Optimal Irrigation Management for Sloping Blocked-End Borders

Jorge Jose Escurra
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OPTIMAL IRRIGATION MANAGEMENT FOR SLOPING, BLOCKED-END BORDERS

by

Jorge José Escurra

A dissertation submitted in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY in

Irrigation Engineering

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2008
ABSTRACT

Optimal Irrigation Management for Sloping and Blocked-end Borders

by

Jorge José Escurra, Doctor of Philosophy
Utah State University, 2008

Major Professor: Dr Gary P. Merkley
Department: Biological and Irrigation Engineering

A robust mathematical model of one-dimensional flow for sloping, blocked-end border irrigation was developed using the four-point implicit method to solve the Saint-Venant equations, the volume-balance solution method, and the implementation of new algorithms to avoid numerical instability and solution divergence. The model has the capability of successfully simulating all surface irrigation phases in blocked-end borders for a range of inflow rates (0.01 – 0.05 m³/s per m), longitudinal slopes (up to 1.00%), and border lengths (100 – 500 m).

To achieve numerical stability over the specified parameter ranges, the model was divided into three parts: (1) advance-phase simulation which uses the four-point implicit solution method of the Saint-Venant equations, with an algorithm that changes the spatial and temporal weighting, in addition to an algorithm that handles the water depth profile at the blocked-end downstream boundary upon completion of the advance phase; (2) simultaneous advance-recession-phase calculations using a hybrid algorithm to solve the governing equations; and (3) recession-phase simulation using the four-point implicit method until (and if) divergence occurs, then the volume-method is applied to complete the simulation. The three parts also involve the use of computational grid management algorithms and a parabolic equation which defines the Chezy coefficient as a function of water depth.

The model incorporates the downhill simplex optimization method to determine the recommended inflow rate and irrigation cutoff time, maximizing a composite irrigation efficiency.
(water requirement efficiency and application efficiency). Different optimum values of inflow rate and irrigation cutoff time for a range of longitudinal slopes, border lengths, and soil types were generated. Most of the optimum values are for relatively high inflow rate and rapid cutoff time. In addition, exponential relations were developed, based on the simulation results, to determine the best irrigation time for maximization of the composite irrigation efficiency for specified, non-optimal inflow rates. The exponential relations are particularly useful in practice when it is not feasible to use the optimum inflow rate due to constraints at the water source, or because of irrigation scheduling issues.
I want to express my enormous gratitude to the Biological and Irrigation Engineering Department at Utah State University, which has supported me financially during the six years of my MS and PhD studies. I would like to deeply thank Dr. Gary P. Merkley who gave me an opportunity to be a doctoral candidate; he provided me with both moral and technical support during the development of my dissertation. His confidence and guidance made my dissertation possible. In addition, I would like to express my gratitude to Dr. Ronald Sims for his invaluable support and advice, which helped me reach the final goal of completing my dissertation. I am also grateful to Drs. Andrew A. Keller, Gilberto E. Urroz, Robert T. Pack, and Kelly L. Kopp for their time serving on my graduate committee.

To my father and mother, Jorge Escurra Cabrera and Elsa Aguirre de Escurra, I would like to acknowledge their encouragement in pursuing my educational goals. Also, to my sister and brother, Guadalupe del Rosario Escurra and Aldo F. Escurra, who always provided me their best wishes and moral support. To my daughter, Fatima Lucia Escurra, the reason to accomplish this dissertation, and to God, in whom I have always trusted and prayed.

J. J. Escurra
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NOTATION

The following symbols are used in this dissertation:

\[ \alpha = \text{constant equal to 1;} \]
\[ \beta = \text{constant equal to 2;} \]
\[ \beta_{\text{max}1}, \beta_{\text{max}2} = \text{weighting factors for application and water requirement efficiencies;} \]
\[ \Delta h_2, \Delta h_3 = \text{depth of water stored in nodes 2 and 3 (m);} \]
\[ \Delta t = \text{time increment (s);} \]
\[ \Delta t_a, \Delta t_r, \Delta t_{a-r} = \text{advance, recession, advance-recession time increments (s);} \]
\[ \Delta V = \text{volume of water stored in the border (m}^3\text{ per m);} \]
\[ \Delta x = \text{distance between two nodes (m);} \]
\[ \gamma = \text{constant equal to 0.5;} \]
\[ \tau = \text{intake opportunity time (m);} \]
\[ \theta = \text{temporal weighting factor;} \]
\[ \phi = \text{spatial weighting factor;} \]
\[ a, k, f_o = \text{Kostiakov parameters (m min}^{-a}, \text{m min}^{-1}); \]
\[ A = \text{cross-sectional area (m}^2\text{);} \]
\[ C = \text{Chezy coefficient;} \]
\[ C_d, n_f = \text{discharge coefficients;} \]
\[ C_{\text{max}}, C_{\text{min}} = \text{maximum and minimum Chezy coefficients;} \]
\[ D = \text{drag term (m}^2\text{);} \]
\[ \text{dsn} = \text{downstream node;} \]
\[ E_a = \text{application efficiency (\%);} \]
\[ E_v = \text{evaporation loss (m);} \]
F = function value;

\( f(a), f(b), f(c), f(x) \) = function using a, b, c; x;

\( F_{ds} \) = function at downstream boundary;

\( f_{tol} \) = fractional converge tolerance;

\( F_r \) = Froude number;

\( g \) = gravity \((m/s^2)\);

\( h \) = water depth \((m)\);

\( h_c \) = depth from the water surface to the centroid of the flow area \((m)\);

\( h_{be} \) = height of the downstream dike \((m)\);

\( h_d \) = water depths at border-end \((m)\);

\( h_{max}, h_{min} \) = maximum and minimum water depths \((m)\);

\( h_n(1), h_n(2), h_n(3) \) = water depths at nodes 1, 2, and 3 \((m)\);

\( I \) = infiltration rate subsurface area \((m^2/s)\);

\( i_{\text{max}} \) = maximum allowable number of iterations;

\( J \) = Jacobian matrix;

\( \text{MAD} \) = management allowed deficit \((\%)\);

\( \text{MaxAdvTime} \) = maximum advance time \((s)\);

\( \text{Maxtco}, \text{Mintco} \) = maximum and minimum cutoff time \((s)\);

\( n \) = Manning roughness coefficient;

\( N \) = number of dimensions;

\( \text{OBF, OBF1, OBF2} \) = objective functions;

\( P \) = pressure term \((m^2)\);

\( Q \) = inflow rate \((m^3/s)\);

\( q_{in} \) = inflow rate flowing into the border \((m^3/s \text{ per } m)\);

\( q_2, q_3 \) = discharge entering to nodes 2 and 3 \((m^3/s \text{ per } m)\);
R = hydraulic radius (m);
S = volume of cracks per unit area (m);
size_{qn}, size_{tco} = grid sizes for the cutoff time and inflow rate;
S_o = longitudinal bed slope (m/m);
S_f = energy loss gradient (m/m);
t = elapsed time (s);
T = top width (m);
t_{aL} = advance time until water reaches the border-end (s);
t_{co} = cutoff time (s);
usn = upstream node;
V = velocity (m/s);
Vol A = permissible water volume in the border (m³);
Vol B = water volume outside of the border boundaries (m³);
Vol C = total water volume inflow to the border (m³);
V_D = volume of water deficit (m³);
V_{in} = volume of water entering the border (m³ per m);
V_{out} = volume of water flowing out of the border (m³ per m);
V_{SRD} = volume of surface runoff (m³);
V_{ZR} = volume of water stored in the root zone (m³);
W_a = water holding capacity (mm/m);
W_p = wetted perimeter (m);
WRE = water requirement efficiency (%);
X_e = new trial point due to expansion;
X_g = centroid of the two points;
X_{N+1} = starting distance;
\(X_r, X_{rr}\) = trial points;

\(Z\) = infiltrated depth of water (m);

\(Z_{req}\) = net infiltration depth (m);

\(Z_{rt}\) = root depth (m);

\(Z_2, Z_3\) = infiltration depths in nodes 2 and 3 (m);

\(B2D\) = Two-Dimensional Model for Basin Irrigation System;

\(FAO\) = Food and Agriculture Organization of the United Nations;

\(NRCS\) = Natural Resource Conservation Service;

\(SIRMOD\) = Surface Irrigation Simulation, Evaluation, and Design Software;

\(SURVED\) = Surface Irrigation Software;

\(U.S.\ SCS\) = United States Soil Conservation Service; and,

\(WinSRFR\) = Surface Irrigation Analysis, Design, and Simulation Software.

**SUBSCRIPTS**

\(i\) = row;

\(j\) = column;

\(R, L, J, K\) = four nodes bounding a computational cell;

\(x\) = horizontal axis; and,

\(y\) = vertical axis.
CHAPTER 1
INTRODUCTION

Background

Water resources have always played a central role in human society and are a key to sustainable economic growth and poverty alleviation. Water is also a non-renewable resource, typically with annual and monthly supply variations, and rational uses of it need to be implemented in the interests of water management in general. In particular, knowledge of crop water consumption and leaching requirements is a key to the rational use of water for agricultural irrigation. Irrigation efficiency, in its various forms, is a primary factor that describes water use in terms of benefit to achieve equilibrium between irrigation water supply and demand. High irrigation efficiencies are often the best way to obtain desirable on-farm water management and rational water use by farmers and irrigators.

The Food and Agriculture Organization (FAO 2002) of the United Nations forecasts that surface irrigation will continue being the most common irrigation method until at least the year 2030. In the United States, surface irrigation accounts for approximately 65% of the twenty million irrigated hectares in that country. In the world, surface irrigation accounts for more than 90% of the total irrigated area, and border irrigation covers more than 20% of the surface-irrigated agricultural regions (World Bank Group 2005). In Asia, Africa, and South America, border irrigation covers much of the irrigated areas in highland regions where the mountains and scarcity of roads complicate the use of machinery and the adoption of furrow irrigation.

Border irrigation typically supplies water for cereals, corn, alfalfa, pastures and other crops that need large quantities of water, such as rice and banana. The principal advantages of border irrigation are: minimal required understanding of how to operate and maintain the system, acceptability by agriculturalists who received knowledge of the irrigation system from past generations, lower required investment (compared to furrow irrigation), sedimentation does not affect irrigation performance, runoff is reduced, and flooding is reduced.
It has been said that even if surface irrigation typically has a lower application efficiency than pressurized irrigation methods, surface irrigation represents a significant alternative due to low cost, easy implementation and resistance of farmers to change due to the complexity of pressurized irrigation system, until the irrigation system transference from surface to pressurized is accomplished around the world. Consequently, research on surface irrigation should continue being developed, as is the trend at some other universities and research institutions.

Some of the current known border irrigation simulation models are: (1) SIRMOD (Walker 2003), which solves the governing equations using a variant of the Eulerian control-volume approach; (2) WinSRFR (Arid-Land Agricultural Research Center 2006), which combines three MS-DOS software applications (SRFR, BASIN, and BORDER), and uses a combination of the zero-inertial and kinematic-wave approaches; and, (3) SURVED (Jurriëns et al. 2001), which combines a variant of the Eulerian control-volume approach and a modified zero-inertia method for modeling the advance phase of surface irrigation events.

Most of the well-known one-dimensional hydraulic simulation models for border irrigation use the complete form of the Saint-Venant equations. However, these models have limitations that, in many cases, require a trial-and-error approach to successfully conclude a simulation. This complicates the implementation of optimization algorithms designed to search for improve irrigation efficiency. This study was oriented to find the highest combined water requirement efficiency and application efficiency under border irrigation for a specific inflow rate and irrigation time (cutoff time) to support crop production and to help reduce agricultural non-point source pollution. To achieve this goal, the use of mathematical algorithms (such as relation of the water depth with the roughness coefficient) that model the physical behavior of water movement on the soil, with a combination of the Saint-Venant equations and a volume-balance method, were implemented. These implementations make the model more robust; this means that it does not suffer from solution divergence due to numerical instability within a given range of parameters. Consequently, the subsequent addition of an optimization algorithm was possible, producing an
optimal irrigation management scheme blocked-end border tool which can lead to improved water management practices, supporting the rational use of irrigation water.

**Statement of the Problem**

Many agricultural areas currently suffer from a lack of water, thereby making it necessary to improve water management in irrigation systems, most of which rely on surface irrigation methods. There is a need for improved best management practices in irrigated lands, not only for efficiency reasons, but also because of a growing concern about environmental effects due to runoff. Mathematical hydraulic modeling is one way to improve the management of surface-irrigated agricultural fields; the hydraulic modeling can lead to improved system design and improved operations. In addition, optimization algorithms can be implemented to develop improved management guidelines for surface-irrigated fields.

SIRMOD is among the most prominent one-dimensional model for the design and evaluation of surface irrigation systems such as furrow, border, and basin methods, and has been periodically improved in order to be able to handle more complex situations. However, the numerical scheme used to solve the governing hydraulic equations (Saint-Venant equations) has some drawbacks in some cases. For example, it is known that when the model is applied to cases involving steep longitudinal ground slopes and low inflow rates on blocked-end border irrigation systems, the simulation can become unstable, requiring a change in the time step size, and a restart of the simulation. In other cases, when the model is run for longitudinal slopes up to 1% on blocked-end border irrigation, artificial hydraulic waves tend to appear spontaneously during the recession phase. Part of this is due to limitations of the Saint-Venant equations for relatively high longitudinal field slopes, but another significant reason is the inherent instability of the numerical solution to these equations when confronted with problematic boundary conditions. In addition, it is assumed that high efficiencies are possible with the border irrigation method, but they are rarely obtained in practice due to the difficulty of balancing the advance and recession phases of water application through management of irrigation time and inflow rate.
The trial-and-error time step variation implemented by hand in SIRMOD to input the design values to avoid numerical divergence could be improved using an algorithm which can automatically change the time step. The last implementation may help to optimize the best time of cutoff and inflow rate to determine the highest irrigation efficiency. Also, another technique that could make the model more robust is to use an algorithm that determines the best values of the weighting factors ($\phi$ and $\theta$ used in the four-point implicit solution method to the governing equations) from many simulations which do not produce numerical instability. In addition, the use of mathematical formulations that describe the water depth behavior with the roughness coefficient during its movement on the soil and its interface with the blocked-end condition could help to make the model robust. Finally, the implementation of the use of volume-balance method and the Saint-Venant equations, to simulate the physical water movement when recession and advance phase are given at the same time, may helps to the results to converge better because the Saint-Venant equations are used in fewer nodes.

**Objectives**

The principal objective of the proposed research was to develop a completely robust mathematical model of one-dimensional flow in border-irrigated agricultural fields for a given range (one that encompasses most situations in practice) of border conditions. To reach the robustness of the model, it was necessary that the model should have the capability to successfully simulate all surface irrigation phases, including advance, ponding, depletion, and recession. During the development of the model robustness the implementation of algorithms, which generate minimum numerical oscillations modifying the weighting factors, describe the water depth behavior with the roughness coefficient during its movement on the soil, and implement the volume-balance method, were tested.

The specific objectives of this research were:

1. To develop a new mathematical model for the robust hydraulic simulation of all phases of irrigation events in sloping, blocked-end borders, with longitudinal field slopes up to 1%;
2. To determine a viable and systematic approach to more accurately calculate values of the temporal and spatial weighting factors (θ and φ), as applied in the four-point implicit solution to the governing hydraulic equations;

3. To conduct trials a border-irrigated field for comparison with selected model-generated hydraulic parameters;

4. To develop an algorithm and write computer code to calculate the optimal time of cutoff and inflow rate for blocked-end, sloping borders; and,

5. To develop guidelines for the irrigation management of block-end borders.
Surface irrigation involves the movement of water over a field surface in the form of open-channel flow. Border irrigation is one of the most complicated and the second-most common surface irrigation method in the world. The primary design factors for surface irrigation systems are longitudinal field slope and border length, soil moisture deficit at the time of irrigation, stream size per unit border width, degree of flow retardation by the crop, and soil intake rate (Walker and Skogerboe 1987). In the specific case of border irrigation, the water moves as shallow overland flow over a smoothed soil surface. Border irrigation has perhaps been studied more extensively than other surface irrigation methods, and flow analysis has been more fully developed because of the simpler mathematics in modeling the geometry and boundary conditions. Border irrigation involves the following characteristics (Walker 1989):

- The soil surface is permeable to water;
- The length of the border strip in the direction of flow is usually greater than the width of the border;
- The longitudinal slope of a border is close to zero, and is usually level in the transverse direction; and,
- The land surface may be vegetated.

As soon the water enters the system, it spreads across the border width to allow uniformity of stream size in all the units of width. The water then advances down to the border in overland flow behind a distinct wetting front. As flow occurs, some water enters the soil through a process of infiltration. The infiltration rate usually decreases with time at each point in the field. Therefore, if the inflow stream size is held constant with time, the surface flow rate and water depth at any point will tend to increase with time. On the other hand, the flow rate at any time decreases with greater downhill distance along the border. The rate of advance of the wetting front down the channel will decrease with time as more soil area becomes available to absorb water, reducing the size of the surface stream.
Under normal conditions flow continues in the above manner until the advancing front reaches the downhill end of the border. At this time water begins either to leave the field as surface runoff or to form surface storage if runoff is prevented by blocking the end. At some later time, presumably when sufficient water has been introduced to meet the gross water requirement, the inflow is stopped. Surface flow continues, but water depth and velocity decrease, beginning at the upper end of the field. When combined surface flow and infiltration reduce the depth of water at the upper end to zero, a recession or drying front is formed. The recession front proceeds downstream until it reaches the lower end of the border. At this time no water remains on the field surface, and the irrigation is complete. As irrigation proceeds, the depth of water that has entered the soil at a given point increases, but at a decreasing rate. It is possible that a greater depth of water will enter the soil than can be held in the root zone. This excess water is considered lost to plants as deep percolation (Jensen 1980).

Border irrigation is characterized by shallow flow on a plane sloping bed using unsteady flow models which can be classified in four groups (Walker and Skogerboe 1987):

1. **Hydrodynamic models**: these models are based on the full form of the Saint-Venant equations of continuity and momentum, and are the most complete of all the models;

2. **Zero-inertia models**: these are the first level of simplification of the Saint-Venant equations by neglecting the “inertia” (time dependent) term in the momentum equation;

3. **Kinematic-wave models**: these represent the next level of simplification where the momentum equation is replaced by a unique relation between mean velocity and water depth. This model was introduced in 1955 and has been used in many hydraulic modeling applications; and,

4. **Storage models**: these are also referred to as volume-balance models and are the simplest type.
Phases of a Typical Surface Irrigation Event

A complete surface irrigation event is composed of four phases (Fig. 1): advance, ponding, depletion, and recession (Sabillón 2003). These phases are briefly described in the following:

a) **Advance Phase**: During this time water advances in overland flow from the upper field boundary toward the lower field boundary.

b) **Ponding Phase**: The time between the end of advance and inflow shut-off. In the case that shut-off occurs first, this phase is of zero duration.

c) **Depletion Phase**: The time between inflow shut-off and the beginning of recession at the upper field boundary. It is also known as recession lag time.

d) **Recession Phase**: The time between the beginning of recession at the upper field boundary and the disappearance of water from the field surface.

Of course, in observing surface irrigation events in the field, the ponding, depletion and or recession phases may sometimes be of negligible duration.

![Fig. 1. Typical advance and recession phases in surface irrigation](image-url)
Infiltration

Generally, defined as the vertical one dimensional flow of water into the soil, infiltration is one of the critical parameters affecting overland flow. In agricultural soils, infiltration decreases with time and depends on many factors (initial soil moisture content, hydraulic gradients, and others). Several equations are used to express the infiltration rate; the Richards, Green-Ampt, and Philip equations are well-known examples (Merkley 2002). The modified empirical Kostiakov equation developed by the U.S. SCS (Soil Conservation Service, now known as the NRCS), commonly used in irrigation systems evaluation and design, was adopted for this analysis. The extended Kostiakov equation can be written as (Walker and Skogerboe 1987):

\[ Z = k \tau^a + f_o \tau \]  

(2.1)

where \( a, k, \) and \( f_o \) are empirical constants; \( Z \) is infiltrated depth of water (m); and, \( \tau \) is intake opportunity time (min).

Rayej and Wallender (1988) employed an even more extended form of the Kostiakov equation:

\[ Z = k \tau^a + f_o \tau + S \]  

(2.2)

where \( S \) represents the volume of cracks per unit surface area (m).

Surface Irrigation Simulation Variables

For the simulation of overland flow in surface irrigation, it is useful to separate constants and parameters from variables. When a factor can be assumed to have one value throughout a single irrigation and for all other irrigations, it is considered to be a constant. If the value of a factor can change within irrigations or between irrigations, it is treated as a variable. If the value of a variable during an irrigation is determined before the study begins, the variable is considered independent. If the value of a variable cannot be determined in advance, but results from the irrigation process, the variable is considered to be dependent.

Examples of dependent variables include velocity of advance and recession, time of advance and recession, depth and velocity of surface flow, depth and volume of water infiltrated,
volumes of surface runoff and deep percolation, recession lag time, efficiencies, and total irrigation time. Examples of independent variables include: inflow rate, land surface slope, hydraulic roughness, and soil infiltration characteristics. Downward slopes are usually considered to be positive. A zero slope is admissible in certain simulation schemes depending on whether normal (uniform flow) water depths are assumed in any of the computations (Walker and Skogerboe 1987).

**Hydraulic Equations**

Surface water flow in irrigation can be described by the equations of Saint-Venant (Chow, 1959). Simulations of irrigation can be obtained by using the equations in their complete form or under certain simplifications.

The equations describing the flow of water over a soil surface express two physical principles, conservation of mass and Newton’s second law, force equal mass times acceleration. These well known partial differential equations are known as the Saint-Venant equations and are derived in texts and papers on open channel flow. The mass conservation equation, or equation of continuity, is defined for on-dimensional incompressible flow as:

\[
\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} + I_x = 0
\]  

(2.3)

And, Newton’s second law or equation of motion (4) is:

\[
\frac{1}{g} \frac{\partial V}{\partial t} + \frac{V \partial V}{g \partial x} + \frac{\partial y}{\partial x} = S_o - S_f + \frac{V I_x}{2gA}
\]  

(2.4)

where Q is the discharge (m³/s per m); A is the cross-sectional area per unit width (m²); Iₓ is the infiltration rate (m²/s per m); S₀ is the constant longitudinal bed slope (m/m); Sᵣ is the energy loss gradient (m/m); g is gravity; x is the distance in the direction of flow (m); V is the velocity (m/s), and, t is elapsed time (s).
Derivation of the Equation of Continuity

Volume-balance is synonymous with mass balance for incompressible flow, and the equation to represent this condition can be derived from easily understood principles. Consider a cubic liquid element, as shown in Fig. 2, to explain the development of a form of the equation of mass balance.

In equation form, this can be represented as follows:

\[ \Delta \text{Volume} = \frac{\partial y}{\partial t} \frac{\partial x}{\partial t} T + \frac{\partial Z}{\partial x} \frac{\partial x}{\partial t} + T \frac{\partial E_v}{\partial x} \frac{\partial x}{\partial t} = -\frac{\partial Q}{\partial t} \]  

where \( T \) is the top width at the water surface (m); \( Q \) is flow rate (m\(^3\)/s); \( Z \) is soil water infiltration (m\(^2\)); \( E_v \) is the equivalent depth of evaporation loss at the water surface (m); and, \( t \) is elapsed time (s).

Let \( T \delta y = \delta A \), and divide by \( \delta x \delta y \),

\[ \frac{\partial A}{\partial t} + \frac{\partial Z}{\partial t} + \frac{\partial Q}{\partial x} + E_v T = 0 \]  

(2.6)
which is the equation of continuity for incompressible, one dimensional flow, where the flow due to seepage loss or gain is essentially vertical. The net mass flow rate entering the same volume is:

\[
\rho AV - \left[ \rho AV + \frac{\partial (\rho AV)}{\partial x} \right] = -\rho \frac{\partial (AV)}{\partial x} \frac{\partial x}{\partial x}
\]  

(2.7)

And, the net change in mass \((V = \delta x / \delta y)\) is:

\[
\rho AV \frac{\partial x}{\partial x} - \left[ \rho AV + \frac{\partial (\rho AV)}{\partial t} \right] = -\rho \frac{\partial A}{\partial t} \frac{\partial x}{\partial t}
\]  

(2.8)

Adding the term that is the mass flow rate leaving a liquid volume as infiltration \((\rho \text{ density kg/m}^3)\):

\[
-\rho l_x - \rho \frac{\partial (AV)}{\partial x} - \rho \frac{\partial A}{\partial t} = 0
\]  

(2.9)

\[
l_x + \frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0
\]  

(2.10)

**Derivation of the Equation of Motion**

Figure 3 shows a side view of an open canal with the force components which act on it in the horizontal direction, and downward due to the weight of the liquid element over a distance \(\Delta x\).

The following derivation does not consider the infiltration term because it has a small magnitude.

**Fig. 3.** Sketch of a liquid element for deriving the momentum equation

Considering that,
\[
\sum F_x = ma_x \tag{2.11}
\]
\[
\gamma A_y - \left[ \gamma A_y + \frac{\partial (\gamma A_y)}{\partial x} \Delta x \right] - \tau_o W_p \cdot \Delta x + \gamma A \Delta x \sin \alpha = pA \Delta x \frac{DV}{Dt} \tag{2.12}
\]
\[
\frac{DV}{Dt} = V \frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} \tag{2.13}
\]
\[
\frac{\partial (A_y)}{\partial y} = A \tag{2.14}
\]
\[
-\gamma A \frac{\partial y}{\partial x} - \tau_o W_p + \sin \alpha = \frac{V}{g} \frac{\partial V}{\partial x} + \frac{1}{g} \frac{\partial V}{\partial t} \tag{2.15}
\]

Recognizing that \(\gamma = \rho g\),
\[
-\frac{\partial y}{\partial x} - S_f + S_o = \frac{V}{g} \frac{\partial V}{\partial x} + \frac{1}{g} \frac{\partial V}{\partial t} \tag{2.16}
\]

where \(S_f\) is equal to energy loss gradient or friction loss; and, \(S_o\) is the longitudinal bed slope.

The Saint-Venant Equations

The Saint-Venant equations embody a system of mass and energy conservation laws that provides a one-dimensional model of open-channel flow. According to Chaudhry (1993), the following assumptions are made in the derivation of the Saint-Venant equations:

- The pressure distribution is hydrostatic;
- The channel bottom slope is small, so that the flow depths measured normal to the channel bottom and measured vertically are approximately the same;
- The flow velocity over the entire channel cross section is uniform;
- The channel is prismatic, the channel cross section and the channel bottom slope do not change with distance. The variation in the cross section or bottom slope may be
taken into consideration by approximating the channel into several prismatic reaches; and,

- The head losses in unsteady flow may be simulated by using the steady-state resistance laws, such as the Manning or Chezy equations.

The Saint-Venant equations of continuity and motion are presented below.

**Equation of Continuity**

For incompressible, one-dimensional flow, this equation can be given as:

\[
\frac{\partial Q}{\partial x} + \frac{\partial h}{\partial t} + \frac{\partial Z}{\partial t} = 0
\]  (2.17)

where \(Q\) is flow rate (m\(^3\)/s or cfs); \(A\) is flow cross-sectional area (m\(^2\) or ft\(^2\)); \(Z\) is a seepage loss term (m\(^2\) or ft\(^2\)); \(x\) is longitudinal distance in the direction of flow (m or ft); and \(t\) is elapsed time (s). The value of \(Z\) is positive for seepage outflow and negative for inflow.

**Equation of Motion**

The following form of the equation of motion is in terms of flow rate, \(Q\), and cross-sectional area, \(A\):

\[
\frac{\partial Q}{\partial t} + \frac{2Q}{A^2g} \frac{\partial Q}{\partial x} (1 - F_r^2) \frac{\partial h}{\partial x} = S_o - S_f
\]  (2.18)

where \(g\) is the ratio of weight to mass (9.81 m/s\(^2\), or 32.2 ft/s\(^2\)); \(F_r\) is the Froude number; \(h\) is water depth (m or ft); \(S_o\) is longitudinal bed slope (m/m); and, \(S_f\) is the energy loss gradient.

**Classical Solution Methods**

**The Method of Characteristics**

The Method of Characteristics has been acknowledged to be the most accurate and reliable of all numerical solution methods for hyperbolic partial differential equations; it is the standard by which other methods are often judged. However, it is difficult to apply when steep
wave fronts are present (Montes 1998). This method is a general solution method for hyperbolic PDEs and could be applied for the Saint-Venant equations. The continuity equation, the equation of motion, and the changes in depth and velocity equations (which indicate the total change in depth and velocity are equal to the sum of the partial changes in depth and velocity due to distance and time) are solved simultaneously for $\frac{\delta y}{\delta x}$. Because of the discontinuous surface profile, the water surface breaks and the slope $\frac{\delta y}{\delta x}$ has two values. Since the two surface slopes do not bear any definite relationship to each other, the value of $\frac{\delta y}{\delta x}$ must be determinate; or mathematically, $\frac{\delta y}{\delta x} = 0/0$. When the denominator is set to zero:

$$
\frac{dx}{dt} = (V + c)dt
$$

where $dx$ is the distance increment; $dt$ is the time increment, $V$ is velocity; and, $c$ is equal to the root square of the ratio of weight to mass (9.81 m/s$^2$, or 32.2 ft/s$^2$) time water depth for wide channels. When the numerator is set to zero Eq (2.20) is used:

$$
d(V \pm 2c) = g(S_o - S_f)dt
$$

where $S_o$ is the longitudinal bed slope (m/m); and, $S_f$ is the energy loss gradient (Chow 1959).

Equations 2.19 and 2.20 are known as the characteristic equations. The method for deriving these equations was given by Massau (1900). He developed a trial-and-error procedure for applying these equations to problems of unsteady flow. The characteristic equations may be written as:

$$
\frac{dx}{dt} = V + c
$$

$$
d(V + 2c) = g(S_o - S_f)dt
$$

$$
\frac{dx}{dt} = V - c
$$
Finite Difference Methods

Chaudhry (1993) and Montes (1998) describe several explicit and implicit methods widely used to solve the Saint-Venant equations. The most common explicit schemes are: Lax or Diffusive, Lax-Wendroff, MacCormack, Lambda, and Gabutti. Among the implicit schemes, the most popular are the four- and six-point implicit methods.

Four-point implicit method In solving the Saint-Venant hyperbolic partial differential equations (PDEs) for the simulation of unsteady open channel flow, this method can be applied in which unweighted averages in both time and space are used on each group of four nodes in the x-t plane, from time j to time j+1, and from distance i to distance i+1. Accuracy decreases as the temporal weighting factor (θ) goes from 0.5 to 1.0. It is recommended to start operations with a value of the spatial weighting factor (ϕ) equal to 0.6. For values of weighting factors equal to zero the method is “fully explicit,” and when less than or equal to unity the method is “fully implicit.” Explicit models are simpler to program because each node can be solved one at a time, while implicit models involve simultaneous solutions to a system of equations.

Six-point implicit method This solution is nearly the same as the four-point implicit method, but the spatial difference expressions become central differences across nodes i-1 and i+1 for the two time steps j and j+1, and the temporal differences are taken at node i only. The six-point implicit method involves less truncation error due to the central difference expressions for spatial derivatives, but a simpler (e.g. four-point implicit) method must be used at the upstream and downstream boundaries.

Surface Irrigation Simulation and Optimization Models

Many models have been developed to simulate and optimize sloping border irrigations. Shankar (1982) developed kinematic-wave and zero-inertia model for border irrigation which are found compatible for freely draining borders but zero-inertia models are superior to kinematic-wave for closed-end borders.

\[ d(V - 2c) = g(S_o - S_i) \, dt \]  

(2.24)
The prediction of three models: Walker and Humphreys (1983) model, Jaynes (1986) model and Ross (1986) model were selected to examine the effect of recession criteria on the prediction of recession times (Maheshwari 1992). The results showed that “for a given recession criterion the recession time varied considerably with the events and indicated that other factors affecting flows in border irrigation should also be taken into account while selecting a recession criterion.”

An analytical model for the entire irrigation cycle of blocked-end borders was simulated by Singh and Yu (1989a) in five phases: advance, storage, vertical recession, horizontal recession, and impounding recession using a volume-balance approach. The model verification was done using 15 sets of experimental data. According to the author, “the average relative error was less than 8% in predicted advance, below 2% in predicted vertical recession, under 5% in predicted horizontal recession, and within 6% for impounding distance” (Singh and Yu 1989b).

Valiantzas (2000) presented an equation describing the time dependence of the surface shape factor to predict the advance phase in surface irrigation. Generally, it is assumed in volume-balance models that the average depth of water on the ground surface is constant. This basic assumption may cause significant errors in the computation of water advance rate. Comparison tests indicated that the suggested variable surface shape factor volume-balance equation produced essentially the same results as the kinematic wave numerical method. The suggested equation is important for the case of level or borders with relatively small longitudinal slopes.

Khanjani and Barani (1999) studied an optimization model for a border irrigation system using the Hook-Jeev pattern search optimization method in conjunction with a general mathematical model of border irrigation used to maximize irrigation application efficiency. In the analysis the border irrigation storage and distribution efficiencies, border slope and length, inflow rate, cutoff time, and the Manning roughness coefficients are used as constraints. The model was compared to field-measured data. As result from this comparison, it is shown that a proper
choice of system parameters (length, inflow rate, and cutoff time) can lead to a maximum value of application efficiency.

Faribotz et al. (2003) simulated all phases of border and furrow irrigation systems using (ZIMOD), a zero-inertia model. The zero inertia model was compared to SIRMOD and field experimental data. It was proved that the Manning roughness coefficient is one of the most important parameters for describing water flow over the ground.

Models for surface irrigation need to determine a combination of parameters that can be used as indicators to measure effectively the irrigation performance. The evaluation of performance uses the following factors (Burt et al. 1997):

1. Uniformity of water application;
2. Application efficiency;
3. “Water requirement” efficiency; and,
4. Water spilled over the top of the blocked end (dike).

Model Sensitivity

A total of six models of border irrigation, viz. Jobling-Turner, Strelkoff, Walker, Jaynes, Schmitz and Ross, were evaluated (Maheshwari et al. 1990) for their sensitivity to the following input variables: field parameters (longitudinal slope, Manning roughness value, parameters of the infiltration equation, and inflow rate), time step, and computational grid size. The analyses showed that, for the variation of the parameters considered, the models were not sensitive to the solution parameters. However, they were sensitive, to varying degrees, to the field parameters.

Numerical Problems with the Solution of the Saint-Venant Equations

There is a large amount of documentation about numerical limitation of the weighted four-point implicit method of solution for the Saint-Venant equations, or Preissmann’s general schemes. However, most of them do not consider all the numerical properties of the model such as dispersion, stability, and dispersion; therefore, hypotheses have been assumed. For instance, Samuels and Skeels (1990) present demonstration carried out by using Fourier. However, their
contributions refer only to the conventional scheme centered in space. Lyn and Goodwin (1987) use the four-point implicit solution, but they do not consider the energy loss term.

Venutelli (2002) presents a numerical analysis of Preissmann’s general schemes applied to the complete Saint-Venant equations written for one dimensional flow. His analysis indicates that the influence of a progressive increase of the friction term, also increase the dispersion errors, which are manifested with numerical oscillations. Also, he proposed the most suitable values for the time-weighting coefficient, greater than 0.5 and less than 1.0, which is able to produce sufficient dissipation for stabilizing the numerical solution.

**Optimization Algorithms**

**Golden Search**

The golden search method is the simplest optimization algorithm to determine the minimum value of a function. The algorithm works using the bisection method that finds roots of functions in one dimension. The golden section search works with a triplet of points (a < b < c) where \( f(b) \) is less than both \( f(a) \) and \( f(c) \). Consequently, it is known that there is a minimum in the interval (a, c). The analog of bisection is to choose a new point \( x \), either between \( a \) and \( b \), or between \( b \) and \( c \). Then, \( f(x) \) is evaluated. If \( f(b) < f(x) \), then the new bracketing triplet of points is (a, b, x). On the other hand, if \( f(b) > f(x) \), then the new bracketing triplet is (b, x, c). In all cases the middle point of the new triplet is the abscissa whose ordinate \( f() \) is the best (lowest) minimum achieved. The process continues until the distance between the two outer points of the triplet is tolerably small. The optimal bracketing interval (a, b, c) has its middle point b a fractional distance 0.38197 from one end (say, a) and 0.61803 from the other end (say, c). These fractions are called the “golden mean” or “golden section.” This optimal method of function minimization, the analog of the bisection method for finding zeros, is known as the golden section search method (Press et al. 2007).
**Downhill Simplex Method**

The downhill simplex method is an optimization algorithm developed by Nelder and Mead (1965) that requires only function evaluations, not the calculation of function derivatives. In an N-dimensional space, a simplex is a polyhedron with N+1 vertices, or points. Thus, N+1 points are chosen to define an initial simplex. The method iteratively updates (moves) the worst point (the farthest point from the optimum value) by four operations: reflection, expansion, one-dimensional contraction, and multiple contractions (Fig. 4).

Reflection consists in moving the worst point of the simplex (where the value of the objective function is the highest) to a point reflected through the remaining N points. If this point is better than the best point, then the method attempts to expand the simplex along this line in what is referred to as “expansion.” On the other hand, if the new point is not much better than the previous point, then the simplex is contracted along one dimension from the highest point. This operation is called “contraction.” Finally, if the new point is worse than the previous points,
the simplex is contracted along all dimensions toward the best point and steps down the N-dimensional valley. Repeating all these procedures, the method is designed to ultimately find the optimal solution, but in some cases it can terminate at a local rather than a global minimum.
CHAPTER 3
MODEL DEVELOPMENT

General Model Description

The robust mathematical model of one-dimensional flow in border-irrigated agricultural fields developed herein is based on a combination of numerical solutions of the Saint-Venant equations, which include the equations of motion and continuity (Eqs. 2.17 and 2.18). Through the solution to the governing equations, water depth and discharge can be calculated for each simulated increment of time and spatial location in what is known as a step-wise numerical solution. The model uses the four-point implicit method to approximate the solution to an integrated form of the Saint-Venant equations. The simulation of surface irrigation in a border is the purpose of the model, and three parts were studied individually, then joined together to apply an optimization algorithm:

1. Advance Phase: This phase involves the simulation of water advance in overland flow from the upper field boundary toward the lower field boundary; upon completion of this phase either a free-draining or a blocked-end downstream boundary condition will occurs.

2. Recession Phase: This phase involves the simulation when the water starts to disappear along the complete field surface. The study of this phase considers both downstream boundary conditions free-draining and blocked-end.

3. Advance-Recession Phase: This part involves the simulation of the water after the time of cutoff and has not yet reached the downstream end of the border; consequently, the model simulates advance and recession simultaneously. This phase was studied for both downstream boundary conditions.

The model has the capability to be robust; this means that it does not fail (numerically diverge) within certain parametric limits of inflow rate, border length and longitudinal slope. The intelligent combination of the three parts explained above will make the model robust and allow the implementation of an optimization algorithm.
**Boundary Conditions**

Various upstream and downstream boundary conditions are used during simulations to divide the model in these three phases of study. Consequently, there are two different upstream boundary conditions. The first is a specified discharge at the head end of the border, occurring at simulation times less than the time of cutoff for the advance phase. This boundary condition can persist for a few time steps after the time of cutoff whereby the inflow rate is zero, but before recession begins. The second upstream boundary condition is for the recession phase after the time of cutoff, which begins when the water depth at the head end of the border decreases to a specified threshold value.

As a downstream boundary condition, the model presents two criteria. The first considers that the downhill end of the border is free draining. The second criteria considers that the border has a blocked downhill end in which a dike is used to avoid surface runoff.

**Governing Equations**

The robust mathematical model uses the water depth-discharge form of the two governing equations. The model executes calculations of water depth and discharge per unit width; this means that the considered border width is equal to 1 m. The two governing equations of the Saint-Venant are the equation of motion and continuity described in details in Chapter 2.

The infiltration rate, $I$, is defined as a function of the intake opportunity time, $\tau$, from the Kostiakov-Lewis equation, and is shown in Eq. (2.1). The pressure term, $P$, is defined as:

$$P = \int_0^y A \, dy = h_c A = \frac{h^2}{2}$$

where $y$ is a variable of integration and $h_c$ is the depth from the water surface to the centroid of the flow area (m). The pressure term is based on the geometric centroid of the flow area and is valid for incompressible flow. Because of the unit width consideration, the pressure term is simplified as a function of the water depth. This definition of the pressure term involves the
presence of vertical accelerations. The drag term, $D$, is the product of area and energy loss gradient, which is defined using the Chezy equation as:

$$D = AS_f = W_p \left( \frac{Q}{CA} \right)^2 = \left( \frac{Q}{Ch} \right)^2$$

(3.2)

where $W_p$ is equal to unity (1.0 m) due to the unit border width specification; and, $C$ is the Chezy coefficient.

**Numerical Solution of the Saint-Venant Equations**

The four-point implicit method is one of the methods used to solve the governing equations in the hydraulic simulation model for border irrigation systems. Also, the volume-balance method solution based on a simplified version of the Saint-Venant equations is also implemented. The volume-balance solution method is detailed when the new techniques for the recession and advance-recessions parts are mentioned. The model was developed based on two unknowns, or dependent variables: $h$ (water depth), and $Q$ (discharge). Owing to the simple rectangular geometry of border-irrigation cross sections, the model calculates the discharge per unit width of a border strip. The solution covers the entire physical domain at each time step and has an unrestricted time step size. The model uses different algorithms, which uses the condition of unrestricted time step size to avoid numerical divergence of the solution. This solution comes from interpolations within a computational cell (Fig. 5). Each cell is bounded by nodes in the x-t plane; in addition, dependent variables values are yielded by the solution, whereby the integrated forms of the two governing. The four nodes defining each rectangular cell are referred to herein as L, R, J, and K, instead of using the subscripts “x+Δx,” and “t+Δt”.
The continuity equation, with water depth and unit discharge as dependent variables, is defined as:

\[
\frac{\partial}{\partial t} \left( \frac{\partial Q}{\partial x} \right) + \frac{\partial}{\partial x} \left( \frac{\partial h}{\partial t} \right) + \frac{\partial}{\partial x} \left( \frac{\partial Z}{\partial t} \right) = 0
\]  

(S.3)

Simplifying, and considering that \( I = \frac{\partial Z}{\partial t} \),

\[
F_1 = \left[ \theta (Q_R - Q_L) + (1 - \theta) (Q_K - Q_J) \right] \Delta t \\
+ \left[ \phi (h_L - h_J) + (1 - \phi) (h_R - h_K) \right] \Delta x \\
+ \left[ \phi (\theta I_L + (1 - \theta) I_J) + (1 - \phi) (\theta I_R + (1 - \theta) I_K) \right] \Delta x = 0
\]  

(S.4)

The equation of motion, with the same two dependent variables, is defined as:

\[
\frac{1}{g} \int_{x}^{x+\Delta x} \left[ \int_{t}^{t+\Delta t} \frac{\partial Q}{\partial t} \right] \partial x + \int_{t}^{t+\Delta t} \left[ \int_{x}^{x+\Delta x} \frac{\partial}{\partial x} \left( P + \frac{Q^2}{gh} \right) \right] \partial x + \int_{t}^{t+\Delta t} \left[ \int_{x}^{x+\Delta x} \frac{\partial}{\partial x} \left( \frac{1}{2g} V^2 \right) \right] \partial x = 0
\]  

(S.5)

Integrating,
\[ F_2 = \frac{1}{g} \left[ (1 - \phi)(Q_R - Q_K) + \phi(Q_L - Q_J) \right] \Delta x \]
\[ + \theta \left[ \left( P + \frac{Q^2}{gh} \right)_R - \left( P + \frac{Q^2}{gh} \right)_L \right] \Delta t \]
\[ + (1 - \theta) \left[ \left( P + \frac{Q^2}{gh} \right)_K - \left( P + \frac{Q^2}{gh} \right)_J \right] \Delta t \]
\[ - S_o \left\{ (1 - \phi)\left[ \theta h_R + (1 - \theta)h_K \right] + \phi\left[ \theta h_L + (1 - \theta)h_J \right] \right\} \Delta t \Delta x \]
\[ + \left\{ (1 - \phi)\left[ \theta D_R + (1 - \theta)D_K \right] + \phi\left[ \theta D_L + (1 - \theta)D_J \right] \right\} \Delta t \Delta x \]
\[ + \left( \theta \frac{V_{IL}}{2g} + (1 - \theta)\frac{V_{IJ}}{2g} \right) + \left( 1 - \phi \right)\left[ \theta \frac{V_{RL}}{2g} + (1 - \theta)\frac{V_{RK}}{2g} \right] \right) \Delta t \Delta x = 0 \tag{3.6} \]

The parameters \( \phi \) and \( \theta \) are spatial and temporal weighting factors, respectively.

Equation 3.6 is defined to be equal to zero, but this is only true in practice when the values of \( \phi \) and \( \theta \) are correct for a given set of parameter and variable values; therefore, numerical solutions are used to drive the equations to zero. Instability of the numerical solution can occur when the values of \( \phi \) and \( \theta \) are not correct. The most appropriate values of \( \phi \) and \( \theta \), which do not cause numerical oscillation, could be the same for different border distances, longitudinal slopes, Chezy coefficient, and soil type. The model considers a range of \( \phi \) and \( \theta \) values from 0.52 to 0.78.

Canelón (2002) established that the four-point implicit method is unconditionally stable for the linearized form of the Saint-Venant equations for any value of time-to-space increment ratio, as long as \( \phi \) is equal to 0.50 and \( \theta \) is near to 0.60. In addition, SIRMOD (Walker 2003) defines \( \phi \) and \( \theta \) equal to 0.6, but the user is allowed to change these values as desired. However, in spite of the unconditional stability for the Saint-Venant equations themselves, the requisite inclusion of upstream and downstream boundary-condition relationships can cause significant numerical instability, a situation often observed in practice.
In Fig. 6 it is seen that three computational cells have six unknowns and equations, and that there are three nodes and one downstream boundary condition. The last two cells have two equations ($Q$ and $h$) and the first cell has only one equation ($h$). The number of unknowns matches the number of equations; therefore, all of the unknowns are solved for simultaneously and a unique solution can be obtained. The system of six equations for $h$ and $Q$ values in this example using three cells is during the advance phase is calculated as shown in Eq. 3.7 at each node:

$$
\begin{bmatrix}
\begin{bmatrix}
\frac{\partial F_1}{\partial h_1} & \frac{\partial F_1}{\partial Q_2} & \frac{\partial F_1}{\partial h_2} \\
\frac{\partial F_2}{\partial h_1} & \frac{\partial F_2}{\partial Q_2} & \frac{\partial F_2}{\partial h_2} \\
\frac{\partial F_2}{\partial h_1} & \frac{\partial F_2}{\partial Q_2} & \frac{\partial F_2}{\partial h_2} \\
\end{bmatrix} & \\
\begin{bmatrix}
\frac{\partial F_{1t}}{\partial Q_3} & \frac{\partial F_{1t}}{\partial h_3} \\
\frac{\partial F_{2t}}{\partial Q_3} & \frac{\partial F_{2t}}{\partial h_3} \\
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
\frac{\partial h_1}{\partial Q_1} & \frac{\partial h_1}{\partial Q_2} & \frac{\partial h_1}{\partial h_2} \\
\frac{\partial Q_2}{\partial Q_1} & \frac{\partial Q_2}{\partial Q_2} & \frac{\partial Q_2}{\partial h_2} \\
\frac{\partial Q_3}{\partial Q_1} & \frac{\partial Q_3}{\partial Q_2} & \frac{\partial Q_3}{\partial h_2} \\
\end{bmatrix} \\
\begin{bmatrix}
\frac{\partial F_{1t}}{\partial x} & \frac{\partial F_{1t}}{\partial x} & \frac{\partial F_{1t}}{\partial x} \\
\frac{\partial F_{2t}}{\partial x} & \frac{\partial F_{2t}}{\partial x} & \frac{\partial F_{2t}}{\partial x} \\
\end{bmatrix}
\end{bmatrix} =
\begin{bmatrix}
F_{1,1} \\
F_{1,2} \\
F_{2,1} \\
F_{2,2} \\
F_{1t} \\
F_{2t}
\end{bmatrix}
$$

(3.7)
Equation 3.8 shows the system of seven equations after the completion of the advance phase for three computational cells. Each computational cell will have two equations (Saint-Venant equations). $F_{ds}$ is the downstream boundary and is function of $h$ and $Q$, where the subscript “ds” represents the downstream node. This downstream boundary condition can be blocked-end or free draining. If the border has a free-draining downstream end, that boundary corresponds to a uniform-flow equation; on the other hand, when the downstream boundary includes a dike, it corresponds (mathematically) to a kind of weir structure. In this example, there are seven equations and three computational cells, so there is a 7 x 7 Jacobian matrix and two 7-element column vectors (Eq. 3.8).

During the free-draining condition at the downstream boundary, the “advancing tip cell” involves a single computational grid in which only the L-subscript term is non-zero. For this advancing tip cell there are two equations and two unknowns ($h$ and $\Delta x$). The equation of continuity which satisfies this condition is:

$$F_{lt} = \theta Q_L \Delta t + \phi \Delta x (h_L + \theta \Delta t l_L)$$  \hspace{1cm} (3.9)
The equation of motion is expressed as:

$$F_{\text{ilt}} = \frac{\phi Q_L \Delta x}{g} - \theta \left( \frac{Q_L^2}{gA_L} + P_L \right) \Delta t - \theta \phi (S_o A_L - D_L) \Delta x \Delta t$$  \hspace{1cm} (3.10)$$

The unit discharge is related to water depth by the Chezy equation to approximate a uniform-flow condition, and can be expressed as:

$$F_{ds} = Q - CA \sqrt{RS_o} = 0$$  \hspace{1cm} (3.11)$$

where $R$ is the hydraulic radius (m), equal to the cross-sectional flow area divided by the wetted perimeter ($R = A/W_p$); and, $C$ is the Chezy coefficient. The partial derivative of $F_{ds}$ with respect to flow ($Q$) is 1.0, and the partial derivative with respect of depth ($h$) is:

$$\frac{F_{ds}}{\partial h} = -1.5C(hS_o)^{1.5}$$  \hspace{1cm} (3.12)$$

The blocked-end condition at the downstream boundary means that the border is somehow diked. It is known that the dike would be of finite height, so water will spill over it if the depth of water is greater than the dike’s height, whereby it is assumed herein that the dike would not erode during over-topping. Therefore, the discharge is related by a weir equation, which is expressed as:

$$F_{ds} = Q - C_d (h-h_{be})^{n_f} = 0$$  \hspace{1cm} (3.13)$$

where $h$ is the depth of water at the last downstream node (m); $h_{be}$ is the height of the downstream dike (m); the discharge coefficient, $C_d$, is equal to 1.8 (metric units); and, the exponent $n_f$ is equal to 1.5. The partial derivative of $F_{ds}$ with respect to flow ($Q$) is 1.0, and the partial derivative with respect of depth ($h$) is:

$$\frac{F_{ds}}{\partial h} = -C_d n_f (h-h_{be})^{n_f-1}$$  \hspace{1cm} (3.14)$$
After the Saint-Venant equations have been discretized using the four-point implicit finite difference method, a set of non-linear algebraic equations result. These algebraic equations are solved using the Newton-Raphson algorithm, which uses the Jacobian matrix to effectively convert the system of non-linear equations to a set of linear equations, then the model condenses the Jacobian matrix into a four-column matrix of non-zero values. The resulting equations are then solved by a form of Gaussian elimination and backward substitution (Canelón 2002). Then, the Newton-Raphson algorithm is applied as:

\[ \ddot{X}_{\text{new}} = \ddot{X}_{\text{old}} - \partial X \]  

and,

\[ \partial X = J^{-1} F \]  

where \( \partial X \) denotes the vector of \( x_j \) values; \( J \) is a Jacobian matrix; and, \( F \) denotes the vector of function values, \( F_i \).

The new values are calculated and the convergence is analyzed. The convergence algorithm evaluates whether \( \delta x \) for the system of linear equations is close to zero and simulation at that time step is done; if not, initial guesses are updated, and the whole process is repeated until convergence is reached (or until the solution diverges).

**Input Data**

The model, which simulates water movement and infiltration during an irrigation event in a border strip, needs various parameters to complete the Saint-Venant equations and perform a hydraulic simulation. These input data are:

1. Inflow (m\(^3\)/s per m): the total discharge of water entering the border strip per unit width; this means that total discharge should be divided by the width of border strip.
2. Net infiltration depth, \( Z_{\text{req}} \) (m): This is considered to be the maximum value of water depth infiltrated in the border during irrigation. This value is a function of the management allowed deficit, \( \text{MAD} \) (%), water holding capacity, \( W_a \) (mm/m), and root depth, \( Z_{\text{rt}} \) (m).
3. Chezy coefficient: this coefficient is part of the Chezy formula (Table 1), this value is a function of the Reynold’s number and relative solid boundary roughness. In the model, the Chezy coefficient is function of the maximum or minimum Chezy value.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Turbulent flow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Manning n</td>
</tr>
<tr>
<td>Concrete or Asphalt</td>
<td>0.01 - 0.013</td>
</tr>
<tr>
<td>Bare sand</td>
<td>0.01 - 0.016</td>
</tr>
<tr>
<td>Graveled surface</td>
<td>0.012 - 0.03</td>
</tr>
<tr>
<td>Bare clay-loam soil (eroded)</td>
<td>0.012 - 0.033</td>
</tr>
<tr>
<td>Sparse vegetation</td>
<td>0.053 - 0.13</td>
</tr>
<tr>
<td>Short grass prairie</td>
<td>0.1 - 0.2</td>
</tr>
<tr>
<td>Bluegrass sod</td>
<td>0.17 - 0.48</td>
</tr>
</tbody>
</table>

4. Slope (m/m): longitudinal slope of the border strip.

5. Border length (m): length of the border strip.

6. End berm height (m): height of the berm at the end of the border.

7. Kostiakov parameters (a, k, f_0): parameters for the Kostiakov-Lewis infiltration equation. These parameters are a function of the soil texture.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>a_x</th>
<th>k_x (m m^{-3})</th>
<th>f_0 (m m^{-1})</th>
<th>family</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>0.317</td>
<td>0.00383</td>
<td>0.000035</td>
<td>0.10</td>
</tr>
<tr>
<td>Clay loam</td>
<td>0.457</td>
<td>0.00326</td>
<td>0.000088</td>
<td>0.35</td>
</tr>
<tr>
<td>Silty loam</td>
<td>0.529</td>
<td>0.0032</td>
<td>0.000136</td>
<td>0.60</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>0.598</td>
<td>0.00332</td>
<td>0.000212</td>
<td>1.00</td>
</tr>
<tr>
<td>Sandy</td>
<td>0.672</td>
<td>0.00393</td>
<td>0.000337</td>
<td>2.00</td>
</tr>
</tbody>
</table>

- Weir parameters (C_d, n_t): For modeling purposes, C_d = 1.8, and n_t = 1.5. These values respond to the formula of a weir which is taken as downstream boundary condition when the border is blocked at the downhill end and the irrigation water overtops the height of the dike. Again, it is assumed that the dike height remains constant during over-topping.
\[ Q = C_d h^{n_f} \]  

(3.17)

where \( Q \) is the discharge \( (m^3/s) \) and \( h \) is the depth of the water over the weir; \( C_d \) and \( n_f \) are the weir parameters.

- **Side berm height (m):** This is the height of border strip berm, which runs parallel to the main direction of flow.
- **End border height (m):** This is the height of border at the downhill end.
- **Advance time increment (s):** define the numbers of time steps used by the model during the advance phase.
- **Recession time increment (s):** define the numbers of time steps used by the model during the recession phase.
- **Advance and Recession time increment (s):** define the numbers of time steps used by the model between the time of cutoff and the end of the advance phase.

During the development of the model, some restrictions of input data were considered. The maximum inflow rate was 0.05 \( m^3/s \) per m and was set to be lower than the maximum allowable inflow that the border can carry. This value was calculated using the Chezy equation, taking the side berm height as the maximum water depth. If the maximum inflow rate is greater than the maximum allowable inflow, the model will use the maximum allowable permissible inflow. The minimum inflow rate was taken to be 0.01 \( m^3/s \) per m. The maximum and minimum longitudinal field slopes were set at 1.00% and 0.05%, respectively, and the maximum and minimum Chezy coefficient values were fixed at 60 and 10, respectively. The maximum and minimum border lengths were 50 and 500 m, and the maximum and minimum advance-phase time increments were 5 to 120 s. The recession time increment \( (\Delta t_r) \) was 20 s; this value is reduced if the cutoff time is small.
Improving Hydraulic Simulations During the Advance Phase

During simulation of the advance phase, a time increment ($\Delta t_a$) is used for the simulation. The advance time increment is the time step increment used for the solution of the Saint-Venant equations and influences the numbers of computational nodes generated by the model. In each computational node the water depth, discharge, distance, advance time and recession time are calculated along the border length. The value of $\Delta t_a$ is automatically determined by the model basis of the cutoff time and inflow rate (5 to 120 s). In addition, the model also presents the time increment in the recession ($\Delta t_r$) and advance-recession phases (when both phases are simultaneously achieved) ($\Delta t_{ar}$) as needed parameters for the computation in the recession and advance-recession phases.

A criterion to standardize the time increment was developed by executing many simulations with different cutoff times and $\Delta t_a$. Normally, the model considers a value of $\Delta t_a$ equal to 60 s because this advance time increment generates a number of nodes which ensure a normal simulation. It is possible that users can use small values of cutoff times; especially, when the border length is short. The time increments need to be smaller than the cutoff time because it influences the numbers of nodes at each time step. If the time increments are equal or not much lower than the cutoff time, the model crashes because of the small number of nodes and the oscillations from the solution to the Saint-Venant equations.

The criterion dictates that when the cutoff time is lower than 50 s and the inflow is lower than 0.01 m³/s per unit width, $\Delta t_a$ is equal to the cutoff time divided by 10 when the cutoff time is lower than 500 s, but greater than 50 s, $\Delta t_a$ is equal to 30 s. The criterion also involves the incorporation of factors which multiplies $\Delta t_a$ to calculate $\Delta t_r$ and $\Delta t_{ar}$ when low inflows and cutoff times are used.

In the present research several techniques were designed, implemented, and tested to improve the numerical stability of the model during simulations of the advance phase. Most surface irrigation hydraulic simulation models which use the full form of the Saint-Venant equations manifest some numerical instability problems. These problems are divided into two
different groups; the first affects the advancing tip cell, and the second affects the downstream boundary conditions. The problems that affect the advancing tip cell are listed below.

**Problem 1: Divergence Due to the Use of a Constant Roughness Coefficient**

In some surface irrigation simulation models, roughness parameters, such as the Manning or Chezy coefficients, are applied to define the $S_r$ term in the Saint-Venant equations for all phases of an irrigation event. It is observed that this criterion can generate some divergence because of the absence of the relation between water depth and roughness coefficient. From the Swamee-Jain equation it is observed that small water depths generate high Chezy coefficients values and low resistance. On the other hand, relatively high water depths generate high hydraulic resistance. To apply these criteria, a relation between water depth and roughness coefficient was found using the concept of different size of water depths produce variable roughness effect. The model uses Chezy coefficient as parameter of roughness coefficient because it comes from a mathematical formulation related to the kinematic viscosity, roughness height, and water depth, in comparison with the fully empirical Manning equation.

**Solution 1: Chezy Coefficient as a Function of Water Depth**

In the physical environment, it has been observed that the Chezy coefficient is not the same at all times and physical locations during the flow of water along the border length and physical locations along the border length. Small water depths will produce more hydraulic roughness than high water depths under the same soil surface condition. The model has simulated this physical situation relating the Chezy coefficient as a function of water depth in which the changes of water depths generate smooth variation in the Chezy coefficient through a parabolic equation. This parabolic interpolation is based on two fixed points, and a zero slope at one of them. This interpolation has specified maximum and minimum Chezy coefficients, and the user can change them if necessary. The model already has set up the maximum and minimum Chezy coefficients ($C_{\text{max}}$ and $C_{\text{min}}$) equal to 60 and 10, and the maximum and minimum water
depths \( (h_{\text{max}} \text{ and } h_{\text{min}}) \) equal to 0.15 and 0 m, respectively. These values were set for a bare soil, but the user can change them in the model if needed due to the presence of vegetation.

With these two points and slope equal zero, a parabolic equation was generated in the following way (Fig. 7):

The general parabola equation is:

\[
y = ax^2 + bx + c
\]  

Setting the slope equal to zero:

\[
\frac{dy}{dx} = 2ax + b = 0
\]  

The criterion used herein is to set the slope to zero at a maximum water depth of 0.15 m:

![Fig. 7. The Chezy coefficient as a parabolic function of water depth](image-url)
Forcing the parabola to pass through (0, 10) and (0.15, 60), it is observed that:

For the first point (0, 10):

\[ 0 = 2a(0.15) + b \]  
\[ -0.3a = b \]  

For the second point (0.15, 60):

\[ 60 = a(0.15)^2 + b(0.15) + c \]

Finally, Chezy values are calculated as a function of water depth using the following formula:

\[ C = -2222.2h^2 + 666.6h + 10 \]  

where \( h \) is water depth (m); and, \( C \) is the Chezy coefficient. Thus, every time the model calculates a "new" value of water depth, the Chezy value will be changed correspondingly.

**Problem 2: Oscillations due to Incorrect \( \phi \) and \( \theta \) Values**

The model uses the four-point implicit solution method to solve the Saint-Venant equations, and it needs accurate spatial and temporal weighting factors (\( \phi \) and \( \theta \)) to allow the solution to converge. The four-point implicit method represents differential equations like the system \( \partial f / \partial t + \partial G(f) / \partial x = 0 \) in the following way (Cunge et al. 1980):
\[
\frac{\partial f}{\partial t} \approx \frac{\left[ \phi f_{j+1}^{n+1} + (1-\phi) f_{j}^{n+1} \right] - \left[ \phi f_{j+1}^{n} + (1-\phi) f_{j}^{n} \right]}{\Delta t}
\]

(3.25)

and,

\[
\frac{\partial f}{\partial x} \approx \frac{\theta \left( G_{j+1}^{n+1} - G_{j}^{n-1} \right) + (1-\theta) \left( G_{j+1}^{n} - G_{j}^{n} \right)}{\Delta x}
\]

(3.26)

where \( \phi \) and \( \theta \) are space and time weighting coefficients, respectively; and, \( f_{j}^{n} \) and \( G_{j}^{n} \) are the functions of interest in a rectangular grid.

If \( \phi \) is equal to 0.5, the four-point scheme is known as the Preissmann scheme (Chaudhry 1993). The scheme can change from explicit to implicit increasing the value for \( \theta \) from 0.5 to 1. If the four-point implicit solution does not present appropriate \( \phi \) and \( \theta \) values for the specific parameters of slope, discharge, and advance increment time, then oscillations are observed. Oscillations affect the results by generating artificial numerical undulations in the variables. Consequently, the results are not consistent and solutions involve negative or high values then the solution diverges.

For practical purposes many surface irrigation hydraulic models set the values of \( \phi \) and \( \theta \) equal to 0.6. For example, the values for \( \phi \) and \( \theta \) are both taken to be 0.6 in SIRMOD (Walker 2003) and these values are used to generate advance and recession simulations. But, the user has the capability to change these values in the program when the solution diverges.

An algorithm to deal with this issue was developed. This algorithm looks for the best values of \( \phi \) and \( \theta \) which avoid divergence due to numerical oscillations. The best values of \( \phi \) and \( \theta \) were found by developing simulations for different values of slope, inflow rate, border length, soil type, and advance time increment, and the behavior of a range of \( \phi \) and \( \theta \) was analyzed.

**Solution 2: Searching for the Best Values of \( \phi \) and \( \theta \)**

The first part of developing the algorithm was the analysis of the \( \phi \) and \( \theta \) values to avoid divergence during the advance phase. The \( \phi \) and \( \theta \) values are spatial and temporal weighting
coefficients used to solve the Saint-Venant equations with the four-point implicit method, as explained above, and the respective values of these weighting factors affects the existence or absence of numerical oscillations in the results. The criterion to analyze the best φ and θ values was defined by the calculation of the number of depth reversals and depth profile instability from the border inlet to the advancing tip.

The number of reversals symbolizes the quantity of times when the calculated depth does not decrease monotonically with increasing distance during the advance phase, and in consecutive nodes the values of water depth vary generating oscillations. The calculation of the number of reversals begins by determining the difference between two consecutive water depths among all the computational nodes at each time step, then the ratio between these two consecutive differences is calculated. This ratio represents the number of reversals at each time step, and if the sign of this ratio is negative, the model increments the number of reversals. After all the depths along the border length are generated for one time step, the model calculates the number of reversals during that time step.

\[
\text{Difference}_{(\text{node 1 - 2})} = h_n(2) - h_n(1) \tag{3.27}
\]

\[
\text{Difference}_{(\text{node 2 - 3})} = h_n(3) - h_n(2) \tag{3.28}
\]

\[
\text{Reversal} = \frac{\text{Difference}_{(\text{node 1-2})}}{\text{Difference}_{(\text{node 2-3})}} \tag{3.29}
\]

Then, the average of reversals is calculated after the simulation finishes and it is equal to the total number for reversals over the total number of time steps during the simulation.

\[
\text{Average reversal} = \frac{\text{Reversal}}{\text{Total steps}} \tag{3.30}
\]
The profile instability is also calculated by dividing the number of reversals over the total number of nodes in each time step (dsn – usn). Then the profile instability average is generated, adding all the profile stabilities during one simulation, and dividing by the total number of time steps, as seen in Eqs. 3.31 and 3.32.

\[
\text{Profile stability} = \frac{\text{Reversal}}{(dsn - usn)}
\]  
\[
\text{Profile stability Average} = \frac{\text{All profile stabilities}}{\text{Total steps}}
\]

The \( \phi \) and \( \theta \) values that achieve the minimum value of average reversals and the lowest profile instability during the advance phase are considered as optimal values, and they are associated with the “best” pair of \( \phi \) and \( \theta \) values for the advance phase. To obtain the best \( \phi \) and \( \theta \) values, a number of simulations for three different situations were applied by calculating the numbers of reversals and the average profile instability for each simulation.

**Blocked-end Downstream Boundary and Variable Chezy C**

This situation considers that the Chezy coefficient is a function of water depth through the parabolic equation, and that the water reaches the blocked downhill end of the border. For this blocked-end downstream boundary and variable C situation, the model runs for 416 time steps to reach the downstream (tail) end of the border with an advance time increment of 60 s. The border length was 10,000 m, the cutoff time was 25,000 s, the side berm and border end heights were set at 0.12 m, and the soil type was silty clay. The inflow rate was set to 0.05 m\(^3\)/s per m, and was constant for all tests. These simulations were run for different longitudinal bed slopes in the range from 1% to 0.05%, maintaining the other input parameters as constants.

There were a total of 2,700 simulations, and the loop in the model was set up to start with values of \( \phi \) and \( \theta \) equal to 0.52, with an increment of 0.02, up to values of \( \phi \) and \( \theta \) were equal to the imposed upper limit of 0.78, respectively. These limits were obtained from analyzing the concept of the trapezoidal rule to calculate the weighting factors. It is known that 0.5 for phi and
0.79 for theta (weighting factors for the four-point implicit solution method), are factors used to
describe a straight line and curved trajectory. During the advance phase the water has a curved
trajectory between adjacent nodes. Consequently, the considered limits to apply the analysis
represent the trajectory between two nodes along the border length. The longitudinal bed slope
increment was 0.0005, starting from the specified minimum value of 0.05%.

The evaluation criterion was to calculate the number of reversals and profile instability
factor during each step of the simulation. Different \( \phi \) and \( \theta \) values that generated minimum
reversals and the lowest profile instability were considered as optimal values. A range of \( \phi \) and \( \theta \)
values were considered to satisfy the condition of lower profile instability factor.

Simulations were executed for different values of advance time increment for different
values of slope. In this case, 2,025 simulations were run three different advance time increments
were considered; 60, 40, and 25 s. The model was also run with three different longitudinal bed
slopes: 1.00%, 0.20%, and 0.05%. Finally, Initial \( \phi \) and \( \theta \) values were 0.52 each, and there were
fifteen increments of 0.02 up to a maximum of 0.78. The same criterion for evaluating reversals
and the same profile instability factor criterion were applied.

**Advancing Downstream Boundary and Variable Chezy C**

A second situation was implemented by considering that the Chezy coefficient is a
function of water depth and until the number of time steps is equal to 150; consequently,
simulates under free-draining condition. For this case the model runs for each of the five soil
types on the basis of the intake family parameters \( (a_x, k_x, \text{and } f_o) \), also for each advance
increment times; 60, 40, and 20 s, longitudinal slopes; 1.00%, 0.20%, and 0.05%, and the inflow
rates 0.05, 0.03, and 0.01 m\(^3\)/s per m. The input parameters were: border length of 10,000 m,
cutoff time of 25,000 s, and border-end and side berm heights of 0.12 m. The unreasonably long
border length of 10,000 m was used to prevent the advance phase from ending during these
tests. Thus, any simulation failures were due to advance phase conditions, not considering
problems associated with transitions to a zero-flow boundary condition when the water reaches the downstream end of a blocked-end border. This analysis involved 40,500 simulations.

Initial $\phi$ and $\theta$ values were 0.52 each, and there were fifteen increments of 0.02, up to a maximum of 0.78. The number of minimum reversals and a profile instability factor criterion, which was applied for the first situation, were followed to define the values of $\phi$ and $\theta$ which produce lower overall numerical instability magnitudes. Initial $\phi$ and $\theta$ values were 0.52 each, and there were fifteen increments of 0.02 up to a maximum of 0.78, the same reversals and a profile instability factor criterion, which was applied for the first situation, were followed to define the values of $\phi$ and $\theta$ which produce the lowest numerical oscillations.

**Advancing Downstream Boundary and Constant Chezy C**

The model has the capability of using different Chezy coefficients as a function of the water depth generated at each time step. This allows the observation of the generation of oscillations for different values of $\phi$ and $\theta$ with constant Chezy coefficients for all simulations. For this case a different model was developed using constant values of the Chezy coefficient in the system of nonlinear algebraic equations based on the Saint-Venant equations. The model was executed for the five different soil types using time steps of 60, 40, and 20 s, longitudinal bed slopes of 1.00%, 0.55%, and 0.25%, and a constant inflow rate of 0.05 m$^3$/s per m. The border length was 10,000 m, the cutoff time was 25,000 s, and the downhill and side berm heights were 0.12 m. The analysis involved 40,500 simulations and 120 time steps.

The values of $\phi$ and $\theta$ were incremented by 0.02 in each simulation from 0.52 to 0.78, and the same reversal and profile instability criteria were applied. The reversal and profile instability criteria were used in all situations to determine the best $\phi$ and $\theta$ values, that is, those that do not generate numerical oscillations and do not cause the solution to diverge.
Determining Appropriate $\phi$ and $\theta$ Values

After searching for $\phi$ and $\theta$ values which produce the minimum number of reversals and the lowest profile instability for the three different situations, the model generates a text file in which all pairs of $\phi$ and $\theta$, their generated number of reversals and profile instability at each time step, and the average reversal and profile instability during the entire advance phase simulation are recorded. On the basis of these data, the best $\phi$ and $\theta$ values are selected (those that have the minimum reversal and the lowest profile instability values) for certain longitudinal field slopes, advance time increments, and discharges. Then, using those $\phi$ and $\theta$ values, the model uses an algorithm which applies the appropriate values of $\phi$ and $\theta$ to avoid extreme numerical oscillations during the simulation. From the analysis of the best $\phi$ and $\theta$ values, it was found that steep slopes, high inflow rates, and high advance time increment require relatively low values of $\phi$ and $\theta$. In addition, most frequently used values of $\phi$ and $\theta$ close to 0.78 in the majority of cases generate a low number of depth reversals and low profile instability. Because of the wide range of $\phi$ and $\theta$ values, the algorithm which determines the appropriate values of $\phi$ and $\theta$ was separated into groups.

Three different groups of the best $\phi$ and $\theta$ were generated. Next, the model starts by using pairs of the best $\phi$ and $\theta$ values from each group when the four-point implicit method is applied until the solution converges following the order of the number of combinations. If the chosen $\phi$ and $\theta$ values still generate numerical divergence, the model continues by selecting the next $\phi$ and $\theta$ pair indicated by the last number of combinations plus 1, and so on until all the pairs of $\phi$ and $\theta$ from the three groups have been evaluated ($\phi$ and $\theta$ from 0.52 to 0.78). In Table 3, the three groups, separated by color and ordered by the number of combinations, are shown:

The purple group represents average values close to 0.78, and the first combination of $\phi$ and $\theta$ is 0.68 and 0.68, respectively. If these initial values generate solution divergence due to numerical oscillations, the model automatically considers the second combination of $\phi$ and $\theta$ values, which is 0.70 and 0.68, respectively. This process continues until the model finds values
of $\phi$ and $\theta$ with low average number of reversals and low profile instability, and results in a successful simulation. If the model cannot find the correct $\phi$ and $\theta$ values in the yellow group, it automatically passes to the orange group (see Table 3).

The orange group represents the second of values which, on average, are close to 0.66. The first combination in this group is $\phi = 0.58$ and $\theta = 0.58$; this is the thirty-seventh combination considering all the possible combinations inside these limits. If the solution still diverges with this combination it uses the following combination: $\phi = 0.6$, and $\theta = 0.58$ (which is the thirty-eighth combination), and so on, until the model finds values of $\phi$ and $\theta$ that do not result in divergence of the solution. If the model does not reach this goal, it moves on to the green group.

The green group represents the third group with average values close to 0.56. The first combination in this group is $\phi = 0.5$ and $\theta = 0.5$, and is the 122nd combination, considering all of the possible $\phi$ and $\theta$ values. If the model still fails, it uses the next combination of $\phi = 0.52$, and $\theta = 0.5$ (123rd combination), and so on, until the solution converges. If convergence is not obtained the model will automatically generate a reduction in the advance time increment, $\Delta t_{\text{adv}}$, and the simulation starts over from the beginning considering a new $\Delta t_{\text{adv}}$ and a number of combination equal to 1.

**Table 3.** Order of $\phi$ and $\theta$ Combinations Used by the Algorithm for Advance-Phase Simulation

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<th>N° of Combination</th>
<th>$\phi$</th>
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<th>0.54</th>
<th>0.56</th>
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The algorithm that changes $\phi$ and $\theta$ values according to numerical convergence of the governing equations has a maximum of 225 number of combinations. It applies the changes in $\phi$ and $\theta$ at each time step on an as-needed basis. The reason that the model applies this change of $\phi$ and $\theta$ is because of solution divergence due to oscillations in the numerical solution of the Saint Venant equations. After all combinations are tested in the model, it reduces the advance time increment by ten-second intervals, and then the whole simulation process is repeated. The following paragraphs describe the problems that affect the downstream boundary conditions.

**Problem 3: Divergence due to Calculations at the Downstream Boundary**

When the model runs, some problems were observed when water reaches the downstream end of the field. These problems occur when the downstream boundary is blocked-end or free draining, and is due to the limitations of the linear interpolation method used by the model to calculate the water depth and discharge over the downhill dike. This affects the stability of the numerical solution, causing the solution to diverge because due to the presence of negative depth or flow values, or division by zero. To solve this problem, various approaches were developed and tested; these were applied for both downstream free-draining and blocked-end boundary conditions. The solutions are given in the following sections.

**Solution 3A: Only One Computational Node Upstream of the Dike**

When the inflow rate is high and the advance time increment is close to sixty seconds, after the first time step, the distance at the last node is much further than the distance at the dike, and two nodes are generated downstream from the dike, with only one node upstream of the dike. Consequently, the model cannot calculate water depth and discharge over the dike because linear interpolation does not perform well when the last node distance is large and one node exists on the upstream side of the boundary. This effect causes numerical instability and solution divergence. To solve this problem, an algorithm that reduces the advance time increment is applied.
As soon as the program identifies this situation, it reduces the advance time increment by ten seconds until it finds at least two nodes upstream of the dike and automatically restarts the calculations with the same input data. This algorithm is also applied to a free-draining downstream boundary condition.

**Solution 3B: Two Nodes Upstream of the Dike and Excessively Fast Advance at the Last Node**

After the situation with one node upstream of the downstream boundary is solved, the next case is the presence of two nodes upstream from the dike and one node downstream, with an excessively fast advance producing unreasonable advance distances. The linear interpolation and weir equation, used by the model to relate water depth and discharge over the dike, does not function properly. The mathematical interpolation consists in equalizing the area between the penultimate node and the dike with the area of the penultimate node and last node; therefore, if the last area is too large, the value of water depth at the dike will be higher than the dike height. To handle this problem, an algorithm which calculates the water depth at one percent of the border length downstream from the dike, was created. This algorithm is used for blocked-end downstream boundary conditions when water reaches the end of the field during the advance phase. The Figure 8 shows two nodes upstream and one node downstream from the dike. The $h_d$ is the water depth generated by the algorithm and $x_{(3)}$ is the distance at the last node (3).

![Fig. 8. Two nodes upstream from the dike and an excessively fast advance](image)
This algorithm generates a parabolic equation taking these three points and calculates the unknown water depth \( h_u \) at a distance downstream from the dike equal to 4% of the border length using a parabolic equation. This procedure is explained by the following formula.

General parabolic equation:

\[ y = ax^2 + bx + c \]  

(3.33)

where \( a, b, \) and \( c \) are defined as:

\[ a = \frac{(y_1 - y_2)C_1 - y_1 + y_3}{x_3^2 - C_0 - C_1C_2} \]  

(3.34)

\[ b = \frac{y_2 - y_1 - aC_2}{x_2 - x_1} \]  

(3.35)

\[ c = y_2 - x_2(ax_2 + b) \]  

(3.36)

\[ C_0 = x_1^2 \]  

(3.37)

\[ C_1 = \frac{x_3 - x_1}{x_2 - x_1} \]  

(3.38)

\[ C_2 = x_2^2 - C_0 \]  

(3.39)

Using a distance downstream from the dike equal to 4% of the border length (x axis), the water depth at that known distance can be calculated using a parabolic equation (y axis) because after many simulations, it was proved that 4% of the border length can effectively be used to calculate the water depth at the border end. In Fig. 9 three sample points of an advance profile are graphed.
The parabolic equation determines a new water depth which is used in the calculation of depth over the dike though linear interpolation. This algorithm is applied when the inflow rate is high, generating an excessive advance rate at the last node. Discharge values at the end of border are calculated using a uniform flow equation in the case of the free-draining downstream condition. A weir equation is applied if water spills over the dike or discharge equals zero if water does not spill over the dike in the case of blocked-end downstream condition.

Using this algorithm, a new node at the end of the border will be created and the model continues with the calculation process. The method of two nodes after the end of the border helps to get a water depth and distance from a new node located close to the border-end. Therefore, the model can take these depths and distances to calculate the water depth at the dike without divergence and continue with the subsequent time steps.

For the free-draining downstream boundary condition, the model uses the same algorithm which generates a parabolic equation with three points, but it calculates the new node at 0.01 of the border length downstream from the dike. In addition, this algorithm is applied when two or three nodes are located upstream from the dike and one node is located downstream. This means that the algorithm works when it identifies three and four nodes in the advance profile. In the case that there are more than four nodes in the advance profile, the model calculates the water depth over the dike with the parabolic equation using the closest three points to the dike. This water depth over the dike is compared with the same depth generated by the...
linear interpolation calculation. If the water depth generated by the linear interpolation calculation is greater than the dike height or the water depth at the penultimate node, the model will use the water depth generated by the parabolic equation. Otherwise, the water depth from the linear interpolation calculation is used. The solution for a free-draining condition uses a different percentage of distance (0.01L) downstream from the dike because water can run until a maximum distance that is close to the border length (lower than 4% of the border length).

**Solution 3C: Handling the Artificial Advance Beyond the Blocked Downstream Boundary**

One of the problems generated by a solution for a blocked-end downstream boundary condition is when the algorithm, which calculates the water depth at four percent of the border length downstream from the dike, is applied to the situation in which water advances far beyond the downstream boundary. In this case, the simulated volume of water that advances beyond the dike is not considered as water volume that entered into the border. A set of formulas were developed to quantify this unconsidered volume of water when a new depth and distance are created downstream of the dike. This set of formulas is only applied when the system has a blocked-end condition following completion of the advance phase. Figure 10 shows the three volumes considered in the model.

![Fig. 10. Three volume definitions used in the set of formulas](image)
A. Water Volume Permissible in the Border: This is the total water volume available in the border per unit width, this means that more water, entering the border, will spill over the border at each time step (m³ per m).

\[ \text{Vol A} = \frac{L^2 S_o}{2} + h_o L \]  

where, \( L \) is the border length; \( S_o \) is longitudinal slope; and, \( h_o \) is the dike height.

B. Water Volume Downstream from the Border Boundary: This is the water volume below the border length per unit width or the water that spills over the dike at each time step that the model generates nodes beyond the downhill end of the border (m³ per m).

\[ \text{Vol B} = \frac{2}{75} h_{\text{end}} L \]  

where \( h_{\text{end}} \) is the depth at the end of the border; and, \( L \) is the border length.

C. Total Water Volume Inflow: This is the total water volume flowing into the border at each time step per unit width (m³ per m):

\[ \text{Vol C} = q_{\text{in}} \Delta t_a \]  

where \( q_{\text{in}} \) is the inflow rate; and, \( \Delta t_a \) is the advance increment time.

With these volumes the model calculates the quantity of water that has not been considered when a new node was created at 4% of the border length downstream from the dike. The model will determine whether this quantity will be considered as a spilled (runoff) or infiltrated volume.

It is known that the volume of water left in the border is equal to water volume permissible in the border, minus the inflow volume that remains inside the border; this last value is equal to the total inflow volume minus the water volume downstream of the border boundary. If this remaining volume is negative, it means that the volume left in the border will spill over the
downstream dike; consequently, the total water left in the border, plus the water volume
downstream of the border boundary, are considered as runoff volume at each time step
increment.

On the other hand, if the value of volume left into the border is positive but is lower than
the water volume downstream from the border boundary, the model considers the difference as
the quantity that spills over the blocked end, and it will be added to the runoff volume at that
advance time increment. Finally, if the value of volume left in the border is greater than the
volume downstream from the border boundary, the model considers the volume downstream from
the border boundary as the quantity which remains in the border.

**Problem 4: Handling Excessive Nodes and Iterations**

Another frequently occurring problem during the advance phase is the large number of
computational nodes which are generated when many time steps are required during a simulation
and they also cause numerical oscillations. The number of computational nodes increases for
several reasons: when the longitudinal slope is mild, the inflow rate is low, the border length is
large, and the advance time increment is small (close to or equal to 5 s). The model is
programmed to allow a maximum of 1,024 computational nodes and a maximum of 50 iterations
for convergence; this means that if the results obtained from the solution of the Saint-Venant
equations do not converge within 50 iterations, the model will cease calculations and will not
generate any simulation results (the solution diverges). The different methods of grid
management to handle are described below.

**Solution 4A: Recalculating Depths, Flows,
and Distances During the Advance Phase**

A computational node redistribution algorithm was implemented to improve solution
convergence during the advance phase. The numerical solution can manifest oscillations which
produce discontinuity in the vicinity of depth and flow values. These discontinuities can lead to
solution divergence, and a redistribution of the nodes, giving new average values of depth and
flow when oscillations are produced, can help improve numerical stability and avoid divergence.
Calculations are applied at each time step and the number of iterations increases until the solution converges. If the solution does not converge after ten iterations, the model calculates the average of all the depths, flows, and distances at each node during these ten iterations, and assumes this average as new values of depth, flow, and distance at each of the last fifteen nodes.

**Solution 4B: Handling More Nodes than the Maximum Allowable**

The maximum number of computational nodes (1,024), used in the model, is considered to be more than adequate because with increasing number of nodes, the simulation slows considerably. An algorithm that handles an exceedence of nodes which can be greater than the maximum number of computational nodes was developed to avoid model failure and unacceptably slow run times. This algorithm is only applied during the advance phase. During a simulation, if water continues advancing and needs to add a new node in the water velocity profile, the model invokes the algorithm only if advancing is not applied and a new node is not added. The new node is added considering the following criteria; if the depth at the penultimate node is higher than 1 mm, if the ratio of the depth at the penultimate and last node is higher than 1%, and if the velocity between the penultimate and last nodes is higher than 0.1 m/min.

Subsequently, the model checks whether the current number of nodes is greater than the maximum number of computational nodes, minus three. If this condition is true (number of nodes is greater than maximum number of nodes minus three), the algorithm is activated. The algorithm considers as new last node the maximum number of computational nodes minus five; and, based on this new last node, the model generates new values of depth, flow, distance, advance time, and recession time at each node without changing the values in the first (upstream) and last (downstream) nodes.

The algorithm calculates a new distance variation taking the distance at the last node and dividing by the new last node, minus one. Then, the algorithm recalculates the new distance based on the distance variation at each node. Later, it uses a linear interpolation method to
calculate new values of advance time, distance, recession time, depth, and flow rate. Figures 11 and 12 explain the main elements of this algorithm. Figure 11 shows how the algorithm does not generate depths, flows, distances, advance and recession times at the first and last nodes, and the generation of new nodes at equal distances along the border. Figure 12 shows the linear interpolation method used to calculate the depths, flows, distances, advance and recession times for the new (relocated) nodes. The linear interpolation consists in obtaining the difference between the distance at the new node $h_{(1nw)}$ and node $h_{(1)}$, divided by the difference between nodes $h_{(2)}$ and $h_{(1)}$. This relation is multiplied by the difference of depths, flows, distances, advance, and recession time at the existing node(1) and node(2) and added to those values of the existing node (1).

**Fig. 11.** Generating new computational nodes from existing nodes

**Fig. 12.** Linear interpolation for recalculation of depths, flows, distances, advance, and recession time for each new node
Finally, the algorithm returns the values of depths, flows, distances, advance, and recession time, and with these new values the four-point implicit solution of the Saint-Venant equations are reapplied.

**Improving Hydraulic Simulations During the Recession Phase**

During recession-phase computations, a recession time increment ($\Delta t_r$) is needed to establish the increment at each time step, the $\Delta t_i$ is used to calculate the depth, discharge, distance and recession time at each different node. $\Delta t_r$ is a parameter in the Saint-Venant equations and in the volume-balance method; both mathematical approaches are used during the simulation of the recession phase.

During the recession phase, the model considers a $\Delta t_r$ lower than $\Delta t_a$ because of the large reduction of the water depth due to the infiltration and no advancing condition. The model normally considers $\Delta t_r$ equal to 20 s because after many simulations, it was observed that this recession time increment generates a number of nodes which are inside of the limits to ensure a normal simulation. However, if the cutoff time is low, $\Delta t_r$ needs to be reduced; consequently, during the recession phase the criterion applied in the advance phase is also considered, using a factor which gives $\Delta t_r$ as a function of $\Delta t_a$.

If the cutoff time is lower than 50 s and the discharge entering the border is lower than 0.01 m³/s per unit width, $\Delta t_r$ is set equal to 0.9$\Delta t_a$. In addition, if the cutoff time is lower than 500 s, $\Delta t_r$ is set equal to 5 s. This criterion was supported executing many simulations with different cutoff times and $\Delta t_r$ which help to determine the best factor that relates $\Delta t_r$ and $\Delta t_a$ when small cutoff times are used to avoid numerical divergence.

In the recession phase, two numerical instability problems were found. One of these was the inaccuracy of $\phi$ and $\theta$ in solution to the Saint-Venant equations, and the other was the presence of zero water depths, which need to be handled to avoid division by zero.
**Problem 1: Inappropriate Values of $\phi$ and $\theta$**

The Saint-Venant equations, which are discretized by the four-point implicit solution method, need accurate spatial and temporal weighting factors ($\phi$ and $\theta$) to make the solution converge; during the advance part an algorithm to find accurate $\phi$ and $\theta$ was developed. In the recession phase, another approach was implemented whereby the volume-balance method is automatically applied when the solution of the Saint-Venant equations experiences numerical instability.

Different authors, such as Valiantzas (2000) and Walker and Kasilingam (2004), warn that the use of volume-balance method may cause significant errors in advance-phase calculations because it is assumed that the average depth of water is constant; if an intake equation with steady state terms is used, the parameters of the equation are determined outside of the volume-balance analysis; consequently, these values are not very accurate because of the difficulty of measurement in the field. In the model, the volume-balance method is applied in the recession phase as a last resort to achieve a successful simulation.

**Solution 1: Implementation of the Volume-Balance Method**

The volume-balance method expresses the idea of mass conservation and is written as:

$$\Delta V = V_{in} - V_{out}$$

(3.43)

where $\Delta V$ is the volume of water stored in the border (m$^3$ per m); $V_{in}$ is the volume of water that enters the border (m$^3$ per m); and, $V_{out}$ is the volume of water that flows out from the border at the downstream end (m$^3$/ per m).

Recession computations take advantage of nodes that were generated during the advance-phase (Fig. 13), and each cell is bounded by two computational nodes at each time step; consequently, the volume-balance method is applied at each cell where inflow and outflow discharges are calculated using the Chezy equation, using a calculated value of $C$. In this way, the stored volume of surface water at each computational cell is determined.
The model analyzes each cell using the following formula, which is given for cell 2:

\[ q_2 \Delta t - (q_3 \Delta t + 0.5(Z_2 + Z_3)\Delta x) = 0.5(\Delta h_2 + \Delta h_3)\Delta x \]  

where \( q_2 \) is the discharge entering cell 2 (m\(^3\)/s per m); \( \Delta t \) is the time increment; \( q_3 \) is the discharge exiting cell 2 (m\(^3\)/s per m); \( Z_2 \) and \( Z_3 \) are the infiltration depths (m), as calculated from the Kostiakov equation; \( \Delta x \) is the distance between nodes 2 and 3; and, \( \Delta h_2 \) and \( \Delta h_3 \) are the depths of water stored in the cell at nodes 2 and 3.

This formula is first applied at the last node where the depths of stored water (\( \Delta h \)) in the cell at the last and penultimate nodes are considered equal. Consequently, there are the same number of unknowns and knowns; finally, the total stored volume at last cell and \( \Delta h \) at the last and second node are calculated, from there, the other cells continue the same procedure until the \( \Delta h \) at the nodes of the first cell is determined.

The model uses the volume-balance method during the recession phase only if the solution to the Saint-Venant equations does not converge. To do this, the model constantly saves the results of discharges, distances, depths, and advance and recession times from the previous steps; then, when the model applies the volume-balance method these saved results are returned as the current initial input for the volume-balance method, and the calculations are executed as explained above.

**Problem 2: Handling Zero Water Depths**

After completion of the advance phase, the four-implicit solution of the Saint-Venant equations could have produced numerical oscillations. These spurious oscillations in the water
depth profile can result in local calculated depths near zero due to infiltration into the soil during the recession phase. The zero depths generate “division by zero” errors because when the solution of the Saint-Venant equations are applied, water depth is used in the denominator, resulting in mathematical singularities. Of course, this situation results in the abrupt termination of the simulation.

Solution 2: Grid Adjustment to Eliminate Nodes with Zero Results

The presence of zero water depths due to infiltration of water into the soil at some nodes is solved using an algorithm that recalculates the water depths and flows at the nodes where these values are zero. The solution approach varies according to the location of the nodes where the zero water depths are presented.

Location of Nodes with Zero Water Depth Close to the Uphill and Downhill Ends of the Border

The approach to solving this issue consists in removing the nodes only if they are close to the uphill and downhill ends of the border (Fig. 14, Case A); in this case, the algorithm counts the number of nodes with zero depths located at the extremes of the border length, then it removes these nodes and recalculates the number of the first node upstream, which is the first node that has a water depth is greater than zero, and the last node downstream, which is equal to the total number of nodes minus the number of removed nodes.

Location of Nodes with Zero Water Depth Between the Uphill and Downhill Ends of the Border

The approach in this case consists in identifying whether the nodes with zero water depth are located between the uphill and downhill ends of the border (Fig. 14, Case B). The algorithm saves the node numbers where zero water depth occurs, then it makes sure that the node numbers are not equal to upstream node (usn) and downstream node (dsn). Making sure that
the location of nodes with zero water depth are between the uphill and downhill ends, after this, the code recalculates values of depths and flows at the nodes that have zero water depth. The recalculation of these values consists in determining the average of the discharge and water depths from the nodes which are adjacent to the node with zero water depth, and populating those average values of depths and flow in the nodes with zero water depth (Fig. 15).

Fig. 14. Location of the nodes with zero water depth along the border

Fig. 15. Recalculation of previous and new water depths and discharges when nodes with zero water depth are not at the extremes

When there are more than three nodes with zero water depth and are located in consecutive positions within the two extremes of the total number of nodes (extreme upstream...
and downstream nodes), the algorithm automatically removes them, taking as the first upstream node the first node with non-zero water depth downstream of the last removed node; consequently, the total number of nodes is equal to the previous total number of nodes minus the number of removed nodes. This algorithm is also used during the advance-recession part.

Implementation of a Technique to Simulate Simultaneous Advance and Recession

During the simultaneous advance-recession phase, there is a time increment (\(\Delta t_{\text{sr}}\)) given by the model. This value is normally equal to 5 s, but only if the cutoff time is lower than 50 s and the inflow is lower than 0.01 m\(^3\)/s per unit width, the \(\Delta t_{\text{sr}}\) is equal to 0.7 \(\Delta t_a\). The 0.7 value is a factor which relates \(\Delta t_{\text{sr}}\) to \(\Delta t_a\) in the model when there is a small cutoff time; it was determined after many simulations, considering different longitudinal slopes, border lengths, and soil types. The value of \(\Delta t_{\text{sr}}\) is lower than both \(\Delta t_a\) and \(\Delta t\) to stabilize the complexity of the numerical calculations by using both the Saint-Venant equations and the volume-balance method.

During the simultaneous advance-recession phase the time increment for both advance and recession should be the same. The idea was to apply both the Saint-Venant equations and the volume-balance method at the same time, then use an optimization algorithm to obtain the same water depth at the computational node which forms the interface between these two methods. The development of the advance-recession phase involves the application of a new technique developed in this research and no problems were observed with its implementation. This new technique is explained below.

Hybrid Implementation of the Solution to the Saint-Venant Equations and the Volume-Balance Method

The Saint-Venant equations compute the water depths, discharges, distance and advance times during the advance phase, and the volume-balance method is automatically applied to compute water depths, discharges, and recession times during the recession phase. In this advance-recession phase, the model divides the implementation of recession and advance phases at the same time step, identifying the maximum water depth in the water profile (Fig. 16).
The reason for the use of maximum water depth, as a division for the application of the Saint-Venant equations and the volume-balance method, is the almost zero or minimum variation of water depth at this location. The node that corresponds to the maximum water depth is called the interface node.

The interface node has two water depths: (1) calculated from the Saint-Venant equations; and, (2) calculated from the volume-balance method and specified discharge coming from the previous elapsed time increment. If these two results of water depths are not the same, the model cannot continue with the next time step because it does not know which water depth should be taken as the initial guess to continue with the solution. So, it recalculates the water depth at the interface node in the next time step.

![Fig. 16. Saint-Venant equations and volume-balance method in the advance-recession phase](image)

To achieve equalization of depths, the model implements the golden search optimization procedure to obtain the same water depth from the Saint-Venant equations and the volume-balance method. The previous discharge at the interface node is modified during the optimization to obtain the same water depth, wherein the golden search optimization procedure has has a bracketing range of 1% of the previous discharge as minimum value, half of the previous discharge as a middle value, and 1.2 times the previous discharge as the maximum value. These three discharges were applied in the golden search procedure as starting values to find the minimum difference between the water depth from the Saint-Venant equations and that of the volume-balance method, both at the interface node.
The time increment used for the solution to the Saint-Venant equations and the volume-balance method is 5 s; this time can change according to the inflow rate and cutoff time values. During the solution of the Saint-Venant equations, the algorithm which finds the best values of $\phi$ and $\theta$ that do not produce numerical oscillation is also applied. Furthermore, if the hybrid implementation of the Saint-Venant equations and the volume-balance method does not allow convergence of the solution, the model will automatically activate the volume-balance method, switching to the recession phase. This is normally executed when the water depths are so small that the change of $\phi$ and $\theta$ values cannot handle the numerical oscillations which are still produced due to the limitations of the four-point implicit solution method.
CHAPTER 4
MODEL IMPLEMENTATION

Functionality of the Model

As explained in Chapter 3, different techniques were implemented in the model to improve its robustness. Figures 17 and 18 show all the calculation processes and how the new techniques are applied. The model starts by calculating the initial depths using the Newton-Raphson method for the border condition. Then, the model simulates the depth profile along the border in which depths, flows, distances, advance times, and recession times are calculated for different nodes at each time step. In this process, the solution is divided into three parts as detailed in Chapter 3: (1) the advance phase; (2) the advance-recession phase; and (3) the recession phase. The model classifies the phase that will be activated based on the cutoff time and the advancing condition. If the total accumulated time (adding all the time steps already simulated) is less than the cutoff time and the water is advancing, the model uses the advance phase. If the total accumulated time is greater than the cutoff time and the water is advancing, the model uses the advance-recession phase (2). Finally, if the total elapsed time is greater than the cutoff time and the water is not longer advancing, the model uses the recession phase (3).

The advance phase (1) involves the solution of the Saint-Venant equations using the four-point implicit method, resulting in a system of non-linear algebraic equations and the addition of the advance time increment to the total accumulated time. These non-linear algebraic equations are solved by the Newton-Raphson algorithm. If the new values are not close to the initial guesses, the solution is not acceptable and the process is repeated until the solution diverges; on the other hand the solution will converge. In the divergence case, the model will automatically apply an algorithm to change $\phi$ and $\theta$ or to reduce the advance time increment. In addition, other new techniques, such as, an algorithm that handles the water effect with the downstream boundary condition (blocked-end), or an algorithm that handles excessively fast advance at the last node, are applied.
The advance-recession phase (2) involves the use of the four-point implicit solution method of the Saint-Venant equations until the depth (4 mm) needed to apply the hybrid numerical solution of the Saint-Venant equations and volume-balance method is reached. In addition, the advance-recession time increment is added to the total accumulated time. Then, the hybrid numerical solution method is applied. If the hybrid numerical solution method begins to diverge due to numerical instability, the model applies recession phase (3) and the numerical solution of the volume-balance method is activated for all the rest of the simulation process. Grid management is also used to remove computational nodes that present zero depths.

The recession phase (3) involves the four-point implicit solution method of the Saint-Venant equations and the volume-balance method. In addition, the recession time increment is added to the total accumulated time. The solution method for the Saint-Venant equations is first applied until (and if) it diverges, then the volume-balance method is applied for the rest of the simulation process.

At the end of each phase, the advance and recession time, and the distance at each generated node, are recorded in arrays to be used for the water requirement and application efficiency calculations. Then, if the solution has converged, the time step is incremented, and the whole process is repeated. Finally, after the simulation is finished the objective function value is calculated based on the water requirement efficiency, application efficiency, and their respective weighting factors. The optimization procedure and the calculation of the objective function are explained below.

**Application Efficiency (Eₐ) and Water Requirement Efficiency (WRE)**

WRE and Eₐ are key criteria in border irrigation system design and management (Zerihun et al. 2002). WRE is defined as the volume of water stored in the root zone by irrigation, divided by the volume of soil water deficit. The deficit is calculated by multiplying the required irrigation depth, based on the soil water deficit (depth) at the time of irrigation, by the product of border
Fig. 17. Flowchart of the simulation model using the new techniques developed in this research
width and length, thereby resulting in a volume of water. Of the applied irrigation water, some remains in the root zone, some may infiltrate below the root zone (deep percolation), and some may be lost as surface runoff. Other losses, such as surface evaporation, are usually negligible. \( E_a \) is defined as the volume of water retained in the root zone divided by the volume of applied irrigation water at the uphill end of the border. In most cases, the application and water requirement efficiencies should be maintained between 90% and 95%, respectively (Walker 2003). Figure 19 shows the water volumes that form part of the \( E_a \) and WRE calculations, where \( Z_R \) is net infiltration depth, \( V_{ZR} \) is the volume of water stored in the root zone, \( V_{DP} \) is the volume of deep percolation, \( V_D \) is the volume of water deficit, and \( V_{SRO} \) is the volume of surface runoff.

The calculation of WRE and \( E_a \) is important to establish an objective function which acts as a performance indicator for a specific inflow rate, border length, cutoff time, soil type, and...
longitudinal ground slope. The way the model calculates WRE and $E_a$ is by using an algorithm that records all the advance times, recession times, and distances. Then, the model generates the values of advance, recession and distance for 100 points along the field length using linear interpolation. The recorded values of advance time, recession time and distance is reorganized in ascendant order as a function of the distance, and linear interpolation is used to generate these values for 100 points (Fig. 20) along the border length, located at a uniform spacing of $\Delta d$. The $\Delta d$ value is calculated by dividing the last recorded distance by the total number of points, minus one (99).

![Fig. 19. Side view of a surface irrigation event](image)

![Fig. 20. Interpolation of the 100 points of the advance and recession curves](image)
The advance time, recession, and distance at each of the 100 points are used to calculate the total volume of water stored in the root zone and the total volume of infiltrated water into the soil. Both are used to calculate WRE and \( E_a \). The volume of water infiltrated into the soil is produced, adding all the average intake opportunity times between two points, multiplied by the difference of distance between these two points and the border width for all 100 points. The model considers a unit border width of 1.0 m. The intake opportunity time is the difference between the calculated recession and advance times at each spatial node. If the total volume infiltrated between two adjacent nodes is greater than the volume of water deficit in the root zone, the total volume of water stored in the root zone will be calculated by adding only the total volume of water infiltrated in the root zone between two points for all 100 points. On the other hand, it will be calculated by adding the total volume of water infiltrated between two points which is less than the volume of root zone water deficit (Fig. 21).

Fig. 21. Calculation of \( E_a \) and WRE indexes between two points

After WRE and \( E_a \) are calculated, the objective function is formulated using two weighting factors for the WRE and \( E_a \) components.
Data Sets and Ranges

As detailed above, the model needs to be robust to apply an optimization algorithm; therefore, specified ranges of inflow rate, border length, and longitudinal slope were generated and the robustness of the model was tested over these ranges. However, it was found that the model can fail when outside of these parameter ranges, in which case the simulation stops prematurely and complete results are unavailable. The range for inflow rates is from 0.01 to 0.05 m³/s per m, the border length is from 100 to 500 m, and the longitudinal slope ranges from 0.05 to 1.00%. Using the robust hydraulic model already developed with the techniques explained in Chapter 3, the advance time for the minimum inflow rate is calculated. This value will be used as the maximum limit of cutoff time during the generation of grids which uses three vertices for the downhill simplex method, as explained below. To calculate the advance time for the minimum inflow rate (0.01 m³/s per m), the model starts with the calculated cutoff time that permits the water to reach the downhill end of the border. When the border is relatively long, the cutoff time (calculated by the model) may be insufficient to allow water to reach the downhill end of the field; however, the model has the capability to identify this problem and it increases the cutoff time by 200 s increments until it determines the minimum cutoff time which allows completion of the advance phase..

In Fig. 22 (Visual BASIC), it is seen that the model passes the value of maximum advance time from the advance time at the last node to a variable called “MaxAdvTime” if the distance at the last node is equal to the border length; on the other hand, the model increments the cutoff time by 200 s until the distance at the furthest (downstream) node is equal to the border length.

```vbnet
If ds(dsn) = blength Then
    MaxAdvTime = tadv(dsn)
Else
    Do
        maxco += 200
        ObjectiveFunc(maxco, Zreq, minqin, typesoil1, borderlengthparameter, longitudinalslope)
    Loop Until ds(dsn) = blength
    MaxAdvTime = tadv(dsn)
End If
```

Fig. 22. Algorithm to increase the cutoff time by 200 s until water advances to the border-end
Optimization Procedure

The optimization procedure starts with the generation of grids which consists of the objective function, the water requirement efficiency (WRE), and the application efficiency (Ea) for a range of inflow rates and cutoff times. This range starts from the minimum inflow rate and cutoff time (0.01 m³/s per m and 100 s), and it is generated giving an increment to each value as described in Eqs. 4.1 and 4.2:

\[
\text{tco}_i = \frac{\text{Max}_{tco} - \text{Min}_{tco}}{\text{size}_{tco} - 2}
\]

\[
\text{qin}_i = \frac{\text{Max}_{qin} - \text{Min}_{qin}}{\text{size}_{qin} - 2}
\]

where Max_{tco} and Min_{tco} are the calculated maximum advance times for the minimum inflow rate and minimum cutoff time, respectively; and Max_{qin} and Min_{qin} are the maximum and minimum inflow rates.

In addition, size_{qin} and size_{tco} are the grid sizes for the cutoff time and inflow rate which have been given by the user. For the optimizations developed as part of the results, the grid size changes according to the soil type because of the presence of different local minima which can vary the results of the optimization algorithm. The model uses a 24-by-22 grid size for cutoff time and inflow rate in clay and clay loam soils, and a 10-by-10 grid size for cutoff time and inflow rate in sandy, sandy loam, and silty loam soils. The reason for this grid generation is to calculate the initial three vertices needed by the optimization algorithm as input to determine the irrigation time and inflow rate which give the highest weighted average of water requirement efficiency and application efficiency. In other words, it helps ensure a valid starting point for the optimization algorithm. After the grid generation procedure, the values of inflow rate and cutoff time which generate the minimum objective function are recorded, and they are used in the optimization algorithm.

After the objective function is calculated using the specific inflow rate, cutoff time, longitudinal slope, net infiltration depth, border length, and soil type a "target function" is
calculated as 600 minus the objective function value. The concept of minimizing the objective function value, to get high values of water requirement and application efficiencies, changes with the use of the target function. For this part, the model takes the highest value of the target function that is equal to the minimum value of the objective function. Also, all the inflow rates and cutoff times are saved in an array. Therefore, the array dimensions are \( \text{size}_{\text{tco}} \times \text{size}_{\text{qin}} \). Then, the maximum value of the target function and the inflow rate and cutoff time, which produce this maximum value, are recorded. Finally, when the grid generation is finished, the model saves the value of the minimum objective function with the corresponding inflow rate and cutoff time.

The following step of the optimization procedure is the formulation of the downhill simplex method in multiple dimensions (Fig. 23); this method starts with the number of dimensions, plus one point, \((N + 1)\). In this case, the model is optimizing in two dimensions; therefore, three starting points are needed. The first starting point is for the minimum inflow rate and cutoff time \((0.01 \text{ m}^3/\text{s/m and 100 s})\), the second is the inflow rate and cutoff time recorded from the grid generation, and the third point is the second point incremented by 40% (including cutoff-time and inflow rate). In addition, the optimization algorithm needs the fractional convergence tolerance to be achieved in the function \((f_{\text{tol}} = 0.01)\), and the maximum allowable number of iterations \((\text{it}_{\text{max}} = 500)\). If convergence of the optimization takes more than 500 iterations, a message will appear, indicating that the maximum number of iterations has been exceeded. If the fractional range from the highest to lowest objective function is lower than the fractional convergence tolerance, the minimum objective function has been reached and the optimization procedure has found the combination of inflow rate and cutoff time that give the highest composite water requirement efficiency and application efficiency. On the other hand, computations continue until a solution has converged.
Objective Function

The objective function is used by the model to obtain the highest composite water requirement efficiency and application efficiency for a given set of dimensional and hydraulic parameters. The objective function involves weighting factors for the water requirement efficiency and application efficiency in its calculation. The objective function responds to a parabolic equation because it is formulated by combining weighting factors, and water requirement and application efficiencies, in which the equation makes a smooth relationship between the weighting factor and the water requirement efficiency, or application efficiency. This equation was formulated considering the weighting factor on the vertical axis and the water requirement efficiency (WRE, %) or the application efficiency (Ea, %) on the horizontal axis (Fig. 24).

Consequently, the objective function, as used in the model, is formulated as follows:

![Flowchart of the downhill simplex method as applied in this research](image-url)
Fig. 24. Parabolic equation involving the weighting factor and the WRE or $E_a$

$$
\beta_{\text{max}} = \text{Weighting factor for WRE, } E_a
$$

WRE = Water Requirement Efficiency (%)

$E_a$ = Application efficiency (%)

$$
\beta_{\text{max}} \left( \frac{\text{WRE}^2}{10000} - \frac{\text{WRE}}{50} + 1 \right) \text{ or } \beta_{\text{max}} \left( \frac{E_a^2}{10000} - \frac{E_a}{50} + 1 \right)
$$

(4.3)

where $E_a$ is the application efficiency (%); WRE is the water requirement efficiency (%); and, $\beta_{\text{max}}$ and $\beta_{\text{max}}$ are, respectively, the weighting factors for application and water requirement efficiencies. The model uses $\beta_{\text{max}}$ and $\beta_{\text{max}}$ equal to 125 and 375, respectively, where the maximum and minimum values of the objective function are 500 and 0, respectively. The objective function gives more weight to WRE than to $E_a$ because high values of $E_a$ can correspond to short cutoff times and large root zone water deficits, emphasizing adequacy of irrigation. Thus, it is not enough to have high $E_a$ when the irrigation is largely incomplete.

**Bracketing the Solution**

The grid generation procedure defines the three vertices which are used by the downhill simplex method for the optimization calculations. These vertices are used by this method to bracket the solution inside of the boundaries for inflow rate which guarantees the robustness of the model. In addition, it is known that very small or very large cutoff times can cause numerical problems in the model; consequently, the vertices also help to consider an acceptable cutoff time.
As explained above, the number of vertices is equal to the number of dimensions, plus one. One vertex is the minimum inflow rate and cutoff time, the second vertex is the inflow rate and cutoff time from the grid generation that produced the lowest objective function, and the third vertex is equal to the second vertex incremented by 40%.

**Optimization Method**

The model uses the downhill simplex method as an optimization algorithm to determine the cutoff time and inflow rate which produce the minimum value of the objective function (i.e. the highest composite efficiency). The downhill simplex method can be used with a maximum of 20 dimensions. In case of many dimensions (>20) the function often does not converge to the minimum because of the constant use of the contraction in all directions. It requires only function evaluations and not the calculation of derivatives. It is known that the use of many dimensions can complicate the convergence of the function to the minimum (Press et al. 2007).

As described above, the method starts by choosing the first points (N +1). The algorithm then makes its way downhill through the N-dimensional topography until it encounters a minimum. Thus, the iterations stop when a movement in any direction would result in an increased objective function value. The implementation of the downhill simplex method in this research follows these steps:

- The method takes steps where the point which gives the highest value of OBF is moved though the other point which gives the lowest value of OBF producing a trial point. This step is called reflection, and they are constructed to conserve the volume of the simplex. The trial point \(X_r\) is generated by the following formula where \(X_g\) is the centroid of N points; \(X_{N+1}\) are the start points and \(\alpha\) equal to 1.

\[
X_r = X_g + \alpha (X_g - X_{N+1})
\]  

(4.4)

- If the OBF of the trial point is between the second highest and lowest value of the vertices of the simplex, then the highest point is replaced by the trial point \(X_r\). If the OBF of \(X_r\) is lower than or equal to the lowest point (lowest OBF), then the trial point is
checked as $X_r$ to see if the OBF drops further in the direction of this $X_r$. The lower of these two points, $X_r$ and $X_r$, replaces the highest point of the simplex, if the OBF of $X_r$ is greater the simplex expands in direction $X_{rr}$. In this expansion a new trial point ($X_e$) is created using the following formula:

$$X_e = X_r + \beta (X_r - X_g)$$ (4.5)

where $\beta$ is equal to 2.

- If the OBF of $X_r$ is higher than or equal to the value of the highest point, the value is checked as $X_{rr}'$ to see if the OBF is lower between the highest point and the average of points except the highest point ($X_b$). If the OBF of $X_r$ is higher than or equal to the second highest point and lower than the highest point the value is checked as $X_{rr}''$ to see if the OBF is lower between the point $X_b$ and the point $X_r$.

- If the value of OBF of $X_{rr}'$ or $X_{rr}''$ is lower than the highest point of the simplex considering $X_r$ then $X_{rr}'$ or $X_{rr}''$ replaces the highest point.

- If the value of OBF of $X_{rr}'$ or $X_{rr}''$ is higher than the highest point of the simplex, considering $X_r$, the simplex contracts itself around its lowest point. This contraction is applied using Eq. 4.6 where $\gamma$ is equal to 0.5:

$$X_c = X_g + \gamma (X_g - X_{N+1})$$ (4.6)

The downhill simplex method can contract in all directions, and when it does, the method pulls itself around the lowest point. Equation (4.6) is applied to reach this goal. In the implementation used in this research, the model works with two dimensions (inflow rate and cutoff time). One added implementation was that if the generated values (trial points), after expansion or contraction, have a cutoff time lower than 5 s and an inflow rate lower than 0.01 m$^3$/s per m, the model will automatically take the reflected point and start over by ordering the N+1 points (number of dimension plus one, for this case N+1 is three). Low values of cutoff time and inflow rate are generated by the downhill simplex calculations to find the optimum values.
Field Tests in Wellsville, Utah

Field tests were conducted on three borders strips, each with a different inflow rate and cutoff time, to compare with the simulation results from the mathematical model described in Chapter 3. These three border strips were called North, South, and Center due to their respective locations. During the field work, four field tests were developed, three of them in bare soil and the other in soil covered by vegetation. The field work was conducted in Wellsville (Utah) several months before completing the final version of the optimization model, but at that time it was recognized that some verification of the model would be required before proceeding with the development of recommendations based on model runs. Appendix C describes the field experiments and compares the results from the model with that from the field work and the SIRMOD program.
CHAPTER 5
RESULTS AND DISCUSSION

Developing a Robust Model

After creating a robust hydraulic model for sloping blocked-end border irrigation systems, the parametric limits of border length (up to 500 m), inflow rate (0.01 to 0.05 m$^3$/s per m), and longitudinal slope (up to 1%) were used in this research to produce the simulation and optimization results described in this chapter.

Results from the Analysis of $\phi$ and $\theta$

As described in Chapter 3, during the development of the algorithm which determines the best values of $\phi$ and $\theta$ (those that do not produce divergence due to numerical instability) many simulations were run with different values of $\phi$ and $\theta$ during the advance phase to select those that lead to solution convergence, and consequently allow for a successful termination of the simulation. The following presents the results from the different situations where the average number of reversals and profile instability for water depths during the advance phase were calculated.

Blocked-end Downstream Boundary Condition and Parabolic Equation A set of simulations were performed for blocked-end downstream boundary conditions. These simulations include only the new techniques implemented to handle the downstream boundary condition explained in Chapter 3. The simulations were performed considering a range of $\phi$ and $\theta$ values from 0.52 to 0.78. Tables 4, 5, and 6 show the range of $\phi$ and $\theta$ values with the lowest and second-lowest profile instability averages, and minimum average reversals for 1.00%, 0.50%, and 0.05% longitudinal field slopes. It is observed that those values of $\phi$ and $\theta$ that have the minimum average reversals are higher values, near 0.78, particularly for steep field slopes. It means that, following the trapezoidal rule to calculate the weighting factors for the four-point implicit solution (Fig. 25), the trajectory between two nodes has a curved shape rather than a straight line when steep longitudinal slopes are present (higher velocities). The curved trajectory
is due to the advance condition in the last nodes of the water profile along the border. The respective captions for Tables 4 to 10 explain what pairs of $\phi$ and $\theta$ values generate the minimum average reversals, lowest profile instability, and second lowest profile instability. The legend presents three rectangles with different colors and margins. The white rectangle with dark margins symbolizes the pairs of $\phi$ (vertical axis) and $\theta$ (horizontal axis) values that have the lowest average profile instability for the specified conditions (soil types, border length, longitudinal slope, advance time increment, and Chezy coefficient). The dark orange rectangle symbolizes the pairs of $\phi$ and $\theta$ values that have the minimum average reversal for the specified conditions. Finally, the light orange rectangle symbolizes the pairs of $\phi$ and $\theta$ values that have the second higher values of profile instability minimum average reversal for the specified conditions (second-lowest profile instability).

Table 4. Best Values of $\phi$ and $\theta$ for a Longitudinal Field Slope of 1.00%.

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Table 5. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 0.50%.

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Table 6. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 0.05%.

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Fig. 25. Impact of differing values of $\phi$ and $\theta$ (from 0.5 to 0.78) on the water surface profile

It is also observed that the number of average reversals reduces as the longitudinal field slope decreases, and is associated with decreasing $\phi$ and $\theta$ values. The lowest value of slope (0.05 %) generates more pairs of values of $\phi$ and $\theta$ that completes the advance phase with more minimum values of average reversals and the lowest average profile instability. Mild slopes can be better handled by the Saint-Venant equations, and the range of $\phi$ and $\theta$ values which secures the convergence during the advance phase are more extended ($\phi$ and $\theta$ range from 0.6 to 0.78). Steep slopes need values of $\phi$ and $\theta$ close to 0.78 to avoid numerical oscillations.

In Tables 7, 8, 9, and 10, values of $\phi$ and $\theta$ that complete the advance phase and secure convergence of the results were plotted for four advance time increments (60, 40, 20, and 5 seconds) with a longitudinal field slope of 0.20%. It is observed that during the analysis to calculate the best values of $\phi$ and $\theta$, there are more $\phi$ and $\theta$ values that produce second-lowest average profile instability when the advance time increment is 5 s and when the longitudinal slope is 0.05%; on the other hand, there are more pair of $\phi$ and $\theta$ values that produce second-lowest profile instability average when the advance time increment is 25 s and the longitudinal field slope is 0.20%. This means that if the advance time increment is reduced to a small number (say 5 s) when the slope is 0.05%, the simulations will be more stable without making many changes in the $\phi$ and $\theta$ values; furthermore, if the advance time increment is reduced progressively, it will help to reduce numerical oscillations during simulations.
Table 7. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 0.20% and $\Delta t_a$ of 60 s.

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$\Delta t_{adv} = 60$ s, Slope $= 0.2 \%$, Aver $\_PS = 0.00001$

Table 8. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 0.20% and $\Delta t_a$ of 40 s.

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</tbody>
</table>

$\Delta t_{adv} = 40$ s, Slope $= 0.2 \%$, Aver $\_PS = 0.00001$
Table 9. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 0.20% and $\Delta t_a$ of 25 s.

Table 10. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 0.20% and $\Delta t_a$ of 5 s.
Advancing Downstream Boundary Condition and Parabolic Equation. These simulations also include only the new techniques implemented to handle the downstream boundary condition explained in Chapter 3. This analysis is only applied to the advance phase and it is free-draining until the number of time steps is equal to 150. Again the Chezy coefficient is considered as function of water depth. Tables 11, 12, and 13 show a range of $\phi$ and $\theta$ values (0.52 to 0.78) with a minimum average reversal for 1.00%, 0.20%, and 0.05% longitudinal field slopes. The captions of Tables 11 to 13 present only a dark orange rectangle that symbolizes the pairs of $\phi$ and $\theta$ values with minimum average reversals.

Table 11. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 1.00%.

<table>
<thead>
<tr>
<th>Slope = 1 %, Ave PS = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
</tr>
<tr>
<td>0.52</td>
</tr>
<tr>
<td>0.54</td>
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<tr>
<td>0.56</td>
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<tr>
<td>0.58</td>
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<tr>
<td>0.60</td>
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<td>0.62</td>
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<td>0.64</td>
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<tr>
<td>0.66</td>
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<td>0.68</td>
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<td>0.70</td>
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<tr>
<td>0.72</td>
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<tr>
<td>0.74</td>
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<tr>
<td>0.76</td>
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<tr>
<td>0.78</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.52</td>
</tr>
<tr>
<td>0.54</td>
</tr>
<tr>
<td>0.56</td>
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<tr>
<td>0.58</td>
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<tr>
<td>0.60</td>
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<td>0.72</td>
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<td>0.78</td>
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</tbody>
</table>
Table 12. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 0.20%.

<table>
<thead>
<tr>
<th>$\Phi$</th>
<th>0.52</th>
<th>0.54</th>
<th>0.56</th>
<th>0.58</th>
<th>0.60</th>
<th>0.62</th>
<th>0.64</th>
<th>0.66</th>
<th>0.68</th>
<th>0.70</th>
<th>0.72</th>
<th>0.74</th>
<th>0.76</th>
<th>0.78</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.52</td>
<td>0.54</td>
<td>0.56</td>
<td>0.58</td>
<td>0.60</td>
<td>0.62</td>
<td>0.64</td>
<td>0.66</td>
<td>0.68</td>
<td>0.70</td>
<td>0.72</td>
<td>0.74</td>
<td>0.76</td>
<td>0.78</td>
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</tbody>
</table>

Table 13. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 0.05%.

<table>
<thead>
<tr>
<th>$\Phi$</th>
<th>0.52</th>
<th>0.54</th>
<th>0.56</th>
<th>0.58</th>
<th>0.60</th>
<th>0.62</th>
<th>0.64</th>
<th>0.66</th>
<th>0.68</th>
<th>0.70</th>
<th>0.72</th>
<th>0.74</th>
<th>0.76</th>
<th>0.78</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.52</td>
<td>0.54</td>
<td>0.56</td>
<td>0.58</td>
<td>0.60</td>
<td>0.62</td>
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</table>
From Tables 11, 12 and 13, it is observed that there are more pairs of \( \phi \) and \( \theta \) values which produce the minimum average number of reversals when the field slope approaches 0.05%. Consequently, the Saint-Venant equations generates less numerical oscillations when it works under mild slopes (close to 0.05 %) conditions. Also, Tables 14, 15, and 16 show the range of \( \phi \) and \( \theta \) values that generates the minimum average reversal and leads to convergence of the numerical solution for 0.05, 0.03, and 0.01 m\(^3\)/s per m inflow rates. The legend of the following tables present two rectangles of different colors: (1) the orange rectangle symbolizes all the pairs of \( \phi \) and \( \theta \) values that have minimum average reversals; and, (2) the magenta rectangle symbolizes all the pairs of \( \phi \) and \( \theta \) that have converge after the analysis for all the 150 time steps.

**Table 14.** Best Values of \( \phi \) and \( \theta \) for Longitudinal Field Slope of 1.00\%, \( \Delta t_a \) of 60 s, Clay Soil, and Inflow Rate of 0.05 m\(^3\)/s per m.

<table>
<thead>
<tr>
<th>( \Phi )</th>
<th>0.52</th>
<th>0.54</th>
<th>0.56</th>
<th>0.58</th>
<th>0.60</th>
<th>0.62</th>
<th>0.64</th>
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<tbody>
<tr>
<td>( \theta )</td>
<td>0.50</td>
<td>0.52</td>
<td>0.54</td>
<td>0.56</td>
<td>0.58</td>
<td>0.60</td>
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Inflow = 0.05 m\(^3\)/s per m, Ave_PS = 0
Table 15. Best Values of $\phi$ and $\theta$ for Longitudinal Field Slope 1.00%, $\Delta t_a$ of 60 s, Clay Soil, and Inflow Rate of 0.03 m$^3$/s per m.

<table>
<thead>
<tr>
<th>Inflow = 0.03 m$^3$/s per m, Ave_PS = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
</tr>
<tr>
<td>0.52 0.54 0.56 0.58 0.60 0.62 0.64 0.66 0.68 0.70 0.72 0.74 0.76 0.78</td>
</tr>
<tr>
<td>$\theta$</td>
</tr>
<tr>
<td>0.52 0.54 0.56 0.58 0.60 0.62 0.64 0.66 0.68 0.70 0.72 0.74 0.76 0.78</td>
</tr>
</tbody>
</table>

Table 16. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 1.00 %, $\Delta t_a$ of 60 s, Clay Soil, and Inflow Rate of 0.01 m$^3$/s per m.

<table>
<thead>
<tr>
<th>Inflow = 0.01 m$^3$/s per m, Ave_PS = 0</th>
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</thead>
<tbody>
<tr>
<td>$\phi$</td>
</tr>
<tr>
<td>0.52 0.54 0.56 0.58 0.60 0.62 0.64 0.66 0.68 0.70 0.72 0.74 0.76 0.78</td>
</tr>
<tr>
<td>$\theta$</td>
</tr>
<tr>
<td>0.52 0.54 0.56 0.58 0.60 0.62 0.64 0.66 0.68 0.70 0.72 0.74 0.76 0.78</td>
</tr>
</tbody>
</table>
From this analysis for different inflow rates, it is observed that low inflow rates generate more values of \( \phi \) and \( \theta \) that have a corresponding minimum average number of reversals. Consequently, low inflow rates tend to produce less numerical oscillations, thereby providing greater numerical stability and model robustness. The same analysis is shown in Appendix A for the five different soil types (clay, clay loam, silty loam, sandy loam, and sandy). It is observed that the soil type with highest infiltration rate (sandy) generates more pairs of \( \phi \) and \( \theta \) values that produce the minimum reversal average; this means that the numerical solution of the Saint-Venant equations is more stable with this soil type.

**Free-Draining Downstream Boundary Condition and Constant Chezy Coefficient** In this last situation, these simulations include only the new techniques implemented to handle the downstream boundary condition explained in Chapter 3. The analysis was applied to the advance phase and the model runs until the number of time steps is equal to 120. In addition, in this particular case, the Chezy coefficient (C) was constant for all simulations. Again, the respective captions of Tables 17 to 23 present two rectangles of different colors: (1) the orange rectangles symbolize all the pairs of \( \phi \) and \( \theta \) values that have the minimum average number of reversals; and, (2) the magenta rectangles symbolize all the pairs of \( \phi \) and \( \theta \) that have converged after the analysis for all 120 time steps.

In Tables 17 to 19, a range of \( \phi \) and \( \theta \) values (0.52 – 0.78) are shown with the minimum average number of reversals, and values of \( \phi \) and \( \theta \) that result in successfully-completed simulations for 1.00%, 0.55%, and 0.25% longitudinal field slopes. From these tables, it is seen that with a constant Chezy coefficient, there are more pairs of \( \phi \) and \( \theta \) values that produce the minimum average reversal when the longitudinal field slope approaches a value of 0.25%. Consequently, the numerical solution of the Saint-Venant equations experiences less instability problems with mild slopes, in spite of a constant value of the Chezy coefficient.
### Table 17. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 1.00%, $\Delta t_a$ of 60 s, Clay Soil, and C of 25.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>0.52</th>
<th>0.54</th>
<th>0.56</th>
<th>0.58</th>
<th>0.60</th>
<th>0.62</th>
<th>0.64</th>
<th>0.66</th>
<th>0.68</th>
<th>0.70</th>
<th>0.72</th>
<th>0.74</th>
<th>0.76</th>
<th>0.78</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.52</td>
<td>0.54</td>
<td>0.56</td>
<td>0.58</td>
<td>0.60</td>
<td>0.62</td>
<td>0.64</td>
<td>0.66</td>
<td>0.68</td>
<td>0.70</td>
<td>0.72</td>
<td>0.74</td>
<td>0.76</td>
<td>0.78</td>
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</table>

There is a color-coded chart showing the values that converge and the minimum average reversal.

### Table 18. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 0.55 %, $\Delta t_a$ of 60 s, Clay Soil, and C of 25.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>0.52</th>
<th>0.54</th>
<th>0.56</th>
<th>0.58</th>
<th>0.60</th>
<th>0.62</th>
<th>0.64</th>
<th>0.66</th>
<th>0.68</th>
<th>0.70</th>
<th>0.72</th>
<th>0.74</th>
<th>0.76</th>
<th>0.78</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.52</td>
<td>0.54</td>
<td>0.56</td>
<td>0.58</td>
<td>0.60</td>
<td>0.62</td>
<td>0.64</td>
<td>0.66</td>
<td>0.68</td>
<td>0.70</td>
<td>0.72</td>
<td>0.74</td>
<td>0.76</td>
<td>0.78</td>
</tr>
</tbody>
</table>

There is a color-coded chart showing the values that converge and the minimum average reversal.
Table 19. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 0.25 %, $\Delta t_a$ of 60 s, Clay Soil, and C of 25.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>0.52</th>
<th>0.54</th>
<th>0.56</th>
<th>0.58</th>
<th>0.60</th>
<th>0.62</th>
<th>0.64</th>
<th>0.66</th>
<th>0.68</th>
<th>0.70</th>
<th>0.72</th>
<th>0.74</th>
<th>0.76</th>
<th>0.78</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.52</td>
<td>0.54</td>
<td>0.56</td>
<td>0.58</td>
<td>0.60</td>
<td>0.62</td>
<td>0.64</td>
<td>0.66</td>
<td>0.68</td>
<td>0.70</td>
<td>0.72</td>
<td>0.74</td>
<td>0.76</td>
<td>0.78</td>
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</tbody>
</table>

In Tables 20 and 21, an analysis for difference advance time increments for a constant Chezy coefficient was developed. From Tables 17 (for a slope of 1.00%), 20, and 21, it is observed that there are more pairs of $\phi$ and $\theta$ values which produce the minimum average number of reversals and result in a successful simulation after 120 time steps, with advance time increments equal to 20 s. This means that reducing the advance time increment helps to diminish the numerical oscillations during a simulation without considering the Chezy coefficient as function of the water depth (i.e. a constant value of Chezy C).

Other important analyses for different Chezy coefficients were also developed. Tables 22 and 23 show the range of $\phi$ and $\theta$ values analysis which produce the minimum average number of reversals and lead to solution convergence after 120 time steps for three different Chezy coefficient values (25, 40, and 60). From Tables 17, 22, and 23, it is seen that the relatively high Chezy coefficient has more pairs of $\phi$ and $\theta$ values that do not generate numerical oscillations; however, using a low Chezy coefficient
Table 20. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 1.00 %, $\Delta t_a$ of 40 s, Clay Soil, and C of 25.

| $\Delta t_{adv}$ = 40 s, C = 25, Aver_PS = 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\theta$ | 0.52 | 0.54 | 0.56 | 0.58 | 0.60 | 0.62 | 0.64 | 0.66 | 0.68 | 0.70 | 0.72 | 0.74 | 0.76 | 0.78|
| 0.52 |   |   |   |   |   | * |   | * |   |   | * |   |   |   |
| 0.54 |   |   |   |   |   | * |   | * |   |   | * |   |   |   |   |
| 0.56 |   |   |   |   |   | * |   | * |   |   | * |   |   |   |   |
| 0.58 |   |   |   |   |   | * |   | * |   |   | * |   |   |   |   |
| 0.60 |   |   |   | * |   | * |   | * |   |   | * |   |   |   |   |
| 0.62 |   | * |   | * |   | * |   | * |   |   | * |   |   |   |   |
| 0.64 |   | * |   | * |   | * |   | * |   |   | * |   |   |   |   |
| 0.66 |   | * |   | * |   | * |   | * |   |   | * |   |   |   |   |
| 0.68 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0.70 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0.72 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0.74 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0.76 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0.78 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

Table 21. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 1.00 %, $\Delta t_a$ of 20 s, Clay Soil, and C of 25.

| $\Delta t_{adv}$ = 20 s, C = 25, Aver_PS = 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\theta$ | 0.52 | 0.54 | 0.56 | 0.58 | 0.60 | 0.62 | 0.64 | 0.66 | 0.68 | 0.70 | 0.72 | 0.74 | 0.76 | 0.78|
| 0.52 |   |   |   |   |   | * |   | * |   |   | * |   |   |   |   |
| 0.54 |   |   |   |   |   | * |   | * |   |   | * |   |   |   |   |
| 0.56 |   |   |   |   |   | * |   | * |   |   | * |   |   |   |   |
| 0.58 |   |   |   |   |   | * |   | * |   |   | * |   |   |   |   |
| 0.60 |   |   |   | * |   | * |   | * |   |   | * |   |   |   |   |
| 0.62 |   | * |   | * |   | * |   | * |   |   | * |   |   |   |   |
| 0.64 |   | * |   | * |   | * |   | * |   |   | * |   |   |   |   |
| 0.66 |   | * |   | * |   | * |   | * |   |   | * |   |   |   |   |
| 0.68 |   | * |   | * |   | * |   | * |   |   | * |   |   |   |   |
| 0.70 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0.72 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0.74 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0.76 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0.78 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
produces more pairs of \( \phi \) and \( \theta \) values that lead to convergence after 120 time steps. Therefore, the Saint-Venant equations have more probability to simulate undulated water depths when applied to borders with vegetation or high surface roughness, but when the Saint-Venant equations use high Chezy coefficients there is more probability of avoiding water depth instability according to the analysis of \( \phi \) and \( \theta \) values.

Finally, the use of free-draining downstream boundary condition generates more pairs of \( \phi \) and \( \theta \) values that converge and do not produce numerical oscillations. Based on all the above information, as a way to make the model more robust, groups of \( \phi \) and \( \theta \) values were classified. This classification was done according to the number of times that a pair of \( \phi \) and \( \theta \) values generated the minimum average number of reversals for all the analyzed cases. As explained in Chapter 3, the pink group was made up of the pairs of \( \phi \) and \( \theta \) values which did not generate numerical oscillations in almost all the analyzed cases. The orange group was defined by the pairs of \( \phi \) and \( \theta \) values which have generated numerical oscillations or “undulations” for some cases during all the analysis. Finally, the green group was defined by pairs of \( \phi \) and \( \theta \) values that have usually generated numerical oscillations or have not made the results converge for all the analyzed cases. Then, Table 3 with this classification was used as algorithm to avoid the “undulations” when the four-point implicit solution of the Saint-Venant equations was used. It was seen that \( \phi \) and \( \theta \), equal to 0.72 and 0.74, respectively, are (in almost all cases) the weighting factors that generate the minimum average number of reversals.
Table 22. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 1.00 %, $\Delta t_a$ of 60 s, Clay Soil, and C of 40.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\Phi$</th>
<th>$\delta$ and $\Phi$ values that converge</th>
<th>Minimum average reversal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.52</td>
<td>0.54</td>
<td>0.56</td>
<td>0.58</td>
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<tr>
<td>0.52</td>
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<td></td>
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<tr>
<td>0.54</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>0.56</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.60</td>
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<td></td>
</tr>
<tr>
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<tr>
<td>0.64</td>
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<td></td>
<td></td>
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<tr>
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<tr>
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<tr>
<td>0.74</td>
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<tr>
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<tr>
<td>0.78</td>
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</tbody>
</table>

Table 23. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 1.00 %, $\Delta t_a$ of 60 s, Clay Soil, and C of 60.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\Phi$</th>
<th>$\delta$ and $\Phi$ values that converge</th>
<th>Minimum average reversal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.52</td>
<td>0.54</td>
<td>0.56</td>
<td>0.58</td>
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<tr>
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<tr>
<td>0.56</td>
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<td></td>
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<tr>
<td>0.58</td>
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<tr>
<td>0.60</td>
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<tr>
<td>0.62</td>
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<tr>
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<tr>
<td>0.66</td>
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<tr>
<td>0.68</td>
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<td></td>
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<tr>
<td>0.70</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.72</td>
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<tr>
<td>0.74</td>
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<td>0.76</td>
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<td></td>
</tr>
<tr>
<td>0.78</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Impact of Changing $\phi$ and $\theta$ Values

The model modifies the $\phi$ and $\theta$ values, choosing those that do not generate significant oscillations. One of the concerns was the impact of creating an algorithm that changes $\phi$ and $\theta$ values when it diverges due to “undulation” problems; consequently, the model was run under the same conditions but with different $\phi$ and $\theta$ values to observe the presence of prominent variations in the results. This involved the simulation of the water depth and the distance after 150 time steps. The model considers a clay soil, a longitudinal slope equal to 1.00%, an advance time increment equal to 60 s, an inflow rate equal to 0.05 m$^3$/s per m, and a border length of 10,000 m.

In Fig. 26 it is seen that the results are not significantly impacted when $\phi$ and $\theta$ are changed. The maximum percentage of difference, among the distance at the furthest downstream node for the three used pairs of $\phi$ and $\theta$, is 0.1; in addition, it is noticed that the maximum difference among the water depth at each node for the three pairs of $\phi$ and $\theta$ is 2 mm.

![Fig. 26. Comparison among different values of $\phi$ and $\theta$ under same border conditions](image-url)
Optimum Irrigation Management in Sloping and Blocked-end Borders

The following paragraphs describe all the results from the optimization procedure to determine the best irrigation management in sloping and blocked-end border irrigation system. This model was run in the Geomatics laboratory at Utah State University using five computers simultaneously to reduce the overall computational time.

Grid Generation for Different Values of $Z_{req}$

As explained in Chapter 4, the model performs a grid generation procedure that defines the three vertices for the downhill simplex optimization method. Using this grid generation procedure, a three-dimensional graph is generated. The three-dimensional graph considers the results from the objective function (z axis), the inflow rate (y axis), and the cutoff time (x axis). Three such graphs were generated using different values of net infiltration depths (10, 7, and 4 cm). For the grid generation procedure, the model runs using the following parameters: sandy loam soil, longitudinal field slope equal to 0.05%, border length of 100 m, and advance time increment equal to 60 s under a blocked-end downstream boundary condition.

From Figs. 27, 28, and 29, it is observed that an extended range of the lowest objective function values are found in the graph where $Z_{req} = 10$ cm. In addition, the range of inflow rates when the cutoff time is high produce the lowest objective function values where $Z_{req} = 10$ cm. On the other hand, during the grid generation for $Z_{req} = 4$ cm, it was found that a short range of inflow rate values, when cutoff time is high, produces the lowest objective function. In addition, it is known that small $Z_{req}$ values are more susceptible to deep percolation losses when the inflow rate is incremented because the net infiltration depth is smaller; consequently, the net infiltration depth along the border is going to be filled with lower volume of water than high of values of net infiltration depths or $Z_{req}$.

Also, in the three graphs, there are some inverted spikes (local minimums) located over the smooth shape of the surface topography in the graph. During the grid generation the advance time (time of the water to reach the border end) was recorded for each inflow rate and
cutoff time. The inflow rates and cutoff time, which start to generate the inverted spikes, had an advance time value close to the time of cutoff. Consequently, it is observed that when the cutoff time approaches the advance time, the value of the objective function suddenly increments. Then, it reduces again when the cutoff time approaches 130% of the advance time. A backwater condition is manifested when the water reaches the downhill end of the border until the water spills over the end dike; consequently, the suddenly increment of the objective function value is attributed to a backwater condition.

Finally, the crops with small root depth and soils with low water-holding capacity will have smaller range of inflow rates and cutoff time that obtain lower objective function values that included the lowest (global optimum) value. This range is made up of values of low inflow rate and high cutoff time, and high inflow rate and low cutoff time. On the other hand, crops with deep roots and soils with high water holding capacity will benefit by a greater range of inflow rates and cutoff times that result in low objective function values. This range is comprised of values of medium and high inflow rates, and medium and high cutoff times.

![Figure 27](image.png)

**Fig. 27.** Objective function versus $t_{co}$ and $q_{in}$ for $Z_{req}$ of 10 cm, sandy loam soil, and longitudinal field slope = 0.05%
Fig. 28. Objective function versus $t_{co}$ and $q_{in}$ for $Z_{req}$ of 7 cm, sandy loam soil, and longitudinal field slope = 0.05%

Fig. 29. Objective function versus $t_{co}$ and $q_{in}$ for $Z_{req}$ of 4 cm, sandy loam soil, and longitudinal field slope = 0.05%
Presence of Spikes (Local Minimums)

An analysis of spikes formation, during the implementation of the grid generation procedure was studied. These spikes produce local minimums which can change the result of the optimization method of finding the optimum global value. Therefore, this is one of the reasons why a grid generation procedure is implemented to find the starting vertices which are close to the optimum global value.

In this part, an analysis which involves the calculation of water requirement efficiency, application efficiency, and the objective function values using three different objective function formulas. During the analysis, the model was run under the following conditions: border length is equal to 300 m, inflow rate equal to 0.01 m$^3$/s per m, advance time increment equal to 30 s, longitudinal slope equal to 0.20 %, $Z_{req} = 10$ cm, and soil type is silty clay. These conditions were simulated for different values of cutoff time to produce objective function values based on water requirement efficiency and application efficiency.

Fig. 30. $E_a$ and WRE variations using OBF, OBF$_2$, and OBF$_3$
Figure 30 shows the objective functions (OBF), objective function 2 (OBF$_2$), objective function 3 (OBF$_3$), application efficiency ($E_a$), and water requirement efficiency (WRE) values plotted for different values of cutoff time. Objective functions 2 and 3 are formulas that involve different weighting factors for $E_a$ and WRE are shown in Eqs (5.1) and (5.2):

\[
OBF_2 = 500 - 2E_a - 3WRE
\]  
\[
OBF_3 = 200 - E_a - WRE
\]  

These objective functions were tested to see if they produce a strong impact when $E_a$ and WRE change suddenly in small amounts due to the implementation of the volume-balance method, or due to changes in $\phi$ and $\theta$ values. It was observed that the objective function, as used in the model, amplifies $E_a$ and WRE variations more than for OBF$_2$ and OBF$_3$; however, the minimum objective function value corresponds to the highest WRE and the higher $E_a$. On the other hand, OBF$_2$ and OBF$_3$’s minimum objective value was associated with the highest $E_a$ and medium-high WRE. Consequently, the minimum objective function value generated by the model gives a more accurate representation of the best irrigation efficiency because it ensures that most of the required infiltration depth along the border length is satisfied. Furthermore, it is known that high $E_a$ values do not necessarily correspond to high crop productivity when the net infiltration depth along the length is not completely irrigated. Finally, the problem of amplification of $E_a$ and WRE variations by the objective function is solved by locating the starting vertices close to the optimum global value using the grid generation procedure.

**Evaluation of Optimization Results for Different Soil Types, Border Lengths, and Slopes**

Many simulations were performed to find the optimum inflow rate and cutoff time to obtain the best irrigation efficiency. These simulations were developed for the five soil types. Three border lengths (100, 250, and 500 m) were simulated for four of the five soil types, five border lengths (100, 175, 250, 375, and 500 m) were simulated for one type of soil (sandy loam) of the five soil types, three longitudinal slopes (0.05, 0.5 and 1 %) for four of the five soil type, five
longitudinal slopes (0.05, 0.15, 0.25, 0.5 and 1%) for one of the five soil types (sandy loam), with $Z_{req}$ equal to 0.1 m, and advance time increment equal to 60 s. The obtained results were the optimum inflow rate and cutoff time which generates the lowest objective function value with their $E_a$ and WRE, respectively. In addition, the results from the grid generation were also obtained; these results show the different inflow rates, cutoff times, and their objective function, $E_a$, and WRE values, respectively.

Exponential Relation Between Inflow Rate and Cutoff Time As explained in Chapter 4, the grid size for the clay and clay loam soil types were 24 x 22 because they exhibit a greater number of downward spikes, or local minimums, thereby requiring more topographical detail to assist in the search for the global optimum. On the other hand, silty loam, sandy loam, and sandy soil types used a 10 x 10 grid size due to a smoother topography with fewer local minimums. The grid generation contains a range of inflow rates and cutoff times in which each inflow rate and cutoff time present a calculated objective function value, water requirement efficiency, and application efficiency for a determined border length, longitudinal slope and soil type. A range of cutoff time belongs to each inflow rate. The lowest objective function value generated at each range (inflow rate with several cutoff times) was grouped and then plotted to observe their mathematical tendency. It was found that some of them, especially those that have mild slopes and short to medium border length, have an exponential mathematical relationship. This means that it is possible to find the best irrigation time to obtain the highest combined water requirement efficiency and application efficiency for non-optimal inflow rates. This is very useful, especially if the farmer or irrigator is unable to use the optimum inflow rate or irrigation time due to constraints at the water source, or because of irrigation scheduling issues. Figures 31 to 35 show an exponential relationship between the inflow rates and the cutoff time which conform to the lowest objective function value for a specific inflow rate and a range of cutoff times.
Fig. 31. Exponential relationship between $q_{in}$ and $t_{co}$ for length of 250 m, slope of 0.05 %, and sandy soil

Fig. 32. Exponential relationship between $q_{in}$ and $t_{co}$ for length of 100 m, slope of 0.05 %, and sandy loam soil
Fig. 33. Exponential relationship between $q_{in}$ and $t_{co}$ for length of 250 m., slope of 0.05 %, and silty loam soil.

Fig. 34. Exponential mathematical relation between $q_{in}$ and $t_{co}$ for length of 100 m, slope of 0.05%, and clay loam soil.
Fig. 35. Exponential relationship between $q_{in}$ and $t_{co}$ for length of 250 m, slope of 0.05%, and clay soil

From Figs. 31 to 35, it is observed that the optimum value for all five soil types are located in a range when the inflow rates are high (> 0.03 m$^3$/s per m) and the cutoff time is low (> 900 s). Also, lower objective function values (high OBF > 3) are located in the same range. After the exponential relation was observed in the sandy, sandy loam and silty loam soils, they repeated the grid generation procedure using a 24 by 22 number of grids. In basis of the results, it was found in the exponential equations for the five types of soils that coefficient values vary from 8.9 to 5.9, and go in descendent order from sandy to clay soil; only with the exception of silty loam soil that has a coefficient value equal to 9.3, which has the lowest coefficient of determination (0.76). In addition, the exponential values vary from 0.83 to 0.92; however, there is no discernible trend from sandy to clay soil types.

**Example Use of the Exponential Equations** As detailed above, different exponential relationships between the inflow rates and the cutoff time which conform to the lowest objective function value for a specific inflow rate and a range of cutoff times (Figs. 31 to 35). These
exponential relationships are very useful for irrigators to find maximum combine water requirement and application efficiencies for a current inflow rate. Two examples are given to explain the use of these exponential curves:

**Case 1**
- Type of soil: Sandy loam
- Slope: 0.05 %
- Border length: 100 m.
- Current inflow rate: 0.012 m³/s per m

For Case 1:

\[
q_{in} = 6.34t_{co}^{-0.92} = \left( \frac{0.012}{6.34} \right)^{-0.92} = 911.34 \approx 912
\]  

(5.3)

**Case 2**
- Type of soil: Clay
- Slope: 0.05 %
- Border length: 250 m.
- Current inflow rate: 0.024 m³/s per m

For Case 2:

\[
q_{in} = 5.94t_{co}^{-0.90} = \left( \frac{0.024}{5.9} \right)^{-0.90} = 456.52 \approx 457
\]  

(5.4)

In these examples, both fields cannot be irrigated with more water than the current inflow because they are the maximum allowable inflow rates permissible by the source (well) to avoid over-exploitation. Consequently, the farmer can use the exponential relationship to obtain the best irrigation cutoff time to improve the composite irrigation efficiency.

For Case 1:

Irrigation time: 912 s

For Case 2:

Irrigation time: 457

Finally, the calculated irrigation times in both cases, according to the exponential relations developed in this research, will give the best combined water requirement and application efficiencies for those inflow rates (0.012 and 0.024 m³/s per m).
**Evaluation of the Optimum Objective Function Values** As explained above, the lowest objective function value was found for different values of border length, longitudinal slope, and soil type. During optimization simulations for clay and clay loam soils was observed that the model took more than one day on a computer to obtain the optimum inflow rate and cutoff time; on the other hand, optimization simulations for silty loam, sandy loam and sandy took less that one day of continuous calculations to reach the optimum values.

The optimum values for sandy soils were determined, and Table 24 shows nine optimum values of inflow rate and cutoff time for sandy soil. These optimizations were run for three longitudinal field slopes (0.05, 0.50, and 1.00%), and three border lengths (100, 250, 500 m,) with a Z<sub>req</sub> of 10 cm. Each optimum value has its calculated objective function, water requirement efficiency (WRE), and application efficiency (E<sub>a</sub>). Also, the ratio (t<sub>cot</sub>/t<sub>al</sub>) between the advance time (time of water to reach the end) and cutoff time is calculated. From Table 29, it is found that the optimum inflow rates are higher values and cutoff time are relatively low. In a few cases, it is observed that the inflow rate is relatively low and the cutoff time is high when the slope is 0.5% for 250- and 500-m border lengths.

The t<sub>cot</sub>/t<sub>al</sub> ratio is close to unity in most cases, but there are a few cases when the t<sub>cot</sub>/t<sub>al</sub> ratio is greater than 1.5, especially when the longitudinal field slope is 0.50 or 1.00%, and the border length is not the minimum.

**Table 24. Optimum Values of Inflow rate and Cutoff time for Sandy Soils**

<table>
<thead>
<tr>
<th>Optimum Values</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutoff Time (s)</td>
<td>Inflow rate (m²/s/m)</td>
</tr>
<tr>
<td>----------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>232</td>
<td>0.048</td>
</tr>
<tr>
<td>150</td>
<td>0.051</td>
</tr>
<tr>
<td>150</td>
<td>0.053</td>
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<tr>
<td>578</td>
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<tr>
<td>1142</td>
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<td>438</td>
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<tr>
<td>1655</td>
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<tr>
<td>5228</td>
<td>0.011</td>
</tr>
<tr>
<td>1274</td>
<td>0.037</td>
</tr>
</tbody>
</table>

Soil type: Sandy
Z<sub>req</sub> (m): 0.1 m.
The optimum values for sandy loam soils were determined, and Table 25 shows twenty-three optimum values of inflow rate and cutoff time for sandy loam soils. These optimizations included five longitudinal field slopes (0.05, 0.15, 0.25, 0.50, and 1.00%) and five border lengths (100, 175, 250, 325, and 500 m) with a $Z_{req}$ of 10 cm. However, for the border length of 500 m only three longitudinal slopes were used (0.05, 0.50, and 1.00%). From Table 25, it is also seen that the optimum inflow rates are higher than the average, and cutoff times are relatively low. In a few cases, it is observed that the inflow rate is relatively low and the cutoff time is high when the longitudinal field slope is 0.50% for a border length of 500 m, or for slopes of 0.25% and 0.15% with a 250-m border length.

The $t_{co}/t_{sl}$ ratio was also calculated for this soil type. It was observed that most of the values are close to unity; there are also a few cases when the $t_{co}/t_{sl}$ ratio is greater than 1.5, especially when the slope is 0.50 or 1.00% and the border length is not the minimum. However, there is a special case when the $t_{co}/t_{sl}$ is equal to 2.2 and the slope is 0.15 % for a 175-m border length.

In this particular case where there are more border lengths and slopes, the objective function values become greater, and the water requirement efficiency and application efficiency values decrease as the slope becomes steeper. This is because with steep slopes it is more difficult to cover all the net infiltration depth along the border length, and more water is lost due to deep percolation.

Table 26 shows nine optimum values of inflow rate and cutoff time for silty loam soils. These optimizations were run for three longitudinal slopes (0.05, 0.50, and 1.00%) and five border lengths (100, 250, and 500 m) with a $Z_{req}$ of 10 cm. From Table 26, it is again found that the optimum inflow rates are higher values and cutoff times are relatively low. The $t_{co}/t_{sl}$ ratio values are close to unity; one $t_{co}/t_{sl}$ ratio greater than 1.5 is found for a slope equal to 0.5 and a border length equal to 250 m. It is also observed that the objective function values increase, and water requirement efficiency and application efficiency values decrease, as the longitudinal field slope increases.
Table 25. Optimum Values of Inflow rate and Cutoff time for Sandy Loam Soils.

<table>
<thead>
<tr>
<th>Cutoff Time (s)</th>
<th>Inflow rate (m³/s/m)</th>
<th>slope (m/m)</th>
<th>Border length (m)</th>
<th>$E_a$ (%)</th>
<th>WRE (%)</th>
<th>$OBF_{Lowest}$ (%)</th>
<th>Model advance time (s)</th>
<th>Ratio t₁/₄₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>298</td>
<td>0.034</td>
<td>0.0005</td>
<td>100</td>
<td>99</td>
<td>100</td>
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<td>0.0015</td>
<td>100</td>
<td>88</td>
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<td>0.1</td>
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<td>60</td>
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<td>0.005</td>
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<td>240</td>
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<td>0.01</td>
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<td>1.02</td>
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<td>1262</td>
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<td>0.0025</td>
<td>250</td>
<td>87</td>
<td>66</td>
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<td>900</td>
<td>1.40</td>
</tr>
<tr>
<td>1150</td>
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<td>0.005</td>
<td>250</td>
<td>62</td>
<td>53</td>
<td>100.9</td>
<td>620</td>
<td>1.85</td>
</tr>
<tr>
<td>380</td>
<td>0.05</td>
<td>0.01</td>
<td>250</td>
<td>60</td>
<td>51</td>
<td>110.0</td>
<td>300</td>
<td>1.27</td>
</tr>
<tr>
<td>789</td>
<td>0.038</td>
<td>0.0005</td>
<td>325</td>
<td>97</td>
<td>91</td>
<td>3.1</td>
<td>840</td>
<td>0.94</td>
</tr>
<tr>
<td>669</td>
<td>0.053</td>
<td>0.0015</td>
<td>325</td>
<td>73</td>
<td>81</td>
<td>22.7</td>
<td>560</td>
<td>1.19</td>
</tr>
<tr>
<td>650</td>
<td>0.05</td>
<td>0.0025</td>
<td>325</td>
<td>67</td>
<td>74</td>
<td>39.0</td>
<td>520</td>
<td>1.25</td>
</tr>
<tr>
<td>635</td>
<td>0.06</td>
<td>0.005</td>
<td>325</td>
<td>61</td>
<td>61</td>
<td>76.1</td>
<td>400</td>
<td>1.59</td>
</tr>
<tr>
<td>613</td>
<td>0.045</td>
<td>0.01</td>
<td>325</td>
<td>63</td>
<td>52</td>
<td>103.5</td>
<td>420</td>
<td>1.46</td>
</tr>
<tr>
<td>1243</td>
<td>0.04</td>
<td>0.0005</td>
<td>500</td>
<td>92</td>
<td>93</td>
<td>2.6</td>
<td>1320</td>
<td>0.94</td>
</tr>
<tr>
<td>2822</td>
<td>0.02</td>
<td>0.005</td>
<td>500</td>
<td>59</td>
<td>67</td>
<td>61.9</td>
<td>1300</td>
<td>2.17</td>
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<tr>
<td>1479</td>
<td>0.025</td>
<td>0.01</td>
<td>500</td>
<td>69</td>
<td>61</td>
<td>69.1</td>
<td>980</td>
<td>1.51</td>
</tr>
</tbody>
</table>

Soil type: Sandy loam  Z₀  req : 0.1 m.

Table 26. Optimum Values of Inflow rate and Cutoff time for Silty Loam Soils.

<table>
<thead>
<tr>
<th>Cutoff Time (s)</th>
<th>Inflow rate (m³/s/m)</th>
<th>slope (m/m)</th>
<th>Border length (m)</th>
<th>$E_a$ (%)</th>
<th>WRE (%)</th>
<th>$OBF_{Lowest}$ (%)</th>
<th>Model advance time (s)</th>
<th>Ratio t₁/₄₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>203</td>
<td>0.049</td>
<td>0.0005</td>
<td>100</td>
<td>97</td>
<td>75</td>
<td>23.6</td>
<td>200</td>
<td>1.02</td>
</tr>
<tr>
<td>147</td>
<td>0.05</td>
<td>0.005</td>
<td>100</td>
<td>99</td>
<td>86</td>
<td>7.4</td>
<td>120</td>
<td>1.23</td>
</tr>
<tr>
<td>360</td>
<td>0.023</td>
<td>0.01</td>
<td>100</td>
<td>65</td>
<td>56</td>
<td>87.9</td>
<td>200</td>
<td>1.80</td>
</tr>
<tr>
<td>573</td>
<td>0.044</td>
<td>0.0005</td>
<td>250</td>
<td>99</td>
<td>95</td>
<td>1.0</td>
<td>580</td>
<td>0.99</td>
</tr>
<tr>
<td>600</td>
<td>0.049</td>
<td>0.005</td>
<td>250</td>
<td>56</td>
<td>68</td>
<td>62.6</td>
<td>380</td>
<td>1.58</td>
</tr>
<tr>
<td>404</td>
<td>0.049</td>
<td>0.01</td>
<td>250</td>
<td>79</td>
<td>62</td>
<td>59.7</td>
<td>300</td>
<td>1.35</td>
</tr>
<tr>
<td>1866</td>
<td>0.022</td>
<td>0.0005</td>
<td>500</td>
<td>98</td>
<td>81</td>
<td>13.6</td>
<td>1860</td>
<td>1.00</td>
</tr>
<tr>
<td>718</td>
<td>0.062</td>
<td>0.005</td>
<td>500</td>
<td>62</td>
<td>51</td>
<td>108.1</td>
<td>620</td>
<td>1.16</td>
</tr>
<tr>
<td>719</td>
<td>0.05</td>
<td>0.01</td>
<td>500</td>
<td>60</td>
<td>42</td>
<td>146.2</td>
<td>600</td>
<td>1.20</td>
</tr>
</tbody>
</table>

Soil type: Silty loam  Z₀  req : 0.1 m.

Table 27 shows nine optimum values of inflow rate and cutoff time for clay loam soils.

These optimizations were run for three longitudinal slopes (0.05, 0.50, and 1.00%) and five border lengths (100, 250, and 500 m,) with a Z₀  req  of 10 cm. From Table 27, it is again found that
the optimum inflow rates are higher values and cutoff times are relatively low. The \( \frac{t_{co}}{t_{al}} \) ratio values are close to unity; a few \( \frac{t_{co}}{t_{al}} \) ratios greater than 1.5 are found when the longitudinal field slope is 0.50% for a border length equal to 250 m and slope is 1.00% for a border length equal to 100 m. It is also observed that the objective function values increase as the slope gets steeper; on the other hand, water requirement efficiency and application efficiency values decrease as the longitudinal field slope becomes steeper.

**Table 27.** Optimum Values of Inflow rate and Cutoff time for Clay Loam Soils.

<table>
<thead>
<tr>
<th>Cutoff Time ( (s) )</th>
<th>Inflow rate ( (m^3/s/m) )</th>
<th>slope ( (n/m) )</th>
<th>Border length ( (m) )</th>
<th>( E_a ) ( (%) )</th>
<th>WRE ( (%) )</th>
<th>( \text{OBF}_{\text{Lower}} ) advance time ( (s) )</th>
<th>( \frac{t_{co}}{t_{al}} ) Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>267</td>
<td>0.037</td>
<td>0.0005</td>
<td>100</td>
<td>99</td>
<td>97</td>
<td>0.35</td>
<td>220</td>
</tr>
<tr>
<td>146</td>
<td>0.06</td>
<td>0.005</td>
<td>100</td>
<td>89</td>
<td>66</td>
<td>44.9</td>
<td>120</td>
</tr>
<tr>
<td>336</td>
<td>0.033</td>
<td>0.01</td>
<td>100</td>
<td>63</td>
<td>52</td>
<td>103.5</td>
<td>160</td>
</tr>
<tr>
<td>580</td>
<td>0.043</td>
<td>0.0005</td>
<td>250</td>
<td>99</td>
<td>96</td>
<td>0.6</td>
<td>560</td>
</tr>
<tr>
<td>681</td>
<td>0.036</td>
<td>0.005</td>
<td>250</td>
<td>67</td>
<td>63</td>
<td>65.0</td>
<td>420</td>
</tr>
<tr>
<td>380</td>
<td>0.049</td>
<td>0.01</td>
<td>250</td>
<td>67</td>
<td>63</td>
<td>65.0</td>
<td>420</td>
</tr>
<tr>
<td>1664</td>
<td>0.031</td>
<td>0.0005</td>
<td>500</td>
<td>78</td>
<td>88</td>
<td>11.5</td>
<td>1480</td>
</tr>
<tr>
<td>689</td>
<td>0.048</td>
<td>0.005</td>
<td>500</td>
<td>56</td>
<td>47</td>
<td>129.5</td>
<td>700</td>
</tr>
<tr>
<td>638</td>
<td>0.05</td>
<td>0.01</td>
<td>500</td>
<td>45</td>
<td>44</td>
<td>155.4</td>
<td>580</td>
</tr>
</tbody>
</table>

Table type: Clay loam \( Z_{req} \) \( (m) \): 0.1 m.

Table 28 shows nine optimum values of inflow rate and cutoff time for clay loam soils. These optimizations included three longitudinal slopes (0.05, 0.50, and 1.00%) and five border lengths (100, 250, and 500 m), with \( Z_{req} \) of 10 cm. It is again found that the optimum inflow rates are relatively high values, and the best cutoff times are relatively low. The \( \frac{t_{co}}{t_{al}} \) ratio values are close to unity. Also, it is found the highest value of the \( \frac{t_{co}}{t_{al}} \) ratio (equal to 3) was greater than that of the other soil types; and the highest \( \frac{t_{co}}{t_{al}} \) ratio is found when the slope is equal 0.50 and 1.00% for a border length equal of 100 m. It is also observed that the objective function values increase as the slope becomes steeper; on the other hand, the water requirement efficiency and application efficiency values decreases as the slope steepens.
Table 28. Optimum Values of Inflow rate and Cutoff time for Clay Soils

<table>
<thead>
<tr>
<th>Optimum Values</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutoff Time (t_{co}) (s)</td>
<td>advance time (t_{al}) (s)</td>
</tr>
<tr>
<td>Inflow rate (I_{co}) (m³/s/m)</td>
<td>(E_a) (%)</td>
</tr>
<tr>
<td>660</td>
<td>0.021</td>
</tr>
<tr>
<td>606</td>
<td>0.021</td>
</tr>
<tr>
<td>890</td>
<td>0.035</td>
</tr>
<tr>
<td>716</td>
<td>0.03</td>
</tr>
<tr>
<td>406</td>
<td>0.048</td>
</tr>
<tr>
<td>1004</td>
<td>0.052</td>
</tr>
<tr>
<td>710</td>
<td>0.065</td>
</tr>
<tr>
<td>677</td>
<td>0.047</td>
</tr>
</tbody>
</table>

Soil type: Clay \(Z_{req}\) (m): 0.1 m.

Finally, according to all the evaluations of the \(t_{co}/t_{al}\) ratio among the optimum values of inflow rate and cutoff time for all soil types, it is observed that the average ratio is 1.1 among all these values for the five soil types. This is without considering \(t_{co}/t_{al}\) ratios higher than 1.5 because those results could be influenced by spikes which produce local minimums or numerical oscillations that generate some variations in water depth profile producing and impact the calculation of the water requirement efficiency and application efficiency. According to the average \(t_{co}/t_{al}\) ratio, it is advisable that when the farmers have the global optimum inflow rate to obtain the best efficiency (combined water requirement and application efficiency), the irrigation cutoff time should be 1.1 times the advance time, which is the time until the water reaches the end of the border to obtain the best irrigation efficiency. Also, the results indicate that a relatively high inflow rate and a low time of cutoff provide the best combined efficiency, in most cases. In addition, these results were compared with \(E_a\) and WRE as obtained from SIRMOD (Walker 2003) simulations, in which the maximum difference was 40%, and the average difference was 13%, compared to the model developed in this research.

However, nine optimization results from the Tables 24 to 28 show impractical water requirement and application efficiency values (< 70%). These results are observed when the borders present longitudinal slope higher or equal than 0.50% and border length higher or equal than 250 m. The reason is steep slopes and long border lengths require the use of more range of \(\phi\) and \(\theta\) values and the implementation of different advance time increments to avoid...
convergence. These changes in the $\phi$ and $\theta$ values and in the advance time increment generate variations in the water requirement and application efficiencies and those are amplified by the parabolic equation which relates the water requirement and application efficiencies with their weighting factors, respectively. In addition, borders with steep longitudinal field slopes cannot produce low values of the objective function because the root zone is not irrigated properly and deep percolation occurs at the downhill end because of the fast advance of the water to reach the border-end.
Summary

The research involved meticulous investigations about improvements to the numerical solution of the Saint-Venant equations for border irrigation hydraulic simulations. One of the initial investigations was an implementation of an artificial viscosity term (Von Newman and Richtmyer 1949) to the momentum equation to handle shock-wave computations (Caramana et al. 1998); unfortunately, the artificial viscosity term did not have a significant impact in the robustness of the model and it was abandoned. This was followed by a review of the optimization in a fertilization model for furrow irrigation that was developed by Sabillon (2003). Sabillon’s model uses the four-point implicit approach for the solution of the area-discharge form of the Saint-Venant equations and the solute mass balance equation. However, the numerical scheme used to solve the governing hydraulic equations in his model does not perform as well as expected in all cases. In fact, all known hydraulic models of surface irrigation experience numerical instabilities under some common irrigation conditions, and are unsuitable for inclusion in an optimization scheme because the solution sometimes diverges.

After gathering information about the solution to the Saint-Venant equations, a new simulation model was designed and developed and a new model for sloping and blocked-end borders was developed. After performing many simulations for different situations, problems with the numerical scheme were found, especially with steep longitudinal field slopes and long border lengths. Different computational techniques, such as the implementation of an artificial viscosity term, incorporation of the Swamee-Jain equation, and the implementation of the six-point implicit solution method was applied to make the model robust. Later, it was observed that these implementations did not solve all of the numerical stability problems in the model. Consequently, new techniques and ideas were applied to solve the numerical instability problems, and the four-point-implicit weighting factors (φ and θ) were studied.
It was found that changing these weighting values influences the robustness of the model, and was implemented with the incorporation of an algorithm that changes the two weighting factors during the simulation of the advance phase. To handle simultaneous recession and advance phases, a hybrid implementation of the numerical solution to the Saint-Venant equations, together with the volume-balance method, was developed to provide greater numerical stability over a wide range of simulation conditions. This technique proved to be very successful in eliminating numerical divergence in many of the previously problematic cases.

Subsequently, the exclusive use of the volume-balance method was implemented in the model when the solution of the Saint-Venant equations experiences numerical instability during the recession phase, in those cases where the advancing water reached the downstream end of the border. In addition, the use of a parabolic equation which defines the Chezy coefficient as function of water depth was added to the model. At the same time, field work at a Utah State University experimental field in Wellsville, Utah, was performed. The field trials included infiltration tests, elevation measurements, advance tests, and inflow rate measurements using a Parshall flume. However, the field data were not useful for purposes of validating the model.

Many optimization procedures in multiple dimensions were reviewed, and the downhill simplex method was selected for use. The optimization method increased the model complexity, and different corrections and modifications were implemented to obtain the final version of the model. An optimization model was developed to calculate the best inflow rate and irrigation time, maximizing a composite irrigation efficiency (water requirement efficiency and application efficiency). Many optimizations were performed for different soil types, border lengths, and longitudinal field slopes. The optimization model developed in this research simulates the hydraulics of irrigation water movement along the length of a blocked-end border and includes two main characteristics: (1) a numerically robust model; and, (2) calculation of optimal water management scenarios for block-end borders.

The development of a robust model yielded interesting and useful results, such as the identification of the best weighting factors (φ and θ) for the four-point implicit solution of the Saint-
Venant equations during advance phase, an algorithm to handle the convergence due to the effect of the blocked-end downstream boundary condition, the hybrid implementation of a solution method for the Saint-Venant equations and the volume-balance method, and an algorithm that defines the Chezy coefficient as a function of water depth. These results were entirely successful for the limited ranges of inflow rate, border length and longitudinal slopes used in this research.

This research also produces useful information for the farmers to optimize the irrigation management in sloping and blocked-end border irrigation systems. The implementation of a multi-dimensional optimization method which generates optimum global values of inflow rate and irrigation time (cutoff time) to determine the highest obtainable irrigation efficiency, the generation of exponential relations between the inflow rates and the cutoff time which conform to the lowest objective function, and, the calculation of the best $t_{c}/t_{a}$ ratio when the optimum global flow rate is present were a complete success. They can be used by farmers to help improve agricultural productivity and reduce non-point source pollution by managing water better and avoiding excessive deep percolation.

**Conclusions**

The optimization model developed in this research simulates the hydraulics of water movement along the length of a sloping and blocked-end border strip. From the model development two main results were obtained. Following are the conclusions achieved during this research.

*Developing a Robust Model*

The model has been shown to be robust over a limited range of longitudinal slopes, border lengths, and inflow rates, and was developed implementing an algorithm that automatically changes the $\phi$ and $\theta$ weighting factors for the four-point implicit solution method of the Saint-Venant equation. Additionally, the volume-balance method was applied within the model under certain conditions, and the Chezy coefficient was determined as function of water depth.
From the results of the analysis for the implementation of the algorithm that changes $\phi$ and $\theta$ values during the advance phase, it is concluded that the best range of values of $\phi$ and $\theta$, contributing to the numerical convergence of the solution is from 0.68 to 0.78; on the other hand, the range of values of $\phi$ and $\theta$ which tend to diverge due to numerical instability is from 0.5 to 0.6.

In addition, as seen in the results, borders with a mild slope (lower than 0.30%), low surface roughness, and high infiltration rate soil types are less prone to numerical divergence in the model due to minimum numerical oscillations. A backwater condition, which occurs when the border is blocked at the downstream end, is an issue which contributes to numerical instability of the model when recession phase starts. It was found that volume-balance method often handles backwater conditions more successfully than the four-point implicit method solution of the Saint-Venant equations for this specific case. Thus, it is concluded that an intelligent combination of both methods in a border irrigation simulation model can provide better overall numerical stability and robustness.

From the results, it is also concluded that low values of the time-step increment reduce numerical oscillations due to the increment of nodes along the water depth profile; consequently, the reduction of the values of increment time-step, when the divergence is faced, is a way to handle numerical instability for the four-point implicit solution of the Saint-Venant equation.

The model uses different values of $\phi$ and $\theta$ during the four-point implicit solution method of the Saint-Venant equations to eliminate the presence of numerical oscillations. The change of $\phi$ and $\theta$ values applied by the algorithm do not produce significant variation in the results from the four-point solution. The water depths, inflows, and distance for each node along the border length present minimum difference when those are achieved using different values of $\phi$ and $\theta$.

The values of maximum and minimum Chezy coefficients, used in the parabolic equation which defines the Chezy coefficient as a function of water depth, work properly for bare soil conditions. This is concluded from the comparison of simulation results from maximum and minimum Chezy coefficients (60 and 10) and the Manning coefficient (0.025) that was used for bare soil conditions. However, only a few simulations results were compared between maximum
and minimum Chezy coefficients and Manning coefficient used for border with vegetation. Consequently, more simulations are required to adjust these maximum and minimum Chezy values for cases which border presents vegetation.

The improvements developed to the four-point implicit solution method of the Saint-Venant equations, make all the solutions converge for within certain parametric limits of border length, inflow rate and longitudinal slope. However, numerical oscillations are still allowed in the water depth profile influencing in the accuracy of the model, especially, when the border inflow rate is low (close to 0.01 m$^3$/m per m) or the longitudinal slope is steep (equal to or greater than 0.5%).

**Optimum Irrigation Management Guidelines**

The objective function gives the best approach to obtain the best irrigation efficiency from other two possible functions, meaning the best combine water requirement efficiency and application efficiency. However, from the results, it was observed that the objective function amplifies the small variation of water requirement and application efficiencies due to changes of $\phi$ and $\theta$ values and application of volume-balance method. Thus, the used objective function could be improved to diminish the amplification effect but it gives the best combine irrigation efficiency result.

Based on the research results, the guidelines for optimal irrigation management are as follows:

1. Crops with large root depth and soils with high water holding capacity, which correspond to a high value of net infiltration depth, will benefit from a large range of inflow rates and long cutoff times, to obtain a range of lower objective function values which include the lowest (global optimum) value.
2. With increasing the values of longitudinal slopes for a specific border length and soil type, it is concluded that the highest combine water requirement efficiency and application efficiency starts to relatively decrease, making the lowest objective function value
increases; consequently, to obtain the best irrigation efficiency is important to consider mild longitudinal slopes.

3. In most of the cases for the five soil types, the optimum values (global optimum) represent relatively high inflow rates and low cutoff time. In only a few cases are the optimum values for low inflow rates and high cutoff time. Therefore, the best way to irrigate blocked-end border irrigation systems is using a high inflow rate and a short irrigation time to obtain the best irrigation efficiency.

4. If farmers have the optimum global inflow rate to obtain the best combine irrigation efficiency, the irrigation cutoff time should be 1.1 times the advance time. This is because it was shown that the average $t_{oc}/t_{sl}$ ratio is 1.1 in almost all the simulations for the five soil types and the objective function gives more weight to the water requirement efficiency than the application efficiency. Consequently, the model has a primordial action to secure the irrigation of the net infiltration depth along the border length.

5. A relationship was found between the best irrigation time to obtain the highest combined water requirement efficiency and application efficiency for non-optimal inflow rates. The relation is given by a series of exponential equations for different soil types, border lengths, and longitudinal slopes studied in this research. The exponential equations have coefficients from 8.9 to 5.9, and they are arranged in descendent order from sandy to clay soils, with the exception of the silty loam soil. The exponents are close to 0.90 and there is no trend of variation from sandy to clay soils. In basis of these exponential relations, farmers can obtain high combine irrigation efficiency although the optimum global value of inflow rate is not available for many reasons such as, high cost of the water for low profit crop, constrains at the water source or because of irrigation scheduling issues. Finally, using simple equations, it is possible to find the best irrigation time to obtain the highest combined water requirement efficiency and application efficiency for non-optimum inflow rate.
Recommendations for Future Research

The results obtained from the model are satisfactory; however, new techniques can be implemented to improve the accuracy and robustness of the hydraulic simulation model. The optimization procedure finds in almost all the cases the global optimum value of inflow rate and cutoff time. However, there are a few cases where the optimization procedure fell in the spikes and found local optimum values of inflow rate and cutoff time. The reason is because the model still permits some numerical oscillations during the simulations. Thus, further research can be developed to totally avoid numerical oscillations in the calculations, even outside of the parameter ranges used in this research.

Another recommendation is to develop an analysis of the best values of $\phi$ and $\theta$, which reduce or eliminate numerical oscillations for the recession phase, its results can improve the effect of the algorithm which changes $\phi$ and $\theta$ values in this research. Also, after simulations for border with different surface roughness, it is recommended to set different $C_{\text{max}}$ and $C_{\text{min}}$ values used in the parabolic equation (which defines the Chezy coefficient as a function of water depth) for different surface characteristics.

It was difficult to validate the model because the periodically submerged-flow condition in the entrance of the border and the presence of cross-sectional slope during the field work in the experimental field at Wellsville; therefore, it is recommended that a second attempt to validate revalidate the model in a field where these disadvantages are not present.

The present model is for border irrigation systems; an extension of the model to furrow irrigation using these new techniques can be also implemented. The model is ready to be extended by modifying the cross-sectional area. A comparison of their analysis and functionality is interesting as a starting point to build a new robust and accurate surface irrigation model.
LITERATURE CITED


Ross, P. J. (1986). “Zero-inertia and kinematic wave models for furrow and border irrigation”.

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APPENDIX A: ANALYSIS OF $\phi$ AND $\theta$ FOR FIVE SOIL TYPES
This appendix shows the results for the analysis for searching for the best $\phi$ and $\theta$ values, used in the four-point implicit solution of the Saint-Venant equations, for five different types of soil; clay, clay loam, silty loam, sandy, and sandy loam. The graphs show the pairs of $\phi$ and $\theta$ values which generate minimum average reversal during the simulation; consequently, these $\phi$ and $\theta$ values will avoid numerical oscillations.

**Table A.1.** Best Values of $\phi$ and $\theta$ for Longitudinal Slope 0.20 %, $\Delta t_0$ of 60 s, Soil type Clay, and Inflow of 0.05 m$^3$/s per m

<table>
<thead>
<tr>
<th>Soil type = Clay, Ave_PS = 0</th>
<th>$\phi$</th>
<th>0.52</th>
<th>0.54</th>
<th>0.56</th>
<th>0.58</th>
<th>0.60</th>
<th>0.62</th>
<th>0.64</th>
<th>0.66</th>
<th>0.68</th>
<th>0.70</th>
<th>0.72</th>
<th>0.74</th>
<th>0.76</th>
<th>0.78</th>
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</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.52</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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**Table A.2.** Best Values of $\phi$ and $\theta$ for Longitudinal Slope 0.20 %, $\Delta t_0$ of 60 s, Soil type Clay Loam, and Inflow of 0.05 m$^3$/s per m

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Table A.3. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 0.20%, $\Delta t_o$ of 60 s, Soil type Silty Loam, and Inflow of 0.05 m$^3$/s per m.

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Table A.4. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 0.20%, $\Delta t_o$ of 60 s, Soil type Sandy Loam, and Inflow of 0.05 m$^3$/s per m.

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Table A.5. Best Values of $\phi$ and $\theta$ for Longitudinal Slope 0.20 %, $\Delta t_a$ of 60 s, Soil type Sandy and Inflow of 0.05 m$^3$/s per m

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Minimum average reversal
APPENDIX B: OBJECTIVE FUNCTION VERSUS $t_{co}$ and $q_{in}$ FOR THREE SOIL TYPES
The appendix shows three dimensions graph (objective function versus cutoff time and inflow rate) for three different soil types; clay loam, silty loam, and sandy. The presence of spikes (local minimums) is observed, especially, for the clay loam soil.

**Fig. B.1.** Objective function versus $t_{co}$ and $q_{in}$ for $Z_{req}$ of 10 cm, clay loam soil, border length equal to 400, $\Delta t_a$ equal to 30 s, and longitudinal field slope = 0.02%
Fig. B.2. Objective function versus $t_{co}$ and $q_{in}$ for $Z_{req}$ of 10 cm, silty loam soil, border length equal to 400, $\Delta t_a$ equal to 30 s, and longitudinal field slope = 0.02%.

Fig. B.3. Objective function versus $t_{co}$ and $q_{in}$ for $Z_{req}$ of 10 cm, sandy soil, border length equal to 400, $\Delta t_a$ equal to 30 s, and longitudinal field slope = 0.02%.
APPENDIX C: FIELD TESTS AND DATA ANALYSIS
Area Description

The field trials took place in June and July of 2007 at a Utah State University experimental field in Wellsville, Utah.

This area is located at 3550 South 100 West, next to Hwy 89. The Natural Resource Conservation Service (NRCS) classifies the soil as a Nibley Silty clay loam with longitudinal field slopes from 0 to 3% (Fig. C.2). The depth to the water tables is usually between 0.8 and 1.2 m. The NRCS also indicates that it is not a saline soil because the electrical conductivity is lower than 2 mmhos/cm and the water holding capacity is 0.28 m (10.9 inches).

Field Work at Utah State University Experimental Field in Wellville, Utah.

Fig. C.1. Map of the north, center, and south borders in Wellville, Utah.
The experimental area was disked and bordered in late April 2007, in preparation for the field trials described herein. There were a total of six borders with widths from 6.9 to 15.6 m, and lengths from to 220 to 242 m. All of the borders were diked at the downhill end with a height equal to 0.4 m. The field where the experiments were conducted was cultivated with barley; it is populated with cattle and sheep for winter grazing. The average ground slope in the direction of flow was 0.31%.

Field Experiments

The field experiment was designed to collect data to be used in the validation of the hydraulic simulation model. As detailed above, the border-irrigated field was prepared for four irrigation events: the first irrigation event was developed in the South border, the second irrigation event was developed in the North border, and the third and fourth were developed in the Center border. The following activities were undertaken:

Infiltration Test Before the first irrigation event on June 14, 2007, infiltration measurements were performed using three ring infiltrometers, three representative points were chosen, one located in the head of the border, the other in the middle part of the border and the last one in the tail. These rings are metal cylinder inserted into the soil to isolate a volume of soil and prevent horizontal infiltration, and the cylinder dimensions were 30 cm in diameter and 50 cm
in height (Fig. C.3). Also, three representative points were chosen tests before the second irrigation event on June 15, 2007; finally, for the third and fourth irrigation event on June 16 and July 30, 2007 two representative points were chosen to perform the infiltration measurement, one was located in the head and the other in the tail of the border.

A total of 14 infiltration measurements were performed before the last irrigation event (From June 05 to July 29, 2007); unfortunately, three of them were discarded because of the presence of rain which modifies all these Kostiakov-Lewis intake parameters. It is important to indicate that the ring did not compact the soil to avoid the situation in which the water does not find preferential flow paths. Also, an earthen ditch surround the ring was constructed to avoid horizontal flow movement. Figure C.4 shows how the installation of the cylinders was performed.
Tables C.1, C.2, C.3, and C.4 show the different Kostiakov-Lewis intake parameters obtained from the infiltration measurements during the field work; it is observed that a total of ten infiltration measurements were considered.

**Table C.1.** Measured Kostiakov-Lewis Intake Parameters for the First Irrigation Event (South border)

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**Table C.2.** Measured Kostiakov-Lewis Intake Parameters for the Second Irrigation Event (North border)

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Table C.3. Measured Kostiakov-Lewis Intake Parameters for Third Irrigation Event (Center border)

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Table C.4. Measured Kostiakov-Lewis Intake Parameters for the Fourth Irrigation Event (Center border)

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</tbody>
</table>

To obtain the Kostiakov –Lewis intake parameters which were used for each irrigation event, the average of the infiltrated depths at each time for all the representative location at the first and fourth irrigation event were calculated, then the Kostiakov-Lewis intake parameters were achieved for each event (Table C.5). The second and third irrigation events use as final Kostiakov-Lewis intake parameters values of location one because they give better results compared with those from the advance test.

Table C.5. Averaged Kostiakov-Lewis Intake Parameters for the Four Irrigation Events

<table>
<thead>
<tr>
<th>Irrigation Event</th>
<th>Date</th>
<th>Border</th>
<th>Kostiakov-Lewis Intake Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>a</td>
</tr>
<tr>
<td>1</td>
<td>14-Jun-07</td>
<td>South</td>
<td>0.52</td>
</tr>
<tr>
<td>2</td>
<td>15-Jun-07</td>
<td>North</td>
<td>0.61</td>
</tr>
<tr>
<td>3</td>
<td>16-Jun-07</td>
<td>Center</td>
<td>0.53</td>
</tr>
<tr>
<td>4</td>
<td>30-Jul-07</td>
<td>Center</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Longitudinal Slope Measurement. In this research, the longitudinal ground slopes of the three borders (North, Center and South) were measured using differential leveling. An electronic digital level (Topcon DL-102), telescopic rod, tape, field notebook, pencil, calculator, flags for turning points, and hammer for installing turning points (Fig. C.5) were used in the field.
Fig. C.5. Field work for longitudinal slope measurement.

From each setup, a rod reading back to a point of known elevation and a reading forward to a point of unknown elevation was taken. For this research, the linear regression of all the corrected heights in every turning point was calculated to obtain the longitudinal ground slope of the borders. In the three borders, the correlation coefficient was greater than 0.97 and the value of slope was 0.30 % (Figs. C.6, C.7, and C.8). In addition, the side ground slope was measured along the three borders, and the correlation coefficient was greater than 0.5 with a cross slope of 0.20 % (Fig. C.9).

Also, the respective lengths of the three borders were measured. The length of the North border was 241 m, that of the Center border was 231 m, and was 226 m for the South border.
Fig. C.6. Average longitudinal slope and linear regression in the north border

Fig. C.7. Average longitudinal slope and linear regression in the south border
Fig. C.8. Average longitudinal slope and linear regression in the center border

**Linear regression:**
Field slope = 0.0029 m/m
R² = 0.97

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Fig. C.9. Average side slope and linear regression through the north, center, and south borders

**Linear regression:**
Field slope = 0.0020 m/m
R² = 0.52
Advance Test, Inflow Rate, and Cutoff Time

As part of the field work, the advance test was performed during each irrigation event. In addition, the inflow rate was measured along all the time of the advance test and the cutoff time was taken (irrigation time). To perform the advance test during the first irrigation event, the border was staked each 20 ft (6.1 m) for a total of 21 flagged stations (0-121 m), in the second irrigation event, the border was also staked each 20 ft for a total of 18 flagged stations (0-104 m), in the third and fourth irrigation event a total of 19 and 14 flagged stations (0-110 and 0-80 m) were staked each 20 ft. Advance data was recorded from the beginning and for duration of water advance; the time that water advances until the flag and the distance from the beginning to each flag were written down. Inflow data was also recorded along all the cutoff time; a Parshall flume (6" width) was used to measure the inflow rate and it was located at 34 m upstream of the end of the ditch that irrigated the three borders (Fig. C.10). The Parshall flume was correctly leveled in the two dimensions to enable application of the standard calibration parameters, and the flume dimensions are shown in Fig. C.11.

The cutoff time was set to the time when the water was approximately halfway down the field because the model should be able to simulate correctly the situation when advance and recession occurs at the same time when advance time is greater than cutoff time. The recession

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**Fig. C.10.** Parshall flume (6" width) and inflow rate being recorded
time was also recorded; unfortunately, the presence of the cross slope and the irregularity of the ground surface made it difficult to measure the recession time at each flag location.

**Fig. C.11.** Dimensions of the Parshall flume (6" width).
Field Experiment Results

A set of data was generated for the hydraulic parameters along the three irrigated borders that were tested. As detailed above, four irrigation events were performed and their respective field experiment tests were developed.

Results for the First Irrigation Event

The graphical results of the analysis on the hydraulic data from the first irrigation event developed on the South border are shown in Fig. C.13, in which the curves for the advance phase represent the results from the field characteristics, SIRMOD, and the model developed in this research. The parameters used to run the two models were: border length of 226 m, longitudinal slope of 0.00305 m/m, Kostiakov “a” of 0.49, Kostiakov “k” of 0.00920 m/min^a, and Kostiakov “f_o” of 0.00047 m/min. The cutoff time was 11,192 s, and the inflow rate was 0.002 m^3/s per m. However, the inflow rate in the two models was not the same as the inflow rate which was measured by the Parshall flume (6” width). The problem was that the inflow rate from the Parshall flume was not accurate because the Parshall flume operated under submerged-flow conditions at some times. The downstream depth at the Parshall flume was not measured; consequently, the flow rate calculations were not accurate. To calculate the inflow rate, SIRMOD was used; it was repeatedly executed, changing the inflow rate, until the results were close to the field advance test results.

Figure C.12 shows a large quantity of water entering the border eight minutes after the first irrigation event starts. During the irrigation events it was difficult to control the entrance of the water from the secondary canals. Another difficulty was the presence of a cross slope equal to 0.002 m/m, which runs downhill towards the south.
Furthermore, it was observed that during the field advance test the water runs until 121 m, or 53% of the total border length. SIRMOD and the model developed herein simulated a maximum distance of 124 and 118 m, respectively. There is a difference of 2 and 3 m compared to the maximum distance from field advance curve. The simulated advance curves from SIRMOD and the new model are close. One of the most influential parameters affecting the advance curve is the longitudinal slope, among the inflow, soil type and border shape. From the border bed elevation is observed that the slope is not smooth and the general slope is 0.00305, this explains why the advance phase from the field characteristics is larger (211 min) than that from SIRMOD and the developed model (196 and 186 min).

SIRMOD uses Manning’s equation; consequently, the roughness is represented by the Manning coefficient. The first irrigation event (Fig. C.14) was developed on a bare soil and the Manning coefficient was 0.025 researched due to its soil condition. On the other hand, the model uses the Chezy equation, consequently, the roughness is represented by the Chezy coefficient; in addition, as it was explained in chapter 3, the model uses a parabola equation to relates the water depth and the Chezy coefficient, the used $C_{\text{max}}$ and $C_{\text{min}}$ were 60 and 10, respectively, and $h_{\text{max}}$ and $h_{\text{min}}$ were 0.15 and 0 m, respectively. Consequently, the results from both models should have some variation.
Results for the Second Irrigation Event

The graphical results of the analysis on the hydraulic data from the second irrigation event, developed on the North border, are shown in Fig. C.15, in which the curves for the advance phase represents the results from the field characteristics, the SIRMOD and the present developed model. The parameters used to run the two models were: border length equals to 241 m, longitudinal slope equals to 0.00305 m/m, the used Kostiakov parameters “a” equals to 0.61, “k” equals to 0.0052 m/min a and “fo” equals to 0.0006 m/min, cutoff time equals to 4245 s, and the inflow rate per meter was 0.009 m³/s per m. As, it was explained above, the used inflow rate in the two models is not the same that the inflow rate which was measured by the Parshall flume (6” width) and SIRMOD was used to calibrate the inflow rate.

It is observed that during the field advance test the water runs until 104 m, this means 43 % of the total border length. The SIRMOD and the developed model simulate a maximum distance of 96 and 109 m, respectively. There is a difference of 5 and 8 m compared to the maximum distance from field advance curve. From the border bed elevation is observed that the slope is also not smooth and the general slope is 0.00305, this again explains why the advance phase from the field characteristics (77 min) differs from the advance curve of SIRMOD and the developed model (74 and 78 min).

This second irrigation event (Fig. C.16) was also developed on a bare soil and the Manning coefficient was also 0.025. For the parabola equation, the model uses a C_max = 60, C_min = 10, h_max = 0.15 m and h_min = 0 m. Also, the problem of cross sectional slope equal to 0.002 m/m and the use of other inflow from the Parshall flume make that the advance curve from the field characteristics is not close to the advance curve from SIRMOD and the model.

Results for the Third Irrigation Event

The graphical result for the third irrigation event developed on the Center Border is shown in Fig. C.17. The parameters used to run the two models were; border length equals to 231 m, longitudinal slope equals to 0.00305 m/m, the used Kostiakov parameters “a” equals to 0.53, “k” equals to 0.0089 m/min a and “fo” equals to 0.0004 m/min, cutoff time equals to 3600 s,
and the inflow rate per meter was 0.002 m³/s per m. Again, SIRMOD was used to calibrate the inflow rate.

It was observed that during the field advance test the water runs until 110 m, this means 47% of the total border length. SIRMOD and the developed model simulate a maximum distance of 109 and 121 m, respectively. There is a difference of 1 and 11 m compared to the maximum distance from field advance curve. From the border bed elevation is observed that the slope is not smooth; consequently, there is a variation among the time of advance phase from the field characteristics (102 min) and the time of the advance curve of SIRMOD and the developed model (72 and 70 min).

This third irrigation event (Fig. C.18) was developed on soil with vegetation. Consequently, SIRMOD used a Manning coefficient equals to 0.04. For the model, the parabola equation used a \( C_{\text{max}} = 30, C_{\text{min}} = 10 \), the \( h_{\text{max}} = 0.15 \) m and \( h_{\text{min}} = 0 \) m. In this case, the roughness coefficient makes that both advance curves from SIRMOD and the model a little apart, this is improved reducing the value of \( C_{\text{max}} = 25 \). Also, the problem of cross sectional slope equal to 0.002 m/m and the use of other inflow from the Parshall flume make that the advance curve from the field characteristics is not close to the advance curve from SIRMOD and the new model.

**Results for the Fourth Irrigation Event**

The graphical results for the fourth irrigation event, also developed on the Center Border, are shown in Fig. C.19. The parameters used to run the two models were: border length equals to 231 m, longitudinal slope equals to 0.00305 m/m, the used Kostiakov parameters “\( a \)” equals to 0.29, “\( k \)” equals to 0.0041 m/min\(^a\) and “\( f_{\text{o}} \)” equals to 0.00012 m/min, cutoff time equals to 4571 s, and the inflow rate per meter was 0.007 m³/s per m. Also, SIRMOD was used to calibrate the inflow rate.

It is observed that during the field advance test the water runs until 79 m, this means 34% of the total border length. SIRMOD and the developed model simulate a maximum distance of 98 and 87 m, respectively. There is a difference of 8 and 19 m compared to the maximum distance from the field advance curve. Also, there is a variation among the time of advance
phase from the field characteristics (94 min) and the time of the advance curve of SIRMOD and the developed model (98 and 97 min) due to the topographical surface.

This fourth irrigation event (Fig. C.20) was developed on a border with large vegetation (greater than 0.6 m). Consequently, SIRMOD used a Manning coefficient of 0.045. For the model, the parabolic equation used a $C_{\text{max}} = 5$, $C_{\text{min}} = 5$, the $h_{\text{max}} = 0.15$ m and $h_{\text{min}} = 0$ m. In this case, the roughness coefficient results in a better match for from SIRMOD and the model. Finally, the problem of cross sectional slope equal to 0.002 m/m and the use of other inflow from the Parshall flume generate advance curves which differ from the standard parabolic advance curve trajectory such as those obtained from SIRMOD and the present model.
Fig. C.13. The first irrigation event during advance phase on south border, USU-Wellsville Farm

Fig. C.14. Field work during the advance test at the first irrigation event
Fig. C.15. The second irrigation event during advance phase on north border, USU-Wellsville Farm

Fig. C.16. Field work during the advance test at the second irrigation event
Fig. C.17. The third irrigation event during advance phase on center border, USU-Wellsville Farm

Fig. C.18. Field work during advance test at the third irrigation event
Fig. C.19. The fourth irrigation event during advance phase on center border, USU-Wellsville Farm

Fig. C.20. Field work during the advance test at the fourth irrigation event
VITA

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(October 2008)

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Computer Programming Languages:

Visual Basic for Applications (VBA) in Excel and ArcGIS 9.1, Microsoft Visual Studio.NET, MATLAB.

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From 2003-2005: Graduate Research Assistant for the International Irrigation Center. Training of Water Management Courses with Emphasis on on-farm water management.

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Interest: