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**A THEORETICAL INQUIRY INTO ASPECTS OF THE
STRUCTURE AND THE MANAGEMENT OF
ECOLOGICAL-ECONOMIC SYSTEMS**

by

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May 2000

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ABSTRACT

We use the theory of continuous-time Markov chains (CTMCs) to analyze hitherto unstudied questions about jointly determined ecological-economic systems. Two specific questions are examined. First, on the methodological front, we show how the theory of CTMCs can be used to effectively model dynamic and stochastic ecological-economic systems. Then, given recent concern about the sustainability of desirable states and lock-in into undesirable states, we partition the state space of our stylized ecological-economic system into good and bad states, and demonstrate the formal relationship between these two sets of states. Second, we discuss a way of looking at the task of managing ecological-economic systems that captures this formal link between the good and the bad states, and has other desirable properties.

Key words: Ecological-economic system, Markov process, optimal management

A THEORETICAL INQUIRY INTO ASPECTS OF THE STRUCTURE AND THE MANAGEMENT OF ECOLOGICAL-ECONOMIC SYSTEMS¹

1. Introduction

Discussions about the environmental consequences of economic activities have increasingly come to dominate academic and public debate in most parts of the world. Two principal issues have been articulated by researchers working in this area. First, Holling (1995), Perrings (1996), and Batabyal (1998a, 1998b, 1999a) have argued that ecological and economic systems are jointly determined. This means that the evolution of the joint system reflects the nature of the links between each sub-system. In particular, any alteration in conditions generates two interconnected sets of effects: a set of ecological effects that affect the evolution of the relevant ecological system and a set of economic effects that involve the economic system. On account of this joint determination, the above authors have argued that ecological-economic systems ought to be studied as *one* system.

Second, Dasgupta (1993), Levin *et al.* (1998), and Perrings (1998) have pointed out that although ecological-economic systems can exist in multiple states, not all of these states are equally valuable. The concern here relates to the effects that economic activities have on the evolution of stochastic ecological-economic systems. Specifically, the worry is that with the continuance of economic activities it might *not* be possible to sustain the desirable states of an ecological-economic system. Put differently, ecological-economic systems may get locked-in to undesirable states.

The formal study of these issues is very much in its infancy. Consequently, there are a number of hitherto unstudied questions in this area that warrant research attention. As Perrings

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(1998) has noted, many of these questions are methodological in nature. As such, this paper has two objectives. First, on the methodological front, we show how the theory of continuous-time Markov chains (hereafter CTMCs)² can be used to effectively model a dynamic and stochastic ecological-economic system. Then, given recent concern about the sustainability of desirable states and lock-in into undesirable states, we partition the state space of our stylized ecological-economic system into good and bad states, and demonstrate the formal relationship between these two sets of states. Second, we discuss a way of looking at the task of managing ecological-economic systems that captures this formal link between the good and the bad states, and has other desirable properties.

The rest of this paper is organized as follows. Section 2 presents a CTMC-theoretic model of a stylized ecological-economic system. Section 3 analyzes this model and discusses some of the management implications of this model of an ecological-economic system. Section 4 concludes and offers suggestions for future research.

2. A Stylized Ecological-Economic System

2.1. Preliminaries

Recently, Perrings (1998, p. 517) has suggested that researchers use “a Markov approach...” to study the behavior of jointly determined ecological-economic systems. There are at least three reasons for this suggestion. First, looked at over time, ecological-economic systems are stochastic processes. Second, the Markov approach focuses on the parameters of the underlying stochastic process in a way that accounts for the essential features of important stability concepts in ecology such as persistence and resilience. Third, this approach allows one to construct reasonable

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²A CTMC is a stochastic process with certain specific properties. For more on CTMCs, see Ross (1996, pp. 231-294; 2000, pp. 313-362) and Taylor and Karlin (1998, pp. 333-417).

managerial objectives that account for the ecological *and* the economic aspects of the problem of managing ecological-economic systems. On account of these reasons, in this paper, we follow Perrings's suggestion and model a jointly determined ecological-economic system as an ergodic³ CTMC. We then study this stylized ecological-economic system from a long run perspective.⁴

2.2. The Model

As a result of natural events (fires, hurricanes) and the continuance of economic activities (grazing, fishing), an ecological-economic system will generally change states over time. This means that this ecological-economic system may exist in multiple states. For example, depending on the intensity of grazing, a rangeland ecological-economic system may exist in more or less degraded states. Similarly, depending on the number of fires and the extent of timber logging, a forest ecological-economic system may exist in alternate states. We now formalize these observations.

Following Ross (1996, pp. 251-294), let the states (the state space) of our stylized ecological-economic system $\{Z(t): t \geq 0\}$ be the set of non-negative integers. Next, to capture the change of states, let us say that when the state is i at time t , our ecological-economic system will make a transition to state j with probability $P_{ij}(t)$.⁵ Further, when in state i , this ecological-economic system will leave the i th state at rate v_i . Using this information, let $s_{ij} = v_i P_{ij}$ be the transition rate of our ecological-economic system from state i to state j . Finally, because we are studying this stylized ecological-economic system from a long run perspective, it will be necessary to specify the

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³Ergodic means that all the states of the CTMC communicate with each other and that all the limiting or stationary probabilities are positive. More on this in what follows. For additional details, see Ross (1996, p. 253).

⁴The study of jointly determined ecological-economic systems from a long run perspective is not without precedent. For more on this, see Perrings (1998), Batabyal (1999a, 1999b), and Batabyal and Beladi (1999a, 1999b).

⁵In what follows, when there is no risk of confusion, we shall suppress the time argument in these transition probabilities.

stationary probabilities for this ecological-economic system. Given the transition probability P_{ij} , the stationary probability that our ecological-economic system will be in state j is P_j , where $P_j = \lim_{t \rightarrow \infty} P_{ij}(t)$. As noted in Ross (1996, p. 251), the P_j 's are the unique non-negative solution of

$$\forall j, P_j = \sum_{\forall i} P_i s_{ij}, \quad (1)$$

and

$$\sum_{\forall i} P_i = 1. \quad (2)$$

This completes the description of our model. We now analyze this model and discuss some of the management implications of this model of an ecological-economic system.

3. Analysis and Management Implications

3.1. The Issues

As noted in section 1, an important concern expressed by students of the environmental consequences of economic change is that it might not be possible to sustain the desirable states of an ecological-economic system, and that ecological-economic systems may get locked-in to undesirable states. From the standpoint of the management of ecological-economic systems, this concern can be addressed in the following way. First, identify the “good” and the “bad” states of an ecological-economic system. At any particular time, if the ecological-economic system happens to be in the bad set of states, then take those steps that will move this ecological-economic system to the good set of states and (hopefully) keep it in this set. Possible examples of such steps include moratoriums on grazing and bans on the logging of timber.

On the other hand, if the ecological-economic system currently happens to be in the good set of states, then a different course of action is called for. Now the ecological-economic system

manager's task is to take those steps that will keep the ecological-economic system in this good set and (hopefully) prevent it from moving to the bad set of states. Possible examples of such steps include the frequent monitoring of fish stocks in a fishery and regulated hunting on hunting grounds.

The reader will note that in this view of the management of ecological-economic systems, the critical variable that the manager seeks to control is *time*. In other words, if the ecological-economic system is currently in the good set of states, then the manager's goal is to take actions to *maximize* the *time* that this ecological-economic system spends in the good set. On the other hand, if the ecological-economic system is currently in the bad set of states, then the manager's goal is to take steps that move the ecological-economic system from the bad set to the good set and, once again, *maximize* the *time* that this ecological-economic system spends in the good set. However, because these times are random variables, what the manager actually wishes to maximize is their expectation.

Two other important aspects of this time-based view of management have received scant attention in the literature; consequently, these aspects deserve some comment here. First, this time-based view is consistent with actual management practices. As Batabyal and Beladi (1999b) and Batabyal *et al.* (1999a, 1999b, 1999c) have pointed out, the use of time restrictions—hunting and fishing season length limitations and grazing period restrictions on rangelands—to manage natural resources is pervasive in North America.

Second, there is an interesting link between time-based managerial objectives and the ecological stability property known as persistence. Persistence refers to “how long a variable lasts before it is changed to a new value” (Pimm, 1991, p. 14). An implication of this definition is that persistence is measured in time units. What this means for our purpose is that by maximizing the expected amount of time that the ecological-economic system spends in the good set of states, the

manager will also be maximizing the *persistence* of this good set of states. This is the ecological side of the management task. To see the economic side, recall the section 1 discussion about the sustainability of good states and lock-in into bad states. By maximizing the expected amount of time during which the ecological-economic system is in the good set of states, the manager will also be *sustaining* these good states. Let us now analyze these issues formally.

3.2. Analytics

To capture the notion of good and bad states, let us partition the state space of our ecological-economic system into two subsets, the good states, denoted by G , and the bad states, denoted by B . Suppose that the ecological-economic system initially is in a state in B . Then, natural events, economic activities (if they are allowed to continue), and most importantly, managerial actions⁶ together result in the ecological-economic system probabilistically moving to a state in G . This means that the manager's actions make it more likely—but they do not ensure—that the ecological-economic system will move to a state in G . Given that the ecological-economic system has just entered G from B , let the random variable T_{BG} denote the amount of time it spends in G .

Now suppose that our ecological-economic system initially is in a state in G . Then, the manager takes actions to keep this ecological-economic system in G for as long as possible. However, because the ecological-economic system in G is affected not only by the manager's actions but also by natural events and ongoing economic activities, it is possible that despite the manager's attempts, this ecological-economic system will slip into B . To this end, let the random variable T_{GB} denote the remaining time in G , given that the ecological-economic system is initially in G . Our task now is to characterize T_{BG} and T_{GB} formally.

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⁶Of the sort discussed in section 3.1.

We first focus on T_{BG} . It is difficult to provide a closed-form expression for the expectation of T_{BG} , $E[T_{BG}]$, directly. However, we do know that T_{BG} takes on non-negative values only. Consequently, we can use this fact and characterize T_{BG} by deriving the Laplace transform⁷ of its distribution function. In order to derive the Laplace transform, we will need to (i) provide an additional definition and (ii) compute the probability that our ecological-economic system is in state i , $i \in G$, given that this ecological-economic system has just entered G . Mathematically, we want to compute $\text{Prob}\{Z(t)=i/Z(t-) \in G, Z(t-) \in B\}$.

First, the definition. Suppose that our ecological-economic system is in state $i \in G$. Then let T_i denote the amount of time until the ecological-economic system enters B , given that it is currently in state i . Define the Laplace transform of the distribution function of T_i by $\hat{F}_{T_i}(\alpha)$, where $F_{T_i}(\cdot)$ is the distribution function of T_i . Now, the required probability of the previous paragraph. This probability is given in

$$\text{LEMMA 1: } \text{Prob}\{Z(t)=i/Z(t-) \in G, Z(t-) \in B\} = \sum_{j \in B} P_j s_{ji} / \sum_{j \in B} \sum_{m \in G} P_j s_{jm}.$$

PROOF: See the Appendix.

As with the definition of the Laplace transform of the distribution function of T_p , let us define the Laplace transform of the distribution function of T_{BG} to be $\hat{F}_{T_{BG}}(\alpha)$, where $F_{T_{BG}}(\cdot)$ is the distribution function of T_{BG} . This Laplace transform is given by the product of the summation of the Laplace transform of the distribution function of T_p over all i , and the probability in Lemma 1. In symbols, we have

$$\hat{F}_{T_{BG}}(\alpha) = \frac{\sum_{i \in G} \hat{F}_{T_i}(\alpha) \sum_{j \in B} P_j s_{ji}}{\sum_{j \in B} \sum_{m \in G} P_j s_{jm}}. \quad (3)$$

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⁷For more on the Laplace transform, see Ross (1996, pp. 19-20) and Taylor and Karlin (1998, p. 311).

We now have a closed-form expression for the Laplace transform of the distribution function of T_{BG} . Note that because the Laplace transform characterizes the distribution function of a non-negative random variable *uniquely*, equation (3) describes the random variable T_{BG} uniquely. This means that in principle, a manager who finds our ecological-economic system to be in a state in B , can use equation (3) to formulate an objective function, which when maximized, will lead to the attainment of a goal that is consistent with the time-based approach to the management of ecological-economic systems that we have been discussing in this paper.

What if our manager finds the ecological-economic system to be initially in a state in G ? In this case, the manager will want to focus on the expectation of T_{GB} , $E[T_{GB}]$. So, as in the case of T_{BG} , let us now derive the Laplace transform $\hat{F}_{T_{GB}}(\alpha)$ of the distribution function of T_{GB} . To undertake this derivation, we will use equation (3) and three other results. We now briefly discuss and then state these three results as three lemmas.

The first result concerns the probability that our ecological-economic system is in state i , $i \in G$, given that this ecological-economic system has just entered G . Mathematically, we want to compute $\text{Prob}\{Z(t)=i/Z(t) \in G\}$. This probability is given in

LEMMA 2: $\text{Prob}\{Z(t)=i/Z(t) \in G\} = P_i / \sum_{j \in G} P_j$.

PROOF: See the Appendix.

The second result demonstrates the relationship between the Laplace transforms $\hat{F}_{T_i}(\alpha)$ and $\hat{F}_{T_j}(\alpha)$, for the random variables T_i and T_j , in states (i,j) . We state this result as

LEMMA 3: $\sum_{i \in B} \sum_{j \in G} P_i s_{ij} \{1 - \hat{F}_{T_j}(\alpha)\} = \alpha \sum_{i \in G} P_i \hat{F}_{T_i}(\alpha)$.

PROOF: See the Appendix.

The third and final result demonstrates the link between the stationary probabilities of our ecological-economic system and the expected amount of time this ecological-economic system

spends in the good set of states G . Formally, we have

$$\text{LEMMA 4: } \left\{ \sum_{i \in B} \sum_{j \in G} P_i s_{ij} \right\} E[T_{BG}] = \sum_{j \in G} P_j.$$

PROOF: See the Appendix.

We are now in a position to derive the Laplace transform of the distribution function of T_{GB} .

Using Lemma 2, we get

$$\hat{F}_{T_{GB}}(\alpha) = \frac{\sum_{i \in G} P_i \hat{F}_{T_i}(\alpha)}{\sum_{j \in G} P_j}. \quad (4)$$

Using Lemma 3, equation (4) can be simplified to

$$\hat{F}_{T_{GB}}(\alpha) = \frac{\sum_{i \in B} \sum_{j \in G} P_i s_{ij} \{1 - \hat{F}_{T_j}(\alpha)\}}{\alpha \sum_{j \in G} P_j}. \quad (5)$$

It will be convenient to use equation (3) to rewrite equation (5). This gives

$$\hat{F}_{T_{GB}}(\alpha) = \frac{\sum_{i \in B} \sum_{j \in G} P_i s_{ij} \{1 - \hat{F}_{T_{BG}}(\alpha)\}}{\alpha \sum_{j \in G} P_j}. \quad (6)$$

Finally, equation (6) can be simplified using Lemma 4. This simplification gives

$$\hat{F}_{T_{GB}}(\alpha) = \frac{1 - \hat{F}_{T_{BG}}(\alpha)}{\alpha E[T_{BG}]}, \quad (7)$$

where $\hat{F}_{T_{BG}}(\alpha)$ is given by equation (3). Equation (7) gives us a closed-form expression for the Laplace transform of the distribution function of T_{GB} . Because of the one-to-one correspondence between distribution functions and Laplace transforms, equation (7) describes the random variable T_{GB}

uniquely. Consequently, a manager who finds our ecological-economic system to be in a state in G , can use equation (7) to formulate an objective function, which when maximized, will lead to the attainment of a goal that is consistent with the time-based approach to the management of ecological-economic systems that is the subject of this paper.

Equation (7) also points to an interesting relationship between the random variables of interest here, i.e., between T_{BG} and T_{GB} . Given that the objective of the manager of an ecological-economic system is to maximize either $E[T_{BG}]$ or $E[T_{GB}]$, it is useful to express this relationship in terms of these two expressions. In order to do this, we shall use a result about the distribution function of T_{GB} . We now state this result as

LEMMA 5: $Prob\{T_{GB} \leq t\} = \int_0^t (Prob\{T_{BG} > \alpha\} d\alpha / E[T_{BG}])$.

PROOF: See the Appendix.

From the definition of the expectation of a random variable,⁸ we have

$$E[T_{GB}] = \int_0^{\infty} t dF_{T_{GB}}(t). \quad (8)$$

With Lemma 5, the right-hand-side (RHS) of equation (8) can be simplified. This gives

$$E[T_{GB}] = \int_0^{\infty} t \int_t^{\infty} \frac{dF_{T_{BG}}(x)}{E[T_{BG}]} dt = \int_0^{\infty} \int_0^x \frac{t dt dF_{T_{BG}}(x)}{E[T_{BG}]}. \quad (9)$$

Now, let us rewrite the last double integral in equation (9) in terms of the expectations of T_{BG} and T_{GB} .

We get

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⁸See equation 1.3.1 in Ross (1996, p. 9).

$$E[T_{GB}] = \frac{E[T_{BG}^2]}{2E[T_{BG}]}. \quad (10)$$

Note that the variance of T_{BG} , $\text{Var}[T_{BG}]$, is non-negative. This means that $E[T_{BG}^2] \geq \{E[T_{BG}]\}^2$. In turn, this tells us that

$$\frac{E[T_{BG}^2]}{2E[T_{BG}]} \geq \frac{E[T_{BG}]}{2}. \quad (11)$$

Now combining equations (10) and (11), we get

$$E[T_{GB}] = \frac{E[T_{BG}^2]}{2E[T_{BG}]} \geq \frac{E[T_{BG}]}{2}. \quad (12)$$

With time in mind, equation (12) gives us the exact relationship between the good states and the bad states of our ecological-economic system. From equation (12) we see that the expected amount of time that an ecological-economic system already in G spends in G is bounded below by one-half of the expected amount of time that a system just moving to G from B spends in G . By choosing actions appropriately, our manager will be able to influence the parameters of the ecological-economic system, i.e., the transition probabilities ($P_{ij}'s$) and the rate of movement between states (v_i). Note that this approach to the management of ecological-economic systems involves the accomplishment of an *integrated* ecological-economic objective. Looked at from an ecological perspective, our manager is attempting to increase the *persistence* of the good set of states. Looked at from an economic perspective, the manager is seeking to *sustain* the desirable set of states. We now sketch the outlines of an optimization problem that shows how this paper's approach to the management of ecological-economic systems might be operationalized.

3.3. Optimization

The CTMC-theoretic approach to the management of ecological-economic systems that we have been discussing thus far is quite comprehensive. This general approach calls for the manager of our ecological-economic system to choose one or more control variables to maximize either $E[T_{GB}]$ or $E[T_{BG}]$. However, at this level of generality, the manager's optimization problem is rather complex. Moreover, it is difficult to show how one might solve such a complex optimization problem. Consequently, in order to illustrate the approach of this paper, we now discuss a simplified version of the (in general) more complex optimization problem.⁹

As before, let $Z(t)$ denote the state of our ecological-economic system and let c denote the manager's single control variable. Further, in this sub-section, we suppose that the evolution of $Z(t)$ is governed by a stochastic differential equation. That equation is

$$dZ = f(Z, c)dt + \sqrt{h(Z, c)}dW, \quad (13)$$

where $f(\cdot, \cdot)$ and $h(\cdot, \cdot)$ are exogenous and known to the manager and dW is the increment of a standard Wiener process. We suppose that initially, our ecological-economic system is in the good set of states G . In this sub-section, we shall denote G by the interval (g_1, g_2) on the real line. The manager's objective is to keep the ecological-economic system in this interval for *as long as possible*.

Formally, we have

$$T_{GB}(z) = \min_t \{t: Z(t) \notin (g_1, g_2) / Z(0) = z\}. \quad (14)$$

Using equation (14), it can be shown¹⁰ that $E[T_{GB}(z)]$, the expectation that we are interested in,

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⁹The remainder of this subsection is based in part on Mangel (1985, pp. 61-64).

¹⁰See Mangel (1985, p. 62).

satisfies a particular differential equation. That equation is

$$f(z,c)\frac{dE[T_{GB}(z)]}{dz} + \frac{1}{2}h(z,c)\frac{d^2E[T_{GB}(z)]}{dz^2} + 1 = 0. \quad (15)$$

Our ecological-economic system manager's optimization problem can now be stated. This manager solves

$$\max_{\{c\}} E[T_{GB}(z)]. \quad (16)$$

This maximization problem can usually be solved by first writing down the appropriate dynamic programming equation and then using a procedure described in Mangel (1985, pp. 58-61). The use of this procedure will enable the manager to solve explicitly for the control variable as a function of the state variable. We have now shown how this paper's approach to the management of dynamic and stochastic ecological-economic systems might actually be operationalized.

4. Conclusions

In this paper, we analyzed two hitherto unstudied questions about the structure and the management of jointly determined ecological-economic systems. First, following the suggestion in Perrings (1998), we showed how the theory of CTMCs can be used to model and study the structural properties of dynamic and stochastic ecological-economic systems. Second, we discussed a time-based approach to the management of ecological-economic systems. In this approach, a manager maximizes the amount of time that an ecological-economic system spends in the good set of states. This results in the accomplishment of an integrated ecological-economic objective. We now briefly discuss how the analysis of this paper would be affected (i) by the need to partition the state space of an ecological-economic system into *more than* two subsets of states, and (ii) by the presence of

a *keystone species* in the ecological-economic system.

As discussed in Stoddart *et al.* (1998), Batabyal *et al.* (1999a) and Batabyal (2000), rangeland ecological-economic systems in the USA typically exist in three to five “condition classes.” So, would the analysis of this paper change when it is necessary to partition the state space of an ecological-economic system into more than two subsets of states? The answer is that although the basic thrust of this paper’s time-based approach to management would not be altered, it would now make more sense to first identify the *specific* subset of states in which the manager would like the ecological-economic system to spend the highest proportion of time. Once this specific subset of states has been identified, a reasonable objective for the manager would be to focus on the *probability* of exit from this identified subset of states. This is because minimizing the probability of exit is, in many ways, analogous to maximizing the time until exit.

As noted in Batabyal (1999c), keystone species play a particularly salient role in the functioning of an ecological-economic system. Consequently, when a keystone species is present in an ecological-economic system, the time-based approach to management that we have discussed in this paper may need to be modified. Specifically, what this means is that instead of attempting to maximize the persistence of a particular subset of states, it may now make more sense to maximize the *persistence* of the *keystone species*.

The analysis of this paper can be extended in a number of different directions. In what follows, we suggest two possible extensions. First, our present state of knowledge permits us to say very little about the *ways* in which managerial actions affect the evolution of an ecological-economic system. Consequently, if the Markovian approach of this paper is appropriate, then additional research is needed to comprehend the effects that alternate managerial actions—such as moratoriums on grazing and fishing season length restrictions—have on the parameters (the P'_{ij} s and the v_i) of

an ecological-economic system.

Second, in this paper we have studied the general case in which the ecological-economic system is ergodic. This means that *all* the states of the ecological-economic system communicate with each other and that *all* the stationary probabilities are positive. As noted in Perrings (1998), on occasion, ecological-economic systems will *not* be ergodic. In particular, it is possible that as a result of specific managerial actions, an ecological-economic system will never exist in a particular subset of states. As such, it would be useful to study what effects these “zero limiting probability” states have on the two managerial objectives $E[T_{BG}]$ and $E[T_{GB}]$ that we have discussed in this paper. Studies that incorporate these aspects of the problem into the analysis will provide additional insights into the structure and the management of jointly determined ecological-economic systems.

Appendix

This appendix contains the proofs of the five lemmas that are stated in the text of the paper.

LEMMA 1: $\text{Prob}\{Z(t)=i/Z(t) \in G, Z(t-) \in B\} = \sum_{j \in B} P_{ij} s_{ji} / \sum_{j \in B} \sum_{m \in G} P_{jm} s_{jm}$.

PROOF: $\text{Prob}\{Z(t)=i/Z(t) \in G, Z(t-) \in B\} = \text{Prob}\{Z(t)=i, Z(t-) \in B\} / \text{Prob}\{Z(t) \in G, Z(t-) \in B\}$. This ratio equals $[\sum_{j \in B} \text{Prob}\{Z(t-)=j\} \text{Prob}\{Z(t)=i/Z(t-)=j\}] / [\sum_{j \in B} \text{Prob}\{Z(t-)=j\} \text{Prob}\{Z(t) \in G/Z(t-)=j\}]$. Now using the transition rates and the stationary probabilities, the preceding ratio of probabilities can be

simplified to $\sum_{j \in B} P_{ij} s_{ji} / \sum_{j \in B} \sum_{m \in G} P_{jm} s_{jm}$. $\square$

LEMMA 2: $\text{Prob}\{Z(t)=i/Z(t) \in G\} = P_i / \sum_{j \in G} P_j$.

PROOF: Simplifying the conditional probability on the left-hand-side (LHS) and using the stationary probabilities, we get $P_i / \sum_{j \in G} P_j$. $\square$

LEMMA 3: $\sum_{i \in B} \sum_{j \in G} P_i s_{ij} \{1 - \hat{F}_{T_j}(\alpha)\} = \alpha \sum_{i \in G} P_i \hat{F}_{T_i}(\alpha)$.

PROOF: (i) Let $P_{ij} = s_{ij} / \sum_{j \in G} s_{ij}$. Now recall the description of T_i from the body of the paper. The

Laplace transform of the distribution function of T_i , $\hat{F}_{T_i}(\alpha) = \{v_i / (v_i + \alpha)\} \{ \sum_{j \in G} \hat{F}_{T_j}(\alpha) P_{ij} + \sum_{j \in B} P_{ij} \}$. (ii)

Note that in any time t , the number of transitions of the ecological-economic system from B to G must be within one transition of the number of transitions from G to B . Consequently, the stationary

rate at which transitions occur from B to G , i.e., $\sum_{i \in B} \sum_{j \in G} P_i s_{ij} = \sum_{i \in G} \sum_{j \in B} P_i s_{ij}$, the stationary rate at which transitions occur from G to B . From (i) we conclude that $\sum_{j \in G} \hat{F}_{T_j}(\alpha) s_{ij} + \sum_{j \in B} s_{ij} = (\alpha + v_i) \hat{F}_{T_i}(\alpha)$.

We now multiply this equality by P_i , sum over all $i \in G$, use (ii), and then use equation 5.5.3 in Ross

(1996, p. 251). This gives $\sum_{i \in B} \sum_{j \in G} P_i s_{ij} + \sum_{j \in G} \hat{F}_{T_j}(\alpha) \{v_j P_j - \sum_{i \in B} P_i s_{ij}\} = \sum_{i \in G} P_i (\alpha + v_i) \hat{F}_{T_i}(\alpha)$. Algebraically manipulating this last equality, we get $\sum_{i \in B} \sum_{j \in G} P_i s_{ij} \{1 - \hat{F}_{T_j}(\alpha)\} = \alpha \sum_{i \in G} P_i \hat{F}_{T_i}(\alpha)$. $\square$

LEMMA 4: $\{\sum_{i \in B} \sum_{j \in G} P_i s_{ij}\} E[T_{BG}] = \sum_{j \in G} P_j$.

PROOF: From Lemma 3 and equation (3), we get $\{\sum_{i \in B} \sum_{j \in G} P_i s_{ij}\} \{1 - \hat{F}_{T_{BG}}(\alpha)\} = \alpha \sum_{i \in G} P_i \hat{F}_{T_i}(\alpha)$. Now divide both sides of this equality by α and then let $\alpha \rightarrow 0$. This gives $\{\sum_{i \in B} \sum_{j \in G} P_i s_{ij}\} E[T_{BG}] = \sum_{j \in G} P_j$. $\square$

LEMMA 5: $\text{Prob}\{T_{GB} \leq t\} = \int_0^t (\text{Prob}\{T_{BG} > \alpha\} d\alpha / E[T_{BG}]).$

PROOF: The Laplace transform of the distribution function of T_{GB} is given by equation (7).

Consequently, if the true distribution function of T_{GB} is as indicated in Lemma 5, then we should

be able to obtain this Laplace transform by using the distribution function given in Lemma 5. Using

this procedure, we get $\hat{F}_{T_{GB}}(\alpha) = \int_0^\infty \exp(-\alpha t) \text{Prob}\{T_{BG} > t\} dt / E[T_{BG}]$. The RHS of this equality can be

simplified. We get $\hat{F}_{T_{GB}}(\alpha) = \int_0^\infty \int_0^\infty \exp(-\alpha t) dt dF_{T_{BG}}(x) / E[T_{BG}]$. Now using the definition of the Laplace

transform (see Ross, 1996, p. 19 or Taylor and Karlin, 1998, p. 311), the preceding equality

simplifies to $\hat{F}_{T_{GB}}(\alpha) = \{1 - \hat{F}_{T_{BG}}(\alpha)\} / \alpha E[T_{BG}]$. We see that the distribution function in Lemma 5 yields

the correct Laplace transform. Moreover, we know that the Laplace transform represents the

distribution function of a non-negative random variable uniquely. Hence Lemma 5 follows. $\square$

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