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Economic Research Institute Study Paper

ERI #96-34

**THE TIMING OF LAND DEVELOPMENT:
AN INVARIANCE RESULT**

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September 1996

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Amitrajeet A. Batabyal

ABSTRACT

The Arrow-Fisher-Henry (AFH) analysis of land development under uncertainty has been conducted in a two-period model. Recently, Capozza and Helsley in 1990 and Batabyal in 1995 have addressed the question of land development under uncertainty in a many-period setting. In this paper, I extend aspects of this literature by analyzing the land development question in a Markov decision theoretic framework. *Inter alia*, I show that the timing of land development is invariant to the manner in which a landowner uses information about the consequences of development.

JEL classification: D81, Q24, R14

Key words: land, development, timing, information, uncertainty

THE TIMING OF LAND DEVELOPMENT: AN INVARIANCE RESULT¹

1. Introduction

Resource economists and urban economists have been interested in the question of land development under uncertainty—with the development decision being potentially irreversible—at least since Weisbrod (1964) and Shoup (1970). Since then, Arrow and Fisher (1974), and Henry (1974) have shed considerable light on this development question. *Inter alia*, these researchers have identified a concept known as option value. The so-called Arrow-Fisher-Henry (AFH) concept of option value—also called quasi-option value—tells us that when development is both indivisible and irreversible, a landowner who ignores the possibility of obtaining new information about the consequences of such development will invariably underestimate the benefits of preservation and hence skew the binary choice development decision in favor of development.

This simple and yet powerful result has been shown to hold in its most general form in a two-period setting. However, the result typically does not hold in more general settings. It has already been shown by Epstein (1980) and Hanemann (1989) that when the development decision is *divisible*, this bias toward development need not arise; indeed it will not arise unless the development benefit function is of a rather specific form.

Similarly, one can ask about the nature of the development decision when this decision is made in a many-period setting. Because the AFH analysis is conducted in a two-period model, the relevant development question is "Do I develop today or tomorrow?" In a many-period setting, this

¹This research was supported by the Utah Agricultural Experiment Station, Utah State University, Logan, UT 84322. Approved as journal paper no. 4832.

question must be changed to "When do I develop?" This follows from the fact that a landowner's decision problem is now not over two periods but over a much longer time horizon. A number of researchers have been interested in this "When to develop land" question. Markusen and Scheffman (1978), Arnott and Lewis (1979), and Capozza and Helsley (1989) have all analyzed this question in a deterministic environment. However, when the underlying development decision is irreversible, as are most development decisions involving the conversion of land from agricultural to urban use,² the use of a certainty framework will bias results about when land should be developed. Indeed, as we have learned from the investment under uncertainty literature,³ uncertainty will typically (i) impart an option value to undeveloped land, and (ii) delay the development of land from, say, agricultural to urban use. As such, if we are to understand when land should be developed in the presence of irreversibilities, it is imperative that we explicitly account for uncertainty.

Recently, Titman (1985), Capozza and Helsley (1990), and Batabyal (1995) have analyzed the development of land under uncertainty. The bulk of Titman's analysis is conducted in a two-period framework; as such, this paper does not really address the timing of land development question in full generality. The works of Capozza and Helsley (1990) and Batabyal (1995) are most closely related to the analysis of this paper. Capozza and Helsley pose the timing of land development question as a "first hitting time" problem.⁴ In their model, urban land rent, R , follows a Brownian motion process with drift. Capozza and Helsley specify a level of rent, R^* , at which it is optimal to convert land from agricultural to urban use. They then use standard methods to solve

²See Capozza and Helsley (1990, p. 187) for more details.

³See McDonald and Siegel (1986), Pindyck (1991), Dixit and Pindyck (1994), and Hubbard (1994).

⁴For more on this, see Ross (1983, pp. 190-192).

for the optimal time of development, t^* , which is also the first time at which the Brownian motion process hits R^* . In contrast with this “first hitting time” approach, Batabyal (1995) has used the theory of Markov decision processes⁵ to provide an answer to the “When do I develop land?” question. To the best of my knowledge, this approach to land development is novel; further, the approach is able to provide an analytical solution to the timing question under study. However, this novelty notwithstanding, Batabyal (1995) has limited the applicability of his model by adopting a restrictive assumption about the manner in which a landowner uses information about the development decision. As such, the purpose of my paper is to continue this line of inquiry, and to cast the land development problem in a more general framework than the one used by Batabyal (1995). In doing so, I shall show that Batabyal's result is actually much stronger than his analysis suggests. Specifically, I shall show that the timing of land development is invariant to the manner in which a landowner uses information about the consequences of development.

It should be noted that this “When to develop land” question is one that is often faced by land developers and urban planners. Indeed, an analysis of this kind has been used (i) to study the value of optimally developed land in Canadian metropolitan areas (Arnott and Lewis 1979), (ii) to comprehend why land has been underutilized in west Los Angeles despite the existence of high land prices (Titman 1985), and (iii) to study the gap between land prices in Vancouver and the value of land in neighboring agricultural areas (Capozza and Helsley 1989).

The rest of this paper is organized as follows. Section 2 briefly discusses Batabyal's (1995) modeling framework and his information use assumption. Section 3 develops a more general framework and provides an answer to the question of the timing of land development in this setting.

⁵For more on this, see Dreyfus and Law (1977, pp. 172-187).

This section also provides a simple numerical example to illustrate the use of the underlying formal argument. Finally, section 4 offers concluding comments.

2. The Batabyal (1995) Theoretical Framework

Batabyal uses the infinitesimal look ahead stopping rule (ILASR) and a theorem contained in Ross (1970, p. 188) which provides conditions under which it is optimal to stop a stochastic process using this rule.⁶ Intuitively, the ILASR can be thought of as a policy which stops a stochastic process precisely in those states for which stopping immediately yields a higher payoff than continuing an additional time h .⁷ The key aspect of the Ross theorem lies in the requirement that the set of states S , for which stopping immediately is better than continuing, be a closed set. In other words, once a stochastic process enters S , the process cannot exit S . Formally, if P_{ij} denotes the transition probability of the stochastic process moving from state i to state j , then it is optimal to stop in state i , if $P_{ij} = 0$ for $i \in S, j \notin S$.

The specific decision rule employed by Batabyal is of the following form: develop the land when the revenue from development is at least as big as some threshold level of revenue. The revenues are a continuous, one-to-one, and monotonic function of learning by the landowner about the consequences of undertaking development activities. In turn, this learning depends on any development-related information that the landowner is able to obtain. Alternately put, Batabyal uses a one-to-one function $f(\bullet)$, to map information about development to revenue from development.

⁶I shall only discuss those aspects of the Batabyal (1995) modeling framework that are directly relevant to my paper. For more details, see Batabyal (1995).

⁷Many economic problems can be analyzed in this kind of optimal stopping framework. For example, in a problem involving investment under uncertainty, stopping corresponds to making the investment, continuing corresponds to waiting. See Dixit and Pindyck (1994) for more such examples.

He then proceeds to analyze the landowner's decision problem using revenue as the relevant state variable. To ensure that the above decision rule involves a set of states which is closed, Batabyal assumes that information is not ephemeral, i.e., any information received by the landowner need not be used immediately. Instead, this information may be stored and used at a later date to decide whether land should or should not be developed.

To understand this information storability assumption, consider a case in which information *is* ephemeral, i.e., if the information is not used relatively quickly, then this information will cease to have value. Suppose that a private developer approaches our landowner with information about an offer to develop land. The landowner will typically have to act on this information—accept or decline the offer—fairly quickly. Assume that the landowner declines the offer. The question that arises is this: will this offer be lost forever, or can the offer and the relevant information be used by the landowner at a future date? If the answer is that the offer will be lost, then this information is ephemeral. The information storability assumption rules out this kind of situation. Further, because there is a one-to-one functional relationship⁸ between information about development and revenue from development, the information storability assumption implies that the “revenues” from development are also storable. The fact that the revenues are storable in this sense tells us that it is optimal to use the ILASR along with the Ross (1970, p. 188) theorem, to determine when land should be developed.

In the next section, I shall show that the timing of land development is *invariant* to whether information—and hence revenue—is storable or not. In other words, the timing of land development

⁸By means of the function $f(\bullet)$.

is unaffected by the increased inflexibility arising from the case in which information must be used immediately.

3. The Markov Decision Theoretic Framework

3a. *Model Essentials*

My model is closely related to that in Batabyal (1995). I shall use an optimal stopping rule, specifically the one stage look ahead policy (OSLAP)⁹ as developed in Ross (1970, pp. 134-139), to provide an answer to the “When to develop land” question.

Consider a landowner who owns a plot of agricultural land. The decision problem faced by this owner concerns when to develop his plot of land. Following AFH, I shall assume that this development decision is indivisible. The landowner solves his problem in a dynamic and stochastic framework. The framework is stochastic because the decision to develop depends fundamentally on the availability of information regarding the consequences of development. This information is produced according to an independent and identically distributed stochastic process $\{X(t): t \geq 0\}$. That is, information of a particular quality is received at time t with a certain probability, independent of information received at any other time. The specific source of information production is not critical to my analysis. It could be the result of in-house R&D activities by the landowner or it could be the result of research undertaken by other public and/or private agencies. In any event, from the perspective of the landowner, information is costly to acquire. As such, in what follows, I shall incorporate this cost in the overall decision problem faced by the landowner.

⁹The OSLAP is the discrete analog of the ILASR.

Upon acquiring information, the owner decides whether to develop his land or to preserve it and wait for additional information. This decision is based on the revenue obtainable from development. Following Batabyal (1995), let $f(\bullet)$ be the continuous, one-to-one, and strictly monotone function which maps information about development to revenue from development. That is, if $x(t)$ is the information received by time t , then $F(t)=f\{x(t)\}$ denotes the revenue from developing, given that a decision to develop has been made. Note that because $f(\bullet)$ is a continuous, one-to-one, and strictly monotone transformation of $X(t)$, $\forall t$, $\{F(t):t \geq 0\}$ retains all the properties of the stochastic information production process.¹⁰ In other words, revenue of amount j is received at time t with probability P_j , independent of any other revenue realization. Mathematically, $Pr\{F(t) = j\} = P_j$.

Should the landowner choose not to develop his land, he incurs benefits and costs. The benefits are the obvious AFH type benefits; the landowner preserves the flexibility to act on new information in the future. The costs arise from the fact that the developer has to pay to obtain information, and he loses the revenue from development. I will denote the net benefit from not developing (preserving), by B .

Let the state at any time t be denoted by $[t, F(t)]$, where $F(t)$ is the revenue that will be received, should the landowner choose to develop at time t . Two facts are worth noting. First, with this specification of the state, I have a two-action—develop or preserve—Markov decision process. This is because (i) the OSLAP is a stationary policy in which the action chosen at any time t depends only on the state of the process $\{F(t):t \geq 0\}$ at t , and (ii) the sequence of states $\{F(t):t=0,1,2,\dots\}$ forms a discrete time Markov chain. Second, the above specification of states does not permit me

¹⁰See Wolff (1989, p. 26) for further details.

to use a stopping rule of the form: develop the land when the revenue from development equals some threshold value directly, because the set of potential stopping (developing) states is not closed. This is because the $F(t)$ (the revenues) can decline in amount. To get around this difficulty, I shall adopt the Batabyal (1995) revenue storability assumption *temporarily*. With this assumption, the state at any time t will be the maximum revenue to be obtained from developing at that time.¹¹

I can now specify the transition probabilities for the countable state Markov chain $\{F(t): t \geq 0\}$.

At any time t , these probabilities are given by¹²

$$\begin{aligned} P_{ij}(t) &= 0 \text{ if } i > j, \\ P_{ij}(t) &= \sum_{r=0}^{r=i} P_r, \text{ if } i = j, \\ P_{ij}(t) &= P_j, \text{ if } i < j. \end{aligned} \tag{1}$$

The first line of (1) tell us that under the OSLAP, the Markov chain will not go to state j if the revenue in state j is less than the revenue in state i . Hence, $P_{ij}(t)=0$. The second line of (1) tells us that if the revenues in states i and j are equal, then under the OSLAP, the probability of making a transition from state i to state j is simply the sum of all the probabilities of receiving revenues of amount 0 through I . Finally, the third line of (1) tells us that if the revenue in state j exceeds the revenue in state i , then under the OSLAP, the Markov chain will move to state j with transition probability $P_{ij}(t) = P_j$.

I shall now characterize the set of states for which developing the land in state i is at least as good as preserving and continuing for exactly one more time period and then developing. This set

¹¹From my earlier discussion, recall that in order for the use of stopping rules like the ILASR or the OSLAP to be optimal, the set of stopping states has to be closed.

¹²Note that the i, j correspond to specific revenue amounts in state i and state j , respectively.

is given by

$$S = \{i: i \geq i \sum_{j=0}^{j=i} P_j + \sum_{j=i+1}^{\infty} jP_j + B\}. \quad (2)$$

The first term on the RHS of the weak inequality in (2) is obtained from the second line of equation (1); this term denotes the expected revenues in states 0 through i . Similarly, the second term is obtained from the third line of equation (1); this term represents the expected revenues in state $i + 1$ through state ∞ . Finally, the third term is the net benefit from not developing the land currently. In total, the three terms on the RHS of the weak inequality denote the expected revenue from delaying development for an additional time period, i.e., preserving in the current time period. After some algebra, (2) can be further simplified to

$$S = \{i: 0 \geq E[\max(F-i, 0)] + B\}. \quad (3)$$

In equation (3), $E[\bullet]$ denotes the expectation operator, $\max(\bullet)$ denotes the maximum operator, and F denotes revenue. Note that because $\max(F-i, 0)$ decreases in i , the maximum revenue obtainable from development cannot decrease with time. Hence, S is a closed set. Now assuming that the revenue Markov chain $\{F(t): t \geq 0\}$ is stable,¹³ the OSLAP requires the landowner to develop land at the first instance in which the revenue from developing is at least i^* , where i^* solves

$$i^* = \min\{i: 0 \geq E[\max(F-i, 0)] + B\}. \quad (4)$$

In this connection it is important to note that under the OSLAP, an optimal policy *never* uses a past revenue amount. Hence, I can dispense with the revenue storability assumption and conclude that the policy described by equations (3) and (4) must be optimal for my original problem in which revenue from development is ephemeral and hence must be acted upon immediately in the

¹³See Ross (1970, p. 135) for a necessary and sufficient stability condition.

development decision. To intuitively see why this must be the case, note that the maximal return to the landowner when revenue is ephemeral cannot be larger than the case in which past revenue realizations can be used to decide whether or not to develop. I have just demonstrated

Theorem 1: When the landowner's problem is modeled as a Markov decision process, the timing of land development is invariant to the storability of information and revenue.

Theorem 1 contains the surprising result that the transience of information and the corresponding development revenues has no bearing on the timing of land development. Further, this theorem complements the investment under uncertainty literature's "value of waiting" result because the theorem tells us that, from a practical standpoint, a landowner need not make his development decision less conservative simply because information, if not used immediately, will be lost. I now provide a simple numerical example¹⁴ to illustrate the above argument.

3b. A Numerical Example

Our landowner is considering whether to develop or preserve his plot of agricultural land. Suppose that at the beginning of each time period, this owner receives information about alternate development prospects which he maps to revenues, i.e., $F(t)$, by means of the $f(\bullet)$ function. Upon every realization of this revenue stochastic process, the landowner must take one of two possible actions, i.e., he must either develop or preserve his land. Information and, hence, the revenues are ephemeral in the sense of the discussion in section 2. Suppose that there are only three states, states 0, 1, and 2, in which the revenues are \$38,000, \$40,000, and \$42,000, respectively. Let the probabilities of obtaining these revenues be $P_0=1/2$, $P_1=3/8$, and $P_2=1/8$, respectively. Suppose that

¹⁴This example is based in part on Dreyfus and Law (1977, p. 186).

a net benefit of $B = -\$50$ is incurred by the owner each time period that the land remains preserved. Finally, suppose that our landowner uses a discount factor of $\beta = 0.99$ in making his decision.

To provide an answer to the "When to develop" question, I shall suppose that once a decision to develop has been made, the corresponding Markov decision process goes to state ∞ , and that it stays there indefinitely, accruing a net benefit of $B = 0$ per time period. Now standard but tedious calculations tell us¹⁵ that in this example, the landowner's optimal policy calls for preserving land when the revenue is \$38,000, and developing the land when the revenue is either \$40,000 or \$42,000. Now equation (4) tells us that the landowner should develop his land in state 1; in this state, the revenue from development is \$40,000.

4. Conclusions

In this paper I modeled the land development question in a dynamic and stochastic framework. In this setting, I provided an answer to the "When to develop land?" question. This answer involved a comparison of the revenue obtainable from developing at time t , i.e., $F(t)$ with the expected revenue to be obtained by preserving and waiting for new information beyond time t . This answer is more general than the answer provided in Batabyal (1995) because the answer demonstrates the invariance of the timing decision to the storability of information and revenue.

The analysis of this paper can be generalized in a number of directions. I suggest two possible extensions. First, in a context similar to that of this paper, one could analyze the development decision when this decision is divisible. This extension will enable one to determine whether the possibility of acquiring new information (learning) truly skews the development decision in favor of

¹⁵See Dreyfus and Law (1977, pp. 274-275) for the details.

increased preservation in this more general case. Second, one could consider the impact on the timing decision when the landowner uses nonstationary and/or randomized policies. A study of the properties of such policies will permit richer analyses of the connections between information production and the timing of land development.

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