Comparisons in the design and implementation of multi-microphone acoustic probes

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Many designs exist for multi-microphones probes used to estimate acoustic active intensity and acoustic energy density. Of these, four microphone cubic designs have found wide use. However, there exist 12 ways to use cubic probes to estimate energy density and 16 ways to estimate intensity. This comparative study is a computational investigation of the errors associated with each design. The frequency range of 0 to 1.4 *ka* is considered. Results are given for only plane wave fields and all angles of incidence are examined. Depending on which quantity is to be estimated (i.e. intensity magnitude, intensity direction, or energy density), a different design is found to perform best. However, the best designs are shown to outperform the other designs by only small amounts.

I. INTRODUCTION

Active intensity and energy density are acoustic energy quantities useful for characterizing sound fields and are used for such applications as sound source localization and active noise control. To calculate these quantities at a given point in space the pressure and particle velocity at that point must be known. Pressure can be measured by a microphone and the particle velocity is typically estimated using the finite-difference technique between multiple microphones.[1] With two microphones the particle velocity can be estimated in one dimension. With three, two dimensions can be calculated and with four or more it is possible to get a complete three-dimensional estimation of the velocity.

Such multi-microphone probes have been in wide use since the 1980s and come in a variety of designs. The most common three-dimensional multi-microphones probes include the four microphone tetrahedral design,[2,3,4] the four microphone cubic design,[5,6] and the six microphone design.[7,8] In this work only the cubic design will be investigated. The cubic design consists of four microphones arranged with one microphone at an "origin" position with the other three microphones equidistant from the first microphone along the three coordinate axes as seen in Figure 1a.

While many multi-microphones probes are simply microphones suspended in space near each other, Elko suggested that the microphones be embedded on the surface of a hard sphere.[9] He found that for a two-microphone case the acoustic scattering off of the sphere had beneficial high-frequency effects in measuring acoustic energy quantities. Locey used this idea for the cubic probe design making a probe as shown in Figure 1b.[10]
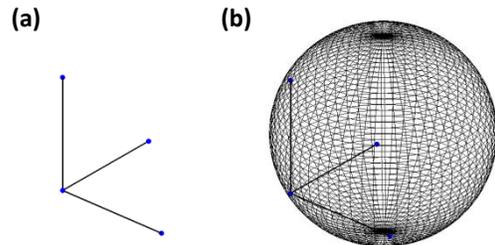

FIG. 1. Suspended (a) and spherical (b) four-microphone cubic probe designs

A thorough study has not been done to investigate whether the "spherical" or "suspended" designs result in more accurate measurements. Also unclear in using cubic probes is how the estimate of the pressure should be estimated. The pressure of the "origin" microphone, the average of the pressures from the four microphones, and a "weighted" average of the pressures favoring the origin microphone have all been used.[6,11,12] There are two ways the particle velocity has been estimated. Commonly it has been estimated by simply taking the finite-difference approximation between each of the two-microphone pairs along the x, y, and z axes thereby estimating an x, y, and z velocity. However, this results in the velocity being estimated at three points in space. To offset this problem, it has been suggested that a first-order Taylor approximation of the velocity be estimated at the centroid of the four microphones by using the finite-difference result of all six two-microphone pairs.[10]

The intensity can also be estimated by considering the cubic probe as three one-dimensional probes. The intensity in each orthogonal direction is calculated using only the pressures measured by the two microphones

along that direction. The total intensity is then the Euclidean norm of the three estimates.

The two ways of estimating the particle velocity combined with the three ways of estimating the pressure and the question of embedding the microphones on a sphere or not leads to 12 total designs for estimating energy density. Considering the probe as three one-dimensional probes leads to four more designs (for a total of 16) for estimating intensity. This work will investigate the errors of each of these designs in measuring the magnitude of active intensity, the angle of active intensity, and the energy density.

II. METHODS

A computational model calculated the measurement errors associated with each of the cubic probe designs. The model assumed that the four microphones were point sensors that were perfectly phase calibrated. For simplicity, the model also only modeled the error that would be seen if the probe was exposed to plane-waves. Results are given from 0 to 1.4 ka (which corresponds to around 6,000 Hz for a 1 inch diameter probe) where k is the wavenumber and a the distance from each microphone to the center of a sphere that is circumscribed by the four points. However, for the spherical probe cases a is defined as greater than the typical radius of the sphere because Elko showed that a 3/2 correction factor was needed to account for the spherical scattering in order to get a correct measurement of the particle velocity.[9] Thus, in order to be able to directly compare the two, ka for the spherical designs is equal to 3/2 times ka of the suspended designs.

An infinite sum is needed to exactly calculate the scattered pressure off a hard sphere.[13] It was found that 25 terms was more than enough for accurate results up to ka=1.4 and so is used here. For the suspended designs, scattering was neglected as it would vary dependent on the size and configuration of the microphones and holders.

The measurement error of any probe design is also dependent on the angle of incidence of the travelling plane wave in relation to the probe. Thus, the average error seen over all incidence angles as well as the maximum error are used for comparison. Certain angles of incidence corresponded to underestimation of the acoustic quantities while others to overestimation. As both are undesirable, only the magnitudes of the errors were examined. Therefore the average error was a measure of how much error (be it positive or negative) one would on average expect if the probe was randomly oriented in a sound field while the maximum error was the worst possible error that would be seen.

As intensity is a vector quantity, both magnitude and direction error were calculated. The magnitude error was expressed in dB according to the equation Error (dB) = 10*log($I_{estimated}$/I_{exact}) while the errors in direction were given in degrees.

Energy density, a scalar, was expressed in dB error and calculated in the same way as intensity magnitude error.

III. INTENSITY MAGNITUDE ERRORS

Each probe's performance in measuring the magnitude of acoustic intensity was plotted. Figure 2 shows the average errors of the suspended designs. In this all following figures the left graph corresponds to the particle velocity being estimated at three locations in space (hereafter referred to as "Three Points") whereas the right graph corresponds to the particle velocity as estimated at the origin by the first-order Taylor approximation (hereafter referred to as "Origin"). Then within each graph for intensity the three estimations of pressure plus the result of considering the probe as three one-dimensional probes are shown.
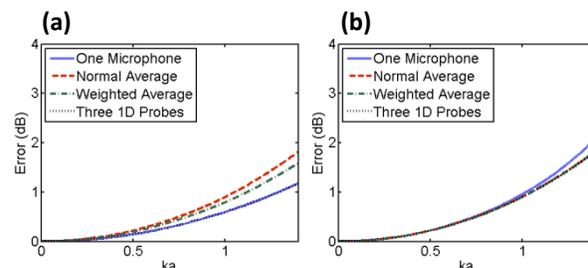

FIG. 2. Average intensity magnitude errors for suspended designs with particle velocity estimated at three points (a) and estimated at the origin (b).

For the Three Points designs we see that Figure 2a shows the lowest average error coming from the One Microphone design. However, for the Origin designs the One Microphone design gives the worst error. In general, we see lower average errors from the Three Points designs than from the Origin designs.

Figure 3 shows the maximum errors for the suspended designs. The One Microphone and Three 1D Probes designs have the lowest maximums for Three Points designs but the highest for the Origin designs, similar to situation for the average errors. And again, the Three Points designs outperform slightly the Origin designs.
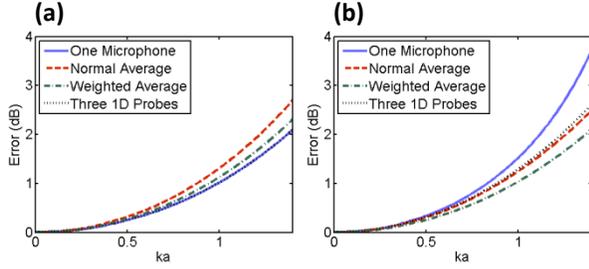
FIG. 3. Maximum intensity magnitude errors for suspended designs with particle velocity estimated at three points (a) and estimated at the origin (b).

Results for the spherical designs are shown in Figures 4 and 5 and show similar results with the best combination being the One Microphone pressure estimate and the Three Points velocity estimate. However, in all cases the differences between the designs at the highest frequency (ka=1.4) is less than one dB.
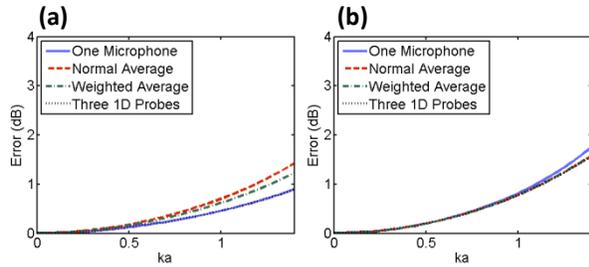

FIG. 4. Average intensity magnitude errors for spherical designs with particle velocity estimated at three points (a) and estimated at the origin (b).
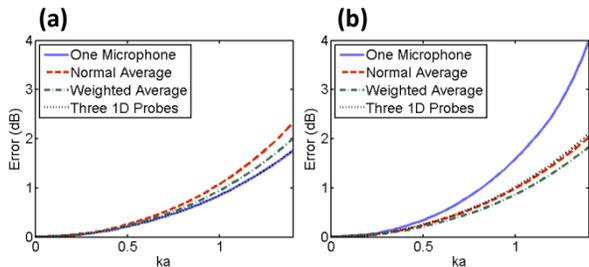

FIG. 5. Maximum intensity magnitude errors for spherical designs with particle velocity estimated at three points (a) and estimated at the origin (b).

Comparing the suspended to spherical results reveals that, overall, the spherical designs perform slightly better than their suspended counterparts. For intensity magnitude the scattering effects of the hard sphere are beneficial. The best design for estimating intensity magnitude is shown to be the One Microphone, Three Points, spherical design.

IV. INTENSITY DIRECTION ERRORS

The errors in estimating the intensity direction are important because the directions errors tend to render a probe unusable at a lower upper-frequency limit than do the magnitude errors. The errors are given in degrees, referring to difference between the three-dimensional angle of incidence of the plane wave and the angle estimated by the probe. Suspended design errors are plotted in Figures 6 and 7.
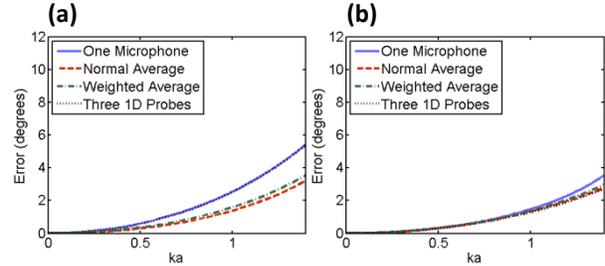

FIG. 6. Average intensity direction errors for suspended designs with particle velocity estimated at three points (a) and estimated at the origin (b).
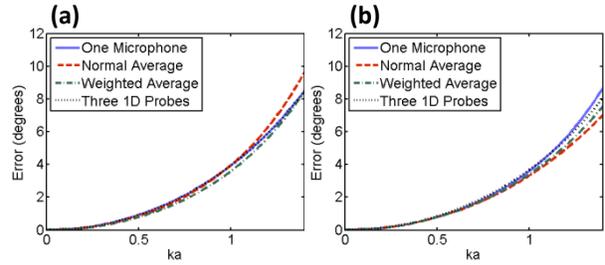

FIG. 7. Maximum intensity direction errors for suspended designs with particle velocity estimated at three points (a) and estimated at the origin (b).

All designs show similar results, except that the One Microphone design shows the largest error in most all cases. In contrast to the intensity magnitude results, the Origin designs outperform the Three Points designs.

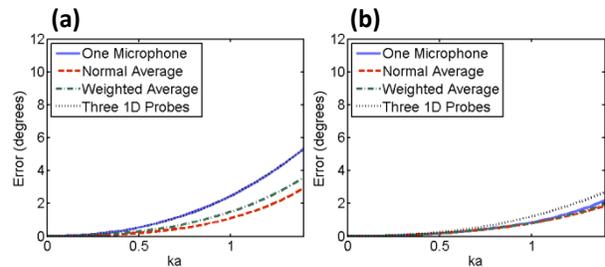The direction errors for spherical designs are shown next in Figures 8 and 9.

FIG. 8. Average intensity direction errors for spherical designs with particle velocity estimated at three points (a) and estimated at the origin (b).
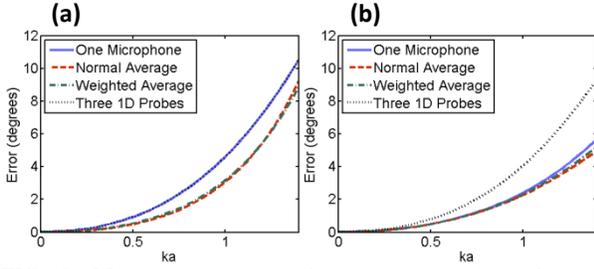
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FIG. 9. Maximum intensity direction errors for spherical designs with particle velocity estimated at three points (a) and estimated at the origin (b).

For both Three Points and Origin designs, the Normal and Weighted Average have the lowest error. The Origin designs show a few degrees less error than the Three Points designs. Comparing all four figures, the spherical designs are all at least as good as or better than the corresponding suspended ones. The best design for estimating intensity direction is the Normal Average, Origin, spherical probe design.

V. ENERGY DENSITY ERRORS

As opposed to intensity, energy density is not a vector quantity so only the magnitude results are given and the Three 1D Probes design does not apply. Figure 10 shows the average errors and Figure 11 the maximum errors for the suspended designs.
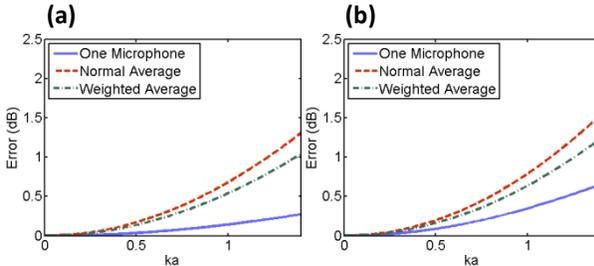

FIG. 10. Average energy density errors for suspended designs with particle velocity estimated at three points (a) and estimated at the origin (b).
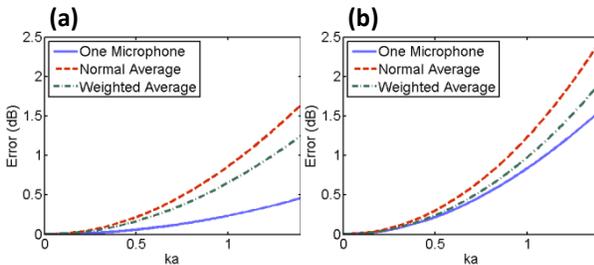

FIG. 11. Maximum energy density errors for suspended designs with particle velocity estimated at three points (a) and estimated at the origin (b).

The One Microphone design clearly has the lowest error. This is as expected because for a suspended case the magnitude of the One Microphone pressure estimate is perfect under our assumptions. And since energy density is calculated from the magnitude of the pressure (as opposed to using any pressure phase information), the best estimate is obtained from just using one microphone.

For particle velocity estimation, the Three Points method designs have lower error.
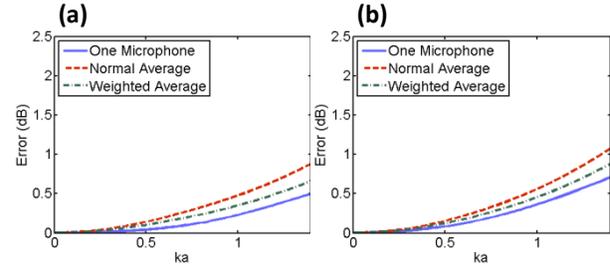

FIG. 12. Average energy density errors for spherical designs with particle velocity estimated at three points (a) and estimated at the origin (b).
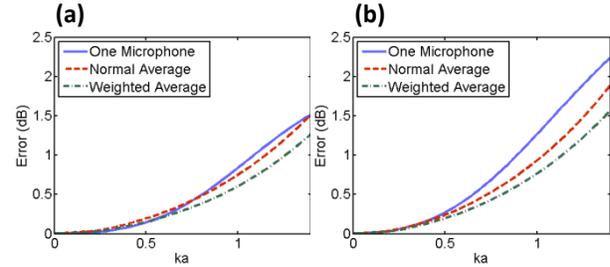

FIG. 13. Maximum energy density errors for spherical designs with particle velocity estimated at three points (a) and estimated at the origin (b).

For spherical designs the One Microphone design has the best average errors but as there are some incidence angles that create a large overpressure on the origin microphone, it has the worst maximum errors. As with the suspended designs, the Three Points method of estimating velocity outperforms the Origin method.

There is no clear trend in whether the designs are better in a suspended or a spherical configuration. The best overall design for estimating energy density is the One Microphone, Three Points, suspended design.

VI. CONCLUSIONS

Depending on the quantity of interest, a different design is calculated to have the lowest measurement error. However, most all of the intensity magnitude and

energy density errors were within one to two dB of each other at the highest frequency considered. This represents a fairly negligible amount of error. For intensity direction, the best designs were about four degrees better than the worst designs, which is also close to negligible, but more significant than the magnitude error spread. Thus the Normal Average, Origin, spherical probe design is concluded to be the most desirable design. However, its superiority is found to be fairly insignificant in the frequency range considered.

VII. ACKNOWLEDGMENTS

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