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## Water Resources Systems Analysis - Course Notes

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**CEE 6410**

# **Water Resources Systems Analysis**

**Course Notes**

by

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Fall, 1999

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# PREFACE

## SUPPLEMENTAL READING

There are now many good systems analysis textbooks available, which focus on civil engineering applications. One example is by Richard de Neufville (1990). It was not selected as a required text for this class because most of the examples are not water resources problems. The chapters on mathematical programming, economics, and philosophical basis for decision making, however, would make excellent supplemental reading to these course notes.

Two texts that do focus entirely on water applications are Loucks et al. (1980) and Mays and Tung (1992). In previous years, the text by Loucks et al. (1980) has been recommended reading for this initial course in water resources systems analysis, as well as the required text for other classes. Unfortunately, it is now out of print. The text by Mays and Tung (1992) is therefore suggested as supplemental reading. It and other reference materials will be placed on reserve in the Merrill Library. The lectures for this course, however, will be focused primarily upon these course notes.

Other texts with civil engineering applications are Stark and Nicholls (1972) and Jewell (1986). Students interested in more rigorous development of mathematical theory that supports the optimization algorithms are referred to Hadley (1962 and 1964), Wagner (1975), Ossenbruggen (1984), and Beightler et al. (1979).

## BEWARE THE PITFALLS

Because the focus in these course notes is primarily upon optimization algorithms and optimization model applications to water resources problems, an important caveat is needed. A student who takes only this initial class in water resources systems analysis will be in danger of obtaining an exaggerated perception of both:

- the importance of optimization relative to other analysis tools in the water resource planning arena
- the credence one should place on the validity of an answer one gets from a single solution to an optimization problem

While optimization is an extremely powerful mathematical tool for water applications, it is usually used as a screening device, the objective of which is to greatly reduce the number of alternatives which should be considered before making a final decision by subsequent analyses (either with or without a formal, mathematical).

For any complex water planning problem, a common sequence in the decision making process is that the results from an optimization model will either become input to a more detailed simulation model, or the optimization model will be revised and run iteratively to observe the effect of

changes in the structure of the objective function or constraints. The point is that a reality check is always needed to determine:

How well does the system you modeled match the system you hoped you were modeling? If the solutions differ from intuitively reasonable solutions, chances are the computer did only what it was told to do, rather than what you hoped it would do.

How well does the solution from any model match the goals of the system planner? The manager of a water system is often more interested in risk aversion and robustness of a solution than in the rigorous maximization of a mathematical function which can never precisely match the real array of (probably multiple) objectives.

In short, an optimization model is only one of several tools in our array of water resource planning approaches; a solution to an optimization problem usually represents only one piece of information of value in the planning process.

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