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Amitrajeet A. Batabyal
Utah State University

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DEVELOPING COUNTRIES AND ENVIRONMENTAL PROTECTION: THE EFFECT OF BUDGET BALANCE AND POLLUTION CEILING CONSTRAINTS

by

AMITRAJEET A. BATABYAL

Department of Economics
Utah State University
3530 University Boulevard
Logan, UT 84322-3530

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Amitrajeet A. Batabyal, Assistant Professor
Department of Economics
Utah State University
3530 University Boulevard
Logan, UT 84322-3530

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DEVELOPING COUNTRIES AND ENVIRONMENTAL PROTECTION: THE EFFECT OF BUDGET BALANCE AND POLLUTION CEILING CONSTRAINTS

Amitrajeet A. Batabyal

ABSTRACT

I analyze the problem faced by an asymmetrically informed supranational government authority (SNGA) with limited financial resources who wishes to design an International Environmental Agreement (IEA). The SNGA cannot contract directly with polluting firms in the various LDCs, but he must deal with such firms through their governments. I study this tripartite hierarchical interaction and focus on the properties of the optimal ex post contracts (IEAs), which can be implemented by the SNGA, in turn, in the case where governments and firms in each nation do not collude and then in the case where governments and firms do collude. I find that the monetary transfers necessary to induce optimal behavior by governments and firms are not very sensitive to the presence of collusion. However, because the optimal contracts satisfy budget balance, and because there is a ceiling on the amount of pollution reduction that an IEA can require, the level and pattern of pollution abatement are never ideal. My analysis suggests that IEAs are not inherently doomed due to a basic monitoring and enforcement problem arising from national sovereignty. However, the success of IEAs is fundamentally contingent on the funds available for environmental protection and the pollution reduction ceiling negotiated by the SNGA and the LDC government.

JEL Classification: D62, D82, Q25

Key words: environmental, agreement, LDCs, budget, ceiling
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1. Introduction¹

With the passage of time, it has increasingly been recognized that environmental protection is an international issue. As noted by Bernauer (1995, p. 354), the scope and significance of this issue have been amply demonstrated by the events of the 1992 Earth Summit in Rio. At this Summit, it became clear that if the northern countries of the world wanted "... the environment to be secured for future generations, [then they would] have to radically assist the South in choosing a different road to development than the one they [had] currently [been] traveling on" (Rogers 1993, p. 27). Indeed, to combat the twin evils of poverty and environmental degradation, developing countries (LDCs) have demanded the transfer of resources and technology from developed countries. In such a contentious setting, the success or failure to protect the environment will depend crucially on the ability of international institutions to craft effective international environmental agreements (IEAs).² Given this, the key question becomes "How can international institutions, which necessarily respect the principle of state sovereignty, contribute to the solution of difficult global problems?" (Keohane, Haas, and Levy 1993, p. 6). This is the central question that I propose to analyze in this paper.

¹This paper has benefited from the comments of seminar participants at the College of William and Mary and at the University of California, Berkeley; three anonymous referees have also assisted me in improving the quality of this paper. The usual disclaimer applies.

²In this paper I shall use the terms IEA and contract interchangeably.
On the academic front, researchers have only recently begun to study issues relating to global environmental protection in a systematic manner. As a result, many specific questions remain unanswered. What kinds of pollution abatement patterns can one expect to observe in economic environments in which an asymmetrically informed supranational governmental authority (SNGA) contracts with governments and polluting firms in individual countries? What kinds of monetary transfers will be necessary to get sovereign nations to voluntarily participate in IEAs? How does the SNGA's inability to monitor pollution abatement in the individual countries affect the contract design question? What are the effects of pollution reduction ceiling constraints? Finally, how does the limited availability of funds affect the SNGA's IEA design question? These are the specific questions that I shall address in this paper.

Although my analysis is, in principle, applicable to any country, the hierarchical interaction that I shall analyze is particularly relevant to LDCs; as such, the reader should note that it is these countries that I have in mind in all of the subsequent analyses.\footnote{The countries I have in mind are those which would be eligible to receive monetary transfers under the Global Environmental Facility's (GEF) standard of per capita income of $4,000 or less. For more details, see Rogers (1993, p. 155).} I now discuss the nascent literature on IEAs and then move on to discuss my model in detail.

2. International Environmental Agreements: A Brief Synopsis

Barrett (1994) has modeled IEAs as games between different countries. While Barrett's analysis is not in the design framework, Barrett makes the important point that for IEAs to work at all, they must be self-enforcing. However, this research has not addressed fundamental questions arising from asymmetrically held information, and the heterogeneity of the contracting countries.
Hoel (1992) has analyzed an IEA in which countries uniformly reduce their pollution emissions. Hoel argues against the institution of such uniform emissions reduction policies in international agreements. He shows that other policies yield higher levels of global welfare.

Shogren, Baik, and Crocker (1992, hereafter SBC), and Sandler and Sargent (1995, hereafter SS) have addressed the question of the minimal number of countries needed to sustain an IEA. In a multiplayer strategic setting, SBC show that countries will sometimes join IEAs because the expected gains from such action outweigh the gains from not joining. However, beyond a critical threshold value, some countries will prefer to free ride and not join the IEA, whereas the IEA members will want nonparticipants to join. SS (p. 152) show that the attainment of international coordination by a “minimal-sized group” is fundamentally dependent on “... how individual pollution activities add to the total pollutants experienced ... [by nations].” While these papers have certainly advanced our understanding of some aspects of “... the multi-faceted design ... problem” (Black, Levi, and de Meza 1993, p. 281), many other important questions—which I discussed in section 1—remain unanswered. As such, I now discuss my modeling approach to the IEA design question.

I shall model the international environment as a multiforked, three-tiered hierarchy. Occupying the topmost tier of the hierarchy is the relevant international institution or SNGA. This SNGA could be an organization such as the World Bank, or the Commission on Sustainable Development (CSD), created in Agenda 21 at the Rio Earth Summit. The second and third tiers of the hierarchy consist of the government and a representative polluting firm in each LDC. Each fork

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4Specifically in its role as an administrator of the Global Environmental Facility (GEF).
of the hierarchy corresponds to a single LDC, and there are $N$ such countries. Three-tiered hierarchies have been studied by Tirole (1986, 1988) and by Kofman and Lawarree (1993), among others. These researchers have studied the effects of potential collusion between the various players in their three-tier hierarchies. However, to the best of my knowledge, the problem of hierarchical contracting with budget balance and pollution reduction ceiling constraints has not been studied to date.

As such, I shall build on and apply the theory of hierarchies to study \textit{ex post} contracting between the SNGA, national governments, and polluting firms in the various LDCs. The rationale for the actual contracting stems from issues including, but not limited to, the harmful atmospheric effects of sulphur and/or nitrogen emissions. The incidence of pollution may be domestic or transboundary. The key element of uncertainty stems from the SNGA's lack of knowledge about the pollution abatement technology/capability available in each country. Whereas the firm in the LDC always knows its technology and the government does in some states of nature, the SNGA is never privy to this information. The random variable denoting the private information about pollution abatement capability is \textit{uncorrelated} across countries. This rules out the possibility of the SNGA engaging in relative performance evaluation. Because most LDCs are very heterogeneous, and because it is difficult to imagine a practical instance in which a SNGA would want to design contracts involving relative performance evaluation, this assumption of uncorrelatedness appears not to be

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5The reader will note that in this modeling scheme, I have conferred, on the SNGA, the role of principal. As such, there is a distinct asymmetry in the assumed power of the SNGA as opposed to that of governments and firms. Given that I am interested in LDCs that typically have limited bargaining power in their dealings with international organizations owing to the fact that their monetary contributions to the budgets of such organizations are minimal, this hierarchical modeling scheme appears to be appropriate. For more on the power of SNGAs over LDCs, see Mosley, Harrigan, and Toye (1991).

6See Crane (1993) and Paarlberg (1993) for a discussion of the relevance of international institutions when the incidence of an environmental externality is domestic.
restrictive. In other words, my analysis holds for any finite set of countries, with the
SNGA/government/firm interaction in one country being independent of the SNGA's dealings with
some other country. As such, without loss of generality, I shall focus on an arbitrary country, say
country $j$, in the finite set of countries. The SNGA's task is to design an incentive-compatible and
collusion-proof budget-balanced contract which will lead to optimal pollution abatement in each
country.

The rest of this paper is organized as follows. In section 3, I describe the model in detail, and
I study the properties of the first best optimum. In section 4, I study the above-described three-tiered
hierarchy with no collusion by the representative firm and the government. In section 5, I study the
case of collusion by the government and the polluting firm. In both sections, I analyze ex post
contracts which can be implemented by the SNGA in a Bayes-Nash equilibrium.

The reasons for wanting to study collusion between the polluting firm and the LDC
government are rather obvious. The government and the firm receive monetary transfers from the
SNGA for their roles in abating pollution. Further, both these players know that the SNGA cannot
monitor their activities owing to sovereignty or, for that matter, enforce the terms of the IEA in the
event of a contractual breach. As such, there will be circumstances in which there are incentives for
the government and the firm to collude to maximize the transfers received from the SNGA. Thus,
an important part of this paper will consist of analyzing collusion-proof contracts.

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7The exact nature of these roles is described in section 3.

8See Peterson (1993) for a discussion of some practical instances of possible government/firm collusion in an
international setting.
3. The Theoretical Framework

3a. Description of the Model

Subscripts \(i=1,2,3,4\) will refer to the state of nature, and superscripts \(j=1,\ldots,N\) will refer to the country. Let \(\theta\) denote the uncertainty about abatement technology/capability that is currently available; \(\theta\) has binary support \([\theta^L, \theta^H]\), where \(0<\theta^L<\theta^H\), and \(\Delta \theta=\theta^H-\theta^L\). I shall refer to \(\theta^L\) as the low abatement capability parameter and to \(\theta^H\) as the high abatement capability parameter.

The risk-averse firm produces clean air, where output and value are denoted by \(x_\theta, x \in \mathbb{R}_+\). The firm chooses a level of pollution abatement \(a \in \mathbb{R}_+\), and \(\theta\) denotes the uncertainty about the pollution abatement technology. The firm’s cost of abatement is \(g(a)\), where \(g'(\cdot)>0\), \(g''(\cdot)>0\), and \(g(0)=0\). The firm has a differentiable net payoff from abatement function \(B[T_r,g(a)]\) with \(\partial B[\cdot]/\partial T_r \in (0,\infty), \forall T_r, T_r \in \mathbb{R}\), is the state \(i\) monetary transfer made by the SNGA to the firm for abating pollution. The firm’s reservation payoff is \(B_r\), where \(B_r B[T_r]\) and \(T_r\) is the reservation transfer. \(B_r\) and \(T_r\) are common knowledge.

The LDC government is risk-averse. It has a strictly concave and differentiable utility function \(V(G_i)\), where \(G_i \in \mathbb{R}_+\), is the state \(i\) monetary transfer made by the SNGA to the government for its role in participating in the IEA. Note that the government’s utility is independent of the level of clean air produced by the firm. The government’s reservation utility \(V_r V(G_r)\), where \(G_r \in \mathbb{R}_+\), is the

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9In my formulation, the production of clean air is uncertain; the cost of abatement is not. Further, uncertainty affects production additively. Alternately, one could make the cost of abatement uncertain; more specifically, one could make the uncertainty term enter the relevant function multiplicatively. While additive uncertainty in the cost function will not materially affect my results, multiplicative uncertainty in either the production function or the cost function will certainly affect my results. To see why, note that, while additive uncertainty leads to first-order conditions with no random variable(s), multiplicative uncertainty results in first-order conditions with the random variable(s). As a result, without additional assumptions about the magnitudes of these random variables in the various states or the magnitudes of the probabilities themselves, it is not possible to obtain concrete results.
reservation transfer, and \( V'(G_i) \in (0, \infty) \), \( \forall G_i \). The government performs two important functions. First, in concert with the SNGA, it negotiates a ceiling, \( \hat{x}_j \), on the amount of abatement that can contractually be required of the firm in its country. This \( \hat{x}_j \) is assumed to be the endogenous outcome of a bargaining game between the SNGA and the LDC government, the details of which I shall not be concerned with. For the purpose of this paper, I shall require that the SNGA treat this pollution ceiling \( \hat{x}_j \) as an exogenous constraint.\(^{10}\) In practical terms, the existence of this constraint means that although the LDC government recognizes the value of participating in an IEA, it is unwilling to give free rein to the SNGA to dictate pollution abatement levels to domestic firms. Second, by employing a monitoring device, the government receives a signal \( s \) from the firm regarding its private information and then it (the government) sends a report \( r \) to the SNGA indicating what it observed about the firm's pollution abatement capability parameter.\(^{11}\) In some states of nature, this monitoring device malfunctions, and, hence in these states, the government will be unable to provide the SNGA with a useful report. Upon receiving \( r \), the SNGA offers the government a transfer \( G_i \in \mathbb{R} \). The reader should note that making reporting a key government function is consistent with the government/SNGA interaction proposed for one specific SNGA, namely, the Commission on Sustainable Development. As noted by Rogers (1993, p. 310), a key aspect of this interaction involves the "... Commission's ... considering information provided by governments ... ."

The SNGA is risk-neutral and has a welfare function defined over clean air which takes the form \( U = \sum_{j=1}^{N} (x_j - \theta_j - G_j - T_j), \), \( j=1,...,N \), where the index \( j \) runs over \( N \), the total number of countries. The

\(^{10}\)Although I am not modeling the bargaining game explicitly, the assumption that \( \hat{x}_j \) is the endogenous outcome of this game means that \( \hat{x}_j \) is not set at an unreasonably high or low level. This in turn means that these pollution ceiling constraints will typically bind in equilibrium. See the appendix for more details.

\(^{11}\)Since the main objective of this paper is not to study domestic monitoring, I shall assume that the use of this monitoring device is costless.
quantity of clean air produced by the firm in country \(j\) is \(x^i.a^r.j.\theta^j\). As stated, the SNGA's welfare is the difference between total clean air and the sum of government and firm transfers. In the rest of the paper, I shall suppress the country superscript; it is understood that the focus is on country \(j\).

The SNGA designs the contract which he offers to the government and the firm. The contract can only be conditioned on what the SNGA actually observes, i.e., the government's report, \(r\), and the firm's production of clean air, \(x\).

In each country, there are four states of nature, each state occurring with probability \(p > 0\), where \(\Sigma_{w} p_r^1\). The SNGA, the government, and the firm sign the contract holding asymmetric information about \(\theta\). The contract is *ex post*, i.e., it is signed after the firm observes the actual realization of \(\theta\). The firm always observes \(\theta\) before choosing its abatement level. The government, on the other hand, may or may not observe the firm's private information. This depends on whether the government's monitoring device functions or malfunctions. In other words, the government's signal, \(s\), may or may not be informative. I can now characterize the four states as follows:

* State 1: The firm and the government both observe \(\theta^L\).
* State 2: The firm observes \(\theta^L\) and the government observes nothing.
* State 3: The firm observes \(\theta^H\) and the government observes nothing.
* State 4: The firm and the government both observe \(\theta^H\).

In state 1, the firm and the government both observe the low abatement capability parameter. The government's monitoring device works and, hence, yields useful information. In state 2, the firm observes the low abatement capability parameter but the government observes nothing. In this state, the government's monitoring device malfunctions. In state 3, the firm observes the high abatement
capability parameter, and, once again, the government's monitoring device malfunctions. Finally, in state 4, the firm and the government observe the high abatement capability parameter.\footnote{\textsuperscript{12}}

Before the SNGA/government/firm game is played, the SNGA and the LDC government negotiate the pollution reduction ceiling $x$. Next, the three-player game is played. The timing of this game is as follows. First, the firm observes the actual realization of $\theta$ and the government receives its signal $s$. Second, the SNGA offers the contract to the government and the firm. Third, the firm chooses $a$. Fourth, clean air $x$ is produced by the firm and the government sends its report $r$ to the SNGA. Fifth, the SNGA compensates the government and the firm with transfers $G(x,r)$ and $T(x,r)$.

In the remainder of this paper I shall assume that the SNGA can verify the veracity of the government's report $r$. In other words, if the government's signal $s$ is noninformative, then the corresponding report $r$ reflects this fact, and the SNGA can verify that the true facts are indeed as they have been reported. In symbols, $s \rightarrow r \rightarrow 0$. On the other hand, to keep the SNGA's design problem interesting and to allow for the possibility of government/firm collusion, I permit the government to lie and report that its signal is noninformative when in fact such is not the case.\footnote{\textsuperscript{13}} That is, $s \rightarrow r \in \{0,0\}$.

\footnote{\textsuperscript{12}I have assumed that the government always knows when its monitoring device malfunctions. This formulation keeps the SNGA's problem interesting, while maintaining analytical tractability. More involved formulations, in which the government does not know for sure when its monitoring device has malfunctioned, are possible. In these formulations, it becomes necessary to introduce a second random variable into the analysis in order to explicitly model the government's uncertainty about the reliability of the information received by it. The introduction of this second random variable increases the number of states and, hence, the number of constraints on the SNGA's overall problem. In this way, these alternate formulations significantly complicate the underlying analysis. Further, these additional modeling features make it virtually impossible to obtain unambiguous results. Also see footnote 13.}

\footnote{\textsuperscript{13}The reader will note that I have restricted the government's message space in certain states. Specifically, the government can lie only in states 1 and 4. The government can also announce the wrong state, but, in my setup, making such an announcement is equivalent to obtaining a noninformative signal. While, in principle, this restriction can be relaxed by allowing for an expanded range of governmental reporting options, from a practical standpoint, such an action would make it exceedingly difficult to obtain concrete results. This is because relaxing the above restriction would lead to an increased number of states and, hence, to more constraints on the SNGA's overall optimization problem.}
This completes the description of my model. I now consider the benchmark case in which perfect information is acquired by the SNGA.

3b. The First Best Optimum

In this case, the SNGA observes $\theta$ and the firm's pollution abatement choice. When this happens, the SNGA bypasses the government and contracts with the firm directly. The government receives its reservation transfer $G_r$ and, hence, its reservation utility $V_r$ in all states. The SNGA solves $\max_{a\theta-g(a)}$. The first-order necessary condition requires that $g'(a_\theta)\geq 1, \forall \theta$. In other words, in the first-best optimum, the marginal cost of abatement is set equal to the marginal welfare from abatement. The optimal level of abatement $a_\theta$ is the same in all states. The firm receives a transfer for undertaking pollution abatement, which is independent of the state of nature. This transfer equals $T_r g_\theta$, where $T_r$ is the reservation transfer and $g_\theta = g(a_\theta)$.

I now discuss the more interesting cases in which the SNGA cannot determine the realization of $\theta$ or the actual abatement undertaken by the firm.

4. The No Government/Firm Collusion Case

In this section, I shall disallow the possibility of collusion between the government and the firm. When the government is paid its reservation transfer $G_r$, it obtains its reservation utility $V_r$ and, hence, it is fully insured. Further, since I am not allowing for the possibility of collusion between the government and the firm and because the SNGA can verify the government's report, by paying $G_r$, the SNGA obtains the government's information at least cost. In terms of the design of the main contract, this means that the three-tiered hierarchy effectively reduces to a two-tiered hierarchy in
which the government plays a passive role.

In this setting, the SNGA's problem is to solve

$$\max_{(T,\theta)} \sum_{\nu i} P_i(a_i, \theta, T_i)$$

(A)

subject to (A1) $B[T_i g(a_i)] \geq B, \forall i$, (A2) $x \geq a_i \theta, \forall i$, (A3) $T_3 g(a_3) \geq T_2 g(a_2) - \Delta \theta$, (A4) $T_2 g(a_2) \geq T_3 g(a_3) - \Delta \theta$, and (A5) $\hat{M} \sum_{\nu i} (\mu_i^j, T_i), \forall i$.

The four constraints in (A1) are the ex post participation constraints. They tell us that it must be individually rational for the firm to contract with the SNGA in every state. This stems from the fact that in this international setting, the SNGA cannot compel the firm to abate pollution. The four constraints in (A2) are the SNGA/government-negotiated “ceiling on pollution reduction” constraints. They tell us that the SNGA cannot, in any state, design a contract which requires that the firm abate pollution in excess of the agreed upon ceiling $x$. Constraints (A3) and (A4) are the incentive compatibility constraints. These constraints arise because the SNGA has imperfect information about $\theta$ in these two states. These are also the states in which the government's signal $s$ is noninformative. Constraint (A3) says that in state 3, the firm should not claim that the state is actually 2. Similarly, (A4) tells us that in state 2, the firm should not claim that the state is actually 3. Finally, (A5) denotes the SNGA's budget constraints. They tell us that irrespective of state, the total sum of transfers paid to the government and the firm in the various countries cannot exceed the SNGA's available budget $\hat{M}$, for environmental protection. I can now solve the SNGA's problem (A), with constraints (A1)-(A5). I am led to

\[14\] I shall not discuss the manner in which the SNGA raises funds. One possibility would be to conform to the text of the Rio Earth Summit agreement known as Agenda 21. According to this agreement, developed countries are supposed to contribute 0.7% of their GNP for the purposes of environmental protection. For more details, see Rogers (1993, pp. 151-160).
Theorem 1: The optimal contract in the three-tiered hierarchy is equivalent to the optimal contract in a two-tiered hierarchy in which (i) the SNGA obtains the government's information at least cost, (ii) the government's reward equals $G_r$ in all states, (iii) the abatement levels satisfy $a_i \geq a_j \quad \forall i$, and $A_1g'(a_1)A_2g'(a_2)A_3g'(a_3)A_4g'(a_4)$, (iv) the payoffs to the firm satisfy $T_T T_{rg}(a_i), \quad \forall i$, and (v) at the optimum, all the constraints except (A4) bind.

Proof: See the Appendix.

Theorem 1 describes the level and pattern of pollution abatement that one may expect to observe in my stylized $N$ country world in which the SNGA contracts with the government and the firm in each LDC independently. First, parts (i) and (ii) tell us that because the SNGA bypasses the government, irrespective of state, the government earns only its reservation transfer. This is as we would expect. Because the government’s reporting task is essentially irrelevant in this context, the government does not earn any informational rents from its private information. Alternately put, the government is paid the very minimum that will insure that it participates in the SNGA’s incentive scheme.

Second, part (iii) tells us that the level of pollution abatement is never first best, and that the optimal contract equalizes the weighted marginal cost of abatement in states 1, 3, and 4. That is, $A_1g'(a_1)A_2g'(a_2)A_3g'(a_3)A_4g'(a_4)$ holds, where $A_1, \ldots, A_4$ are weights.\footnote{See step 7 of the proof for the exact representation of these weights.} The level of abatement is never first best because the budget constraints \textit{and} the pollution reduction ceiling constraints bind at the optimum. It is important to note that this result is not due to the existence of the ceiling constraints \textit{per se}. Even if these constraints did not exist, the above result would still hold because the budget constraints bind in equilibrium. From the proof of Theorem 1, it is clear that the attainment of the
first-best level of abatement is closely linked to the SNGA's shadow value of funds and to the shadow value of the ceiling constraints. The larger these shadow values, i.e., the larger \( \gamma_i \) and \( \delta_i \), the greater the distortion between \( a_i \) and the levels of abatement specified by the contract. Indeed, it is easy to verify—see Step 6 of the proof—that as \( \gamma_i \to 0 \) and \( \delta_i \to 0 \), for \( i \in \{2, 3\} \), \( a_i \to a^* \).

Intuitively, we would expect \( a_1 - a_3 - a_4 \) to hold because the SNGA has complete information about \( \theta \) in states 1 and 4. This notwithstanding, Theorem 1 tells us that the budget balance constraints and the pollution ceiling constraints together drive a wedge between the first-best level of abatement and the level of abatement that the SNGA can contractually require of the firm in these two states. Given that the SNGA cannot require the first-best level of abatement in states in which he does have complete information, it should not be surprising that the SNGA is unable to require that clean air be produced at the first-best level in states 2 and 3. These are the states in which he has incomplete information about \( \theta \). In these two states, the SNGA has to make sure that the firm does not abate pollution in state 3 at the level that is appropriate for state 2, and vice versa. As such, the budget balance constraints, the pollution ceiling constraints, and the requirements of incentive compatibility preclude the SNGA from requiring that \( a_2 - a_3 - a_4 \), hold in equilibrium.

Third, parts (iv) and (v) tells us that at the optimum, all the \textit{ex post} participation constraints bind. This means that the optimal contract offered by the SNGA must respect the fact that the firm cannot be compelled to abate pollution, if doing so would be individually irrational. Part (v) also tells us that all the pollution reduction ceiling constraints bind at the optimum. Intuitively speaking, these constraints bind because the SNGA's welfare function is linear and increasing in the level of abatement. As such, the SNGA will always want to set the level of abatement at its upper bound.
In this case, the upper bound is given by the pollution ceiling constraints.\textsuperscript{16}

The fact that these ceiling constraints bind means that the LDC government’s negotiations with the SNGA are critical to the nonattainment of the first best level of abatement. In particular, because $x$ is the result of negotiations between the SNGA and the LDC government, we can think of the level of $x$ as a measure of the negotiating power of the government. Thus, from a welfare perspective, a high $x$ implies a higher level of welfare for the SNGA and a lower payoff for the polluting firm. This is because the SNGA will always want a high $x$ as its welfare function is increasing in the level of abatement. On the other hand, the LDC government will typically want a low $x$ because the government is representing the polluting firm in its country, and this polluting firm’s net payoff from pollution abatement is decreasing in the level of abatement. In specifying the levels of pollution abatement, the contract described in Theorem 1 optimally accounts for the conflicting interests of the SNGA and the polluting firm.

I now proceed to consider the effects of government/firm collusion on the optimal \textit{ex post} contract designed by the SNGA.\textsuperscript{17}

\section*{5. The Government/Firm Collusion Case}

Recall that because countries are sovereign, the SNGA is unable to either monitor the actions of the government and the firm or enforce the terms of the IEA in the event of a contractual breach. Since the SNGA can never acquire the firm's private information and must rely on the government's

\textsuperscript{16}See footnote 10 as well.

\textsuperscript{17}For more on the regulation of pollution with asymmetric information in a closed economy, see Batabyal (1995) and the many sources cited therein. Also see Demsiki and Sappington (1987).
report to design the optimal contract, an efficient contract must not only be individually rational and incentive compatible but it must be collusion-proof\(^8\) as well.

I shall model collusion between the government and the firm as follows. Before the resolution of the uncertainty regarding abatement capability, the firm and the government sign a secondary contract which entails the offer and acceptance of a bribe from the firm to its government. Naturally, this secondary contract is *unobservable* by the SNGA. The bribe \(b(\cdot,\cdot,\cdot)\), can only be conditioned on what the firm and the government both observe, i.e., the government's report \(r\) and clean air \(x\). With the offer and acceptance of this bribe, the firm's total transfer becomes \(\{T(\cdot)-b(r,x)\}\) and the government's total transfer becomes \(\{G(\cdot).b(r,x)\}\). I shall not be concerned with the question of how the surplus from the bribe is divided. For my purpose, it is only necessary to stipulate that the bribe is actually paid by the firm to its government.

Collusion by the firm and the government fundamentally alters the incentives of the various parties and, as we shall see, the nature of the optimal contract that can be implemented by the SNGA. To see why the firm might want to bribe its government in my four-state world, consider state 4. In state 4, the government is indifferent between reporting that it has observed \(\theta^H\) and reporting that it has observed \(\theta^O\). However, the firm would prefer that the government report \(0\). This is one instance in which a clear rationale exists for the firm to bribe its government.

In order to formulate and solve the SNGA's problem when there is collusion, I shall follow Tirole (1986, pp. 192-197; 1988, pp. 461-462).\(^{19}\) Tirole's method involves imposing constraints in addition to the usual participation and incentive compatibility constraints. These additional

\[^{18}\text{See footnote 8 as well.}\]

\[^{19}\text{For a somewhat different approach to modeling collusion, see Kofman and Lawarree (1993).}\]
constraints are designed to preclude government/firm collusion and, hence, make the main contract collusion-proof.

I can now formulate the SNGA’s problem. The SNGA solves

$$\max_{(\bar{a}_i, \bar{T}_i)} \sum_{i=1}^{n} p_i (a_i \cdot \theta_i - \bar{G}_i - \bar{T}_i)$$

subject to (A1)-(A5) with $T_i$ replaced with $\bar{T}_i$, (B1) $\nu(\bar{G}_i) \geq \nu_r$, (B2) $\bar{G}_1 + \bar{T}_1 - g(a_1) \geq \bar{G}_2 + \bar{T}_2 - g(a_2)$, (B3) $\bar{G}_4 + \bar{T}_4 - g(a_4) \geq \bar{G}_3 + \bar{T}_3 - g(a_3)$, (B4) $\bar{G}_3 + \bar{T}_3 - g(a_3) \geq \bar{G}_2 + \bar{T}_2 - g(a_2 - \Delta \theta)$, and (B5) $\bar{G}_2 + \bar{T}_2 - g(a_2) \geq \bar{G}_3 + \bar{T}_3 - g(a_3, \Delta \theta)$.

The constraints in (B1) are the government's participation constraints. Constraints (B2) and (B3) are the core collusion constraints. Recall that in states 1 and 4, the government's signal $s$ is informative. In these two states, the government can hide this fact. Given this, constraints (B2) and (B3) tell us that should the firm bribe its government, then the total sum of the transfers less the cost of pollution abatement in states 1 and 4 cannot be less than the corresponding totals in states 2 and 3, respectively. Constraint (B4) tells us that the government should not be able to bribe the firm to abate—in state 3—at the level that is appropriate for state 2. Similarly, (B5) tells us that the government should not be able to bribe the firm to claim that the state is 3 when it is 2. Solving the SNGA's problem (B) subject to (A1)-(A5), and (B1)-(B5), I can state

**Theorem 2:** The optimal contract in the three-tiered hierarchy when there is government/firm collusion is one in which (i) $a_i > a_i, \forall i$, and $\bar{a}_1 g'(a_1) - a_3 g'(a_3) - a_4 g'(a_4) > a_2 g'(a_2)$, (ii) $G_r - \bar{G}_i, \forall i$, (iii) $\bar{T}_i - T_r, g(a_i), \forall i$, and (iv) at the optimum all the constraints except (A4) and (B5) bind.

**Proof:** See the Appendix.

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20The collusion-proof transfers to the government and the firm will be denoted by $\bar{G}$ and $\bar{T}$, respectively.
Intuitively, in order to verify that the contract described in Theorem 2 is indeed collusion-proof, I have to show that constraints (A1)-(A5) and constraints (B1)-(B5) are satisfied. By part (iv) of Theorem 2, (A1), (A2), (A3), (A5), (B1), (B2), (B3), and (B4) are satisfied. The proof of Theorem 2 tells us that (A4) and (B5) hold as strict inequalities. Thus, the equilibrium contract is collusion-proof. Note that the SNGA is typically worse off when the government and the firm collude. This is because in this collusion case, the number of binding constraints exceeds the number of binding constraints in the no-collusion case. However, if the SNGA does indeed offer the contract with the characteristics described in Theorem 2, then his monetary obligations will be as described in the Theorem. This is because the equilibrium contract is collusion-proof. The SNGA offers the best contract possible from the set of feasible ex post contracts that are constrained to be budget balancing and collusion-proof.

A comparison of the optimal ex post contracts without and with collusion can be made with the aid of Table 1. First, we see that the level of pollution abatement is never first best. In the no collusion case, we have seen that: (i) asymmetric information, (ii) the requirements of incentive compatibility in states 2 and 3, and (iii) the budget balance and the pollution ceiling constraints prevented the SNGA from requiring that pollution be abated at the first best level. Unlike the no collusion case, in this collusion case, the SNGA’s decision problem is characterized by asymmetric information in all four states. To account for this, the SNGA will typically want to design the firm and government transfers in such a way so that incentive compatibility is maintained and no collusion occurs. However, in designing the firm and government transfers, the SNGA does not have free rein because the budget balance constraints prevent the SNGA from offering large sums of money to the
Table 1. *Ex Post* Contracting Without and With Collusion

<table>
<thead>
<tr>
<th>Contracting</th>
<th>Without Collusion</th>
<th>With Collusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pollution abatement</td>
<td>$a_i &gt; a_i, \forall i$</td>
<td>$a_i &gt; a_i, \forall i$</td>
</tr>
<tr>
<td>Level and pattern</td>
<td>$\frac{A_1 g'(a_1) - A_3 g'(a_3) - A_4 g'(a_4)}{A_2 g'(a_2)} &gt; \frac{\tilde{A}_1 g'(a_1) - \tilde{A}_3 g'(a_3) - \tilde{A}_4 g'(a_4)}{\tilde{A}_2 g'(a_2)}$</td>
<td>$\frac{A_1 g'(a_1) - A_3 g'(a_3) - A_4 g'(a_4)}{A_2 g'(a_2)} &gt; \frac{\tilde{A}_1 g'(a_1) - \tilde{A}_3 g'(a_3) - \tilde{A}_4 g'(a_4)}{\tilde{A}_2 g'(a_2)}$</td>
</tr>
<tr>
<td>Clean air production</td>
<td>$\hat{x}, \forall i$</td>
<td>$\hat{x}, \forall i$</td>
</tr>
<tr>
<td>Transfers to the government</td>
<td>$G_r \cdot V^{-1}(V_r), \forall i$</td>
<td>$G_r \cdot V^{-1}(V_r), \forall i$</td>
</tr>
<tr>
<td>Net payoffs to the polluting firm</td>
<td>$T_i - T_r \cdot g(a_i), \forall i$</td>
<td>$\tilde{T}_i - T_r \cdot g(a_i), \forall i$</td>
</tr>
</tbody>
</table>

government and the firm. As well, the ceiling constraints prevent the SNGA from inducing the firm to abate at high levels. These features of the problem combine to make it impossible for the SNGA to design a contract in which the first best level of abatement obtains.

More formally, the level of pollution abatement is never first best because the budget constraints and the pollution ceiling constraints bind. As in the no collusion case, the attainment of the first best level of abatement in the collusion-proof contract is closely related to the magnitude of:

(i) the SNGA's shadow value of funds, and (ii) the shadow value of the ceiling constraints. It is important to note that these shadow values collectively prevent the attainment of the first-best level of abatement. That is, if either the budget constraints or the ceiling constraints were eliminated, the SNGA would still be unable to design a first-best contract. Note that collusion has no qualitative impact on the pattern of abatement. However, the quantitative impact will almost certainly be different because, in general, the weights $A_1$ and $\tilde{A}_1$, $A_3$ and $\tilde{A}_3$, etc., will not be the same.
Second, the level of clean air production is the same in both contracting scenarios because the pollution reduction ceiling constraints bind. This tells us that the government’s negotiations with the SNGA has a powerful impact on the optimal contract. In the analysis of both instances, the pollution reduction ceiling constraints bind. As such, in no state can the SNGA require that pollution be abated at the first-best level. To fix ideas, consider the following numerical example. Suppose that the pollutant in question is sulphur dioxide (SO₂) emissions and that the firm is a coal-fired electric power plant. Further, suppose that the LDC government and the SNGA agree on a SO₂ emissions floor\textsuperscript{21} of 2.5 pounds per million British thermal units (mmBTUs) of electricity generated. In other words, the power plant cannot be required to reduce SO₂ emissions below this 2.5 mm BTU figure. Then my analysis tells us that the optimal contract will require that this power plant reduce its emissions to exactly 2.5 pounds of SO₂ per mm BTUs.

As discussed in section 4, this floor can be thought of as a measure of the LDC government’s negotiating power. Further, the floor can also be given a welfare interpretation. To see how, consider an emissions floor of 1.2 pounds of SO₂ per mm BTUs. Since the government is representing the firm in its nation and the firm’s payoff is decreasing in abatement, the government will want to negotiate a relatively high floor on emissions (low ceiling on clean air production). On the other hand, the SNGA’s welfare is increasing in abatement, so the SNGA will want to negotiate a low floor on emissions (high ceiling on clean air production). Given this, compared to the 2.5

\textsuperscript{21}Because this example is in terms of emissions directly and clean air production only indirectly, we have a floor. However, this should not confuse the reader, because a floor on pollution emissions is the same thing as a ceiling on clean air production.
figure, the 1.2 figure would imply a lower level of governmental negotiating power, a lower payoff for the polluting firm, and higher welfare for the SNGA.22

Third, the transfers to the government remain unchanged; in both regimes, the government earns its reservation transfer in all states. This is the expected result in the no collusion case. The government plays a completely passive role, and, hence, it is paid only its reservation transfer and no more. In the collusion case, the government plays an active role. Despite this, the government earns no informational rents because the SNGA is successful in inducing the government to reveal its private information truthfully at least cost. More formally, the government earns no rents because all four participation constraints bind.

Fourth, the net payoff to the firm exhibits the same pattern in both contracting regimes. In other words, the possibility of government/firm collusion does not result in any qualitative difference in the monetary transfers that are to be paid to the polluting firm. However, it should be noted that in the collusion case, the SNGA designs the firm transfers so that the equilibrium contract is collusion-proof.

6. Conclusions

In this paper I analyzed the question of environmental protection for developing countries within the framework of the directives set forth in the various agreements reached at the 1992 Rio Earth Summit. I modeled the institutional setting for the underlying problem as a three-tiered hierarchy with $N$ forks and then I studied the nature of the optimal, budget-balanced, ex post contracts, without and with collusion. Three significant policy conclusions emerge.

22 Also, see the textual discussion immediately preceding section 5.
First, whether or not there is collusion, the participation constraints for the government and
the firm bind at the optimum. This means that the SNGA is forced to pay a price in order to obtain
voluntary participation by the players in the individual nations. Several observers, such as Rogers
(1993, p. 236), have worried that many of the Earth Summit directives “... offer a back door option
by which signatories can excuse themselves at a later date if the going gets too tough.” The
implementability of ex post contracts should diminish such concerns because an ex post contract can
be viewed as a limited liability contract. In this sense, as compared to an ex ante contract, an ex post
contract is more likely to be renegotiation-proof.

Second, money and the LDC government's negotiating power matter. That is, the level and
pattern of equilibrium abatement is never first best because the budget and the ceiling constraints bind
at the optimum in both scenarios analyzed. In other words, in the kind of environment analyzed in
this paper, the SNGA will never be able to design an IEA which requires that pollution be abated at
the first-best level. Further, the attainment of the first-best level of pollution abatement is closely
linked to the SNGA's shadow value of funds and to the shadow value of the ceiling constraints. We
have seen that as these shadow values approach zero, the optimal contract can induce the firm to
abate pollution at the first-best level in three of the four states.

Third, if given the option, the SNGA will, in general, prefer ex ante contracting to ex post
contracting. Because ex ante contracting involves optimization with fewer constraints, the SNGA's
expected welfare with ex ante contracting is typically at least as high as his expected welfare with ex
post contracting. However, in the context of LDCs, unless the SNGA can limit the ex post liability
of the players, nations may well refuse to participate in ex ante contracting schemes.
The line of research pursued in this paper brings good news and bad news. The good news is that the SNGA can indeed circumvent the monitoring and enforcement problem stemming from national sovereignty by designing collusion-proof *ex post* contracts. The bad news is that the SNGA cannot design *ex post* IEA's in which first-best levels of pollution abatement obtain.

With talk of rising disparity between the South and the North and the increasingly acrimonious nature of international discussions regarding the use of environmental resources, the design question studied in this paper takes on particular significance. This is in no small measure due to the fact that the implementation of such agreements will do more to engender and maintain international security than will most strategic or unilateral policy measures.
References


Appendix

In this appendix, I provide the proofs of the two Theorems stated in the text of the paper. Both proofs involve Kuhn-Tucker analysis.

Proof of Theorem 1: I shall proceed by means of seven steps. The Lagrangian is:

$$\mathcal{L} = \Sigma_i p_i (x_i - T_i) + \Sigma_i \alpha_i (B[i] - B_i) + \Sigma_i \delta_i \{x^i - x_i\} + \beta \{T_3 - g(\ast) - T_2 - g(\ast)\} + \Sigma_i \gamma_i \left[\dot{M} - \Sigma_j \{G_j \cdot T_j^f\}\right]$$  \hspace{1cm} (a)

where $a_i$, $\delta_i$, $\gamma_i$, $i = 1, 2, 3, 4$ are the multipliers corresponding to (A1), (A2), (A3), and (A5), respectively. The first-order necessary conditions are:

1. $a_1 (\partial B[i]/\partial T_1) - \gamma_1 \cdot p_1$
2. $\alpha_2 \{\partial B[i]/\partial T_2\} - \beta \cdot \gamma_2 \cdot p_2$
3. $\alpha_3 \{\partial B[i]/\partial T_3\} - \gamma_3 \cdot \beta \cdot p_3$
4. $\alpha_4 \{\partial B[i]/\partial T_4\} - \gamma_4 \cdot p_4$
5. $a_1 B'[\ast] g(a_1) - p_1 - \delta_1$
6. $a_2 B'[\ast] g'(a_2) - \beta g'(a_2 - \Delta \theta) \cdot p_2 - \delta_2$
7. $a_3 B'[\ast] \cdot \beta g'(a_3) - p_3 - \delta_3$
8. $a_4 B'[\ast] g'(a_4) - p_4 - \delta_4$

Step 1: The firm participation constraints bind at the optimum.

Proof: I have to show that $a_i > 0$, $\forall i$. From (a1) $\alpha_1 - \gamma_1 \cdot p_1$, a contradiction. Thus $\alpha_1 > 0$.

From (a2) $a_2 - \gamma_2 \cdot (p_2 \cdot \beta)$, a contradiction. Thus, $a_2 > 0$. From (a3) I get $\partial B[i]/\partial T_3 = (\gamma_3 - \beta \cdot p_3)/a_3$. Since $\partial B[i]/\partial T_3 \epsilon (0, \infty)$, it follows that $\alpha_3 > 0$. From (a4) $\alpha_4 - \gamma_4 - p_4$, a contradiction. Thus $\alpha_4 > 0$.

Step 2: The budget constraints bind at the optimum.

Outline of Proof: I have to show that $\gamma_i > 0$, $\forall i$. I shall demonstrate the validity of this claim for $i = 1$ only. The argument is similar for $i = 2, 3, 4$. $\gamma_1 = 0$ implies that $T_1$ can be lowered without violating $\alpha_1 > 0$.

This is a contradiction. Thus, $\gamma_1 > 0$.

Step 3: The pollution reduction ceiling constraints bind at the optimum.

\[^{23}\text{I shall check later to see that (A4) is satisfied.}\]
Outline of Proof: I have to show that $\delta_i > 0$, $\forall i$. I shall demonstrate the validity of this claim for $i = 3$ only. The argument is similar for $i = 1, 2, 4$. $\delta_3 < 0$ implies that $a_3$ can be raised without violating $a_3 > 0$. This is a contradiction. Thus, $\delta_3 > 0$. ■

Step 4: At the optimum, (A3) holds with equality.

Proof: $\beta < 0$ implies that $T_3$ can be lowered and $a_3$ can be raised without violating $a_3 > 0$. This is a contradiction. Thus, $\beta > 0$. ■

Step 5: $T_i = T_i \cdot g(a_i)$, $\forall i$.

Proof: This follows from the fact that $a_i > 0$, $\forall i$. ■

Step 6: $a_i > a_i$, $\forall i$.

Proof: From (a1) and (a5) I get $0 < g'(a_i) - (p_i - \delta_i)/(p_i + \gamma_i) < 1 - g'(a_i)$, $a_i > a_i$. From (a2) and (a6) I get $0 < g'(a_2) - (p_2 - \delta_2)/(p_2 + \gamma_2) < 1 - g'(a_2)$, $a_2 > a_i$. From (a3) and (a7) I get $0 < g'(a_3) - (p_3 - \delta_3)/(p_3 + \gamma_3) < 1 - g'(a_3)$, $a_3 > a_i$. Finally, from (a4) and (a8) I get $0 < g'(a_4) - (p_4 - \delta_4)/(p_4 + \gamma_4) < 1 - g'(a_4)$, $a_4 > a_i$. ■

Step 7: $A_1 g'(a_1) - A_3 g'(a_3) - A_4 g'(a_4) > A_2 g'(a_2)$, where $A_i = (p_i + \gamma_i)/(p_i + \delta_i)$, $i = 1, \ldots, 4$.

Proof: From the proof to Step 6, $(p_2 + \gamma_2)/(p_2 + \delta_2) g'(a_2) < 1 - (p_i + \gamma_i)/(p_i + \delta_i) g'(a_i)$, $i = 1, 3, 4$. ■

Finally, I shall check to see that (A4) is satisfied. Since $a_i > 0$, $i = 1, 3$, I conclude that $T_2 - g(a_2) - T_3 - g(a_3) > T_3 - g(a_3 + \Delta \theta)$. This completes the proof of Theorem 1. ■■

Proof of Theorem 2: I shall proceed by means of thirteen steps. The Lagrangian is\(^{24}\)

\[
\mathcal{L} = \sum_i p_t(x_i - \bar{G}_i - \bar{T}_i) + \sum_i a_t(B^* - B_t) + \sum_i \delta_i (V^* - V_t) + \sum_i \delta_i (\ddot{x}^* - x_t) + \beta \{T_3 - g(\bar{g}) - T_2 + g(\bar{g})\} + \\
\varepsilon_1 \{G_1 - \bar{G}_1 - g(\bar{g}) - \bar{G}_2 - g(\bar{g})\} + \varepsilon_1 \{G_4 - \bar{G}_4 - g(\bar{g}) - \bar{G}_2 - g(\bar{g})\} + \\
\varepsilon_2 \{G_3 - \bar{G}_3 - g(\bar{g}) - \bar{G}_2 - g(\bar{g})\} + \varepsilon_2 \{G_4 - \bar{G}_4 - g(\bar{g}) - \bar{G}_2 - g(\bar{g})\} + \\
\varepsilon_2 \{G_3 - \bar{G}_3 - g(\bar{g}) - \bar{G}_2 - g(\bar{g})\} + \sum_i \gamma_i [\dot{\theta} - \sum_j \dot{\theta}_j] \}.
\]

\(^{24}\)I will check later to see that (A4) and (B5) are satisfied.
where \( \alpha, \beta, \epsilon, \kappa, \gamma, i=1,2,3,4, l, j \), are the multipliers associated with (A1), (B1), (A2), (A3), (B2), (B3), (B4), and (A5), respectively. The first-order necessary conditions are (b1)
\[
\begin{align*}
&v_1'v(G_1)-p_1\epsilon_1\gamma_1, \\
&(b2) \quad v_2'v(G_2)-p_2\epsilon_1\kappa_2, \\
&(b3) \quad v_3'v(G_3)-p_3\epsilon_2\gamma_3, \\
&(b4) \quad v_4'v(G_4)-p_4\epsilon_2\gamma_4,
\end{align*}
\]
(b5) \[
\begin{align*}
&\alpha_1(\partial B[\ast]/\partial T)\cdot p_1\epsilon_1\gamma_1, \\
&(b6) \quad \alpha_2(\partial B[\ast]/\partial T)\cdot p_2\epsilon_1\kappa_2, \\
&(b7) \quad \alpha_3(\partial B[\ast]/\partial T)\cdot p_3\epsilon_2\kappa_3, \\
&(b8) \quad \alpha_4(\partial B[\ast]/\partial T)\cdot p_4\epsilon_2\kappa_4.
\end{align*}
\]
Step 1: The state 1 government and firm participation constraints bind at the optimum.
Proof: From (b1) I get \((p_1-\epsilon_1\gamma_1)/v_1\cdot v'(G_1) > 0 \rightarrow v_1 > 0\). From (b1) and (b5) I get
\[
v_1'v(G_1)\geq \alpha_1B[\ast].
\]
Now \(\alpha_1 > 0\) or \(v_1'v(G_1) > 0\), a contradiction. Thus \(\alpha_1 > 0\). •
Step 2: The state 2 government and firm participation constraints bind at the optimum.
Proof: From (b2) \(v_2 > 0\), which is impossible. Thus, \(v_2 > 0\). From (b2) and (b6)
\[
v_2'v(G_2)\geq \alpha_2B[\ast].
\]
Now \(\alpha_2 > 0\) or \(v_2'v(G_2) > 0\), a contradiction. Thus, \(\alpha_2 > 0\). •
Step 3: The state 3 firm and government participation constraints bind at the optimum.
Proof: From (b7) I get \((p_3-\beta\epsilon_2\gamma_3)/v_3\cdot v'(G_3) > 0 \rightarrow v_3 > 0\). From (b3) \(v_3 > 0\), \(\beta\epsilon_2\gamma_3\). Substituting this value of \(\epsilon_2\gamma_3\) in (b7) I get \(\beta\epsilon_2\gamma_3 > 0\), a contradiction. Thus, \(v_3 > 0\). •
Step 4: The state 4 government and firm participation constraints bind at the optimum.
Proof: From (b4) I get \((p_4-\epsilon_2\gamma_4)/v_4\cdot v'(G_4) > 0 \rightarrow v_4 > 0\). From (b4) and (b8) I get
\[
v_4'v(G_4)\geq \alpha_4B[\ast].
\]
Now \(\alpha_4 > 0\) or \(v_4'v(G_4) > 0\), a contradiction. Thus, \(\alpha_4 > 0\). •
Step 5: \(G_1G_2G_3G_4\).
Proof: This follows from the fact that \(v_i > 0\), \(\forall i\). •
Step 6: For every state, the budget constraint binds at the optimum.
Proof: Suppose not. Then $\gamma_i > 0$, $\forall i$ and for any $i$, $i-1, \ldots, 4$, the SNGA can lower $\vec{g}_i$ and $\vec{T}_i$, without violating $v > 0$, and $\alpha > 0$ for that same $i$. This is a contradiction. Thus, $\gamma_i > 0$, $\forall i$.

Step 7: The pollution reduction ceiling constraints bind at the optimum.

Proof: Suppose not. Then $\delta_i > 0$, $\forall i$ and for any $i$, $i-1, 2, 3, 4$, the SNGA can raise $a_i$ without violating $\alpha_i > 0$ for that same $i$. This is a contradiction. Thus $\delta_i > 0$, $\forall i$.

Step 8: $a_i > a_p$, $\forall i$.

Proof: (b5) and (b9) yield $0 < g'(a_1) - (p_i - \delta_i)/(p_i + \gamma_i) < 1 - g'(a_1) - a_1 < a_i$. From (b6) and (b10) I get $0 < g'(a_2) - (p_i - \delta_i)/(p_i + \gamma_i) < 1 - a_2 < a_i$. From (b7) and (b11) I get $0 < g'(a_3) - (p_i - \delta_i)/(p_i + \gamma_i) < 1 - a_3 < a_i$. Finally, from (b8) and (b12) I get $0 < g'(a_4) - (p_i - \delta_i)/(p_i + \gamma_i) < 1 - a_4 < a_i$.

Step 9: Constraint (A3) binds at the optimum.

Proof: $\beta > 0$ and (A4) yield $0 < g'(a_1) - (p_i - \delta_i)/(p_i + \gamma_i) < 1 - a_1 < a_i$. This is a contradiction. Thus, $\beta > 0$.

Step 10: Constraint (B4) binds at the optimum.

Proof: Suppose not. Then $\kappa > 0$ and (B4) reduces to $\vec{g}_i > \vec{g}_p$, $\forall i$. This tells us that $\vec{g}_i$ can be lowered without violating $\alpha_i > 0$. This is a contradiction. Thus, $\kappa > 0$.

Step 11: $\tilde{A}_i g'(a_1) - \tilde{A}_i g'(a_2) - \tilde{A}_i g'(a_3) - \tilde{A}_i g'(a_4) > \tilde{A}_i g'(a_2)$, where $\tilde{A}_i (p_i - \delta_i)/(p_i - \gamma_i)$, $i-1, \ldots, 4$.

Proof: From the proof to Step 8, it follows that $\tilde{A}_i g'(a_2) < 1 - \tilde{A}_i g'(a_1) - \tilde{A}_i g'(a_3) - \tilde{A}_i g'(a_4)$.

Step 12: $\tilde{T}_i T_i g(a_i)$, $\forall i$.

Proof: This follows from the fact that $a_i > 0$, $\forall i$.

Step 13: Constraints (B2) and (B3) bind at the optimum.

Proof: This follows upon substitution of the results of Steps 5 and 12 into (B2) and (B3), respectively.
I now check to see that (A4) and (B5) are satisfied. The fact that (A4) is satisfied can be verified in a manner analogous to that employed in the proof of Theorem 1. Having shown that (A4) is satisfied, to verify that (B5) is satisfied, it suffices to note that from Step 5, $\bar{G}_2 \bar{G}_3$. This completes the proof of Theorem 4. ■■