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DEVELOPING COUNTRIES AND ENVIRONMENTAL PROTECTION: THE EFFECTS OF EX ANTE VERSUS EX POST CONTRACTING

by

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DEVELOPING COUNTRIES AND ENVIRONMENTAL PROTECTION: THE EFFECTS OF EX ANTE VERSUS EX POST CONTRACTING

Amitrajeet A. Batabyal

ABSTRACT

In the 1992 Rio Earth Summit, developing countries (DCs) were adamant that, in order to protect the environment for the future, new institutions were needed which would channel resources from the wealthy developed countries to the poor DCs. With this backdrop, I analyze the problem faced by an asymmetrically informed supranational governmental authority (SNGA) who wishes to design an International Environmental Agreement (IEA). The SNGA cannot contract directly with polluting firms in the various DCs; instead, he must deal with such firms through their governments. I study this tripartite hierarchical interaction and focus on the properties of the optimal ex ante and ex post IEAs, which can be implemented by the SNGA in two different scenarios. My analysis suggests that IEAs are not inherently doomed due to a basic monitoring and enforcement problem stemming from national sovereignty. Further, desirable levels of pollution abatement can result in a number of contractual settings.

JEL Classification: O19, Q28

Key words: international environmental agreement, developing country, hierarchical
1. Introduction

With the passage of time, it has increasingly been recognized that environmental protection is a global issue. The significance of this issue has been amply demonstrated by the events of the 1992 Rio Earth Summit. At this Summit, it became clear that if the developed countries of the world wanted "... the environment to be secured for future generations, [then they would] have to radically assist the South in choosing a different road to development than the one they [had] currently [been] travelling on" (Rogers, 1993, p. 27). Indeed, to combat the evils of poverty and environmental degradation, developing countries (DCs) have demanded the transfer of resources from developed countries. In such a contentious setting, the success or failure to protect the environment will depend crucially on the ability of international organizations to craft effective international environmental agreements (IEAs).\(^1\) Given this, a key question becomes "How can international institutions, which necessarily respect the principle of state sovereignty, contribute to the solution of difficult global problems?" (Keohane, Haas, and Levy, 1993, p. 6). This is the central question that I propose to analyze in this paper.

On the academic front, researchers have begun to study issues relating to global

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\(^1\)I have benefited from the comments of Larry Karp, Stefan Reichelstein, and seminar participants at Dartmouth College, Resources for the Future, UC Riverside, University of Guelph, University of Toronto, and the 1994 winter meetings of the North American Econometric Society. I acknowledge financial support from the Giannini Foundation, the Faculty Research Grant program at Utah State University, and the Utah Agricultural Experiment Station, Utah State University, Logan, UT 84322-4810, by way of project UTA 024. Approved as journal paper No. 5014. The usual disclaimer applies.

\(^2\)In this paper I shall use the terms IEA and contract interchangeably.
environmental protection in a systematic manner only very recently.\textsuperscript{3} As a result, many specific questions remain unanswered. What kinds of pollution abatement patterns can one expect to observe in situations in which an imperfectly informed supranational governmental authority (SNGA) contracts with governments and polluting firms in individual countries? What kinds of monetary transfers will be necessary to get sovereign nations to voluntarily participate in IEA’s? What is the effect of contractual procedure, i.e., \textit{ex ante} versus \textit{ex post} contracting\textsuperscript{4} on the nature of pollution abatement patterns across countries? Finally, how does the SNGA’s inability to monitor pollution abatement in the individual countries affect the IEA design question? These are some of the specific questions that I shall address in this paper.

In particular, I shall build on the economics of hierarchies to study the global pollution control question as a problem in mechanism design.\textsuperscript{5} This perspective not only highlights the effect of key informational asymmetries on the design of contracts, but it also provides insights into the kinds of pollution control arrangements one might expect to observe in an inherently hierarchical and noncooperative international environment. As far as the regional aspects of this pollution control question are concerned, there are two issues to note. The first issue concerns the potentially deleterious effects of excessive compliance costs stemming from multiple levels of regulations. Second, there is the possibility that regions (nations) will compete among themselves to attract

\textsuperscript{3}See Keohane, Haas, and Levy (1993) for a more detailed corroboration of this claim.

\textsuperscript{4}By \textit{ex ante}, I mean contracting that takes place with all parties holding symmetric but imperfect information about the pollution abatement technology of firms. By \textit{ex post}, I mean contracting that takes place with the parties holding asymmetric information about the pollution abatement technology of the same firms.

\textsuperscript{5}For more on hierarchical analyses in regional science, see Kohsaka (1986), West, Ryan, and von Hohenbalken (1988), and Crihfield and Panggabean (1996).
business investment by weakening their regulatory requirements. With regard to these two issues, my hierarchical approach implicitly sides with those who have called for a centralized approach to environmental regulation (see Peltzman and Tideman (1972), Cumberland (1979, 1981), and Markusen, Morey, and Olewiler (1993, 1995)).

Although my analysis is, in principle, applicable to any country, the hierarchical interaction that I shall analyze is particularly relevant to DCs; as such, the reader should note that it is these countries that I have in mind in all of the subsequent analyses.

2. International Environmental Agreements: A Brief Synopsis

Barrett (1994) has modeled IEAs as games between countries. While Barrett’s analyses are not in the design framework, he makes the important point that for IEAs to work at all, they must be self-enforcing. Hoel (1992) argues against the institution of uniform emissions reduction policies in international agreements, showing that other policies yield higher levels of global welfare. Petrakis and Xepapadeas (1996) show that a large enough group of environmentally conscious countries can make self-financing side payments to a group of less environmentally conscious countries so as to produce a stable coalition which leads to lower overall pollution emissions. While these papers have certainly advanced our understanding of some aspects of “... the multi-faceted design ... problem,” (Black, Levi, and de Meza, 1993, p. 281), many other important questions,

6Closely related to this issue is the “industrial flight” hypothesis. This hypothesis says that firms in developed countries will attempt to take advantage of lower environmental standards in developing countries by relocating to such countries. The relevance of this hypothesis for the design of IEAs and the extent to which this hypothesis is true is the subject of debate. For more on this matter, see Weidenbaum (1980), Leonard (1984), and Kleindorfer, Kunreuther, and Hong (1996).

7The countries I have in mind are those which would be eligible to receive monetary transfers under the Global Environmental Facility’s (GEF) standard of per capita income of $4,000 or less. For more details, see Rogers (1993, p. 155).
which I discussed in section 1, remain unanswered. As such, I now discuss my modeling approach to the IEA design question.

I shall model the international environment as a multi-forked, three-tiered hierarchy. Occupying the top tier of the hierarchy is the relevant international institution, which I shall call a SNGA. This SNGA could be an organization such as the World Bank, or the Commission on Sustainable Development (CSD) created in Agenda 21 at the Rio Earth Summit. The second and third tiers of the hierarchy consist of the government and a representative polluting firm in each DC. Each fork of the hierarchy corresponds to a single DC, and there are \( N \) such forks/countries. Three-tiered hierarchies have been studied by Tirole (1986), Kofman and Lawarree (1993), and Batabyal (1996, forthcoming). Batabyal (forthcoming) compares a two-tiered hierarchy with a three-tiered hierarchy and shows that IEAs are greatly affected by the existence of budget balance and pollution reduction ceiling constraints negotiated by the SNGA and a DC government. However, to the best of my knowledge, the problem of \textit{ex ante} versus \textit{ex post} contracting with possible government/firm collusion has not been studied to date.

As such, I shall apply the theory of hierarchies to study \textit{ex ante} and \textit{ex post} contracting between the SNGA, national governments, and polluting firms in the various countries. The rationale for such contracting stems from issues such as the harmful atmospheric effects of carbon

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\(^8\)Specifically in its role as an administrator of the Global Environmental Facility (GEF).

\(^9\)The reader will note that in this modeling scheme, I have conferred, on the SNGA, the role of principal. As such, there is a distinct asymmetry in the assumed power of the SNGA as opposed to that of governments and firms. Given that I am interested in DCs, which typically have limited bargaining power in their dealings with international institutions owing to the fact that their monetary contributions to the budgets of such organizations are minimal, this hierarchical modeling scheme would appear to be appropriate. For more on the power of SNGAs over DCs, see Mosley, Harrigan, and Toye (1991).
The incidence of pollution may be domestic or transboundary. The key element of uncertainty stems from the SNGA's lack of knowledge about the pollution abatement technology/capability available in each country. This lack of knowledge about abatement capability is the source of imperfect information. Whereas the firm in the DC always knows its technology and the government does too in some states of nature, the SNGA is never privy to this information. The random variable denoting the private information about pollution abatement capability is uncorrelated across countries. My analysis holds for any finite set of countries, with the SNGA/government/firm interaction in one country being independent of the SNGA's dealings with some other country. Hence, without loss of generality, I shall focus on an arbitrary country, say $j$, in the finite set of countries. The SNGA's task is to design incentive compatible and collusion-proof ex ante and ex post IEAs, which can be implemented in a Bayes-Nash equilibrium.

The rest of this paper is organized as follows. In section 3, I describe the model in detail, and I study the properties of the first-best optimum. In section 4, I study ex ante and ex post contracting, with no collusion by the firm and the government. In section 5, I study ex ante and ex post contracting with possible collusion by the government and the representative polluting firm. The ex ante versus ex post distinction is important. Theoretically, this distinction reflects the symmetric (but imperfect) information versus asymmetric information cases. In a practical setting, the relevant issue concerns the possible need to limit the ex post liability of the players in the various

10See Crane (1993) and Paarlberg (1993) for a discussion of the relevance of international institutions when the incidence of an environmental externality is domestic.

11Because I am analyzing a Bayesian game, i.e., a static game of incomplete information, the appropriate equilibrium concept is that of a Bayes-Nash equilibrium. This is standard usage in game theory. For more details, see Gibbons (1992, Chapter 3).
nations in order to get them to voluntarily participate in the IEA. Thus, we can think of ex post contracting as a case of contracting with limited liability constraints. Alternately, we can think of the ex ante versus ex post distinction as a reflection of the power of the SNGA. If the SNGA is sufficiently powerful to get a country to contract before the resolution of the uncertainty regarding pollution abatement capability, then the contract is ex ante. Presumably, this is what an SNGA would like to do. However, it is possible that some countries will refuse to participate until the resolution of the uncertainty because ex ante participation might result in losses to polluting firms. In this case, an individually rational contract will be ex post.

The reasons for studying collusion between the polluting firm and the DC government are threefold. First, while the DC government participates in the IEA because it recognizes the value of such international participation, this government also acts as the polluting firm’s advocate. This aspect of the problem will give rise to scenarios in which government/firm collusion becomes a desirable option. Second, the government and the firm receive monetary transfers from the SNGA for their roles in abating pollution. Further, both of these players know that the SNGA cannot monitor their activities owing to sovereignty or, for that matter, enforce the terms of the IEA in the event of a contractual breach. As such, there will be circumstances in which there are incentives for the government and the firm in each country to collude to maximize the transfers received from the SNGA. Third, as Mookherjee and Png (1995) have noted, corruption is an endemic part of public life in many developing countries. This suggests a need for explicitly modeling the activities of potentially corruptible players. Due to these three reasons, an important part of this paper will

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12See Peterson (1993) for a discussion of some instances of possible government/firm collusion in an international setting.
consist of analyzing collusion-proof contracts.¹³

3. THE THEORETICAL FRAMEWORK

**Description of the Model**

Subscript $i = 1, 2, 3, 4$ will refer to the state of nature, and superscript $j = 1, \ldots, N$ will refer to the country. $\theta$ denotes the uncertainty about the pollution abatement technology/capability that is currently available; $\theta$ has binary support $[\hat{\theta}, \hat{\theta}]$, where $0 < \hat{\theta} < \hat{\theta}$, and $\Delta = \hat{\theta} - \hat{\theta}$. I shall refer to $\hat{\theta}$ as the low-abatement capability parameter and to $\hat{\theta}$ as the high-abatement capability parameter.

The risk-averse firm produces clean air, whose output and value are denoted by $x, x \in \mathbb{R}_+$. This firm chooses abatement $a, a \in \mathbb{R}_+$. The firm’s cost of abatement is $g(a)$, where $g'(\bullet) > 0$, $g''(\bullet) > 0$, and $g(0) = 0$. The firm has a differentiable net payoff from abatement function $B[T_i - g(a_i)]$ with $B \{ \bullet \} \in (0, \infty), \forall i$. $T_i \in \mathbb{R}$ is the state $i$ monetary transfer made by the SNGA to the firm for abating pollution. The firm’s reservation payoff is $B_r = B[T_r]$, and $T_r \geq 0$ is the reservation transfer. $B_r$ and $T_r$ are common knowledge.

The DC government is risk-averse; it has a strictly concave and differentiable utility function $V(G_i)$, where $G_i \in \mathbb{R}_+$ is the state $i$ monetary transfer made by the SNGA to the government for participating in the IEA. The government’s reservation utility is $V_r = V(G_r)$, where $G_r \in \mathbb{R}_+$ is the reservation transfer, and $V(G_i) \in (0, \infty), \forall G_i$. By employing a monitoring device, the government receives a signal, $s$, from the firm regarding its private information and then it sends a report, $r$, to

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¹³I do not discuss the manner in which the SNGA raises revenue. One possibility would be to conform to the text of Agenda 21. According to this document, developed countries are supposed to contribute 0.7% of their GNP for the purpose of environmental protection. For more details, see Rogers (1993, pp. 151-160). Second, I do not impose a budget balance constraint on the SNGA’s decision problem. This issue has been addressed in Batabyal (forthcoming).
the SNGA indicating what it observed about the firm’s pollution abatement capability parameter. In some states, this monitoring device malfunctions, and, hence in these states, the government will not be able to provide the SNGA with a useful report. On receiving $r$, the SNGA offers the government a transfer $G_i \in \mathbb{R}_+$. Making the government’s central task one of reporting is consistent with the government/SNGA interaction proposed for one SNGA, the Commission on Sustainable Development. As noted by Rogers (1993, p. 310), a key aspect of this interaction involves the “... Commission’s... considering information provided by governments...”

The SNGA is risk-neutral and he has a welfare function which takes the form

$$U = \sum_j (a_j + \theta_j - G_j - T_j), \quad j = 1, \ldots, N,$$

where $j$ runs over the total number of countries. Clean air produced in country $j$ is $x_j = a_j + \theta_j$. As stated, the SNGA’s welfare is the difference between total clean air and the sum of government and firm transfers. In the rest of this paper, I shall suppress the country superscript; it is understood that the focus is on country $j$. The contract can only be conditioned on what the SNGA actually observes, i.e., the government’s report, $r$, and the firm’s production of clean air, $x$.

There are four states of nature, each occurring with probability $p_j > 0$, where $\sum_j p_j = 1$. I shall consider ex ante and ex post contracting. In the ex ante case, the SNGA, the government, and the firm sign the contract holding symmetric but imperfect information about $\theta$. In the ex post case, the signing of the contract takes place after the resolution of the uncertainty. The firm always observes $\theta$ before choosing its abatement level. The government, on the other hand, may or may not observe the firm’s private information. This depends on whether the government’s monitoring device functions or malfunctions. Hence, the government’s signal, $s$, may or may not be informative. I can now characterize the four states as follows:
*State 1: The firm and the government both observe $\Theta$.

*State 2: The firm observes $\Theta$ and the government observes nothing.

*State 3: The firm observes $\Theta$ and the government observes nothing.

*State 4: The firm and the government both observe $\Theta$.

In state 1, the firm and the government observe the low-abatement capability parameter. The government's monitoring device works and hence yields useful information. In state 2, the firm observes the low-abatement capability parameter, but the government observes nothing. In this state, the government's monitoring device malfunctions. In state 3, the firm observes the high-abatement capability parameter and the government observes nothing. Once again, the government's monitoring device malfunctions. Finally in state 4, the firm and the government observe the high-abatement capability parameter.

The timing of the game between the SNGA, the government, and the firm in the *ex ante* case is as follows. First, the SNGA offers a contract to the government and the firm. Second, the firm observes $\Theta$ and the government receives its signal, $s$. Third, the firm chooses abatement, $a$. Fourth, clean air, $x$, is produced by the firm and the government sends its report, $r$, to the SNGA indicating what it observed. Finally, the SNGA compensates the government and the firm with transfers, $G(x, r)$ and $T(x, r)$. When contracting is *ex post*, the uncertainty is resolved first and then the contract is signed by the players.

I shall assume that the SNGA can verify the veracity of the government report, $r$. In other words, if the government's signal, $s$, is noninformative, then the corresponding report, $r$, reflects that fact and the SNGA can verify that the true facts are as they have been reported. In symbols, $s = 0 \rightarrow r = 0$. On the other hand, to keep the SNGA's design problem interesting and to allow for
the possibility of government/firm collusion, I shall permit the government to lie and report that its signal is noninformative, when, in fact, such is not the case. That is, \( s = \theta \rightarrow r \in \{\theta, 0\} \). This completes my description of the model. I now consider the benchmark case in which perfect information is acquired by the SNGA.

**The First-Best Optimum**

In this case, the SNGA observes the abatement capability parameter and the firm's pollution abatement choice. Hence, the SNGA bypasses the government and contracts with the firm directly. Since the government has no role to play, it receives its reservation transfer, \( G_r \), and hence its reservation utility, \( V_r \), in all states. The SNGA solves \( \max_a [a + \theta - g(a)] \). The first-order necessary condition is \( g'(a) = 1, \forall \theta \). In other words, in the first-best optimum, the marginal cost of pollution abatement is set equal to the SNGA's marginal welfare from abatement. The optimal level of abatement, \( a^* \), is independent of the state. The firm receives a transfer for abating pollution; this transfer too is independent of the state. The total monetary transfer equals, \( T_r + g^* \), where \( T_r \) is the reservation transfer and \( g^* = g(a^*) \). I now move on to the more interesting cases in which the SNGA cannot observe \( \theta \) or the actual abatement undertaken by the firm.

4. **THE NO-GOVERNMENT/FIRM COLLUSION CASE**

**Ex Ante Contracting**

Since the contracting is *ex ante*, the SNGA, the government, and the firm share imperfect but symmetric information about \( \theta \). For the time being, I shall disallow the possibility of government/firm collusion. When the government is paid, \( G_r \), it obtains its reservation utility, \( V_r \), and hence it is fully insured. Further, since I am not allowing for collusion between the government
and the firm as yet, and, because the SNGA can verify the government’s report by paying $G_r$, the SNGA obtains the government’s information at least cost. This means that the three-tiered hierarchy reduces to a two-tiered hierarchy in which the government plays a completely passive role.

The SNGA’s problem now is to solve

$$\max_{(T, a_i)} \sum_{i \epsilon I} p_i (a_i + \theta_i - T_i)$$

subject to (1a) $\sum_{i \epsilon I} p_i B[T_i - g(a_i)] \geq B_r$, (1b) $T_3 - g(a_3) \geq T_2 - g(a_2 - \Delta \theta)$, and (1c) $T_2 - g(a_3) \geq T_3 - g(a_3 + \Delta \theta)$.

Constraint (1a) is the firm’s individual rationality constraint. Because the contracting is ex ante, there is a single probabilistically weighted constraint. Constraints (1b) and (1c) are the firm’s incentive compatibility constraints. These constraints arise because the SNGA has imperfect information about $\theta$ in these two states. These are also the states in which the government’s signal, $s$, is noninformative. Constraint (1b) says that in state 3, the firm should not claim that the state is 2. Similarly, (1c) says that in state 2, the firm should not claim that the state is 3. I can now solve the SNGA’s problem as stated in (1)-(1c). I am led to

**Theorem 1:** The optimal IEA is one in which (i) the SNGA obtains the government’s information at least cost, (ii) the government’s reward is $G_r$ in all states, (iii) the abatement levels satisfy $a_1 = a_3 = a_4 = a_4 > a_2$, (iv) the firm transfers satisfy $T_3 > T_1 = T_4 > T_2$, and (v) all the constraints except (1c) bind at the optimum.

**Proof:** See the Appendix.

Theorem 1 describes the pattern of abatement one may expect to observe in my stylized $N$ country world in which the SNGA contracts independently with the government and the firm in each DC. Since the SNGA acquires the government’s information in states 1 and 4 and because this
information is verifiable, the firm is required to abate pollution at the first-best level. The optimal IEA then specifies equal rewards to the firm in these two states. On the other hand, when the state is 2 or 3, the SNGA’s information is imperfect. The optimal IEA now specifies a higher transfer in state 3, to prevent the firm from lying about its true pollution abatement capability. The optimality of the IEA stems in part from the feature that the SNGA rewards high pollution abatement with a high transfer and “punishes” low pollution abatement with a low transfer. The level of abatement in state 2 is lower than the level in the other states. This makes it less desirable to abate pollution at a low level in state 3.

**Ex Post Contracting**

I now consider the case where the SNGA is unable to contract with the government and the firm until the resolution of the uncertainty about the abatement capability parameter. Once again, I shall disallow the possibility of government/firm collusion. With no collusion, the government plays a passive role. It receives its reservation utility and hence it is fully insured. In this case, the SNGA’s problem is to solve

\[
\max_{(a_i, T_i)} \sum_{i} p_i (a_i + \theta_i - T_i)
\]

subject to (2a) \( B[T_i - g(a_i)] \geq B, \forall i \), and constraints (1b) and (1c).

As opposed to the *ex ante* case, in the *ex post* case, it must be individually rational for the firm to contract with the SNGA in every state. As a result, the participation constraint must be satisfied in each state. Further, this setting is characterized by asymmetrically held information. That is, whereas the SNGA does not know the state of nature, the firm does. The timing of the game now is such that the uncertainty is resolved first and then the players contract. The optimal contract now has the properties stated in
**Theorem 2:** The optimal *ex post* IEA is one in which (i) the SNGA obtains the government’s information at least cost, (ii) the government’s transfer is $G_r$, in all four states, (iii) $a_1 = a_3 = a_4 = a_\ast > a_2$, (iv) $T_3 > T_1 = T_4 > T_2$, and (v) at the optimum, all the constraints bind except (1c) and (2a, for $i = 3$).

**Proof:** See the Appendix.

I now comment on some aspects of the optimal *ex ante* and *ex post* contracts. Inspecting Table 1, we see that there is no material difference in the two IEAs. In particular, because the government’s report is verifiable, and because it does not collude with the firm, optimal insurance for the firm in both IEAs require that $T_1 = T_4$. Second, in these no-collusion cases, incentive problems are limited to states 2 and 3. In these states, the optimal IEA must reward truth telling. As such, in both IEAs we have $T_3 > T_2$. Finally, in the *ex ante* and in the *ex post* IEAs, $T_3 > T_1$. This is because, in both the contracting scenarios, the SNGA has to make a higher than zero profit transfer in the favorable state (state 3) so as to preserve the incentive for the firm to not lie and abate pollution at the level appropriate for state 2. Moreover, in the *ex post* case, in state 3, the incentive

<table>
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<th>TABLE 1: The No-Government/Firm Collusion Case: <em>Ex Ante</em> versus <em>Ex Post</em> IEAs</th>
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compatibility constraint is always more restrictive than the participation constraint. This explains why the former constraint binds and the latter constraint is slack in equilibrium. I now consider the effects of government/firm collusion on the optimal IEA designed by the SNGA.

5. THE GOVERNMENT/FIRM COLLUSION CASE

Ex Ante Contracting

Recall that because countries are sovereign, the SNGA is unable to either monitor the actions of the government and the firm or enforce the terms of the IEA in the event of a contractual breach. Since the SNGA can never acquire the firm’s private information and must rely on the government’s report to design the optimal contract, an efficient contract must allow for and preclude the possibility of the government and the firm colluding to maximize the transfers received from the SNGA. In other words, an efficient contract must be individually rational, incentive compatible, and collusion-proof.

I shall model collusion between the government and the firm as follows. In the ex ante case, before the resolution of the uncertainty about abatement capability and at the time of signing the contract between the SNGA, the government, and the firm, the firm and the government sign a side contract which entails the offer of a bribe from the firm to its government. Naturally, this secondary contract is unobservable by the SNGA. The bribe, \( b(r, x) \), can only be conditioned on what the firm and the government observe, i.e., the report, \( r \), and clean air, \( x \). With the offer and acceptance of this bribe, the firm’s transfer becomes \( \{ T(r) - b(r, x) \} \) and the government’s transfer becomes \( \{ G(r) + b(r, x) \} \).
Collusion by the firm and the government alters the incentives of the various parties and, as we shall see, the nature of the optimal contract offered by the SNGA. To see why the firm might want to bribe its government, consider state 4. In this state, the government is indifferent between reporting that it has observed $\hat{\theta}$ and reporting that it has observed $0$. However, the firm would prefer that the government report $0$. This is one instance in which a clear rationale exists for the firm to bribe its government.

In order to solve the SNGA’s problem when there is collusion, I shall appeal to the equivalence principle (Tirole, 1986, p. 195) and restrict myself to contracts that are collusion-proof. Tirole’s method involves imposing constraints in addition to the usual participation and incentive compatibility constraints for the government and the firm. These additional constraints are designed to preclude government/firm collusion and hence make the main contract collusion-proof. I can now formulate the SNGA’s problem. Denoting the collusion-proof transfers to the government and the firm by $\bar{G}$ and $\bar{T}$, the SNGA solves

$$\max_{(\bar{G},\bar{T},a)} \sum_{i} p_{i}(a_{i}+\theta_{i}-\bar{G}_{i}-\bar{T}_{i})$$

subject to (1a)-(1c), (3a) $\sum_{i} p_{i}V(\bar{G}_{i}) \geq V_{r}$, (3b) $\bar{G}_{1}+\bar{T}_{1}-g(a_{1}) \geq \bar{G}_{2}+\bar{T}_{2}-g(a_{2})$, (3c) $\bar{G}_{4}+\bar{T}_{4}-g(a_{4}) \geq \bar{G}_{3}+\bar{T}_{3}-g(a_{3})$, (3d) $\bar{G}_{3}+\bar{T}_{3}-g(a_{3}) \geq \bar{G}_{2}+\bar{T}_{2}-g(a_{2}+\Delta \theta)$, and (3e) $\bar{G}_{2}+\bar{T}_{2}-g(a_{2}) \geq \bar{G}_{3}+\bar{T}_{3}-g(a_{3}+\Delta \theta)$.

Constraint (3a) is the government’s participation constraint. Constraints (3b) and (3c) are the core collusion constraints. Recall that in states 1 and 4, the government’s signal, $s$, is informative. In these two states, the government can hide this fact. Given this, constraints (3b) and (3c) tell us that should the firm successfully bribe its government, the total sum of the transfers less the cost of abatement in states 1 and 4 cannot be less than the corresponding totals in states 2 and...
Finally, constraints (3d) and (3e) tell us that it must not be possible for the government to bribe the firm. More specifically, (3d) tells us that in state 3, the government should not be able to bribe the firm to abate at the level that is appropriate for state 2. Similarly, (3e) tells us that the government should not be able to bribe the firm to claim that the state is 3 when it is actually 2. Solving the SNGA's problem (3) subject to (1a)-(1c), (3a)-(3e), I can state

**Theorem 3:** The optimal IEA in the three-tiered hierarchy with government/firm collusion is one in which (i) \( a_1 = a_3 = a_4 = a_* > a_2 \), (ii) \( \bar{G}_4 > \bar{G}_1 > \bar{G}_2 = \bar{G}_3 \), (iii) \( \bar{T}_3 > \bar{T}_4 > \bar{T}_1 > \bar{T}_2 \), (iv) \( \bar{G}_4 + \bar{T}_4 = \bar{G}_3 + \bar{T}_3 \), and (v) all constraints except (1c), (3b), and (3e) bind at the optimum.

**Proof:** See the Appendix.

The first thing to note is that the SNGA is worse off when the government and the firm collude. This is because, in the collusion case, the number of binding constraints exceeds the number of binding constraints in the no-collusion case. However, if the SNGA does offer the IEA with the characteristics described in Theorem 3, then his total monetary transfers cannot be altered by changing the government's report or the firm's abatement level.

I now comment on some of the noteworthy features of the IEA described in Theorem 3. From Theorem 3(i) and Table 2, we see that, like the no-collusion case, the first best level of abatement can be required of the firm in states 1, 3, and 4. The optimal level of pollution abatement in the low-abatement capability state is lower than the first best level. This tells us that collusion *per se* has no effect on the pattern of equilibrium pollution abatement.
**TABLE 2: Ex Ante Contracting Without and With Collusion**

<table>
<thead>
<tr>
<th>IEA</th>
<th>Without Collusion</th>
<th>With Collusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pollution abatement pattern</td>
<td>$a_1 = a_3 = a_4 = \alpha &gt; a_2$</td>
<td>$a_1 = a_3 = a_4 = \alpha &gt; a_2$</td>
</tr>
<tr>
<td>Transfers to the government</td>
<td>$G_r\quad \bar{G}_4 &gt; \bar{G}_1 &gt; \bar{G}_2 = G_3$</td>
<td></td>
</tr>
<tr>
<td>Transfers to the polluting firm</td>
<td>$T_3 &gt; T_4 = T_1 &gt; T_2$</td>
<td>$\bar{T}_3 &gt; \bar{T}_4 &gt; \bar{T}_1 &gt; \bar{T}_2$</td>
</tr>
</tbody>
</table>

Part (ii) and Table 2 tell us that, in the collusion case, the government is rewarded for the usefulness of its report. In states 2 and 3, the government reports truthfully. Thus, optimal insurance requires that transfers to the government in these two states be equal. On the other hand, in state 4, the government’s reward must be high. A similar line of reasoning applies to state 1. Further, the transfers in states 4 and 1 exceed those in states 2 and 3 because, in states 4 and 1, the government may lie, and hence the SNGA has to create the right incentives to induce truth telling. This is in contrast to the no-collusion case in which the government plays a passive role and receives its reservation transfer in all four states.

Part (iv) tells us that the total transfers from the SNGA to the government and the firm in states 3 and 4 are equal. However, by part (iii), the transfers to the firm between these two states vary. Why is this so? In state 3, the firm can lie about the abatement capability parameter that it has observed, and the government will not be able to tell the difference between truth telling and lying because its signal is noninformative. In order to prevent lying, the firm’s reward in state 3 must be higher. In state 4, the government’s signal is informative. Now the government has to be induced to report truthfully with a higher transfer, and the firm’s reward is correspondingly lower. From
Table 2, we see that in the no-collusion case, \( T_4 = T_1 \), because the SNGA acquires the government’s verifiable information at least cost and because the government reports truthfully. On the other hand, in the collusion case, \( \tilde{T}_4 > \tilde{T}_1 \). This is because the SNGA must create incentives so that the dual objectives of preventing collusion and encouraging the firm to act truthfully in the high abatement capability state are achieved.

Finally, part (v) tells us that (3b) is slack at the optimum. This is because when the firm observes the low-abatement capability parameter, the government’s report does not make a difference since the firm voluntarily prefers to abate pollution at the low level.

**Ex Post Contracting**

I now study the case in which there is government/firm collusion but where the SNGA is unable to get the parties to contract until the uncertainty about the abatement capability parameter has been resolved. I denote the government and the firm transfers by \( \tilde{G} \) and \( \tilde{T} \), respectively. The SNGA now solves

\[
\max_{\{\tilde{G}, \tilde{T}, a_i\}} \sum_{s_i} p_i (a_i + \theta_i - \tilde{G}_i - \tilde{T}_i)
\]

subject to (1b), (1c), (2a), (3b)-(3e), and (4a) \( V(\tilde{G}_i) \geq V_r, \forall i \). The optimal IEA has the properties stated in

**Theorem 4:**

(i) \( a_1 = a_3 = a_4 = a_* > a_2 \), (ii) \( \tilde{G}_4 > \tilde{G}_1 = \tilde{G}_2 = \tilde{G}_3 = \tilde{G}_r \), (iii) \( \tilde{T}_3 > \tilde{T}_1 > \tilde{T}_2 \) and \( \tilde{T}_4 > \tilde{T}_1 > \tilde{T}_2 \), (iv) \( \tilde{G}_3 + \tilde{T}_3 = \tilde{G}_4 + \tilde{T}_4 \), and (v) at the optimum all the constraints bind except (1c), (2a, for \( i = 3,4 \)), (3e), and (4a, for \( i = 4 \)).

**Proof:** See the Appendix.

A comparison of the optimal *ex ante* and *ex post* contracts with government/firm collusion
can be made with the aid of Table 3. There are three essential differences. First, while the *ex ante* contract specifies $\widetilde{G}_4 > \widetilde{G}_1 > \widetilde{G}_2 = \widetilde{G}_3$, the *ex post* contract specifies $\widetilde{G}_4 > \widetilde{G}_1 = \widetilde{G}_2 = \widetilde{G}_3 = G_r$. In both cases, the government’s signal is noninformative in states 2 and 3. Hence, the optimal *ex ante* and *ex post* contracts specify $\widetilde{G}_2 = \widetilde{G}_3$. Further, in both cases the government can lie about its signal in states 1 and 4. In order to induce truth telling by the government, the optimal contract specifies $\widetilde{G}_4 > \widetilde{G}_1$. However, in the *ex post* case, $\widetilde{G}_1 = \widetilde{G}_2$. This is because in both these states the participation constraints bind at the optimum. Second, in the *ex ante* case $\widetilde{T}_3 > \widetilde{T}_4 > \widetilde{T}_1 > \widetilde{T}_2$, whereas in the *ex post* case $\widetilde{T}_3 > \widetilde{T}_4 > \widetilde{T}_2$ and $\widetilde{T}_4 > \widetilde{T}_1 > \widetilde{T}_2$. In both cases, $a_4 = a_1 > a_2$. As such, both contracts specify $\widetilde{T}_1 > \widetilde{T}_2$. Note that whereas in the *ex ante* case $\widetilde{T}_3 > \widetilde{T}_4$, in the *ex post* case, a direct comparison of $\widetilde{T}_3$ and $\widetilde{T}_4$ cannot be made. This is because the participation constraints in these two states are slack in equilibrium.\(^{14}\) Third, while in the *ex ante* case, three constraints—(1c), (3b), and (3e)—are slack in the optimum; in the *ex post* case, five constraints—(1c) (2a, for $i = 3, 4$), (3e), and (4a, for $i = 4$)—are slack. This tells us that the SNGA will prefer *ex ante* contracting to *ex post* contracting when there is government/firm collusion.

Finally consider Table 4, which describes the differences in the optimal *ex post* IEA, without and with collusion. We see that collusion has no impact on the pattern of equilibrium abatement. In both scenarios, the government earns its reservation transfer in three of the four states. This is because, in the no-collusion case, the government plays a passive role. In the collusion case, the government’s active role notwithstanding, the government typically earns no rents because three of the four participation constraints bind. In the no-collusion case, the passive government’s reward in state 4 is $G_r$. As contrasted to this, in the collusion case, the SNGA precludes collusion in state 4

\(^{14}\)Also see the Appendix for step 11 in the proof of Theorem 4.
TABLE 3: The Government/Firm Collusion Case: Ex Ante versus Ex Post Contracting

<table>
<thead>
<tr>
<th>IEA</th>
<th>Ex Ante</th>
<th>Ex Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pollution abatement pattern</td>
<td>( a_1 = a_3 = a_4 = a_5 &gt; a_2 )</td>
<td>( a_1 = a_3 = a_4 = a_5 &gt; a_2 )</td>
</tr>
<tr>
<td>Transfers to the government</td>
<td>( G_4 &gt; G_1 &gt; G_2 = G_3 )</td>
<td>( G_4 &gt; G_1 = G_2 = G_3 = G_r )</td>
</tr>
<tr>
<td>Transfers to the polluting firm</td>
<td>( \bar{T}_3 &gt; \bar{T}_4 &gt; \bar{T}_1 &gt; \bar{T}_2 )</td>
<td>( \bar{T}_3 &gt; \bar{T}_1 &gt; \bar{T}_2 )</td>
</tr>
<tr>
<td>SNGA’s total transfers in states</td>
<td>( \bar{G}_4 + \bar{T}_4 = \bar{G}_3 + \bar{T}_3 )</td>
<td>( \bar{G}_4 = G_r = \bar{T}_3 - \bar{T}_4 )</td>
</tr>
<tr>
<td>3 and 4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE 4: Ex Post Contracting Without and With Collusion

<table>
<thead>
<tr>
<th>IEA</th>
<th>Without Collusion</th>
<th>With Collusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pollution abatement pattern</td>
<td>( a_1 = a_3 = a_4 = a_5 &gt; a_2 )</td>
<td>( a_1 = a_3 = a_4 = a_5 &gt; a_2 )</td>
</tr>
<tr>
<td>Transfers to the government</td>
<td>( G_r )</td>
<td>( G_4 &gt; G_1 = G_2 = G_3 = G_r )</td>
</tr>
<tr>
<td>Transfers to the polluting firm</td>
<td>( T_3 &gt; T_4 = T_1 &gt; T_2 )</td>
<td>( \bar{T}_3 &gt; \bar{T}_1 &gt; \bar{T}_2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \bar{T}_4 &gt; \bar{T}_1 &gt; \bar{T}_2 )</td>
</tr>
</tbody>
</table>

by setting \( \bar{G}_4 > G_r \). Along the same lines, the SNGA alters the firm transfers in the collusion case.

When there is no collusion, \( T_1 = T_4 \). In the collusion case, the SNGA makes the optimal IEA collusion-proof by setting \( \bar{T}_4 > \bar{T}_1 \).
6. CONCLUSIONS

In this paper I analyzed the question of environmental protection for developing countries within the framework of the directives set forth in the agreements reached at the 1992 Rio Earth Summit. I modeled the institutional setting for the underlying problem as a three-tiered hierarchy with $N$ forks, and then I studied the nature of the optimal ex ante and ex post IEAs, without and with collusion. A number of policy conclusions emerge.

First, the pattern of pollution abatement is generally insensitive to the form of contracting. That is, irrespective of whether contracting is ex ante or ex post, and irrespective of whether there is collusion, the first best level of abatement can be required by the SNGA in three of the four states. In a practical setting, this tells us that the contractual procedure and the SNGA’s inability to monitor the actions of the government and the firm make no difference to the basic design question as long as the SNGA can (i) get individual nations to participate in the contracting process, and (ii) create incentives so that the relevant parties reveal their private information truthfully. Indeed, this paper suggests that problems stemming from an SNGA’s lack of monitoring and enforcement powers are far less significant than is generally believed. Alternately put, national sovereignty appears to be less of an issue than is commonly believed. As I have shown, in a number of circumstances, the pattern of pollution abatement is almost ideal.

Second, the SNGA will, in general, prefer ex ante contracting to ex post contracting. This is because ex ante contracting (i) is characterized by symmetric, as opposed to asymmetric, information, and (ii) involves fewer binding constraints. However, it should be noted that, in the context of DCs, unless the SNGA can limit the ex post liability of the players, nations may well refuse to participate in ex ante contracting schemes.
Third, as compared to the *ex ante* transfers, the *ex post* transfers are somewhat more difficult to pin down definitively. Several observers, such as Rogers (1993, p. 236), have worried that many of the Earth Summit directives “... offer a back door option by which signatories can excuse themselves at a later date if the going gets too tough.” The implementability of *ex post* contracts should diminish such concerns because an *ex post* contract is like a limited liability contract. In this sense, as compared to an *ex ante* contract, an *ex post* contract is more likely to be renegotiation-proof.

Fourth, by making the contracting process country specific, particularly the award of transfers, I have emphasized the heterogeneity of developing regions (countries). However, this approach precludes the possibility of holding similar regions (countries) to similar environmental standards. Further, by not imposing a budget balance on the SNGA’s decision problem, I have finessed the possibility of DCs competing among themselves for a greater share of the SNGA’s budget for environmental protection. To address these aspects of the problem adequately, one would have to model the IEA design problem in a framework which permits relative performance evaluation. A start in this direction has been made in Batabyal (forthcoming).

With talk of rising disparity between the developing and the developed countries, and the increasingly acrimonious nature of international discussions regarding the use of environmental resources, the design question studied in this paper takes on particular significance. This is in no small measure due to the fact that the implementation of such mechanisms will do more to engender and maintain international security than will most strategic or unilateral policy measures.
REFERENCES


APPENDIX

In this appendix, I provide the proofs of Theorems 1 and 4. Both proofs involve the Kuhn-Tucker analysis. The proofs of Theorems 2 and 3 are available from the author on request.

Proof of Theorem 1: I shall proceed by means of five steps. Omitting (1c) temporarily, the Lagrangian for problem (1) is

\[ \mathcal{L} = \sum_{i \neq i} p_i (a_i + \theta_i - T_i) + \alpha \left\{ \sum_{i \neq i} p_i B \left[ T_i - g(a_i) \right] - B_r \right\} + \beta \left\{ T_3 - g(a) - T_2 + g(a_2 - \Delta \theta) \right\}, \] (A1)

where \( \alpha \) and \( \beta \) are the multipliers corresponding to (1a) and (1b), respectively. For \( i = 1,3,4 \), this Lagrangian depends on \( a_i \) only through \((a_i - T_i)\) and \( \{T_i - g(a_i)\}\). Thus, for \( i = 1,3,4 \), it suffices to maximize \( \{a_i - g(a)\} \) over \( a_i \). This yields \( g'(a_i) = 1 \Rightarrow (g')^{-1}(1) = a_*, a, i = 1,3,4 \). The remaining first-order conditions are (A1a) \( \alpha \{ \partial B[\bullet]/\partial T_1 \} = 1 \), (A1b) \( \alpha \{ \partial B[\bullet]/\partial T_2 \} = 1 + \beta/p_2 \), (A1c) \( \alpha \{ \partial B[\bullet]/\partial T_3 \} = 1 - \beta/p_3 \), (A1d) \( \alpha \{ \partial B[\bullet]/\partial T_4 \} = 1 \), and (A1e) \( 1 + (\beta/p_2) g''(a_2 - \Delta \theta) = \{ 1 + \beta/p_2 \} g''(a_2) \).

Step 1: At the optimum, (1b) binds.

Proof: Suppose not. Substituting \( \beta = 0 \) in (A1e), we see that the first-best optimum results. But this is incentive incompatible for the firm. Thus, \( \beta > 0 \).

Step 2: \( a_2 < a_* \).

Proof: (A1e) tells us that \( g'(a_2) < 1 \). But \( g''(\bullet) > 0 \) and \( g'(a_i) = 1, i = 1,3,4, a_2 < a_* \).

Step 3: \( T_1 = T_4 \).

Proof: This follows from a comparison of (A1a) and (A1d).

Step 4: \( T_3 > T_2 \).

Proof: From step 1, \( \beta > 0 \). This, combined with the fact that

\[ a_3 > a_2 \Rightarrow g(a_3) > g(a_2 - \Delta \theta) \Rightarrow T_3 > T_2. \]
Step 5: \( T_3 > T_1 \).

Proof: Compare (A1a) and (A1c), recalling that \( \partial B[\bullet] / \partial T_i > 0 \), and that
\[
\{(1/\alpha) - (\beta/\alpha p_3)\} > 0.
\]

Finally, I have to show that (1c) is satisfied. This is equivalent to showing that 
\[
g(a_3 + \Delta \theta) - g(a_3) > g(a_2) - g(a_2 - \Delta \theta),
\]
This last inequality holds because \( a_3 > a_2 \), \( \Delta \theta > 0 \), 
\( g'(\bullet) > 0 \), and \( g''(\bullet) > 0 \). This completes the proof of Theorem 1. ■

Proof of Theorem 4: I shall proceed by means of eleven steps. Omitting (1c) and (3c) temporarily, 
the Lagrangian to problem (4) is
\[
\mathcal{L} = \sum_{i,p} \alpha_i (a_i + \theta_i - \bar{G}_i - \bar{T}_i) + \sum_{\nu_i} \alpha_i \{B[\bar{T}_i - g(a_i)] - B_{\nu_i}\} + \sum_{\nu_i} \nu_i \{V(\bar{G}_i) - V_{\nu_i}\} + \beta \{\bar{T}_3 - g(a_3) - \bar{T}_2 + g(a_2)\} + \epsilon_1 \{\bar{G}_1 + \bar{T}_1 - g(a_1) - \bar{G}_2 - \bar{T}_2 + g(a_2)\} + \epsilon_2 \{\bar{G}_4 + \bar{T}_4 - g(a_4) - \bar{G}_3 - \bar{T}_3 + g(a_3)\} + \kappa \{\bar{G}_3 + \bar{T}_3 - g(a_3) - \bar{G}_2 - \bar{T}_2 + g(a_2 - \Delta \theta)\},
\]
where the \( \alpha_i \), the \( \nu_i \), \( \beta \), \( \epsilon_1 \), \( \epsilon_2 \), and \( \kappa \) are the multipliers corresponding to (2a), (4a), (1b), (3b), (3c), and (3d), respectively. As in the proof of Theorem 1, for \( i \neq 2 \), I get \( q = a_+ \). The remaining nine first-order conditions are 
(A2a) \( (\nu_1/p_1) V'(\bar{G}_1) = 1 - \epsilon_1/p_1 \), (A2b) 
(A2c) \( (\nu_3/p_3) V'(\bar{G}_3) = 1 + (\epsilon_2 - \kappa) \), (A2d) 
(A2e) \( (\alpha_1/p_1) \{\partial B[\bullet] / \partial \bar{T}_1\} = 1 - \epsilon_1/p_1 \), (A2f) 
(A2g) \( (\alpha_2/p_2) \{\partial B[\bullet] / \partial \bar{T}_2\} = 1 + (\beta + \epsilon_1 + \kappa)/p_2 \), (A2h) 
(A2i) \( (\alpha_4/p_4) \{\partial B[\bullet] / \partial \bar{T}_4\} = 1 - \epsilon_2/p_4 \),
\[
1 + \{ (\beta + \kappa) \}/p_2 \} g'(a_2 - \Delta \theta) = \{ 1 + (\beta + \kappa) \}/p_2 \} g'(a_2).
\]

Step 1: \( a_2 < a_* \).

Proof: From (A2i) it follows that \( g'(a_2) < 1 \Rightarrow a_2 < a_* \). ■

Step 2: Constraints (1b) and (3d) bind at the optimum.
Proof: I have to show that $\beta > 0$, and that $\kappa > 0$. Suppose not. There are four cases to consider:

(i) $\beta = 0$, (ii) $\beta > 0$, $\kappa = 0$, (iii) $\beta = \kappa = 0$, and (iv) $\beta > 0$, $\kappa > 0$. Now (A2i) can be written as (A2j)

$$\{1 - g'(a_2)\}/\{g'(a_2) - g'(a_2 - \Delta \theta)\} = \{\beta + \kappa\}/p_2.$$  

If case (i) were true, then from (A2j) I get

(A2k) $\kappa = p_2 \{1 - g'(a_2)/\{g'(a_2) - g'(a_2 - \Delta \theta)\} \}$. 

If, on the other hand, case (ii) were true, then from (A2j), I get

(A2l) $\beta = p_2 \{1 - g'(a_2)/\{g'(a_2) - g'(a_2 - \Delta \theta)\} \}$. Since the RHSs of (A2k) and (A2l) are equal, we must have $\beta = \kappa$. This rules out cases (i) and (ii). If case (iii) were true, then the first-best optimum would result. However, this outcome is incentive-incompatible for the firm; hence, case (iii) is ruled out. I conclude that $\beta > 0$, and that $\kappa > 0$. 

Step 3: Constraint (4a, for $i = 3$) binds at the optimum.

Proof: I have to show that $\psi_3 > 0$. Suppose not. Then $\psi_3 = 0$ and (A2c) imply that $\kappa = p_3 + \epsilon_2$. Using this value of $\kappa$ in (A2g), I get

$$\alpha_3 = -\beta/\partial B[\cdot]/\partial \bar{T}_3 < 0.$$ 

However, this is impossible; hence, $\psi_3 > 0$. 

Step 4: At the optimum, $\bar{G}_2 = \bar{G}_3 = G_r$.

Proof: Using $\beta > 0$, $\kappa > 0$, $\psi_3 > 0$ with (3d) gives the claimed result. $\bar{G}_2 = G_r$ means that $\psi_2 > 0$. 

Step 5: Constraint (2a, for $i = 2$) binds at the optimum.

Proof: I have to show that $\alpha_2 > 0$. From (A2f) $\alpha_2 = 0 \Rightarrow p_2 = -\beta + \epsilon_1 + \kappa$, which is impossible irrespective of whether $\epsilon_1 \geq 0$. Thus $\alpha_2 > 0$. 

Step 6: Constraint (2a, for $i = 3$) is slack at the optimum.

Proof: $\beta > 0$, $\alpha_2 > 0$ tell us that

$$\bar{T}_3 - g(a_3) = \bar{T}_2 - g(a_2) + \{ g(a_2) - g(a_2 - \Delta \theta) \} = T_r + \{ D \}, D > 0.$$ 

In turn this tells us that $\bar{T}_3 - g(a_3) > T_r \Rightarrow \alpha_3 = 0$. 

Step 7: Constraints (2a, for $i = 1$) and (4a, for $i = 1$) bind in equilibrium.

Proof: (A2a) says that $\epsilon_1 = \psi_1 = 0$ cannot hold. (A2a) and (A2e) say that either (i) $\alpha_1 = \psi_1 = 0$
or (ii) \( \alpha_1 > 0 \) and \( v_1 > 0 \). If (i) holds, then (3b) is slack and \( \epsilon_1 = 0 \). But this is impossible. Hence, the claimed result follows. ■

**Step 8:** Constraints (2a, for \( i = 4 \)) and (4a, for \( i = 4 \)) are slack at the optimum.

**Proof:** (A2d) and (A2h) say that either (i) \( \alpha_4 > 0 \) and \( v_4 > 0 \) or (ii) \( \alpha_4 = v_4 = 0 \). If (i) holds, then (3c) is violated. Thus \( \alpha_4 = v_4 = 0 \). ■

**Step 9:** \( \bar{G}_4 > \bar{G}_1 = \bar{G}_2 = \bar{G}_3 = G_r \).

**Proof:** This follows from the facts that \( v_1 > 0, v_2 > 0, v_3 > 0 \), and \( v_4 = 0 \). ■

**Step 10:** \( \bar{T}_3 > \bar{T}_1 > \bar{T}_2 \) and \( \bar{T}_4 > \bar{T}_1 > \bar{T}_2 \).

**Proof:** The claimed result holds because \( \alpha_1 > 0, \alpha_2 > 0, \alpha_3 = \alpha_4 = 0 \), and \( \alpha_1 = \alpha_3 = \alpha_4 = \alpha_4 > \alpha_2 \). ■

**Step 11:** At the optimum, (3b) binds; (3c) binds iff \( \bar{G}_4 - G_r = \bar{T}_3 - \bar{T}_4 \).

**Proof:** Constraint (3b) binds because \( \alpha_1 > 0, \alpha_2 > 0, v_1 > 0, v_2 > 0 \). The statement \( \bar{G}_4 + \bar{T}_4 - g(\alpha_4) = \bar{G}_3 + \bar{T}_3 - g(\alpha_3) \) and the facts \( \alpha_3 = \alpha_4 = v_4 = 0 \) and \( v_3 > 0 \) tell us that \( \bar{G}_4 - G_r = \bar{T}_3 - \bar{T}_4 \). It is easy to verify that \( \bar{G}_4 - G_r = \bar{T}_3 - \bar{T}_4 \), \( \alpha_3 = \alpha_4 = v_4 = 0 \), and \( v_3 > 0 \) tell us that (3c) binds. ■

I now have to show that (1c) and (3e) are satisfied. The fact that (1c) is satisfied can be verified in a manner analogous to that used in the proof of Theorem 1. Given that (1c) is satisfied, to see that (3e) is satisfied, note that from Step 9, \( \bar{G}_2 = \bar{G}_3 \). This completes the proof of Theorem 4. ■