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DETRENDING TIME-AGGREGATED DATA

by

DAVID AADLAND

**Department of Economics
Utah State University
3530 Old Main Hill
Logan, UT 84322-3530**

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David Aadland, Assistant Professor

**Department of Economics
Utah State University
3530 Old Main Hill
Logan, UT 84322-3530**

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DETRENDING TIME-AGGREGATED DATA

David Aadland

ABSTRACT

This paper examines the combined influences of detrending and time aggregation on the measurement of business cycles. The approximate band-pass filter of Baxter and King (1999) performs relatively well in the sense that it retains the basic shape of disaggregate spectra and cospectra when applied to time-aggregated data and is straightforward to apply across sampling intervals. Simulation of a simple weekly RBC model confirms the theoretical results.

JEL Codes: C1, E3

DETRENDING TIME-AGGREGATED DATA

1 Introduction

One of the most challenging and controversial aspects of business-cycle research is attempting to disentangle the cyclical and growth components of macro time series. Kuznets' work in the 1960's is a classic example of how seemingly innocuous data transformations can lead to spurious results, such as his discovery of 20-year long swings in economic activity (Adelman (1965); Howrey (1968)). Another example is provided by the influential work of Nelson and Kang (1981), who show that the residuals from a regression of a random walk on time display spurious periodicity. Although researchers, in part due to the aforementioned studies, are now more aware of the possible pitfalls of detrending data, the debate over which method is "best" continues. Some of the more commonly used detrending methods include: regression on polynomials in time, the first difference (FD) filter, the Hodrick-Prescott (HP) filter, and the Baxter-King (BK) approximate band-pass filter, to mention a few.

The HP filter introduced by Hodrick and Prescott (1980) has, in particular, received considerable attention. This flexible detrending method trades off deviations from trend against an adjustable smoothness criterion. The HP filter has been criticized for generating spurious cycles in difference stationary data (Harvey and Jaeger (1993); Cogley and Nason (1995b)); altering the persistence, variability and co-movement of time series (King and Rebelo (1993)); and (similar to the FD filter) for passing through too much high-frequency or "irregular" variation. Nevertheless, the HP filter has withstood two decades of use and remains a leading method for detrending time series data. The BK filter is, however, gaining in popularity (e.g., see Stock and Watson (1999)) and is more consistent with the Burns and Mitchell (1946) definition of the business cycle as containing cycles that last between six and 32 quarters.

In addition to filtering out a trend, most macro time series are also implicitly passed through a time-aggregation (TA) filter. For institutional reasons, business-cycle researchers work almost exclusively with quarterly data. Since data-generating processes for economic data are generally thought to operate at continuous or very short discrete-time intervals, observed quarterly data are therefore some sort of transformation of the high-frequency data. If the variable is a flow, then it is typically summed (or averaged) over shorter time intervals. If it is a stock, then it is

typically systematically sampled. Ideally, we would like time aggregation to preserve as many of the underlying properties of the high-frequency data-generating process as possible. This and other aspects of time aggregation have been extensively explored both in the time domain (e.g., Tiao (1972); Weiss (1984); and Rossana and Seater (1992)) and in the frequency domain (e.g., Sims (1971) and Granger and Siklos (1995)).

The unique contribution of this paper is to examine how the *joint* application of detrending and TA filters influence how we look at business cycles. As mentioned above, there are many studies that examine the individual effects of detrending and time aggregation, but despite the fact that detrending and time aggregation are almost always joint phenomena, few have examined their combined effects. The results in this paper suggest that the interaction between the two filters is important for how we measure business cycles and test our models. In particular, the BK filter performs relatively well in the sense that when applied to time-aggregated data it (1) retains the basic shape of the disaggregate spectra and cospectra and (2) is easy to adjust across data frequencies. The FD and HP filters, on the other hand, either tend to distort the shape of the disaggregate spectra or are not straightforward to adjust across data frequencies. Simulations from a simple real business cycle (RBC) model confirm the theoretical results.

Section 2 briefly reviews spectral analysis as it relates to detrending and TA filters. Section 3 investigates the practical importance of the detrending and TA filters by applying the analysis to a RBC model. Section 4 concludes.

2 Spectral Analysis

The most transparent method for investigating the effects of detrending and TA filters is through spectral analysis. As a result, I begin this section by briefly reviewing several aspects of spectral analysis as it relates to filtering theory. I then focus on the properties of the detrending and TA filters individually, as well as their combined or “cascaded” effect.¹

Assume $\{x_t\}_{t=-\infty}^{\infty}$ is a covariance-stationary time series with absolutely summable autocovariances given by $\gamma_j = E[(x_t - \mu_x)(L^j x_t - \mu_x)]$, where $L^j x_t = x_{t-j}$, $\mu_x = E[x_t]$, and $E[\cdot]$ is the

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¹See Priestley (1988) and Hamilton (1994) for more thorough discussions of spectral analysis.

mathematical expectation operator. Cramér's spectral representation theorem (Priestley (1988)) guarantees that x_t can be decomposed into a weighted sum of sine and cosine waves according to

$$x_t = \mu_x + \int_{-\pi}^{\pi} [\alpha_1(\omega) \sin(\omega t) + \alpha_2(\omega) \cos(\omega t)] d\omega, \quad (1)$$

where the weights $\alpha_1(\omega)$ and $\alpha_2(\omega)$ are serially and mutually uncorrelated random variables over the angular frequencies ω measured in radians. Using the autocovariance generating function (ACGF) for x_t , $g_x(z) = \sum_{j=-\infty}^{\infty} \gamma_j z^j$, De Moivre's theorem and some simple trigonometry, one can generate the population spectrum

$$s_x(\omega) = (2\pi)^{-1} \{ \gamma_0 + 2 \sum_{j=1}^{\infty} \gamma_j \cos(\omega j) \}. \quad (2)$$

For covariance-stationary time series, $s_x(\omega)$ will be symmetric about $\omega = 0$ with a period equal to 2π . As a result, all relevant information regarding the spectrum is contained within frequencies between 0 and π . Furthermore, we can recover the k^{th} autocovariance for x using $s_x(\omega)$ according to

$$\gamma_k = \int_{-\pi}^{\pi} s_x(\omega) \cos(\omega k) d\omega. \quad (3)$$

Equation (3) indicates the practical use of the spectrum. For example, we are able to decompose the fraction of the variance in x (γ_0) due to cycles with frequencies between ω_1 and ω_2 by integrating the area under the spectrum between ω_1 and ω_2 (i.e., $\int_{\omega_1}^{\omega_2} s_x(\omega) d\omega$). In particular, this allows us to isolate the amount of variation in a stationary series that is due to cycles at business-cycle frequencies.

Time-invariant linear filters can be introduced by pre-multiplying x_t by a polynomial in the lag operator L , $h(L) = \sum_{j=-\infty}^{\infty} h_j L^j$ with h_j absolutely summable. This generates

$$y_t = h(L)x_t, \quad (4)$$

where y_t is the filtered version of x_t . Using (4) and $g_x(z)$ it can be shown that a linear filter has the effect of multiplying the spectrum for x by $f(\omega) = h(e^{-i\omega})h(e^{i\omega})$, which is often referred to as

the squared gain or squared modulus of the transfer function. Therefore, the spectrum for y is given by $s_y(\omega) = f(\omega)s_x(\omega)$. If $f(\omega) > 1$, then the filter magnifies the variation in x at frequency ω and if $f(\omega) < 1$, then the filter diminishes variation in x at frequency ω .

Linear filters can also be expressed in their polar form. Writing $h(\omega)$ in its polar form produces

$$h(\omega) = R \exp(-i\varphi(\omega)), \quad (5)$$

where $R = \sqrt{f(\omega)}$ represents the gain of the filter and $\varphi(\omega)$ represents the phase shift introduced by the filter $h(L)$. The HP filter, for example, is a symmetric filter and does not introduce a phase shift (i.e., $\varphi(\omega) = 0$, for all ω) while temporal summing and systematic sampling are typically asymmetric filters, so they are likely to introduce a phase shift.

Business-cycle researchers are also interested in the co-movement between variables. In order to examine the impact of filtering on the co-movement between two variables, let $h(L)$ be a 2×2 matrix filter

$$\begin{bmatrix} h_1(L) & 0 \\ 0 & h_2(L) \end{bmatrix} \quad (6)$$

that operates on the vector time series $x_t = (x_{1,t}, x_{2,t})'$. Then the (2×2) matrix spectrum for $y_t = (y_{1,t}, y_{2,t})'$ can be written as $s_y(\omega) = h(e^{-i\omega})s_x(\omega)h(e^{i\omega})'$. The main-diagonal elements are the spectra for y_1 and y_2 , while the off-diagonal elements are the cross-spectrum between y_1 and y_2 . For example, the cross-spectrum between y_1 and y_2 ($s_{y_{12}}(\omega)$) can be written in terms of the cross-spectrum between x_1 and x_2 as $s_{y_{12}}(\omega) = h_1(e^{-i\omega})h_2(e^{i\omega})s_{x_{12}}(\omega)$. As expected, both $h_1(L)$ and $h_2(L)$ are important in determining the nature of the co-movement between the filtered variables y_1 and y_2 . The cross spectra $s_{y_{12}}(\omega)$ and $s_{x_{12}}(\omega)$ are generally complex numbers and can be decomposed as

$$s_{x_{12}}(\omega) = c_{x_{12}}(\omega) + iq_{x_{12}}(\omega) \quad (7)$$

where $c_{x_{12}}(\omega)$ is referred to as the cospectrum and $q_{x_{12}}(\omega)$ as the quadrature. Since $q_{x_{12}}(\omega)$ integrates to zero over $(-\pi, \pi]$, the contemporaneous covariance between x_1 and x_2 is given by $\int_{-\pi}^{\pi} c_{x_{12}}(\omega) d\omega$.

2.1 Temporal Aggregation Filters

I use the term temporal aggregation to refer to a mapping of either a continuous or short-interval discrete time series process to a longer-interval discrete time series process. Although this paper focuses exclusively on discrete-time processes, the results could be generalized to handle underlying continuous-time processes (e.g., see Sims (1971), Geweke (1978) or Sargent (1987)). Time-aggregated flow data are most often either summed or averaged over time while stock data are typically systematically sampled at regularly spaced discrete-time intervals. Time aggregation, whether a flow or a stock variable, can be thought of as involving a two-step process (Lippi and Reichlin (1991)). In step one, the variable is passed through an aggregation filter. For example, under temporal summing the filter is $h_{TS}(L) = 1 + L + L^2 + \dots + L^{n-1}$, where n indicates the number of disaggregate (often referred to as *basic*) periods being aggregated; and under systematic sampling the aggregation filter is $h_{SS}(L) = L^k$, where $k \in \{0, 1, \dots, n-1\}$.² Step two then involves systematically sampling every n^{th} observation from the $h_{TS}(L)$ - or $h_{SS}(L)$ -filtered data to form a series of non-overlapping aggregates.

In terms of their spectra, a temporally aggregated series can be related to its disaggregated counterpart using the z and Fourier transforms of $h(L)$ and the folding operator (Sims (1971); Granger and Siklos (1995)) which is given by

$$F[s_x(\omega)] = \sum_{j=-I}^I s_x(\omega + 2\pi j/n) \quad (8)$$

where $F[s_x(\omega)]$ is defined over $\omega \in (-\pi/n, \pi/n]$ and I is the largest number such that $\omega + 2\pi j/n$ falls in the range $(-\pi, \pi]$. The folding operator reflects the well-known *aliasing* identification problem, whereby harmonics at various frequencies cannot be distinguished from one another in sampled data.³ In essence, aliasing dictates that frequencies outside the $(-\pi/n, \pi/n]$ range of

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²For example, when aggregating a monthly flow variable to an annual frequency, $n = 12$. Aggregating a weekly flow variable to an annual frequency, gives $n = 52$. As $n \rightarrow \infty$, one can think of the underlying data generating process as operating in continuous rather than discrete time.

³When sampling from a continuous time process, I is set to ∞ as frequencies larger than π in magnitude are also aliased (Sims (1971)). Since I am treating the underlying process as one set in discrete time, conceptually, one can imagine that frequencies higher than π have been previously aliased in the sampling from an underlying

the time-aggregated process get successively folded back into the $(-\pi/n, \pi/n]$ range. The initial folding point, π/n in this context, is sometimes referred to as the Nyquist frequency. Alternatively, the basic and time-aggregated series can be related in the time domain according to their ACGFs. These are respectively

$$g_y(z) = \sum_{j=-\infty}^{\infty} \Gamma_j z^j \text{ and } g_Y(z) = \sum_{j=-\infty}^{\infty} \Gamma_{nj} z^j, \quad (9)$$

where the Γ 's are the autocovariances of the $h_{TS}(L)$ - or $h_{SS}(L)$ -filtered data, and lower- and upper-case letters (i.e., y and Y) refer to the pre-sampled and post-sampled series. From (9), it is clear that time aggregation produces an aggregate ACGF which is simply a sampled version of the basic ACGF at equally spaced n -length intervals.

Returning to the frequency domain, the spectra of a time-aggregated series $\{Y\}$ is then

$$s_Y(\omega) = F[s_y(\omega)] = F[f(\omega)s_x(\omega)], \quad (10)$$

where $f(\omega)$ is the squared gain and $s_Y(\omega)$ is defined over $(-\pi/n, \pi/n]$. Alternatively, one could redefine $s_Y(\omega)$ in terms of aggregate time and write it as $s_Y(\tilde{\omega})$, where $\tilde{\omega} = \omega n \in (-\pi, \pi]$. Since the folding operator F is an operator rather than a linear filter, there does not exist a unique transfer function for time aggregation of any covariance-stationary time series. Figure 1, however, shows the squared TA transfer function associated with $h_{TS}(L)/n$ along with the (shaded) aliased frequencies that will be consecutively folded back (first few folds are shown with vertical lines) onto the range $(0, \pi/n]$. Clearly, the systematic sampling step associated with temporal aggregation produces a loss of information as some higher-frequency harmonics in the basic series will become convoluted with lower-frequency harmonics. The extent to which this convolution is important depends on the nature of the underlying time series, and in particular, on the relative power of the disaggregate spectrum at cycles with periods greater than $2n$ basic periods. The practical importance of this aliasing problem will be explored in more detail below.

These results also extend naturally to a vector time series, with each individual series potentially filtered in a different manner. In this case the cross spectrum between the time aggregates Y_1 and

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continuous-time process.

Y_2 is

$$s_{Y_{12}}(\omega) = F[s_{y_{12}}(\omega)] = F[h_1(e^{-i\omega})h_2(e^{i\omega})s_{x_{12}}(\omega)]. \quad (11)$$

2.2 Detrending Filters

As mentioned above, there are many different methods that have been applied in business-cycle research to detrend nonstationary time series. In this paper, I focus on three detrending methods – the FD, HP, and BK filters. The properties of these filters are well documented (e.g., see King and Rebelo (1993) and Canova (1998)), so I will only briefly review their properties.⁴

First, the FD filter is given by $h(L) = 1 - L$, which implies that the corresponding squared transfer function is $f(\omega) = 2(1 - \cos(\omega))$. Since $f(0) = 0$, the FD filter removes variation due to the lowest frequency cycles, but unnecessarily magnifies high-frequency variation as can be seen in Figure 2. Another undesirable property of the FD filter is that since it is an asymmetric filter, it introduces a phase shift into the filtered time series.

Second, the infinite-sample version of the HP filter solves the following problem

$$\min_{\{g_t\}} \left\{ \sum_{t=-\infty}^{\infty} [y_t - g_t]^2 + \lambda \sum_{t=-\infty}^{\infty} [(g_{t+1} - g_t) - (g_t - g_{t-1})]^2 \right\} \quad (12)$$

where λ is an adjustment parameter that governs the growth component g_t by trading off squared deviations from trend against a smoothness constraint. King and Rebelo (1993) show that the filter for the cyclical portion ($y_t - g_t$) of this filter can be written as

$$h(L) = \frac{\lambda(1 - L)^2(1 - L^{-1})^2}{1 + \lambda(1 - L)^2(1 - L^{-1})^2}. \quad (13)$$

Given the four first-difference terms in the numerator of (13), the cyclical portion of the HP filter produces a stationary series for any underlying series integrated up to the fourth degree.

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⁴I do not directly investigate the effects of detrending via regressions of polynomials in time because it is seldom used in modern business-cycle research, in part due to the influential research of Nelson and Plosser (1982) who find that many macro series are consistent with difference rather than trend-stationary specifications.

Furthermore, since (13) is a symmetric filter, it does not introduce any phase shift. When $\lambda = 1600$, the transfer function associated with (13) is an approximate high-pass filter that when applied to quarterly data approximately removes the variation associated with cycles of period longer than 32 quarters. The HP filter for $\lambda = 1600$ is shown in Figure 2.

Third, the BK filter of Baxter and King (1999) is an approximate band-pass filter that is commonly used to eliminate variation associated with cycles of period less than six and more than 32 quarters (i.e., a BP(6,32) filter). The BP(6,32) filter, however, cannot be applied to finite time series because it involves an infinite number of weights in the linear filter.⁵ Nevertheless, Baxter and King (1999) show how to calculate an approximate BP(6,32) filter by minimizing the deviation of the ideal and actual weights subject to the constraints that (1) the weights are truncated at K ; (2) the filter does not introduce a phase shift; and (3) the associated transfer function removes all variation at the zero frequency (i.e., eliminates certain trends). The BK filter ($K = 6$) that solves this problem is depicted in Figure 2. The BK filter provides a good approximation to the ideal BP(6,32) filter with moderate leakage, compression and exacerbation, using the terminology of Baxter and King (1999). However, the BK filter involves a trade-off. Higher values of K , all else equal, lead to better approximations of the ideal BP filter, but it requires a loss of $2K$ observations.

2.3 Cascading the Time-Aggregation and Detrending Filters

The previous two subsections discussed the individual effects of temporal aggregation and detrending on a time series. In the context of business-cycle research, however, these two filters are almost always applied together. Despite this fact, there have been very few studies that examine their joint influence.⁶

Typically macro data are aggregated over time before they are detrended. Accordingly, the spectrum of the detrended, time-aggregated series can be related to the spectrum of the basic series

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⁵The HP filter as described above essentially suffers from the same problem since its interpretation as a high-pass filter are based on an infinite sample size. Baxter and King (1999) discuss the consequences of applying the HP filter to finite samples.

⁶The only papers I am aware of that examine the dual issues of detrending and time aggregation are Lippi and Reichlin (1991); Ravn and Uhlig (1997); Baxter and King (1999); and Maravall and del Rio (2001) – two of which are unpublished manuscripts.

according to

$$s_Y(\omega) = f_{DT}(\omega)F[f_{TA}(\omega)s_x(\omega)], \quad (14)$$

where $\omega \in (-\pi/n, \pi/n]$ and f_{TA} and f_{DT} refer to the squared transfer functions for temporal aggregation and detrending respectively. Expanding (14) using (8) produces

$$s_Y(\omega) = f_{DT}(\omega)f_{TA}(\omega)s_x(\omega) + f_{DT}(\omega)f_{TA}(\omega) \sum_{j=-I, j \neq 0}^I s_x(\omega + 2\pi j/n), \quad (15)$$

where again I is the largest number such that $\omega + 2\pi j/n$ falls in the range $(-\pi, \pi]$. As (15) shows, the cascaded transfer function can be decomposed into two parts – the first operates on the lower frequency range of x (i.e., $\omega \in (-\pi/n, \pi/n]$) and the second operates on the higher frequencies that are aliased with $\omega \in (-\pi/n, \pi/n]$.

Figure 3 depicts the cascaded (squared) transfer functions associated with time aggregation ($n = 3$) and the FD, HP and BK filters. As in Figure 1, the aliased frequencies are shaded and will be sequentially folded onto the range $\omega \in (-\pi/n, \pi/n]$. It is clear from Figure 3, which presents the temporal averaging ($h_{TS}(L)/n$) case, that FD- and HP-filtered time-aggregated data will tend to suffer from the aliasing identification problem in that they pass through aliased frequencies. The BK filter, on the other hand, by only passing through cycles with periods between six and 32 quarters, tends to remove the frequencies that suffer the most from the aliasing identification problem in time-aggregated data. A similar story (figure omitted due to space limitations) emerges from the systematic-sampling filter, $h_{SS}(L) = L^k$, where $k \in \{0, 1, \dots, n-1\}$. Since the squared transfer function for $h_{SS}(L)$ is always $f_{TA} = 1$, the cascaded transfer functions associated with systematic sampling and the three detrending options essentially become an extended (i.e., $\omega > \pi$) version of Figure 2 with the first folding point at π . In general, the aliasing problem is more severe when detrending systematically sampled as opposed to temporally summed (or averaged) data. It is important to note, however, that irrespective of whether the data are temporally summed or systematically sampled, the BK filter tends to pass through less of the variation associated with aliased frequencies than the HP filter or especially the FD filter.

Next, I consider four simple time series processes to highlight how time aggregation and de-

trending operate together to effect the spectra of a basic series. The four processes are assumed to take the form

$$x_t = \tau_t + \epsilon_t \quad (16)$$

$$x_t = \tau_t + 0.5x_{t-1} + \epsilon_t \quad (17)$$

$$x_t = \tau_t + \epsilon_t + 0.5\epsilon_{t-1} \quad (18)$$

$$x_t = \tau_t + 0.5x_{t-1} + \epsilon_t + 0.5\epsilon_{t-1} \quad (19)$$

where $\tau_t = a + bt$ is a deterministic linear trend, and ϵ_t is assumed to be mean-zero, white-noise process. Since all four series are non-stationary, they do not have a spectral decomposition. Application of the detrending filters, however, can be thought to work in two steps (Cogley and Nason (1995b)). In step one, the filter makes the series stationary by linearly detrending, and then in step two, the filter acts upon the stationary series that remains. The four processes are aggregated over time using the filter $h_{TS}(L)/n$ with $n = 4$. Conceptually, one can imagine the data as being generated at quarterly intervals but the researcher only observes time-aggregated annual observations. Since the BK filter is designed to only pass through cycles with periods between eight and 1.5 years, it becomes an approximate high-pass filter because cycles with periods of less than two years cannot be identified in annual data. Lastly, the HP adjustment parameter for annual data is set at $\lambda = 10$ as suggested by Baxter and King (1999). Similar, but slightly smaller, annual values for λ are suggested by Ravn and Uhlig (1997) and Maravall and del Rio (2001).

Figure 4 depicts the spectra for detrended, basic $\{x_t\}$ and detrended, time aggregated $\{x_t\}$ using the FD, HP and BK filters. There are several interesting features of the graphs in Figure 4. First, and most starkly, it is clear that the FD filter, when applied to time-aggregated data, greatly distorts the spectral representation of the basic series. Since the FD filter is being applied to time-aggregated rather than basic data, it takes the form $h(L) = 1 - L^n$ with squared transfer function $f(\omega) = 2(1 - \cos(n\omega))$. As a result, the FD filter applied to time-aggregated data experiences a phase shift relative to the basic FD filter that decreases the length of its periods by a factor of n . Whereas the FD filter applied to the basic data tends to monotonically magnify high-frequency variation (see Figure 2), the FD filter applied to time-aggregated data tends to produce spectra with

multiple peaks. For the case of $n = 4$, the peaks are associated with cycles that have periodicity of approximately 10 and 30 quarters. Consequently, the combined use of time-aggregated data and first-differencing can produce variation at business-cycle frequencies, even when it is absent in the basic data.⁷ Second, there is a moderate amount of aliasing present in the HP-filtered spectra, most of it occurring around the $\omega = \pi/n$ frequency.⁸ Third, at least for time-series processes given by (16) - (19), the BK and HP cascaded filters are surprisingly similar and both act as approximate high-pass filters when applied to time-aggregated data.

The analysis above extends to the co-movement between two series in a similar manner. The cross spectrum between detrended, time-aggregated series Y_1 and Y_2 can be related to the cross spectrum of the unfiltered underlying series x_1 and x_2 according to

$$s_{Y_{12}}(\omega) = f_{12}^{DT}(\omega)f_{12}^{TA}(\omega)s_{x_{12}}(\omega) + f_{12}^{DT}(\omega)f_{12}^{TA}(\omega) \sum_{j=-I, j \neq 0}^I s_{x_{12}}(\omega + 2\pi j/n), \quad (20)$$

where $f_{12}^{DT}(\omega) = h_{1,DT}(e^{-i\omega})h_{2,DT}(e^{i\omega})$ and $f_{12}^{TA}(\omega) = h_{1,TA}(e^{-i\omega})h_{2,TA}(e^{i\omega})$ are the cross transfer functions for detrending and time aggregation respectively. Although variables will typically be detrended in the same fashion, they may be subject to different types of time aggregation. For example, when considering the co-movement between a stock and a flow variable, one series may be systematically sampled while the other is temporally summed. In this case, the time aggregation cross transfer function $f_{12}^{TA}(\omega)$ is likely to involve an imaginary component. Focusing exclusively on the cospectrum (i.e., real part of the cross spectrum) between Y_1 and Y_2 , we can write

$$c_{Y_{12}}(\omega) = \text{Re}[f_{12}^{DT}(\omega)f_{12}^{TA}(\omega)]c_{x_{12}}(\omega) + \text{Re}[f_{12}^{DT}(\omega)f_{12}^{TA}(\omega)] \sum_{j=-I, j \neq 0}^I c_{x_{12}}(\omega + 2\pi j/n) - \left\{ \text{Im}[f_{12}^{DT}(\omega)f_{12}^{TA}(\omega)]q_{x_{12}}(\omega) + \text{Im}[f_{12}^{DT}(\omega)f_{12}^{TA}(\omega)] \sum_{j=-I, j \neq 0}^I q_{x_{12}}(\omega + 2\pi j/n) \right\} \quad (21)$$

⁷This result is related to the finding of Working (1960), who showed that a first-differenced, time-aggregated random walk contains a moving average term.

⁸To see this most clearly, contrast the HP-TA cascaded (squared) transfer function in Figure 3 with the HP aggregate spectra in Figure 4.

where $\text{Re}[z]$ and $\text{Im}[z]$ refer to the real and imaginary parts of z , respectively. The basic quadrature, $q_{x_{12}}$, shows up in the formula for the aggregate cospectra, $c_{Y_{12}}$, because the cascaded cross transfer functions may involve imaginary numbers, which will in turn be multiplied by the imaginary number associated with $q_{x_{12}}$. Figure 5 depicts the real and imaginary parts of the transfer function from $s_{x_{12}}$ to $c_{Y_{12}}$ for two detrended, temporally aggregated time series – one temporally averaged using $h_{TS}(L)/n$ and one systematically sampled using $h_{SS}(L) = L^0$, where in both cases $n = 3$. Again, aliased frequencies are shaded and vertical lines represent folding points. What stands out the most in Figure 5 is the fact that the FD filter, and to a lesser extent the HP filter, behave differently when applied to aggregate data than when applied to basic data. For example, even the non-aliased portion of the HP filter is no longer an approximate high-pass filter. Furthermore, both the FD and HP filters suffer more acutely from the aliasing problem than does the BK filter, both in terms of the real and imaginary components of the cross transfer functions.

3 Application to Real Business Cycle Theory

In this section, I investigate the practical consequences of detrending time-aggregated data within the context of a simple RBC model, similar to the one presented in King, Plosser and Rebelo (1988). Begin by letting the basic decision interval (assumed to be one week) be indexed by t . A representative agent is assumed to maximize an expected stream of discounted utility

$$U(C, L) = E_t \sum_{s=0}^{\infty} \beta^{t+s} \left[\log(C_{t+s}) + \frac{\phi}{\eta} l_{t+s}^{\eta} \right] \quad (22)$$

by choosing consumption and leisure paths, where E_t is the mathematical expectations operator conditional on all information dated time t and earlier, β a subjective discount factor, C_t consumption, ϕ leisure's weight in total utility, $l_t = (N - L_t)/N$ the proportion of endowed time spent toward leisure, N the endowment of time available for leisure and labor, L_t labor hours, and $1/(1 - \eta)$ the intertemporal elasticity of proportional leisure.⁹ Consumption is subject to the

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⁹The steady-state intertemporal elasticity of labor is also $1/(1 - \eta)$ under the assumption that equal proportions of time are spent in leisure and labor activities.

resource constraint

$$C_t \leq Y_t - I_t, \quad (23)$$

where Y_t is output and I_t gross investment into the capital stock K_t . Capital accumulates according to

$$I_t = K_{t+1} - (1 - \delta)K_t, \quad (24)$$

and output is given by the production function

$$Y_t = A_t K_t^\alpha (z_t L_t)^{1-\alpha}, \quad (25)$$

where $z_t/z_{t-1} = \exp(\mu)$ is a deterministic labor-augmenting technology process. Total factor productivity (TFP) follows the process $A_t = A_{t-1}^\rho \exp(\epsilon_t)$, where ϵ_t is a mean-zero, white noise process with standard deviation σ .

The consumption and labor Euler equations for this problem are

$$C_t^{-1} = \beta E_t [C_{t+1}^{-1} (1 + \alpha Y_{t+1}/K_{t+1} - \delta)] \quad (26)$$

$$(1 - \alpha)Y_t/L_t = \phi l_t^{\eta-1} C_t/N. \quad (27)$$

I turn now to calibrating a weekly version of the RBC model. Since weekly data are generally not available, I use an analog version of the model in quarterly time along with some consistency conditions for time aggregation that should be satisfied in the steady state. In other words, I impose that steady-state flow variables in quarterly and weekly time (denoted F and F_*) should obey $F = nF_*$, where $n = 13$. For stocks, the variables should obey $S = S_*$. These constraints are then substituted into the weekly, steady-state version of the RBC model and solved jointly with

the corresponding quarterly steady-state equations to get

$$\begin{aligned}
 \beta_* &= \beta n e^{\mu/n} \left(e^\mu + \beta (e^{\mu/n} n - e^\mu) \right)^{-1} \\
 \delta_* &= (\delta/n) + (1 - e^{\mu/n}) + (1/n)(e^\mu - 1) \\
 \alpha_* &= \alpha \\
 \eta_* &= \eta + \log(\phi/\phi_*)/\log(l) \\
 \mu_* &= \mu/n,
 \end{aligned} \tag{28}$$

where asterisks denote weekly parameters.

There are six unknown parameters in (28) and only five equations. To identify the parameters, I specify that the weekly intertemporal labor supply elasticity is three times as large as its quarterly counterpart, roughly in line with microeconomic evidence (e.g., Kimmel and Kniesner (1998) and MaCurdy (1983)). See Aadland and Huang (2001) for a more detailed discussion of this calibration procedure and the empirical evidence regarding labor supply elasticities across data frequencies. Furthermore, steady-state conditions are not helpful in pinning down weekly values for ρ and σ . As in Chari, Kehoe and McGratten (2000), I specify that the weekly value of ρ is equal to its quarterly value raised to the 1/13th power. The weekly value of σ is chosen to be consistent with the standard deviation for the error in a quarterly, time-aggregated version of the TFP process, which is set at $\sigma = 0.9$. (Further details regarding the calibration of ρ and σ are outlined in a technical appendix, which is available upon request.) Table 1 depicts commonly chosen quarterly parameter values and the implied weekly parameter values.

Table 1. RBC Parameters Values

Decision Interval	Parameters							
	β	$1 - \delta$	α	η	ϕ	μ	ρ	σ
Quarterly	0.9895	0.975	0.34	0.33	1.11	0.005	0.90	0.90
Weekly	0.9992	0.9981	0.34	0.67	1.39	0.0003	0.9919	0.2618

The linearized version of the model, coupled with the calibrated parameter values, is then used

to simulate 100 artificial time series realizations. Given that the labor-augmenting technology process (z_t) is trend stationary, series such as output, consumption, investment and real wages will fluctuate about a linear trend. Accordingly, the artificial weekly data are first detrended and then the resulting series are used to estimate standard deviations, correlations, spectra and cospectra for select series. To investigate the effects of time aggregation, the weekly data are first temporally aggregated in a manner consistent with the U.S. data-collection procedures. Then the time-aggregated artificial quarterly data are detrended and used to estimate similar statistics. Lastly, actual detrended quarterly U.S. data are analyzed in a similar fashion. (A technical appendix, available upon request, provides additional details regarding the US data, detrending procedures, time-aggregation methods, and estimation of the spectra and cospectra.)

Begin by considering standard deviations and correlations in the time domain, which are shown in Table 2. The second-moment properties of the simple RBC model are well known (King and Rebelo (1999)). The model does a good job in predicting the fact that consumption is less volatile than output; that investment is more volatile than output; and that consumption, investment and hours worked are procyclical. However, the simple RBC model tends to produce too little volatility in hours worked and predicts grossly counterfactual positive correlations between average labor productivity (or equivalently real wages) and hours worked. To examine the effects of detrending and time aggregation, it is necessary to contrast the results from the weekly and time-aggregated quarterly RBC models. Begin by noticing that the relative volatility predictions are comparable both across decision intervals and detrending methods, although the differences between the weekly and quarterly RBC models are somewhat larger under first-differencing as compared to either the HP or BK filters.

Next consider the cross correlations for the weekly and time-aggregated quarterly RBC models.¹⁰ The HP and BK cross correlations are quite similar across weekly and quarterly models. Taking this together with the standard deviation evidence suggests that the interaction of time aggregation and either the HP or BK filters produces limited distortion of the basic second moments (within the time domain). The FD cross correlations, however, are both substantially different than the

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¹⁰To make the weekly cross correlations comparable to the quarterly correlations, I sample every 13th weekly cross correlation.

HP and BK correlations and different across weekly and time-aggregated quarterly RBC data. Whereas the HP- and BK-filtered weekly data display cross correlations that die out gradually over time, the FD-filtered weekly data display strong contemporaneous cross correlations and nearly no lagged (or lead) cross correlations. This is largely due to the fact that the HP and BK filters tend to smooth the cyclical component of the data relative to the FD filter. As for the effects of time aggregation and detrending, which is the primary focus of this paper, the time-aggregated quarterly cross correlations clearly do not retain the cross-correlation patterns present in weekly data. In particular, the combined use of the TA and FD filters tends to give the appearance that there is persistence in the cross correlations over the first couple of leads and lags, where none is present in the weekly data. This can be seen even more clearly in Figure 6, which depicts the 32 weekly lead and lag cross correlations for various series (as opposed to only the five sampled correlations shown in Table 2). The sharp peak in the FD cross correlations and the relatively smooth decay of the HP and BK cross correlations, when compared to the quarterly cross correlations in Table 2, confirm that time aggregation tends to distort the cross correlation pattern in FD-filtered data but not in HP- or BK-filtered data.

Figure 7 depicts detrended weekly and quarterly estimated spectra for output in the RBC and US economies. The most remarkable feature in Figure 7 is the difference between first-differenced weekly and quarterly RBC spectra. The spectra for weekly output in the RBC economy is nearly a “flat line”, implying that detrended weekly RBC output is approximately white noise. The fact that standard RBC models produce little persistence in output is a well-known result (Cogley and Nason (1995a)). The surprising result is that detrended quarterly (i.e., weekly summed) RBC output displays variation at business-cycle frequencies. This is similar in spirit to the findings of Cogley and Nason (1995b) regarding the HP filter, except for the fact that the results presented here do not rely on difference-stationary specifications. The oscillatory and gradual downward trend of the quarterly RBC spectra over $\omega \in (0, \pi/13]$ are the result of time aggregation (which tends to magnify low-frequency variation), first-differencing in quarterly time (i.e., applying $h(L) = 1 - L^{13}$ which produces the oscillations), and the aliasing of higher-frequency variation that occurs due to systematic sampling.

The other notable feature of Figure 7 is the surprising similarity in the shapes of the HP-

and BK-filtered spectra for weekly and quarterly output, both across sampling intervals and across filters. These similarities are driven by two forces. First, the spectral shape for filtered RBC output reflects the fact that it is both nearly difference stationary (i.e., has an autoregressive root very near one) and the model fails to propagate the technology shocks through time. This implies that the spectra for RBC output primarily reflect the shape of the asymmetric $S(L)$ filter that remains after first-differencing (see Cogley and Nason (1995b), page 259). Second, the similarity of the HP- and BK- filtered output is due to the fact that output does not display much high-frequency variation and therefore the high-pass HP filter and band-pass BK filter behave similarly. As Baxter and King (1999) mention, this is not true for series such as inflation which display substantially more high-frequency variation. Therefore, according to the criteria that detrending filters do not distort basic spectra when applied to time-aggregated data, these results provide strong evidence in favor of using either the HP or BK filters over first-differencing.

The conclusions are much the same for the cospectra. Figure 8 shows the cospectra between detrended average labor productivity and hours worked in the RBC and US economies. First notice that the primary peak in the cospectra for the US data is below zero, which reflects the negative correlation between the two series, while the peaks in the RBC cospectra are above zero, reflecting the strong positive correlations predicted by the model. This mismatch between theory and observation persists across detrending methods and time aggregation. More importantly, however, notice that the shapes of the weekly and quarterly RBC cospectra are much closer for HP- and BK-filtered data as opposed to first-differenced data. Once again, this provides support for the use of either the HP or BK filters rather than the FD filter when detrending time-aggregated data.

4 Conclusion

This paper attempts to shed light on an important practical problem for business-cycle researchers: “Is time aggregation an important ingredient in the decision of how to detrend macro data?” The answer appears to be a resounding “yes.” In fact, the analysis in this paper suggests an ordinal ranking in terms of the appropriateness of detrending methods given that the criteria are (1) does

the detrending filter (when applied to time-aggregated data) retain the shape of the spectra and cospectra in basic time? and (2) how easy is it to apply the filter across sampling intervals? Of course, there are other criteria for choosing a detrending method such as how one wishes to define business-cycle fluctuations or how robust the detrending method is to the nature of the nonstationarity. That said, according to criteria (1) and (2) and working from the bottom up, the FD filter appears to be the least attractive. While it is straightforward to apply the FD filter across sampling intervals, FD filtering of time-aggregated data greatly distorts the basic spectra and cospectra. This distortion is both an aliasing problem and a direct problem of the FD filter in that it tends to magnify cyclical variation at high frequencies. According to these criteria, the second-best option is the HP filter. It provides limited distortion of the basic series and moderate amounts of aliasing of higher-frequency variation. However, it is not immediately clear how the value of the smoothing parameter should change across sampling intervals. The accepted value for λ in quarterly data is 1600, but there is no widely accepted value for λ in, say, monthly or annual data. Overall, the BK filter appears to best satisfy criteria (1) and (2). When applied to time-aggregated data, the BK filter results in minimal aliasing and retains the shape of the basic spectra and cospectra. In addition, it is easy to apply the BK filter to data at different sampling intervals since it is an approximate band-pass filter, and the passed-through frequencies are naturally adjusted across sampling intervals.

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5 Technical Appendix

This appendix describes the technical details for detrending time-aggregated data in the U.S. and RBC economies. The intent is to be sufficiently thorough that an individual could readily replicate the results of Section 3.

5.1 U.S. Data Definitions and Sources

The data are taken from FRED, the Federal Reserve Economic Database of the St. Louis Federal Reserve Bank and span the first quarter of 1950 through the fourth quarter of 1999. The series selected are as follows, with FRED mnemonic in parentheses: output is measured as real gross domestic product (GDPC96); consumption is measured as real personal consumption expenditures of all goods and services (PCECC96) less consumption expenditures on durable goods (PCEDC96); investment is measured as the sum of real fixed private investment (FPIC1), real public gross investment (DGIC96 + NDGIC96 + SLINVC96), and real personal consumption expenditures on durable goods (PCDGCC96); total hours are measured as the product of average weekly hours in private nonagricultural establishments (AWHNOG) and total nonfarm payroll employees (PAYEMS); and lastly, average productivity is calculated by dividing output by total hours. All the variables have been seasonally adjusted at the source; output, private consumption, investment and government consumption are measured in chain-weighted 1996 dollars; and all but average productivity have been transformed into per-capita terms using the entire civilian, non-institutionalized population that is sixteen and over (CNP16OV).

5.2 Calibrating Weekly ρ and σ

Begin by assuming that total factor productivity (denoted in logarithms) follows a first-order autoregressive process in weekly time

$$A_t = \rho_* A_{t-1} + \epsilon_t, \quad (29)$$

where ϵ_t is a mean-zero, white-noise process with standard deviation σ_* . Repeated substitutions for A on the right-hand-side of (29) produces

$$A_t = \rho_*^{13} A_{t-13} + \sum_{s=0}^{12} \rho_*^s \epsilon_{t-s} = \rho A_{t-13} + \nu_t. \quad (30)$$

Treating (30) as the quarterly process for total factor productivity, then we have a straightforward mapping from the quarterly parameters to the weekly parameters. Given a value for ρ , then the weekly autoregressive coefficient is given by $\rho_* = \rho^{1/13}$. Furthermore, given a measure of the standard deviation of ν_t , σ , then the weekly standard deviation is given by

$$\sigma_* = \sqrt{\sigma^2 \frac{(1 - (\rho_*^2)^{13})}{(1 - \rho_*^2)}}.$$

5.3 Time-Aggregation Procedures

It is important that the time aggregation procedures for the RBC data match the U.S. sampling and aggregation procedures. Series taken from the National Income and Product Accounts (NIPA)—output, consumption, and investment—are collected using a wide range of sources and at varying points in the quarter; see the BEA's website (<http://www.bea.doc.gov/bea/mp.htm>) for more details. The artificial data for these series were aggregated using the temporal aggregation operator, $h_{TS}(L) = 1 + L^1 + \dots + L^{12}$. Series taken from the Bureau of Labor Statistics (BLS)—average weekly hours, employment, and the population—are collected from the household and establishment surveys. The following excerpt is taken from the BLS website (<http://stats.bls.gov:80/>) regarding their sampling methods:

For both surveys, the data for a given month relate to a particular week or pay period. In the household survey, the reference week is generally the calendar week that contains the 12th day of the month. In the establishment survey, the reference period is the pay period including the 12th, which may or may not correspond directly to the calendar week.

In accordance with BLS techniques, average quarterly hours worked and population data generated from the model were thus treated as being systematically sampled from the second week of the month and then averaged over the three months in the quarter, that is, $h_{SS}(L) = (L^3 + L^7 + L^{11})/3$.

5.4 Empirical Detrending Procedures

Before detrending, all U.S. variables were transformed into natural logarithms. The variables within the RBC model are measured as log deviations from their steady state. Therefore, the artificial RBC variables were made comparable to their U.S. counterparts by adding the logged steady-states to the proportional deviations from the steady-state values. While the FD filters for weekly and quarterly data are straightforward to apply (i.e., $h(L) = 1 - L$ and $h(L) = 1 - L^{13}$ respectively), the ideal properties of the HP and BK filters require an infinitely large sample. To apply the HP filter in finite samples, I use the Regression Analysis of Time Series (RATS) procedure *hpfilter.src* with $\lambda = 1600$ for quarterly data and $\lambda = 1600 * 13^4$ for weekly data, as suggested by Ravn and Uhlig (1997). For the BK filter, I apply the RATS procedure *bkfilter.src* available at <http://www.estima.com/procindx.htm>. For quarterly data, the BK filter parameters are set at upper = 6, lower = 32, arpad = 4, and nma = 12. For weekly data, all four parameters are set at 13 times their quarterly values.

5.5 Estimation of Spectra and Cross Spectra

The spectra for the U.S. and RBC artificial data are estimated using the modified Bartlett kernel (Hamilton (1994), pp. 330-332):

$$s_x(\omega) = \frac{1}{2\pi} \left(\hat{\gamma}_0 + 2 \sum_{j=1}^h \left(1 - \frac{j}{h+1}\right) \hat{\gamma}_j \cos(j\omega) \right),$$

where $h = 16$ for quarterly data and $h = 16 \times 13 = 208$ for weekly data and $\hat{\gamma}_s$ indicates the s -lag autocorrelation coefficient for x_t . The cross spectra are estimated similarly using

$$s_{xy}(\omega) = \frac{1}{2\pi} \left(\sum_{j=-h}^h \left(1 - \frac{|j|}{h+1}\right) \hat{\gamma}_{xy}(j) \cos(j\omega) \right),$$

where $\hat{\gamma}_{xy}(s)$ indicates the correlation between x_t and y_{t+s} .

Table 2. Select Second-Moment Statistics for the US and RBC Economies

Standard Deviations Relative to y	US Data			Weekly RBC Model			Time Aggregated Quarterly RBC Model			
	FD Filter	HP Filter	BK Filter	FD Filter	HP Filter	BK Filter	FD Filter	HP Filter	BK Filter	
std(c)	0.662	0.605	0.581	0.340	0.404	0.422	0.385	0.410	0.424	
std(dk)	2.366	2.359	2.238	2.973	2.894	2.867	2.968	2.894	2.865	
std(n)	1.050	1.096	1.125	0.496	0.481	0.475	0.464	0.474	0.473	
std(y/n)	0.727	0.548	0.523	0.505	0.543	0.555	0.531	0.547	0.557	
Correlations	Lags	FD Filter	HP Filter	BK Filter	FD Filter*	HP Filter*	BK Filter*	FD Filter	HP Filter	BK Filter
corr(c,y)	-2	0.257	0.648	0.676	-0.002	0.297	0.444	-0.038	0.329	0.450
	-1	0.388	0.783	0.831	-0.003	0.580	0.738	0.198	0.642	0.739
	0	0.585	0.815	0.859	0.996	0.933	0.921	0.943	0.928	0.919
	+1	0.271	0.642	0.707	0.007	0.760	0.929	0.347	0.833	0.929
	+2	0.091	0.455	0.469	0.007	0.591	0.786	0.111	0.648	0.791
corr(dk,y)	-2	0.217	0.593	0.659	-0.001	0.479	0.694	-0.018	0.532	0.699
	-1	0.339	0.759	0.824	-0.001	0.713	0.923	0.234	0.794	0.923
	0	0.724	0.851	0.865	0.999	0.989	0.985	0.978	0.984	0.984
	+1	0.328	0.701	0.738	-0.004	0.639	0.840	0.178	0.716	0.841
	+2	0.089	0.507	0.511	-0.004	0.357	0.544	-0.075	0.400	0.550
corr(n,y)	-2	0.109	0.365	0.445	-0.001	0.496	0.714	-0.018	0.510	0.659
	-1	0.432	0.656	0.725	-0.001	0.717	0.925	0.138	0.751	0.891
	0	0.750	0.868	0.885	0.999	0.974	0.966	0.914	0.967	0.972
	+1	0.482	0.811	0.842	-0.006	0.605	0.799	0.400	0.772	0.852
	+2	0.259	0.653	0.672	-0.005	0.311	0.487	-0.099	0.425	0.571
corr(y/n,n)	-2	0.019	0.051	-0.072	-0.007	0.208	0.346	-0.136	0.310	0.431
	-1	-0.074	-0.185	-0.295	-0.009	0.515	0.675	0.348	0.674	0.735
	0	-0.407	-0.423	-0.457	0.996	0.909	0.883	0.885	0.911	0.901
	+1	-0.143	-0.467	-0.525	0.001	0.721	0.903	0.179	0.764	0.882
	+2	-0.188	-0.474	-0.510	0.002	0.548	0.755	0.029	0.574	0.712

Notes: The variables c, dk, n, and y/n refer to consumption, investment, labor hours and average labor productivity, respectively. Std(x) refers to the standard deviation of detrended x relative to the standard deviation in detrended y. Corr(x,z) refers to the cross correlation between detrended x and detrended z. Lag j refers to the correlation between contemporaneous x and z lagged j periods.

*The reported cross correlations refer to every 13th weekly lagged cross correlation. That is, the cross correlations for lags=-2,-1,0,+1,+2 correspond to weekly lags=-26,-13,0,+13,+26.

Figure 1. Temporal Aggregation Transfer Functions

Aliased frequencies are shaded, Vertical lines denote folding points

Figure 2. Detrending Transfer Functions

Figure 3. Cascaded Transfer Functions ($n = 3$)

Aliased frequencies are shaded, Vertical lines denote fold points

Figure 4. Detrended Basic and Time Aggregated Spectra (n = 4)

Figure 5. Cascaded TA-SS Cospectra Transfer Functions ($n = 3$)

Aliased frequencies are shaded, Vertical lines denote fold points

Figure 6. Weekly Lagged Cross Correlations for RBC Model

Figure 7. Spectra for Detrended RBC and US Output

Figure 8. Cospectra for Detrended Wages and Hours Worked

Detrending Time-Aggregated Data

David Aadland*

April 2002

Abstract

This paper examines the combined influences of detrending and time aggregation on the measurement of business cycles. The approximate band-pass filter of Baxter and King (1999) performs relatively well in the sense that it retains the basic shape of disaggregate spectra and cospectra when applied to time aggregated data and is straightforward to apply across sampling intervals. Simulation of a simple weekly RBC model confirms the theoretical results.

JEL Codes: C1 and E3.

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*The author is an assistant professor in the Department of Economics, Utah State University, 3530 Old Main Hill, Logan, UT, 84322-3530, 435-797-2322 (phone), 435-797-2701 (fax), aadland@econ.usu.edu.

1 Introduction

One of the most challenging and controversial aspects of business-cycle research is attempting to disentangle the cyclical and growth components of macro time series. Kuznets' work in the 1960's is a classic example of how seemingly innocuous data transformations can lead to spurious results, such as his discovery of 20-year long swings in economic activity (Adelman (1965); Howrey (1968)). Another example is provided by the influential work of Nelson and Kang (1981), who show that the residuals from a regression of a random walk on time display spurious periodicity. Although researchers, in part due to the aforementioned studies, are now more aware of the possible pitfalls of detrending data, the debate over which method is "best" continues. Some of the more commonly used detrending methods include: regression on polynomials in time, the first difference (FD) filter, the Hodrick-Prescott (HP) filter, and the Baxter-King (BK) approximate band-pass filter, to mention a few.

The HP filter introduced by Hodrick and Prescott (1980) has, in particular, received considerable attention. This flexible detrending method trades off deviations from trend against an adjustable smoothness criterion. The HP filter has been criticized for generating spurious cycles in difference stationary data (Harvey and Jaeger (1993); Cogley and Nason (1995b)); altering the persistence, variability and co-movement of time series (King and Rebelo (1993)); and (similar to the FD filter) for passing through too much high-frequency or "irregular" variation. Nevertheless, the HP filter has withstood two decades of use and remains a leading method for detrending time series data. The BK filter is, however, gaining in popularity (e.g., see Stock and Watson (1999)) and is more consistent with the Burns and Mitchell (1946) definition of the business cycle as containing cycles that last between six and 32 quarters.

In addition to filtering out a trend, most macro time series are also implicitly passed through a time-aggregation (TA) filter. For institutional reasons, business-cycle researchers work almost exclusively with quarterly data. Since data-generating processes for economic data are generally thought to operate at continuous or very short discrete-time intervals, observed quarterly data are therefore some sort of transformation of the high-frequency data. If the variable is a flow, then it is typically summed (or averaged) over shorter time intervals. If it is a stock, then it is