Interactive Scenario Analysis of Exhaustible Resource Problems

Arthur J. Caplan
Associate Professor of Economics
Department of Economics
Utah State University
3530 Old Main Hill
Logan, Utah 84322-3530
(435) 797-0775
acaplan@econ.usu.edu

John Gilbert
Associate Professor of Economics
Department of Economics
Utah State University
3530 Old Main Hill
Logan, Utah 84322-3530
(435) 797-2314
jgilbert@econ.usu.edu

Abstract: We provide several interactive models that can be used in an intermediate- or graduate-level natural-resource economics course to numerically solve a host of exhaustible-resource problems, and thereby help to verify the intuition and symbolic solutions typically provided in textbooks. Examples are drawn from Tietenberg (2006).

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1. Introduction

Based on our experience, students often feel intimidated by the leap made from introductory to intermediate level economics courses and beyond. Their sense of intimidation is often driven by a reliance on quantitative techniques in their intermediate and graduate courses (e.g., algebra and calculus) to solve key economic problems previously presented (in graphical format) in the introductory courses. Many times the dimension of an economic problem can broaden to such a degree that the quantitative leap imposed upon the student ultimately becomes a leap of faith. This paper addresses a particular type of 'credibility leap' that students are required to make in natural-resource economics courses due to constraints imposed by the dimensionality of the textbook itself. The constraints emerge in the context of a multi-period exhaustible-resource problem, inducing the textbook author to require of his readers acceptance of the computer-generated numerical solutions provided throughout the text.

To help break these constraints, we offer an interactive analysis of exhaustible resource problems using the Solver add-in in Excel, which is programmed through a

series of macros to facilitate the exploration of a variety of scenarios both graphically and numerically. To help narrow the resulting gaps between the extraction problems' dimensions and solutions, we provide interactive examples that allow the student to verify the numerical solutions provided in the textbook, in addition to considering numerous alternative scenarios.[1] As a result, the student is able to make the leap with greater insight, rather than on faith alone.

The examples we present in this paper are based on various resource extraction problems contained in Tietenberg (2006), a leading textbook for intermediate-level natural-resource and environmental-economics courses. Our motivation for developing these examples is perhaps best captured by the following statement:

"Practically speaking, solving these equations to find the optimal solution is not a trivial matter, but neither is it very difficult… As an exercise, those interested in computer programming might construct a program to reproduce these results" (Tietenberg, 2006, page 147).

This statement follows a presentation of the results for a basic T-period exhaustible-resource extraction problem. The results include (a) the efficient extraction path of a finite resource (until the resource is driven to exhaustion); (b) the associated time path of per-unit prices; and (c) the marginal user cost, which reflects the scarcity-value of an additional unit of the exhausted resource. These results later serve as a point of comparison for the various extensions to the basic decision problem presented throughout

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[1] Each of the specific examples discussed below in Sections 2 – 4, as well as several additional examples, are available from the authors upon request.

the remainder of the chapter – e.g., the availability of a renewable substitute resource and the case of increasing marginal extraction cost (MEC). Although the student is provided with a basis for comparing the numerical outcomes of the various extraction problems presented in the textbook, he is precluded from understanding how the basis itself is derived. Furthermore, the student is not provided with the means to experiment with self- or instructor-devised alternative extensions to the basic model.[2]

In the next section, we present the basic T-period exhaustible-resource extraction problem (e.g., the mining of ores, oil, or minerals) and discuss the two main extensions provided in the textbook – the availability of a renewable substitute resource and increasing MEC. This section demonstrates the way in which the Solver add-in can be used to interactively solve a variety of exhaustible-resource problems. Section 3 discusses the methods used to incorporate this type of interactive analysis into course pedagogy, as well as outcomes from both the students' and the instructor's perspectives. Section 4 concludes.

2. Exhaustible-Resource Extraction Problems

As shown in Tietenberg, the basic (dynamically efficient) T-period exhaustible-resource extraction problem can be written as:

$$\underset{\{q_t\}}{Max} \ \sum_{t=1}^{T}\left(\frac{TB_t - TC_t}{(1+r)^{t-1}}\right) \text{ subject to } TB_t = \int_{s_t=0}^{q_t} p_t(s_t)\,ds, \ TC_t = cq_t, \text{ and } Q = \sum_{t=1}^{T} q_t \quad (1)$$

where q_t is the amount of the resource extracted in period t, $p_t(q_t)$ is society's (stationary) inverse demand function, TB_t is the associated social benefit (i.e., area beneath society's inverse demand curve) based on q_t, TC_t is the social cost of extraction, c is a constant MEC, r is the discount rate, Q is the aggregate amount of the finite resource, and T is the length of the planning horizon.

Initially, the model's parameter set $\{c, r, Q\}$ is ascribed pre-determined numerical values, $p_t(q_t)$ is given a simple linear form, and T is set equal to 2. Students are then re-taught the integral-calculus procedure necessary to solve $\int_{s_t=0}^{q_t} p_t(s_t)ds$, which in turn enables the instructor to introduce the Lagrangian-multiplier method as a solution technique for the simple two-period dynamic extraction problem posed in expression (1). Lastly, first-order conditions are derived and, along with the given inverse demand function $p_t(q_t)$, are simultaneously solved for the efficient extraction path (q_t^*, $t = 1,2$), the corresponding price path (p_t, $t = 1,2$), and the corresponding (present value) marginal user cost, λ, (which is the problem's Lagrangian multiplier).

Tietenberg (2006) provides a considerable amount of intuition for this basic two-period model and then moves the student through a series of extensions, beginning with an increase in T from 2 to what is effectively an open-ended number of years (henceforth the "baseline model"). It is here that the student is asked to make her first leap of faith. A numerical solution for this model – provided in Table 1 below – is presented in the textbook and certain features of the solution are discussed, in particular the fact that the amount extracted is smoothly decreasing over time until the resource is completely exhausted in period $t = 9$.

[INSERT TABLE 1 HERE]

Of course the natural question on the tips of the students' tongues at this point in the lesson is something like, "Is there anyway to verify this solution?" To help answer this question we provide the students with the interactive scenario analysis presented in Figures 1A and 1B. This is a version of the problem built in Excel.

Although several implementations of economic models in Excel have been presented in the literature (particularly at the principles level), this model has several innovative features that are worth briefly outlining. As with other models, we take full advantage of Excel's ability to present data in both tabular and graphical form. In terms of design, we present the model on two sheets, the first representing a baseline and the second the experiment. This enables an easy comparison between any two scenarios. We also use 'forms' to allow the user to easily interact with the sheet using a mouse (making use in the classroom significantly easier). Because we want the model to be able to easily and transparently handle multiple scenarios, we have implemented a macro to handle model selection. Using checkboxes, the user can select an extension, and the macro handles the job of adapting the extraction problem within Solver.[3]

We first consider the baseline case. Begin by noting in Figure 1A that the parameter values chosen in lines 1 – 7 for the experiment correspond exactly to the values for the baseline model (see also the footnote to Table 1). Values for lines 4 and 5, as well as for the model extensions in lines 8 – 12 are for the moment "turned off".[4]

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[3] Further details on using the Solver macro are provided in an online How-To manual (also available upon request from the authors).

[4] To "turn on" the Recycling Model (lines 8 -11), the "Activate Model Extensions" box would need to be checked and the Recycling Model option button would need to be selected. Similarly, to turn on the Substitute Renewable Resource Model (line 12), the Activate Model Extensions box needs to be checked along with the corresponding option button. Since the recycling and substitute resource models are distinct

[INSERT FIGURES 1A AND 1B HERE]

Clicking the "Solve Model" button launches Solver and the results are presented in graphs in Figure 1B for q_t and p_t, which correspond exactly to the values provided in Table 1. In particular, we note the smooth path of extraction to exhaustion and the corresponding negative relationship between q_t and p_t. We also obtain in the Summary Outcomes section of the experiment column (lines 13 − 18 in Figure 1A) the associated values for the total amount extracted, total net benefit, and marginal user cost, each of which correspond to the solution provided in the textbook.

The process of inputting the decision problem into Excel and subsequently being able to verify the numerical solution provided in the textbook narrows the credibility gap (discussed in Section 1) for the students. In the process, the students learn how easy it is to access the Solver commands used to solve for this baseline result as well as how easy it is to effect changes in the parameter values in lines 1 − 7 of Figure 1A and to track the corresponding changes in results.[5]

The approach also allows us to extend the model in various ways. The first extension to the baseline model is increasing MEC. Tietenberg (2006) chooses a simple functional form based on the aggregate of past extraction levels to represent the way in which the MEC increases over time:

$$c_t = \begin{cases} 2 & \text{for } t = 1 \\ 2 + 0.1\left(\sum_{t=1}^{t-1} q_t\right) & \text{for } t > 1 \end{cases} \qquad (2)$$

where now the student is instructed to recognize that equation (2) is substituted into (1) for parameter *c*, and that the MEC is now time-dependent and increasing over time for any $q_t > 0, t > 1$. The student also naturally recognizes how complicated the extraction problem has become relative to the initial two-period model, irrespective of the fact that a computer-generated numerical solution to this problem is again provided in the text.

The graphical results for this extension are provided in Figure 2. To obtain these results, the parameter value for "Step Increase as Fraction of Extraction" in line 5 of the experiment column in Figure 1A has been increased from the baseline of zero to 0.1 in order to correspond with the step increase in (2) for $t > 1$. Note how the graphs retain the baseline result as a point of reference for the new price, MEC, and extraction paths. In this case, the step increases in MEC induce a reduction in the extraction rate in the early periods (as a response to the step increase in marginal cost) and thus lengthen the extraction path from the baseline of nine years to 12 years. The associated marginal user cost and net benefit decrease.

[INSERT FIGURE 2 HERE]

As an alternative to the step-increase functional form of (2), suppose we instead use a simpler form to capture the effect of time on MEC:

$$c_t = 2 + 0.1t \quad for\ t > 0 \tag{2'}$$

i.e., MEC increases at a constant rate over time, rather than as a function of the aggregate of past extraction levels as in (2).[6] The graphical results for this extension are presented in Figure 3. To obtain these results, the parameter value in line 5 of the experiment column in Figure 1A has been reset to zero and the value for the "Step Increase in *c* Each

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[6] Although Tietenberg (2006) does not provide this particular example, its simplified functional form may be more effective in introducing the student to the notion of increasing MEC.

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Period" in line 4 has been increased from the baseline of zero to 0.1 in order to correspond with the step increase in (2') for $t > 0$.

[INSERT FIGURE 3 HERE]

Comparing Figures 2 and 3, the students see that it is now efficient to extract more of the resource in the earlier periods, which induces complete exhaustion of the resource to occur in an earlier period (period nine as opposed to period 12) as well as a more abrupt transition path toward complete exhaustion. This result is counter to the general implication of increasing MEC as discussed in Tietenberg (2006). This is because the only form of increasing MEC discussed in Tietenberg (2006) is represented by (2). As a result, students are led to believe that as a general rule increasing MEC provides incentive to extract *less* of the resource in earlier periods, not more. Our interactive scenario presented in Figure 3 therefore serves as a caveat for the students.[7]

Our final two extensions to the baseline model are the inclusion of (i) a renewable resource that can be perfectly substituted for the exhaustible resource and (ii) disposal costs and a recycling option. Both of these extensions can be "activated" in lines $8-12$ of Figure 1A as experiments with their own corresponding baseline parameter values. Due to space restrictions, we focus our discussion on extension (i).

This extension provokes a few changes to the definitions of TB_t and TC_t in expression (1) and also adds a new set of first-order conditions to account for the renewable substitute's optimal time path. In particular, $TB_t = \int\limits_{\left(s_t^x + s_t^s\right)=0}^{\left(q_t^x + q_t^s\right)} p_t\left(s_t^x + s_t^s\right) d\left(s^x + s^s\right)$ and

$TC_t = cq_t^x + dq_t^s$, where q_t^x now represents the amount extracted of the exhaustible-

resource and q_t^s represents the amount used of the renewable substitute in period t. The parameter d is the constant marginal cost associated with using additional q_t^s.[8]

Similar to the numerical results provided for the baseline extraction problem in Table 1, Tietenberg (2006) also provides the students with the numerical results for this renewable-substitute extension (not shown here), and the intuition for why the results differ. In particular, with a renewable substitute the efficient extraction path for the exhaustible resource is typified by more of the resource being extracted sooner, leading to complete exhaustion of the resource sooner than in the case without the renewable substitute. Figure 4 provides the graphical results for this extension.

[INSERT FIGURE 4 HERE]

To obtain these results, the parameter values in the experiment column of the basic model (lines 1 – 7 in Figure 1A) are identical to those for the baseline model. The "Activate Model Extensions" box has been checked and the "Substitute Renewable Resource Model" option button has been selected, thus "turning on" extension (i). After clicking the "Solve Model" button, graphs for the renewable-resource and baseline models appear, along with summary outcomes for the former model. Comparing the two models leads to a verification of the numerical results presented in the textbook. In the presence of a substitute renewable resource, more of the exhaustible resource is used sooner, leading to a more abrupt (and early) transition to complete exhaustion in Figure 4. Additional information included in the Summary Outcomes (lines 13 – 18 in the experiment column of Figure 1A) that is not provided in the textbook is the associated

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[8] Note that q_t^x and q_t^s share the same price (p_t) because of the perfect-substitutability assumption.

aggregate net benefit, which at $173.77 is larger than that obtained without the renewable substitute.

3. Incorporating Interactive Scenario Analysis into Course Pedagogy

The interactive scenarios presented in Section 2 have been incorporated into our course pedagogy for the past two years, and have generated several insights into their use for both in-class teaching and as evaluative tools of student performance. These efforts are similar to those made by Mercado, et al. (1998) with GAMS as well as by Naevdal (2003), Craft (2003), Mixon and Robson (2001), Fisher (2001), and Cahill and Kosicki (2000). The interactive scenarios have been presented real-time to the students as an in-class teaching tool. This has enabled impromptu modifications to the various parameter values of the different models and close observation of how corresponding outcomes change relative to the baseline model. Student feedback on the methods used to incorporate the interactive scenarios has been positive. Students appreciate the in-class demonstrations to verify the numerical solutions presented in the textbook, and view them as a helpful reinforcement technique.

The interactive scenarios have also been incorporated into homework assignments (e.g., students are required to re-work some of the scenarios with new parameter values, etc., and to turn in their computer disks for grading). Questions based on these scenarios have significantly broadened the scope of the assignments in terms of the problems that can be independently investigated by the students themselves. For example, rather than requiring them to repeatedly work out the tedious calculus for a series of two-period extraction problems, the interactive scenarios enable the students to obtain results for a

wide variety of T-period problems (including the substitute renewable resource and recycling model extensions) and to focus more attention on the meaning of the results themselves. This, in turn, helps build their intuitive reasoning skills to a much greater extent than repeated applications of the two-period model.

A common critique of simulation models such as the one presented here is that they are 'black boxes'. We believe this criticism is unfounded. A simulation model need be no more of a black box than an algebraic model, since in both cases the relations of the model must be explicitly documented (although the form may differ slightly). So long as the relations of the model are adequately described, and related back to the relations in the simpler two-period model which is taught using traditional methods prior to introducing the simulation, we have found that students understand the simulation model well. Simulation then offers the considerable advantages that have been discussed above.

4. Conclusion

In this paper we have provided a new interactive scenario analysis of exhaustible resource problems that can be used in intermediate- and graduate-level natural-resource economics courses to numerically solve a host of extraction problems and thereby help verify the intuition and symbolic solutions provided in the textbook. The specific textbook upon which the scenarios presented in this paper are based is Tietenberg (2006). However, other natural-resource economics texts at the intermediate and graduate levels might similarly be supplemented with these numerical models. The overarching goal of incorporating these models into the course curriculum (as both in-class teaching and student-evaluative tools) is to enable the student to make a more informed leap-of-faith

between the symbolic and numerical results presented in the textbook, on the one hand, and the intuition on the other. This is particularly relevant for the higher-dimension exhaustible resource problems encountered by students at more advanced levels.

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Mercado, Ruben P., David A. Kendrick, and Hans Amman (1998) "Teaching Macroeconomics with GAMS," *Computational Economics*, 12(2): 125-149.

Mixon, Wilson J. Jr. and Gary S. Robson (2001) "A Microsoft Excel-Based Aid for Learning Cost Estimation," *Journal of Private Enterprise*, 17(1): 143-146.

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Table 1. Numerical Solution for the Basic T-Period Extraction Problem.[*]

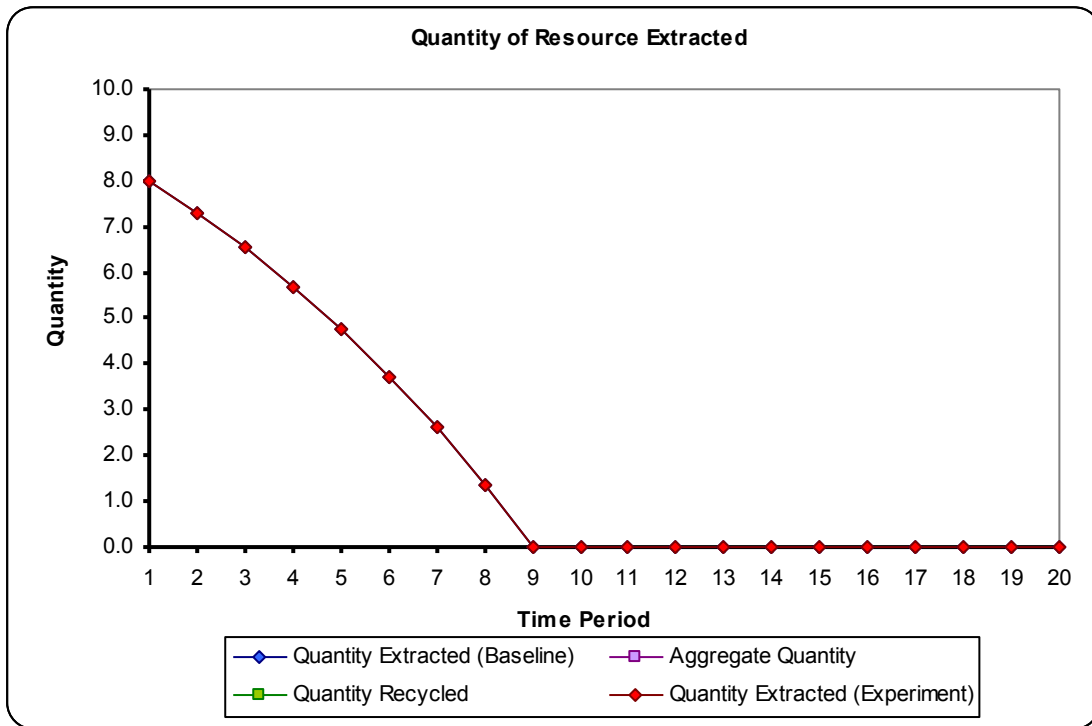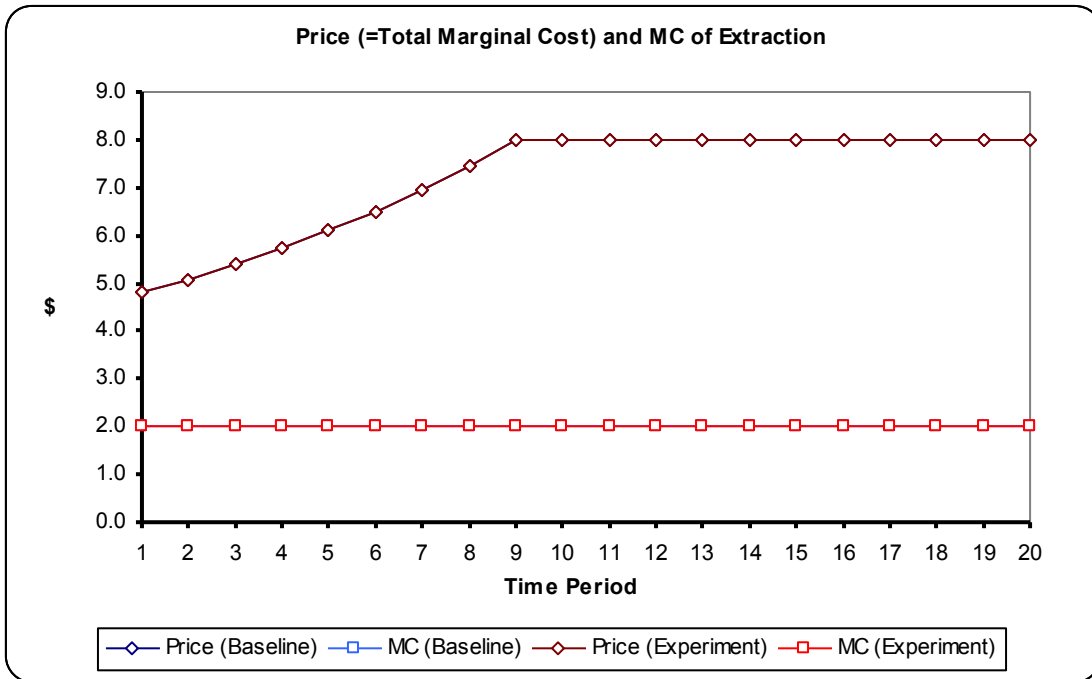| Variable	Value
q_1	8.004
q_2	7.305
q_3	6.535
q_4	5.689
q_5	4.758
q_6	3.733
q_7	2.607
q_8	1.368
$q_t \geq 9$	0.000
λ	2.798

[*]$p_t(q_t) = a - bq_t$, where a = \$8 and b = 0.4;
c = \$2; Q = 40; and r = 0.10.

Figure 1A. Experimental Interface

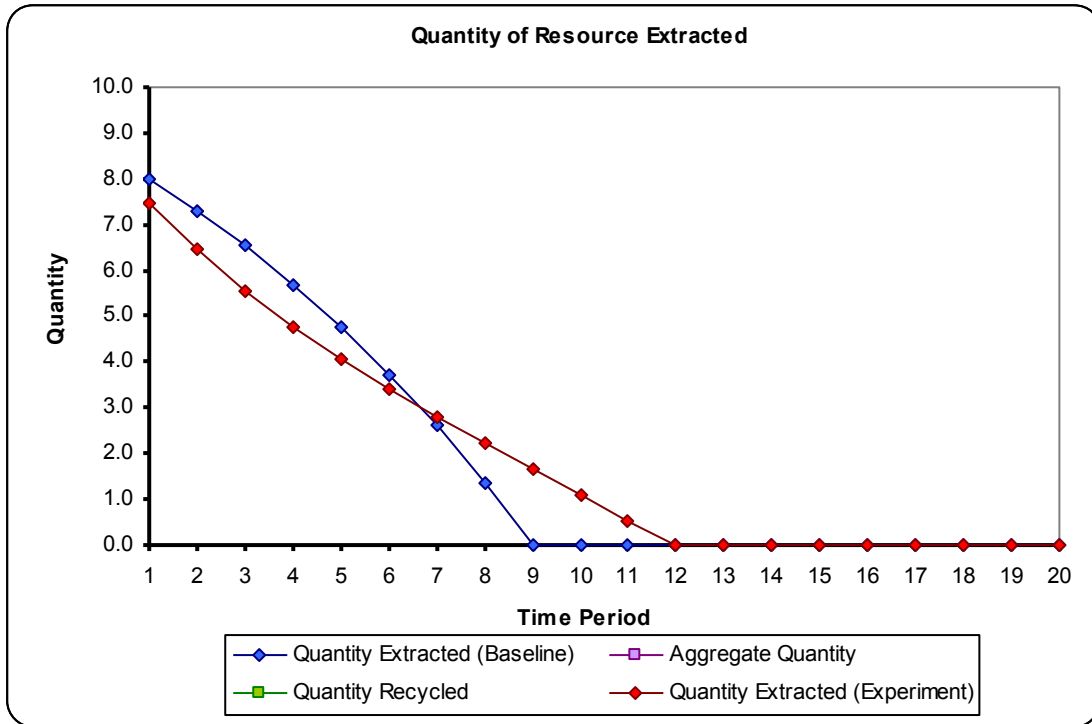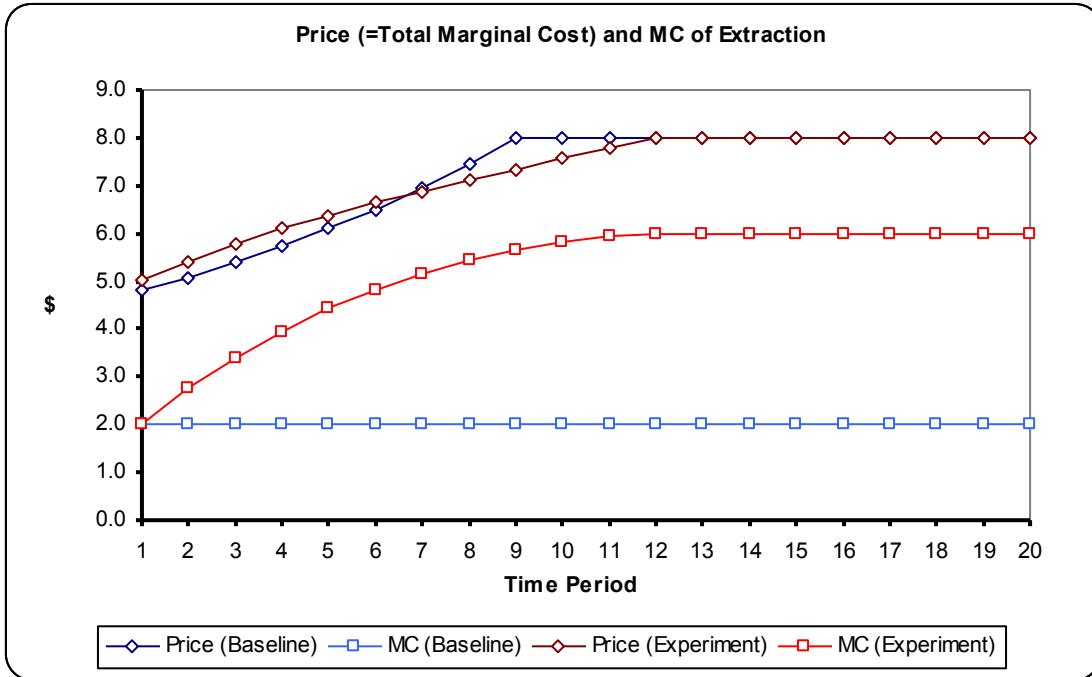| Depletable Resource Extraction Examples - 20 Periods (Experiment)		

Basic Model (N-period, Constant Cost Case)	Baseline	Experiment
Vertical Intercept of Demand Curve (a)	8.00	8.00
Slope of Demand Curve (b)	0.40	0.40
Initial Marginal Cost of Extraction (c)	2.00	2.00
Step Increase in c Each Period	0.00	0.00
Step Increase as Fraction of Extraction	0.00	0.00
Interest Rate (r)	10.0%	10.0%
Resource Quantity Constraint (Q)	40.00	40.00

☐ Activate Model Extensions

○ Substitute Renewable Resource Model		
(Constant) MC of Substitute (d)	6.00	6.00

◉ Recycling Model (Chapter 9)		
Disposal Cost (f)	0.50	0.50
Disposal Cost Avoided Due to Recycling (f)	0.50	0.50
Extraction Cost Avoided Due to Recycling (k)	0.05	0.05
(Constant) Marginal Cost of Recycling (g)	0.40	0.40

Solve Model

Summary Outcomes		
Total Extraction	40.00	40.00
Total Benefit	272.46	272.46
Total Cost	80.00	80.00
Total Net Benefit	192.46	192.46
Present Value of Total Net Benefit	152.15	152.15
Marginal User cost at T=20	2.80	2.80

Figure 1B. Baseline Outcome

Price (=Total Marginal Cost) and MC of Extraction

Legend: Price (Baseline), MC (Baseline), Price (Experiment), MC (Experiment)

Quantity of Resource Extracted

Legend: Quantity Extracted (Baseline), Aggregate Quantity, Quantity Recycled, Quantity Extracted (Experiment)

Figure 2. Step Increase as Fraction of Extraction

Price (=Total Marginal Cost) and MC of Extraction

Legend: Price (Baseline) — MC (Baseline) — Price (Experiment) — MC (Experiment)

Quantity of Resource Extracted

Legend: Quantity Extracted (Baseline) — Aggregate Quantity — Quantity Recycled — Quantity Extracted (Experiment)

Figure 3. Step Increase in Marginal Cost Each Period

Price (=Total Marginal Cost) and MC of Extraction

Legend: Price (Baseline), MC (Baseline), Price (Experiment), MC (Experiment)

Quantity of Resource Extracted

Legend: Quantity Extracted (Baseline), Aggregate Quantity, Quantity Recycled, Quantity Extracted (Experiment)
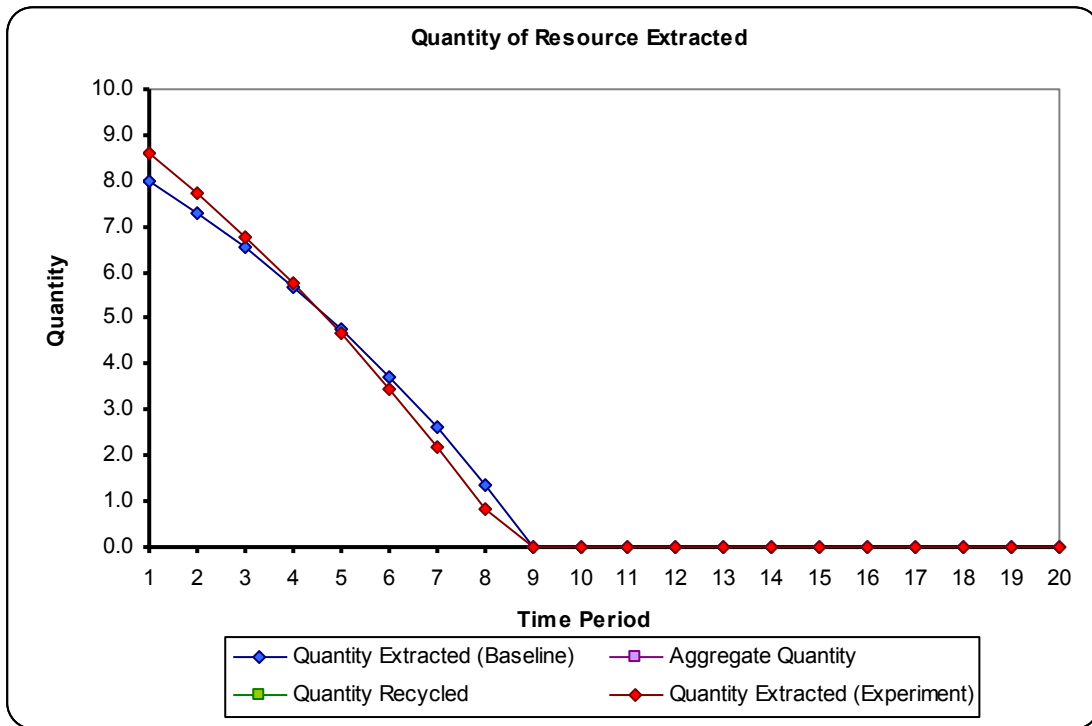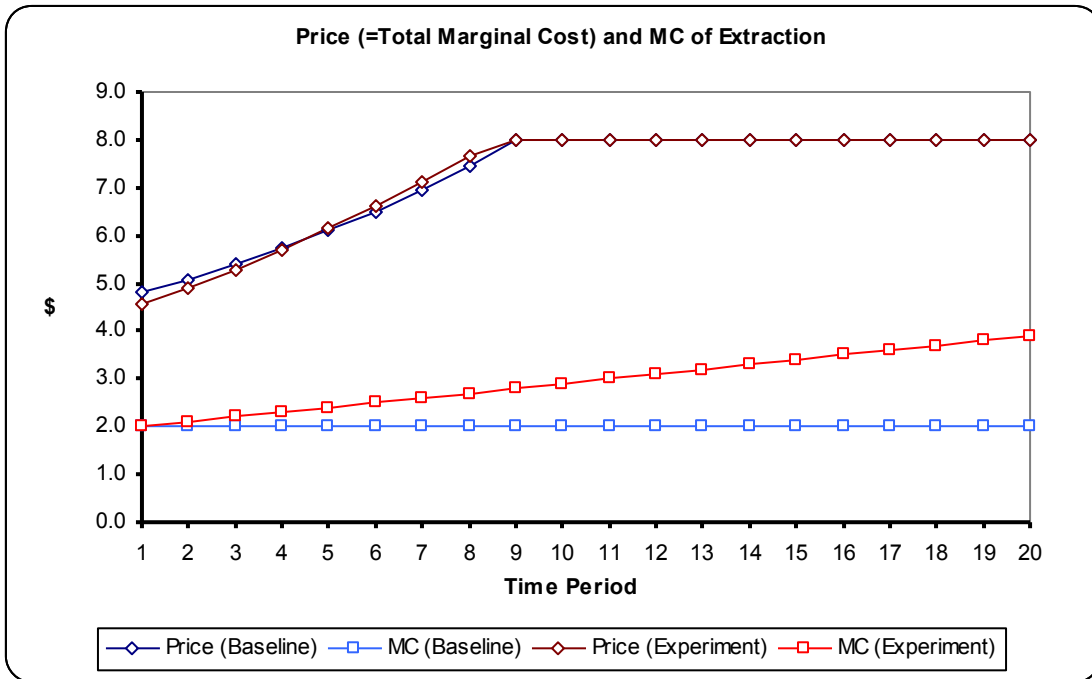
Figure 4: Substitute Renewable Resource

Price (=Total Marginal Cost) and MC of Extraction

Legend: Price (Baseline) — MC (Baseline) — Price (Experiment) — MC (Experiment)

Quantity of Resource Extracted

Legend: Quantity Extracted (Baseline) — Aggregate Quantity — Quantity Recycled — Quantity Extracted (Experiment)