MOTION COMPENSATION OF INTERFEROMETRIC SYNTHETIC APERTURE RADAR

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ABSTRACT

Synthetic aperture radar (SAR) is a digital signal processing technique which enhances the azimuth resolution of a radar image using the target Doppler history created by the motion of the radar platform. If the platform deviates from a constant velocity, straight-line path then image quality is lost and image details become unfocused. Motion compensation (MOCO) is a technique in which the position and attitude of the platform is recorded or estimated and then used to correct the scene's Doppler history as if a straight-line, constant velocity path had been taken. Brigham Young University's interferometric synthetic aperture radar (YINSAR) was flown on a Cessna Skymaster which experienced significant motion due to the aircraft's small frame. But using multiple motion sensors, such as an inertial measurement device and various GPS units, the motion can be compensated for. This report discusses some basic SAR theory, discusses SAR interferometry, gives a brief description of YINSAR and measurement devices, investigates various SAR motion compensation algorithms, and displays selected image results from YINSAR.

INTRODUCTION

In recent years, SAR imaging has become a viable solution to a wide variety of problems in areas such as archeology, topographical mapping, environmental monitoring, and military situations. In order to be useful, the radar images must be accurate. However, conventional algorithms for the creation of SAR images assume that the radar travels on a straight-line path and is moving at a constant velocity. Unfortunately, when these requirements are not met the result is a defocused and deformed image. If radar platform motion is accurately measured then post-processing corrections can be applied to recover image quality.

SAR BACKGROUND

The resolution of a radar image pixel can be broken into the resolution of the range and azimuth (cross-range) directions (see Fig. 1). Resolution in the range direction is directly affected by the bandwidth of the radar transmit pulse and is given by

$$\delta x \approx \frac{c}{2B} \qquad (1)$$

where c is the speed of light and B is the transmit pulse bandwidth. A higher bandwidth implies a shorter pulse length; however, if the transmit pulse is shortened, the received echo's signal-to-noise ratio (SNR) also drops, resulting in noisy received echos and a noisier image. To overcome the balance between bandwidth and SNR, a linear frequency modulated (LFM) chirp pulse shape is used. When a pulse of this shape is correlated with another time-delayed copy, the result is a pulse with both a short duration (high bandwidth) and a high SNR (low noise). This process of creating higher range resolution by correlating the received pulse with the transmit pulse is known as range compression or range matched filtering.
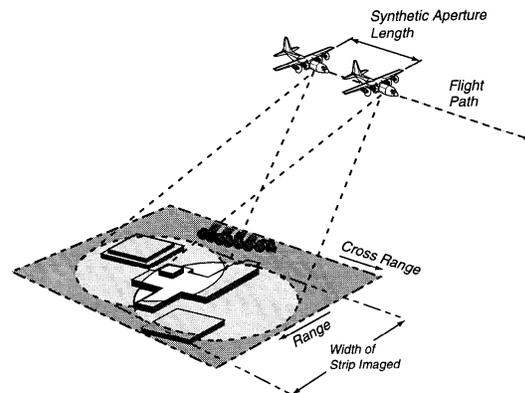

Figure 1: SAR imaging geometry[1]

In the azimuth direction, however, the pulse shape has no effect. The resolution here is directly related to the size of the antenna beamwidth which is directly related to the antenna size by the equation[2]:

$$\beta \approx \frac{\lambda}{D} \qquad (2)$$

where β is the antenna beamwidth, λ is the carrier wavelength, and D is the antenna size in wavelengths. The wider the beamwidth the poorer the azimuth resolution while the narrower the beamwidth, the better the resolution. But having a large antenna on a small airplane is not realizable, thus a different solution is needed. SAR is a signal processing technique that uses many measurements from a small antenna to synthesize a large antenna, subsequently creating higher azimuth resolution. (see Fig. 1) The idea behind SAR is the tracking of the azimuth phase history of a target. If a constant velocity, straight-line path is maintained by the radar as it passes a target, the target's collected phase history will be approximately quadratic. The significance of having a quadratic target phase history means that pulses received by the target will be shifted linearly in frequency as the aircraft flies by the target. This linear frequency shift in the received data is also known as a Doppler shift. With the phase history now viewed as a linear frequency shift or a LFM chirp, the phase data can be correlated in the azimuth direction as done for the range. The technique of increasing azimuth resolution using the SAR technique is also known as azimuth compression or azimuth matched filtering.

But in many cases maintaining a constant velocity, straight-line path is not possible and motion compensation of the data is necessary. YINSAR experiences many motion errors due to wind gusts and piloting errors, thus creating defocused radar images. These motion errors are measured by multiple sensors and then received data is corrected.

INTERFEROMETRY

Interferometry is the use of two channels of radar data to infer the topography of the imaged scene. The idea of determining scene topography originated with the area of stereometry. As seen from Fig. 2, stereometry uses two observations, r_a and r_b (offset by some baseline, b), to infer information in three dimensions about a target. Both observations have slightly differing ranges to the target and using basic trigonometry the height of the target can be inferred.

Both observations have slightly differing ranges to the target and using basic trigonometry, the height of the target can be found using the law of cosines,[3]

$$r_b^2 = r_a^2 + b^2 - 2br_a \sin(\theta - \alpha). \qquad (3)$$

The target height, h, is then found by the equation

$$h = H - r_a \cos(\theta). \qquad (4)$$

Typically, one of the ranges, the length of the baseline, and the height, H, are known exactly. This implies that the accuracy of the target height estimate, h is dependent
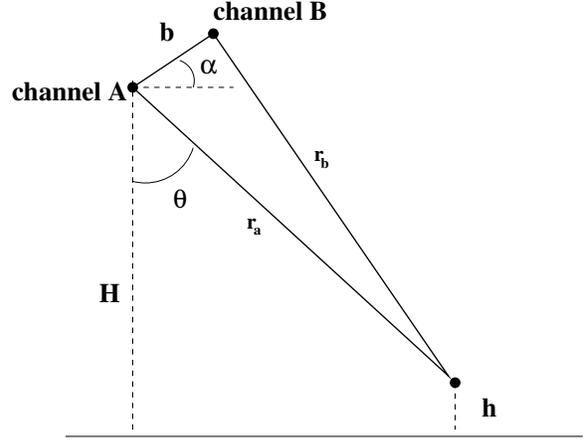

Figure 2: Stereometric geometry model

upon the measurement of the second range, r_a, or upon the change in range Δr, which is defined as

$$\Delta r = r_b - r_a. \qquad (5)$$

Graham[3] first proposed the idea of using the phase difference from two SAR images, slightly offset, to determine the value of Δr. The idea is that if one of the received SAR images has a phase given by

$$\phi_a = A_1 \exp\left(-j\frac{4\pi}{\lambda}r_a\right) \qquad (6)$$

and a second SAR image which is slightly offset from the first has a phase

$$\phi_b = A_2 \exp\left(-j\frac{4\pi}{\lambda}r_b\right) \qquad (7)$$

then the phase difference

$$\Delta\phi = \arg\left[(\phi_a)(\phi_b)^*\right] = \frac{4\pi}{\lambda}\Delta r \qquad (8)$$

from which the range difference Δr can be estimated. The accuracy of the estimate is related to the size of the wavelength and the size of the baseline, b. Two single-look complex (SLC) SAR images are used to make a phase difference image known as an interferogram. Interferograms are initially produce wrapped phase differences between 0 and 2π and require two-dimensional phase unwrapping in order to find the Δr estimate.

YINSAR

YINSAR is an X-band interferometric synthetic aperture radar created at Brigham Young University.[4] The purpose of YINSAR is to demonstrate that accurate, high resolution SAR images can be generated using low-cost

Figure 3: Twin-engine Cessna owned by Utah State University used to fly YINSAR

hardware. The radar platform is a four-passenger, dual-engine Cessna 337M owned and maintained by Utah State University (Fig. 3). The SAR operates at an altitude of 300m and has theoretical resolution of about a meter in both range and azimuth directions. More YINSAR specifications are found in a paper by Doug Thompson.[4]

MOTION MEASUREMENT

The primary motion sensor on board YINSAR is an inertial navigational unit (INU) which uses high-precision gyros and accelerometers and strap-down navigation to measure platform position and attitude. The secondary motion collection sensors include several GPS units: A kinematic GPS, and two differentially corrected GPS units. A description of the different measurement devices and their specifications is found in Table 1. Due to its higher sampling rate and high precision, the INU is used as the primary sensor for detecting platform position and attitude. The INU measurements, however, are subject to biases and tend to drift over time thus needing to have corrections in order to remain accurate. This correction is applied by using a technique[5] based on the slower sampled GPS data to create inertial measurement drift corrections by approximating the drift as a quadratic function.

MOTION ERRORS

Three types of motion errors are present during the collection of SAR data. They are translational motion errors, errors in platform attitude, and irregular azimuth spacing. Translational motion errors are small translational deviations aways from a nominal path or track. These type of deviations, also known as range delay variations, can be seen in Fig. 4 and have two principle ef-

fects upon radar images. First, there is a change in phase of the received pulses. The phase of the received pulse can be written as

$$\phi = \frac{4\pi R}{\lambda} \qquad (9)$$

Any deviations of the aircraft from its ideal path can be seen as a change in the transmission line length and consequently a change in the received pulse phase. These changes corrupt target phase histories and subsequently defocus the image. The other effect of translational errors is upon the drifting of target signatures through range bins. As the platform deviates from a nominal track, the reflectivity information contained in the image pixels drift into neighboring range bins which cause inaccuracies in the image.

The next type of motion errors are those in platform attitude, or variations in roll, pitch, and yaw. These errors effectively steer the beam to a different imaging angle or "squint" angle. Their effects are a Doppler shifting of the power spectrum of the received data. Since a radar's azimuth matched filters (which are bandpass filters) are usually centered around a certain Doppler value (such as zero), a steering of the spectrum away from zero Doppler can result in a lowered image SNR. When seen throughout an image, a light and dark azimuth banding is observed in the SAR image. An important parameter in correcting this problem is known as the Doppler centroid. The Doppler centroid, f_{DC}, is the center frequency of the received power centroid and is often a range-dependent value. The most common way to estimate the Doppler centroid is through frequency domain correlation techniques, but other more efficient time domain methods exist.[6] The greatest importance the Doppler centroid plays in processing is to know when SAR data may be aliased, that is

$$2f_{DC} > \frac{1}{PRF} \qquad (10)$$

where PRF is the radar pulse repetition frequency. Once the Doppler centroid is known, the data can be either Doppler shifted back to zero Doppler, or the azimuth bandwidth can be limited to prevent aliased artifacts to appear in the final images.

The final type of motion error is in azimuth spacing. In order for the phase history to be accurate, the spacing at which the radar pulses are collected must be uniform. This constant spacing is difficult to maintain because of varying cross range platform velocity. This error can be viewed as a making a synthetic array of antennas, but with unequal spacing between elements. The result of these errors is phase history corruption, which defocuses images.

It is important to compensate for all the three types of motion error to preserve image phase. Uncompensated motion can have adverse effects on SAR imagery and

Device	Type	Collection rate(Hz)	Measurement format
Ashtech	Differential GPS	10	Lat/Lon(WGS84)
Satloc	Differential GPS	5	Lat/Lon (WGS84)
Trimble	Kinematic GPS	5	roll,pitch,yaw in CW from N
INU	Gyros	1000	Δangle measurements taken from inertial axes
INU	Accelerometers	500	force measurements taken along inertial axes

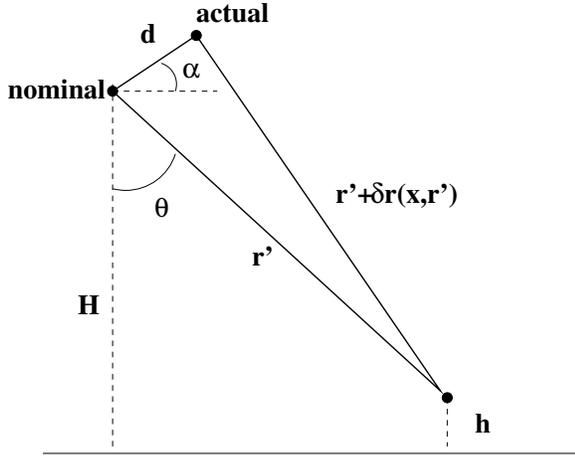**Table 1**: YINSAR motion data collection devices

Figure 4: MOCO geometry

upon interferometry. It is useful to investigate the various motion compensation algorithms available.

MOCO ALGORITHMS

The primary focus of this study was to verify the validity of the motion compensation algorithm used by YINSAR.[7] In the field of synthetic aperture radar, there are three levels of motion compensation available based upon the size of the antenna beamwidth, and the magnitude of translational deviations. All the motion compensation algorithms presented here assume that the cross-track displacement value, $\delta(r, x)$, is already known. This value is typically calculated using the GPS/accelerometer positions of the platform and a rough ground height estimate. Other methods to calculate this value using the raw data also exist.[8]

Narrowbeam MOCO

The first type of motion assumes a narrow azimuth beamwidth and is known as narrowbeam MOCO. The narrow beamwidth assumption makes the motion compensation simple. The algorithm consists of 4 steps applied to range compressed SAR data.

1. The direct motion compensation begins with a slant range dependent phase correction

$$\exp{(j(\omega_0)\frac{2}{c}\delta r(x, r'))}. \qquad (11)$$

2. Next, the drifting of targets into other range bins must be corrected. If there is a negligible amount of slant range dependence on the deviations, such as in a satellite platform, then the approximation $\delta(x, r') \approx \delta(x)$ holds and a simple range bin shift can be applied. This range shift is often coupled with the phase correction and applied as a single multiplication step in the range-Doppler domain by

$$\exp{(j(\omega_0 + \omega)\frac{2}{c}\delta r(x))} \qquad (12)$$

However, in cases where there is a significant amount of slant range dependence, such as in a SAR with a wide range swath, range bins cannot be shifted. They require a resampling which must be done with enough precision to preserve phase information.[7] A good interpolation scheme is a multiple point $sinc()$ interpolator.[1]

3. Next, any variations of Doppler centroid are accounted for by adjusting the center frequency of the azimuth matched filters in the range-Doppler domain to increase the average SNR of the final image.

4. Finally, any unequal spacing due to variations in the along track velocity coupled with a constant PRF are corrected by interpolation in the azimuth direction. Similarly, to maintain high phase accuracy, the interpolation must be a higher order interpolator. This step could also be performed at the beginning of the motion compensation process with minimal error in the results.[9]

This process motion compensates narrow beamwidth SAR data, including data with a slant range dependence. This approach makes the assumption that platform deviations from the nominal track are small enough that any

4

x' dependence can be removed. Unfortunately, in many situations deviations from a nominal track are large and the motion compensation algorithm must include higher order corrections.

Second Order Narrowbeam MOCO

When cross-track deviations become too great, the narrowbeam compensation becomes invalid. For example, as shown in Fig. 5, a target that is located at zero Doppler will require a correction using the displacement, $\delta r(x, r')$, however, a target located at the front edge of the swath will require a slightly different correction as

$$\delta r(x, r') \cos\left(\frac{\theta}{2}\right) \qquad (13)$$

where δr is the zeros Doppler correction and θ is the azimuth beamwidth. So, in a narrow beam SAR system where large deviations are present, a higher order correction is necessary. This motion compensation technique begins by decomposing the deviations into range dependent and range independent components

$$\delta r(x, r') = \delta_{r_0}(x) + \delta_v(x, r'). \qquad (14)$$

which represent first and second order motion corrections. The following algorithm operates upon range compressed data and attempts to make the system appear narrowbeam.

1. The first step in higher order motion compensation is to apply what is known as first order or bulk motion compensation. This correction, called a narrowbeam correction,[10] is a bulk phase and range bin shift with respect to a specific reference range, r_0. A commonly chosen reference range is the center of the image. The correction is reduced down to a phase multiplication

$$\exp\left(j(\omega_0)\frac{2}{c}\delta_{r_0}r(x)\right) \qquad (15)$$

and a range bin shift

$$\exp\left(j(\omega)\frac{2}{c}\delta_{r_0}r(x)\right). \qquad (16)$$

Both operations are typically applied as a single multiply on the range Fourier transformed data. This correction while not completely correct, puts the data in an approximately correct location.[6] The idea is that when bulk deviations are compensated, the system is reduced to a narrowbeam and that any range migrating phase errors are considered negligible.

2. Next, after applying a range migration correction from one of the azimuth processing algorithms, the residual motion deviations, $\delta_v r(x, r')$, are applied. The correction here is simply a multiplication of the data by the phase correction

$$\exp\left(j(\omega_0 + \omega)\frac{2}{c}\delta_v r(x, r')\right). \qquad (17)$$

If the data is slant-range dependent, the bins may also need to be resampled to account for any range dependence.

3. Next, variations of Doppler centroid are accounted for by adjusting the center frequency of the azimuth matched filters in the range-Doppler domain to increase the average SNR of the final image.

4. The final step in this algorithm is to correct any faulty PRF spacing by resampling the motion compensated data in the azimuth direction. As was mentioned in the previous algorithm, this step could have been performed at the beginning of this algorithm.

This algorithm functions fairly well to reduce any range migrating errors that may be introduced into a narrow azimuth beamwidth SAR system by large deviations from a nominal track. However, in some instances the assumption that a SAR model can be reduced to narrowbeam is not valid. It is in these cases that motion compensation needs to be done pixel-by-pixel.

Widebeam MOCO

The assumption of a narrow azimuth beamwidth supports the idea that the deviations in the broadside direction were the same for all targets in the beam. This allows the removal of the target azimuth coordinate dependence from the deviation function and enables easier motion compensation. Unfortunately, in cases where a SAR has a wide azimuth beamwidth, the motion deviation may have a target azimuth position dependence, even when platform deviations are small. This effect is visualized in Fig. 5 where the target at the leading edge of the azimuth beam requires a smaller correction than the target located at zero Doppler. In considering a narrow beam SAR system the motion compensation procedure outlined in the previous section above works acceptably, however in the case of a system with a wide azimuth beamwidth the method must be modified. The following approach is used for widebeam motion compensation.

1. In[11] the problem is approached by applying the same first order motion compensation as shown in Eq. (15) and Eq. (16). This shifts the data to an approximately correct location, minimizing the effects of range-varying phase error.
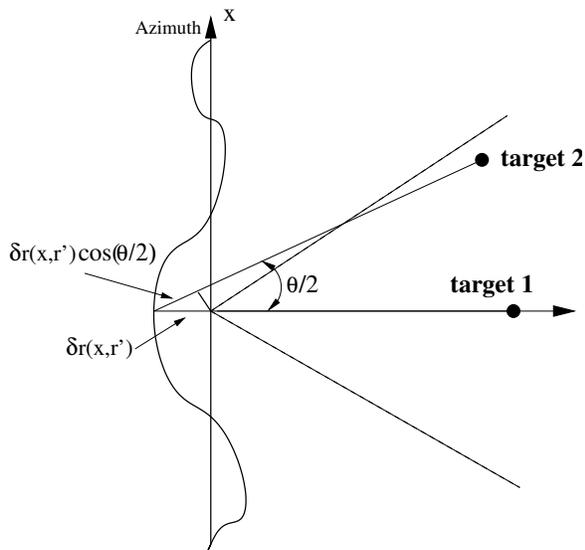
Figure 5: Widebeam SAR geometry

2. The next step is to azimuth compress the image and apply a phase correction. Due to the spatially varying nature of the data this correction is a target dependent filter that is most efficiently applied in the two-dimensional Fourier domain. The filter for a target at (x', r') is given by

$$H(k_{x'}, k_{r'}) = \exp\left(j2k\delta r_v(x)\right) \quad (18)$$

where

$$2k = \sqrt{k_{r'}^2 + k_{x'}}$$
$$x = x' - \frac{k_{x'}}{k_{r'}}r' \quad (19)$$

where $k_{r'}$, $k_{x'}$ are the range and azimuth wavenumbers and where $\delta r_v(x)$ is the residual motion error after the first order motion correction. The primary difficulty behind this algorithm is the large amount of computation required when dealing with a new filter for every target.

In 2001, Madsen[6] slightly modified the approach to help improve the overall computation time. In this newer approach, the same first-order correction is applied, but the target-dependent phase correction is done in patches rather than for each target. The correction for an entire patch is chosen to be the correction for a target at the patch center. All other targets in the patch need a slight phase correction, but this can be done using an analytic expression.

The importance of correcting the phase histories through motion compensation is not only important for creating the images, but also important for improving interferometry results.

Clearly, having accurate motion compensation is important in interferometry, for an accurate phase estimate produces an accurate height estimate. Of all types of motion that affect the interferometric height estimate, the one with greatest impact is the roll of the aircraft,[12] .[13] If not corrected, these roll errors can introduce tens of meters to the height estimates.[13]

YINSAR APPLICATIONS

The BYU synthetic aperture employed a narrowbeam motion compensation algorithm as prescribed by Lundgreen[7] to correct its imagery. While the algorithm produced excellent results for short flights low amounts of motion, it ran into problems on flights of longer duration and large amounts of motion. The first solution to this problem was to try using a higher order motion compensation algorithm. Unfortunately, while improving images, the higher order motion compensation created images that were not as useful because of large amounts of range bin shifting from larger deviations.

Another solution for a longer flight is to use a segmented ideal track. The benefit behind this technique is that a narrowbeam motion compensation technique accurately for the entire flight due to the lowered amount of cross-track deviation for any particular part of the flight.

In some cases targets were better focused in the motion compensated images (see the top part of Figs. 6, 7). Yet, not all targets were focused (see middle part of Figs. 6, 7) to to a highly varying attitude. Future work will include improving SAR processing using autofocus methods (such as the Phase Gradient Autofocus algorithm), and implementing a real-time motion compensation algorithm for YINSAR.

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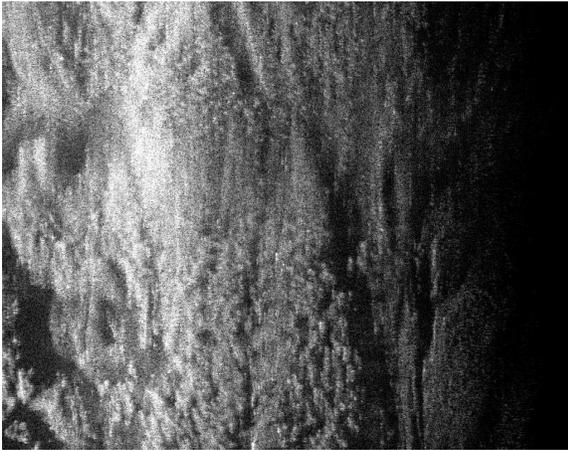
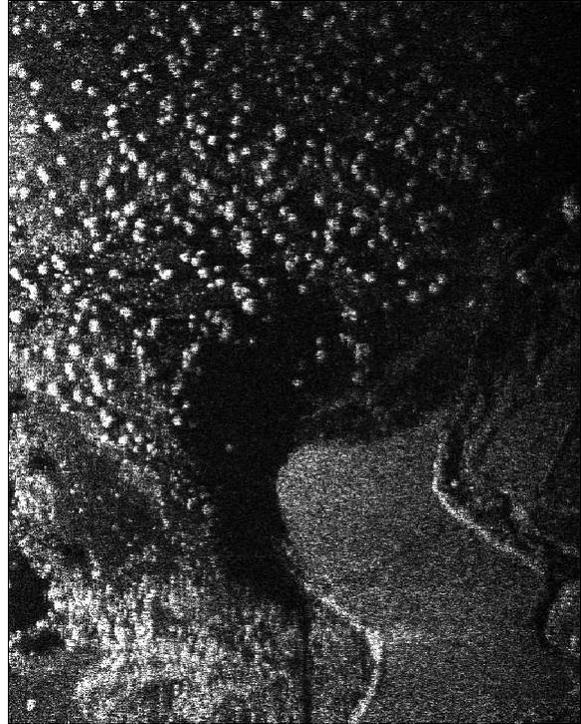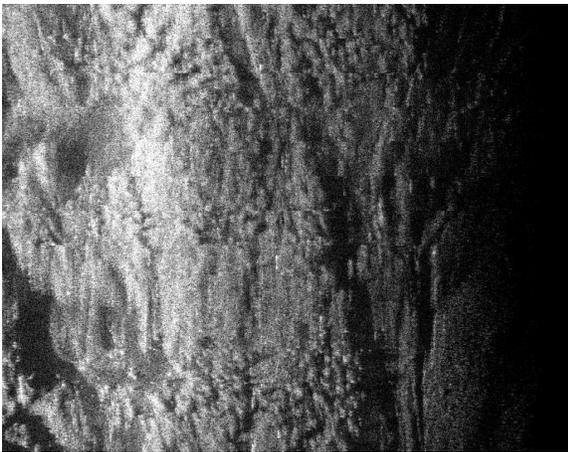**Figure 6**: Slumgullion slide, CO - without motion compensation

Figure 7: Slumgullion slide, CO - with motion compensation. Note the improvement in the detail in which artifacts in the image can be resolved (Bushes, trees, etc.).
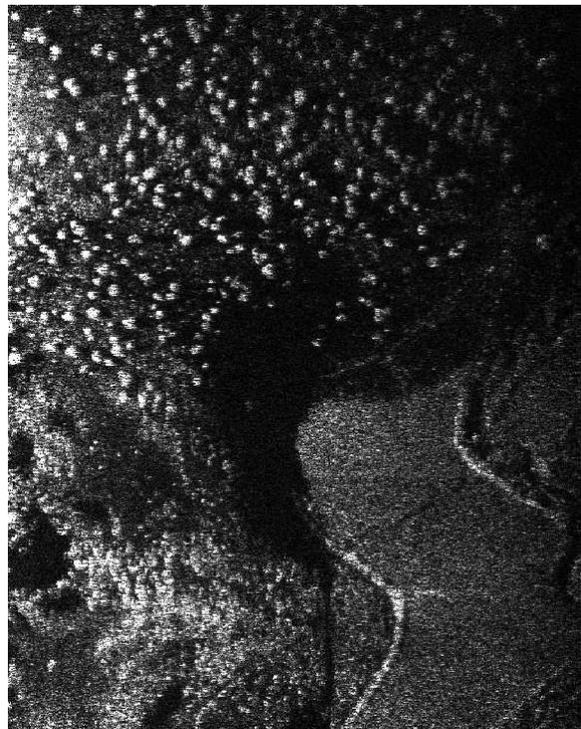

Figure 8: Wolf Creek, UT - without motion compensation

Figure 9: Wolf Creek, UT - with motion compensation.