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**NONRENEWABLE RESOURCE EXTRACTIONS WITH A
POLLUTION SIDE EFFECT: A COMPARATIVE
DYNAMIC ANALYSIS**

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ABSTRACT

In this paper, we present a nonrenewable resource model including environmental pollution as a state variable. The model is analyzed to identify some of the characteristics of the optimal paths. In addition, we present a numerical example on the basis of the algebraic solutions of our qualitative model, and identify some of the characteristics of the optimal time paths for two sets of social costs of the pollutant. These results are consistent with the proposition of the previous literature that levying the shadow cost of the pollution stock reduces the consumption of resource; hence, it slows the accumulation of the pollutants in the atmosphere. One quirk in the results, however, is that extractions will persist longer in the higher pollution cost scenario. The costate variable for the resource stock is decomposed into a scarcity effect and a cost effect and the costate variable for the pollution stock is decomposed into an undesirable abundance effect and a cost effect. Both of these, however, are cost effects.

JEL classification: Q30

Key words: nonrenewable resource, environmental pollution stock, scarcity effect, undesirable abundance effect, cost effect

NONRENEWABLE RESOURCE EXTRACTIONS WITH A POLLUTION SIDE EFFECT: A COMPARATIVE DYNAMIC ANALYSIS

INTRODUCTION

There are few subjects in economics that have been discussed as extensively as the problem of environmental pollution. Following Pigou's initial insight on this subject (1920), a numerous of studies have been undertaken to design environmental policies for pollution abatement. In a static model analysis, it has been significantly suggested that if a regulatory agency imposes the value of marginal social damage incurred by environmental pollution as a Pigouvian tax, then the Pareto optimality in a society would be attained (Baumol 1972, Baumol and Oates 1988). In this analysis, the value of marginal social damage is denoted as the sum of the value of marginal disutility of consumers and the marginal cost of firms with respect to the increment of environmental pollution. On the other hand, as concerns about the spillover effect of pollution in economic growth process have increased (Mishan, 1969, IPCC 1990) two approaches have been directed to observe the side effect of pollution on the optimal endogenous variables in the model. One approach has modified the optimal growth model to reflect environmental pollution (Forster 1973; Gruver 1976; Nordhaus 1992, 1993; Selden and Song 1995) and the other one has changed the nonrenewable resource model to include environmental pollution stock as a state variable (Forster 1980, Kolstad and Toman 2001).

The main result of the modified optimal growth model is that the rate of both the optimal consumption and capital at stationary state are lower than when environmental pollution is not considered (Forster 1973, Selden and Song 1995). A modified nonrenewable resource model has shown that the optimal extraction of resource is slowed in responding to the accumulation of pollution stock (Forster 1980, Kolstad and Toman 2001). Similar to the suggestion in a static model analysis, dynamic analyses considering environmental pollution have also proposed that levying the shadow cost of environmental pollution stock as an optimal tax reduces the rate of consumption of goods and extraction of resource stock over time; thereby, slowing the accumulation of environmental pollution in the future (Nordhaus 1992, 1993; Kolstad

and Toman, 2001).

In this paper we present an optimal control nonrenewable resource model that includes a pollution externality, and analyze the optimal path using the first order necessary conditions. In addition, we identify using the first order necessary conditions the optimal pollution tax to be applied to the sale of the extracted resource. Our results are consistent with those stated above that the optimal tax will slow the depletion of the resource and the accumulation of the pollutant. One quirk in the results, however, is that extractions will persist longer in the higher pollution cost scenario. The costate variable for the resource stock can be decomposed into the scarcity effect and the cost effect; and the costate variable for the pollution stock can be decomposed into the undesirable abundance effect and the cost effect. Both of these, however, are cost effects. We then present a numerical example and discuss the characteristics of the resulting numerical optimal paths. We will call our resource fossil fuels and our pollution externality will be thought of as carbon dioxide buildup in the atmosphere. The model, however, includes only the bare essentials; hence, with a little imagination it can be applied to other pollution problems.

NONRENEWABLE RESOURCE MODEL WITH A POLLUTION STOCK

The objective of this problem is to maximize the present value of the net surplus stream subject to the constraints. These constraints are the laws of motion for the nonrenewable resource stock, fossil fuels, and the environmental pollution stock, atmospheric carbon. We use y as the instantaneous extraction and consumption of the resource, x as resource stock, and z as the pollution stock. Net surplus is given as

$$NS(y, x, z) = \int_0^y D(v) dv - C(z) - c(y, x)$$

where $D(y)$ is the instantaneous demand function for the extracted resource, fuel, $C(z)$ is social cost function associated with the stock of pollution, atmospheric carbon, and $c(y, x)$ is the extraction cost function. The demand function is assumed to be differentiable and negatively sloping, and the two cost functions are assumed to be twice continuously differentiable and

convex. The cost function $C(z)$ is posited to contain the additional costs to firms and individuals because of the level of z , and also to include any loss of consumers' surplus because the level of z directly affects their level of utility. In addition, extraction costs are posited to increase as the extraction increases, $c_y > 0, c_{yy} \geq 0$, and as the resource stock gets smaller $c_x < 0, c_{xx} \geq 0$. Because of these concavity assumptions the net surplus functional is concave, and this concavity together with the linear laws of motion, which are discussed below, imply that the necessary conditions are also sufficient.

The resource stock decreases by the amount of the extraction, and the increase in the stock of the pollutant is posited to be proportional to the extraction, which is equal to the consumption of the resource. That is, the consumption of a unit of fossil fuels causes specific increase, σ , in atmospheric carbon. The dynamic optimization problem is to maximize the present value of the net surplus stream

$$(1) \quad W = \int_0^T e^{-rt} NS(y, x, z) dt + e^{-rT} S(z(T))$$

subject to

$$\frac{dx(t)}{dt} = -y(t)$$

$$\frac{dz(t)}{dt} = \sigma y(t) \quad \sigma > 0$$

$$x(0) = x^0 \text{ given, } z(0) = z^0 \text{ given}$$

$$y(t), x(t), z(t) \geq 0$$

The real market rate of interest, r , is treated as a constant to simplify the problem. The terminal (scrap) value function at time T is $S(z(T))$. The terminal time, T , is endogenous to the problem, and is the time when either the resource stock is exhausted or the extraction ceases.

The present value Hamiltonian with two state variables is

$$(2) \quad H(y, x, z, \lambda_1, \lambda_2) = e^{-rt} NS(y, x, z) + \lambda_1(t) \{-y(t)\} + \lambda_2(t) \sigma y(t)$$

where $\lambda_1(t)$ and $\lambda_2(t)$ are the present value costate variables for the nonrenewable resource stock

and the environmental pollution stock, respectively. The non-negativity constraint on $x(t)$ is handled by the transversality conditions stated below, and the non-negativity constraint on $z(t)$ is automatically satisfied and ignored. The non-negativity constraint on $y(t)$ is included by maximizing the Hamiltonian subject to $y(t) \geq 0$; hence, we use the Lagrangean function

$$L(y, x, z, \psi_1, \psi_2, v) = H(y, x, z, \psi_1, \psi_2) + v(t)y(t)$$

We use the optimality theorem for the Hestenes Bolza problem as stated in Long and Voutsden (1977, pp 11-34) in Theorem 1. In the terminology of this theorem, we have three control parameters. They are T , the stopping time for extractions, $x(T)$, the nonrenewable resource stock at that time, and $z(T)$, the environmental pollution stock at that time. In addition, the rate of extraction, $y(t)$, is the only control variable. The present value necessary conditions for the optimality of Equation (1) are

$$e^{-rt} D(y^*(t)) = e^{-rt} c_y(y^*(t), x^*(t)) + \lambda_1^*(t) - \sigma \lambda_2^*(t) - v^*(t), \quad v^*(t)y^*(t) = 0$$

$$\frac{d\lambda_1^*(t)}{dt} = e^{-rt} c_x(y^*(t), x^*(t))$$

$$\frac{d\lambda_2^*(t)}{dt} = e^{-rt} C'(z^*(t))$$

$$\frac{dx^*(t)}{dt} = -y^*(t)$$

$$\frac{dz^*(t)}{dt} = \sigma y^*(t)$$

$$x^*(0) = x^0, \quad z^*(0) = z^0$$

And the present value transversality conditions are

$$\lambda_1^*(T^*) \geq 0, \quad \lambda_1^*(T^*)x^*(T^*) = 0$$

$$\lambda_2^*(T^*) = e^{-rT^*} S'(z^*(T^*)) \quad \text{for } z^*(T^*) > 0$$

$$re^{-rT} S(z^*(T^*)) - L(y^*(T^*), x^*(T^*), z^*(T^*), \lambda_1^*(T^*), \lambda_2^*(T^*), v^*(T^*)) = 0$$

where the super asterisk (*) denotes optimal values. Define the current value costate variables, $\psi_i(t)$, as $\psi_i(t) = e^{rt} \lambda_i(t)$ ($i = 1, 2$) and $\zeta(t) = e^{rt} v(t)$. Then, the current value necessary conditions are

$$(3) \quad D(y^*(t)) = c_y(y^*(t), x^*(t)) + \psi_1^*(t) - \sigma \psi_2^*(t) - \zeta^*(t) \quad \zeta^*(t) y^*(t) = 0$$

$$(4) \quad \frac{d\psi_1^*(t)}{dt} = r \psi_1^*(t) + c_x(y^*(t), x^*(t))$$

$$(5) \quad \frac{d\psi_2^*(t)}{dt} = r \psi_2^*(t) + C'(z^*(t))$$

$$(6) \quad \frac{dx^*(t)}{dt} = -y^*(t)$$

$$(7) \quad \frac{dz^*(t)}{dt} = \sigma y^*(t)$$

$$(8) \quad x^*(0) = x^0, \quad z^*(0) = z^0$$

And the current value transversality conditions are

$$(9) \quad \psi_1^*(T^*) \geq 0, \quad \psi_1^*(T^*) x^*(T^*) = 0$$

$$(10) \quad \psi_2^*(T^*) = S'(z^*(T^*)) \text{ for } z^*(T^*) > 0$$

$$(11) \quad \int_0^{y^*(T^*)} D(v) dv - c(y^*(T^*), x^*(T^*)) - \psi_1^*(T^*) y^*(T^*) + \psi_2^*(T^*) \sigma y^*(T^*) + \zeta^*(T^*) y^*(T^*) = 0$$

An important piece of information used in deriving Equation (11) is that $S(z^*(T^*))$ is the present value of cost of $z^*(T^*)$ from T^* onward:

$$(12) \quad S(z^*(T^*)) = - \int_{T^*}^{\infty} e^{-r(t-T^*)} C(z^*(T^*)) dt = -C(z^*(T^*)) \int_{T^*}^{\infty} e^{-r(t-T^*)} dt = -C(z^*(T^*)) / r$$

Equation (3) states that along the optimal path the value in consumption of the resource extraction, $D(y^*(t))$, is equal to the sum of two costs and the shadow value of the resource. The two costs are the extraction costs, $c_y(y^*(t), x^*(t))$, and the pollution costs associated with the consumption of the extraction, $-\sigma \psi_2^*(t)$. The costate variable $\psi_2^*(t)$ is the value of a unit of the atmospheric carbon; hence it is negative, implying that $-\sigma \psi_2^*(t)$ is the positive cost associated with an additional unit of consumption of the resource. It is easy to see that taking this externality cost into account decreases the consumption of the resource; therefore, it slows the accumulation of the pollutant. Because the atmosphere is a common property resource, a market economy will not automatically achieve the optimal path; however, if a tax equal to $-\sigma \psi_2^*(t)$ is

levied per unit of $y(t)$ sold the optimal path will be attained. In addition, Equation (4) shows that the shadow value of resource stock is consistent with Hotelling's rule (1931).

The transversality conditions, Equations (9), (10), and (11), determine the optimal levels of the two costate variables at the optimal stopping time, and the optimal stopping time, T^* . Equations (3) through (8) determine the optimal paths of the control variable, y , and the state variables and costate variables on the time horizon $t = 0$ to $t = T^*$. To illustrate how this works we present a simple algebraic model and its solution. This also allows us to illustrate some additional characteristics of the optimal path.

THE CHARACTERISTICS OF THE COSTATE VARIABLES

The shadow value of the resource exists because of a cost effect and maybe a scarcity effect also. A scarcity effect will exist only if the resource is exhausted. The shadow value can be written as (see Lyon (1999) for a development of this concept)

$$\psi_1^*(t) = e^{-r(T^*-t)}\psi_1^*(T^*) - \int_t^{T^*} e^{-r(s-t)} c_x(y^*(s), x^*(s)) ds$$

In this

$$e^{-r(T^*-t)}\psi_1^*(T^*) \quad \text{is the scarcity effect}$$

and

$$- \int_t^{T^*} e^{-r(s-t)} c_x(y^*(s), x^*(s)) ds \quad \text{is the cost effect.}$$

The scarcity effect insures that the resource has the same present value in each time period, which by Equation (9) will be zero if the resource stock is not exhausted. Note that because c_x is negative the cost effect is positive, and that as t approaches T^* the cost effect approaches zero. The injection of an additional unit of the resource at time t will affect the marginal unit in every time period from then to T^* , and will result in cost savings all along that path. The cost effect is simply the sum of those savings. Viewing this in Equation (3) ψ_1^* is one of three costs of consuming the marginal unit of y at t .

The costate variable for the stock of pollution, $\psi_2(t)$, is the current value of the change of the solution value of Equation (1) per unit change in environmental pollution stock at time t , $\frac{\partial W^*}{\partial z(t)} = \psi_2^*(t)$. As above we can separate $\psi_2(t)$ into two effects. The solution to the law of motion (Equation (5)) for $\psi_2(t)$ with boundary condition Equation (10) can be written:

$$\psi_2^*(t) = e^{-r(T^*-t)}\psi_2^*(T^*) - \int_t^{T^*} e^{-r(s-t)} C'(z^*(s)) ds$$

In this

$$e^{-r(T^*-t)}\psi_2^*(T^*) \quad \text{is the undesirable abundance effect,}$$

and

$$- \int_t^{T^*} e^{-r(s-t)} C'(z^*(s)) ds \quad \text{is the cost effect.}$$

The cost effect shows the present value of the costs that the marginal unit of extraction (consumption) of the resource will impose on society from time t to T^* , and the undesirable abundance effect shows the present value of the costs imposed from T^* on. This follows because

$$\psi_2^*(T^*) = -C'(z^*(T^*)) / r$$

by Equations (10) and (12). The undesirable abundance effect and the scarcity effect from above have the same general appearance; however, the former is negative and the latter is positive. In addition, the pollutant is anything but scarce.

If there is private ownership of the exhaustible resource and the sellers are price takers, then the value of the resource, $\psi_1^*(t)$, will be competed into the market price of y ; hence, we can generate the equality in Equation (3) by imposing an optimal tax of $\sigma\psi_2^*(t)$ per unit of y . This amount of optimal tax results in a reduced rate of fossil fuel extraction and thereby a reduction in the accumulation of environmental pollution. We now present our algebraic model, and then the numerical model.

THE ALGEBRAIC MODEL AND ITS SOLUTION

We use the following equations:

$$(13) \quad D(y) = a_0 - b_0 y \quad a_0, b_0 > 0$$

$$(14) \quad C(z) = a_1 z + .5b_1 z^2 \quad a_1, b_1 > 0$$

$$(15) \quad c(y, x) = c_1 (x^0 - x)y \quad c_1 > 0$$

With these three definitions the equations to be solved to find the optimal path are:

$$(3a) \quad a_0 - b_0 y(t) - c_1 (x^0 - x(t)) - \psi_1(t) + \sigma \psi_2(t) + \zeta(t) = 0 \quad \zeta(t)y(t) = 0$$

$$(4a) \quad \frac{d\psi_1(t)}{dt} = r\psi_1(t) - c_1 y(t)$$

$$(5a) \quad \frac{d\psi_2(t)}{dt} = r\psi_2(t) + a_1 + b_1 z(t)$$

$$(6a) \quad \frac{dx(t)}{dt} = -y(t)$$

$$(7a) \quad \frac{dz(t)}{dt} = \sigma y(t)$$

$$(8a) \quad x(0) = x^0, \quad z(0) = z^0$$

$$(9a) \quad \psi_1(T) \geq 0, \quad \psi_1(T)x(T) = 0$$

$$(10a) \quad \psi_2(T) = -[a_1 + b_1 z(T)]/r \quad \text{for } z(T) > 0$$

$$(11a) \quad a_0 y(T) - .5b_0 y(T)^2 - c_1 (x^0 - x(T))y(T) - \psi_1(T)y(T) + \sigma \psi_2(T)y(T) + \zeta(T)y(T) = 0$$

We now examine the endogenous variables at the terminal time. First we show that

$y^*(T^*) = 0$. The solution to Equation (3) at the terminal time is:

$$-[a_0 - b_0 y^*(T^*) - c_1 (x^0 - x^*(T^*)) + \zeta^*(T^*)] = -\psi_1^*(T^*) + \sigma \psi_2^*(T^*)$$

with $\zeta^*(T^*)y^*(T^*) = 0$. Substituting this into the solution to Equation (11a) yields:

$$.5b_0 y^*(T^*)^2 = 0$$

which has only one solution, $y^*(T^*) = 0$. Actually for continuous functions both $\zeta^*(T^*)$ and

$y^*(T^*)$ will equal zero. Using these and Equation (10a) we can now write the solution to

Equation (3) at the terminal time as:

$$a_0 - c_1 (x^0 - x^*(T^*)) - \psi_1^*(T^*) - \sigma [a_1 + b_1 z^*(T^*)]/r = 0$$

We see that the differential equations in Equations (6a) and (7a) are linearly dependent; hence we can eliminate one of them. We choose to eliminate (7a) by combining it with (6a) as follows:

$$\frac{dz}{dt} = -\sigma \frac{dx}{dt} \quad \text{with} \quad x(0) = x^0, z(0) = z^0$$

This initial value problem has the solution:

$$(16) \quad z(t) = \sigma (x^0 - x(t)) + z^0$$

Hence,

$$(3aT) \quad a_0 - c_1(x^0 - x^*(T^*)) - \psi_1^*(T^*) - \sigma\{a_1 + b_1[\sigma(x^0 - x^*(T^*)) + z^0]\}/r = 0$$

From Equation (9a) we know that at least one of $x^*(T^*)$ and $\psi_1^*(T^*)$ will equal zero, and from (3aT) we see that at least one of them is non-zero. Hence, Equation (3aT) can be used to solve for the one that is non-zero.

Suppose $x^*(T^*)$ is non-zero implying $\psi_1^*(T^*) = 0$; hence, the solution to Equation

(3aT) is:

$$(17) \quad x^*(T^*) = \frac{-r(a_0 - c_1x^0) + \sigma(a_1 + b_1(\sigma x^0 + z^0))}{rc_1 + b_1\sigma^2}$$

This requires

$$(18) \quad a_0 < c_1x^0 + \sigma(a_1 + b_1(\sigma x^0 + z^0))/r$$

This will be satisfied if extraction costs and externality (pollution) costs are high as the resource stock approaches zero relative to demand. The parameter a_0 determines the level of demand at each price, and the terms on the right hand side of this last expression determine the two costs as x approaches zero.

Now suppose $x^*(T^*) = 0$ implying $\psi_1^*(T^*) \geq 0$; hence, the solution to Equation (3aT) is:

$$(19) \quad \psi_1^*(T^*) = a_0 - c_1x^0 - \sigma(a_1 + b_1(\sigma x^0 + z^0))/r$$

This requires that the inequality in Equation (18) be reversed. This will be satisfied if demand is

high relative to extraction costs and externality (pollution) costs as the resource stock approaches zero.

To solve for the endogenous variables on the time horizon $t \in [0, T^*)$ first solve Equation (3a) for $y(t)$ yielding:

$$y(t) = (a_0 - c_1(x^0 - x(t)) - \psi_1 + \sigma\psi_2) / b_0$$

which will be positive, so we can ignore $\zeta(t)$. Then substitute this result into Equations (4a) and (6a), and because we are eliminating Equation (7a), we substitute Equation (16) into (5a). This gives three linear differential equations in three endogenous variables:

$$(4b) \quad \frac{d\psi_1(t)}{dt} = r\psi_1(t) - c_1(a_0 - c_1(x^0 - x(t)) - \psi_1 + \sigma\psi_2) / b_0$$

$$(5b) \quad \frac{d\psi_2(t)}{dt} = r\psi_2(t) + a_1 + b_1(\sigma(x^0 - x(t)) + z^0)$$

$$(6b) \quad \frac{dx(t)}{dt} = -(a_0 - c_1(x^0 - x(t)) - \psi_1 + \sigma\psi_2) / b_0$$

These have initial the condition, $x(0) = x^*$, and the terminal condition

$\psi_2(T) = -[a_1 + b_1(\sigma(x^0 - x(T)) + z^0)] / r$ which is a combination of Equation (10a) and (16). In

addition, for parameter values that satisfy Equation (18) they have the terminal conditions,

$\psi_1^*(T^*) = 0$ and Equation (17). When the parameter values satisfy Equation (18) with the

inequality reversed the terminal conditions include $x^*(T^*) = 0$ and Equation (19).

THE NUMERICAL MODEL AND OPTIMAL PATHS

To illustrate conclusions generated above we present the solution paths for two sets of parameter values in two scenarios. These values will present a comparative dynamic analysis of two economies with different costs associated with the pollutant. Both will have the following parameter values $a_0 = 32, b_0 = 0.5, b_1 = 0.05, c_1 = 0.6, \sigma = 1, r = 0.1, x^0 = 25, z^0 = 2$. Scenario E (for exhaustion) will have $a_1 = 0$ which will give sufficiently low pollution costs that the resource stock will be exhausted. Scenario N (for non-exhaustion) will have $a_1 = 0.8$. This gives higher pollution costs than does Scenario E and results in not all of the resource stock being

consumed. These values were selected to illustrate the effects of the pollution cost function on the solution time paths. As expected the higher pollution costs slow the consumption of the resource, and for these values result in some of the resource stock being left in the ground. It seems logical that it will also result in the resource being consumed for a shorter period of time. As will be seen below, however, this conjecture is wrong, at least for these equations. The extractions in Scenario E will take place in a finite time period, but Scenario N will have positive extractions from now on, T^* is infinite. This is depicted in Figure 1, which shows optimal extractions for the two scenarios. Note that initially Scenario E's extractions are larger than those for Scenario N; however, the two extraction curves cross, depicting higher extractions for Scenario N.

Figure 2, which identifies the resource stock over time for the two scenarios, shows, as expected, that the resource stock is always higher for Scenario N than Scenario E. Thus, while the

time horizon of consumption of the resource is longer for Scenario N, the remaining stock also larger at each point in time.

The higher initial extractions for Scenario E than for Scenario N indicate that price is initially higher for Scenario N. This is due to a higher shadow value for the pollutant, $\psi_2(t)$, which results from the higher costs of pollution for this scenario. This is illustrated in Figure 3,

which shows shadow values over time. The absolute value of $\sigma\psi_2(t)$ is the marginal cost of the pollutant per unit of resource consumption, $y(t)$; hence, this figure depicts the higher costs for Scenario N. Also note that the upper portion of this figure illustrates that the marginal value of the resource, $\psi_1(t)$, is higher for Scenario E. The lower value for Scenario N is once again the result of the higher pollution costs for this scenario. These same facts are illustrated in a different way for both scenarios in Figure 4.

Equation (3) states that along the optimal path price of the extracted resource, $D(y(t))$, is equal to the sum of the three costs—marginal extraction costs, $c_y(y(t), x(t))$, shadow value of the resource, $\psi_1(t)$, and the marginal pollutant cost, $\sigma\psi_2(t)$. The two price curves cross, just as the two extraction curves in Figure 1 cross, and the crossing represents exactly the same information. As indicated above, we did expect not this crossing. The three curves that terminate at about $t = 6.6$ (actually $T^* = 6.5773$) are for Scenario E which is the time when the resource is exhausted. These three curves depict for Scenario E the relative importance of the different costs, because the vertical distance between the Scenario E curves shows the cost that is being added. Initially the shadow value of the resource and the shadow value of the pollutant are approximately equally important, with marginal extract being zero; however, by the time t approaches T^* marginal extraction cost and shadow value of pollutant are about equally important, with the shadow value of the resource being of lesser importance. For Scenario N, initially the shadow value of the pollutant is significantly larger than the shadow value of

the resource with the marginal extraction cost being zero. As t gets large, however, the shadow value of the resource goes to zero, the shadow value of the pollutant continues to be dominant, and the marginal extraction cost is of significant size. These two costs are the dominant reason why extractions are very small as t becomes large.

SUMMARY

We have analyzed a natural resource model with a non-renewable resource and a pollution side effect of the consumption of the resource. We refer to the resource as fossil fuels and the pollutant as atmospheric carbon; however, the methodology is general enough that other applications will also fit. An optimal control model was presented and analyzed to identify some of the characteristics of the optimal paths. In addition, we present a numerical example on the basis of the algebraic solutions of our qualitative model, and identify some of characteristics of the optimal time paths for two sets of social costs of the pollutant. These results are consistent with the proposition of the previous literature that levying the shadow cost of pollution stock reduces the consumption of resource; hence, it slows the accumulation of pollutant in the atmosphere. One quirk in the results, however, is that extractions will persist longer in the higher pollution cost scenario. The costate variable for the resource stock is decomposed into a scarcity effect and a cost effect; and the costate variable for the pollution stock is decomposed into an undesirable abundance effect and a cost effect. Both of these, however, are cost effects.

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NONRENEWABLE RESOURCE EXTRACTIONS WITH A POLLUTION SIDE EFFECT: A COMPARATIVE DYNAMIC ANALYSIS

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AND

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Abstract

In this paper, we present a nonrenewable resource model including environmental pollution as a state variable. The model is analyzed to identify some of the characteristics of the optimal paths. In addition, we present a numerical example on the basis of the algebraic solutions of our qualitative model, and identify some of the characteristics of the optimal time paths for two sets of social costs of the pollutant. These results are consistent with the proposition of the previous literature that levying the shadow cost of the pollution stock reduces the consumption of resource; hence, it slows the accumulation of the pollutant in the atmosphere. One quirk in the results, however, is that extractions will persist longer in the higher pollution cost scenario. The costate variable for the resource stock is decomposed into a scarcity effect and a cost effect; and the costate variable for the pollution stock is decomposed into an undesirable abundance effect and a cost effect. Both of these, however, are cost effects.

JEL Classification: Q 30

Keywords: nonrenewable resource, environmental pollution stock, scarcity effect, undesirable abundance effect, and cost effect.

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INTRODUCTION

There are few subjects in economics that have been discussed as extensively as the problem of environmental pollution. Following Pigou's initial insight on this subject (1920), a numerous of studies have been undertaken to design environmental policies for pollution abatement. In a static model analysis, it has been significantly suggested that if a regulatory agency imposes the value of marginal social damage incurred by environmental pollution as a Pigouvian tax, then the Pareto optimality in a society would be attained (Baumol 1972, Baumol and Oates 1988). In this analysis, the value of marginal social damage is denoted as the sum of the value of marginal disutility of consumers and the marginal cost of firms with respect to the increment of environmental pollution. On the other hand, as concerns about the spillover effect of pollution in economic growth process have increased (Mishan, 1969, IPCC 1990) two approaches have been directed to observe the side effect of pollution on the optimal endogenous variables in the model. One approach has modified the optimal growth model to reflect environmental pollution (Forster 1973; Gruver 1976; Nordhaus 1992, 1993; Selden and Song 1995) and the other one has changed the nonrenewable resource model to include environmental pollution stock as a state variable (Forster 1980, Kolstad and Toman 2001).

The main result of the modified optimal growth model is that the rate of both the optimal consumption and capital at stationary state are lower than when environmental pollution is not considered (Forster 1973, Selden and Song 1995). A modified nonrenewable resource model has shown that the optimal extraction of resource is slowed in responding to the accumulation of pollution stock (Forster 1980, Kolstad and Toman 2001). Similar to the suggestion in a static model analysis, dynamic analyses considering environmental pollution have also proposed that levying the shadow cost of environmental pollution stock as an optimal tax reduces the rate of consumption of goods and extraction of resource stock over time; thereby, slowing the accumulation of environmental pollution in the future (Nordhaus 1992, 1993; Kolstad