse solutions, and the matrix $D$ represents a diagonal weighting term that can be used to incorporate prior information regarding the weights.\textsuperscript{19} The problem formulation in (5) is non-convex. The constraint $x_m(i) = |x_c(i)|$ that the magnitude of the complex reflectivity $|x_c(i)|$ be defined equal to the magnitude reflectivity $x_m(i)$ is non-convex. This can be relaxed to $x_m(i) \geq |x_c(i)|$ so that the problem is convex. The accuracy of the results can be checked by simply noting how how tight the inequality constraints are. The problem can now be posed as a second order cone problem (SOCP) which can be solved efficiently. The details of converting this problem into an SOCP can be found in [20]. The terms $w_1, w_2, w_3$ represent general weighting terms. The variable subscripts $p_1$ and $p_2$ on the norms in (5) modify the effective cost function. An appropriate norm should be used based on the assumed statistics of $\|Gx_m\|_{p_2}$ and $\|Fx_m\|_{p_1}$.\textsuperscript{8} The $l_1$ norm is optimal for Laplacian distributions where large tails and outliers are common. The $l_2$ norm is optimal for Gaussian
distributions where the tails decay rapidly and outliers are uncommon. The $l_1$ norm will be selected going forward since in general the magnitude RCS tends toward Laplacian distributions.

5. RESULTS

The proposed processing was applied to the GOTCHA data set which includes eight full circle collections. The circle collects are separated by approximately 0.18 degrees and extend from 43.7 to 45.0 degrees in elevation. The 360 degrees in azimuth was divided into non-overlapping four-degree images resulting in ninety total images. The 2D scene shown in Figure 5(a) was generated by coherently processing all eight vertical collections into the appropriate four-degree segmented azimuth images. Then all ninety magnitude images were combined into a single image. The target of interest is highlighted in Figure 5(a) by a white box. This target is a Ford Taurus station wagon shown in Figure 5(b). The 2D vertical plane that the images lie in corresponds to roughly the ground plane.

The images in Figure 7 and Figure 8 processed all 360 degrees concurrently in a single SOCP. This means the $A$ matrix contained ninety subimages, for illustration Figure 4(a) shows four subimages. The images in Figure 9 processed only 36 degrees concurrently. This requires solving ten SOCPs to cover all 360 degrees in azimuth. Each figure specifies the processing weights $w_1$, $w_2$, and $w_3$ in the caption. There are many ways to divide the scene for processing, which has implications regarding how coherent the scene and target are assumed to be. The method used here was merely selected at random. The type of spatial and aspect filter has a significant affect on the solution. The aspect filter selected was a second order derivate of $[-1 2 -1]$, which gives the frequency response in Figure 6(a). This helps explain the diagonal block pattern seen in Figure 4(b). The spatial filter used was a Laplacian filter and has the magnitude frequency response shown in Figure 6(b).

![Figure 5](http://proceedings.spiedigitallibrary.org/)

(a) (b)

Figure 5. (a) GOTCHA Scene overview. (b) Back-projected reference target.

Figures 7, 8, and 9 modify a single weighting parameter in order to simplify interpretation. Figure 7 modifies only the sparse term $w_3$. Figure 8 modifies only the spatial term $w_3$. And Figure 9 modifies only the aspect term $w_3$. The visual interpretation of Figure 7 is as expected. The heavier the weight used for sparsity the sparser the solution. The spatial weighting is modified in Figure 8 and the results changes appropriately based on the weighting. The heavier the weighting, in this case, the smoother the image. The aspect results shows slightly modified images. Figure 10 examines the individual subimages which cover in azimuth 72 to 108 degrees. This image is composed of the nine 4 degree coherent images that cover the 36 degree extent. Figure 10(a) shows the back-projected subimage data. The glint from the side of the car is the most obvious feature. As the aspect weighting increases the glint lines become cleaner in Figure 10(b) and Figure 10(c). This is exactly what would be expected due to the $l_1$ cost function being employed. This has promising applications to limited aperture collections, and possible ATR.
Figure 6. (a) The aspect filter $F$ frequency response. (b) The spatial filter $G$ magnitude response.

Figure 7. (a) $w_1=10e^{-3}$, $w_2=5e^{-3}$, $w_3=1e^{-3}$. (b) $w_1=10e^{-3}$, $w_2=5e^{-3}$, $w_3=20e^{-3}$. (c) $w_1=10e^{-3}$, $w_2=5e^{-3}$, $w_3=20e^{-3}$.

Figure 8. (a) $w_1=10e^{-3}$, $w_2=1e^{-3}$, $w_3=1e^{-3}$. (b) $w_1=10e^{-3}$, $w_2=2.5e^{-3}$, $w_3=1e^{-3}$. (c) $w_1=10e^{-3}$, $w_2=10e^{-3}$, $w_3=1e^{-3}$.

Figure 9. (a) $w_1=10e^{-3}$, $w_2=5e^{-3}$, $w_3=20e^{-3}$. (b) $w_1=100e^{-3}$, $w_2=5e^{-3}$, $w_3=20e^{-3}$. (c) $w_1=500e^{-3}$, $w_2=5e^{-3}$, $w_3=20e^{-3}$. 
6. CONCLUSION

The formulation derived here has several advantages. The first, is the final problem is convex in the form of an SOCP, which is efficient to solve, guarantees the global optimal solution, and has fast commercial solvers available. Second, the sparse formulation naturally allows the model to scale linearly with each additional pixel instead of exponentially. Any scene size could be processed by appropriately segmenting the scene. The model formulation also naturally extends to compressive sampling and analysis. For example, if a percentage of the pulses were randomly dropped the reconstruction could still be applied. The model would naturally update the expected IPR for each pixel given the actual data being used. The full results could be contrasted against the partial results for comparison and analysis. It is presumable due to the robust nature of the $l_1$ norm that large RCS features could be recovered accurately. Last, the results are demonstrated with both a spatial and aspect filter on actual SAR data as opposed to synthetically generated data. This demonstrates an appropriate robust model that can handle the nonideal peculiarities that synthetic data can often omit.

There are natural extensions of this research and an immediate area of further interest is application to 3D scenes. The preliminary results are encouraging regarding the potential advantages. Another application would be augmenting the model to be parametric. This could be done by incorporating statistics about each pixel. The statistics could be gathered from the back-projected images for each pixel, and its surrounding neighbors, magnitude RCS distribution. This information could then be used to generate individually optimized aspect and spatial filters. This could potentially be done in an iterative manner. The last area of interest is working on combining the spatial and aspect filters in order to understand the potential benefits and direct application to SAR GMTI.

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